Field Theory of the Quantum Kicked Rotor

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The quantum kicked rotor is investigated by field theoretical methods. It is shown that the effective theory describing the long wavelength physics of the system is precisely the supersymmetric nonlinear σ model for quasi-one-dimensional metallic wires. This proves that the analogy between chaotic systems with dynamical localization and disordered metals can indeed be exact. The role of symmetries is discussed. [S0031-9007(96)01666-3]

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Quantum mechanics tends to suppress the chaoticity of classical dynamical systems. The investigation of this phenomenon in *periodically driven* systems, i.e., systems that are governed by a Hamiltonian with periodic time dependence, has led to the discovery of one of the most intriguing parallels between the fields of nonlinear dynamics and disordered solids: In the quantum kicked rotor (QKR), a typical representative of this class of system, the quantum mechanical suppression of chaos exhibits striking similarities to the phenomenon of Anderson localization in disordered metallic wires [1].

By definition, the kicked rotor is a point particle that moves freely on a circle. The particle is kicked periodically in time, where the kick strength depends on the angular position. When a kick strength parameter $k\tau$ exceeds a certain threshold value, the dynamics becomes globally chaotic. In a statistical physicist's language, the chaoticity of the motion manifests itself as follows: An ensemble of particles prepared at time $t = 0$ so as to have definite angular momentum l_0 but arbitrary angular coordinate θ will *diffuse* in l space around the initial condition l_0. In the corresponding *quantum system* ($\theta \to \hat{\theta}, l \to \hat{l}, [\hat{l}, \hat{\theta}] = -i\hbar$), the unbound diffusion in l space is suppressed by localization. Numerical [1] and analytical [2,3] studies have shown that the QKR localization is analogous to the Anderson localization displayed by metallic wires with many channels (quasi-1D wires). In particular, it has been demonstrated numerically [4] that a phenomenological modeling of the QKR by random band matrices can successfully explain essential features of the localization phenomenon. Random matrix models of the same type are known [5], in turn, to describe the universal large distance physics of disordered wires.

However, the equivalence between the rotor and quasi-1D wires still has the status of a conjecture. A rigorous answer to the question whether this analogy is *complete* (and not just restricted to the bilateral appearance of localization) has not yet been given. Can a simple one-dimensional driven system indeed exactly mimic the behavior of disordered electronic conductors, which includes a variety of complex phenomena that have recently been found [6–8]? In the present Letter we are going to show that the answer is positive. This is done by mapping the kicked rotor onto the very same supersymmetric nonlinear σ model that is known to describe the long wavelength physics of disordered wires. The fact that both models can be described by the same effective field theory implies that all that is known about the QKR applies to disordered wires and vice versa.

The QKR is defined by the time dependent Hamiltonian

$$\hat{H} = \frac{\hat{l}^2}{2} + k\cos(\hat{\theta} + a)\sum_{n=-\infty}^{\infty}\delta(n\tau - t),$$

where the particle's moment of inertia has been set to unity and $a \in \mathcal{R}$ is a symmetry breaking parameter whose meaning will be explained below. To elucidate the analogy between this Hamiltonian and disordered electron systems, one may consider the discrete time analog of a four-point Green function in (angular) momentum space:

$$\langle\!\langle l_1|G^+(\omega_+)|l_2\rangle\langle l_3|G^-(\omega_-)|l_4\rangle\rangle_{\omega_0}, \qquad (1)$$

where $G^{\pm}(\omega_{\pm}) := \sum_{n=0}^{\pm\infty}\hat{U}^n e^{i\omega_{\pm}n\tau} = [1 - (\hat{U}e^{i\omega_{\pm}\tau})^{\pm 1}]^{-1}$, $\hat{U} = \exp(i\hat{l}^2\tau/4)\exp[ik\cos(\hat{\theta} + a)]\exp(i\hat{l}^2\tau/4)$ denotes the Floquet operator, i.e., the unitary operator governing the time evolution during one elementary time step, $\omega_{\pm} = \omega_0 \pm (\omega/2 + i0)$, and $\langle\cdots\rangle_{\omega_0} := \tau\int_0^{2\pi/\tau} d\omega_0(\cdots)/2\pi$ is an average over the rotor's quasienergy spectrum.

