Talk 1. Quasi-particle charge and heat currents in *d*-wave superconductor

Talk 2. The Nernst effect in vortex liquid state of cuprates

Talk 3. Magnetization of vortex liquid state

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Supported by NSF-MRSEC, ONR

Boulder, July 7-11, 2008

Talk 1

Quasi-particle charge and heat currents in *d*-wave superconductor

1. **Introduction : Charge and heat currents**
   Hall effect, Nernst effect, Thermal Hall effect

2. **The Hall effect in cuprates**

3. **Quasiparticles and Thermal Hall conductivity**

N. P. Ong, Princeton University

Collaborators:
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Yuexing Zhang, Jeff M. Harris, Y.F. Yan, P.W. Anderson

Supported by NSF-MRSEC, ONR

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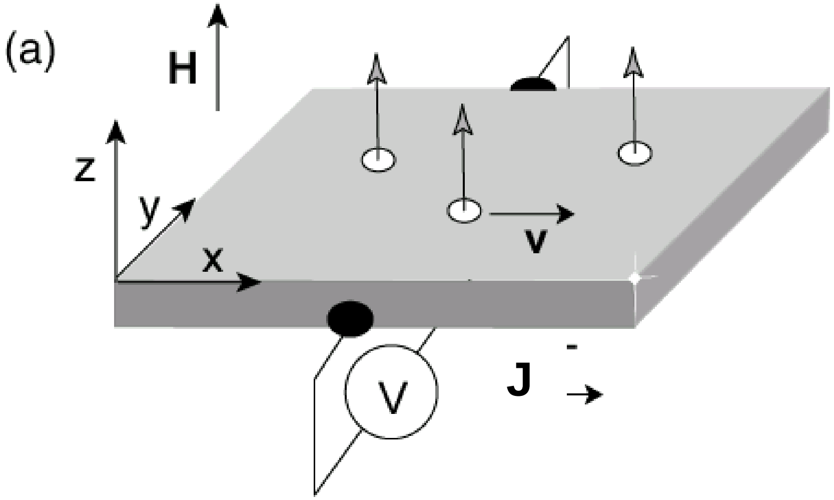
The Hall effect

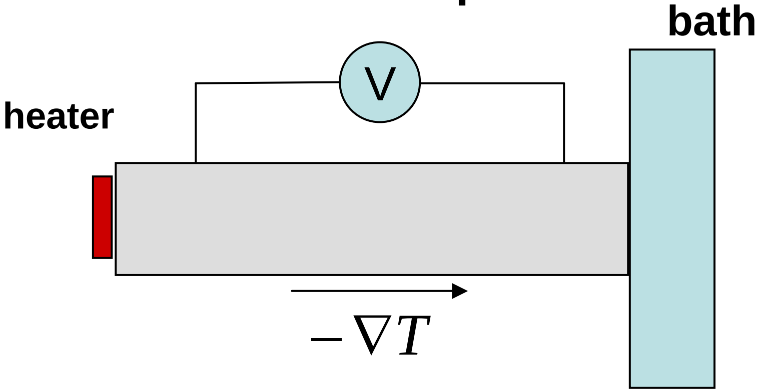


$$J_x = \sigma_{xx} E_x + \sigma_{xy} E_y$$

$$J_y = \sigma_{yx} E_x + \sigma_{yy} E_y$$

$$\mathrm{E} = \overleftrightarrow{\rho} \bullet \mathrm{J}$$

Thermopower



$$J = \sigma E + \alpha(-\partial_x T)$$

$$S = \left(\frac{E}{\nabla T}\right)_{J=0} = \frac{\alpha}{\sigma}$$

Seebeck coef.

Boltzmann-equation expressions for currents

$$\frac{\partial f_k}{\partial \mathbf{k}}.\dot{\mathbf{k}} + \frac{\partial f_k}{\partial \mathbf{x}}.\mathbf{v}_\mathbf{k} = -\frac{g_\mathbf{k}}{\tau}$$

$$f_\mathbf{k} - f_\mathbf{k}^0 = g_\mathbf{k}$$

charge $$\mathbf{J} = 2e\sum_k g_\mathbf{k}\,\mathbf{v}_\mathbf{k}$$

heat $$\mathbf{J}^h = 2\sum_k (\varepsilon_\mathbf{k} - \mu)\, g_\mathbf{k}\,\mathbf{v}_\mathbf{k}$$

Charge current $\mathbf{J} = \sigma\,\mathbf{E}$

$$g_\mathbf{k} = -\frac{\partial f_\mathbf{k}^0}{\partial \varepsilon} e\,\mathbf{E}.\vec{\ell}_\mathbf{k}$$

$$\frac{\partial f_k}{\partial \mathbf{x}} = \frac{\partial f_\mathbf{k}^0}{\partial \varepsilon}\hbar\,\mathbf{v}_\mathbf{k}$$

conductivity $$\sigma = 2e^2\sum_k\left(-\frac{\partial f_\mathbf{k}^0}{\partial \varepsilon}\right)\mathrm{v}_\mathbf{k}\,\ell_\mathbf{k}\cos^2\vartheta_\mathbf{k}$$

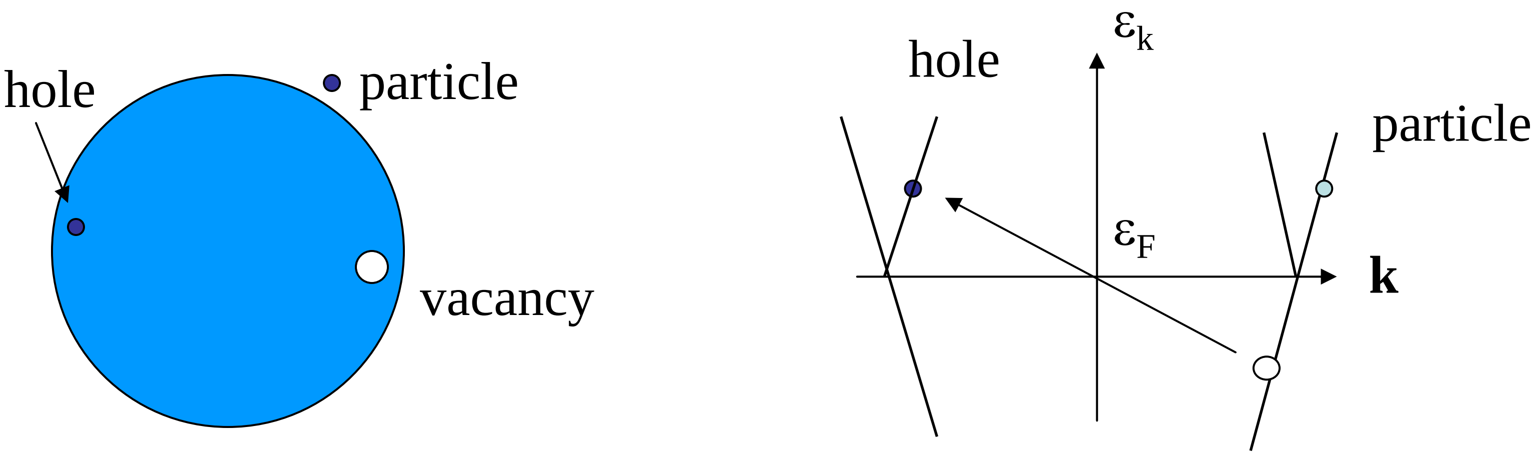
Presence of temp. gradient

Thermoelectric current $\mathbf{J} = \alpha\,(-\nabla T)$

$$g_\mathbf{k} = -\frac{\partial f_\mathbf{k}^0}{\partial \varepsilon}\frac{(\varepsilon_\mathbf{k} - \mu)}{T}\vec{\ell}_\mathbf{k}.(-\nabla T)$$

Thermoelectric cond. $$\alpha = 2e\sum_k\left(-\frac{\partial f_\mathbf{k}^0}{\partial \varepsilon}\right)\frac{(\varepsilon_\mathbf{k} - \mu)}{T}\mathrm{v}_\mathbf{k}\,\ell_\mathbf{k}\cos^2\vartheta_\mathbf{k}$$

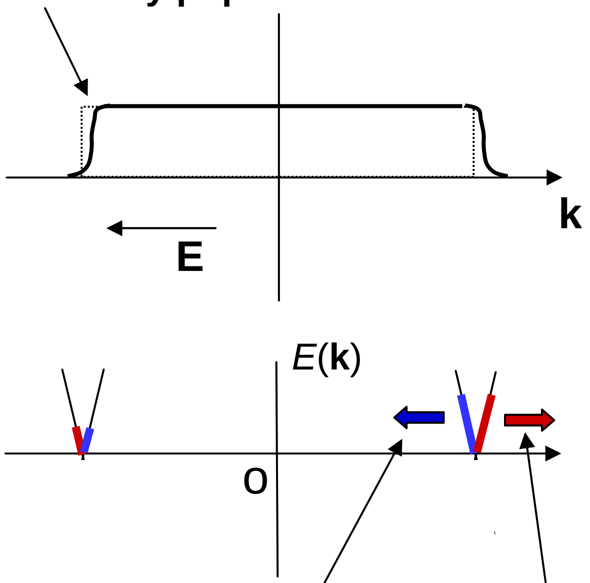
Quasi-particle excitations in normal state of Fermi liquid



$$h_{k\uparrow}^{+} = c_{-k\downarrow}$$

A spin-down vacancy at –k translates to a spin-up hole excitation at k

Charge and heat currents in the "excitation" representation



Large vacancy population

Large left hole current

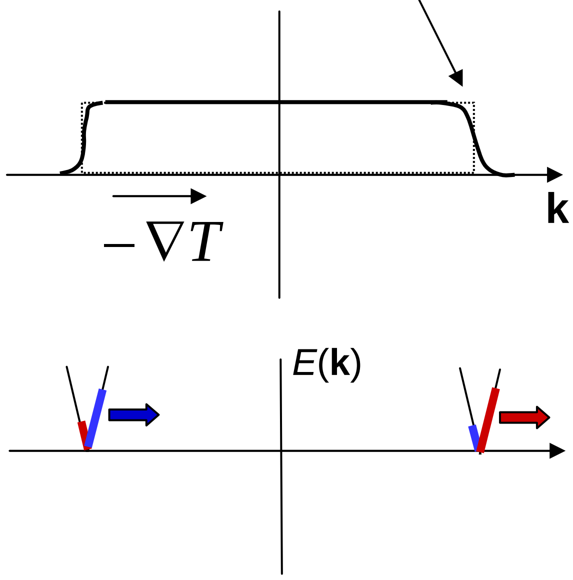
Large right electron current

Charge currents add

$$\mathbf{J} = \sigma \mathbf{E}$$

Mass currents nearly cancel.
Difference is the Peltier heat current

$$\mathbf{J}^h = \tilde{\alpha}\,\mathbf{E}$$



Large vacancy population

Both mass (entropy) currents flow to right

Mass currents add

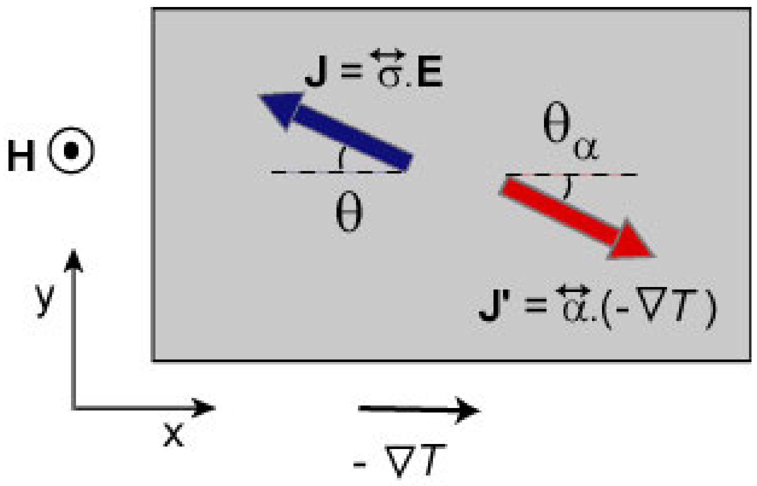
$$\mathbf{J}^h = \kappa_e(-\nabla T)$$

Charge currents nearly cancel.
Difference is the Peltier charge current

$$\mathbf{J} = \alpha(-\nabla T)$$

The Nernst effect (quasiparticles)

Wang et al. PRB ‘01



$$\mathbf{J} = \overleftrightarrow{\sigma}\mathbf{.}\mathbf{E} + \overleftrightarrow{\alpha}\mathbf{.}(-\nabla T)$$

Open boundaries, so set **J** = 0.

$$\mathbf{E} = -\overleftrightarrow{\rho}\mathbf{.}\overleftrightarrow{\alpha}\mathbf{.}(-\nabla T)$$

$$E_y = -\left(\rho\alpha_{yx} + \rho_{yx}\alpha\right)(-\partial_x T)$$

Off-diag. Peltier cond.

$$\alpha_{xy} = 2e^2 \sum_{\mathbf{k}} \left(-\frac{\partial f_{\mathbf{k}}^0}{\partial \varepsilon}\right) \frac{\varepsilon_{\mathbf{k}} - \mu}{T} \ell_y \, \mathbf{v}\times\mathbf{B}\mathbf{.}\frac{\partial}{\partial \mathbf{k}}(\ell_x)$$

Measured Nernst signal

$$e_N \equiv \frac{E_y}{|\nabla T|} = \frac{\pi^2}{3}\frac{k_B^2 T}{e}\frac{\partial \theta}{\partial \varepsilon}$$

Generally, very small because of cancellation between α_{xy} and σ_{xy}

NPO, PRB (1991)

The 2D Hall conductivity σ_{xy}

$$\frac{\partial f_{\mathbf{k}}}{\partial \mathbf{k}} . \dot{\mathbf{k}} = -\frac{g_{\mathbf{k}}}{\tau} \qquad f_{\mathbf{k}} - f_{\mathbf{k}}^{0} = g_{\mathbf{k}}$$

Boltzmann Eq.

$$g_{\mathbf{k}} = -\tau \frac{\partial f_{\mathbf{k}}}{\partial \mathbf{k}} . (e\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Eq. of motion

$$J_y = e \sum_{\mathbf{k}} \left(-\frac{\partial f_{\mathbf{k}}^{0}}{\partial \varepsilon} \right) e\, \mathbf{v} \times \mathbf{B} . \frac{\partial}{\partial \mathbf{k}} (eE . \vec{\ell})\, \mathrm{v}_y \tau$$

Hall current in 2nd order

$$\sigma_{xy} = e^3 B \sum_{\mathbf{k}} \left(-\frac{\partial f_{\mathbf{k}}^{0}}{\partial \varepsilon} \right) \quad \hat{\mathbf{t}} . \nabla_{\mathbf{k}} (\ell_x)\, \ell_y$$

Gauss mapping to …

$$\sigma_{xy} = e^3 B \frac{1}{2} \quad \oint d\vec{\ell} \times \vec{\ell}$$

Area swept out in *ell*-space!

