Theory of the Anderson Transition in the Quasiperiodic Kicked Rotor

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We present the first microscopic theory of transport in quasiperiodically driven environments (''kicked rotors''), as realized in recent atom optic experiments. We find that the behavior of these systems depends sensitively on the value of a dimensionless Planck constant $\tilde{h}$: for irrational values of $\tilde{h}/(4\pi)$ they fall into the universality class of disordered electronic systems and we describe the corresponding localization phenomena. In contrast, for rational values the rotor-Anderson insulator acquires an infinite (static) conductivity and turns into a ''supermetal.'' We discuss the ensuing possibility of a metal-supermetal quantum phase transition.

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Since its introduction in the late 1970s [1], the quantum kicked rotor—a quantum particle moving on a ring under the influence of periodic driving in time—has been one of the most prominent model systems of quantum chaos (see Ref. [2] for review.) The rotor owes its popularity to a combination of nominal simplicity and astonishingly rich phenomenology, resembling the physics of Anderson localization in disordered electronic systems [2–4]. In Ref. [5] this system has for the first time been realized in a cold atom setting and localization phenomena otherwise predicted for quasi-one-dimensional quantum wires have indeed been observed. The next experimental breakthrough occurred in 2008 when an effectively higher-dimensional generalization of the rotor, the so-called quasiperiodic quantum kicked rotor (QQKR), was realized in a gas of cold cesium atoms, and an Anderson-type *transition* has been seen [6]. By now, the critical states emerging at the transition point have been observed [7], and it stands to reason that the QQKR makes for an almost ideal environment to study critical phenomena in the Anderson universality class.

Although the physics of quasi-one-dimensional localization in the kicked rotor has been a subject of theoretical research for more than three decades (see Refs. [2,4] for review), a truly microscopic theory has been formulated only recently [4]. Concerning the physics of QQKR, no first-principle theory of its Anderson transition has been formulated as yet, and this is a gap we aim to fill in this Letter. In the QQKR, d-dimensional behavior is simulated by modulated driving at d different frequencies. Below, we will map the low energy physics of this system onto an effective field theory equivalent to the nonlinear σ model of disordered metallic systems [8]. This construction establishes the connections to d-dimensional disordered metals; for the first time it microscopically explains the observation of Anderson-type criticality. However, the rotor is not a genuine metal, and these differences show in anomalies at certain configurations of its system parameters: it has been known for some time [1] that the one-dimensional kicked rotor displays so-called quantum resonances at rational values $\tilde{h} = 4\pi p/q$, $p, q \in \mathbb{N}$ of its dimensionless [9] Planck constant. At resonance, the system compactifies to a ring of radius q (in angular momentum space), and no Anderson localization exists.

Below we will show that in dimensions $d > 1$ the situation is even more interesting. At rational $\tilde{h}/(4\pi)$ the system compactifies in one direction, e.g., for $d = 2$ a nominally two-dimensional QQKR maps onto the surface of an infinitely long cylinder of finite circumference q (cf. Fig. 1, top). Response and correlation functions dominantly couple to the cylinder's compact extension, and this has a number of interesting consequences: below we consider a (measurable) observable which, in the light of the above analogies to metallic systems, plays a role analogous to an optical conductivity. We find that the system supports three distinct phases in which this observable exhibits qualitatively different behavior. *Localized regime*: In the limit $q \to \infty$, and at low dimensions $d < 3$ (or $d \geq 3$ below the Anderson transition), the system is localized and exhibits a vanishing conductivity, Fig. 1(b). *Metallic regime*: For $d \geq 3$, $q \to \infty$ (or $d \geq 4$, q finite), above the Anderson transition the system behaves as a metal, and the conductivity is finite, Fig. 1(a). *Supermetallic regime*: At finite q and $d < 4$ (or $d \geq 4$ below the Anderson transition), the system is localized in the quasi (d-1) directions along the cylinder axis, Fig. 1(c). In this case, the optical conductivity transverse to the cylinder axis diverges. Varying the rotor's kicking strength, one may change the strength of the effective ''disorder'' and hence drive a metal-insulator Anderson transition ($q \to \infty$), or a metal-supermetal transition (finite q). Importantly, the above phenomenology requires large values $q \gg 1$, where the limiting case of irrational $\tilde{h}/4\pi$ can be realized by sending $p, q \to \infty$. The reason is that the analogies between the rotor and disordered metals are limited to semiclassical regimes, $\tilde{h} = 4\pi p/q \ll 1$, or $q \gg p \geq 1$ [10], implying a