Before turning to a quantitative discussion of the system, let us explain the meaning of the symmetry breaking parameter a. The localization length of a disordered wire depends on the behavior of the Hamiltonian under the time reversal transformation T: $t \to -t$, $p \to -p$, $x \to x$, where p and x are the momentum and the position. In the case of the rotor, where localization takes place in momentum rather than coordinate space, this symmetry operation is irrelevant. However, it has been shown [9] that the transformation T_c: $t \to -t$, $\theta \to -\theta$, $l \to l$ plays a role analogous to T in disordered metals. (Note that T_c differs from T just by the exchange of momentum and position.) To couple the system to a T_c breaking perturbation in a simple way, we put it on an angular lattice of spacing $1/(2\pi L)$, $L \in \mathcal{N}$, thereby giving it the topology of a ring of circumference L in *momentum space* [10]. It will turn out that the symmetry breaking

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parameter a then acts like a T_c breaking Aharonov-Bohm flux piercing the ring.

In the following we deal with the correlator (1) by field theoretical methods. The strategy of our approach is dictated by the experience gained from both the analysis of disordered metals [11] and a recent field theoretical approach [12] to Hamiltonian chaotic systems. Owing to the different formulation of periodically driven systems, however, the actual computational scheme deviates significantly from these cases. To simplify the notation, we temporarily focus on the case of unbroken T_c symmetry, $a = 0$.

Invariance under the transformation T_c, which acts as an *antiunitary* operator in the quantum system, results in the Floquet operator being a symmetric matrix when represented in the l basis (from now on we refer to all operators in l representation). This makes it possible to decompose $U = \{\langle l|\hat{U}|l'\rangle\}$ by $(Ue^{i\omega_\pm\tau})^{\pm 1} = V_\pm V_\pm^T$, where $V_\pm$ does not possess any symmetries other than unitarity. We choose $V_\pm = e^{\pm i\omega_\pm\tau/2}K^{a=0}_{\pm,k/2}$, where $K^a_{\pm,k} := \{\langle l|\exp(\pm i\hat{l}^2\tau/4)\exp[\pm ik\cos(\hat{\theta} + a)]|l'\rangle\}$, and write the Green functions appearing in (1) as [13]

$$G^\pm(\omega_\pm) = \begin{pmatrix} 1 & V_\pm \\ V_\pm^T & 1 \end{pmatrix}^{-1}_{11} =: \tilde{G}^\pm(\omega_\pm)_{11}\,. \qquad (2)$$

In the next step we introduce a superfield $\psi = \{\psi_{\lambda\alpha tl}\}$, $\lambda, \alpha, t = 1, 2$, with complex commuting (anticommuting) components $\psi_{\alpha=1}$ ($\psi_{\alpha=2}$), and consider the generating functional

$$\int \mathcal{D}(\psi,\bar{\psi})\exp[-\bar{\psi}(G^{-1} + J)\psi], \qquad (3)$$

where $G = E^{11}_{\mathrm{AR}} \otimes 1_{\mathrm{BF}} \otimes \tilde{G}^+(\omega_+) + E^{22}_{\mathrm{AR}} \otimes 1_{\mathrm{BF}} \otimes \tilde{G}^-(\omega_-)$, matrices with subscript "AR" ("BF," "T") act in the two-dimensional spaces of λ (α, t) indices [the t indices refer to the matrix structure appearing in (2)] and $(E^{ij}_{\mathrm{X}})_{i'j'} := \delta_{ii'}\delta_{jj'}$, $\mathrm{X} = \mathrm{AR}, \mathrm{BF}, \mathrm{T}$. Here and below, indices that are not indicated explicitly are summed over. Expressions like (1) can readily be obtained from (3) by differentiating twice with respect to matrix elements of the source field J. As we are interested in the general structure of the theory, rather than in the calculation of any particular correlation function, we henceforth omit J.

After a few elementary manipulations, namely matrix transpositions and regrouping of integration variables, the Gaussian integral (3) takes the simple form

$$\int \mathcal{D}(\phi,\chi)e^{-\frac{1}{2}(\bar{\phi}\phi+\bar{\chi}\chi)+\bar{\phi}E^{11}_{\mathrm{AR}}\otimes V_+\chi+\bar{\chi}E^{22}_{\mathrm{AR}}\otimes V_-^T\phi}, \qquad (4)$$

