



Nonequilibrium dynamics in Coulomb glasses near the metal-insulator transition

Dragana Popović

*National High Magnetic Field Laboratory
Florida State University, Tallahassee, FL, USA*

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Lecture I: Metal-insulator transition and complexity in electronic systems

- Modern **technology**: synthetic materials, devices
- Designing materials: **metals vs. insulators**
- **Metal-insulator transition**
- **Coulomb glass**
- **Miscellaneous** complex electronic systems
- Metal-insulator transition in **two dimensions (2D)** - general
- Practical **realizations of 2D** systems
- Metal-insulator transition (**MIT**) in **2D** – some experiments
- **Literature**

Lecture II: Studies of the electron dynamics near the 2D MIT: Relaxations of conductivity

Lecture III: Studies of the electron dynamics near the 2D MIT: Fluctuations of conductivity

Lecture I: Metal-insulator transition and complexity in electronic systems



Modern technology: 1947 - present

Invention of the transistor

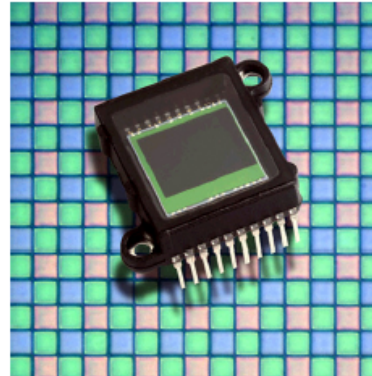
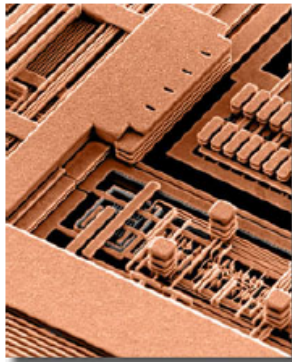
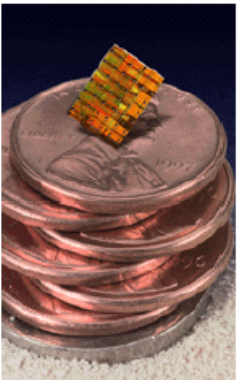
(Bardeen, Brattain, Shockley, Nobel Prize in Physics, 1956)



❖ How does all this work: "The art of electronics"



❖ Ultra small integrated circuits, transistor chips,...



**Invention of
integrated circuits**

**(Kilby, Nobel Prize
in Physics, 2000)**

❖ Devices that control (switch on-off) electrical currents:

Turn conductors into insulators and vice versa

Why semiconductors?

Designing materials: metals vs. insulators



- good metals (Cu, Au, Ag, ...) and (band) insulators (C-diamond, Si, Ge, ...) well understood – see textbooks (band theory)
- many “old” (doped semiconductors – Si:P, ...) and novel materials (high- T_C superconductors, ...) not understood

Turning insulators into metals using doping!

Metals vs. insulators

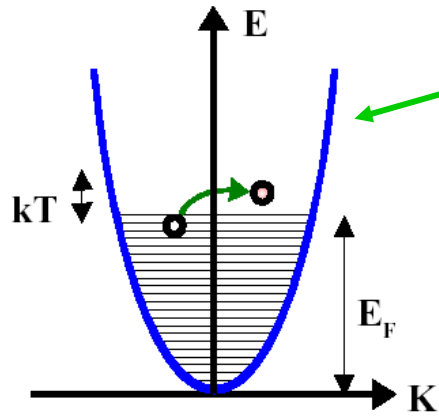
$$\sigma(T=0) \neq 0$$

$$\sigma(T=0) = 0$$

(σ – conductivity; metal-insulator transition at $T=0$ – quantum effects)



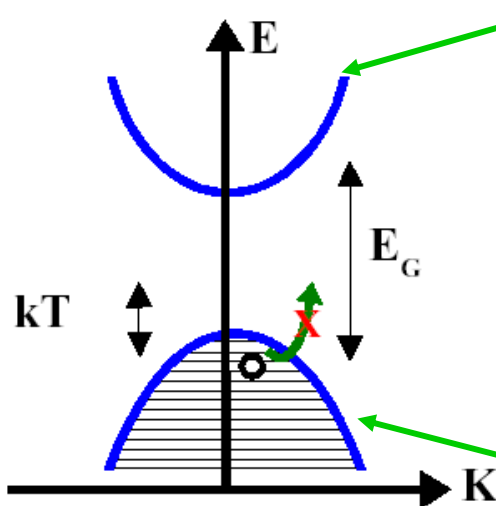
◆ Metals: Pauli principle, large **Fermi** (kinetic) energy
($E_F \sim 10\,000\text{ K}$)



Conduction band (partly filled)

- conductivity $\sigma (T=0) \neq 0$ ($\sigma = 1/\rho$)
- elementary excitations: a few electron-hole pairs (fermions; weakly interacting – “Fermi liquid”)
- **hard to affect**, stable, robust

◆ Insulators: large **energy gap** ($> 5\text{ eV} \approx 50\,000\text{ K}$; room $T \approx 300\text{ K}$)



Conduction band (empty)

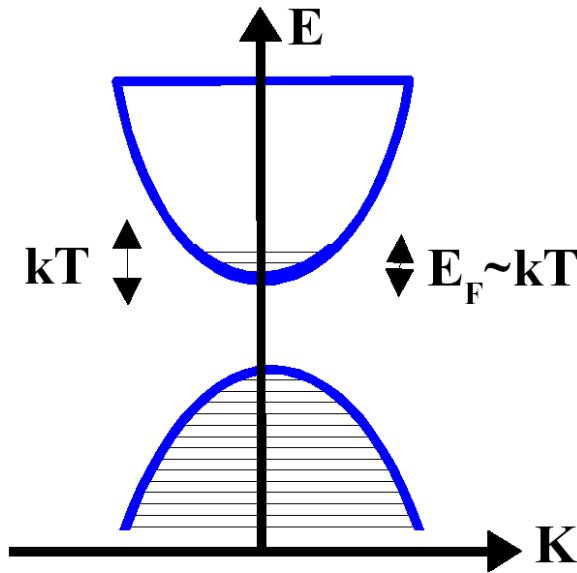
- $\sigma (T=0) = 0$
- elementary excitations: collective modes (phonons, spin waves; typically bosons)
- **hard to affect**, stable, robust

Valence band (filled)



◆ Semiconductors: insulators with $E_G < 1-2 \text{ eV}$ ($\sim 10\,000 \text{ K}$); some electrons in the conduction band at room T as a result of thermal fluctuations

◆ Doped insulators: **small** Fermi energy



- intentional adding of specific impurities (dopant)
→ introducing new charge carriers
- easy (!!!) to affect and control (ρ depends on doping)
- elementary excitations: ???
(no simple picture)

So, are doped semiconductors metals [$\sigma(T=0) \neq 0$]?

Doped semiconductors: Si:P – a classic example (the basis of semiconductor technology!)

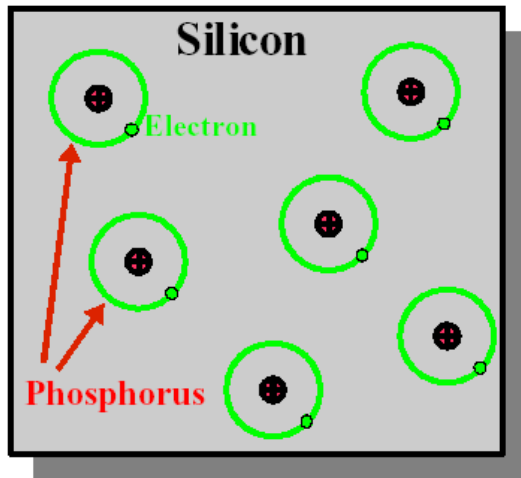


Si – group IV element (four valence electrons)

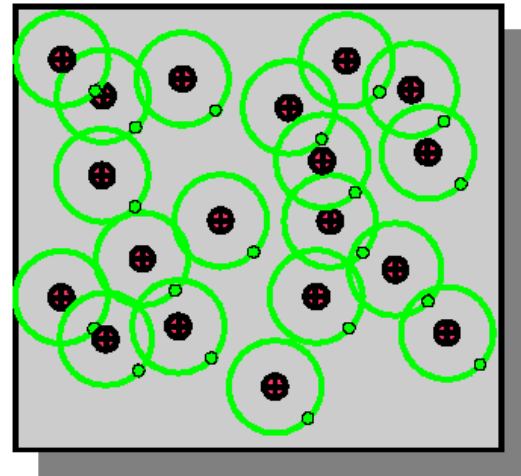
P – group V element (five valence electrons; substitutional impurity)

- the fifth P electron is only **weakly** bound to the P atom

$\sigma(T=0)=0$
Insulator



$\sigma(T=0) \neq 0$
Metal



Carriers but no conductivity at $T=0$!!!