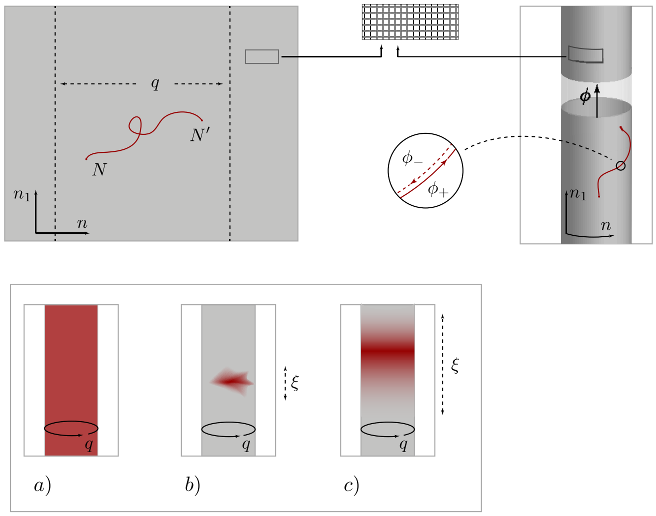

FIG. 1 (color online). The $d = 2$ QQKR at finite q. Left: q-periodic angular momentum space. Right: compactification to cylinder of circumference q, threaded by flux ϕ. Grey shaded areas represent a lattice of angular momentum sites. Transport between sites N and N' takes place through phase coherent propagation of advanced (solid lines) and retarded (dashed lines) quantum amplitudes subject to different flux values $\phi_\pm$. Bottom, the three distinct regimes, (a) unbound diffusion, (b) localization at $\xi < q$, and (c) localization at $\xi > q$.

large number of coupled quantum channels in the transverse direction. A survey of these phases is shown in Table I.

In dimensionless units [9], the Hamiltonian of the system is defined as $\hat{H}'(t) = \frac{1}{2}\tilde{h}^2\hat{n}^2 + K(t)\cos\hat{\theta}\sum_m \delta(t-m)$, where $\hat{\theta}$ and $\hat{n} = -i\partial_\theta$ are angular and angular momentum operator, respectively. The time dependent kicking amplitude, $K(t) \equiv Kf(\cos(\theta_1 + \omega_1 t), \ldots, \cos(\theta_{d-1} + \omega_{d-1}t))$, where $K \in \mathbb{R}$ sets the kicking strength, and f is some function chosen to be smooth and of unit characteristic variation [2]. Finally, the frequencies $\omega_1, \ldots, \omega_{d\text{-}1}$, are incommensurate to the kicking frequency 2π and among themselves, and $\theta_1, \ldots, \theta_{d\text{-}1}$ are arbitrary phases.

Motivated by the experimental setup [6], we consider the spreading of a wave function initially uniform in θ, as described by the correlation function

$$E(t) \equiv \frac{1}{2}\sum_n \overline{|\langle n|\Pi_{m=1}^t \hat{U}'(m)|0\rangle|^2}\, n^2, \qquad (1)$$

where $\hat{U}'(m) \equiv \exp(i\frac{\tilde{h}}{2}\hat{n}^2)\exp(i\frac{K(m)}{\tilde{h}}\cos\hat{\theta})$ and the overline stands for the average over the parameters θ_i. The

TABLE I. Regimes realizable in the QQKR.

