Evolutionary games of condensates in driven-dissipative systems of non-interacting bosons

Johannes Knebel[1], Markus F. Weber[1], Torben Krüger[2], Erwin Frey[1]

[1]Arnold Sommerfeld Center for Theoretical Physics, LMU München; [2]IST Austria

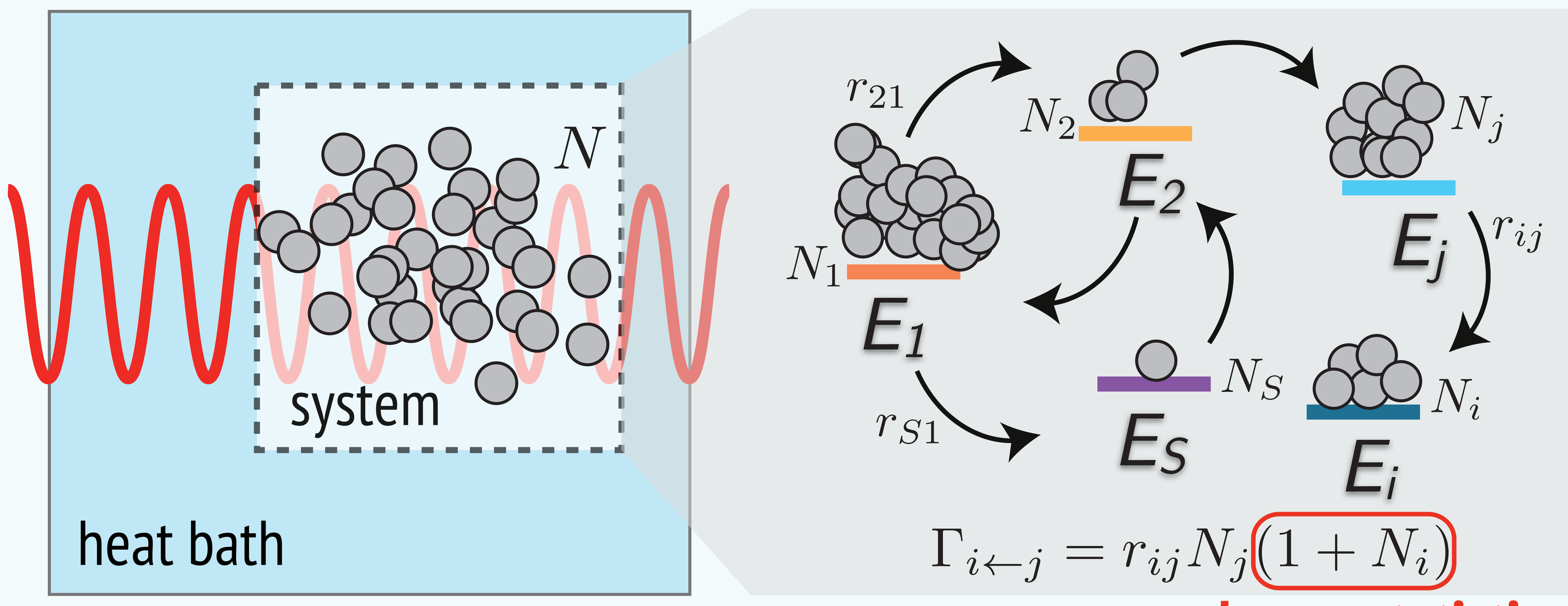
Summary

- Condensation is a collective behavior of particles observed in both classical and quantum physics. For example, when an equilibrated, dilute gas of bosonic particles is cooled to a temperature near absolute zero, the ground state becomes macroscopically occupied (Bose-Einstein condensation). Whether novel condensation phenomena occur far from equilibrium is a topic of vivid research.
- Only recently has it been proposed that a driven and dissipative gas of bosons may condense not only into a single, but also into multiple non-degenerate states[1]. In our work, we applied concepts from evolutionary dynamics to determine the states that become condensates[2]. This condensate selection is guided by the vanishing of relative entropy production.