The 2D Hall conductivity σ_{xy}

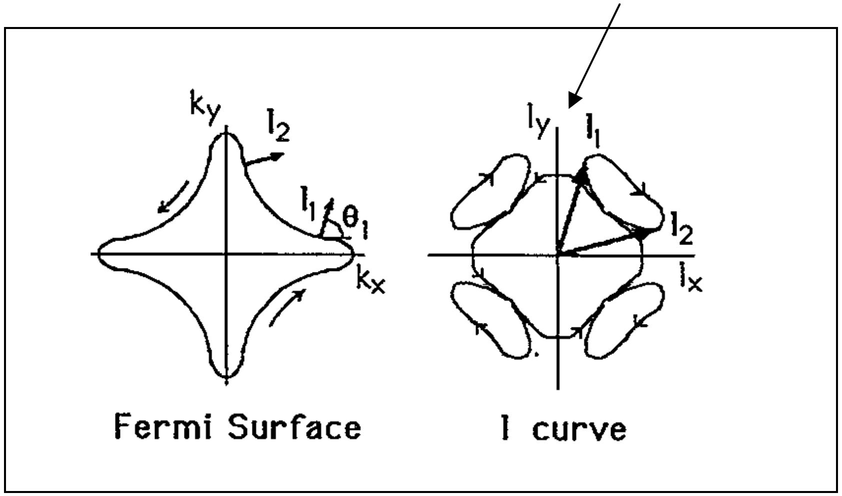
$$\sigma_{xy} = 2(e^3/\hbar)B \sum_{\mathbf{k}} \left[\frac{-\partial f_{\mathbf{k}}}{\partial \varepsilon}\right](v_y \tau_{\mathbf{k}}) \left[v_y \left[\frac{\partial}{\partial k_x}\right] - v_x \left[\frac{\partial}{\partial k_y}\right]\right](v_x \tau_{\mathbf{k}}) ,$$

$$\sigma_{xy} = (e^3/2\pi^2\hbar) \int dk_t |\mathbf{v}|^{-1} [v_y \tau_{\mathbf{k}} (\mathbf{v} \times \mathbf{B}) \cdot \nabla (v_x \tau_{\mathbf{k}})] ,$$

$$\vec{A} = \frac{1}{2} \oint d\vec{\ell} \times \vec{\ell}$$

$$\sigma_{xy} = 2(e^2/h)\frac{\mathbf{B}.\mathbf{A}}{\phi_0}$$

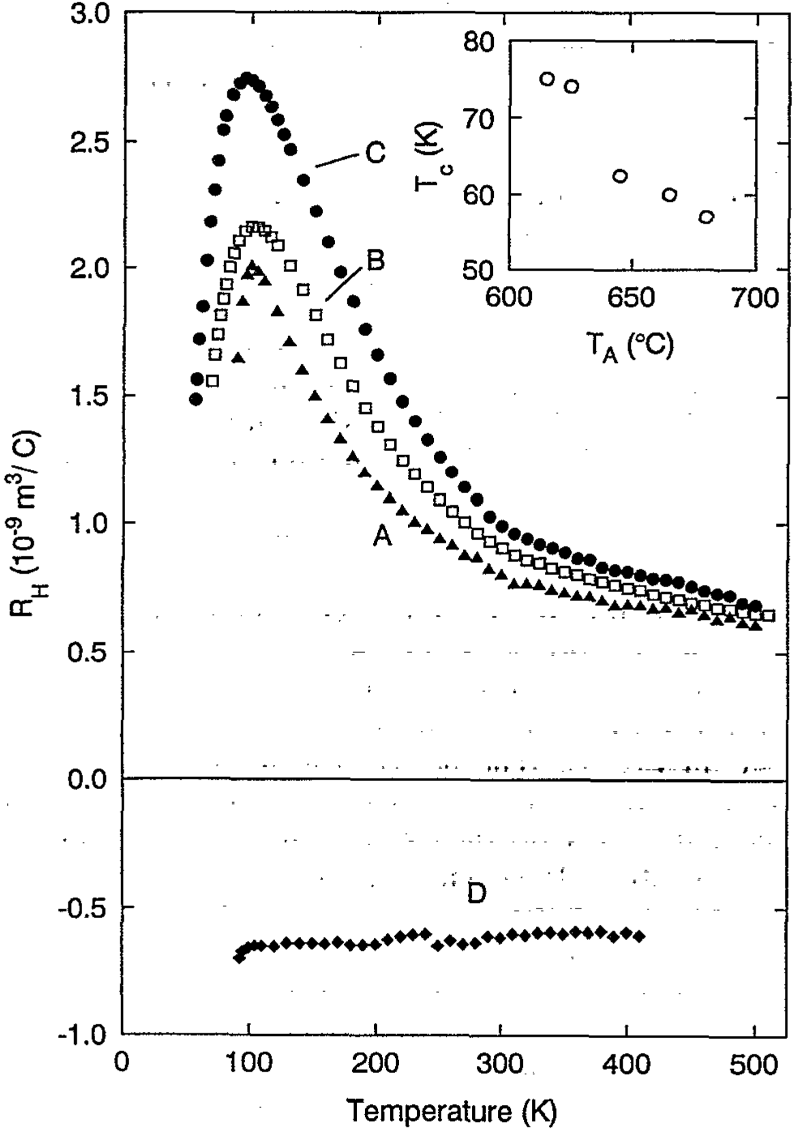
swept area

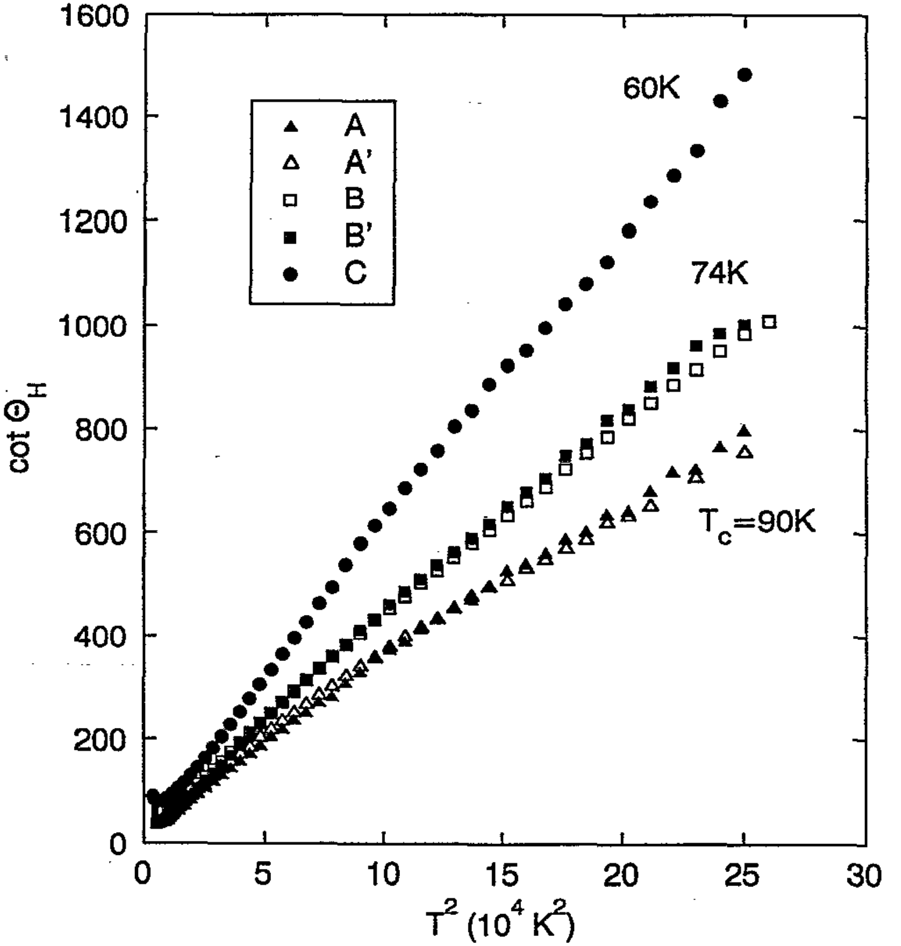


σ_{xy} is the area swept out by mfp (for *arb.* anisotropy)

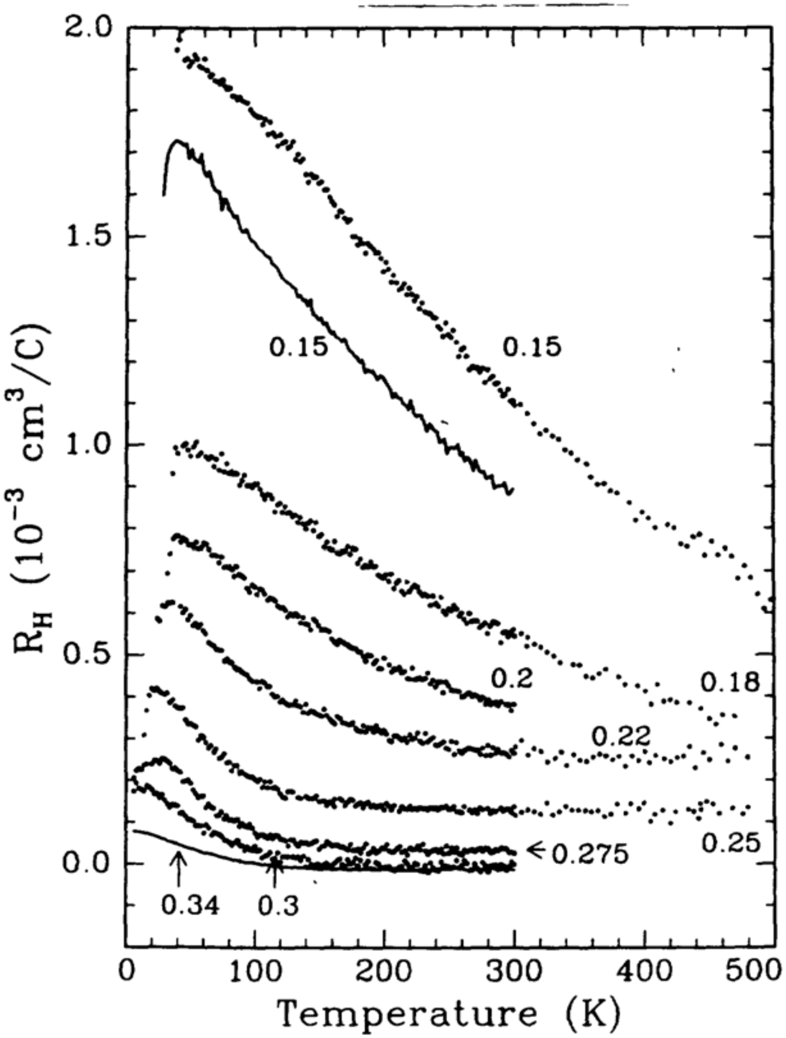
Temp. dependences of Hall coef. and Hall angle in YBCO

Harris, Yan, NPO, PRB '92

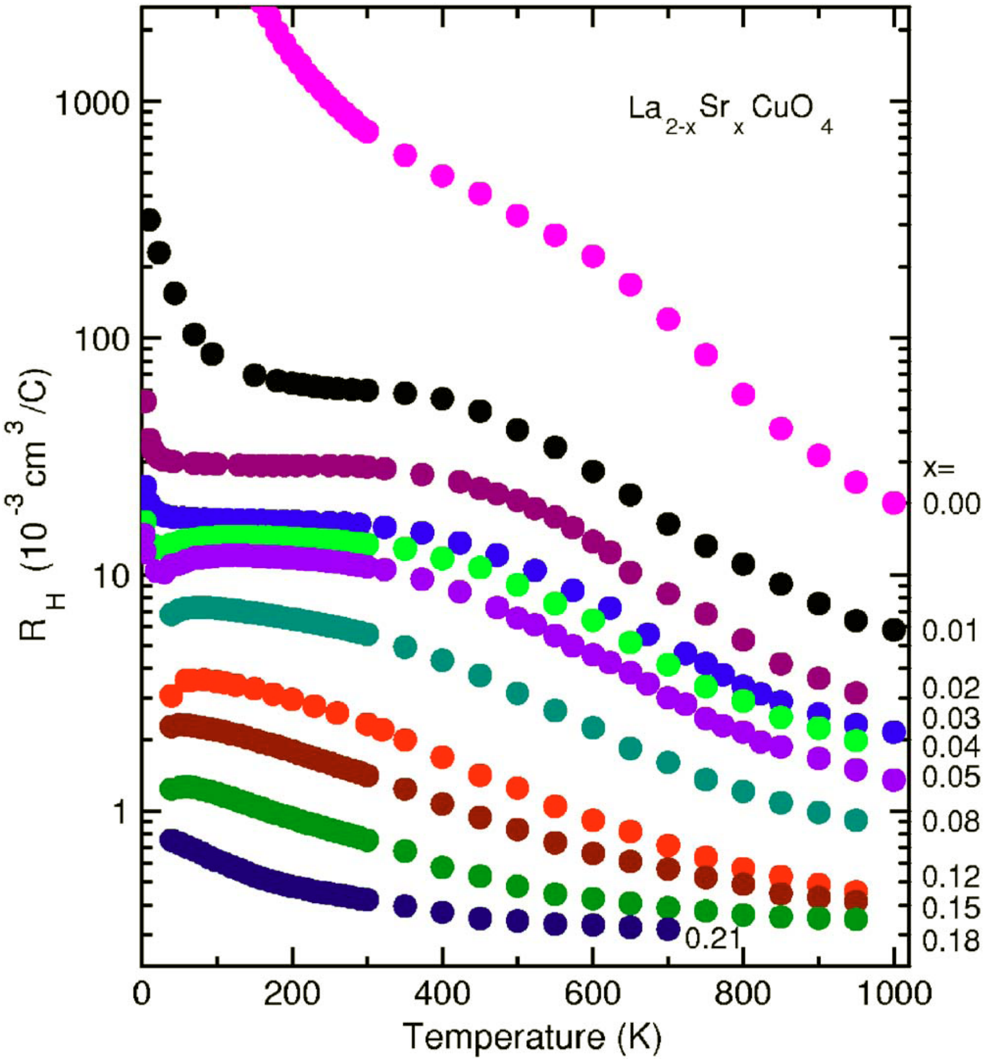




Similar T dependence of R_H seen in LaSrCuO



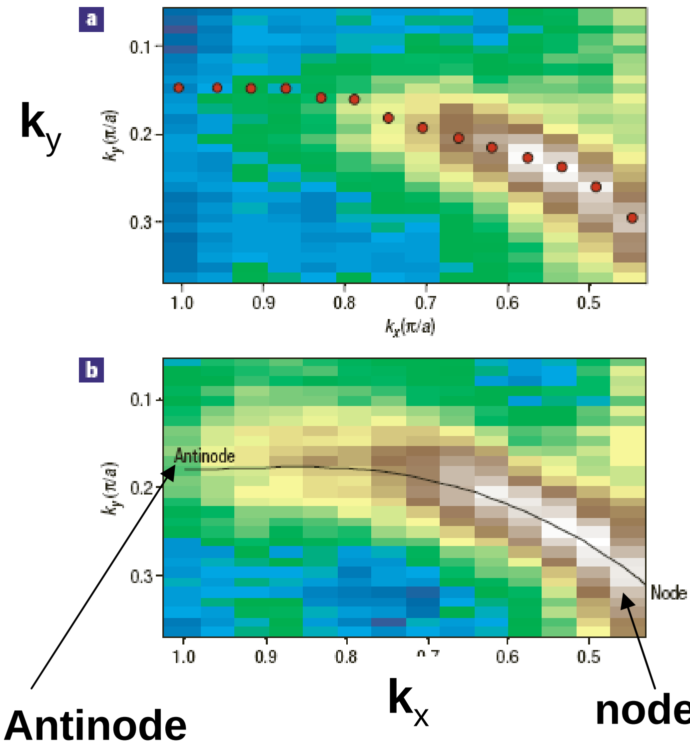
Hwang Batlogg et al., PRL ‘94

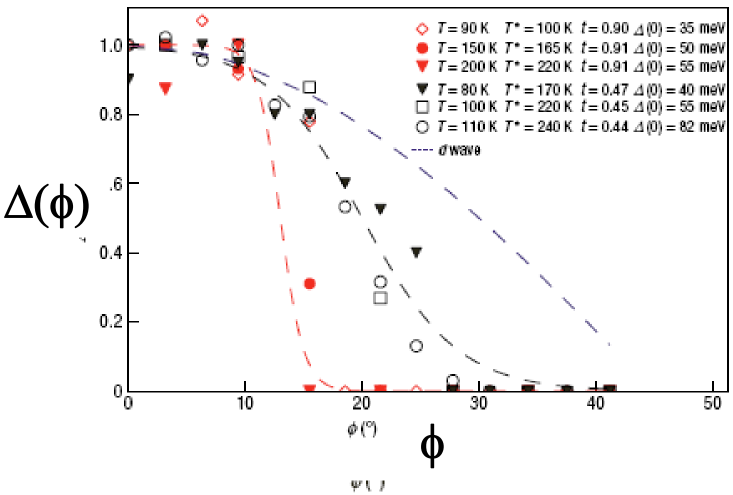


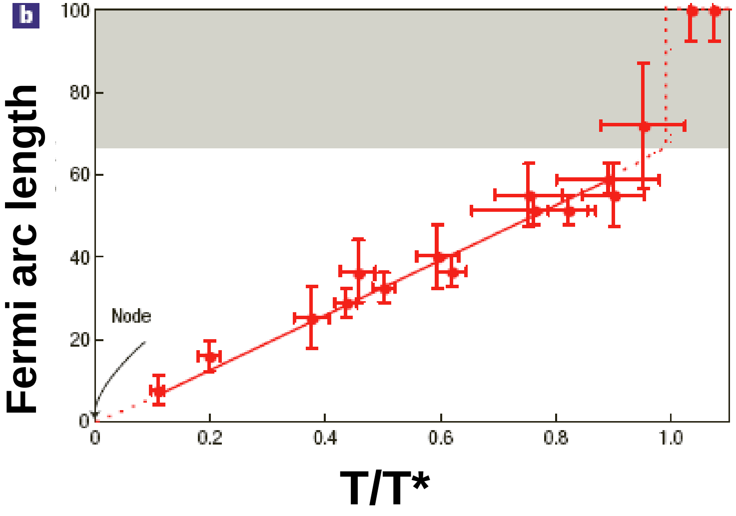
Ono, Komiya, Ando, PRB ‘07

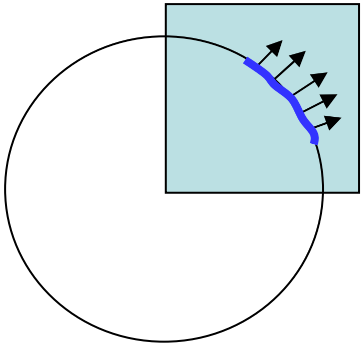
Evolution of the pseudogap from Fermi arcs to the nodal liquid

