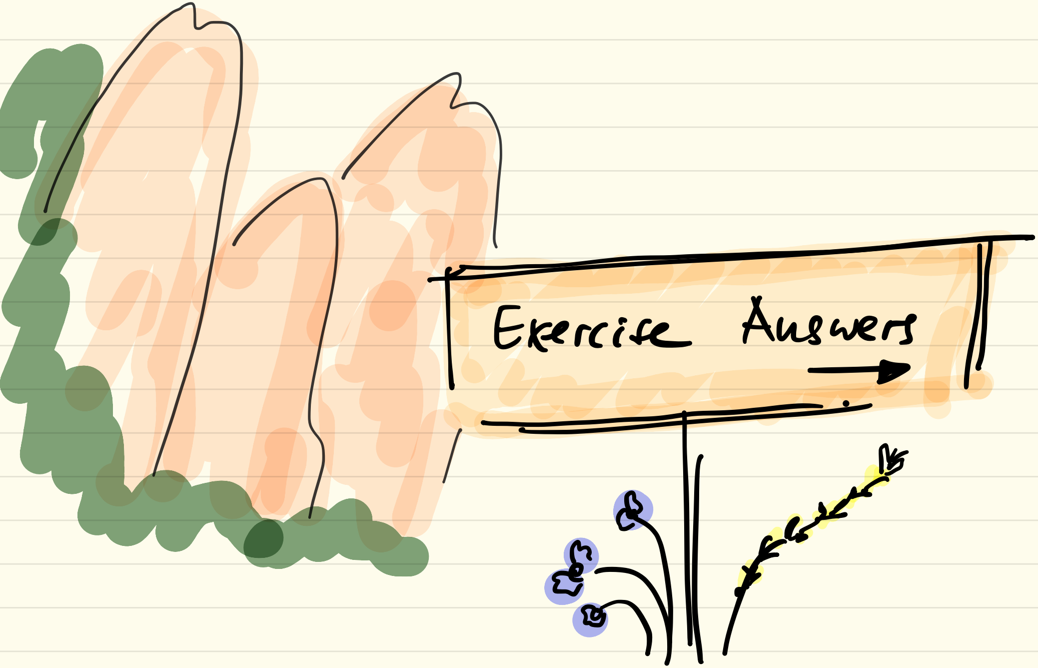
Exercise Answers

Exercise I

$$\rho_S = (I \otimes S)\left(|\Phi_0\rangle\langle\Phi_0| \right)$$

$$\rho_S \longleftrightarrow S$$

$\langle \Phi_k | \rho_S | \Phi_l \rangle \quad \forall k, l$ all info on S

$|\Phi_0\rangle = |00\rangle + |11\rangle$

$|\Phi_1\rangle = |01\rangle + |10\rangle$

$|\Phi_2\rangle = |01\rangle - |10\rangle$

$|\Phi_3\rangle = |00\rangle - |11\rangle$

$$\sum_i \langle \Phi_k | I \otimes P_i \cdot (I \otimes S)\left(|\Phi_i\rangle\langle\Phi_i| \right)(I \otimes P_i) |\Phi_l\rangle$$

$$= \sum_i \langle \Phi_k | P_i \otimes P_i \cdot (I \otimes S)\left(|\Phi_0\rangle\langle\Phi_0| \right) P_i \otimes P_i |\Phi_l\rangle$$

$P_i \otimes P_i |\Phi_k\rangle = \lambda_k^i |\Phi_k\rangle \quad \lambda_k^i = \pm 1.$

$k = l$ case $= \langle \Phi_k | \rho_S | \Phi_k \rangle$ } depolarizing channel

$k \neq l$ case $= 0$ (canceling of signs)

Exercise $\underline{\text{II}}$

$\rho_S \longleftrightarrow S$ S single qubit channel.

diagonalize $\rho_S = \sum_i \lambda_i |\psi_i\rangle\langle\psi_i|$

arb. state $|\psi_i\rangle = (I \otimes B_i)|\Phi_0\rangle$ (prove).

for some B_i

hence $\rho_S = \sum_i (I \otimes \sqrt{\lambda_i} B_i)|\Phi_0\rangle\langle\Phi_0|(I \otimes \sqrt{\lambda_i} B_i^{\dagger})$

hence $A_i = \sqrt{\lambda_i} B_i$ and since ρ_S has at most

4 eigenstates with $\lambda_i \neq 0$, there are at most 4

A_i's which are non-zero.

Exercise III

Consider $P_c F_n^\dagger P_m P_c$

$F_m = \sum_j u_{jm} E_j$

$F_n^\dagger = \sum_i u_{in}^* E_i^\dagger$

Consider $P_c F_n^\dagger F_m P_c = \sum_{ji} u_{jm} u_{in}^* P_c E_i^\dagger E_j P_c$

$= \sum_{ji} u_{jm} u_{in}^* \alpha_{ij} P_c$

$= (U^\dagger \alpha U)_{nm} P_c$

$= D_{nm} P_c$

Exercise IV

Let $[A,B] \propto \mathbb{I}$ like $[\vec{p}, \vec{q}]$.

then $\exp(A+B) = \exp(A)\exp(B)\exp\left(-\frac{[A,B]}{2}\right)$

since $[A,[A,B]] = [B,[A,B]] = 0$

$= \exp(B)\exp(A)\exp\left(+\frac{[A,B]}{2}\right)$

$\Rightarrow \exp(A)\exp(B) = \exp(B)\exp(A)\exp([A,B])$