Kicking strength	Generic $\tilde{h}$	Resonant $\tilde{h}$
$<$ critical value	Insulator	Supermetal
$>$ critical value	Metal ($d \geq 3$)	Metal ($d \geq 4$)

mapping to an effectively higher-dimensional system [11] is accomplished by interpreting $|\Theta\rangle \equiv |\theta_0, \theta_1, \ldots, \theta_{d\text{-}1}\rangle$, $\theta_0 \equiv \theta$ as a d-dimensional coordinate vector, comprising a "real" angular coordinate, and a generalization of the parameters θ_i to "virtual" coordinates. Similarly, we introduce a d-dimensional angular momentum state, $|\tilde{N}\rangle = |n, n_1, \ldots, n_{d\text{-}1}\rangle$, where $\hat{n}_i = -i\partial_{\theta_i}$ is conjugate to θ_i, with eigenvalues $n_i \in \mathbb{Z}$. Defining the operator $\hat{\Phi}(t) \equiv \exp(-it\sum_i \omega_i\hat{n}_i)$, we then pass to a "gauge transformed" Hamiltonian $\hat{H}'(t) \to \hat{\Phi}(t)\hat{H}'(t)\hat{\Phi}^{-1}(t) \equiv \hat{H}(t) \equiv \hat{T}(\hat{N}) + \hat{V}(\hat{\Theta})\sum_m \delta(t-m)$, with the kinetic energy $\hat{T}(\hat{N}) = \frac{1}{2}\tilde{h}^2\hat{n}^2 + \tilde{h}\sum_{i=1}^{d-1}\omega_i\hat{n}_i$ and the *time independent* potential $\hat{V}(\hat{\Theta}) = K\cos\hat{\theta}f(\cos(\hat{\theta}_1), \ldots, \cos(\hat{\theta}_{d\text{-}1}))$. Similarly, the effective Floquet operator $\hat{U}(m) \equiv \hat{\Phi}(m+1)\hat{U}'(m)\hat{\Phi}^{-1}(m)$ becomes time independent, viz. $\hat{U} \equiv \exp(\frac{i}{\tilde{h}}\hat{T})\exp(\frac{i}{\tilde{h}}\hat{V})$. Finally, the higher-dimensional representation of the correlation function reads $E(t) = \frac{1}{2}\sum_{\tilde{N}}|\langle\tilde{N}|\hat{U}^t|0\rangle|^2 n^2$. We have, thus, traded the time dependence of the original problem for an effective extension to a multidimensional Hilbert space with autonomous stroboscopic dynamics.

The effective Floquet operator $\hat{U}$ possesses two fundamental symmetries: time reversal symmetry [12] T: $t \to -t$, $\hat{\Theta} \to -\hat{\Theta}$, $\hat{N} \to \hat{N}$, and invariance under the translation $\hat{n} \to \hat{n} + q$. According to general principles of quantum mechanics, the translational symmetry can be used to reduce the theory to one defined in the "unit cell" $I_q \equiv \{0, \ldots, q-1\}$ of real angular momentum coordinates. To this end, one defines the reduced "Bloch-Floquet" operator $\hat{U}_\phi \equiv e^{(i/\tilde{h})\hat{T}(\hat{N})}e^{(i/\tilde{h})\hat{V}(\hat{\Theta}+\phi)}$, now acting in the "compactified" Hilbert space of states $|N\rangle \equiv |n_0, n_1, \ldots, n_{d\text{-}1}\rangle$, $n_0 = n \bmod q \in I_q$ with periodic boundary conditions in the n direction. The configuration space of the theory thus becomes the d-dimensional generalization of a "cylinder" of circumference q. The shifted angular operator is given by $\hat{\Theta} + \phi \equiv (\hat{\theta} + \phi, \theta_1, \ldots, \theta_{d\text{-}1})$, where the Bloch phase $\phi \in [0, 2\pi/q]$ may be interpreted as an Aharonov-Bohm flux threading the cylinder, cf. Fig. 1. It is then a matter of a straightforward if somewhat lengthy calculation to show that $\langle\tilde{N}|\hat{U}|0\rangle = q\int_0^{2\pi/q}\frac{d\phi}{2\pi}\langle N|\hat{U}_\phi|0\rangle e^{i\phi n}$, entailing

