



Nonequilibrium dynamics in Coulomb glasses near the metal-insulator transition

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Lecture I: [Metal-insulator transition and complexity in electronic systems](#)

Lecture II: [Studies of the electron dynamics near the 2D MIT: Relaxations of conductivity](#)

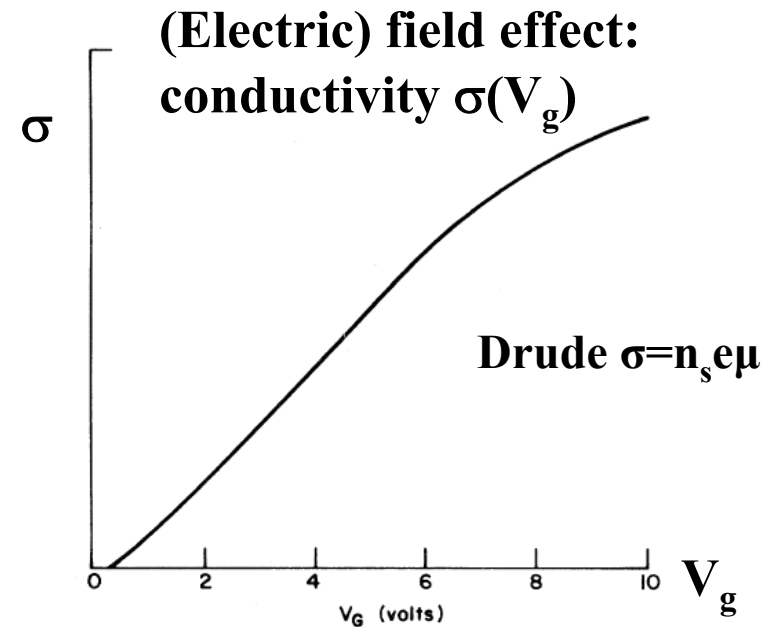
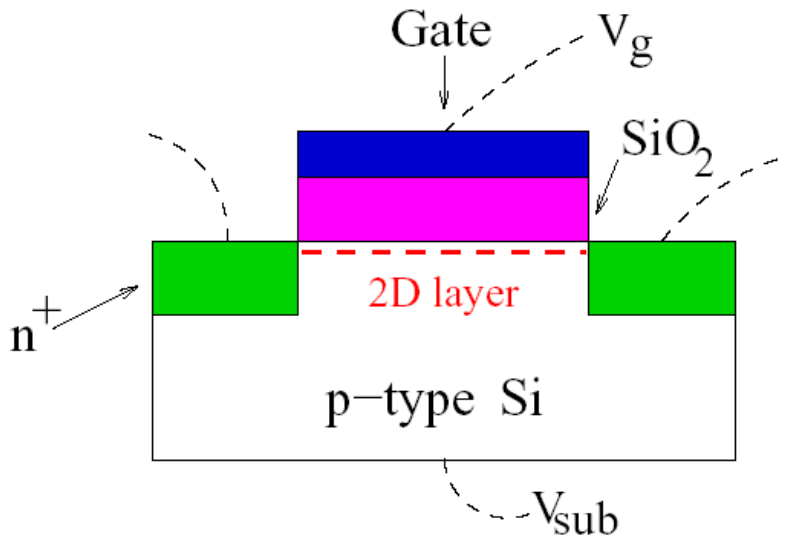
- 2D electron system in Si: **temperature dependence of conductivity**
- **Relaxations** of conductivity after a **rapid change of density**
- Relaxations after a waiting time protocol: **aging and memory**
- **Aging** across the 2D **MIT**

Lecture III: [Studies of the electron dynamics near the 2D MIT: Fluctuations of conductivity](#)

Studies of the electron dynamics near the 2D MIT



- **Problem: strongly interacting electrons in a random potential**
- **2D electron system** in Si MOSFETs; samples with very **different** amounts of **disorder**



- **“peak mobility”** at 4.2 K – rough measure of disorder

- vary density n_s using V_g

**Evidence for phase transition(s)?
MIT, glass transition?**

Resistivity in high-mobility (low disorder) Si MOSFETs



- high-mobility ($\sim 25,000 \text{ cm}^2/\text{Vs}$) Si MOSFETs (Groningen/Delft);

$L=120 \text{ }\mu\text{m}$, $W=50 \text{ }\mu\text{m}$

($d_{0x}=147 \text{ nm}$, Al gates,

$N_a \sim 10^{14} \text{ cm}^{-3}$)

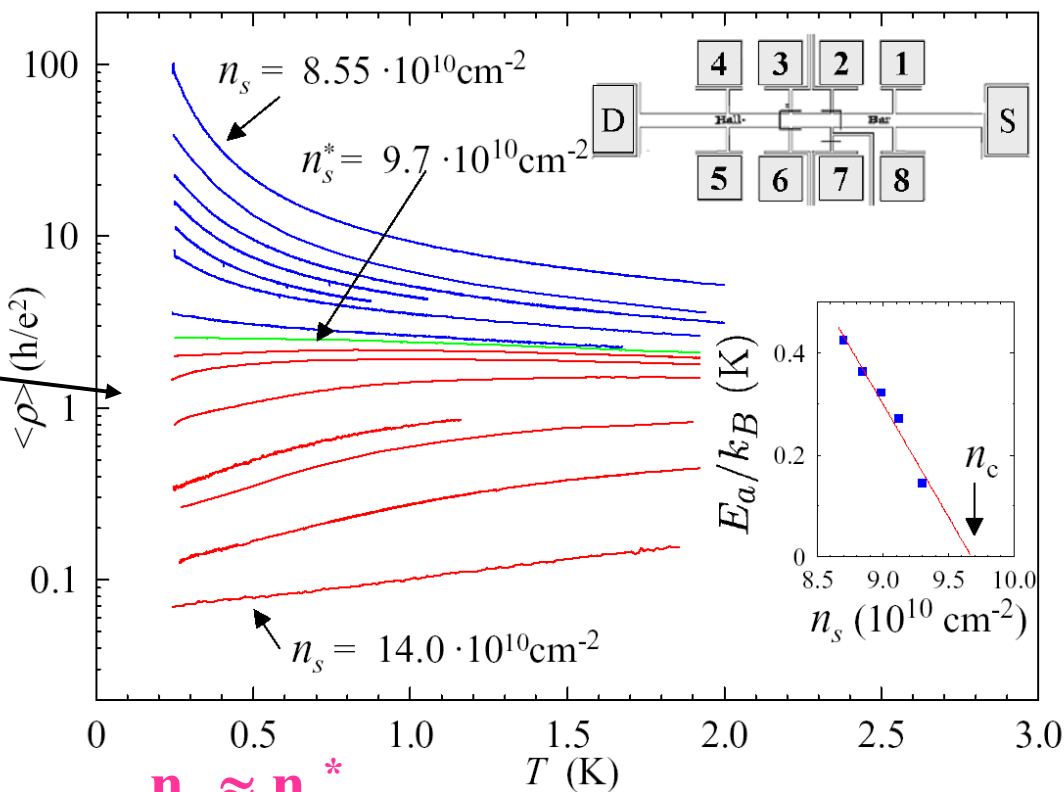
“separatrix” n_s^*

- lowest n_s and T :

$$\langle \rho \rangle \propto \exp(E_a/k_B T)$$

$$\Rightarrow n_c \approx 9.7 \times 10^{10} \text{ cm}^{-2}$$

critical density (where strong localization ends)