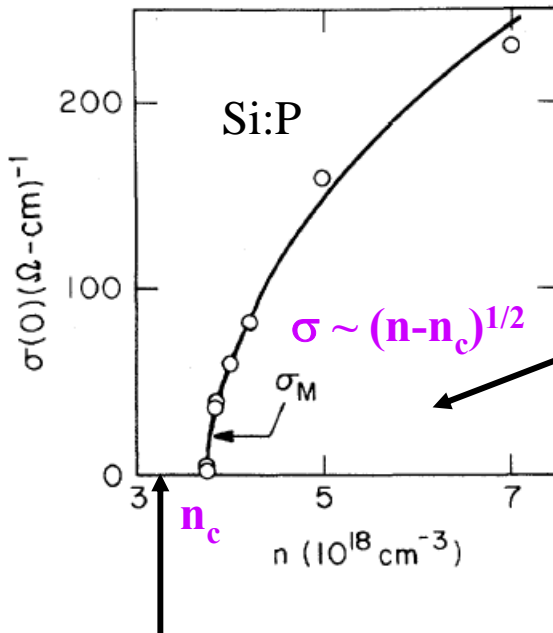
What do the experiments say?

Metal-insulator transition



Low-temperature (< 1 K) conductivity: experiment (Si:P)

(Paalanen, Rosenbaum, Thomas, Bhatt; 1982)



- at low density $n < n_c$, there are carriers, but **no conductivity!!**

$\sigma(n > n_c) \neq 0$:
metal

Metal-insulator transition

at $n = n_c \neq 0$

(quantum, *i.e.* $T=0$ phase transition)

$\sigma(n < n_c) = 0$:
insulator



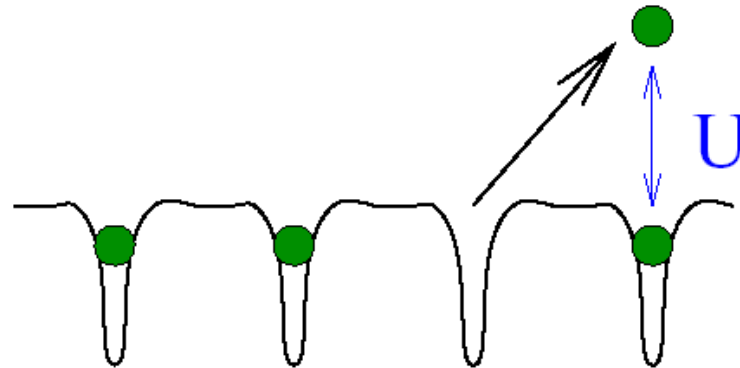
Failure of the single-electron model
(*i.e.* band structure calculations)

What's missing???

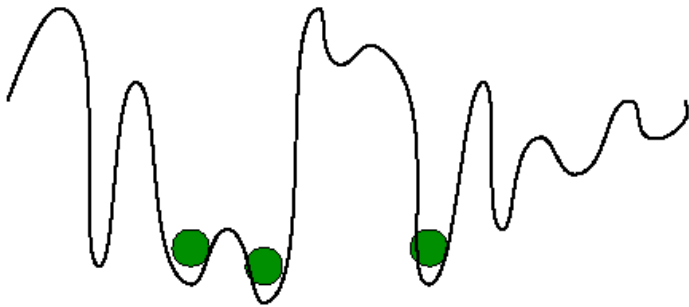


- high density – kinetic energy (**Fermi energy**) dominates
- low density – **potential energy** dominates:

➤ **electron-electron interactions**
(**Mott insulator**)

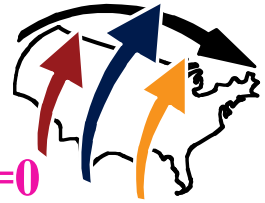


➤ **disorder** due to impurities,
defects (**Anderson insulator**)

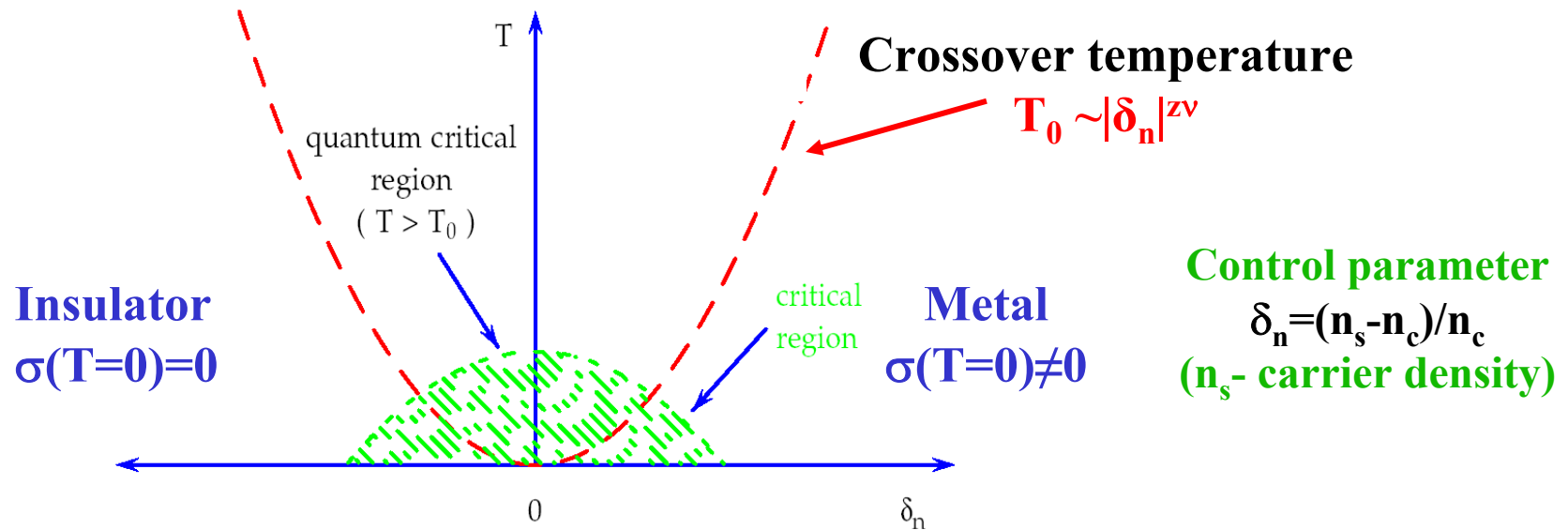


**Result: formation of localized
(bound) states → no conduction**

Metal-insulator transition: a quantum critical point



- Experiments: a **continuous** transition; sharp phase transition at **T=0**



- “dynamical scaling” in the critical region: $\sigma(n_s, T) \propto T^x f(T/\delta_n^{z\nu})$
- power-law critical behavior: $\sigma(n_s, T=0) \propto \delta_n^\mu$

Theoretical problems: **no broken symmetry; order parameter? No small parameter; elementary excitations? Standard approaches fail**

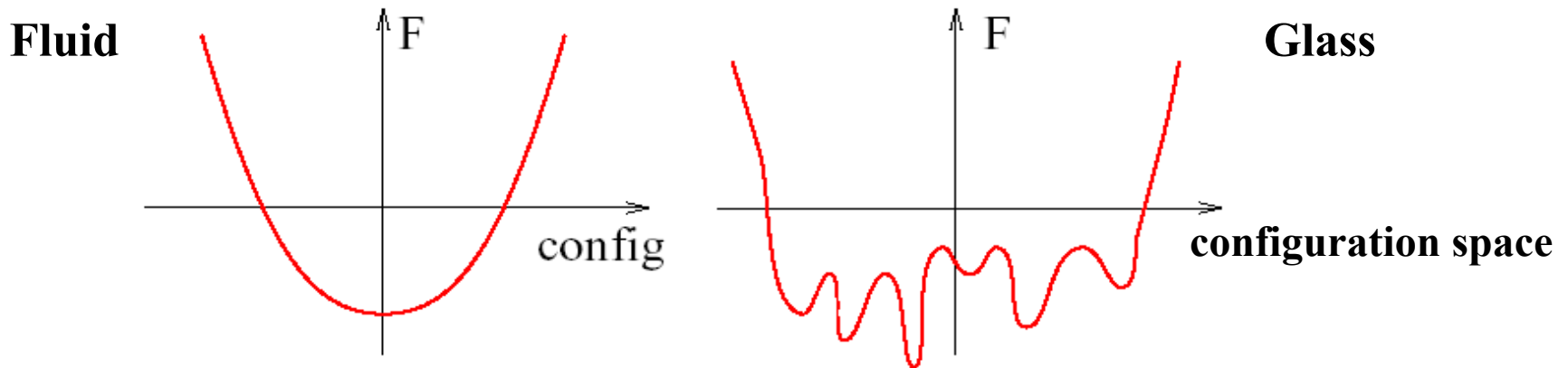
It gets even more complicated...