Kanigel, Campuzano et al. Nature Phys. 2007

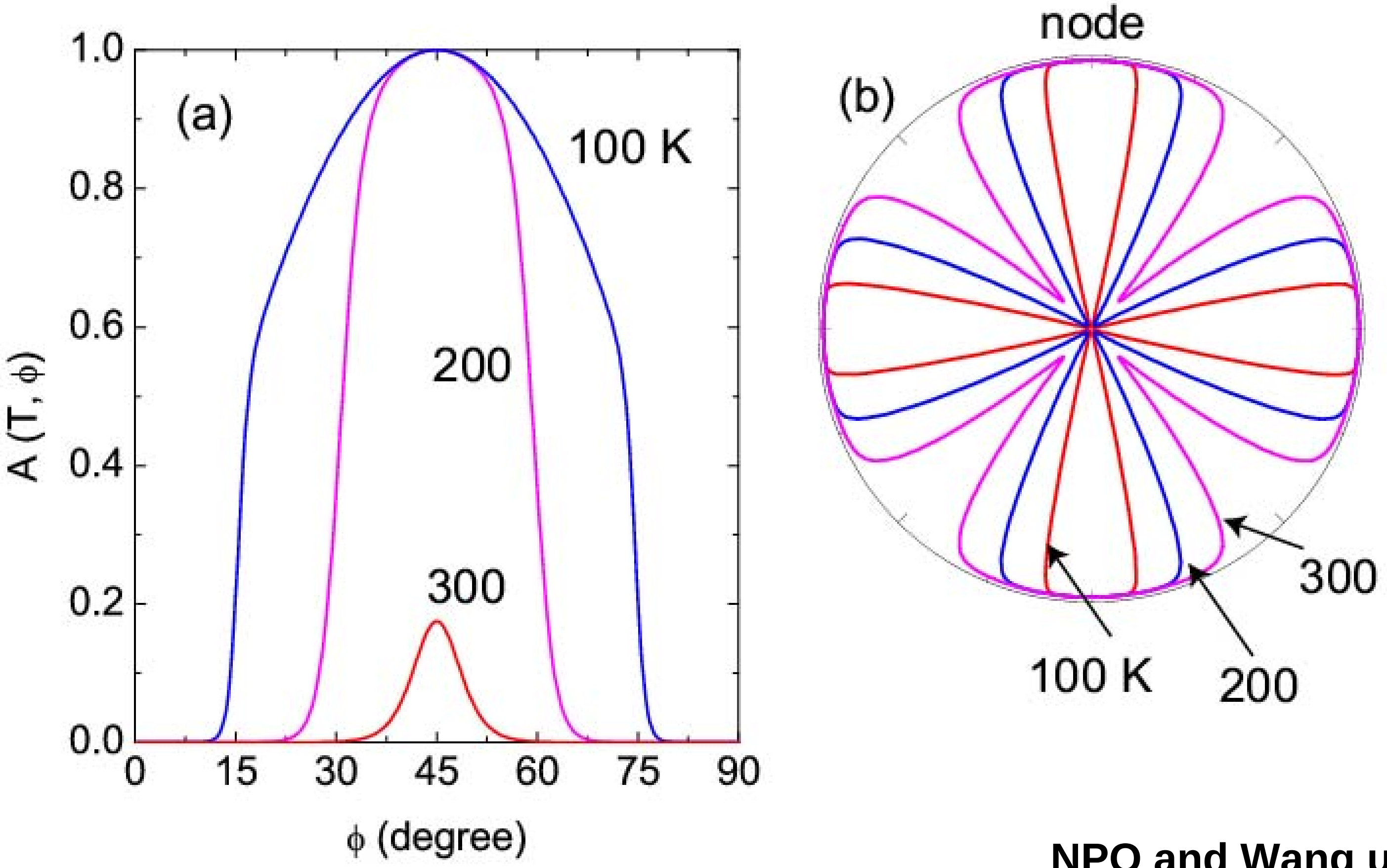






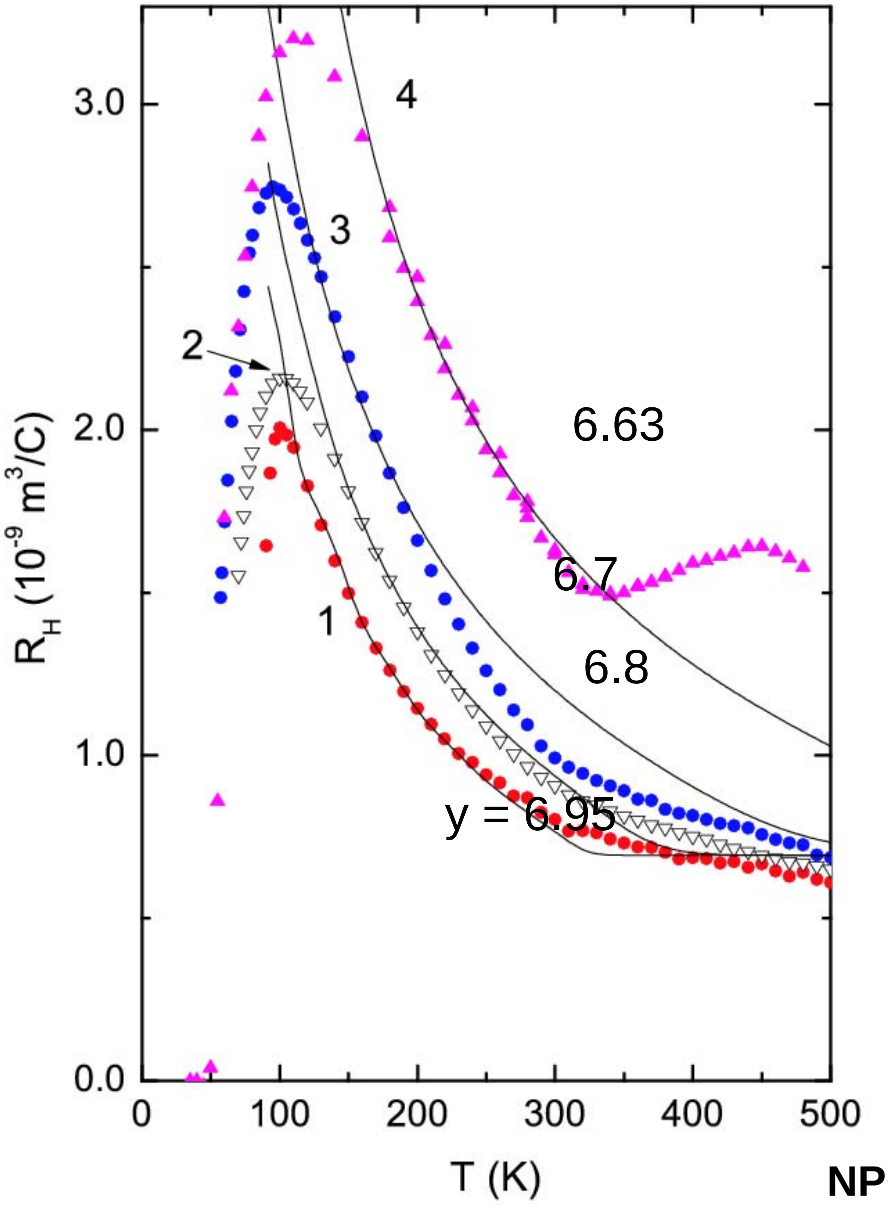


Model: Only states on Fermi arc have long mean free path



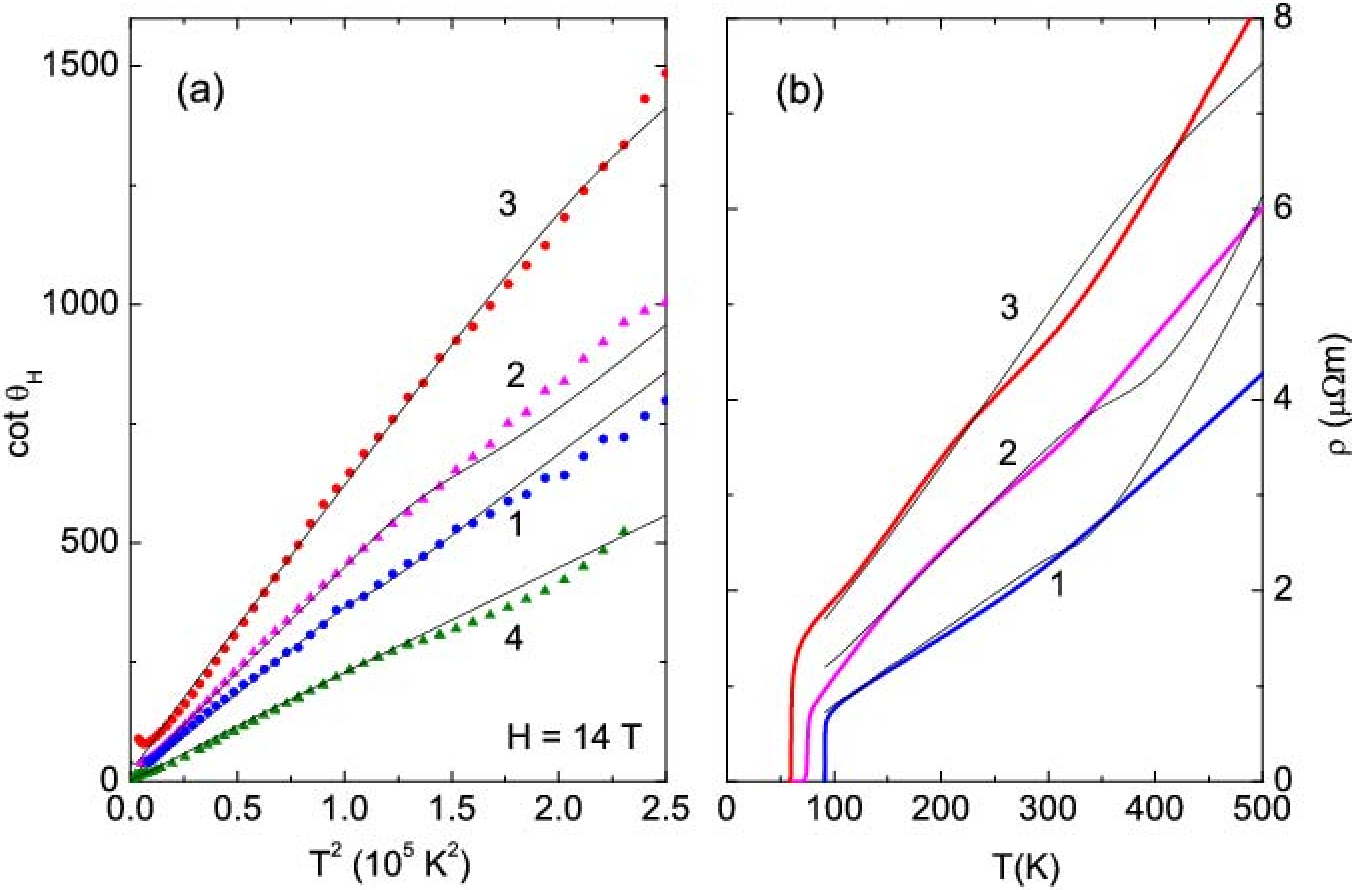
NPO and Wang unpubl.

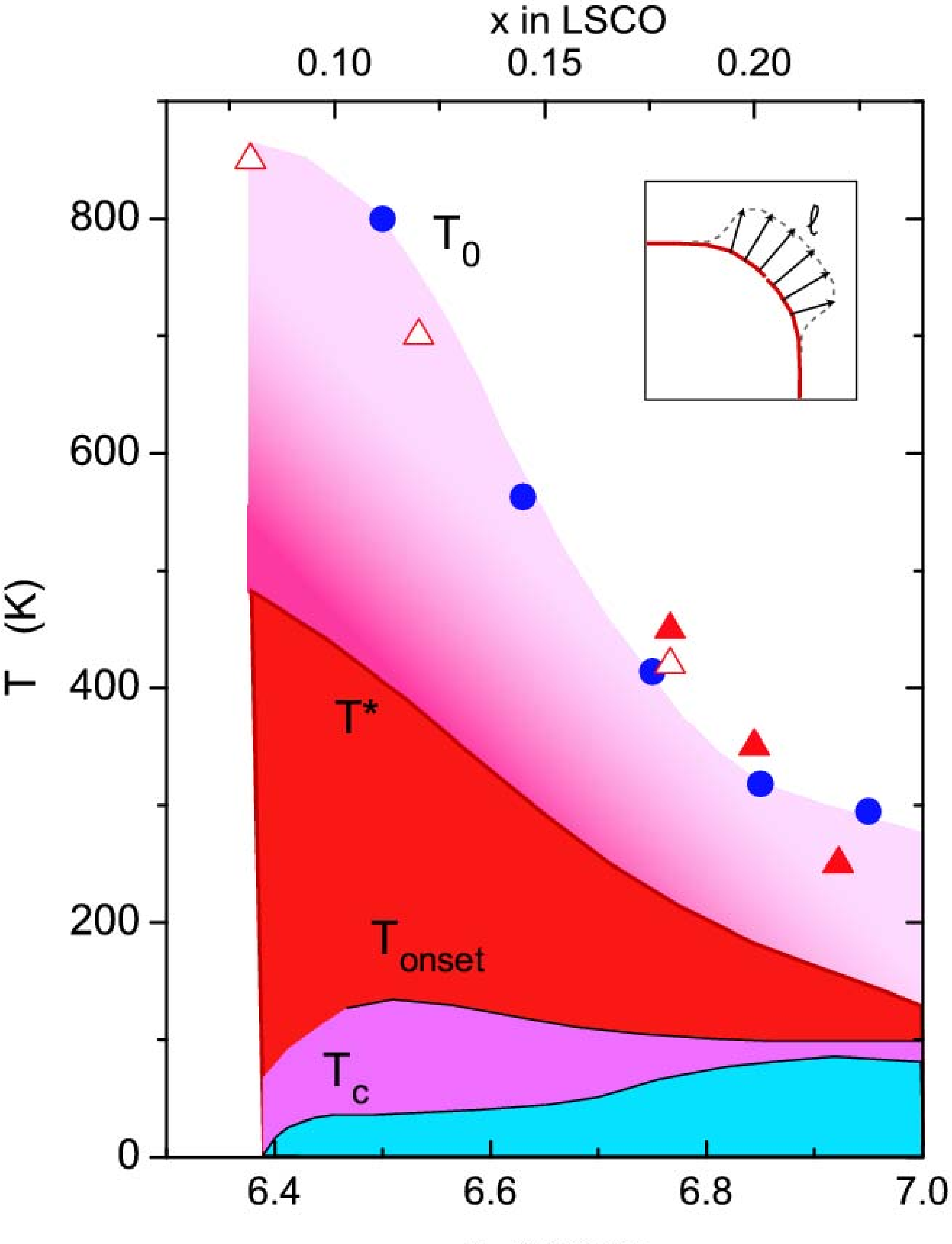
Fits to T dependence of R_H in YBCO



NPO and Wang unpubl.

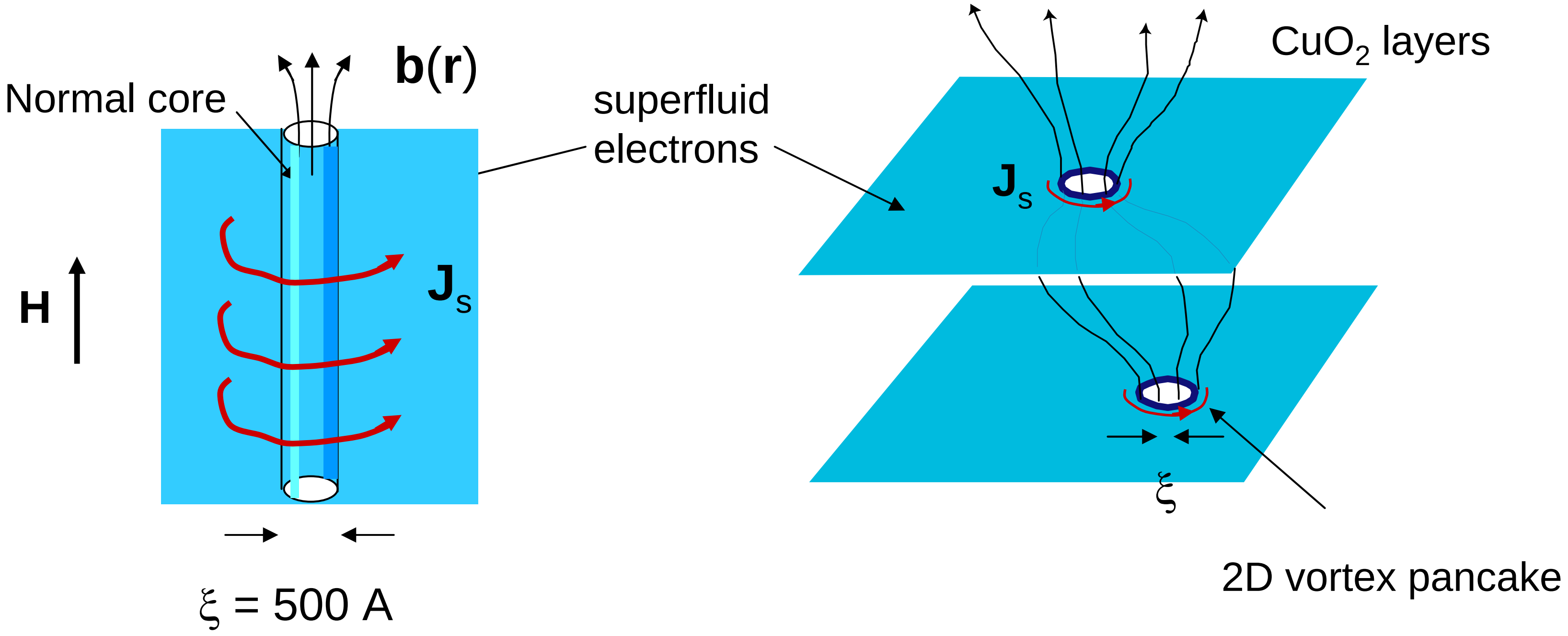
Fits to cotθ and resistivity ρ



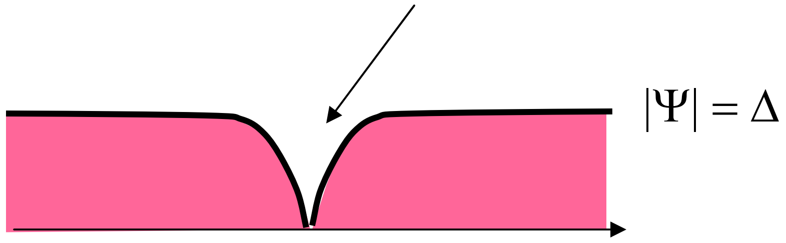

x in LSCO
0.10
0.15
0.20
T (K)
800
600
400
200
0
T$_0$
T*
T$_{onset}$
T$_c$
ℓ
6.4
6.6
6.8
7.0
y in YBCO$_y$