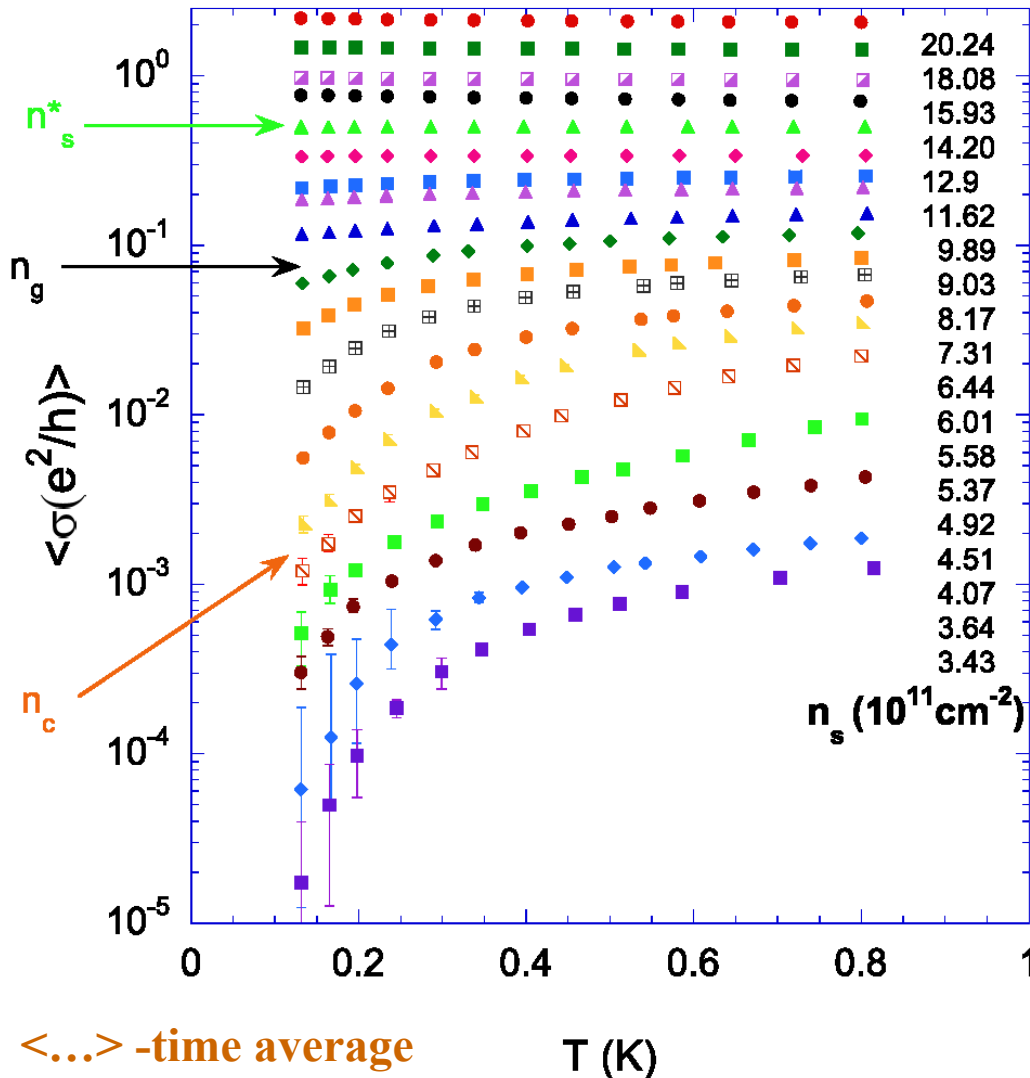
$$n_c \approx n_s^*$$

$$n_c \leq n_g$$

glass transition

Will show this later

Conductivity in low-mobility (high disorder) Si MOSFETs



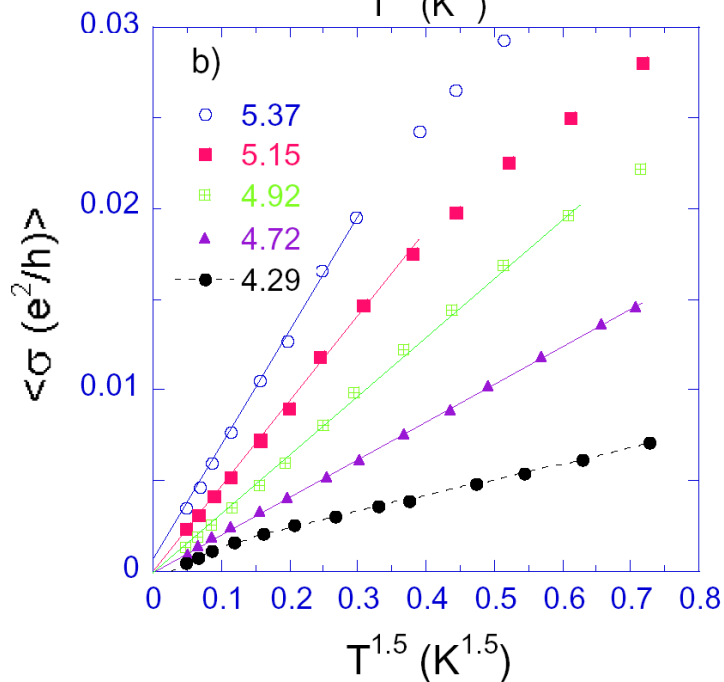
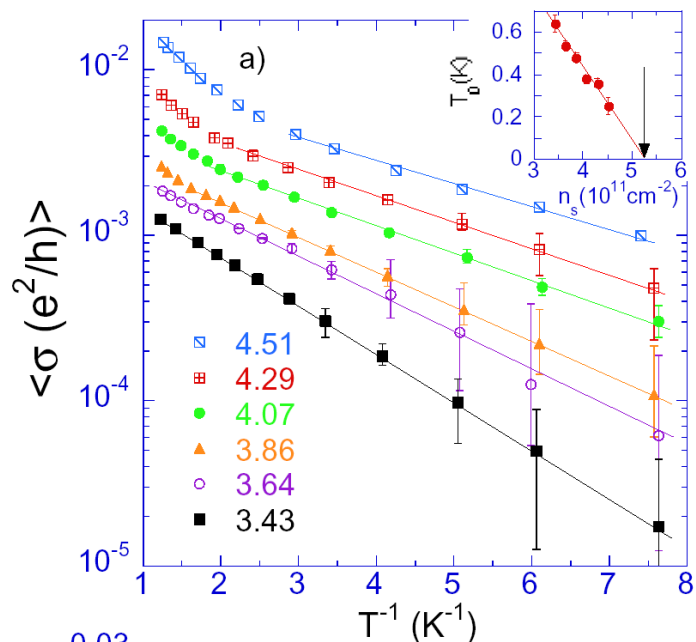
- **low-mobility** ($\sim 600 \text{ cm}^2/\text{Vs}$) Si MOSFETs (IBM);
 $L \times W$: 1×90 and $2 \times 50 \mu\text{m}^2$
 $(d_{\text{ox}} = 50 \text{ nm}, \text{poly-Si gates}, N_a \sim 10^{17} \text{ cm}^{-3})$
- **metallic** $\langle \sigma(T) \rangle$ at high n_s
- **$d\langle \sigma \rangle/dT = 0$** at $n_s^* = 12.9 \times 10^{11} \text{ cm}^{-2}$
 (“separatrix”)
- **metal-insulator transition:**
 $n_c = (5.0 \pm 0.3) \times 10^{11} \text{ cm}^{-2}$
- **glass transition (will show later):**
 $n_g = (7.5 \pm 0.3) \times 10^{11} \text{ cm}^{-2}$

→ metallic glass

$k_F l < 1$ (“bad” metal)



[S. Bogdanovich and D. Popović,
PRL 88, 236401 (2002)]



- at the lowest n_s , strongly localized:

$$\langle \sigma \rangle \propto \exp(-T_0/T),$$

$$\longrightarrow n_c = (5.0 \pm 0.3) \times 10^{11} \text{ cm}^{-2},$$

$$n_c \ll n_s^*$$

- just above n_c (metallic glass):

$$\langle \sigma \rangle = a(n_s) + b(n_s)T^x, \quad x \approx 1.5$$

non-Fermi liquid behavior

(not a good metal)

$$\langle \sigma(n_c, T) \rangle \propto T^{3/2} \quad (\text{a power law, as it should be for the MIT})$$

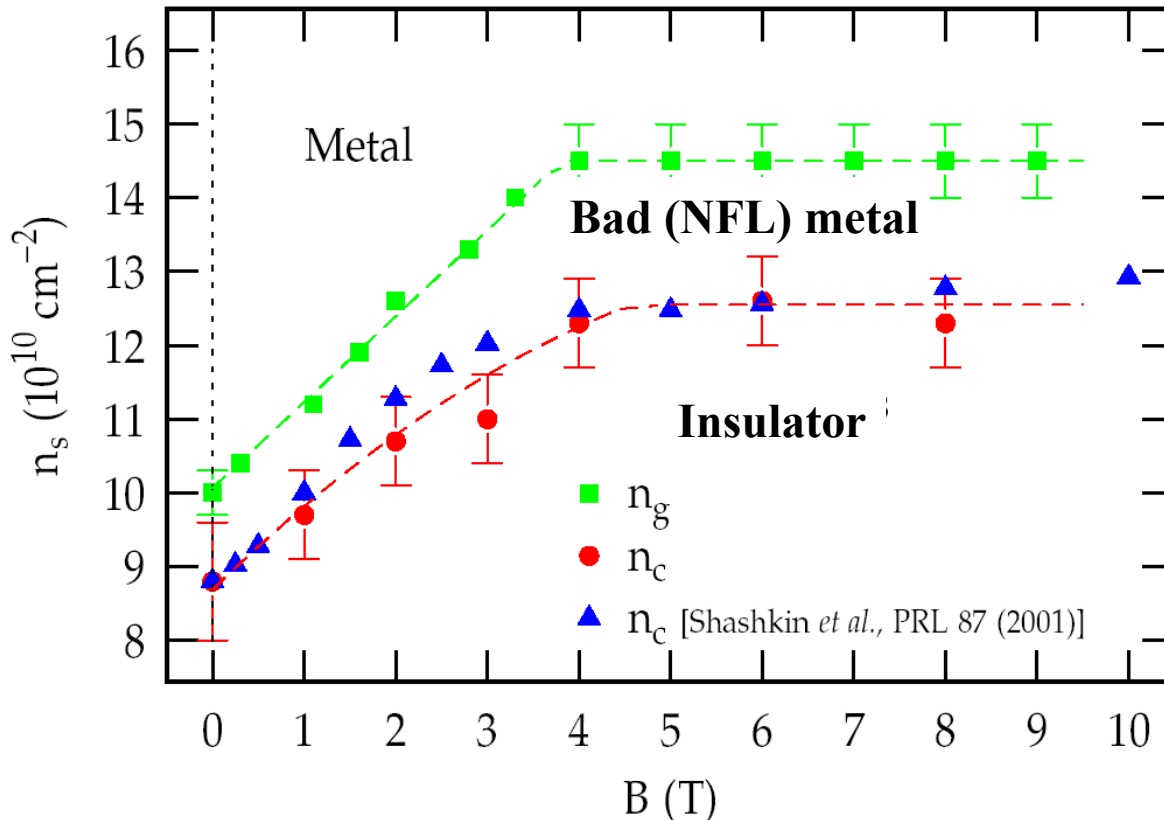
(consistent with V. Dalidovich and V. Dobrosavljević,
PRB 66, 081107 (2002), for the metallic glass phase)

**Back to high-mobility samples;
apply parallel magnetic field B**

(no orbital effect;
B couples only to spins)



B → 2DES spin-polarized



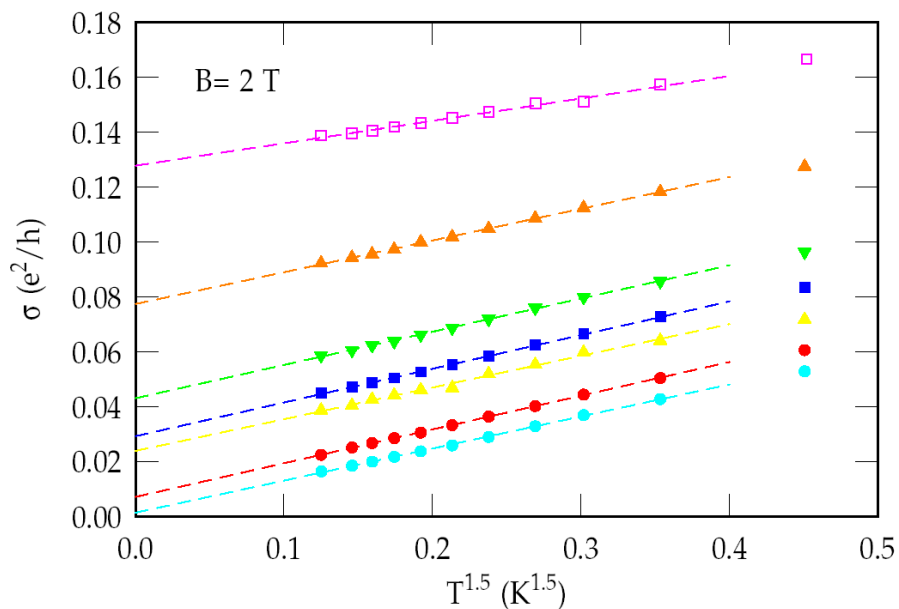
B=0:
(almost) no intermediate phase

Apply B:
emergence of intermediate phase with the same $\sigma(T)$ as in samples with high disorder

(suppression of screening by parallel B \Rightarrow effective disorder increases)

[Jaroszyński, Popović, Klapwijk, PRL 92, 226403 (2004)]

Intermediate metallic phase



$$\sigma(n_s, T, B) = \sigma(n_s, T=0, B) + b(n_s, B) T^{3/2} !$$

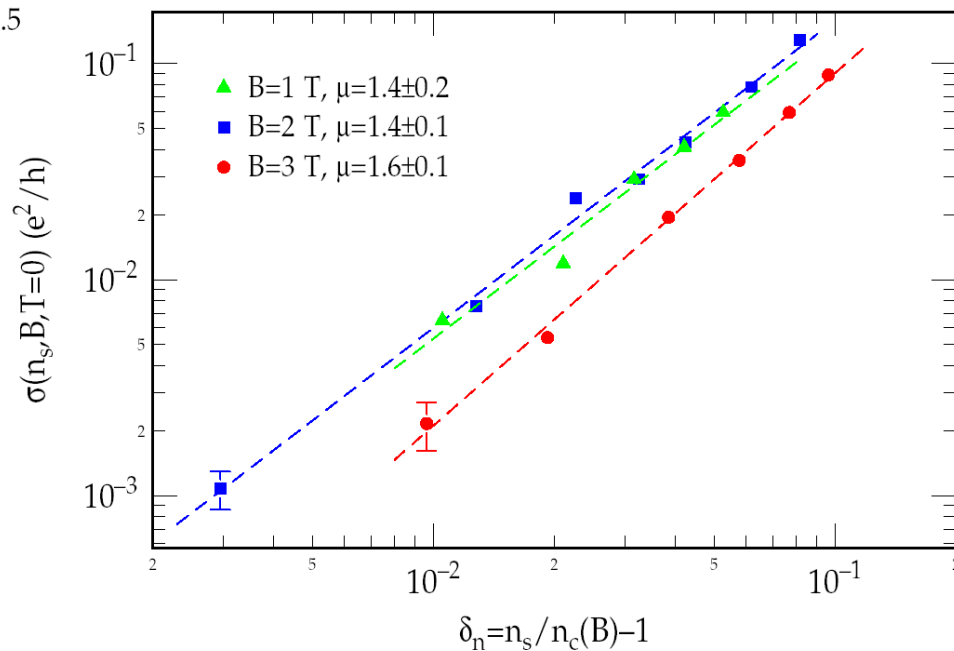
(same as in low-mobility samples at $B=0$!)