- Coulomb repulsion: keep electrons apart (**uniform** density)
- Random potential: **nonuniform** density
- competition between Coulomb interactions and disorder

 **Frustration!** (can't make everyone happy)

 emergence of (**exponentially**) many **metastable** states with similar (free) energy



Experimental signature: **slow, out-of-equilibrium dynamics**

- **Slow nonequilibrium dynamics – similarities to other glasses?**

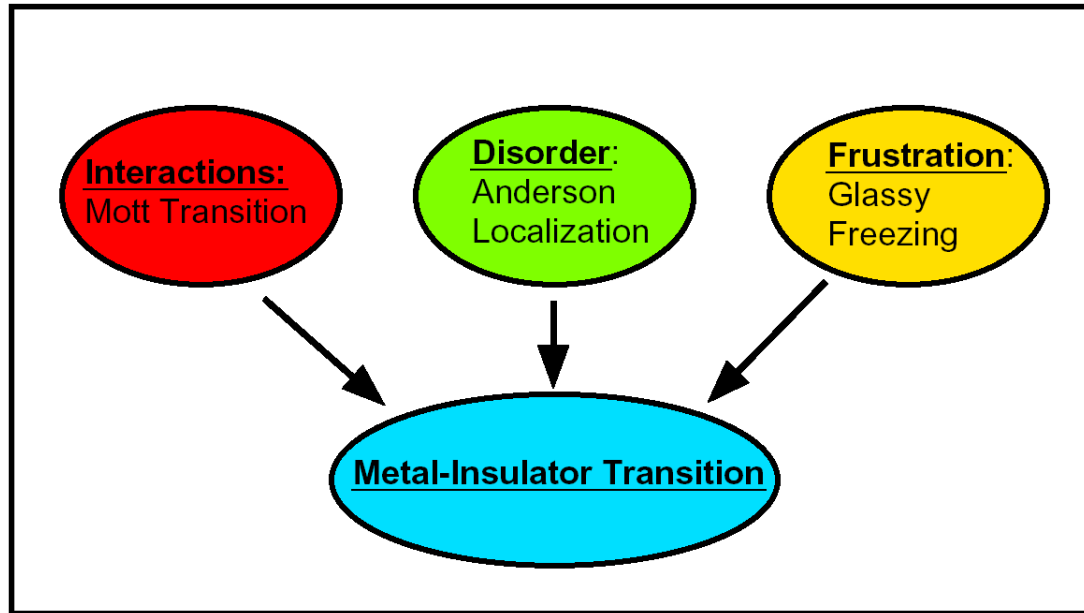


- **Unifying ideas, concepts?**

Relevance for the MIT?



Three basic mechanisms for electron localization:



Metal-insulator transition and glassiness –
two of the **most fundamental** problems in condensed matter physics

Coulomb glass

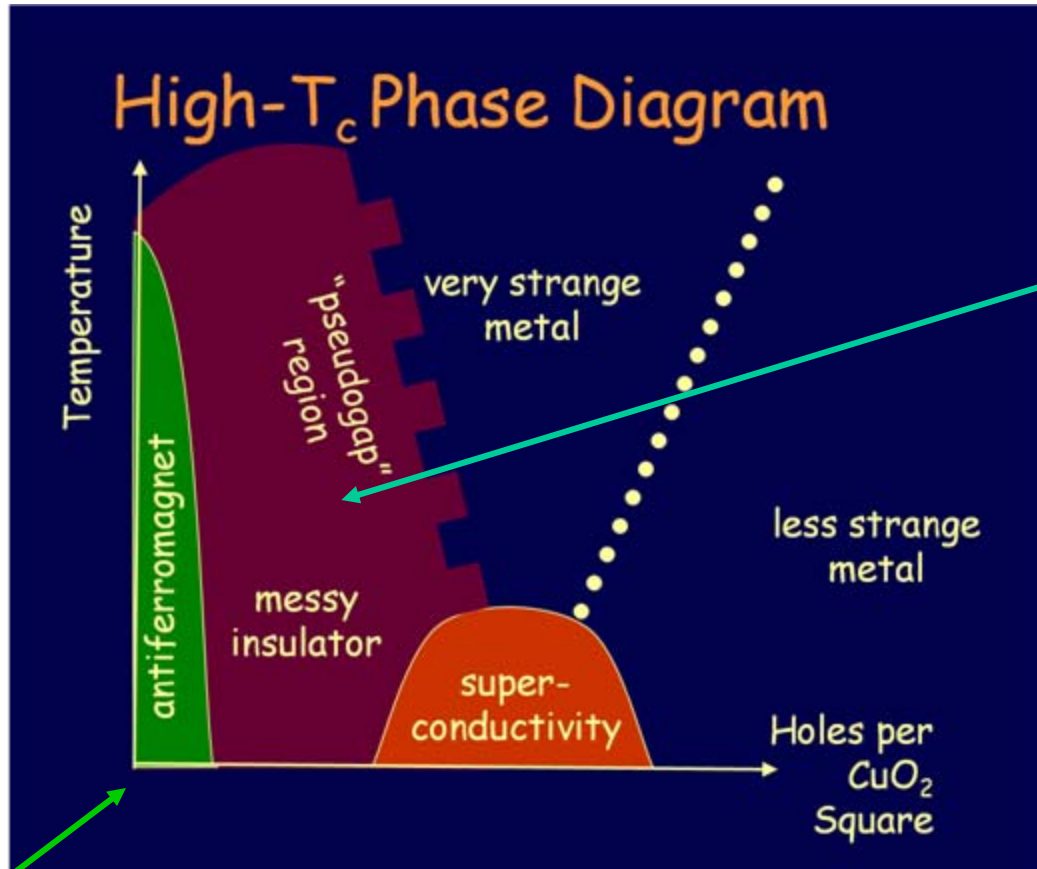


- expected in Anderson insulators with strong electron-electron interactions [M. Pollak (1970); Efros, Shklovskii (1975); Davies, Lee, Rice (1982,84)]

Observations of glassiness in electronic systems – very few:

- slow relaxations in GaAs capacitance (Monroe *et al.*)
 - slow relaxations and thermal hysteresis in conductivity of granular films (Goldman *et al.*, Wu *et al.*, Frydman *et al.*)
 - slow relaxations of photoconductivity in $\text{YH}_{3-\delta}$ (Lee *et al.*)
 - slow relaxations, aging, memory in conductivity of InO_x (Ovadyahu *et al.*) and granular Al (Grenet *et al.*)
 - 2D electrons in Si (DP *et al.*): slow relaxations, aging, memory; slow, correlated dynamics – from insulating to (poorly) metallic
 - lightly doped cuprates (DP *et al.*)
- } my work

Complex behavior of high- T_C superconductors



Emergence of
**nanoscale
inhomogeneities!**

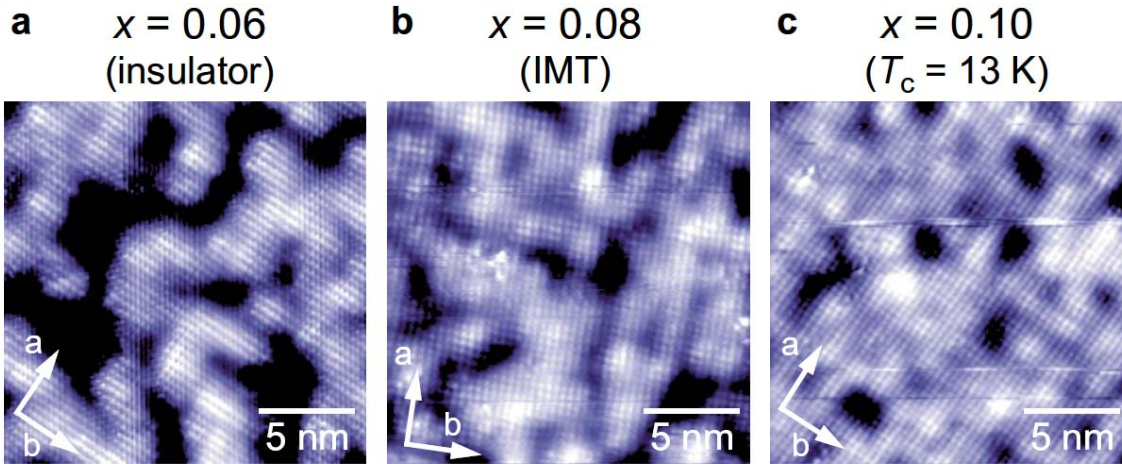
- only a few per cent of dopants cause a transition from an insulating to a (super) conducting state

- undoped parent compound (e.g. La_2CuO_4): **Mott insulator**, not a metal
- single-electron **band theory** of solids **fails** also here