Non-interacting bosons in driven-dissipative systems[1]

- Many-body open quantum system
  - > non-interacting bosons
  - > dissipative (weakly coupled to a heat bath)
  - > driven (periodically in time, Floquet system)

incoherent (classical) dynamics on a coarse-grained timescale



$$\Gamma_{i\leftarrow j} = r_{ij} N_j \boxed{(1+N_i)}$$

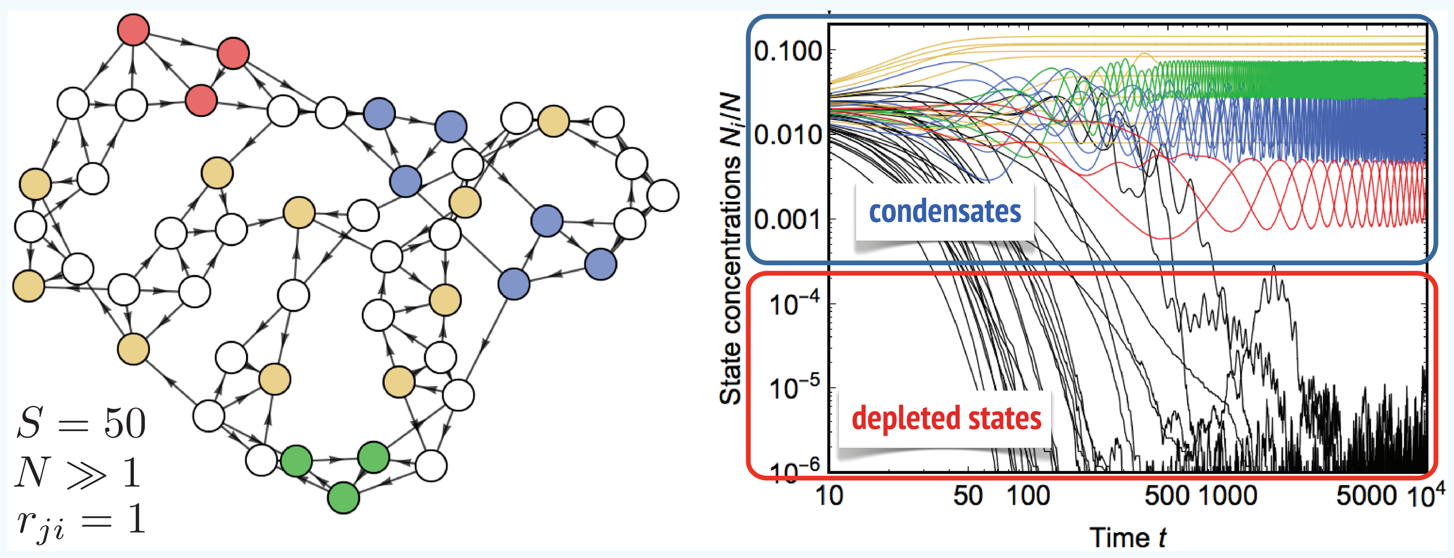
boson statistics

- Coarse-grained dynamics captured by a classical master equation

$$\partial_t P(\mathbf{N},t) = \sum_{j\neq i}^{S} \Big(\Gamma_{i\leftarrow j}(N_i - 1, N_j + 1) P(\mathbf{N} - \mathbf{e}_i + \mathbf{e}_j, t) - \Gamma_{i\leftarrow j}(N_i, N_j) P(\mathbf{N}, t) \Big)$$

Condensation into multiple quantum states[1,2]

- Starting from a random network of states, multiple states may be macroscopically occupied.



- Question: Which of the states become the condensates?
- Correspondence to evolutionary dynamics

Quantum (Floquet) states	⟷	Strategies
Non-interacting bosons	⟷	Interacting agents
Transitions	⟷	Reactions and switches
Multiple condensates	⟷	Multiple winning strategies

Condensation is captured by the antisymmetric Lotka-Volterra equation[2]

- On the leading order timescale, condensation is captured by a coupled sytem of nonlinear ODEs; $x_i = N_i/N$, $N \gg 1$

$$\frac{\mathrm{d}}{\mathrm{d}t} x_i = x_i \sum_j (r_{ij} - r_{ji}) x_j = x_i (A\mathbf{x})_i$$

, also known as the replicator equation for zero-sum games in evolutionary dynamics.

