Quantum Error Correction

3 x 11.00 - 12.30

I
- Intro : theory versus experiment
- Quantum Error Correcting Conditions

II
- Bosonic Codes
- Stabilizer Codes (including continuous-variable codes)

III
- Decoding
- Decoding for GKP code : path integral... .. minimum action.

(• Decoding for surface-GKP code)

Short QEC timeline

1995 Shor 9-qubit code, correcting 1 error
1996 Shor fault-tolerant quantum computing

1996 Quantum error correction conditions

(Knill / Laflamme, Bennett / DiVincenzo / Smolin / Wootters)

5 qubit code, correcting 1 error

1997 Gottesman: Stabilizer codes
Kitaev: toric code

lots of theory

2001 Topological Quantum memory (Dennis / Kitaev / Landahl / Preskill)

lots of theory

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Experimental Status

(no bosonic codes on this list)

Code	What has been done
3-bit repetition code. 0→000, 1→111 5-bit repetition code 0→00000, 1→11111	Early liquid-NMR & ion-trap qubit experiments. Life-time preservation of NV-center in diamond qubits (Delft, 2015) & superconducting qubits (Google/UCSB, 2014). Evidence that 5-qubit code performs better than 3-qubit by Google/UCSB.
4-qubit error detection code [[4,2,2]] which can detect a single error	IBM Quantum Experience (IBM & Vuillot, 2017). Ion trap qubits (Maryland/IonQ, 2017)
5-qubit code which corrects a single error [[5,1,3]].	Not yet. Fault-tolerant circuits for code are larger.
7-qubit Steane code [[7,1,3]] which corrects a single error, smallest color code	Ion-trap qubits (Innsbruck, 2014): logic on encoded block, but no evidence for life-time improvement
9-qubit surface code (plus 8 ancilla qubits)	Planned for superconducting qubits by e.g. IBM, Google & Delft (2018+)

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theory versus Experiment : where do your errors come from ?

Depolarizing noise toy model :

$$\rho \longmapsto (1-p)\,\rho + p_x X \rho X + p_y Y \rho Y + p_z Z \rho Z$$

$$1-p+\sum_{i=x,y,z} p_i = 1 .$$

More general toy model :

$$\rho \mapsto S(\rho) = \sum_i E_i \rho E_i^{\dagger} \qquad \sum_i E_i^{\dagger} E_i = I .$$

S : trace-preserving completely-positive map (TCP)

Conversion between toy-models via Pauli twirling

$$\rho \mapsto \sum_{i=0,..,3} P_i \, S(P_i \, \rho \, P_i) \, P_i \quad = \tilde{S}(\rho)$$

$$\boxed{\begin{array}{l} P_0 = I \\ P_1 = X \\ P_2 = Y \\ P_3 = Z \end{array}}$$

$\tilde{S}$: depolarizing channel

Exercise I
Prove this

Hint : use

$$\rho_S = (I \otimes S)(|\Phi_0\rangle\langle\Phi_0|)$$

$$|\Phi_0\rangle = |00\rangle + |11\rangle$$

$\rho_S \longleftrightarrow S$ 1-to-1

Use other Bell states ...

"Zoo of error strengths":

Depol. channel: easy ... probability of error

- Entanglement fidelity $\langle \Psi_0 | (I \otimes S)(|\Psi_0\rangle\langle\Psi_0|) |\Psi_0\rangle$
- Diamond norm $\| I - S \|_\diamond = \max_{\Psi} \| |\Psi\rangle\langle\Psi| - (I \otimes S)(|\Psi\rangle\langle\Psi|) \|_{tr}$

$$\|A\|_{tr} = \mathrm{Tr}\sqrt{AA^\dagger}$$

- Purify $S(\rho) = \sum_i E_i \rho E_i^\dagger \rightarrow U |\psi\rangle_S |0\rangle_E =$

$$\sum_i E_i |\psi\rangle_S |i\rangle_E$$

$|\psi\rangle_S$: system, $|i\rangle_E$: environment

Write $E_i = \alpha_i I + \beta_i E^i_{else}$

$$U |\psi\rangle_S |0\rangle_E = \sum_i \alpha_i |\psi\rangle_S |i\rangle_E + |error(\psi)\rangle$$

$\max_{\psi, \|\psi\|=1} \| |error(\psi)\rangle \|$: total error amplitude

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Advantage: able to extract Z, X etc. error amplitude separately.