Nanoscale charge inhomogeneities



STM images

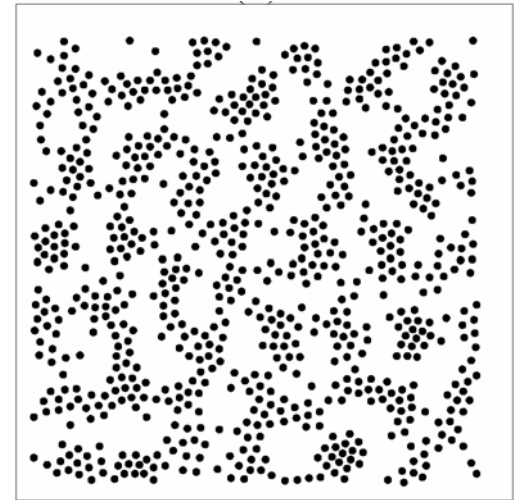


[Kohsaka *et al.*, *Phys. Rev. Lett.* **93**, 097004 (2004)]

- global phase separation not possible because of charge neutrality

- (infinitely?) **many possible arrangements** of nanoscopic ordered regions with comparable energies \implies **charge (Coulomb) glass?**

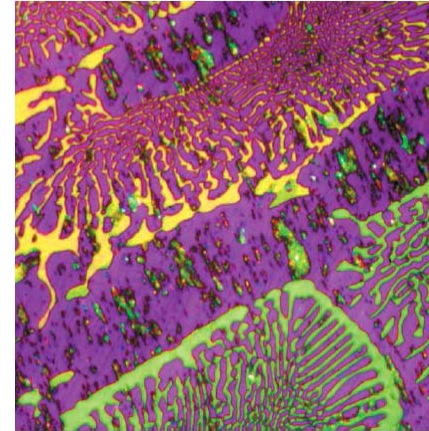
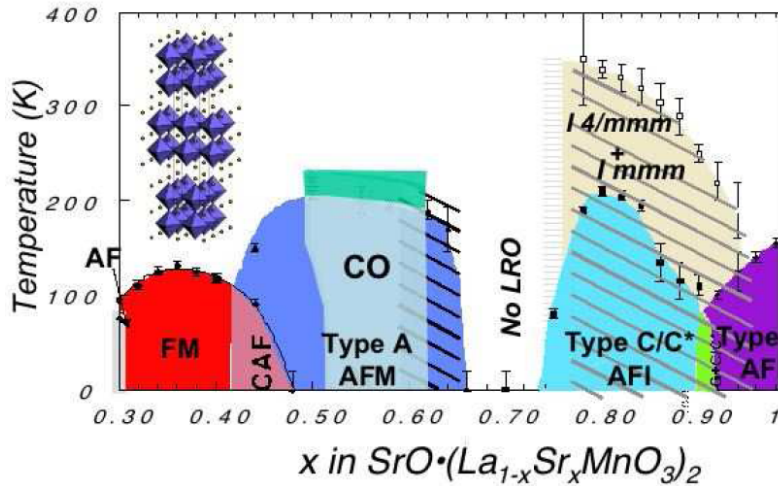
Computer modeling



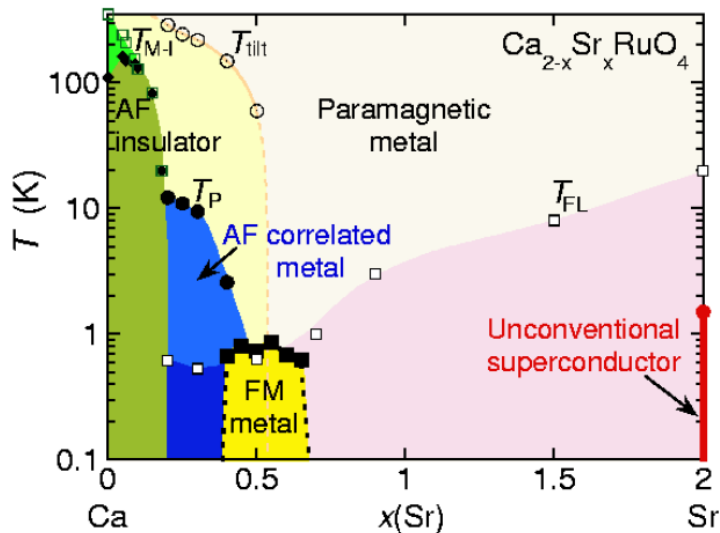
(“stripe- and clump-forming systems”)

[Reichhardt *et al.*, *Europhys. Lett.* **72**, 444 (2005)]

Some other complex, strongly correlated electronic systems



Percolative conduction in half-metallic-FM and insulating-ferroelectric mixture of $(\text{La,Lu,Sr})\text{MnO}_3$ (Park *et al.*, 2004).



Emergence of **intermediate heterogeneous phases** due to the existence of several competing ground states

Dynamics?

- **quantum** effects important

Metal-insulator transition in two dimensions



Is there a true ($T=0$) metallic state in 2D?

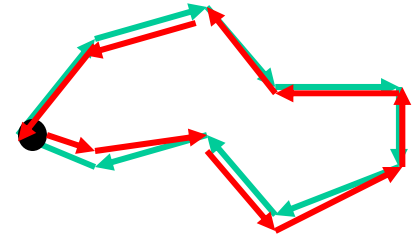
- Theoretical arguments from the 1980s:

no true ($T=0$) metallic state or MIT in 2D

Noninteracting electrons: **always localized**

Strong disorder: $\sigma \sim \exp [-(T/T_0)^{1/p}]$ $p=1, 2, 3$
[strong localization]

Weak disorder: $\sigma = ne^2\tau/m^* + A(e^2/h) \ln (T/T_0)$
[weak localization; Abrahams, Anderson, Licciardello,
Ramakrishnan, PRL 42, 673 (1979)]



Weakly interacting electrons: **always localized (also in T)**

[Altshuler, Aronov, SSC 39, 115, (1979); JETP 50, 968 (1979)]

Strongly interacting electrons:

No disorder: Wigner crystal (insulator)



But:

- mid 1990s – present: **experiments** suggesting a **true MIT in 2D**
- **an active research area in both theory and experiment**

What about *strongly interacting* systems with **weak disorder**?
Can *strong* electron-electron interactions cause delocalization?

Theory:

- early calculations **perturbative** ($\ln T$ – leading corrections)
- a **hint** from theory [Finkel'stein, Z. Phys. B 56, 189 (1984)] that the 2D metal might be possible, **but theory uncontrolled at low T...**
- **Punnoose, Finkel'stein, Science 310, 289 (2005):**
interaction contribution changes sign and **wins** over weak localization!

2D metal possible!

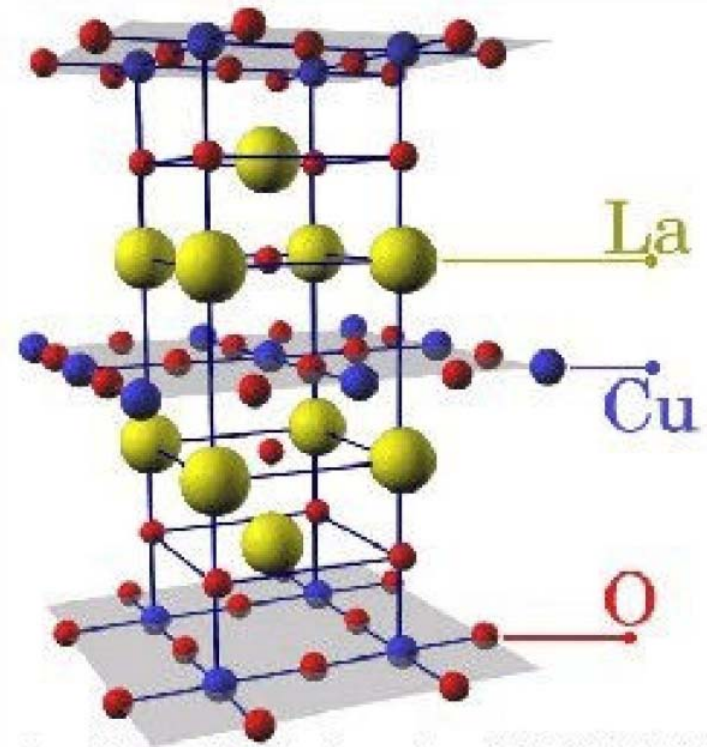
But:

- this theory probably does **NOT** describe experiments

Practical realizations of 2D systems

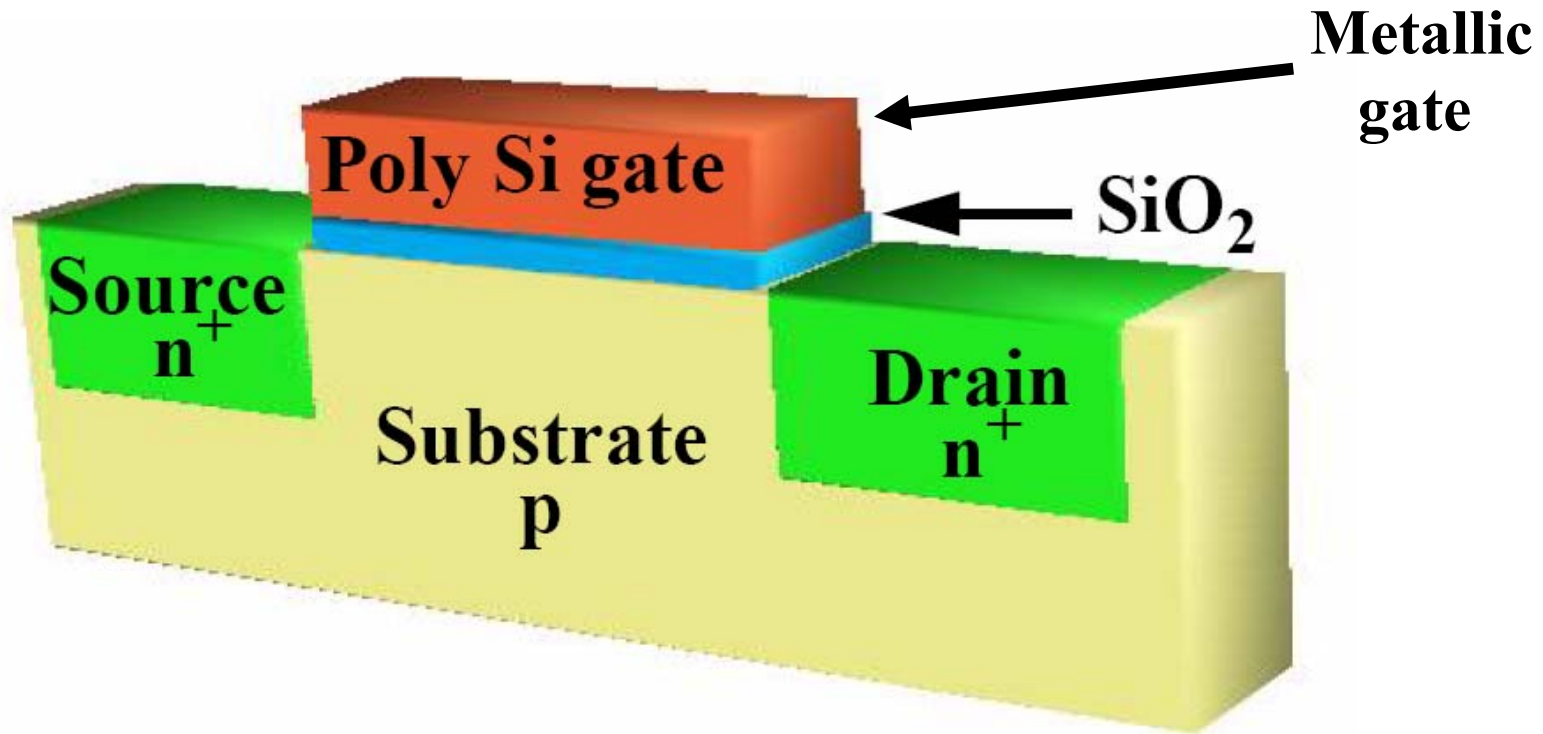


- **thin films**
- semiconductor **heterostructures** at sufficiently low temperatures
(*e.g.* Si MOSFET and AlGaAs/GaAs)
- **quasi-2D systems: layered structures**
(*e.g.* cuprates)



Back to transistors...:

Si MOSFET – the basis of semiconductor technology

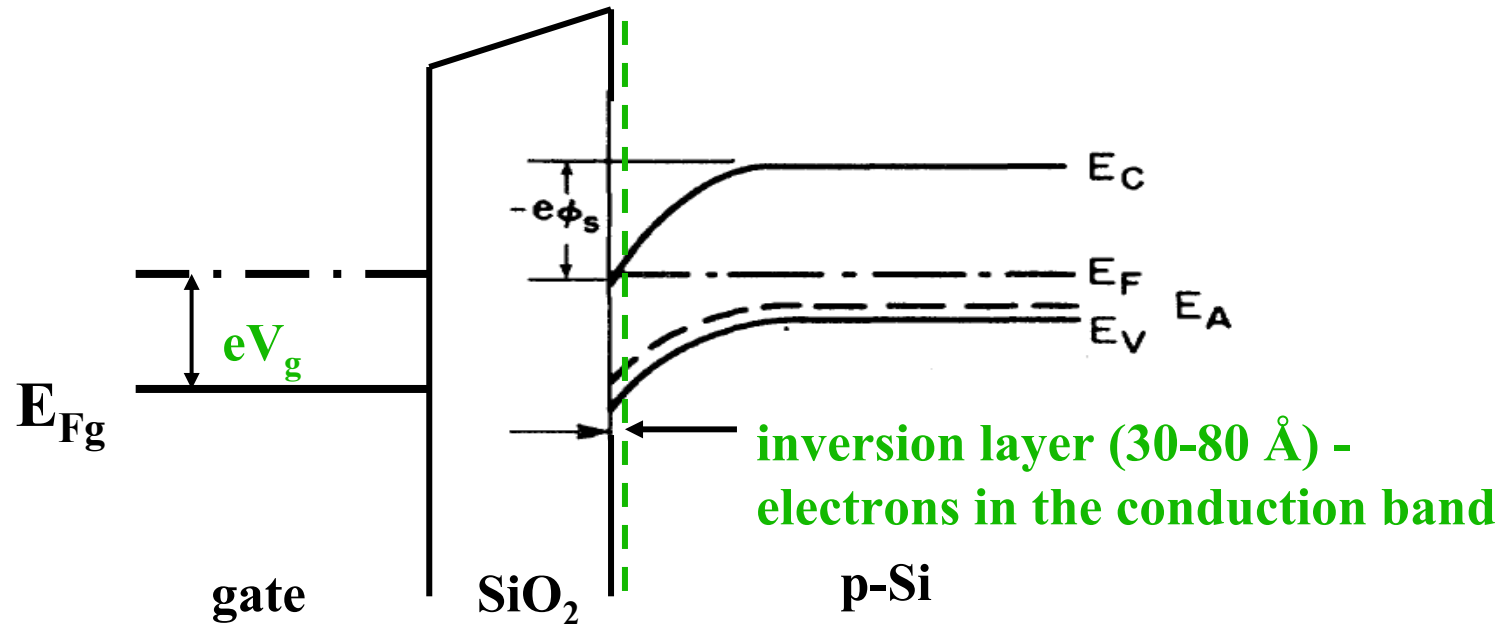
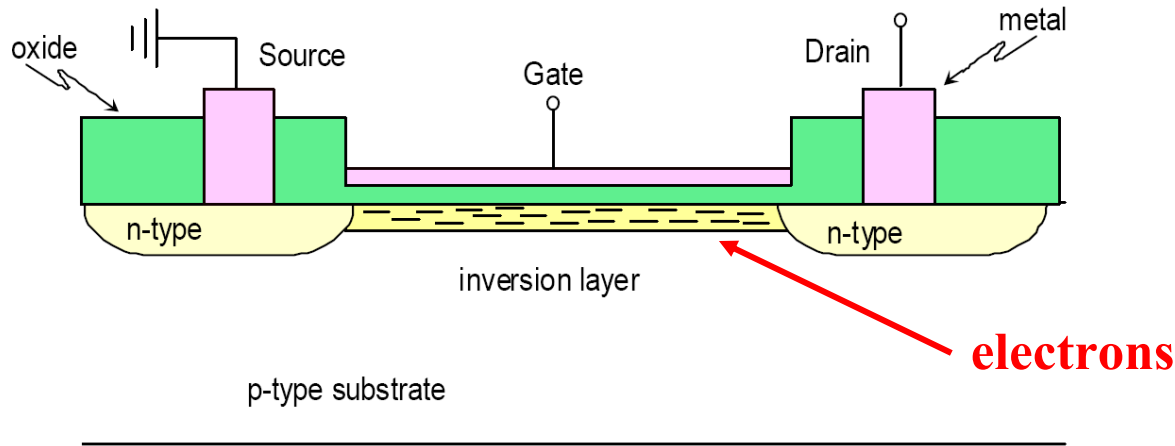