$$E(t) = \frac{q}{2}\int_0^{2\pi/q}\frac{d\phi}{2\pi}\,\partial^2_{\phi_+\phi_-}\mathrm{tr}(\hat{U}^t_{\phi_+}\delta_{\hat{N}0}\hat{U}^{t\dagger}_{\phi_-})|_{\phi_\pm=\phi}. \qquad (2)$$

Here, the trace extends over all states $\{|N\rangle\}$.

Equations (1) and (2) are different (yet equivalent) ways of probing the spreading of angular momentum states. Anticipating a competition of classical diffusion and quantum localization, we expect three qualitatively distinct cases (cf. Fig. 1 bottom and Table I): if the localization length, ξ, is infinitely large, unbound diffusive spreading $n^2 \sim Dt$ characterized by a diffusion coefficient, D, leads to a linear increase $E(t) \sim Dt$ (metal). In contrast, for

$\xi \ll q$ we expect saturation, $E(t) \stackrel{t\gg\xi^2/D}{\longrightarrow}$ const (insulator), up to corrections exponentially small in the ratio q/ξ which cannot be quantitatively resolved by the present theory. Finally, in cases where $\xi \gg q$ the system behaves similar to a finite quantum system of characteristic quasi-level spacing $\sim 1/(q\xi^{d-1})$. For large times, $t \gg q\xi^{d-1}$, individual states of this system can be resolved and a formal decomposition of $\hat{U}_\phi$ in quasienergy states shows that $E(t) \sim t^2$ (supermetal).

Turning to a quantitative description, we define the resolvent operators $\hat{G}^\pm_\phi(\omega_\pm) \equiv [1 - (e^{i\omega_\pm}\hat{U}_\phi)^{\pm 1}]^{-1}$, where $\omega_\pm \equiv \omega_0 \pm \frac{1}{2}(\omega + i0)$. The Fourier transform $E(\omega) = \int_0^\infty dt e^{i\omega t} E(t)$, then assumes the form $E(\omega) = \int_0^{2\pi} \frac{d\phi}{2\pi} \partial^2_{\phi_+\phi_-}|_{\phi_\pm=\phi} Y(\phi_+, \phi_-, \omega)$, where $Y(\phi_+, \phi_-, \omega) \equiv \langle \mathrm{tr}[\hat{G}^+_{\phi_+}(\omega_+)\delta_{\hat{N}0}\hat{G}^-_{\phi_-}(\omega_-)]\rangle_{\omega_0}$ and $\langle \cdots \rangle_{\omega_0} = \int_0^{2\pi} \frac{d\omega_0}{2\pi}$. Notice that this expression resembles the two-particle response functions describing the optical conduction properties of electronic systems.

To make further progress, we describe the correlation function Y in terms of a low energy effective field theory. The technical details of this mapping [13] are nearly identical to those of our earlier treatment of the one-dimensional rotor [4], and we here restrict ourselves to a brief sketch of the principal steps. We start from a Gaussian integral representation [8],

$$Y = \int dN \int D(\bar{\psi}, \psi) \langle e^{-\bar{\psi} G^{-1} \psi} \rangle_{\omega_0} X[\bar{\psi}, \psi], \qquad (3)$$