$$\sigma(n_s, T=0, B) \propto \delta_n^\mu, \quad \mu \sim 1.5$$

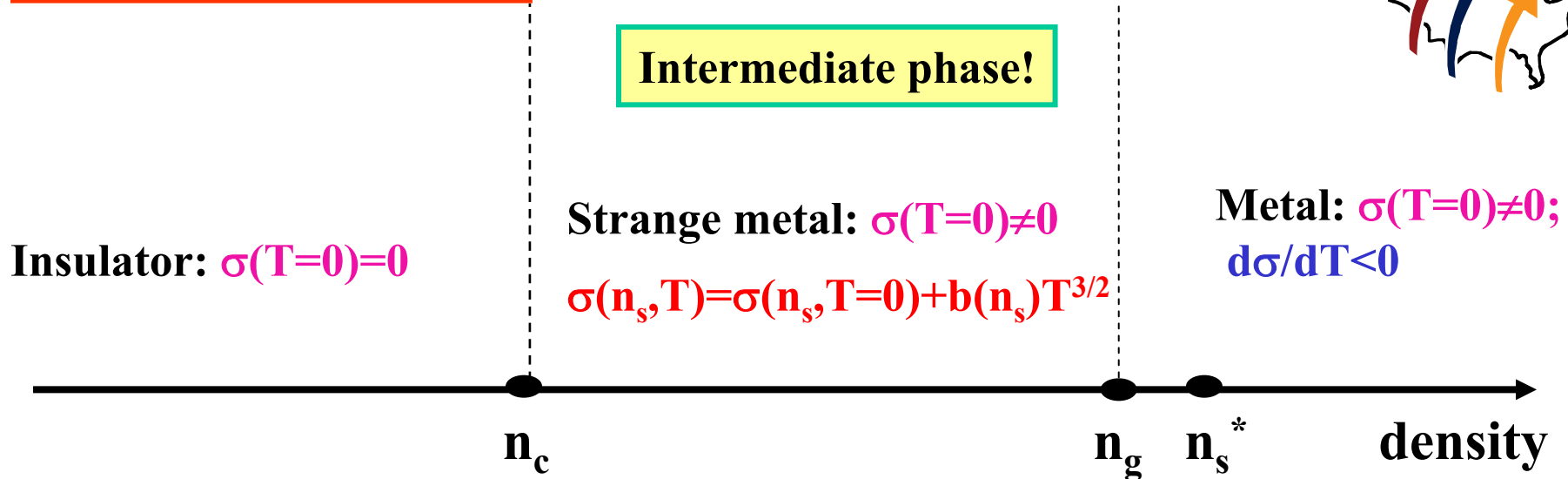
consistent with QPT

$n_s (10^{10} \text{ cm}^{-2}) = 11.9, 11.6, 11.3, 11.2, 11.0, 10.9, 10.7$ from top; $n_c(B=2\text{T}) = 10.67 \times 10^{10} \text{ cm}^{-2}$

- at $B=0$, $\mu \sim 1-1.5$ [Fletcher et al., *Semicond. Sci. Tech.* 16, 386 (2001)]



T=0 phase diagram



n_s^* – separatrix (from transport)

n_g – glass transition (will show later)

n_c – critical density for the MIT from $\sigma(T)$ on both insulating and metallic sides

High disorder (low-mobility devices): $n_c < n_g < n_s^*$

Low disorder (high-mobility devices): $n_c \approx n_s^* \gtrsim n_g$ for $B=0$,

$n_c < n_s^* \gtrsim n_g$ for $B \neq 0$

[Bogdanovich, Popović, PRL 88, 236401 (2002);
 Jaroszyński, Popović, Klapwijk, PRL 89, 276401 (2002);
 Jaroszyński, Popović, Klapwijk, PRL 92, 226403 (2004)]

How to probe glassy dynamics?



- measure **response** of the system **to** some kind of a **perturbation** (e.g. after a rapid cooling; a spin glass in a magnetic field)
- here, perturbation = **change of V_g** ; measure conductivity **σ vs. time t** after the perturbation is switched off



supercooled water

[see also papers by Z. Ovadyahu for similar work in InO_x electron glass deep in the insulating regime]

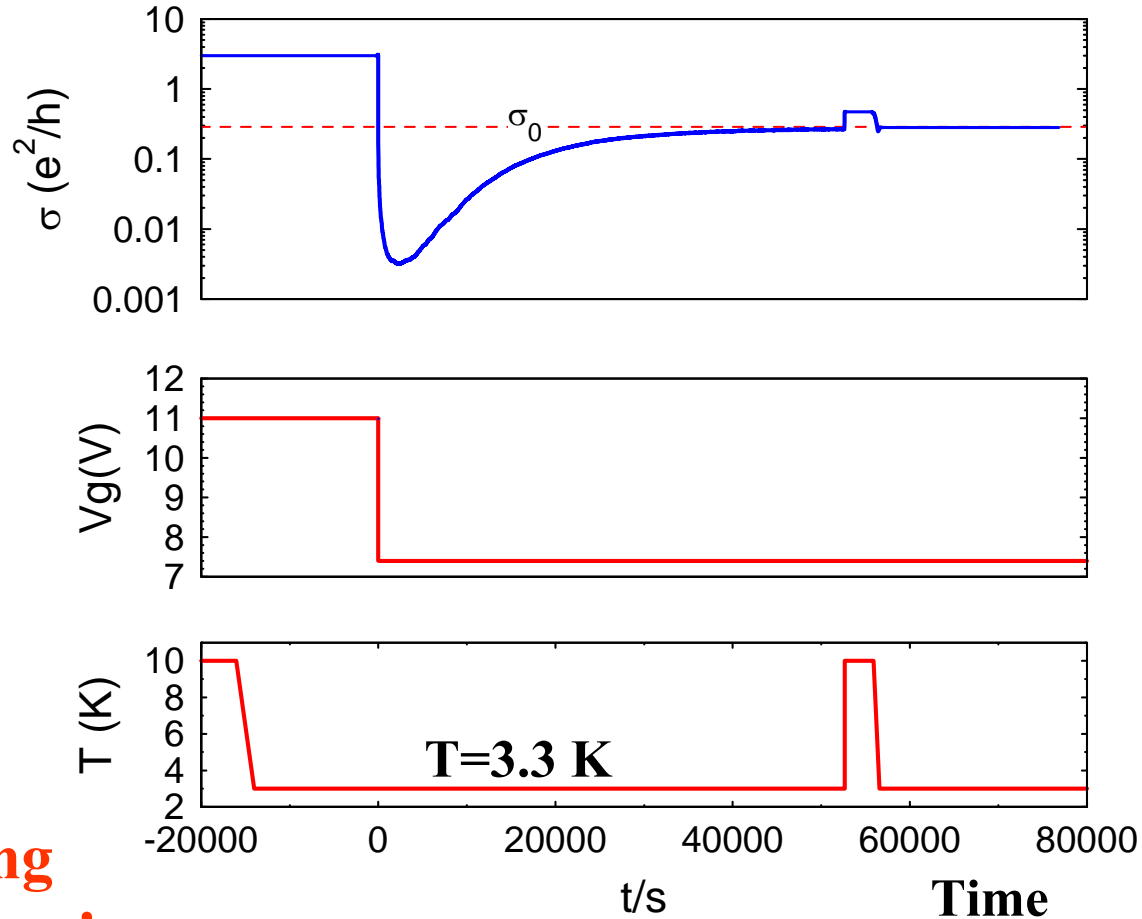
Relaxations of conductivity after a rapid change of n_s



$n_g \approx 7.5 \times 10^{11} \text{ cm}^{-2}$, $n_c \approx 4.5 \times 10^{11} \text{ cm}^{-2}$

Low-mobility samples

Initial
 $n_s (10^{11} \text{ cm}^{-2})$
 $= 20.26 > n_g$
 $k_F l \leq 1$



σ_0 – equilibrium conductivity at T and final n_s

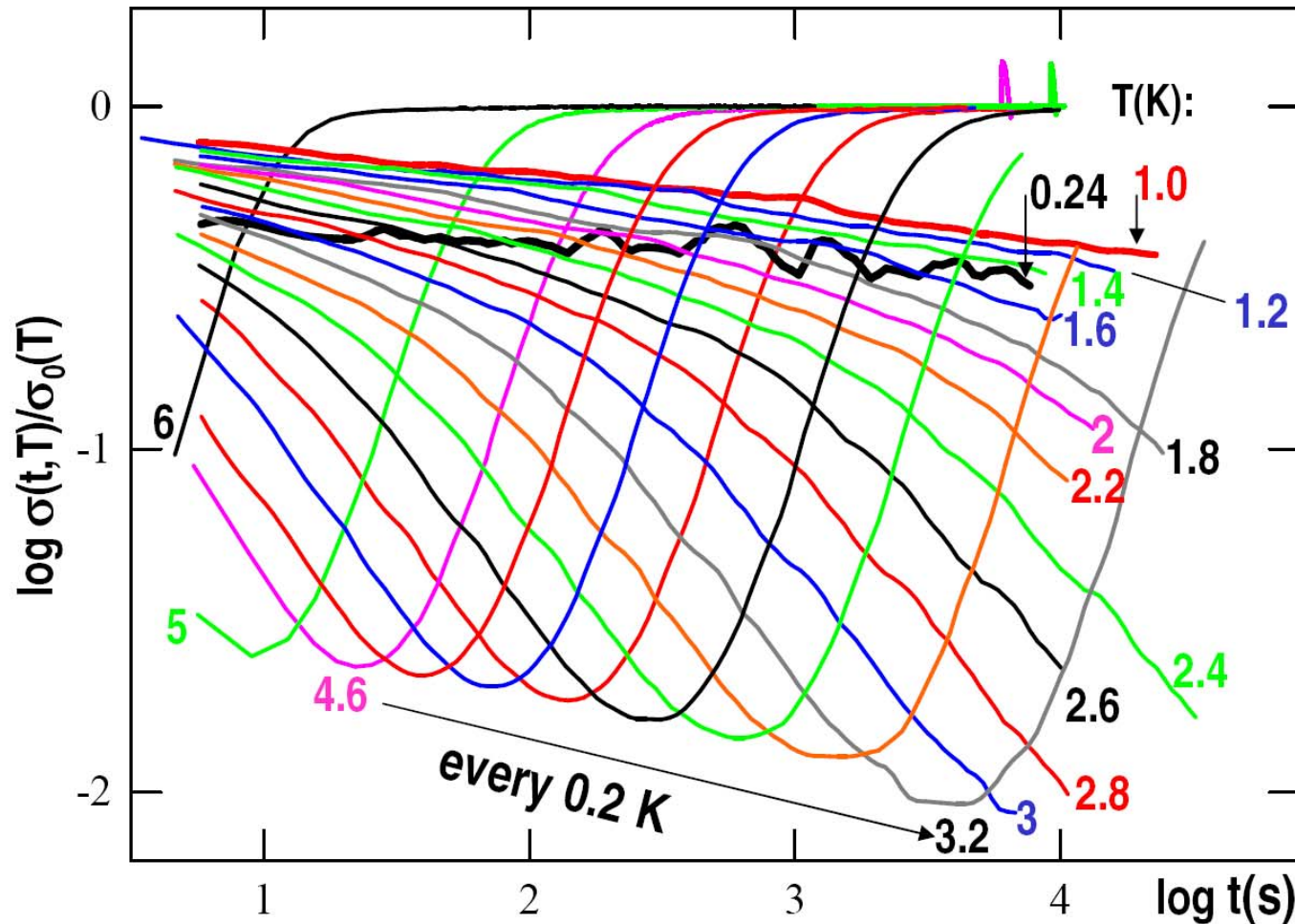
Final
 $n_s (10^{11} \text{ cm}^{-2}) =$
 $= 4.74 \geq n_c$

$\Delta E_F \gg k_B T$

Overshooting of equilibrium!