- Mathematics to determine condensates and depleted states
  1. Find a condensate vector (linear programming theory[3])

     $A\mathbf{c} \le 0$ and $\mathbf{c} - A\mathbf{c} > 0$
  2. Relative entropy of a condensate vector to the state concentrations

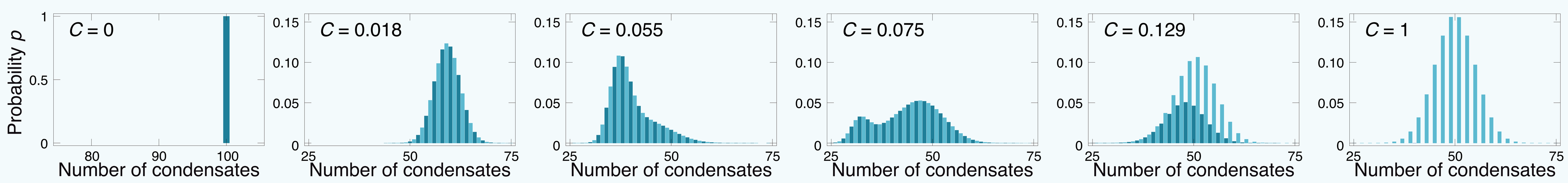
     $$D(\mathbf{c}\|\mathbf{x}) = \sum_{i\in I} c_i \log(c_i/x_i)$$

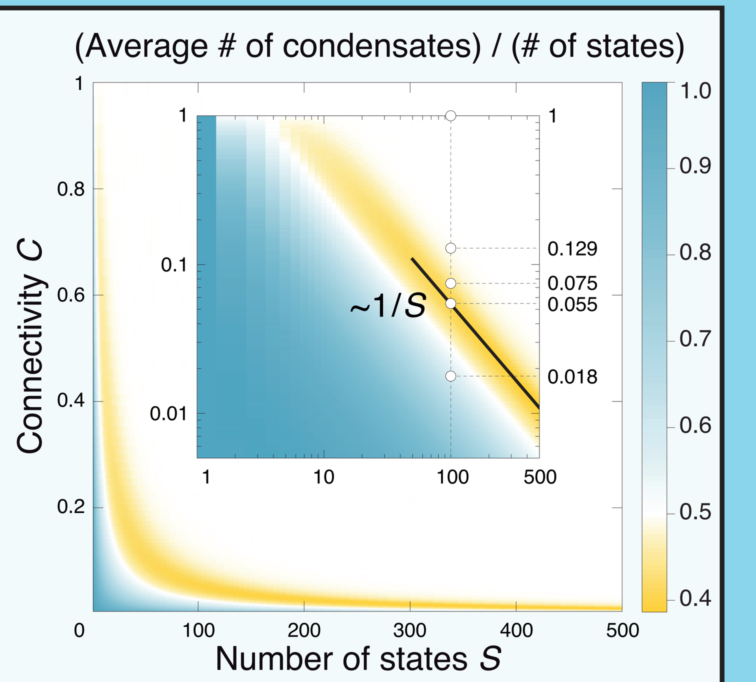
     is a Lyapunov function and determines the condensates.

$$\underbrace{\begin{pmatrix} 0 & r_{12}-r_{21} & \cdots & * \\ & 0 & \ddots & \vdots \\ \text{“}-\text{”} & & \ddots & * \\ & & & 0 \end{pmatrix}}_{\text{Antisymmetric matrix } A} \underbrace{\begin{pmatrix} + \\ \vdots \\ 0 \\ + \end{pmatrix}}_{\text{Condensate vector } \mathbf{c}} = \underbrace{\begin{pmatrix} 0 \\ \vdots \\ - \\ 0 \end{pmatrix}}_{\text{Depletion rates } A\mathbf{c}} \begin{array}{l} \longrightarrow \text{Condensation } x_i(t) \ge \epsilon > 0 \\ \\ \longrightarrow \text{Depletion } x_i(t) \to 0 \end{array}$$

Condensation in large random networks of states shows critical behavior[2]

- Measure the number of condensates depending on the connectivity C of random networks, example for $S = 100$





- Interplay between criticality of random networks and dynamics of condensate selection.

References

[1] D. Vorberg, W. Wustmann, R. Ketzmerick, A. Eckardt, Generalized Bose-Einstein condensation into multiple states in driven-dissipative systems, PRL 111, 240405 (2013).
[2] J. Knebel, M.F. Weber, T. Krüger, E. Frey, Evolutionary games of condensates in coupled birth-death processes, Nat. Comm. 6, 6977 (2015).
[3] A. W. Tucker, Dual systems of homogenous relations, in: H. W. Kuhn and A. W. Tucker, Linear Inequalities and related systems, Prin. Univ. Pr. (1956).