where the superfield $\psi = \{\psi_{N,\lambda,\alpha}\}$, $\alpha = b, f$ distinguishes between commuting and anticommuting components, and $\lambda = \pm$ between retarded and advanced components. The preexponential term is given by $X[\bar{\psi}, \psi] = \psi_{N,+,b}\bar{\psi}_{0,+,b}\psi_{0,-,b}\bar{\psi}_{N,-,b}$ and $G^{-1} = \mathrm{diag}(G^{-1}_{\phi_+}(\omega_+), G^{-1}_{\phi_-}(\omega_-))$ is a matrix block-diagonal in advanced-retarded (AR) space. To make progress with this expression, we apply the color-flavor transformation [14], an integral transform that trades the integral over ψ and ω_0 for the integration over an auxiliary field, Z: $Y(\phi_+, \phi_-, \omega) = \int D(Z, \tilde{Z})(\cdots) \exp(-S[Z, \tilde{Z}])$, where

$$S[Z, \tilde{Z}] = -\mathrm{str}\ln(1 - Z\tilde{Z}) + \mathrm{str}\ln(1 - e^{i\omega}\hat{U}^\dagger_{\phi_-} Z \hat{U}_{\phi_+}\tilde{Z}),$$

where "str" is the supertrace [8], and we have temporarily suppressed the preexponential terms for notational simplicity. Here, $Z = \{Z_{N\alpha,N'\alpha'}\}$ is a bilocal supermatrix field, subject to the constraints $\tilde{Z}_{b,b} = Z^\dagger_{b,b}$, $\tilde{Z}_{f,f} = -Z^\dagger_{f,f}$, and $|Z_{b,b}Z^\dagger_{b,b}| < 1$. The anticommuting blocks $Z_{\alpha,\alpha'}$ and $\tilde{Z}_{\alpha,\alpha'}$, $\alpha \neq \alpha'$ are independent. Physically (cf. Ref. [4] for a more extensive discussion), the field $Z_{N,\alpha;N',\alpha'} \sim \psi_{N,\alpha,+}\bar{\psi}_{N',\alpha',-}$ describes the pair propagation of a retarded and an advanced single particle amplitude at a slight difference in frequency, ω, and Aharonov-Bohm flux $\varphi \equiv \phi_+ - \phi_-$. The structure of the action $S[Z, \tilde{Z}]$ shows that field configurations $\hat{U}^\dagger_{\phi_-} Z \hat{U}_{\phi_+} \sim Z$, near stationary under the adjoint action of the Bloch-Floquet operator, dominantly contribute to the field integral. The identification of these "slow modes" is facilitated by passing to a Wigner representation, $Z_{N_1,N_2} \to Z_{N,\Phi}$, where $N = (N_1 + N_2)/2$, and Φ is dynamically conjugate to N. Because of the fast relaxation of the dynamics in the space of angular variables, Φ, the modes of lowest action $Z_{N,\Phi} = Z_N$ depend only on N. These angular zero modes then produce the low energy representation

$$Y(\varphi, \omega) = -\int dN \int dQ e^{-S[Q]} (Q_N)_{+b,-b} (Q_0)_{-b,+b},$$
$$S[Q] = -\frac{1}{8} \int dN \mathrm{str}\Big(\sum_{i=0}^{d-1} D_i (\partial^\varphi_i Q)^2 + 2i\omega Q \sigma^3_{\mathrm{AR}}\Big), \qquad (4)$$

where the field Q is a matrix in the AR space,

$$Q = \begin{pmatrix} 1 & Z \\ \tilde{Z} & 1 \end{pmatrix} \sigma^3_{\mathrm{AR}} \begin{pmatrix} 1 & Z \\ \tilde{Z} & 1 \end{pmatrix}^{-1},$$

and

$$\sigma^3_{\mathrm{AR}} = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}.$$

