Nonlinear optics of superconductors

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Collective mode vs. single particle physics

Tsuji and Aoki arXiv: 1404.2711

Anderson model perspective

Review: convert to spin operators

$$\hat{\sigma}_k = \frac{1}{2} \psi_k^\dagger \tau \psi_k$$

$$\psi_k = \begin{pmatrix} c_{k\uparrow} \\ c_{k\downarrow}^\dagger \end{pmatrix}$$ τ are Pauli matrices

$$\sigma_z = \frac{1}{2}\left(c_{k\uparrow}^\dagger c_{k\uparrow} + c_{-k\downarrow}^\dagger c_{-k\downarrow}\right)$$

$$\sigma_x = \frac{1}{2}\left(c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger + c_{-k\downarrow} c_{k\uparrow}\right)$$

$$\sigma_y = \frac{i}{2}\left(c_{k\uparrow}^\dagger c_{k\downarrow}^\dagger - c_{-k\downarrow} c_{k\uparrow}\right)$$

$$\sigma_x - i\sigma_y = c_{k\uparrow}^\dagger c_{k\downarrow}^\dagger = \sigma_+$$

$$\sigma_x + i\sigma_y = c_{-k\downarrow} c_{k\uparrow} = \sigma_-$$

Mean field Hamiltonian

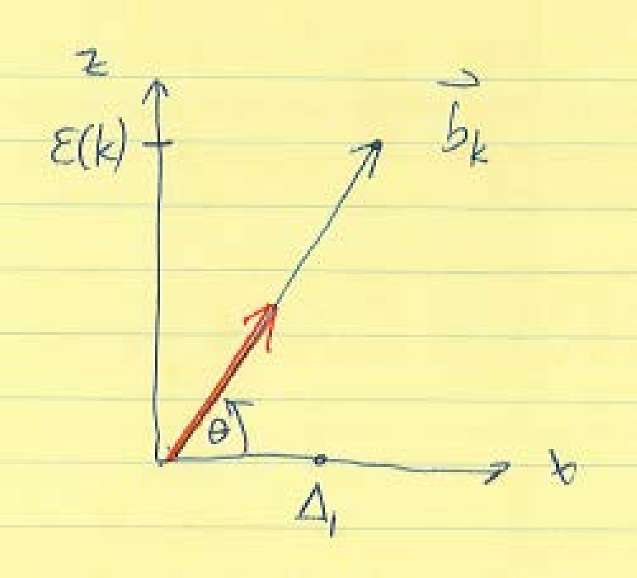
In equilibrium spins align with mean field.

$$\mathcal{H} = \sum_k \hat{\sigma}_k \cdot \vec{b}_k \qquad \vec{b}_k \text{ is the mean field}$$

$$\vec{b}_k = \hat{x}\,\Delta_1 + \hat{y}\,\Delta_2 + \hat{z}\,\mathcal{E}(\vec{k})$$

$$\Delta = \Delta_1 + i\Delta_2$$

$$\left.\begin{aligned} \Delta_1 &= \sum_k \langle \sigma_{kx} \rangle \\ i\Delta_2 &= \sum_k \langle \sigma_{ky} \rangle \end{aligned}\right\} \text{self-consistency relations}$$



The dynamics are governed by the Bloch equation

$$\frac{d\vec{\sigma}_k}{dt} = i\left[\mathcal{H}, \vec{\sigma}_k\right] = 2\,\vec{b}_k \times \vec{\sigma}_k$$

Now apply time-dependent vector potential $\vec{A}(t)$

$$\varepsilon(\vec{k}) \rightarrow \varepsilon(\vec{k} - \tfrac{e}{c}\vec{A})$$

$$\mathcal{H} = \sum_k \vec{\sigma}_k \cdot \vec{b}_k(t) \rightarrow \sum_k \sigma_{k\cancel{z}}\, b_z(t)$$