[J. Jaroszyński and D. Popović, PRL 96, 037403 (2006)]

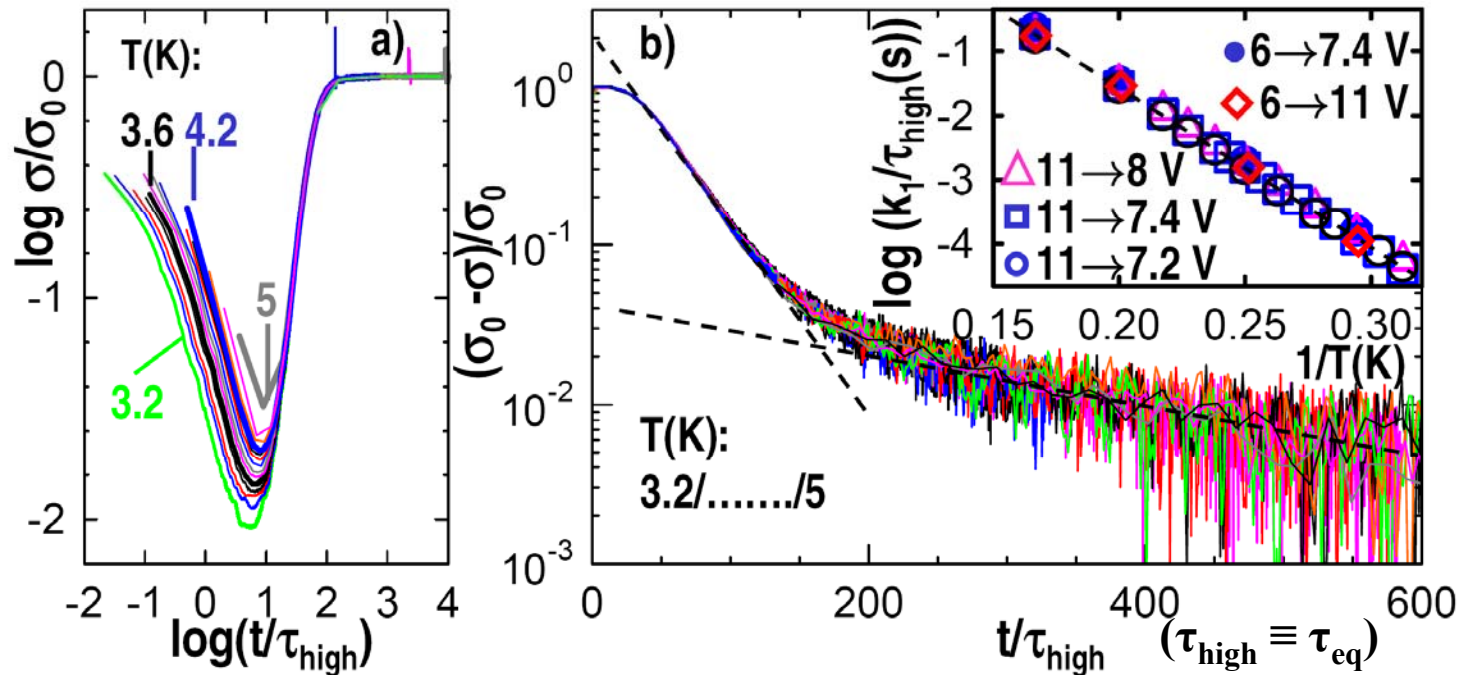
Repeat measurement at (many) different T (after warm-up to 10 K):



- minimum moves to longer times as T decreases – slower relaxations

Approach to equilibrium:

data (for different T) collapse for times after the minimum



- Relaxations exponential

- Characteristic (equilibration) time $\tau_{\text{eq}} \propto \exp(E_A/T)$, $E_A \approx 57$ K
- The system reaches equilibrium after a long enough t

$\tau_{\text{eq}} \rightarrow \infty$ as $T \rightarrow 0$, *i.e.* glass transition $T_g = 0$

[see Gempel, Europhys. Lett. 66, 854 (2004)
for a 2D Coulomb glass; also showed aging!]

Initial relaxation:

data (for different T) collapse for times before the minimum:



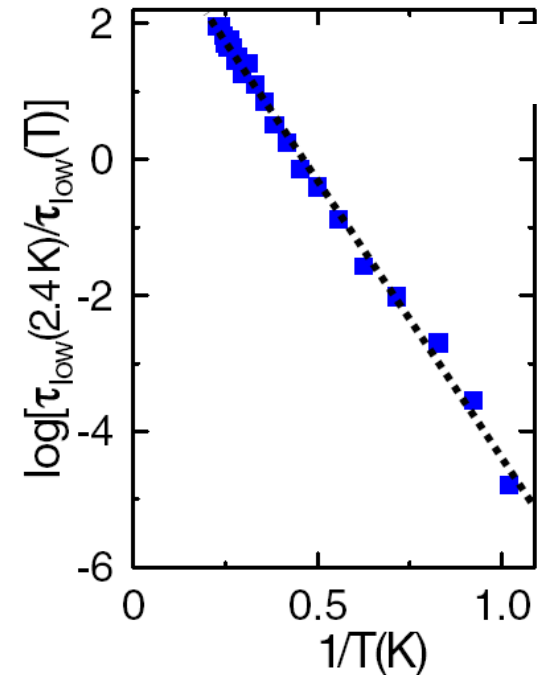
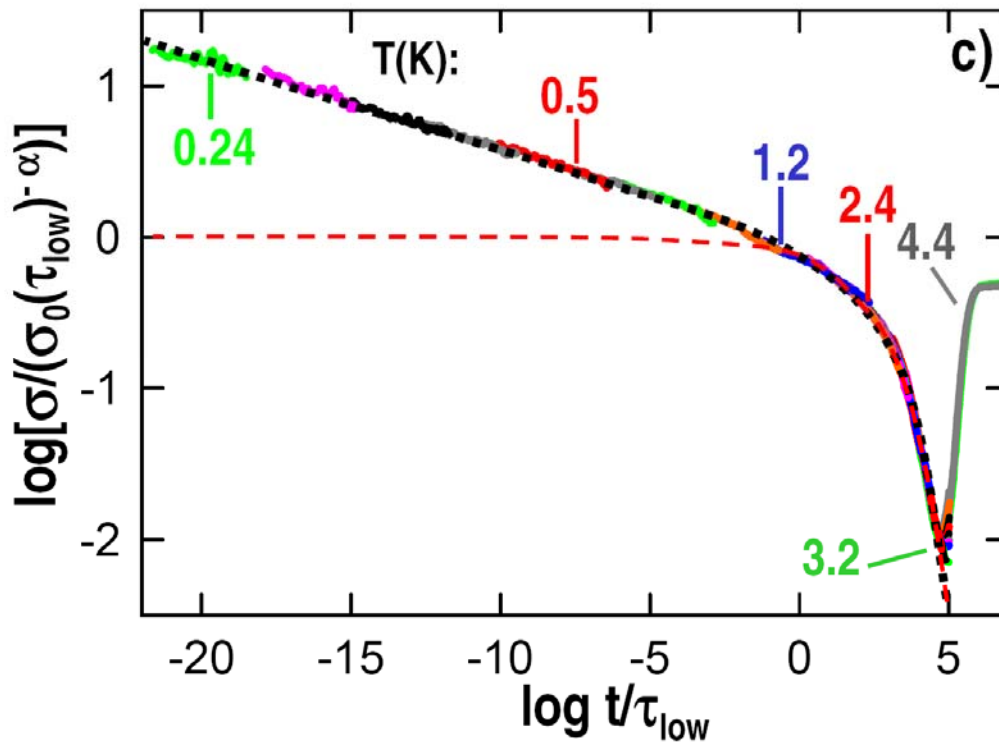
- for short enough $t < \tau_{eq}$,

$$\sigma(t, T) / \sigma_0 \propto t^{-\alpha(n)} \exp\{-[t/\tau_{low}(n_s, T)]^{\beta(n)}\} \quad (\alpha=0.07, \beta < 0.3 \text{ for this } n_s)$$

($n \equiv n_s$)

glassy relaxation

$$\tau_{low} \propto f(n_s) \exp(E_a/T), \quad E_a \approx 20 \text{ K}$$





Repeat everything for many different n_s



$$\tau_{\text{low}} \propto \exp(an_s^{1/2}) \exp(E_a/T), E_a \approx 20 \text{ K}$$

• $T \rightarrow 0$:

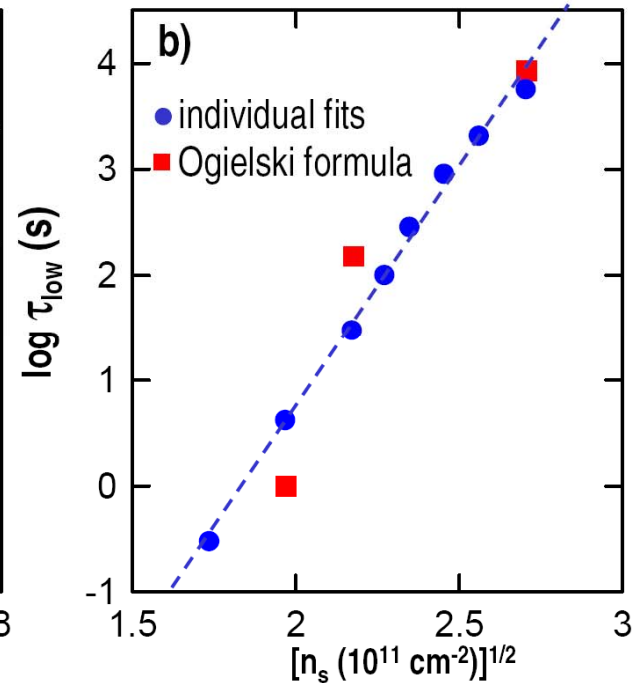
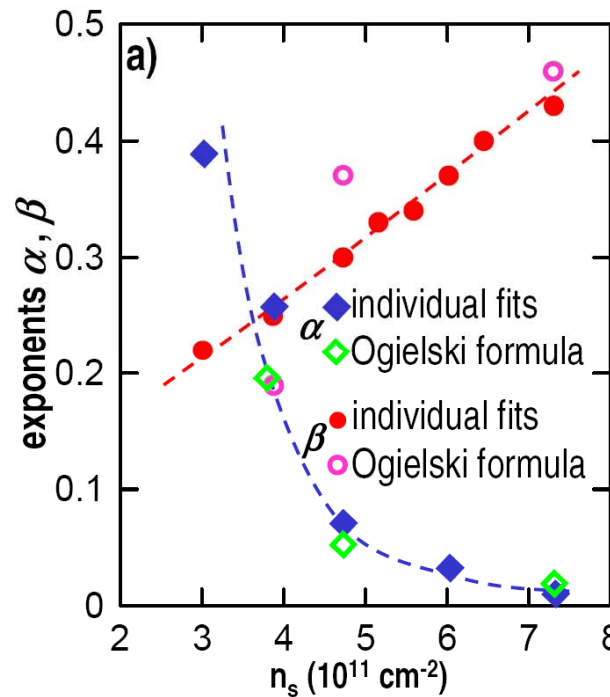
$$\sigma/\sigma_0 \propto t^{-\alpha(n)}$$

as expected for a phase transition at $T=0$

(previous slide: scaling as $T \rightarrow 0$)

$$\alpha(n_s) \rightarrow 0 \text{ as } n_s \rightarrow n_g^-$$

(no slow relaxation for $n_s > n_g$)