where the fields $\phi = \{\phi_{\lambda\alpha tl}\}$ and $\chi = \{\chi_{\lambda\alpha tl}\}$ comprise components of both ψ and $\bar{\psi}$. Instead of displaying the structure of these new quantities explicitly, we merely note two essential features that fix their functionality as integration variables: (i) ϕ and χ are independent of each other and (ii) they possess the symmetry $\bar{Y} = (1_{\mathrm{AR}} \otimes M)Y$, $Y = \phi, \chi$, where $M = E^{11}_{\mathrm{BF}} \otimes \sigma^1_{\mathrm{T}} + E^{22}_{\mathrm{BF}} \otimes (i\sigma^2_{\mathrm{T}})$ and σ^i_{X} ($i = 1, 2, 3$, $\mathrm{X} = \mathrm{AR}, \mathrm{BF}, \mathrm{T}$) denotes the Pauli matrices.

The next step in the construction of the field theory is the average over the phase $\exp(i\omega_0\tau)$, which plays a role similar to the energy average employed in Ref. [12]. In that case, energy averaging led to a quartic [$\sim(\bar{\psi}\psi)^2$] nonlocal contribution to the action of the field theory. The latter was eliminated by means of a matrix valued auxiliary field Q that coupled to the *dyadic* product $\psi\bar{\psi}$. The phase average to be carried out in the present problem produces in addition to the quartic term an infinite series of higher contributions to the action. We have not succeeded in decoupling these terms by elementary means. On the other hand, the experience gained from previous diagrammatic analyses [3] of the QKR suggests that a field coupling to $\psi\bar{\psi}$ should again describe the large scale physics.

The problem of identifying this field is solved by a recently discovered identity [14] that adapts the Hubbard-Stratonovich transformation to averages over *unitary* operators. In the special case under consideration, namely, a phase or U(1) average, this identity reads

$$\langle e^{\bar{\phi}_1 u\eta_1+\bar{\eta}_2\bar{u}\phi_2}\rangle_{\omega_0} = \int \mathcal{D}\mu(Z,\tilde{Z})e^{\bar{\phi}_1 Z\phi_2+\bar{\eta}_2\tilde{Z}\eta_1}, \qquad (5)$$

where $u := \exp(i\omega_0\tau)$, $\eta_1 = V_+|_{\omega_0=0}\chi_1$, $\bar{\eta}_2 = \bar{\chi}_2 V_-^T|_{\omega_0=0}$, all subscripts refer to the λ indices (AR space), $Z = \{Z_{\alpha tl,\alpha' t' l'}\}$ is a nonlocal (in l) 4×4 supermatrix field, $\mathcal{D}\mu(Z,\tilde{Z}) = \mathcal{D}(Z,\tilde{Z})\mathrm{sdet}(1 - Z\tilde{Z})$ with "sdet" the superdeterminant, and $\int \mathcal{D}(Z,\tilde{Z})$ stands for the integral over the matrix elements of Z and $\tilde{Z} := Z^\dagger\sigma^3_{\mathrm{BF}}$.

The proof [14] of (5) makes use of group theoretical concepts and the theory of generalized coherent states [15] and is too lengthy to be reported here. We note however that the field $Z_{ll'}$ takes values in the unrestricted set of 4×4 complex supermatrices. The latter can be interpreted as a space parametrizing the coset space $\mathbf{G}/\mathbf{K}$, $\mathbf{K} = \{k \in \mathbf{G} | k\sigma^3_{\mathrm{AR}} = \sigma^3_{\mathrm{AR}}k\} \subset \mathbf{G}$, where $\mathbf{G}$ is the group of 8×8 supermatrices g subject to the constraint $g^\dagger\eta g = \eta$, $\eta = (\sigma^3_{\mathrm{AR}} \otimes E^{11}_{\mathrm{BF}} + 1_{\mathrm{AR}} \otimes E^{22}_{\mathrm{BF}}) \otimes 1_T$. This coset space is the field manifold of a "*unitary*" supersymmetric σ model that is twice as large as in the usual case [11] on account of the extra T space indices. The relationship between the Z field and this manifold is the first indication of the fact that we will end up with a nonlinear σ model.