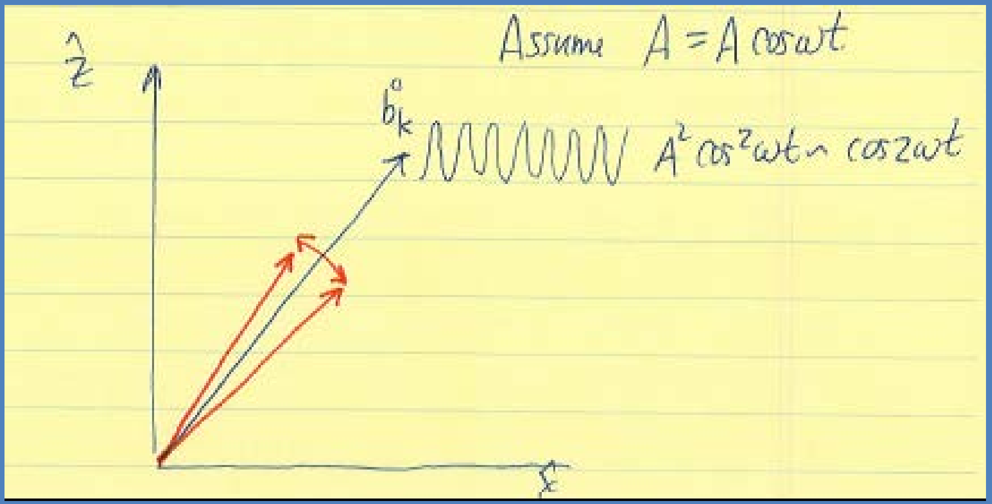
$$= \frac{1}{2}\sum_k \left(c^\dagger_{k\uparrow} c_{k\uparrow} + c^\dagger_{-k\downarrow} c_{-k\downarrow}\right) \varepsilon\left(\vec{p} - \frac{e\vec{A}}{c}\right)$$

When we sum on spins we obtain the result that

$$b_{kz} = \frac{1}{2}\left[\varepsilon\left(\vec{k} - \frac{e\vec{A}}{c}\right) + \varepsilon\left(\vec{k} + \frac{e\vec{A}}{c}\right)\right]$$

For $\varepsilon(k) = \frac{\hbar^2 k}{m}$ the linear order term in $\vec{A}$ vanishes. Now the picture

$$b_{kz} = \varepsilon(\vec{k}) + \frac{e^2}{c^2} \frac{\partial^2 \varepsilon(\vec{k})}{\partial k^2} A^2(t) + O(A^4)$$

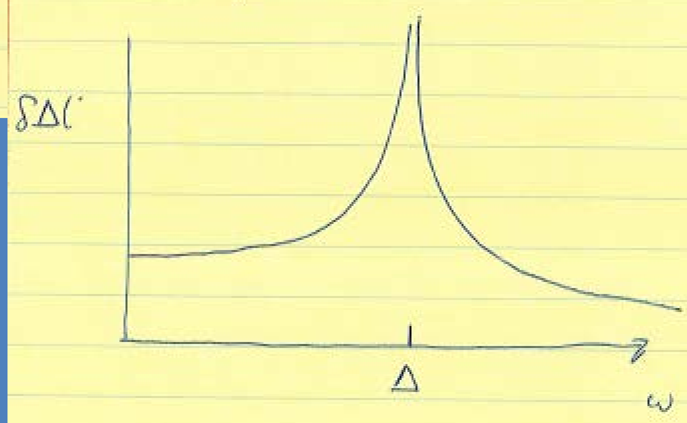


According to this picture the order parameter oscillates at frequency 2ω.

According to Tsuji and Aoki self-consistent solution:

$$\frac{\delta\Delta(t)}{\Delta_0} \propto \frac{\Delta}{\sqrt{\Delta^2-\omega^2}} A^2 \cos 2\omega t \quad \text{for} \quad \omega \ll \Delta$$

$$\propto \frac{\Delta}{\sqrt{\omega^2-\Delta^2}} A^2 \cos 2\omega t \qquad \omega \gg \Delta$$



Assume cosine dispersion relation then

$$J_{NL}(t) \propto A(t) \sum_k \varepsilon(k)\, \delta\sigma_{kz}(t)$$

From linearized Bloch equation

$$\delta\sigma_{kz}(t) = \frac{\Delta}{\varepsilon_k}\, \delta\sigma_{kx}(t)$$

$$J_{NL}(t) \propto A(t) \sum_k \Delta\, \delta\sigma_{kx}(t)$$

$$J_{NL}(t) \propto A(t) \frac{\Delta}{V_0}\, \delta\Delta(t)$$

where V is pairing interaction

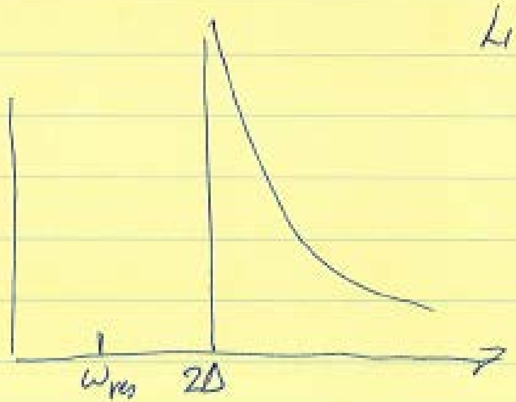
As $\delta\Delta(t) \sim A^2$

we find finally that

$$J_{NL}(t) \propto \frac{e^2 \Delta}{V_0} \frac{\Delta}{\sqrt{\Delta^2 - \omega^2}} \cancel{A^3 \cos^3 \omega t} \, (A \cos \omega t)^3$$