Vortex in Niobium

Vortex in cuprates

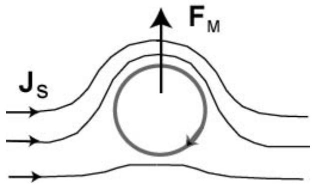


Gap Δ(**r**) vanishes in core



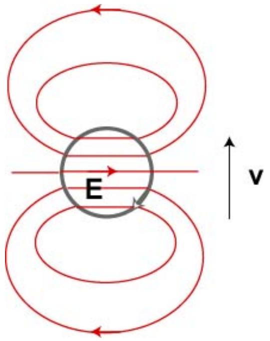
Vortex motion in type II superconductor

(Bardeen Stephen, Nozieres Vinen)

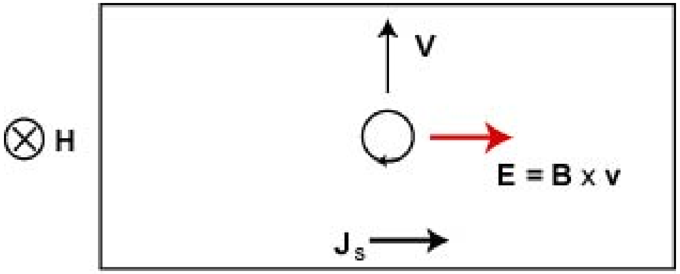


Applied supercurrent J_s exerts magnus force on vortex core

$$\mathbf{F}_M = \mathbf{J}_s \times \vec{\Phi}_0$$



**Velocity gives *induced E*-field in core (Faraday effect)
Current enters core and dissipates (damping viscosity)**

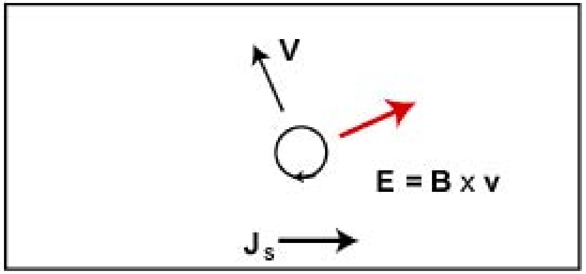


Motion of vortices generates *observed E*-field

$$\boxed{\mathbf{E} = \mathbf{B} \times \mathbf{v}}$$

$$\rho_{xx} = \rho_N \frac{H}{H_{c2}} = B\Phi_0 / \eta$$

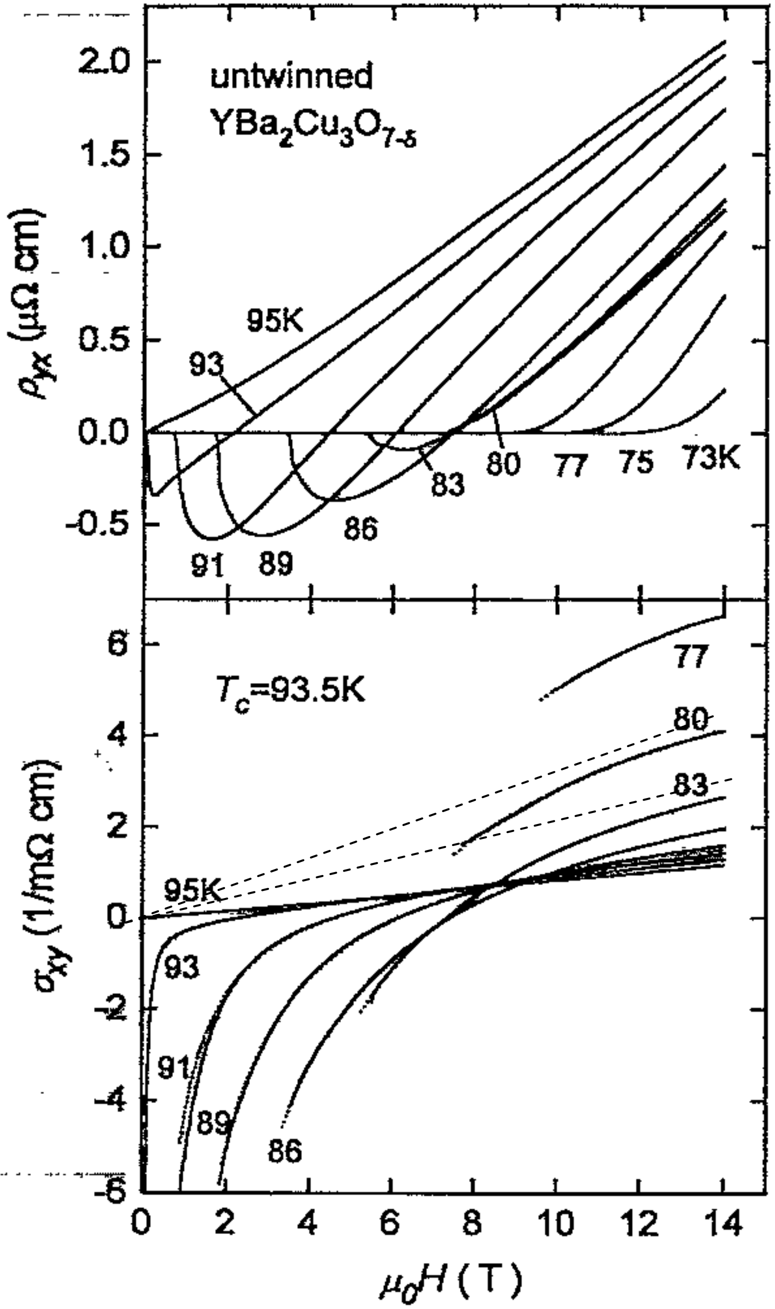
Consequence of Josephson equation



Tilt angle of velocity gives negative vortex Hall effect

In clean limit, vortex v is || - J_s

Vortex Hall current



Harris et al. PRB '95

Vortex Hall σ_{xy} is *negative.*
Appearance is abrupt

Invert matrix

$$\sigma_{xy} = \frac{\rho_{yx}}{\rho_{xx}^2 + \rho_{yx}^2}$$

Quasiparticle and vortex Hall conductivities are *additive*

$$\sigma_{xy} = \sigma_{xy}^N + \sigma_{xy}^S$$

~H

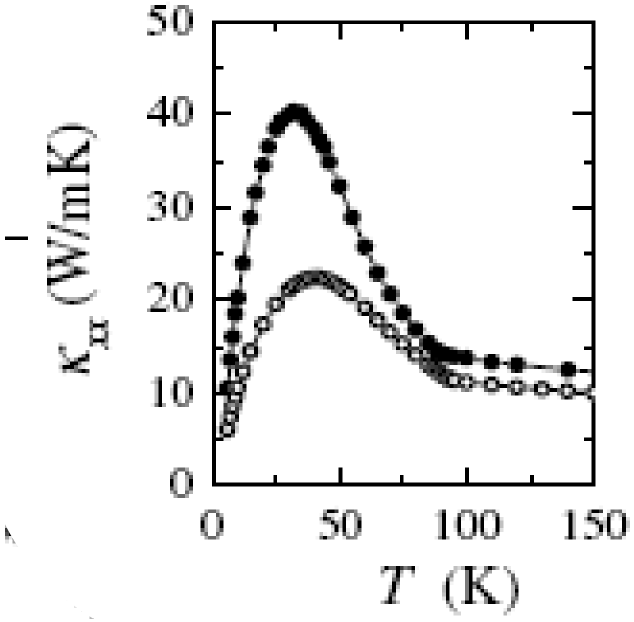
~ -1/H

Thermal Hall conductivity of quasi-particles in cuprates

K. Krishana, Yuexing Zhang, J. M. Harris, NPO

Problem: **Separate the QP current from vortex currents?**

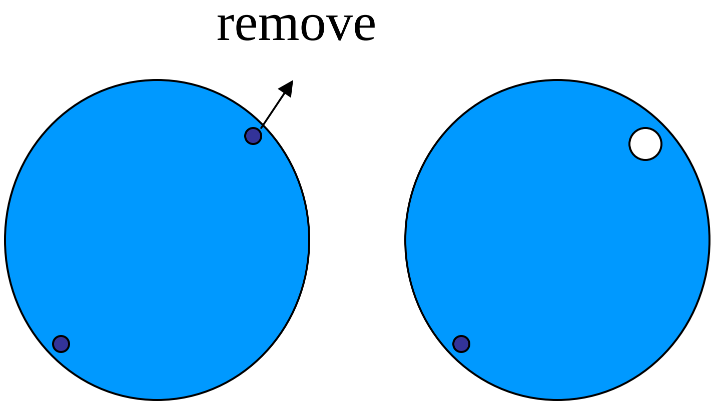
Monitor thermal currents.

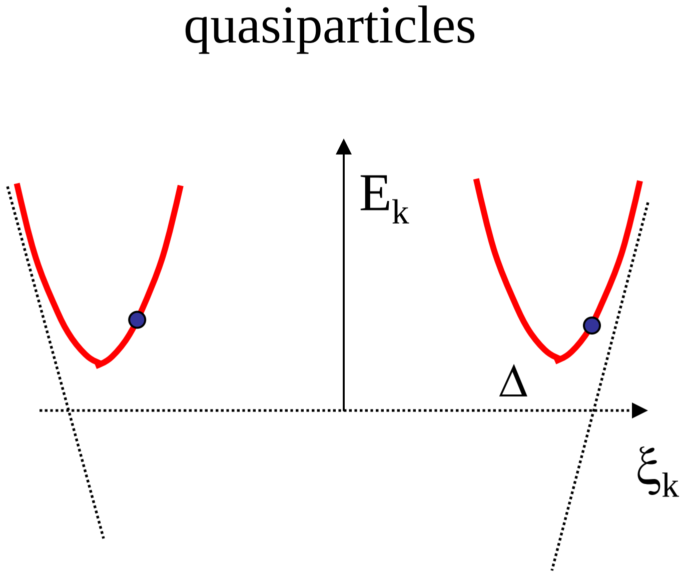


κ_{xx} vs. T in 90-K YBCO
(twinned and untwinned)

Is peak from QP or phonons?

Excitations of an *s*-wave superconductor





Energy cost $E_k = [\ \xi_k{}^2 + \Delta^2\ ]^{1/2}$

$$\gamma^\dagger_k = u_k\, c^\dagger_{k\uparrow} + v_k\, c_{-k\downarrow}$$

QP's cost energy, but *increase* entropy S (lower free energy F)

Heat transport in low-T_c superconductors

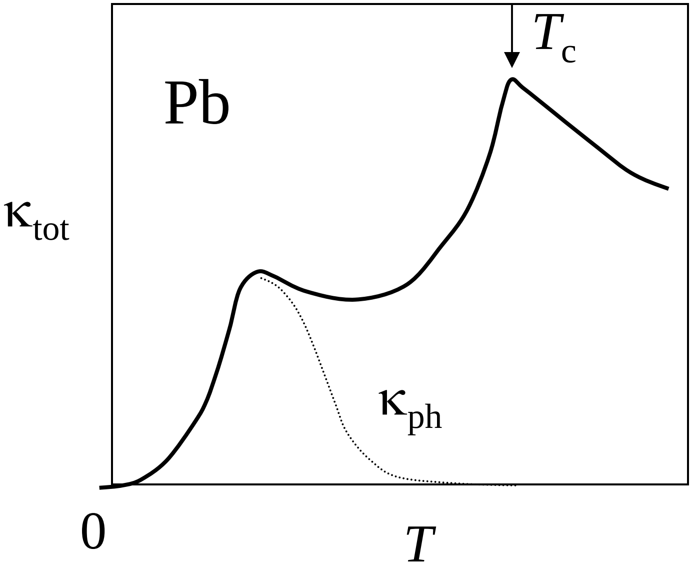
$\mathbf{J}_Q = \kappa_{tot} (- \nabla T)$

Heat-current density

$\kappa_{tot} = \kappa_{el} + \kappa_{ph}$

electrons phonons

Condensate does not carry heat (zero entropy)



Jericho, Odoni, Ott, PRB ‘85

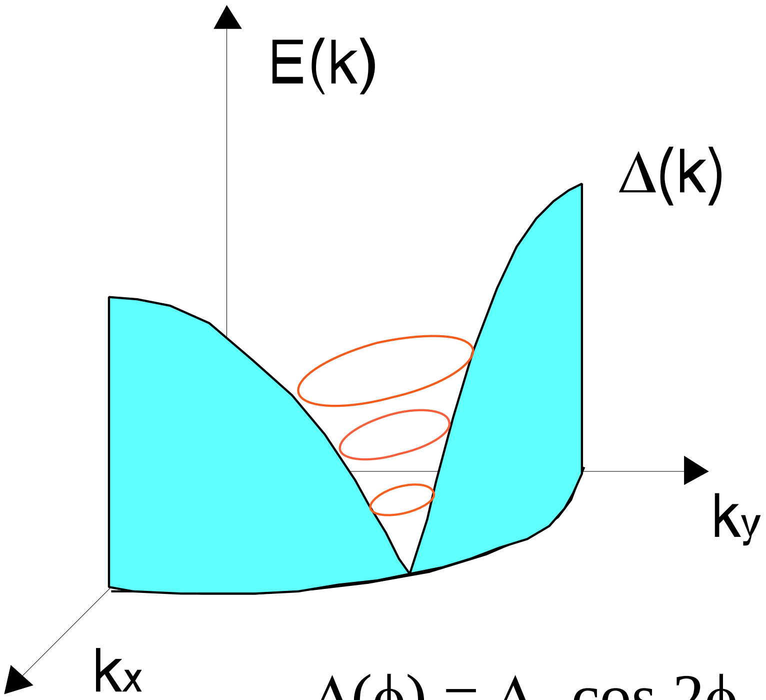
$T > T_c$, κ_{tot} mostly electronic

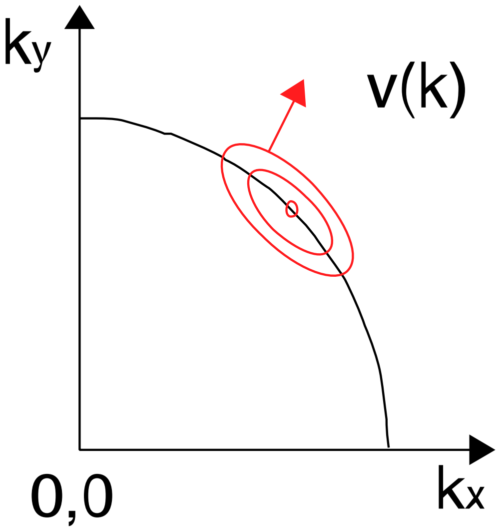
Below T_c, QPs carry heat

$T << T_c$, QP population → 0

Phonons are long-lived

Dirac-like spectrum of QP at nodes

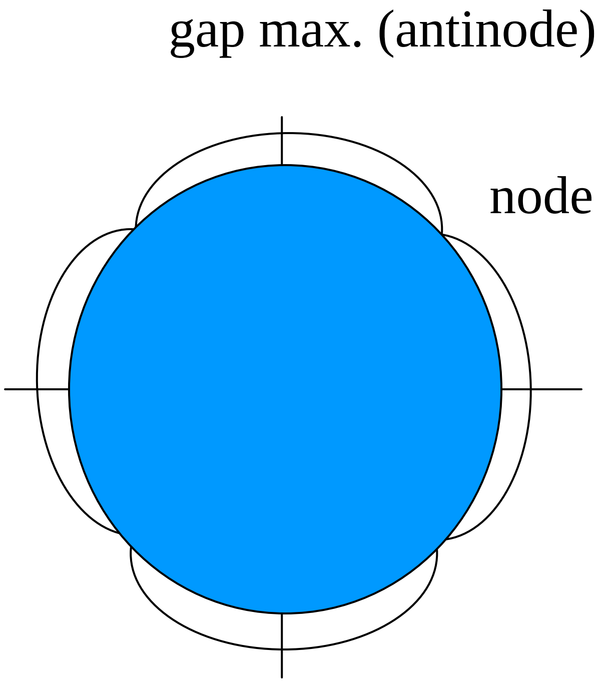




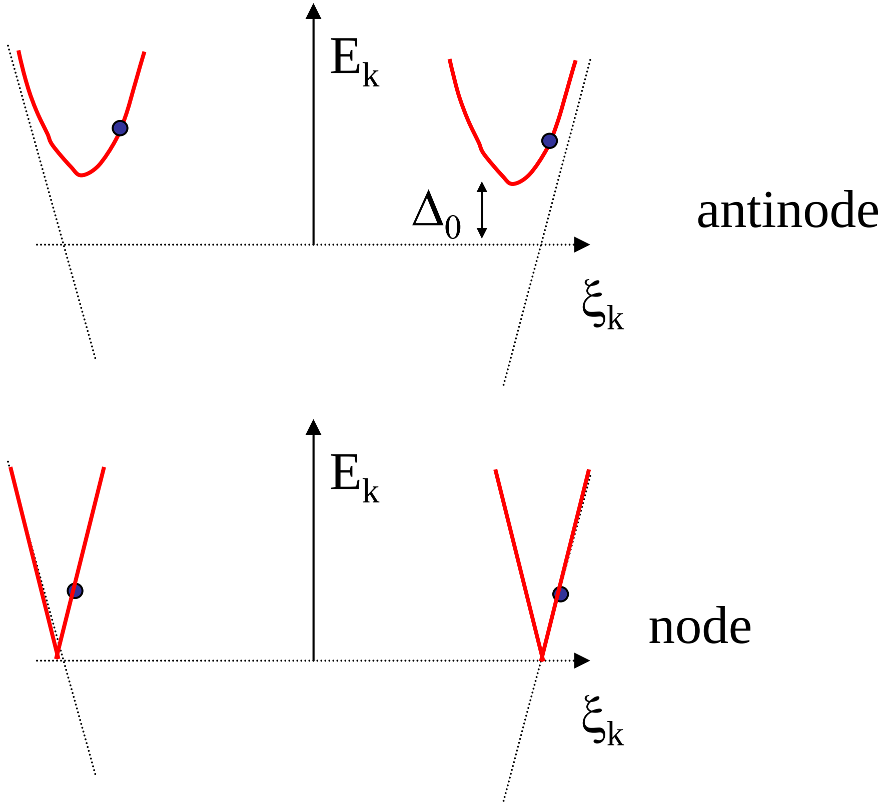
$$\Delta(\phi) = \Delta_0 \cos 2\phi$$

Quasiparticle dispersion *E* vs. k is linear.