Depending on the detailed choice of the kicking potential, the diffusion coefficients may take anisotropic values $D_i = \frac{1}{2\tilde{h}^2}\langle (\partial_{\theta_i} V(\Theta))^2 \rangle_\Theta = (K/\tilde{h})^2 \times O(1)$, where $\langle \cdots \rangle_\Theta$ is an angular average. Much like in a disordered metal, such anisotropy may be absorbed in a rescaling of coordinates, $n_i \to (D/D_i)^{1/2} n_i$, where D is the average value of the diffusion coefficients. This rescaling does not affect the principal conclusions on localization whence we assume $D_i = D$ for notational simplicity. Finally, $\partial^\varphi_i \equiv \partial_{n_i} + i\varphi[\sigma^3_{\mathrm{AR}},]\delta_{i0}$ is a covariant derivative accounting for the coupling to an Aharonov-Bohm flux ϕ in the compact n direction.

Technically, Eq. (4) represents the main result of the present Letter. We have described the low energy physics of the QQKR in terms of a nonlinear σ model mathematically equivalent to that of a disordered metal [15]. The construction is parametrically controlled by the parameters $\tilde{h}/K$, $\omega \ll 1$ and corrections to the effective action are small in these parameters. In the following, we discuss a number of physical predictions of Eq. (4).

Metal.—For $d \geq 3$, $(q = \infty)$ or $d \geq 4$ (q finite) the system supports an Anderson (metal-insulator) transition. Above the transition, $K/\tilde{h} \gg 1$, fluctuations are weak and the action may be expanded to quadratic order in Z. Doing the Gaussian integral over Z, one then obtains

$$Y(\varphi, \omega) = \frac{1}{D_\omega \varphi^2 - i\omega}, \qquad (5)$$

where $D_\omega \simeq D$ is the diffusion constant weakly renormalized by nonlinear and frequency dependent corrections.

Substituting this result into the expression for $E(t)$, we obtain diffusive growth $E(t) \sim Dt$, corresponding to a finite optical conductivity (Table I, bottom).

Insulator.—For $q = \infty$, the system is in a localized phase in low dimensions, $d < 3$, or below the Anderson transition at $K/\tilde{h} = O(1)$ in $d \geq 3$. In these regimes, the diffusion constant is undergoing strong renormalization, $D_\omega \overset{\omega \to 0}{\longrightarrow} i\omega$. This in turn leads to saturation $E(t) \overset{t \to \infty}{\longrightarrow}$ const, and vanishing (static) conductivity (Table I, top left). For $d \geq 3$, the condition of scale invariance at criticality predicts critical scaling, $D_\omega \sim (-i\omega)^{(d-2/d)}$ and $E(t) \sim t^{(2/d)}$, in agreement with experiment [7].

Supermetal.—For $d \leq 3$ and $q < \xi$, the system is localized in the virtual directions and delocalized along the real angular momentum direction [cf. Fig. 1(c)]. Resonant transmission through the discrete levels of the ensuing system of effectively finite size then leads to supermetallic growth $E(t) = Ct^2$ at large time scales corresponding to a diverging conductivity (Table I, top right). Phenomenological reasoning may be applied to estimate both the coefficient, C, and the crossover time, t_ξ, to supermetallic scaling: at short times, the uncertainty in quasilevel resolution, $\sim t^{-1}$, is larger than the characteristic quasilevel spacing $\Delta_t \equiv 1/(qL_t^{d\text{-}1})$ of a fictitious system of size $q \times L_t^{d\text{-}1}$, where $L_t \equiv (D_{t^{-1}}t)^{1/2}$ is the characteristic extension of a diffusive process of duration t in the virtual directions, and $D_{t^{-1}}$ is a shorthand for the diffusion coefficient renormalized down to frequency scales $\omega \sim t^{-1}$. The random mixing of a large number of levels reflects diffusion and a growth behavior $E(t) \sim Dt$. The borderline condition $t_\xi^{-1} = \Delta_{t_\xi}$ marks the crossover to long-time dynamics, $t > t_\xi$, governed by localization effects. In this regime, individual levels are no longer mixed by quantum uncertainty. The coherent propagation through individual states then leads to $E(t) = Ct^2$, where the coefficient C is fixed by the matching condition $D_{t_\xi}t_\xi = Ct_\xi^2$, i.e., $C = D_{t_\xi}/t_\xi$. The application of scaling arguments [8] leads to $t_\xi \sim q^2D$ and $t_\xi \sim \exp(2qD)$ in dimensions $d = 2$ and $d = 3$, respectively. Finally, it is clear as a matter of principle that for $d \geq 4$, the lowering of the bare value of $D \sim (K/\tilde{h})^2$ will trigger an Anderson transition in the (d-1)-dimensional virtual directions. At the critical point, the system undergoes a metal-supermetal transition in the long-time scaling of the observable E.