and therefore has components at ω and 3ω. The nonlinear current on resonance at $\omega = \Delta$ has been observed

Like 2-photon absorption



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Higgs Amplitude Mode in the BCS Superconductors $Nb_{1-x}Ti_xN$ Induced by Terahertz Pulse Excitation

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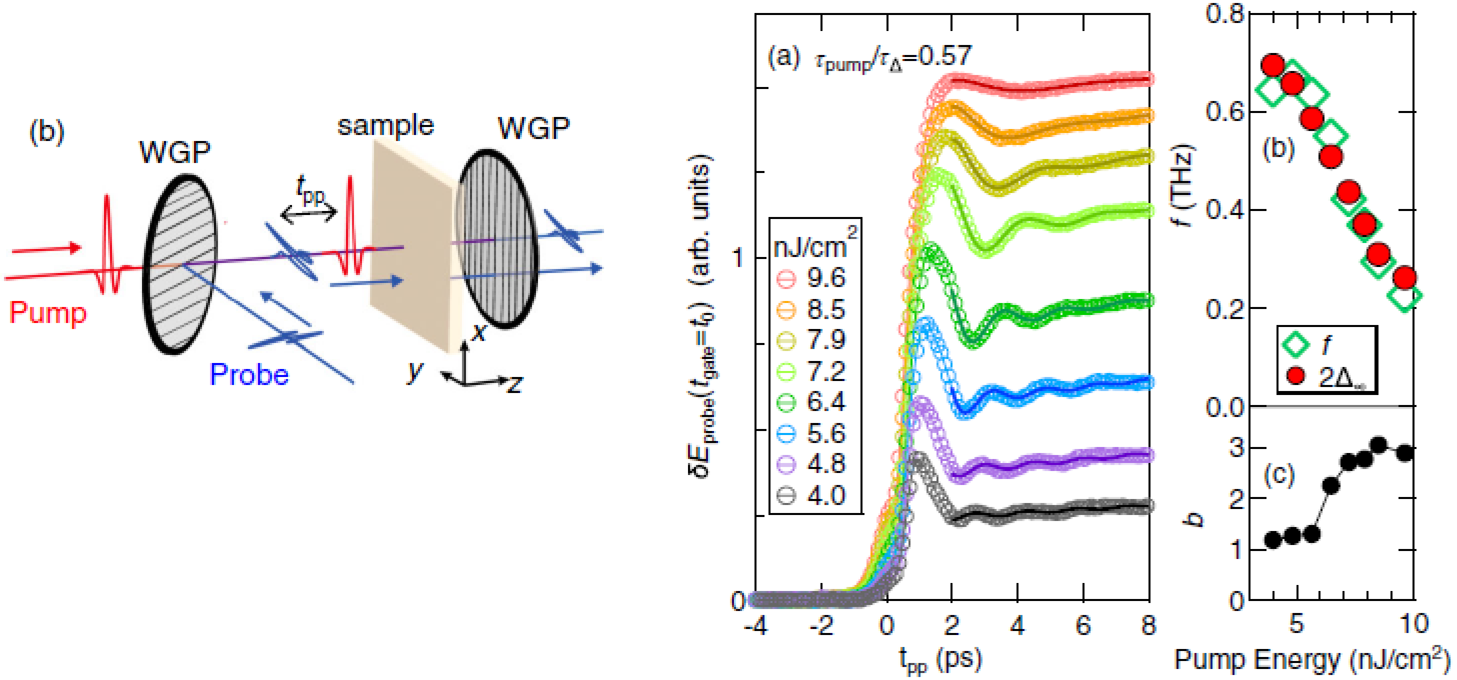

But is the coupling to the condensate the only viewpoint?

How about 2-photon absorption?

$$\sigma^{(3)} \propto \left| \langle 0 | J | n \rangle \langle n | J | m \rangle \langle m | J | 0 \rangle \right|^2$$