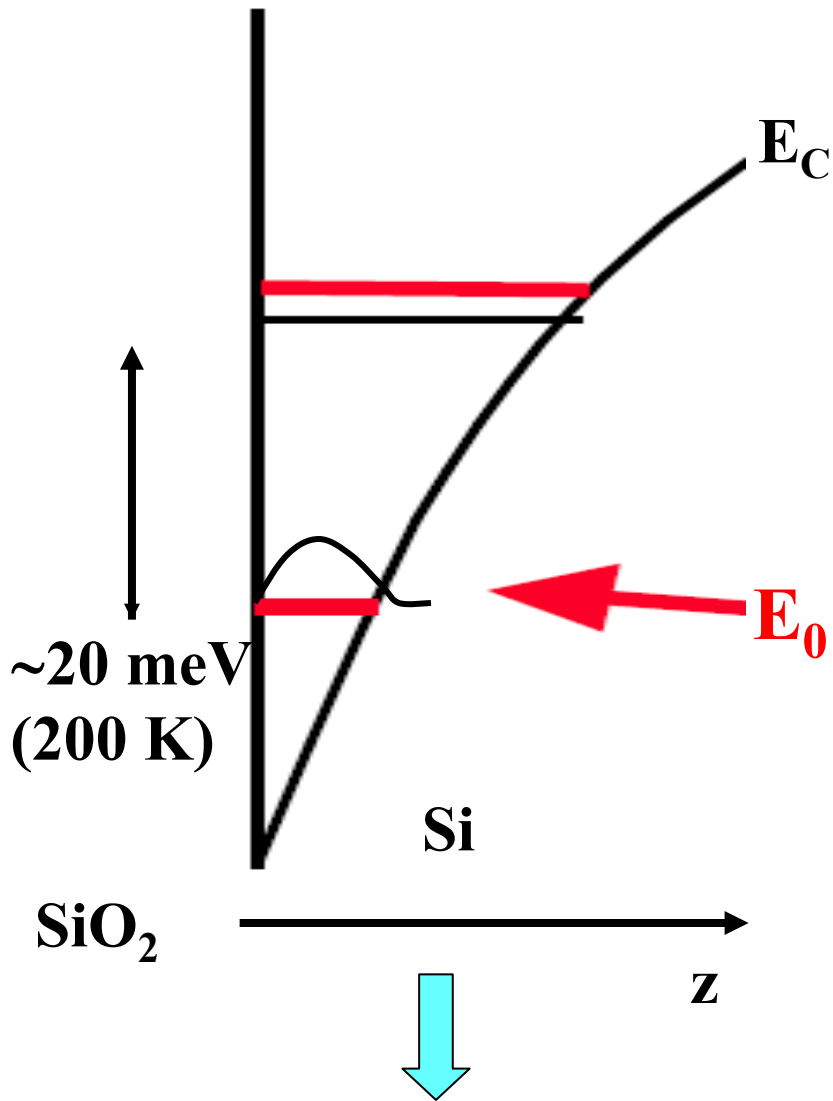
Metal-Oxide-Semiconductor

Field-Effect Transistor – conducts current because of the effect of the electric field (V applied to the gate) at the surface of Si

Gate, source, drain, substrate – all doped Si (in the old days, gate was Al)

Si MOSFET: a capacitor!





- electrons confined in a narrow potential well at the interface
- motion perpendicular to the interface (“z-direction”) is quantized
- discrete energy levels (subbands) for motion in the z-direction; energy levels:

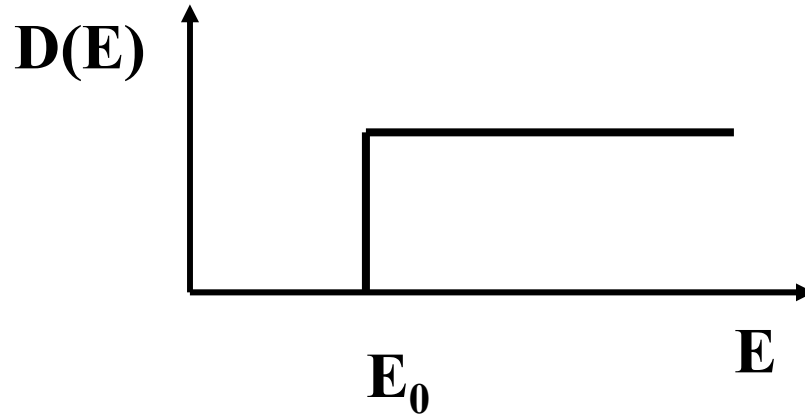
$$E = E_n + \underbrace{(k_x^2 + k_y^2) \hbar^2 / 2m^*}_{\text{in-plane motion}}, \quad n=0,1,2,\dots$$

electrons free to **move in a plane** parallel to the interface

At low enough T, only E_0 is occupied: a **2D** system.



2D density of states: $D(E) = g_s g_v m^* / 2\pi \hbar^2 = \text{const}$
(g_s, g_v – spin and valley degeneracies)

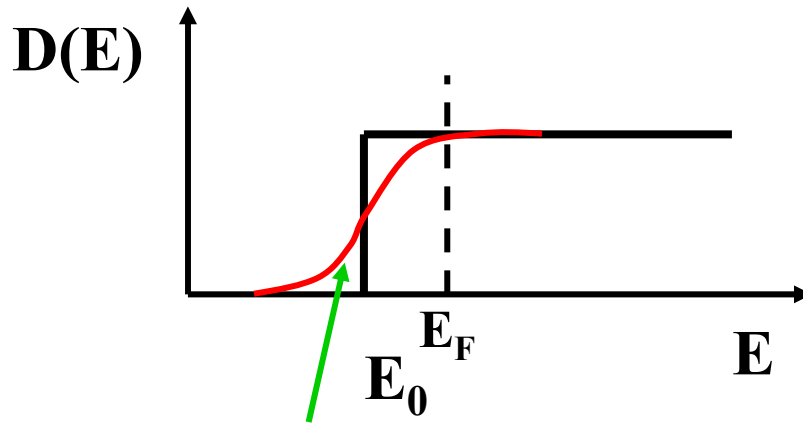


- **2D systems studied extensively since late 1960s**
[Fowler, Fang, Howard, and Stiles, PRL 16, 901 (1966): 2D in Si MOSFETs]
- **dimensionality plays a fundamental role in many phenomena (e.g. integer and fractional quantum Hall effects observed in 2D systems in high magnetic fields – Nobel prizes in physics, 1985 and 1998)**

Room for more???



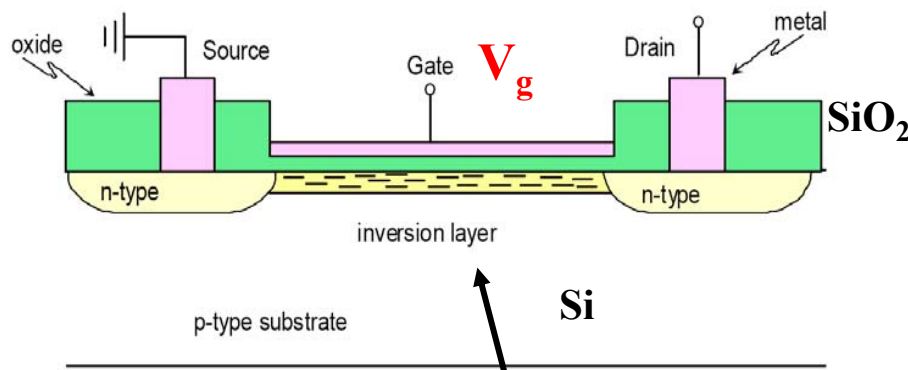
2D density of states: $D(E) = g_s g_v m^* / 2\pi \hbar^2 = \text{const}$
(g_s, g_v – spin and valley degeneracies)



Add disorder

Carrier density n_s ($=D(E)E_F$ at $T=0$; E_F - Fermi energy) can be **tuned continuously over two orders of magnitude (!) by varying V_g**

band tail (strongly localized states)