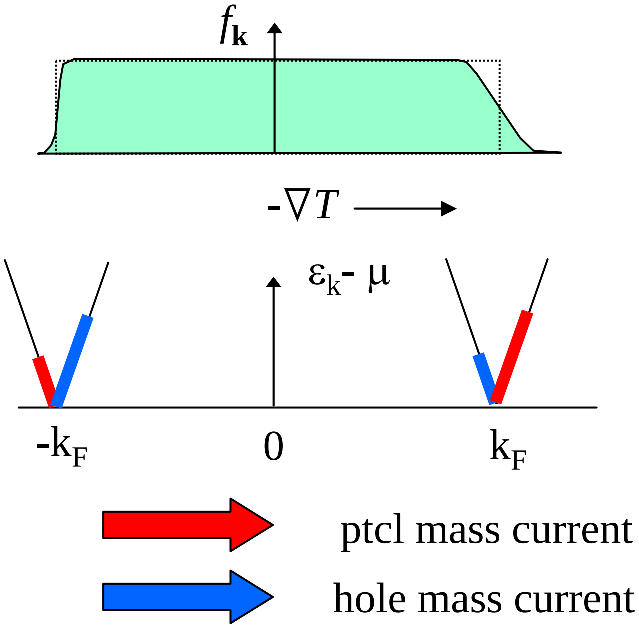
Quasiparticles in a *d*-wave superconductor

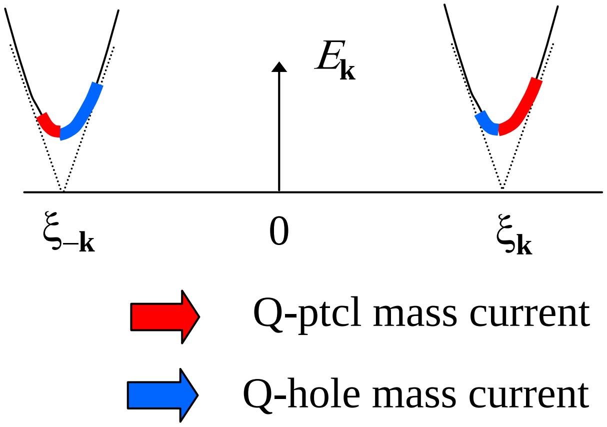


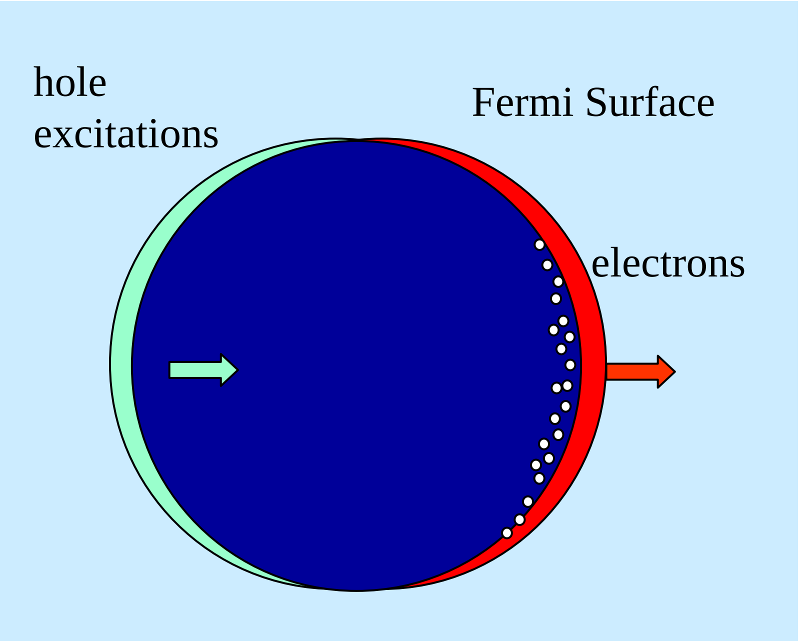
$$\Delta(\phi) = \Delta_0 \cos 2\phi$$

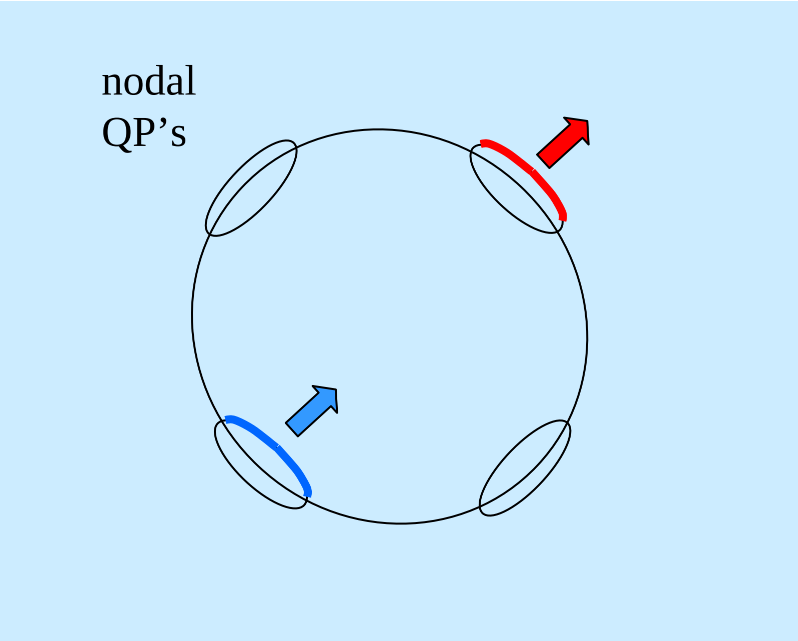


Heat current in normal and superconducting states



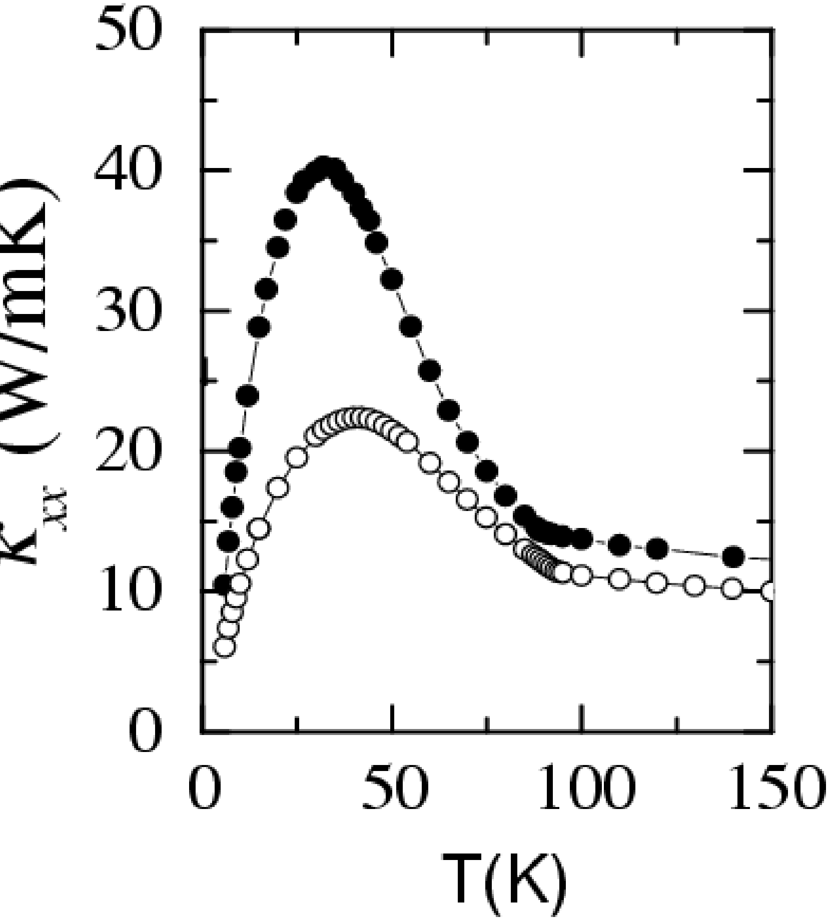


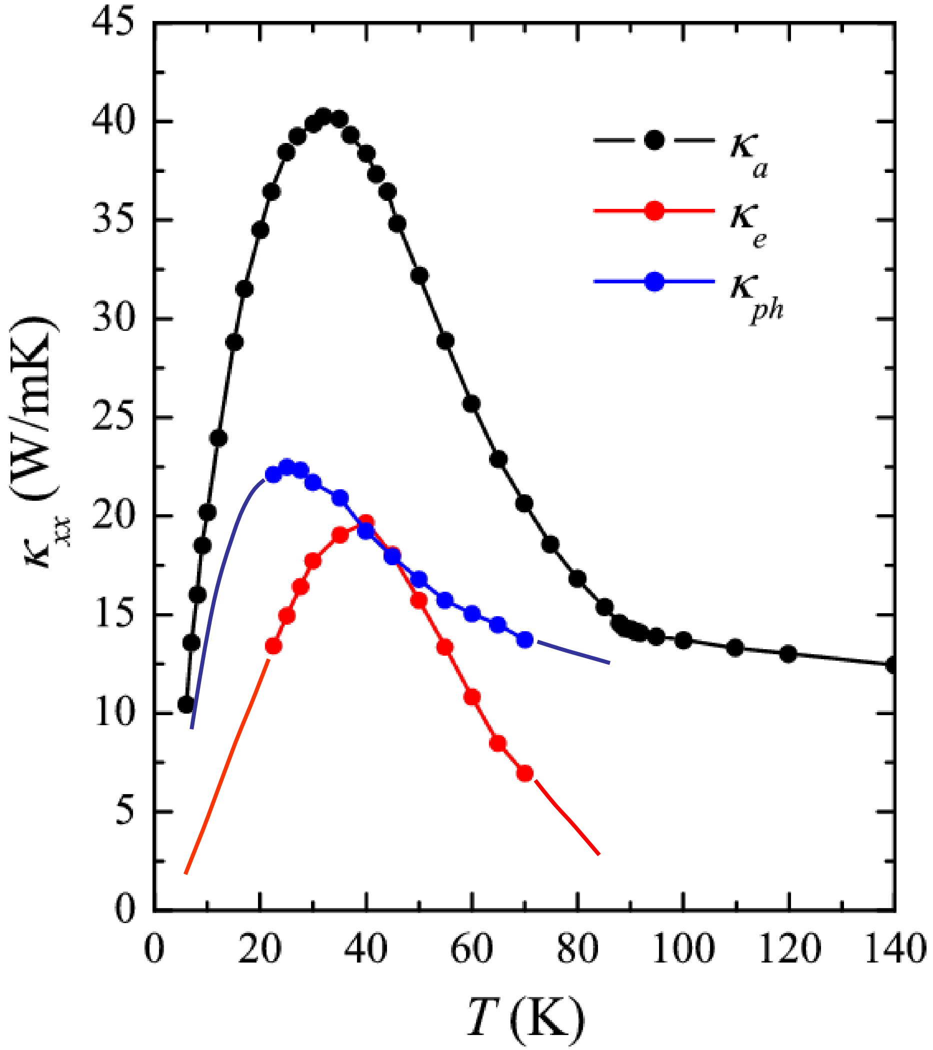




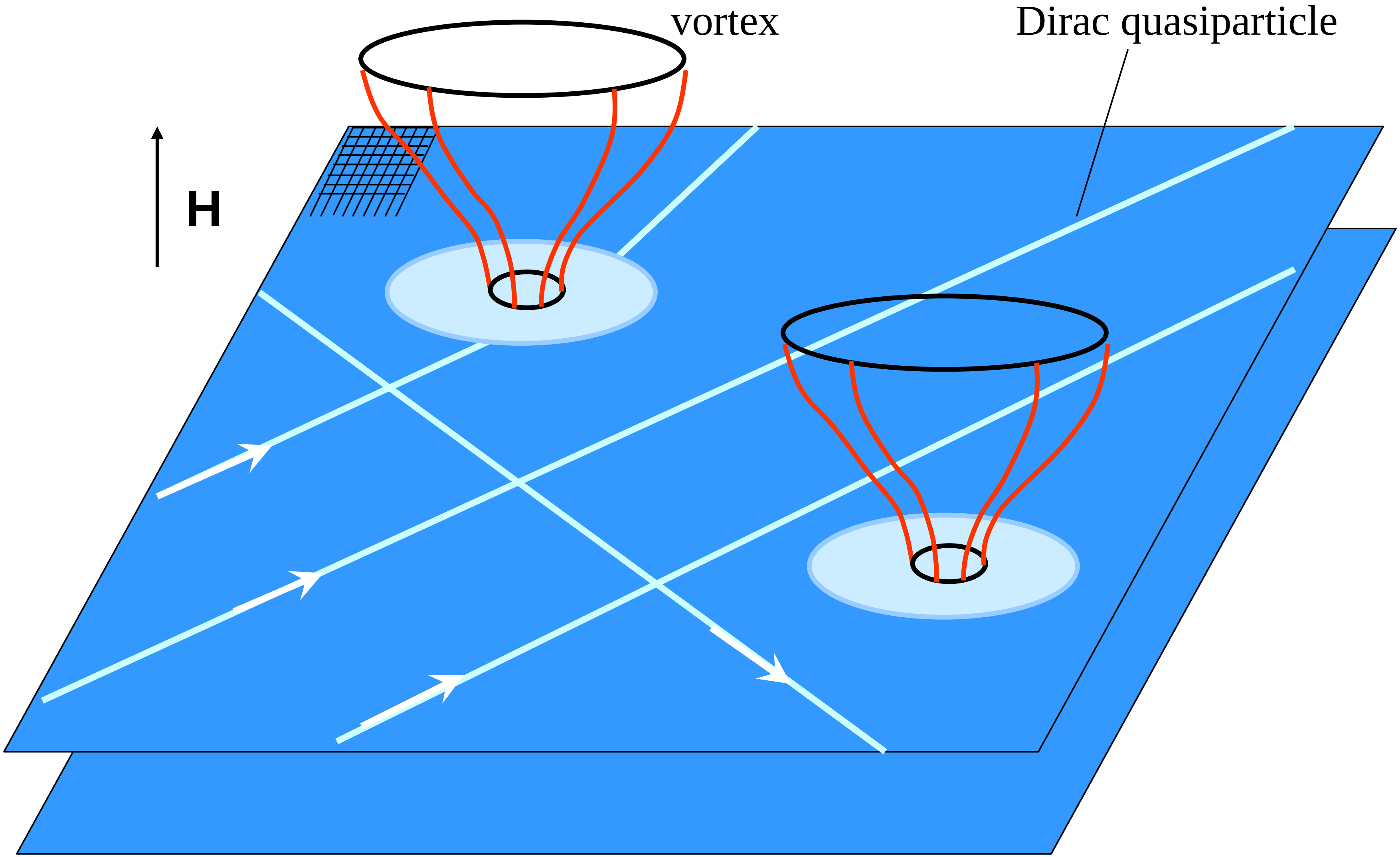
Separating electronic and phonon κ in 93-K $YBa_2Cu_3O_7$