(biased noise, leakage ...)

Example: single-qubit channel, extract Z error amplitude ε_Z

Write $E_i = \sum_{j=0,..,3} \alpha_{ij} P_j$, collect terms which act like Y or Z on qubit..

for $i = 0,.. 3$ (why at most 4 Kraus operators E_i: exercise II)

α: 4x4 matrix

$$\varepsilon_Z = \max_{\psi} \left\| \left(\sum_i \alpha_{i2} |i\rangle_E Y + \sum_i \alpha_{i3} |i\rangle_E Z \right) |\psi\rangle_S \right\|$$

Take $|\psi_S\rangle = |\Phi_0\rangle$ so that $Y|\Phi_0\rangle \perp Z|\Phi_0\rangle$ $\quad \boxed{|\Phi_0\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}}$

$$\Rightarrow \quad \varepsilon_Z = \sqrt{\sum_i |\alpha_{i2}|^2} + \sqrt{\sum_i |\alpha_{i3}|^2} = \sqrt{p_y} + \sqrt{p_z}$$

p_y, p_z ?

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Twirl channel ... $\longrightarrow$ depolarizing channel (p_x, p_y, p_z)

χ - matrix

$$\sum_i E_i \, \rho \, E_i^{\dagger} = \sum_{kj} (\alpha^{\dagger}\alpha)_{kj} \, P_j \, \rho \, P_k$$

χ-matrix of twirled channel is diagonal and $p_y = \sum_i |\alpha_{i2}|^2$

$p_z = \sum_i |\alpha_{i3}|^2$

$$\varepsilon_z^2 = p_y + p_z + 2\sqrt{p_y p_z}$$

"coherent errors"

versus

$$p_y + p_z$$

"stochastic errors"

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Errors can be correlated in space & time ....

Time

$$H = \frac{\omega_0 + \Delta(t)}{2} Z$$

$\Delta(t)$ depends on local electric or magnetic field

$$\langle \Delta(t) \rangle = 0 \qquad \langle \Delta(t) \Delta(t') \rangle = f(t-t')$$

Fourier Trafo ($f(t-t')$) $= \tilde{f}(\omega)$ = noise spectral density $= \frac{1}{\omega}$

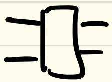
Slowly-varying $1/f$ noise

Dynamical decoupling techniques more useful than QEC via redundancy ....

Spin echo: $X R_z(\phi) X R_z(\phi) = R_z(-\phi) R_z(\phi) = I$

Space

Usual Model

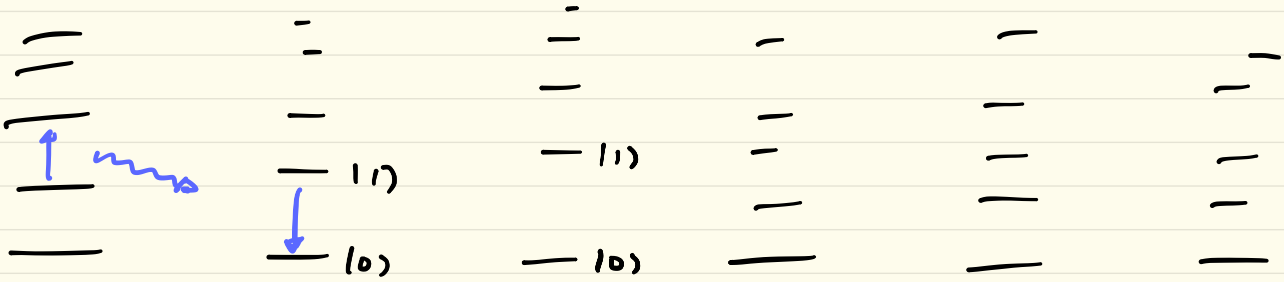


2-qubit gate is assumed to have correlated 2-qubit errors, e.g

But interactions happen due to coupling between qubits in real space and resonances in frequency space:

cross-talk all over

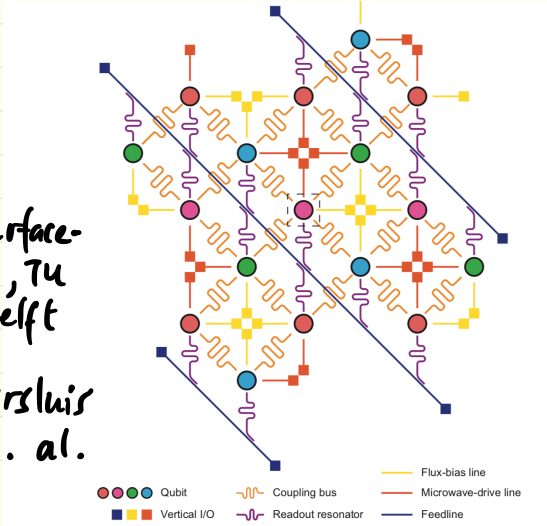
Superconducting Qubits (see DiVincenzo Lectures)

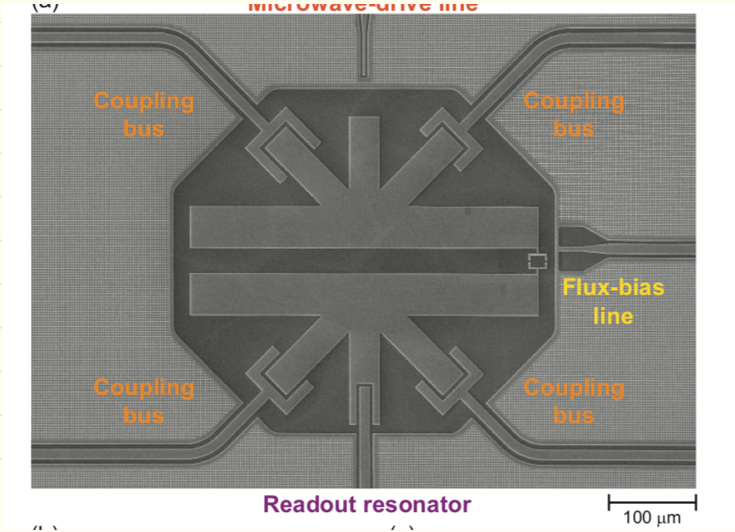


harmonic   qubit 1   qubit 2   ..   harmonic   ...

Surface-17, TU Delft

Versluis et. al.





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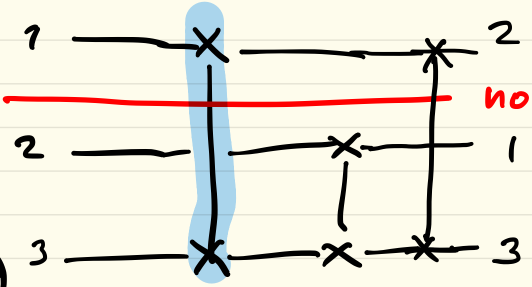
How well do concepts of fault-tolerance work in practice for small codes?

<u>Example</u>:

Fault-tolerant SWAP, avoiding 2 qubit error

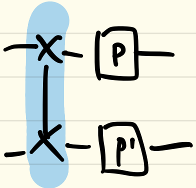


no cross-talk allowed between qubit 1 & 2

But if ....

Let a single SWAP gate fail, i.e.

X
|
X SWAP gate



P, P' : Paulis

(say the first gate) <u>only qubit 1 or 2</u> will have Pauli error

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extra

But if there is cross talk, then this concept may not be useful...

- direct SWAP between qubit 1 and 2 takes 1 SWAP gate. Two-qubit error rate p.
- fault-tolerant SWAP takes 3 SWAP gates and ancilla qubit .... if 1 SWAP gate has cross-talk error rate $p/3$ for error on qubit & spectator qubit, then two-qubit error rate is also p.