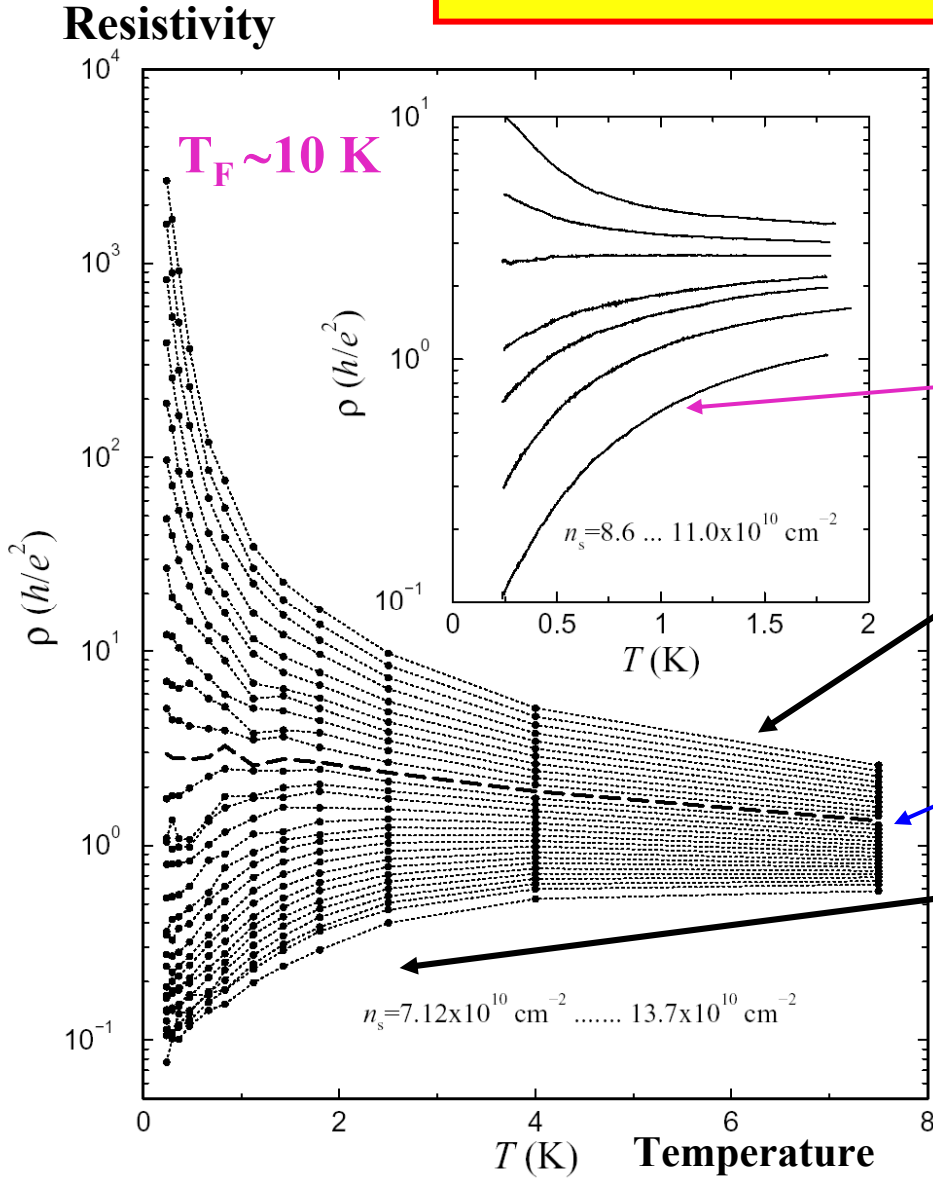


Disorder due to (Na+) ions randomly distributed throughout SiO₂ (frozen out below ~100 K), and to surface roughness

2D electrons move in a smooth random potential

2D at low T

2D metal-insulator transition?



• dramatic change of behavior near “separatrix” at $T < 0.3T_F$

Large resistivity drop! (origin?)

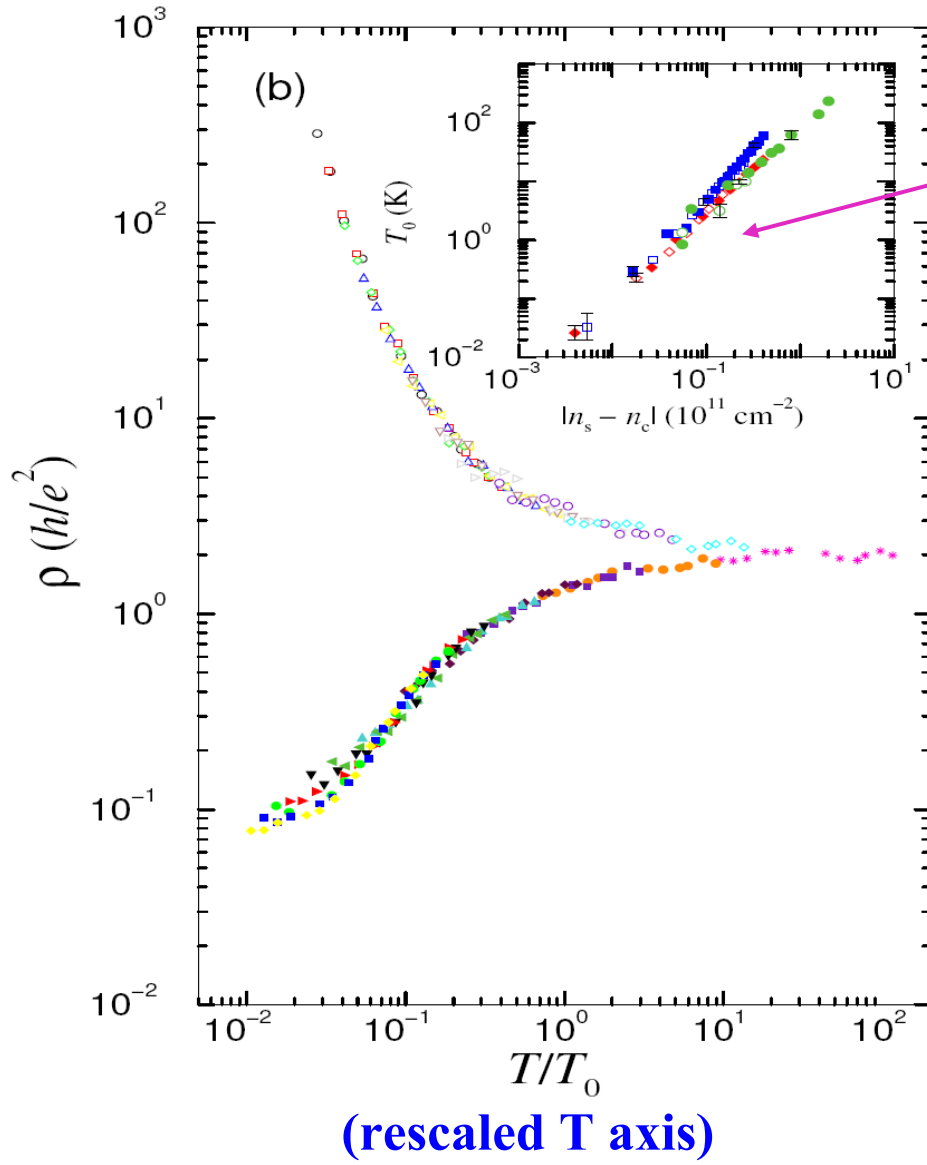
insulating behavior for $n_s < n_s^*$ [exponential $\rho(T)$]

“separatrix” n_s^* ($= n_c$? **NO!**)
 n_c – critical density for the MIT)

metallic behavior for $n_s > n_s^*$

[Kravchenko, Mason, Bowker, Furneaux, Pudalov, D’Iorio, PRB 51, 7038 (1995); 2DES in Si MOSFETs]

The same data:

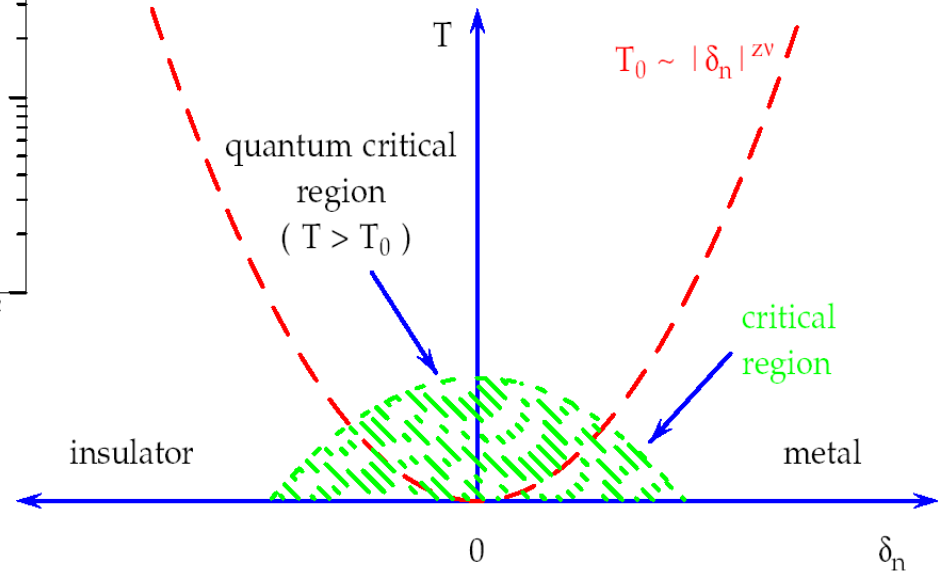


$$T_0 \sim |\delta_n|^{z\nu} \quad (\delta_n = n_s/n_c - 1)$$

“dynamical scaling”:

$$\sigma(n_s, T) = \sigma_c f(T/\delta_n^{z\nu})$$

Metal-insulator transition as a quantum critical point:



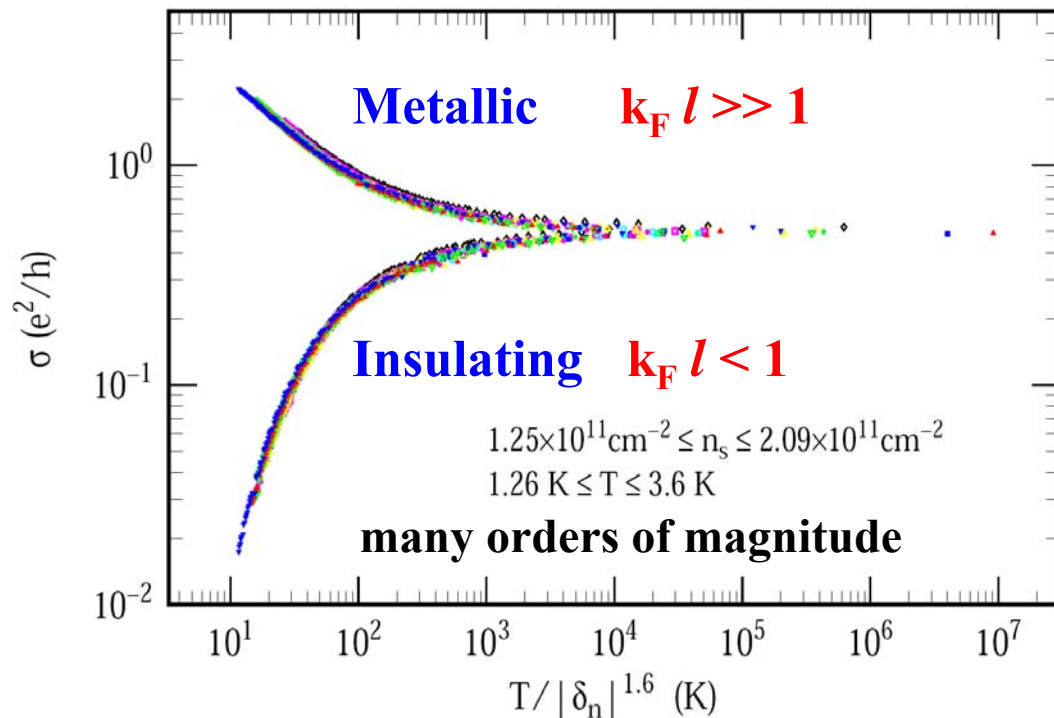
[Kravchenko *et al.*, PRB 51, 7038 (1995)]

Conductivity $\sigma = 1/\rho$



[Popović, Fowler, Washburn;
PRL 79, 1543 (1997); other Si
MOSFETs where disorder was varied]

The race begins!



**Change to metallic behavior
occurs at:**

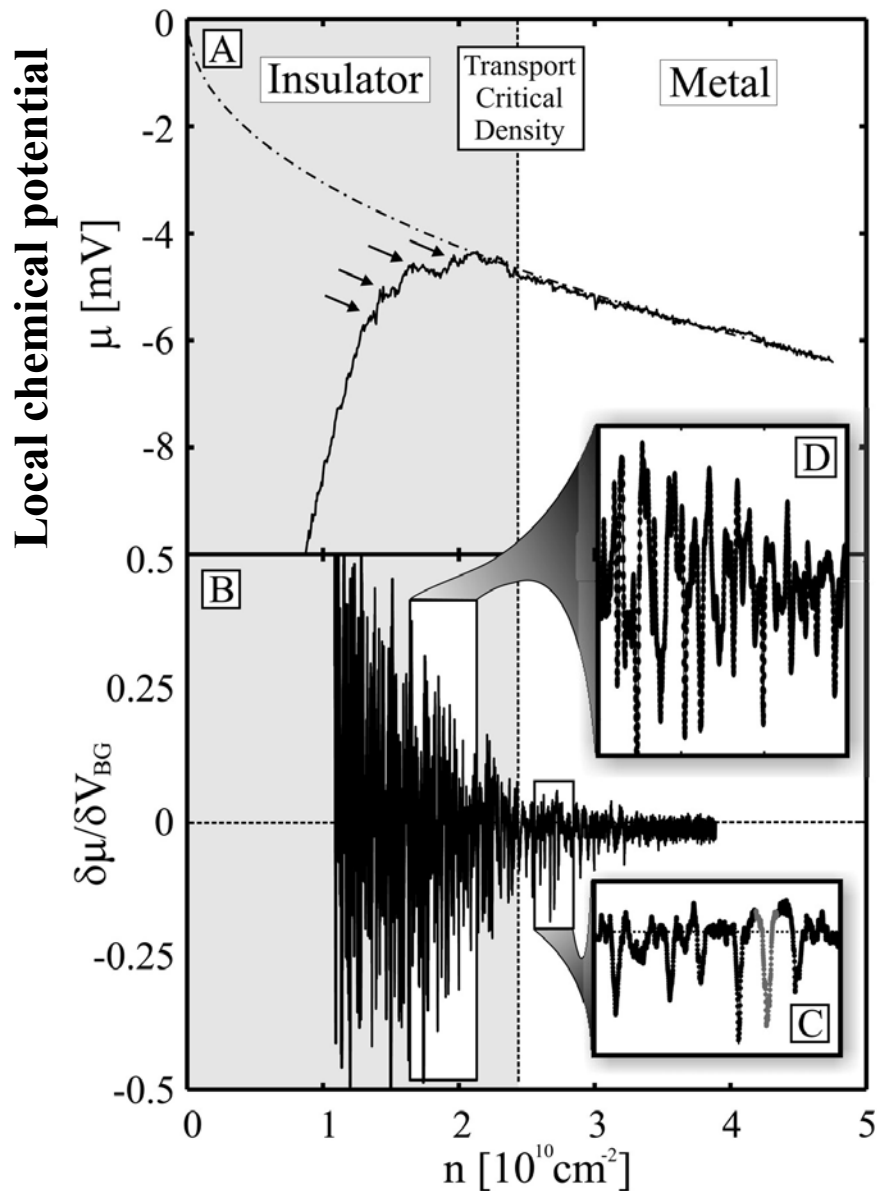
- **low densities ($n_s \sim 10^{11} \text{ cm}^{-2}$)**
- Fermi energy:** $E_F = \pi \hbar^2 n_s / 2m^* \approx 0.6 \text{ meV}$
- Electron-electron interaction energy:**
 $E_{e-e} \sim (e^2/\epsilon)(\pi n_s)^{1/2} \approx 10 \text{ meV}$
- $r_s \equiv E_{e-e}/E_F \propto n_s^{-1/2} \sim 10!$**

- **critical conductivity $\sim e^2/h$**
- $\sigma \sim (e^2/h)(k_F l) \Rightarrow k_F l \sim 1$
- (l – mean free path; k_F – Fermi wave vector)

Strong Coulomb interactions and disorder!



Local compressibility



- as the density approaches n_c from the metallic side, **2DHG fragments** into localized charge configurations that are distributed in space

⇒ **insulating phase is spatially inhomogeneous**

- the structure with sharp spikes **emerges already at $n > n_c$** (on the metallic side of the MIT!)

Dynamics? Coulomb glass?

[Ilani, Yacoby, Mahalu, Shtrikman, Science 292, 1354 (2001); also, PRL 84, 3133 (2000); 2D holes in GaAs]

Summary of Lecture I



- **Metal-insulator transition**: a **fundamental** problem of **relevance** to many interesting materials
- **Problem**: **strongly (Coulomb)** interacting electrons in a **random potential**
⇒ expect **frustration, dynamic inhomogeneities**
- **2D systems** in semiconductors:
 - **easy** to use; can be precisely engineered (**semiconductor technology!**)
 - **control** and vary density (**interactions**) and **disorder** independently
 - “**simple**” – no magnetic or structural degrees of freedom
(**unlike e.g. cuprates**)

Lectures II and III: study electron **dynamics** as n_s is varied **through the MIT**

Evidence of a phase transition? What can we learn about the MIT and out-of-equilibrium dynamics in general?

Literature I



- **Doped semiconductors; strong localization (Anderson and Mott insulators), variable-range hopping transport:**
B.I. Shklovskii and A.L. Efros, *Electronic Properties of Doped Semiconductors* (Springer-Verlag, Berlin, 1984) – out of print; <http://www.tpi.umn.edu/shklovskii/>
- **Disorder and interactions; metal-insulator transition (3D, 2D); inhomogeneous phases; glassy behavior**
E. Miranda and V. Dobrosavljević, *Disorder-Driven non-Fermi Liquid Behavior of Correlated Electrons*, Rep. Prog. Phys. 68, 2337 (2005)
- **E. Dagotto, *Complexity in Strongly Correlated Electronic Systems*, Science 309, 257 (2005) – a very brief (few pages) review**
- **Metal-insulator transition in 2D - basic issues:**
E. Abrahams, S.V. Kravchenko, M.P. Sarachik, *Metallic Behavior and Related Phenomena in Two Dimensions*, Rev. Mod. Phys. 73, 251 (2001)
- **Weak localization and e-e interaction effects in metals:**
P.A. Lee and T.V. Ramakrishnan, *Disordered Electronic Systems*, Rev. Mod. Phys. 57, 287 (1985)
- **2D systems in Si and other semiconductors – basics:**
T. Ando, A.B. Fowler, and F. Stern, *Electronic Properties of Two-Dimensional Systems*, Rev. Mod. Phys. 54, 437 (1982)