$$\kappa_a = \kappa_e + \kappa_{ph}$$

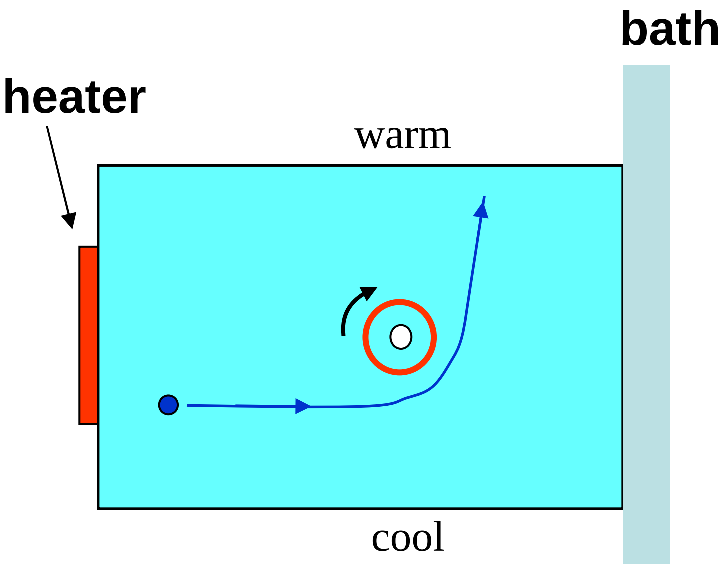




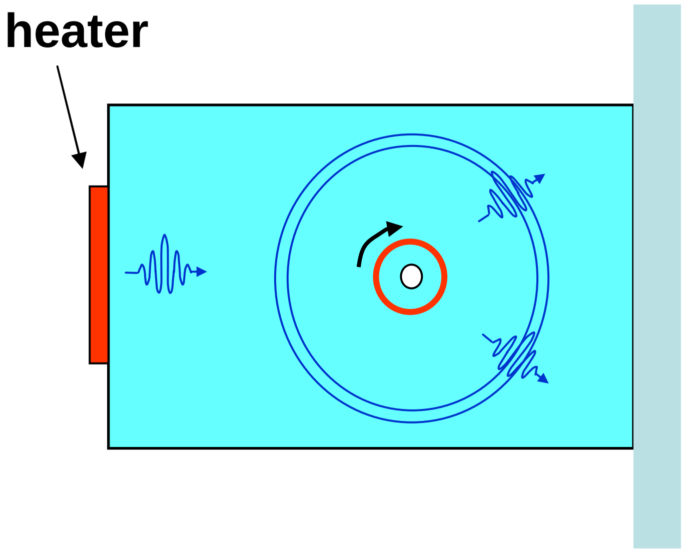
Quasiparticles in the CuO_2 plane



Hall thermal current from asymmetric scattering of QP by vortex



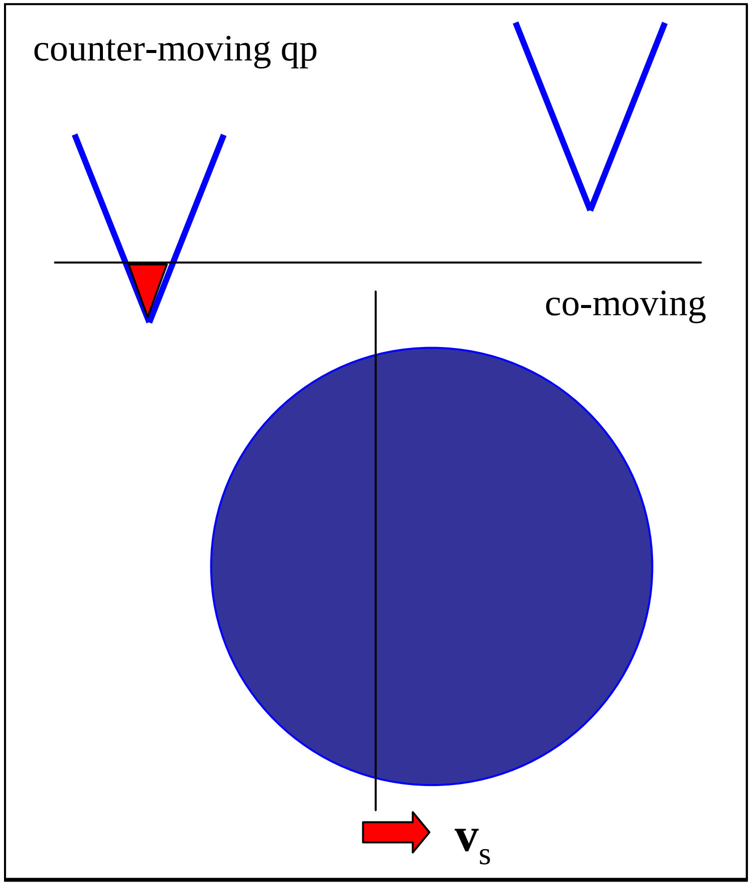
asymmetric scattering of QP by vortex



scattering of phonons: no asymmetry

Doppler shift

QP excitations

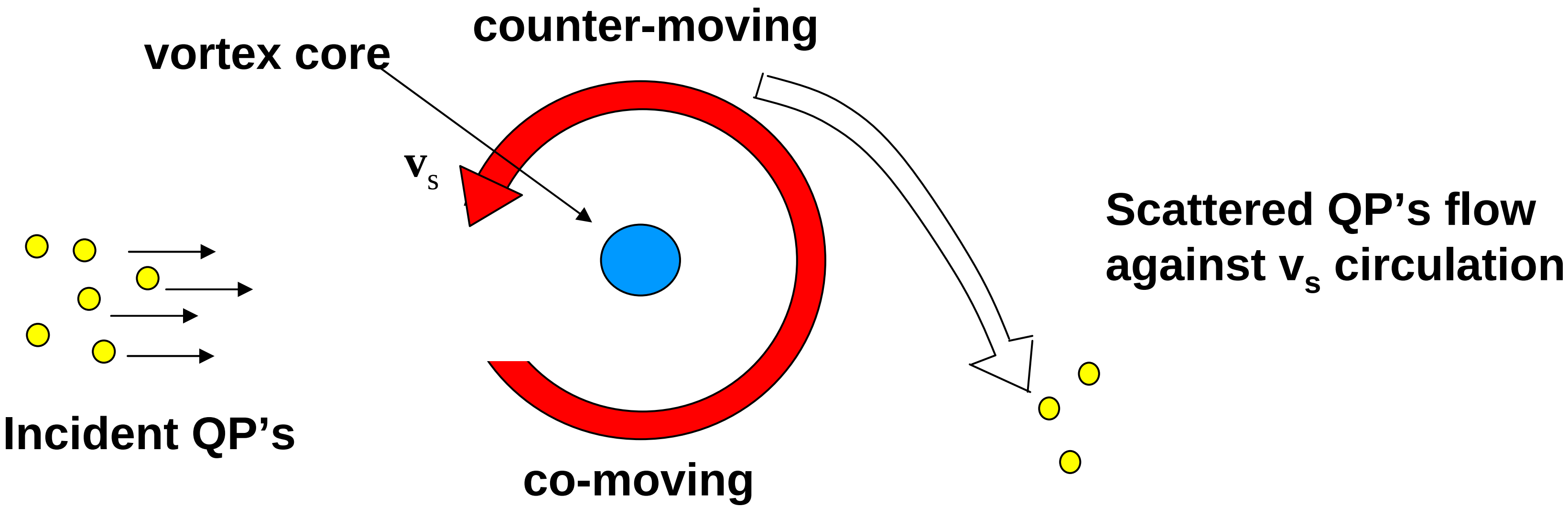


$\mathbf{J} = \mathbf{J}_s + \mathbf{J}_{qp} \qquad (\mathbf{J}_s \parallel -\mathbf{J}_{qp})$

In a supercurrent $\mathbf{v}_s$, energy of counter-moving QP's lowered.

Origin of QP Hall current

Doppler shift lowers energy of counter-moving QPs



vortex core

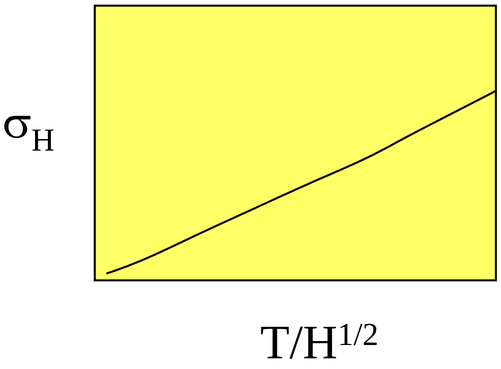
counter-moving

v_s

Incident QP's

co-moving

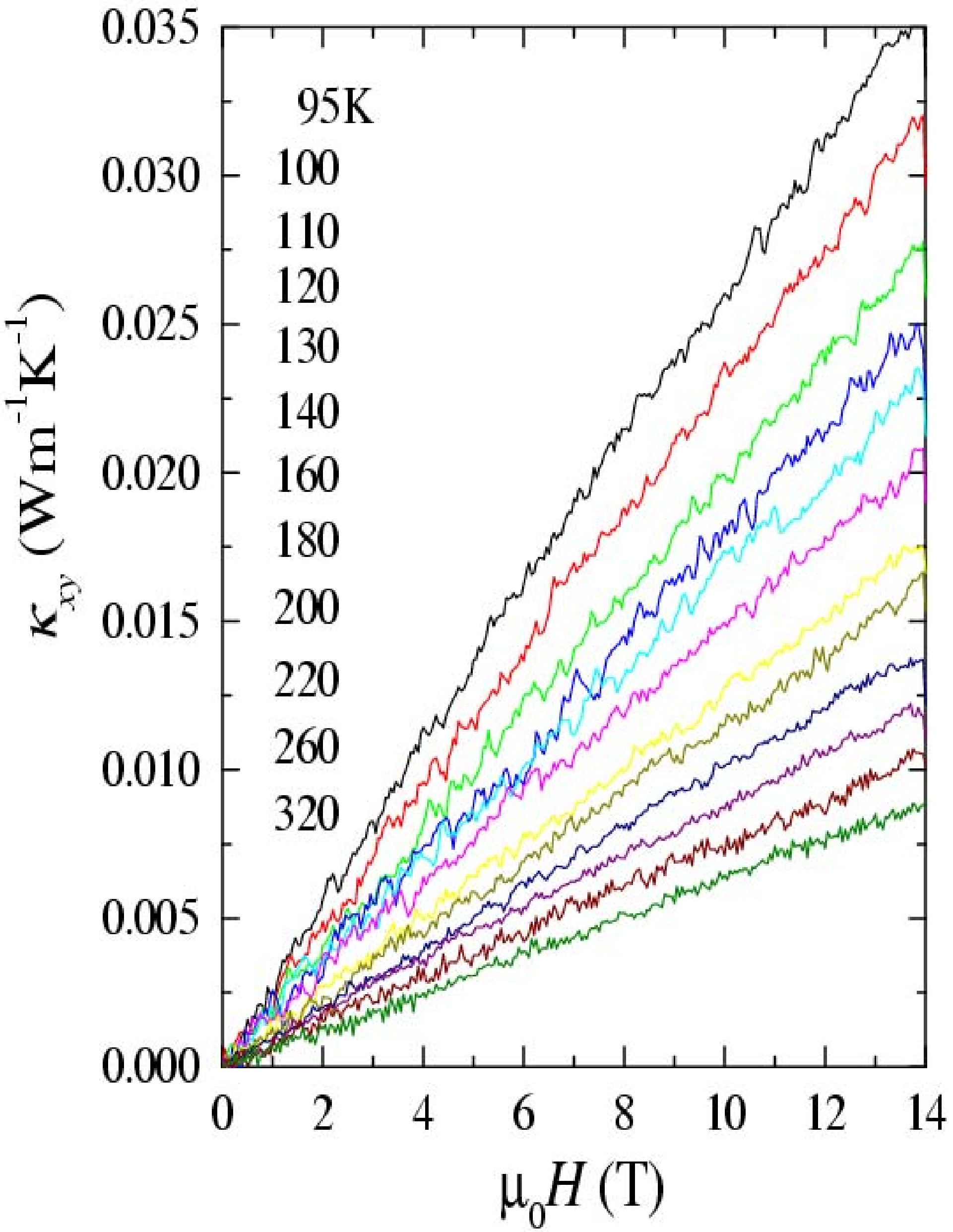
Scattered QP's flow against v_s circulation



Skew scattg cross-section (Durst,Vishwanath, Lee, PRL '03)

$$\sigma_H \sim T / H^{1/2}$$

Thermal Hall Conductivity κ_{xy} in high-purity $YBCO_7$ (normal state)

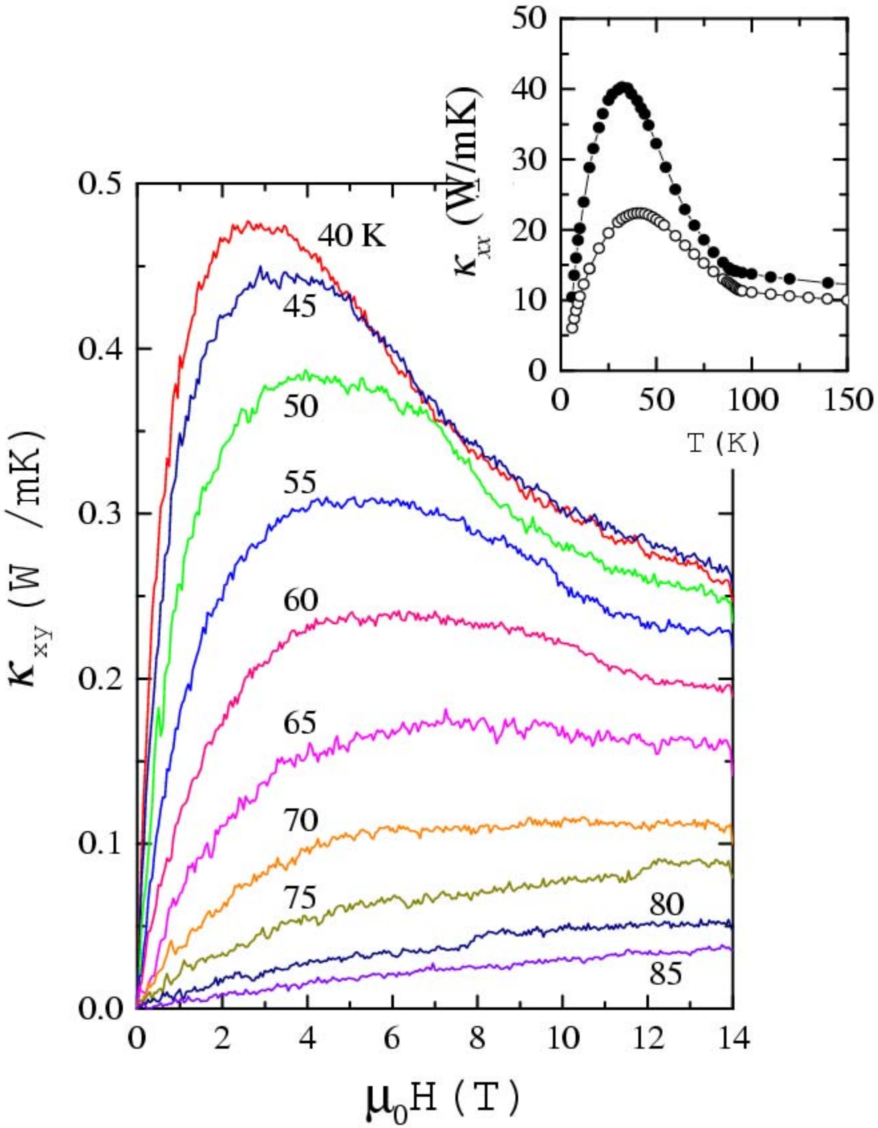


Thermal Hall Conductivity κ_{xy} In high-purity $YBCO_7$

1) Hall signal much larger below Tc

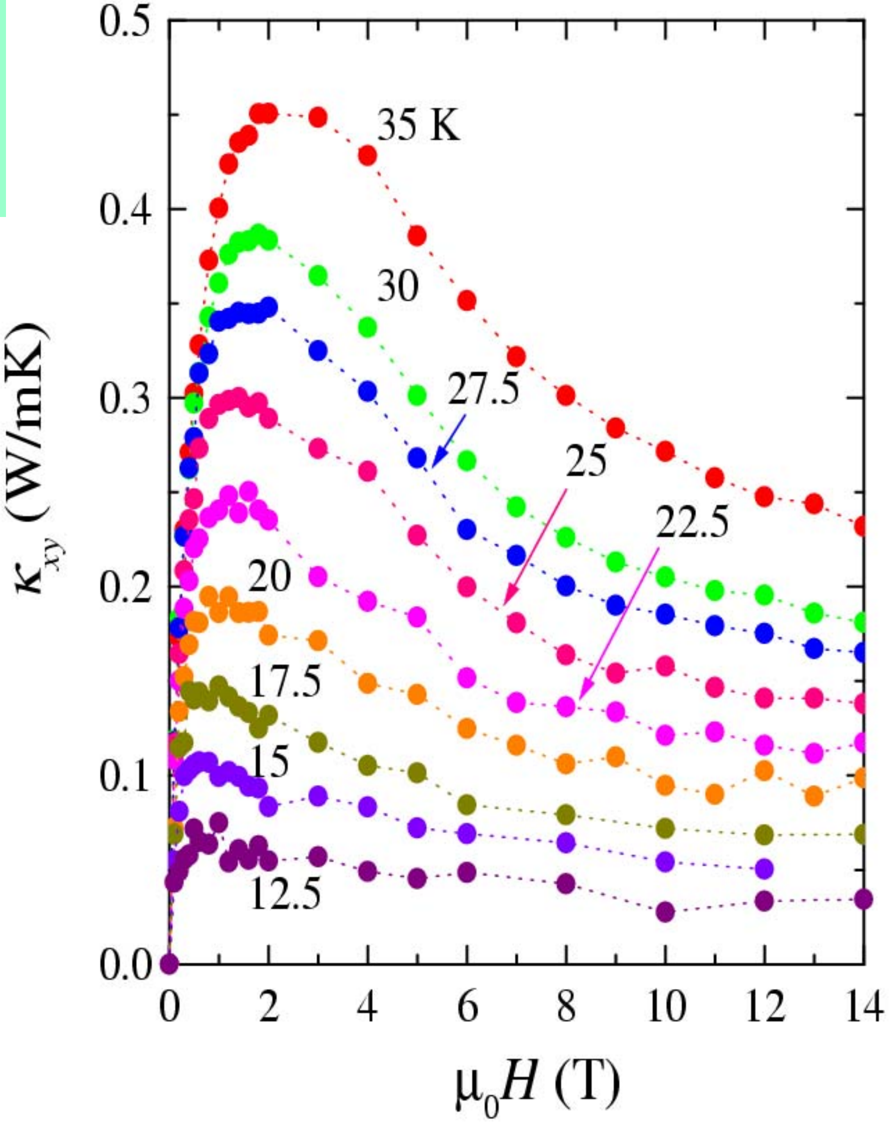
2) Giant increase in initial slope 85 to 40 K