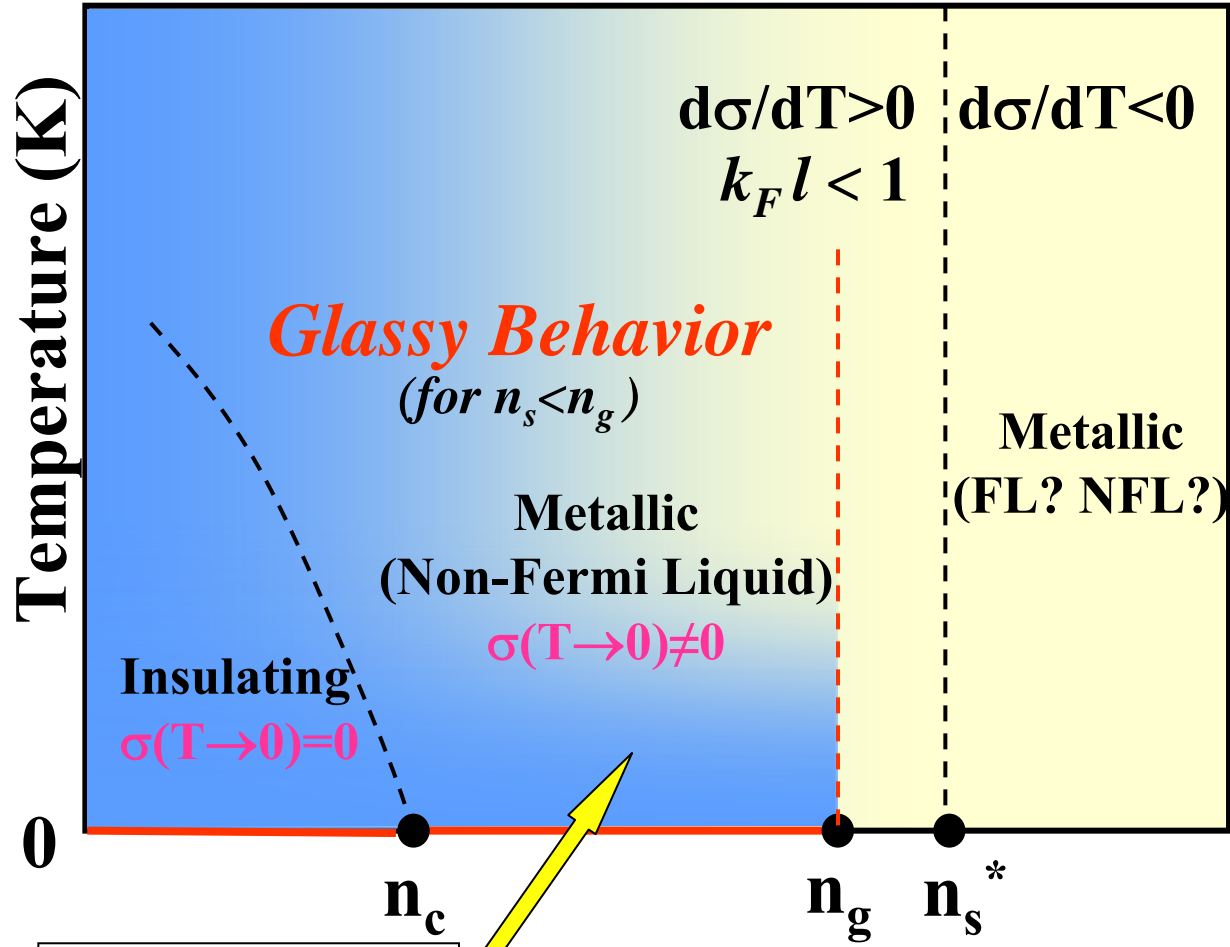
Coulomb interactions in 2D: $E_F/U \sim n_s^{1/2}$

What have we learned from relaxations?



- data strongly suggest $T_g=0$ for $n_s \leq n_g$ in a 2DES in Si
(diverging equilibration time, scaling of nonexponential relaxations, power law as $T \rightarrow 0 \Rightarrow T_g = 0$; similar behavior in spin glasses, where $T_g \neq 0$)
 - at finite T , the system appears glassy for short enough t
(e.g. at $T=1$ K, equilibration time $\sim 10^{13}$ years!
age of the Universe $\sim 10^{10}$ years)
 - Coulomb interactions between 2D electrons – a dominant role in the out-of-equilibrium dynamics
 - as $T \rightarrow 0$, no relaxations for $n_s > n_g$; no relaxations for $k_F l > 1$
- Note:** system equilibrates only after it first goes farther away from equilibrium!

Phase diagram of a 2DES in Si

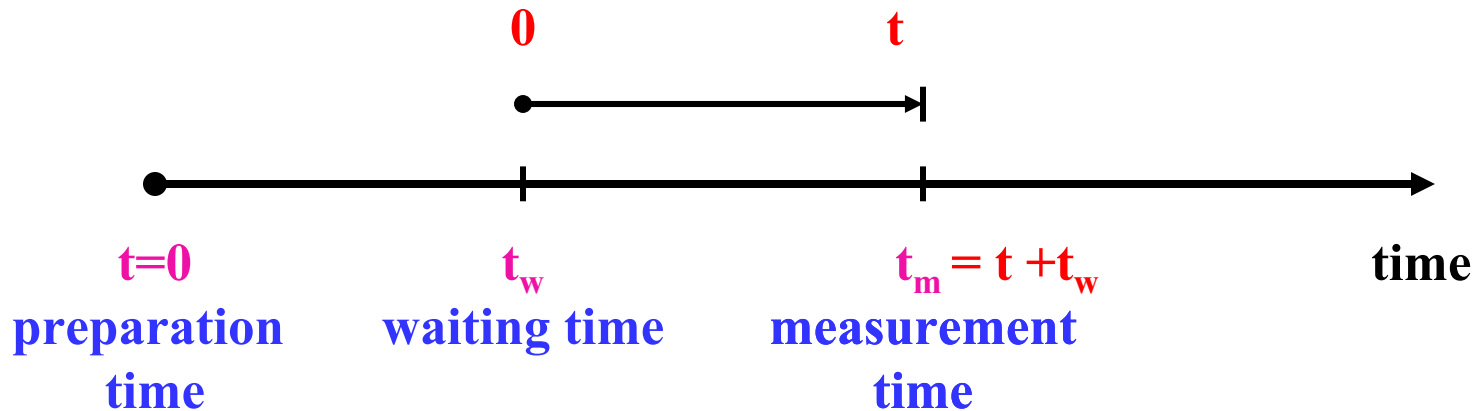


intermediate,
glassy phase

Glassy regime: slow, nonexponential relaxations, diverging equilibration time ($T_g=0$)



Times

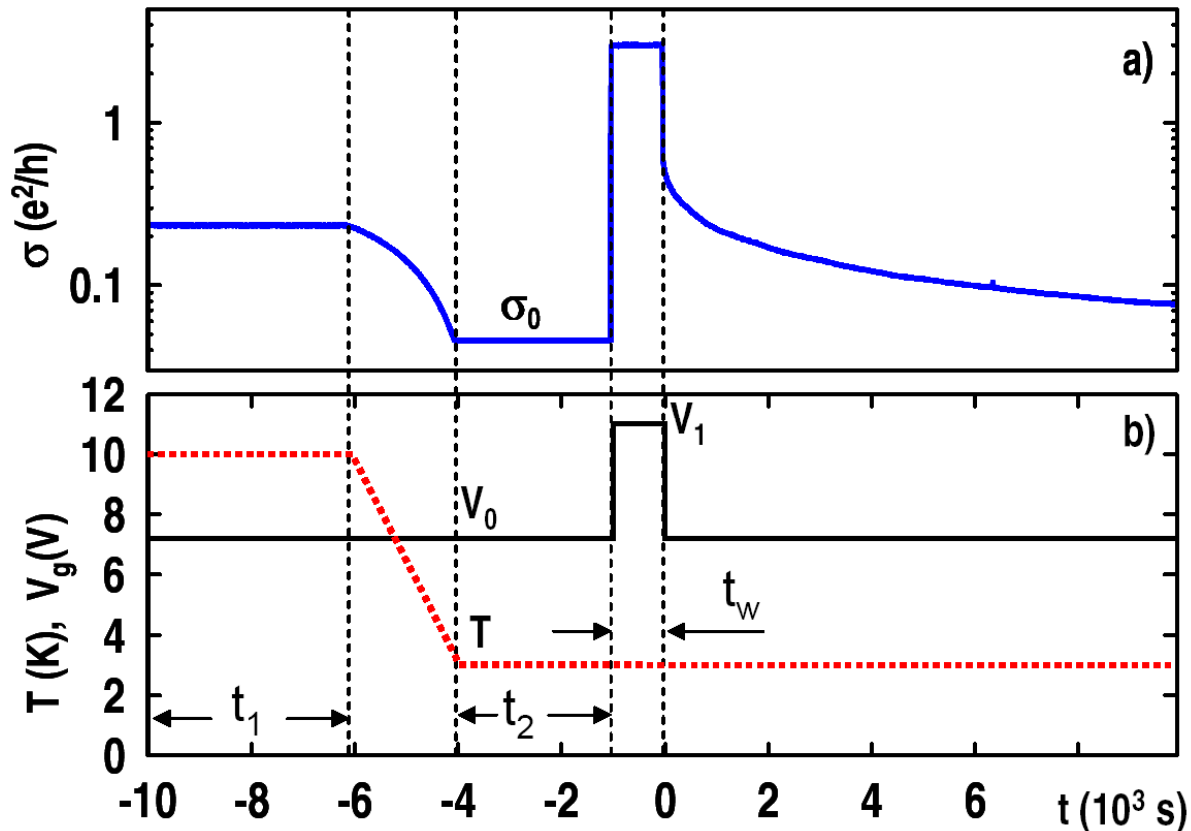


In equilibrium $\tau_{eq} < t_m$
Out of equilibrium $\tau_{eq} > t_m$

Out of equilibrium:
responses and correlations depend
on two times, t and t_w (age) - aging

- *Slow Relaxations and Nonequilibrium Dynamics in Condensed Matter*, edited by J.-L. Barrat, M.V. Feigelman, J. Kurchan, J. Dalibard (Springer, New York, 2003) - Les Houches summer school
- *Ageing and the Glass Transition*, edited by M. Henkel, M. Pleimling, R. Sanctuary (Lecture Notes in Physics, Springer, 2007) – Univ. of Luxemburg summer school