After this comment on the formalism, we proceed to apply (5) to the construction of the field theory for the rotor. To that end we insert (5) into (4) and perform the Gaussian integration over the fields ϕ and χ. As a result we obtain for the generating functional (at $J = 0$),

$$\int \mathcal{D}(Z,\tilde{Z})\exp\mathrm{str}[\ln(1 - Z\tilde{Z}) - \tfrac{1}{2}\ln(1 - Z\tau^{-1}Z^T\tau) - \tfrac{1}{2}\ln(1 - e^{i\omega\tau}\tilde{Z}U_0\tau^{-1}\tilde{Z}^T\tau U_0^\dagger)], \qquad (6)$$

where $U_0 = U|_{a=0}$, $\tau = M\sigma^3_{\mathrm{BF}}$, and the supertrace "str" includes a trace over the l space. So far all manipulations

have been exact. We next restrict the field theory to its infrared limit, which describes the long-time–large-distance physics we are interested in.

The action of the field theory (6) vanishes for fields Z proportional to unity in angular momentum space if $\omega \to 0$ and the constraint

$$\tilde{Z} = \tau^{-1} Z^T \tau \tag{7}$$

is imposed. Field configurations that violate this symmetry are "massive" and cannot contribute to the long-range correlations of the model. We therefore restrict the integration in (6) to the field manifold specified by (7). (The integration over the massive quadratic fluctuations around this manifold yields a factor of unity by the supersymmetry of the model.) Note that the unitary coset space $\mathbf{G}/\mathbf{K}$ subject to the constraint (7) defines the field space of the "*orthogonal*" nonlinear σ model. As a preliminary result, we thus find that our field theory has the same symmetries as the one describing time reversal invariant disordered metals. What happens when this symmetry is gradually broken by the introduction of a small finite value of a? In that case, the decomposition of the time evolution operator has to be generalized to $(Ue^{i\omega_\pm \tau})^{\pm 1} = V^a_\pm V^{-aT}_\pm$, where $V^a_\pm = e^{\pm i\omega_\pm \tau/2} K^a_{\pm,k/2}$. All further steps can be repeated in essentially the same way as before and we again arrive at (6). The only change is that the fields appearing in the action have undergone a "gauge transformation" $Z_{ll'} \to Z^a_{ll'} := \exp(-ial\sigma^3_{\mathrm{T}}) Z_{ll'} \exp(ial'\sigma^3_T)$, and similarly for $\tilde{Z}$.

To carry the analogy to disordered metals further, we need to expand the action around the limit $Z_{ll'} = \delta_{ll'} Z_0$, $\omega = 0$. Details of this somewhat tedious calculation will be presented elsewhere [16]. Here we restrict ourselves to a rough sketch of the main ideas. We first subject the action in (6) to a "semiclassical approximation." The expansion parameter of this approximation is $\hbar/\delta l \ll 1$, where δl is the typical angular momentum scale over which the relevant Z fields fluctuate [17]. As a result, (i) the symbol str in (6) no longer includes a trace over angular momentum space but rather an integral over the phase space coordinates (l, θ) of the *classical* rotor, (ii) $Z(l, l')$ is replaced by its Wigner transform $Z(l, \theta)$ (we temporarily suppress the superscript a in Z^a), and (iii) $U_0 Z U_0^\dagger$ is replaced by Z_u, where $f_u(l, \theta) = f(\theta + \tau(l + k \sin\theta), l + k \sin\theta)$ denotes the classical one time step evolution (standard map) of a phase space function.

The angular variable θ of the standard map is a rapidly relaxing degree of freedom, which leads us to expect that only θ independent field configurations contribute to the long wave limit of the model. To formulate this statement in a quantitative manner we do a Fourier transform, $Z(l, \theta) = \sum_{m=-\infty}^{\infty} Z_m(l) \exp(im\theta)$, and observe that the nonzero modes $Z_{m\neq 0}$ are massive. Integration over these fields in Gaussian approximation yields an effective field theory for the massless zero mode $Z(l) := Z_0(l)$.

We finally expand the action $S[Z]$ in terms of slowly fluctuating fields. This program is carried out most economically in the l-Fourier space, i.e., in angular coordinates [18]. The small parameters of this expansion scheme are $\omega\tau$ and the characteristic "momentum" ϕ (ϕ is an angular variable) of slowly fluctuating fields $Z(\phi)$, which is of order $\phi \ll k^{-1} \ll 1$. Concerning the former, we note that the mean quasienergy level spacing of the model is $\Delta = 2\pi(L\tau)^{-1}$. Since we are interested in small frequencies of $\mathcal{O}(\Delta)$, we have $\omega\tau \sim L^{-1} \ll 1$. To leading order in $\omega\tau$ and ϕ the action of the generating functional reads