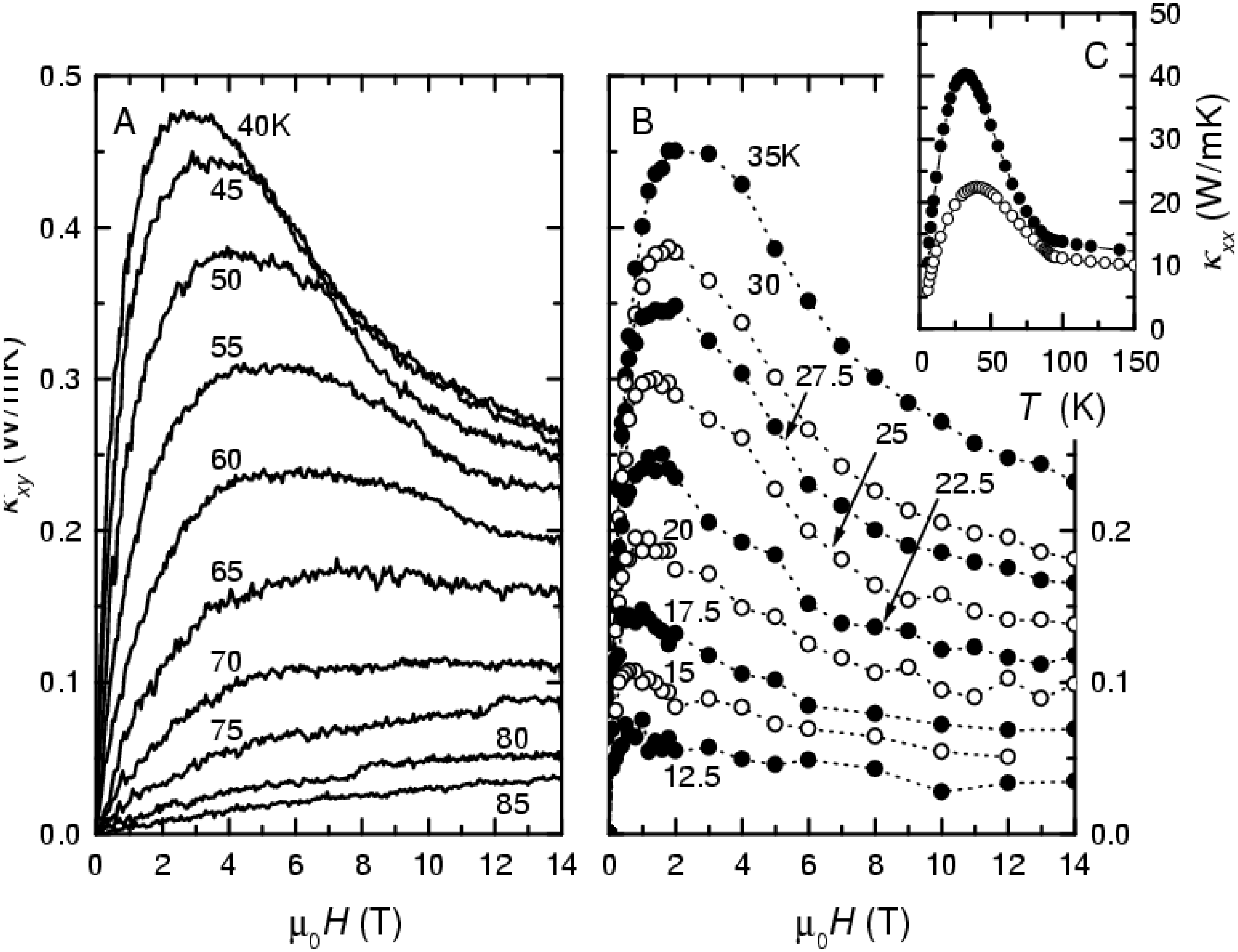
3) Strongly non-linear in H



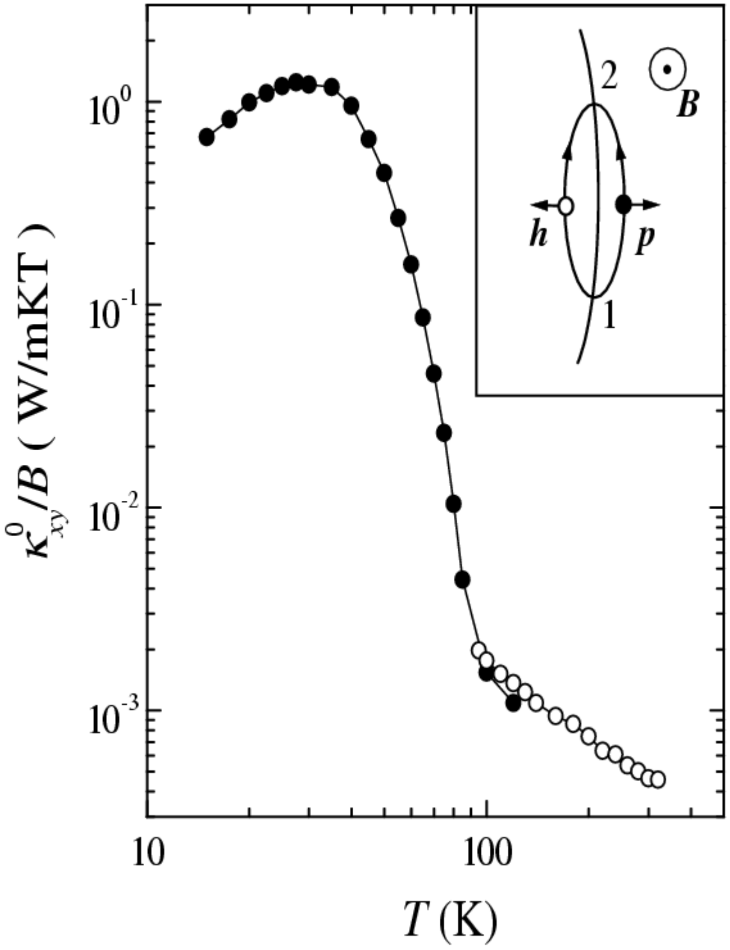
Thermal Hall Conductivity κ_{xy} in high-purity $YBCO_7$

(12.5 to 35 K)




A
B
C
40K
45
50
55
60
65
70
75
80
85
35K
30
27.5
25
22.5
20
17.5
15
12.5
κ_{xy} (W/mK)
$\mu_0 H$ (T)
$\mu_0 H$ (T)
κ_{xx} (W/mK)
T (K)
0.0
0.1
0.2
0.3
0.4
0.5
0
2
4
6
8
10
12
14
0
50
100
150
0
10
20
30
40
50

Plot initial slope $\lim_{B\text{-}0} \kappa_{xy}/B$ vs. T.



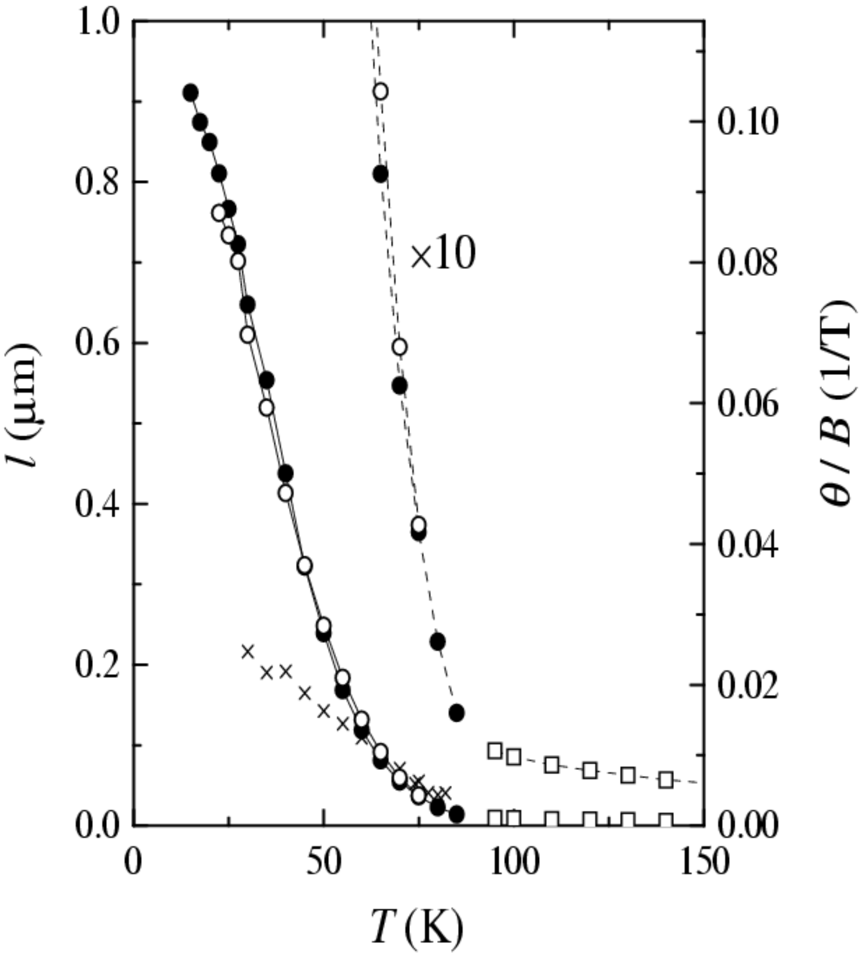
Initial slope increases by 1000 between 85 and 30 K

Steep increase in QP mean-free-path ~ 120

QP mean-free-path l
derived from Hall angle

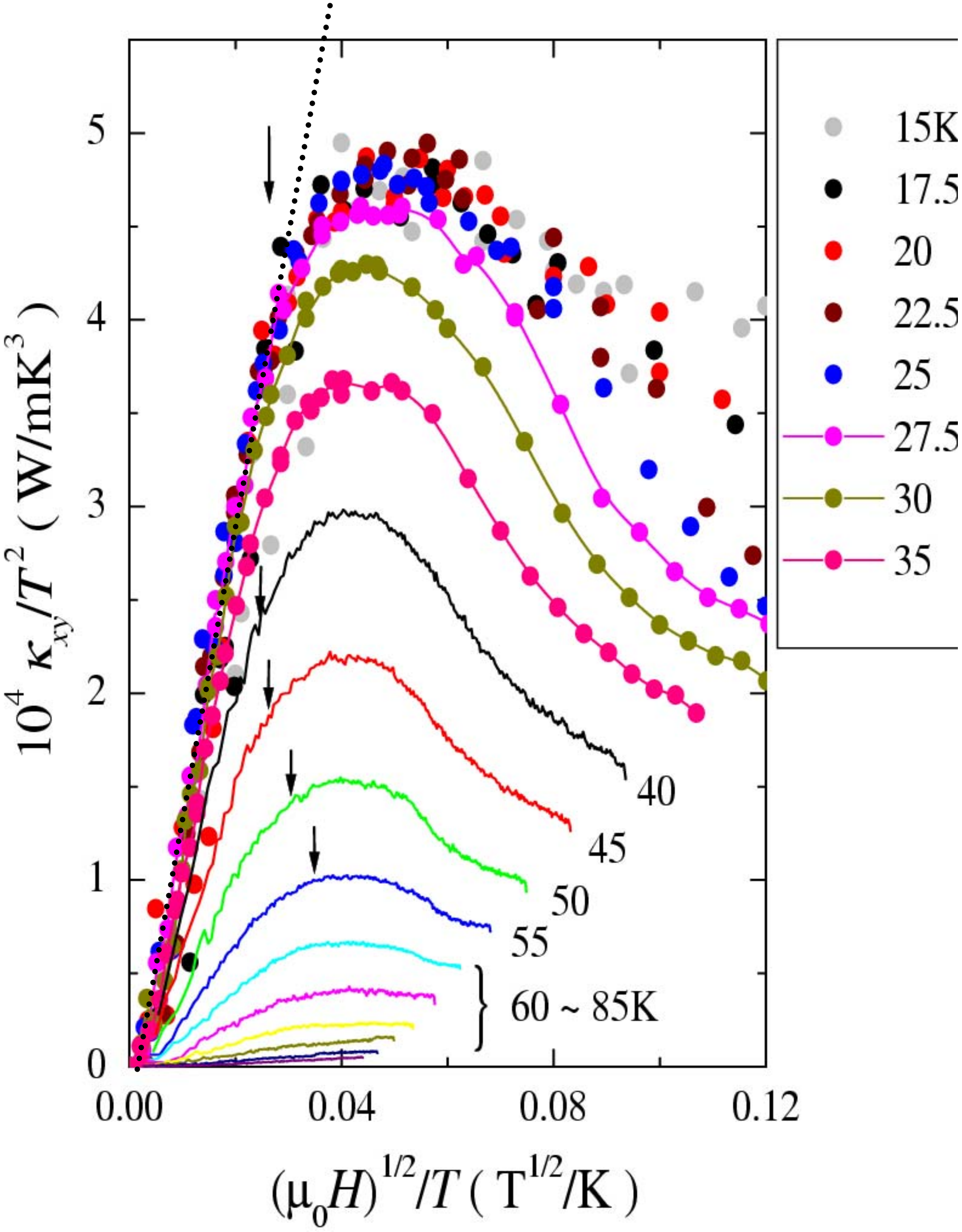
Increases by 120 from
85 to 30 K

Abrupt increase at Tc
(coherence effect?)