Relaxations of conductivity after a waiting time protocol: aging and memory



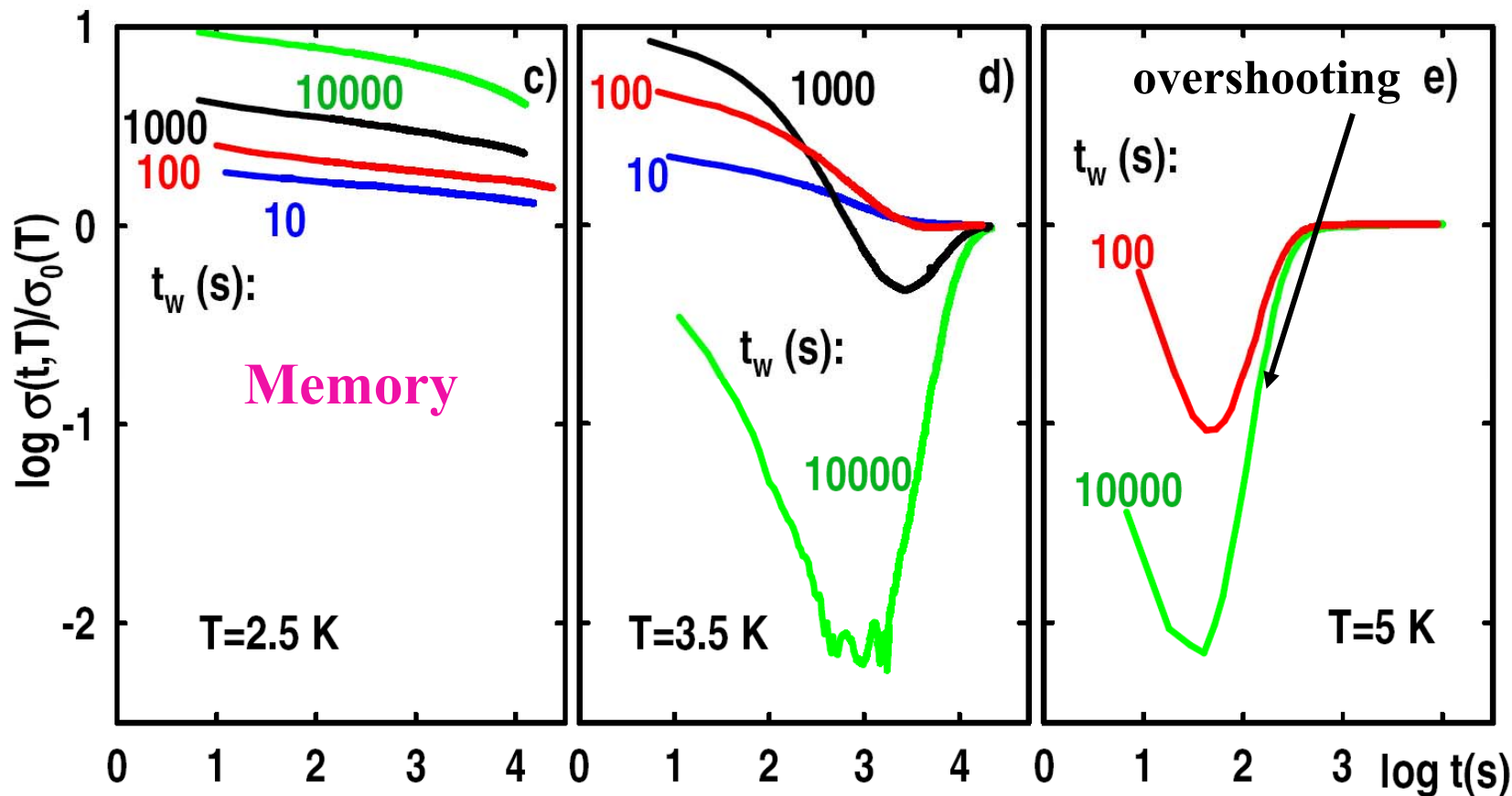
Initial and final
 $n_s(10^{11}\text{cm}^{-2})=3.88 < n_c$;
 density during $t_w=1000$ s:
 $n_s(10^{11}\text{cm}^{-2})=20.26 > n_g$

- change history by varying T and t_w

[J. Jaroszyński and D. Popović, Phys. Rev. Lett. 99, 046405 (2007)]

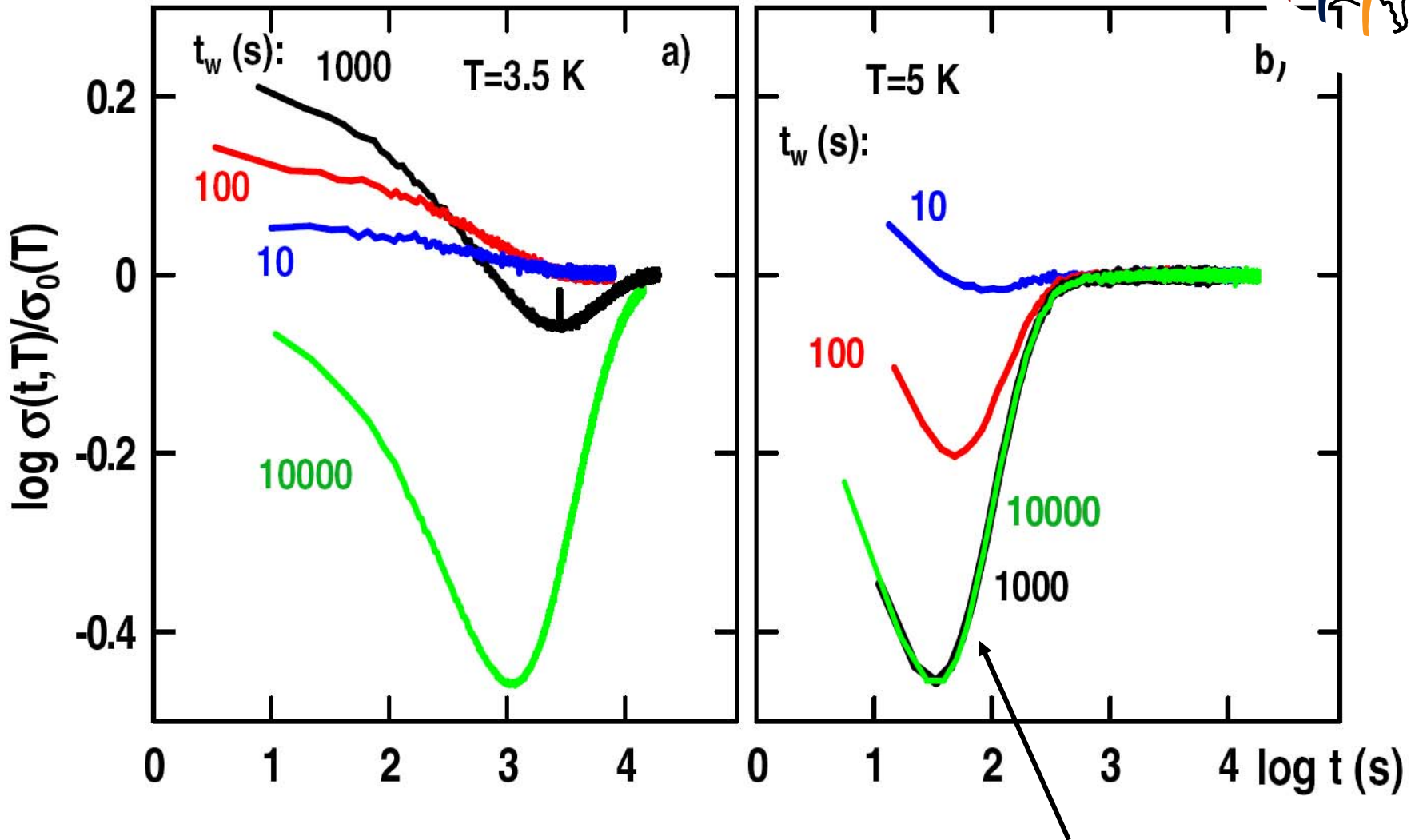


Relaxations for a few different T and t_w :



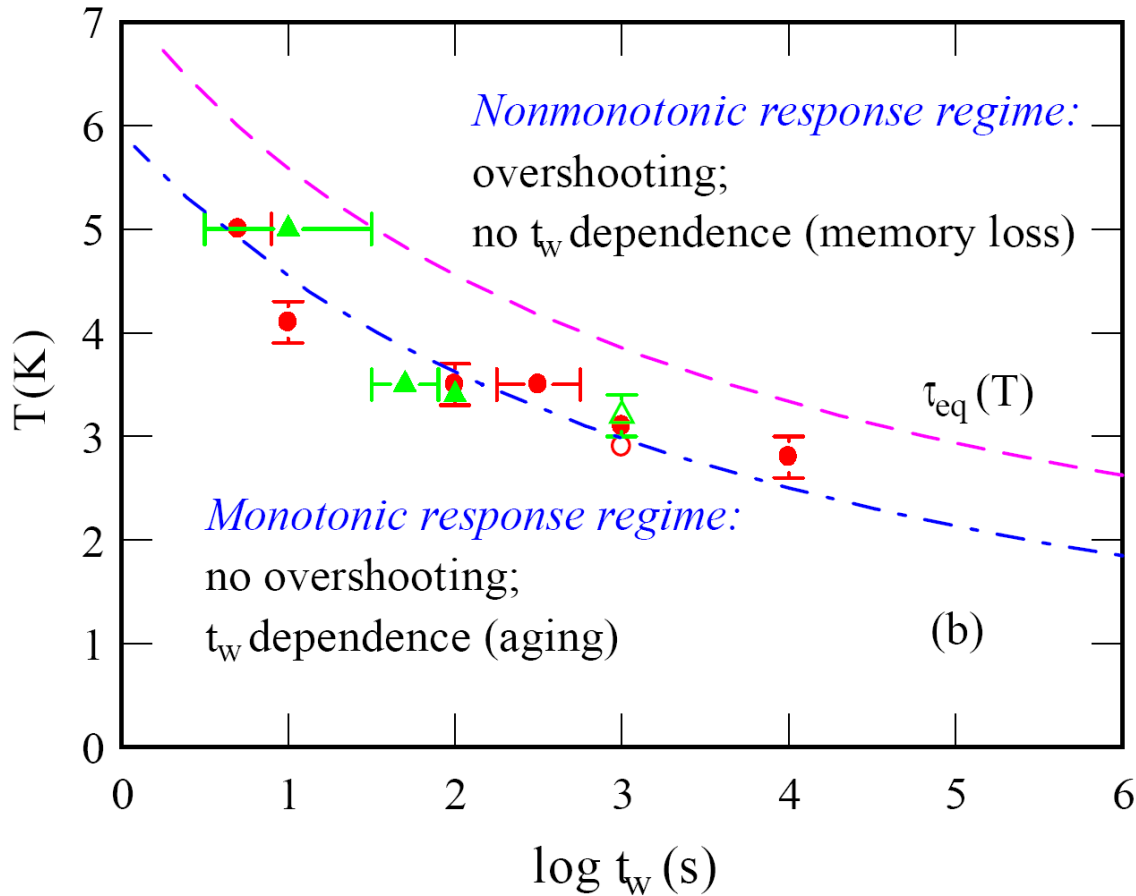
Response (conductivity) depends on the system history (t_w and T) in addition to the time t – *aging* – a key characteristic of relaxing glassy systems.

And a few more... :



Memory loss

When is overshooting observed?