$$S[Z^a, \tilde{Z}^a] \simeq \int dx \left(-\frac{i\omega\tau}{2} \mathrm{str}[1 - Z^a(x)\tilde{Z}^a(x)]^{-1} - \frac{D}{4} \partial_{x'}\partial_x \mathrm{str}\, \ln[1 - Z^a(x)\tilde{Z}^a(x')]\,|_{x'=x} \right), \tag{8}$$

where we have Fourier-transformed back to angular momentum space, taken a continuum limit (i.e., the variable x is a smoothed version of the l index, $\sum_l \to \int dx$) and $D = k^2/2 + \ldots$ is the classical diffusion coefficient of the rotor [19]. (The dots indicate oscillatory corrections [20] to D that result from elimination of the nonzero modes and are smaller than the leading term by powers of k.) Introducing an 8×8 matrix field Q by

$$Q = \begin{pmatrix} 1 & Z \\ \tilde{Z} & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & Z \\ \tilde{Z} & 1 \end{pmatrix}^{-1},$$

we can rewrite the functional integral as

$$\int \mathcal{D}Q \exp \int \mathrm{str} \left(\frac{D}{32} \nabla_a Q \nabla_a Q + \frac{i\omega\tau}{8} Q \sigma^3_{\mathrm{AR}} \right),$$
$$\nabla_a Q = \nabla Q + ia[\sigma^3_{\mathrm{T}}, Q], \tag{9}$$

which is precisely the nonlinear σ model for a quasi-one–dimensional metallic ring in the presence of a T–breaking (Aharonov-Bohm) vector potential of strength a.

Because of its importance for an understanding of the localization physics of wires, the model (9) has been investigated thoroughly [11]. Let us now review some of its essential properties in the terminology of the kicked rotor. For times less than $\mathcal{O}(\tau k^2)$ the kicked particle performs a diffusive motion in momentum space. On larger time scales, quantum localization confines the particle to stay within a volume specified by the localization length $\xi_o = k^2/2$. For "flux" strengths $a \sim L^{-1}$, the orthogonal symmetry of the model is broken and one expects a doubling of the localization length $\xi_o \to \xi_u = 2\xi_o$ [11]. (Note, however, that $a_{\max} = 2\pi/L$ corresponds to one "flux quantum" ϕ_0 penetrating the ring. As the physics of Aharonov-Bohm geometries is ϕ_0 periodic, $a_{\max}$ is the maximum field strength that can be realized in our model.

For systems with $L \gtrsim \xi_o$ this strength does not suffice to cause a crossover from ξ_o to ξ_u.)

A careful look reveals that the localization length ξ_o predicted by our analysis is four times larger than the length ξ_n found in numerical work (cf., e.g., Ref. [21]). We believe that this discrepancy is caused by an ambiguity in the convention of what is called a localization length: The "field theoretical" localization length determines the exponential decay of the *average transition probability* $\langle |G^+(\omega_+; l, l')|^2 \rangle_{\omega_0} \sim \exp(-|l - l'|/\xi_o)$ between two remote states $|l - l'| \gg \xi_o$. In numerical measurements, however, one computes the *average of individual decay constants*, which is to say that one calculates $|l - l'|/\xi_n = \langle -\ln |G^+(\omega_+; l, l')|^2 \rangle_{\omega_0}$. The lengths ξ_o and ξ_n thus defined do not coincide in general. According to Ref. [22] they are related by $\xi_0 = 4\xi_n$ for quasi-1D wires. In view of this, our analytical result does agree with the numerics after all.

To summarize, we have mapped both the unitary and the orthogonal quantum kicked rotor on the supersymmetric nonlinear σ model for quasi-1D wires. This proves the long-standing conjecture that the universal properties of these two classes of system are indeed the same. Our mapping is straightforward and direct and avoids some approximations made in earlier work, namely, (i) the replacement of the deterministic rotor by a stochastic model and (ii) the passage from unitary to Hermitian randomness. Note that the rotor-metal analogy is not restricted to the phenomenon of strong localization. It has recently been shown that quantum interference in metals manifests itself in various *prestages* of localization such as nontrivial wave function statistics [6,7] or the appearence of prelocalized states [8]. The exact correspondence between quasi-1D wires and the rotor suggests that these effects must be observable in the latter, too. In fact, the rotor may be an ideal model system for highly accurate numerical analyses of these prelocalization phenomena, since it can be implemented more efficiently on a computer than can weakly disordered multichannel wires.

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