For $d = 2$ the qualitative discussion above can be backed up by a more sophisticated approach. In this case, the theory is defined on an infinitely long cylinder, $N = (n, n_1) \in I_q \times \mathbb{R}$. Fluctuations of the field Q inhomogeneous in the n direction are gapped, reflecting ergodic chaotic mixing in the transverse direction. For frequency scales smaller than this gap, we are left with the quasi-one-dimensional field $Q_N = Q_{n_1}$ with effective action $S[Q] = -\frac{q}{8}\int dn_1 \mathrm{str}\{D(\partial_{n_1}Q)^2 + D\varphi^2[Q, \sigma_{\mathrm{AR}}^3]^2 + 2i\omega Q\sigma_{\mathrm{AR}}^3\}$. The correlation function $Y(\varphi, \omega)$ of this theory can be computed by adaption [13] of Efetov's transfer matrix technique [8]. The qualitative features mentioned above then follow from scaling properties of the corresponding solutions. For a quantitative discussion including prefactors, we refer to Ref. [13].

Summarizing, we have introduced the first microscopic theory of Anderson localization in the quasiperiodic kicked rotor. For irrational values of $\tilde{h}/(4\pi)$, the system is described by a d-dimensional nonlinear σ model which entails a near perfect analogy to the physics of d-dimensional disordered metals. However, for rational values, its effective topology changes, and a dimensional reduction to a quasi (d-1)-dimensional system takes place. We discussed the ensuing consequences, including the existence of a metal-supermetal quantum phase transition in $d \geq 4$. The results reported above should be applicable to the realistic environment [6,7]. Our analysis analytically describes, for the first time, the mechanisms by which the QQKR shows critical behavior. The tuning to resonance conditions $\tilde{h} = 4\pi p/q$ can be experimentally achieved by variation of the kicking period [5]. We expect that the metal-supermetal transition can be observed in the experimental atom optics setup [6] where the condition $d \geq 4$ at sufficiently large q can be met [16].

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[8] K. B. Efetov, *Supersymmetry in Disorder and Chaos* (Cambridge University Press, Cambridge, England, 1997).
[9] Throughout we will work in dimensionless units where time (kicking strength and Planck's constant) are scaled by (the inverse of) the kicking period. The moment of inertia of particle is set to unity.
[10] For example, in the extreme limit $q = 1$ the rotor is integrable [D. R. Grempel, S. Fishman, and R. E. Prange, Phys. Rev. Lett. **49**, 833 (1982); S. Fishman, D. R. Grempel, and R. E. Prange, Phys. Rev. B **29**, 4272 (1984)] and no longer behaves as a disordered system.
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[15] More precisely, Eq. (4) represents a σ model of unitary symmetry [8], as relevant for systems of broken T invariance. In the present context, T invariance is broken by the phases $\phi_{\pm}$. (It gets restored after integration over all values of $\phi_{\pm}$.) At off resonance, i.e., $q \rightarrow \infty$, the T breaking is so weak that we should actually employ the T-invariant form of the nonlinear σ model (cf. Ref. [4]). However, this extension is not of great consequence and will be discussed elsewhere [13].
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