- **overshooting** only when the system is excited out of a thermal equilibrium ($t_w \gg \tau_{eq}$); no memory
- no OS when excited out of a relaxing (nonequil.) state ($t_w \ll \tau_{eq}$): **aging and memory**

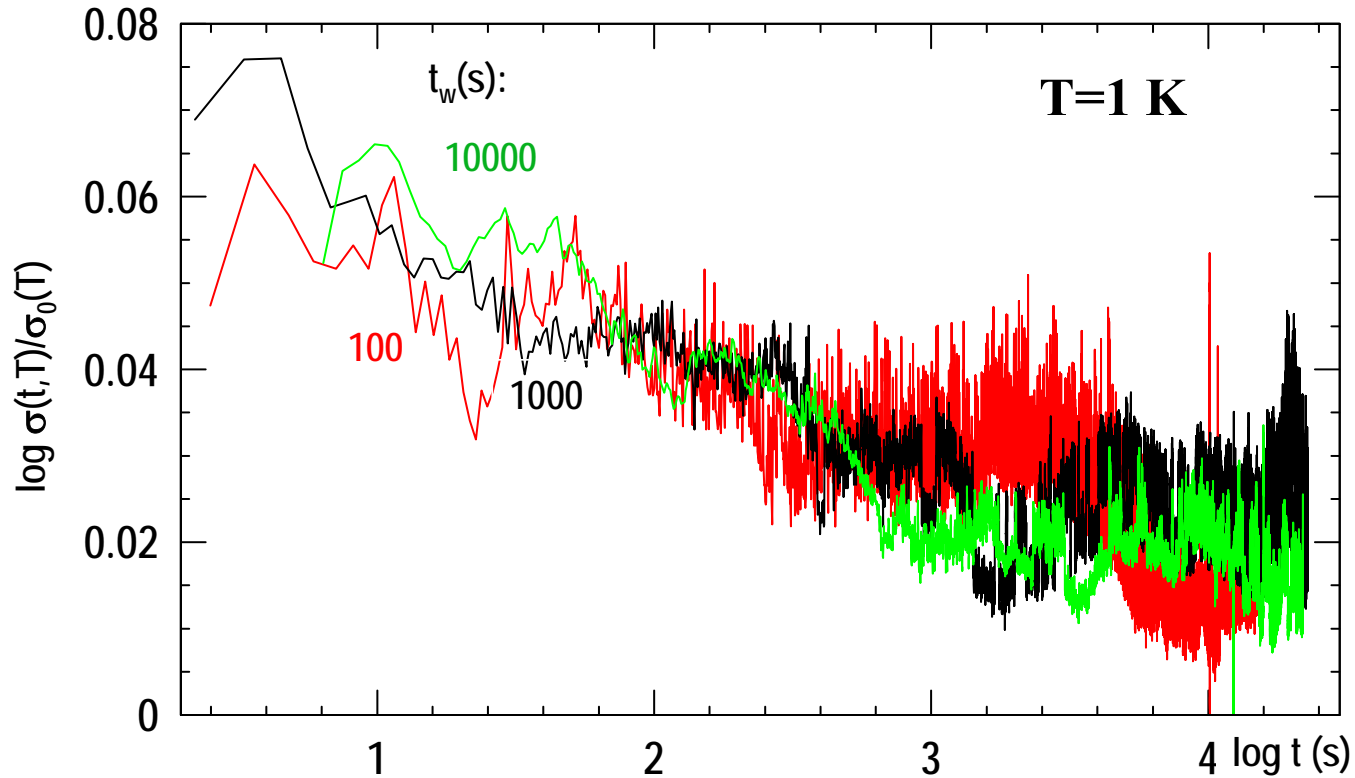
What is the origin of overshooting???



- **observed** in a variety of systems (e.g. insulating granular metals, manganites, biological systems)
- some theoretical **models**
[Morita *et al.*, PRL 94, 087203 (2005); Mauro *et al.*, PRL 102, 155506 (2009)]
- **large** perturbations out of equilibrium?
- here $\Delta E_F \gg T$ should trigger **major charge rearrangements**
(n_s changed up to a factor of 7; in InO_x , density change $\sim 1\%$)



Remove all 2D electrons from the inversion layer during t_w
($V_1 < V_T$):



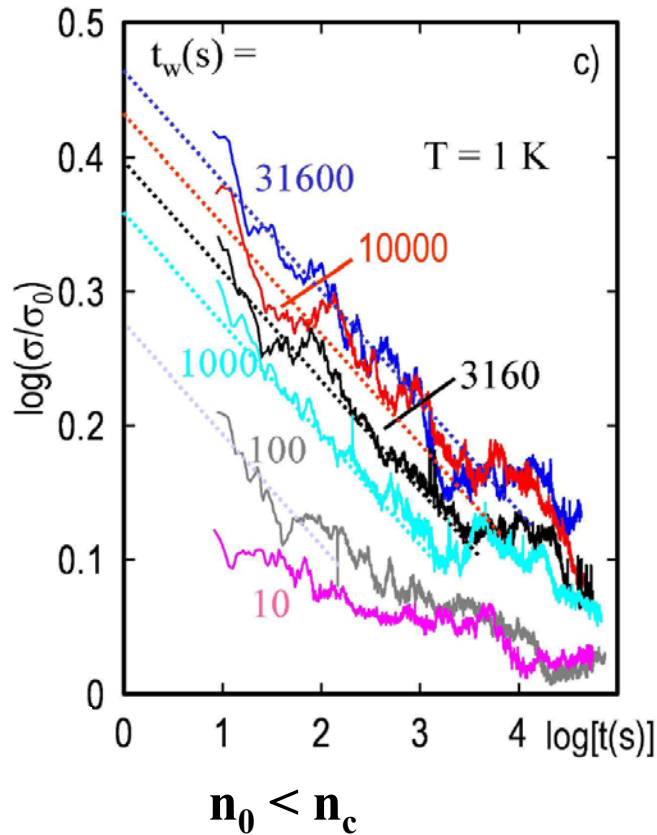
No t_w dependence, *i.e.* no memory!

\Rightarrow Glassiness from 2DES, not from background charges

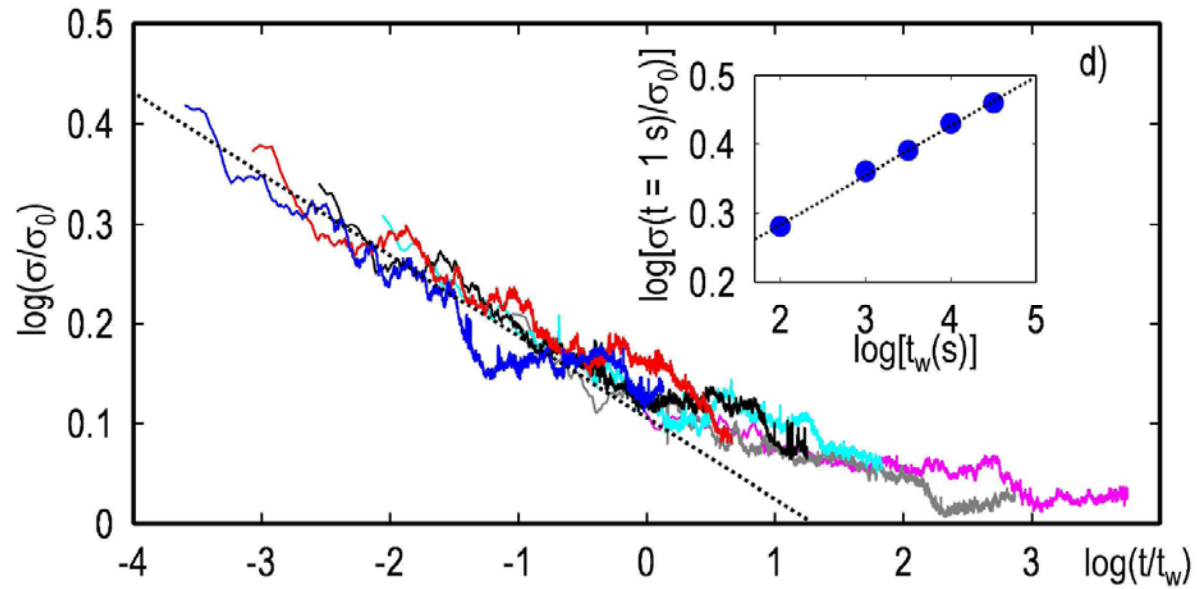
Aging regime (no OS, T=1 K)

[J. Jaroszyński and D. Popović, Phys. Rev. Lett. 99, 216401 (2007)]

(T= 1 K: $\tau_{eq} \sim 10^{13}$ years!
Age of the Universe $\sim 10^{10}$ years)



Full (simple) aging: $\sigma(t/t_w)$



$$\sigma(t)/\sigma_0 \propto (t/t_w)^{-\alpha} \quad \text{for } t \leq t_w$$

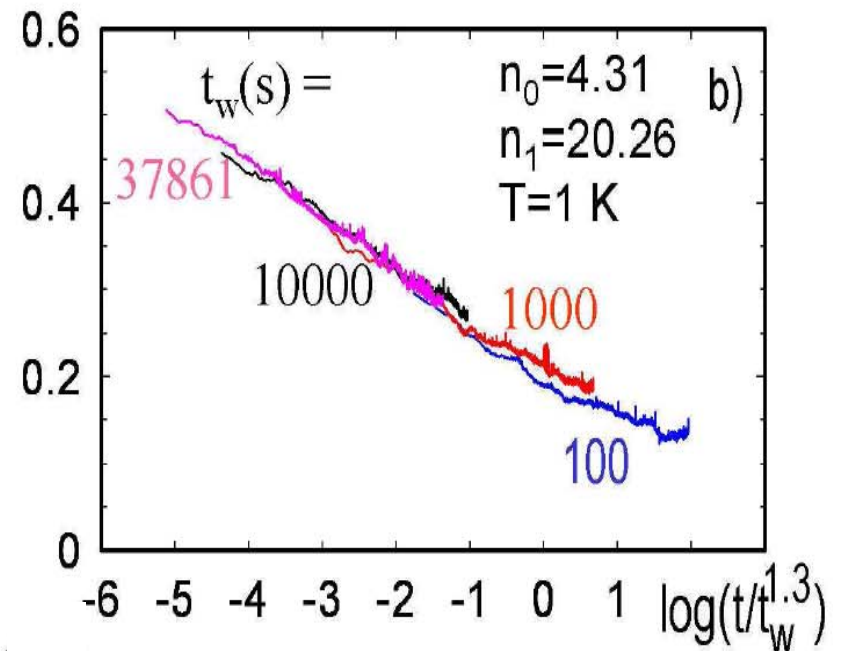
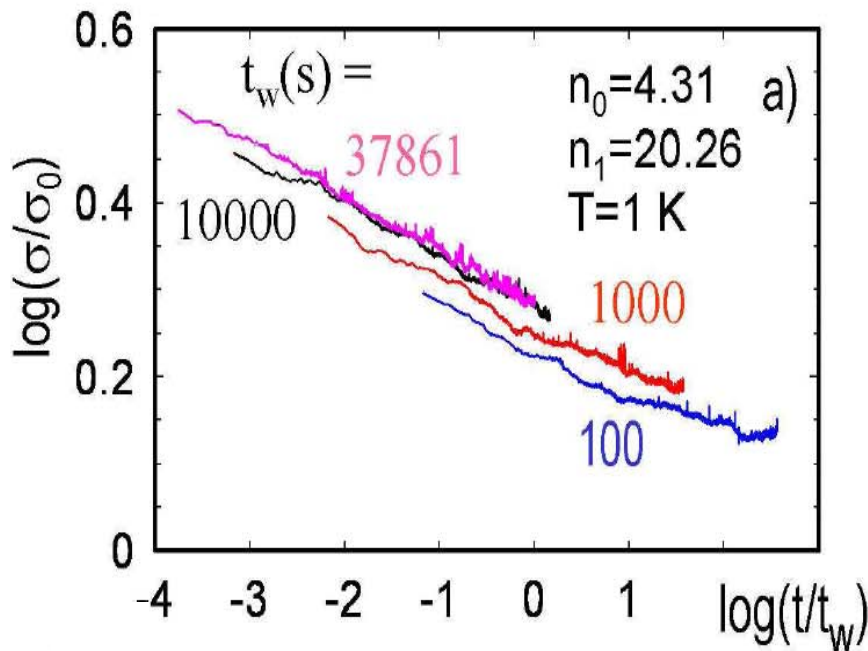
\Rightarrow a memory of t_w is imprinted on each $\sigma(t)$



- $\sigma(t, t_w)$ exhibit **full aging** for $n_s < n_c$
- for $n_s > n_c$, an increasingly strong **departure from full aging** that reaches maximum at n_g

aging function: $\sigma(t/t_w^\mu)$

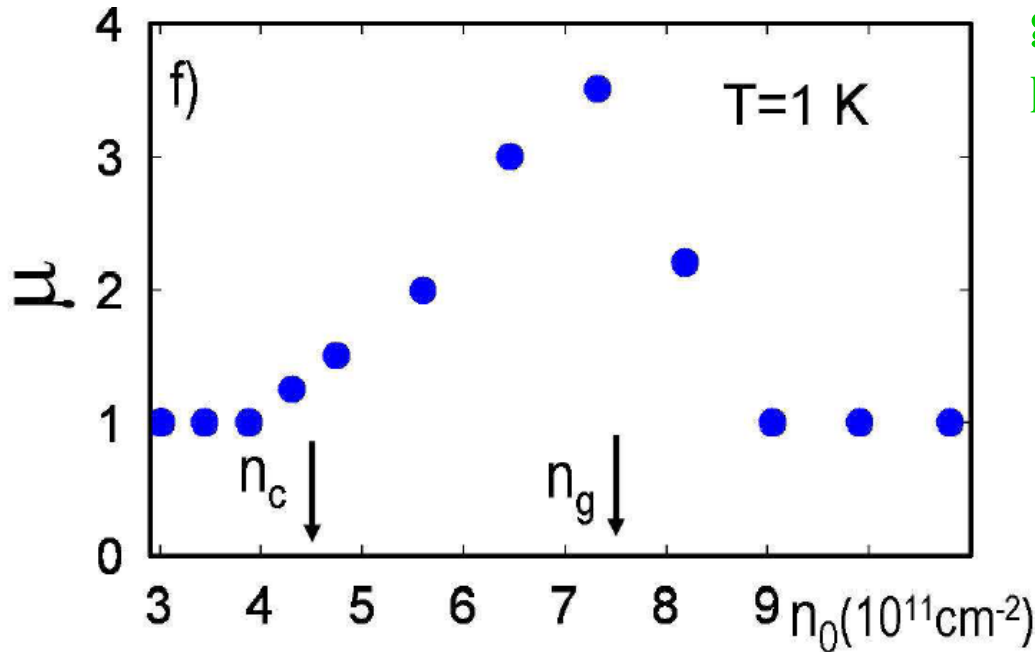
(μ -scaling useful in studies of other glasses; may not have a clear physical meaning)





- $\sigma(t, t_w)$ exhibit **full aging** for $n_s < n_c$
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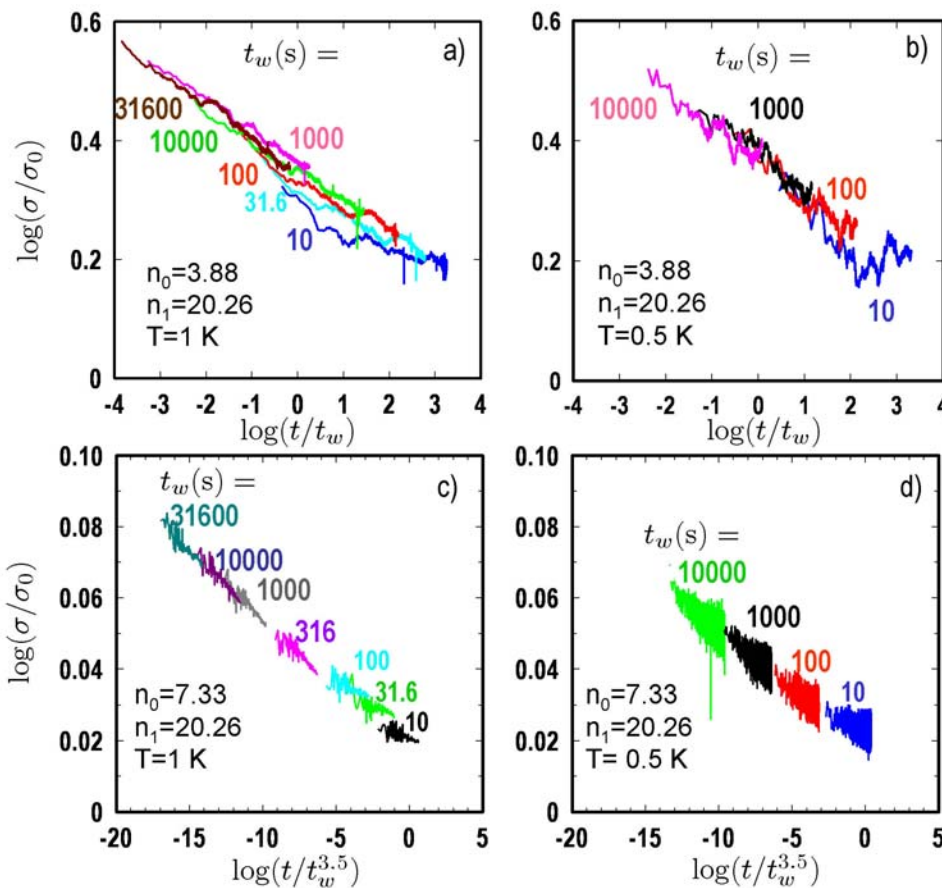


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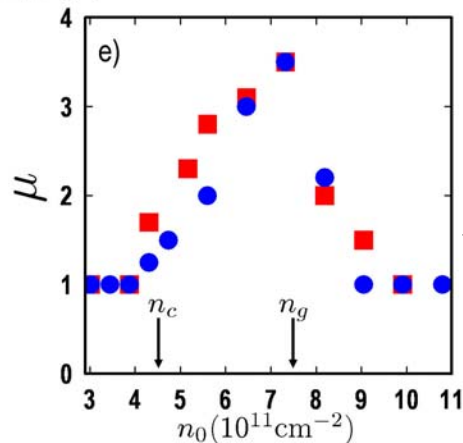
full aging: $\mu=1$

- an abrupt change in aging at the 2D MIT (n_c)
- insulating glassy phase and metallic glassy phase are different!

NOTE: mean-field models of glasses include both those that show full aging and those where no t/t_w scaling is expected.

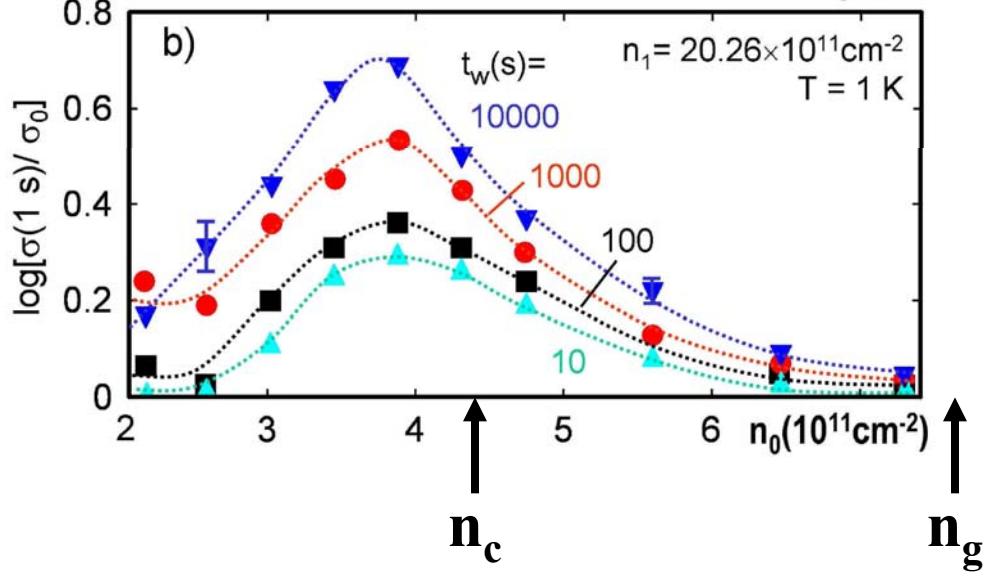
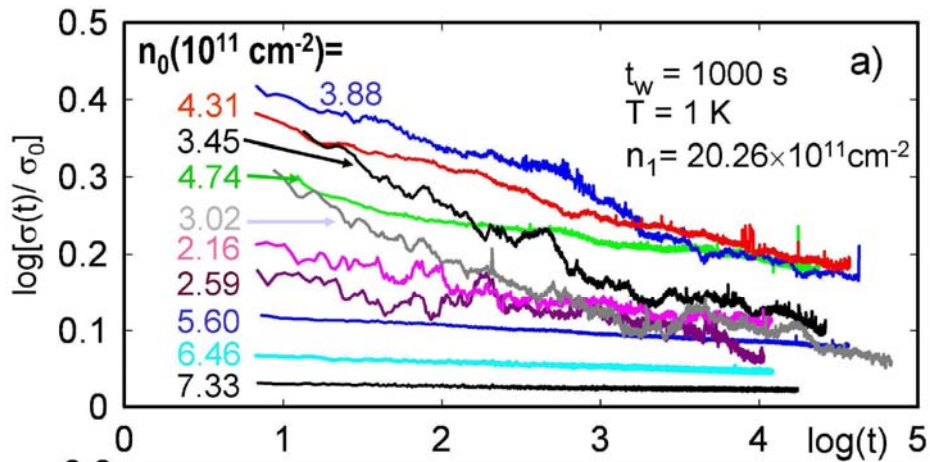


μ does not depend on temperature



two different samples

Fixed t_w and n_1 ; vary n_0



$$\sigma(t)/\sigma_0 = [\sigma(t=1s)/\sigma_0] t^{-\alpha}$$

- both relaxation amplitudes $\sigma(t=1s)/\sigma_0$ and slopes α depend nonmonotonically on n_0
- another change in aging properties at $n_s \approx n_c$

Relaxation amplitudes peak just below n_c , and they are suppressed in the insulating regime!

Summary of Lecture II



- Emergence of an **intermediate, (NFL) metallic phase** ($n_c < n_g$) between the metal and the insulator
- **Glassy behavior for $n_s < n_g$** (in the insulator and in the intermediate phase) – glassy ordering as a **precursor of the MIT in a 2DES in Si**
- abrupt **changes in aging at the MIT**
- **2DES in Si:**
 - **similarities to other glassy systems** (*e.g.* **spin glasses**)
 - a “simple”, **model system** for exploring the dynamics of strongly correlated systems (**free of complications associated with changes in magnetic or structural symmetry**)

Lecture III: other probes of the electron dynamics – fluctuations of σ