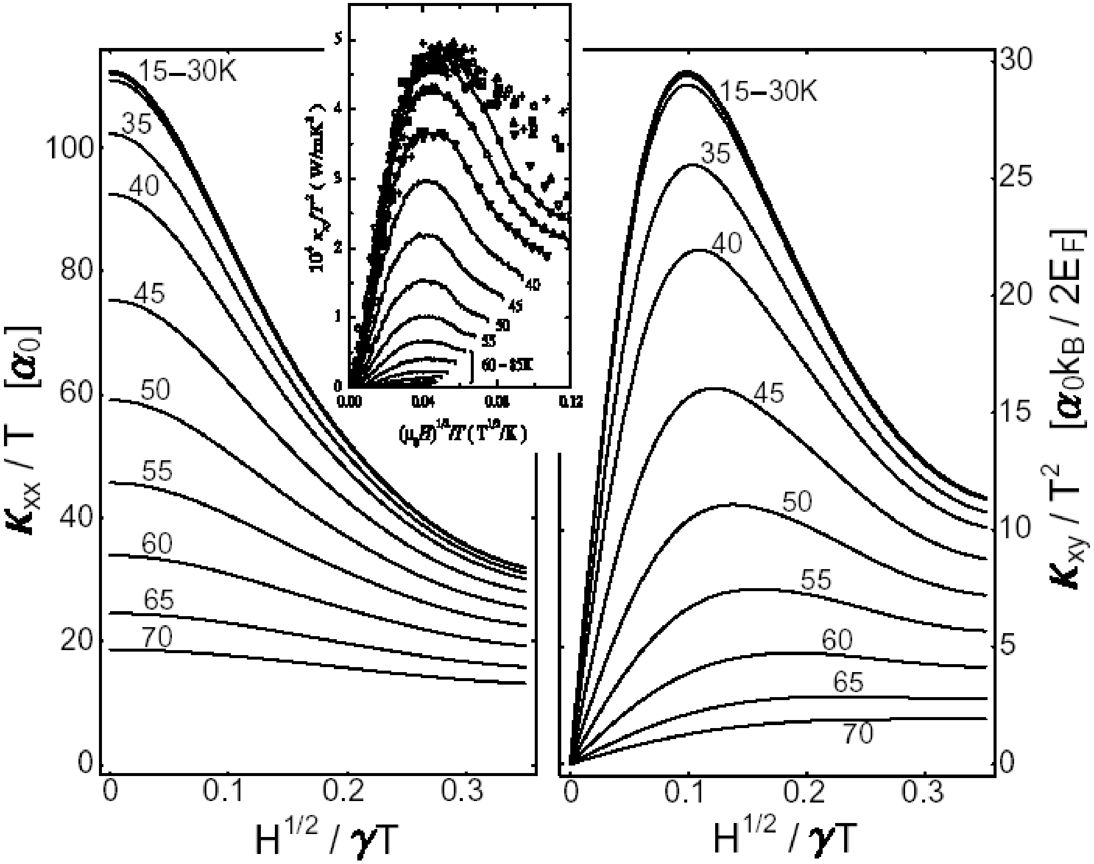
Simon-Lee scaling

$\kappa_{xy} = T^2 \, F(H^{1/2}/T)$



$\kappa_{xy} = C_0 \, (TH)^{1/2}$

$F(x) \sim x$



Calculated fits to K_{xx} and K_{xy}
(Adam Durst, Ashvin Vishwanath, P.A. Lee, 2003)

Durst, Vishwanath, Lee

$\kappa_{xx} \;=\; c_e v_F\, l \sim T^2 T^{-1}$

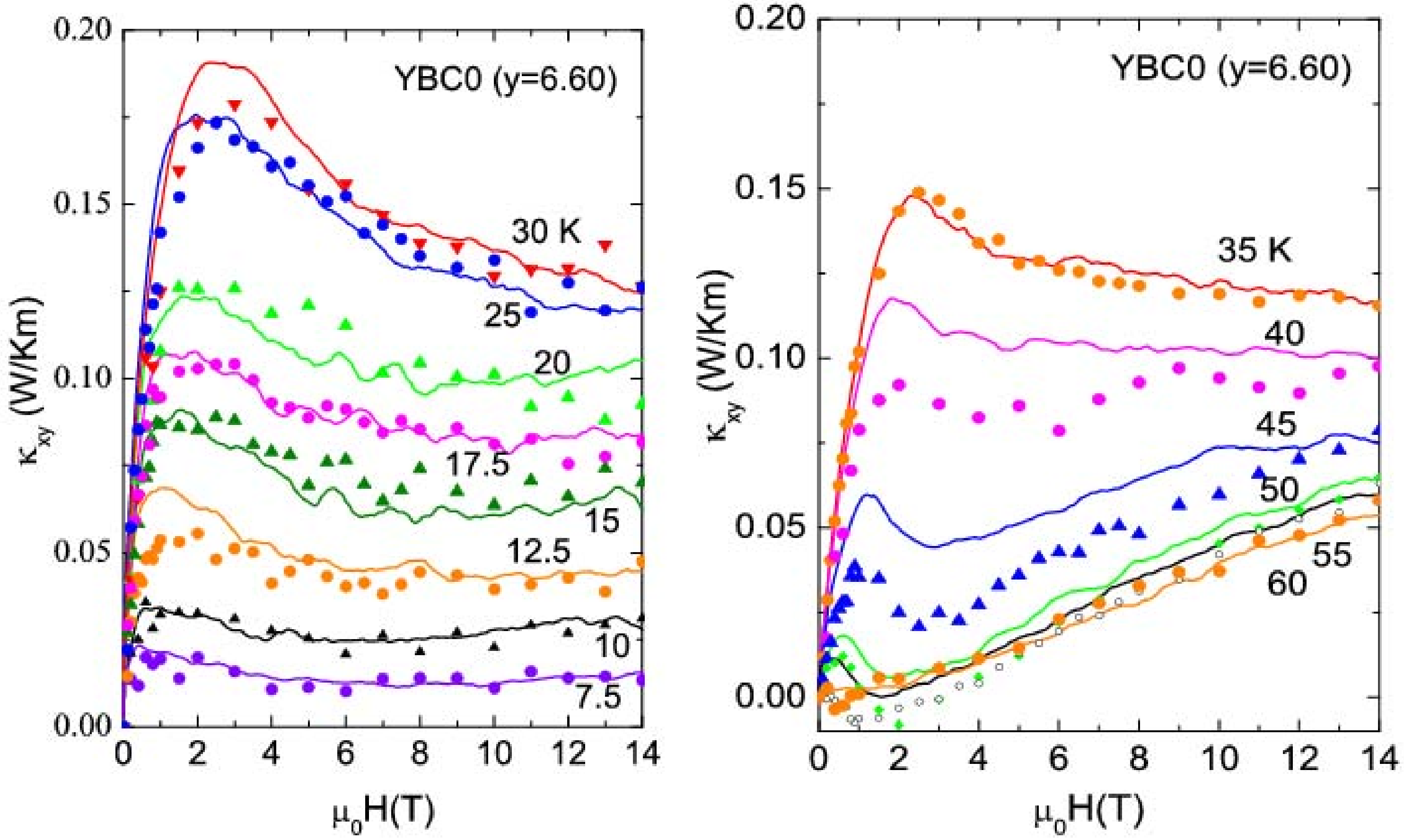
$\kappa_{xy} \;=\; \kappa_{xx} \tan\theta \;=\; \kappa_{xx}\, n_V\, \sigma_H\, l$

$\sim\; T^2 T^{-1}\,.\,H\,.TH^{-1/2}.\,T^{-1} \;\sim\; (TH)^{1/2}$

Explains observation

$\kappa_{xy} = C_0\,(TH)^{1/2}$

Quasi-particles are *hole* like in UD YBCO y=6.60



Summary

Below *T_c*, we observe

- 1000-fold increase in κ_{xy} (weak field)
- 200-fold increase in *QP* mfp *l*.
  (80 Angstrom to 2 microns)
- Giant anomaly in κ_{tot} is entirely from QP.
- *Steep* increase in mfp starts just below *T_c*
  (conflicts with ARPES)
- Intriguing scaling behavior in κ_{xy} (Simon-Lee)
- No evidence (yet) for Landau quantization

References for Lect. 1 (website http://www.princeton.edu/~npo/)

1. N. P. Ong, Phys. Rev. B 43, 193 (1991).
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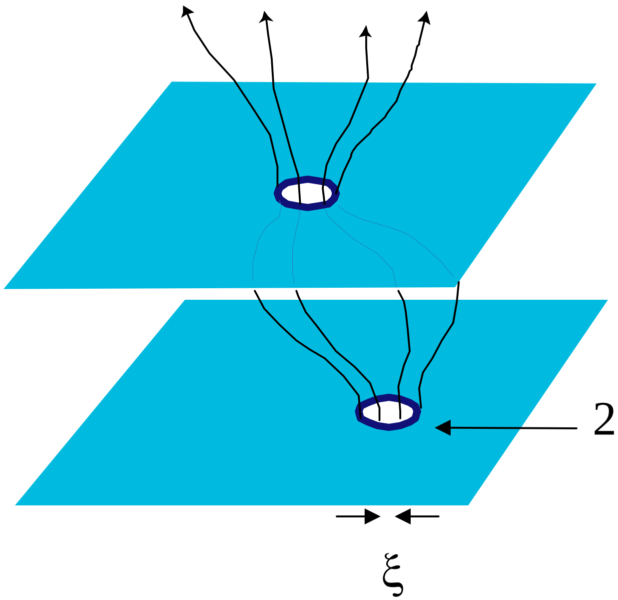
5. K. Krishana, J. M. Harris, and N. P. Ong, Phys. Rev. Lett. 75, 3529 (1995).
6. Y. Zhang, N.P. Ong, Z.A. Xu, K. Krishana, R. Gagnon, and L. Taillefer, Phys. Rev. Lett. 84, 2219 (2000).
7. Y. Zhang, N.P. Ong, P. W. Anderson, D. A. Bonn, R. X. Liang, and W. N. Hardy, Phys. Rev. Lett. 86, 890 (2001).
8. Adam C. Durst, Ashvin Vishwanath, and Patrick A. Lee, Phys. Rev. Lett. 90, 187002 (2003).

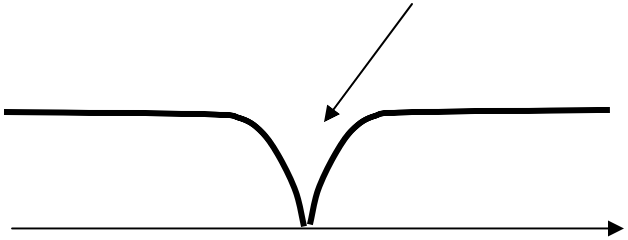
Vortices in cuprates



CuO_2 planes

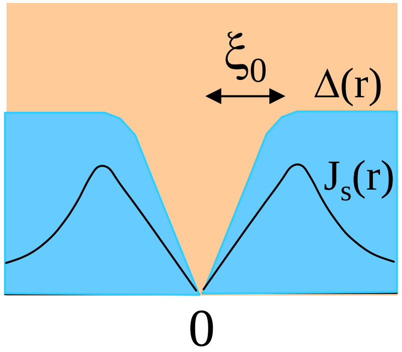
2D vortex pancake

Gap amplitude vanishes in core

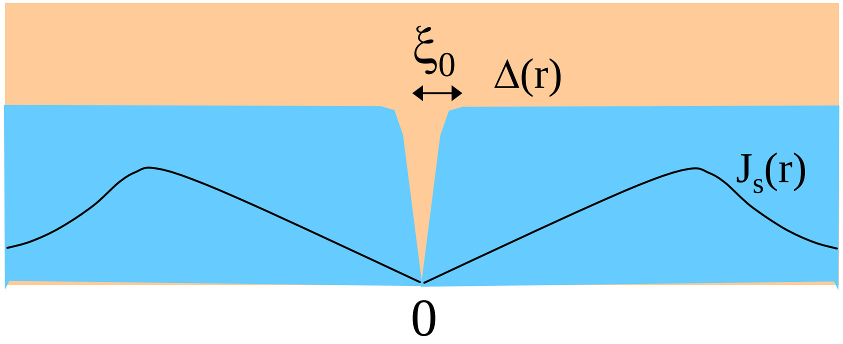


$|\Psi| = \Delta$

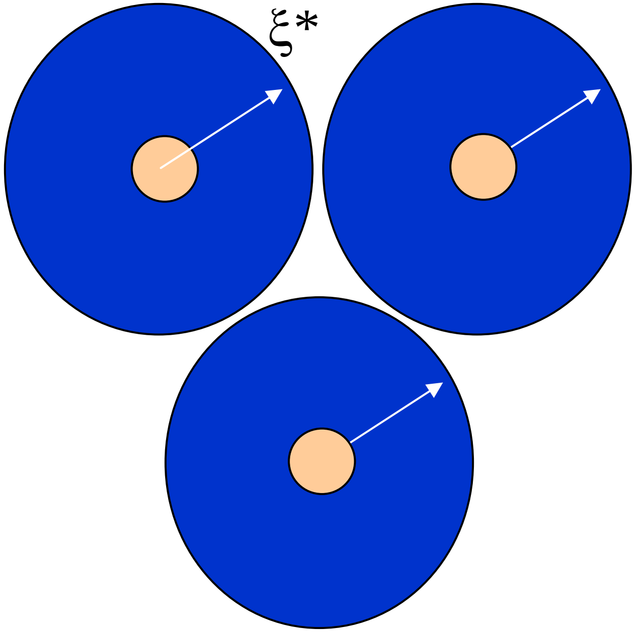
A new length scale ξ*



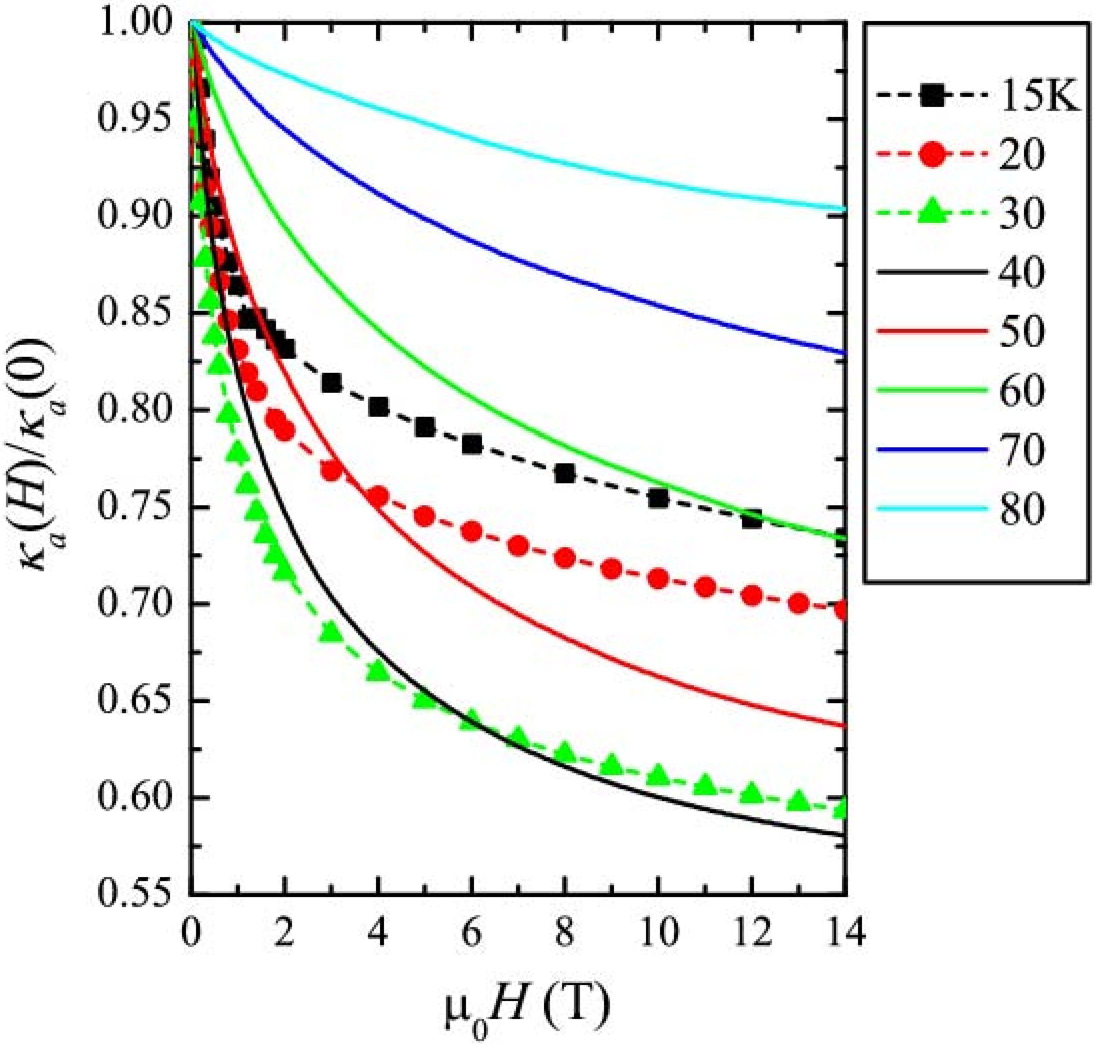
Cheap, fast vortices



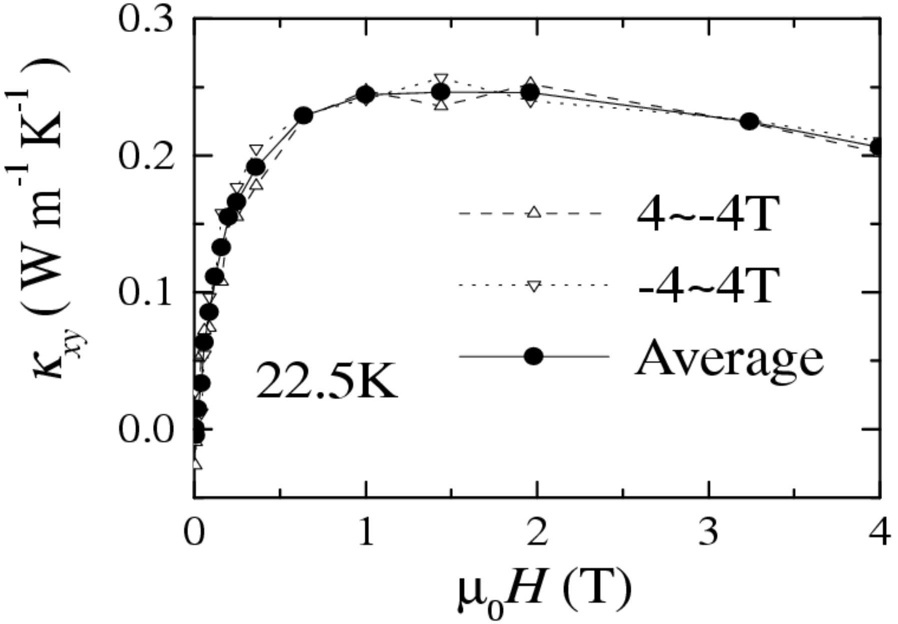
$$H^* = \frac{\phi_0}{2\pi\xi^{*2}}$$



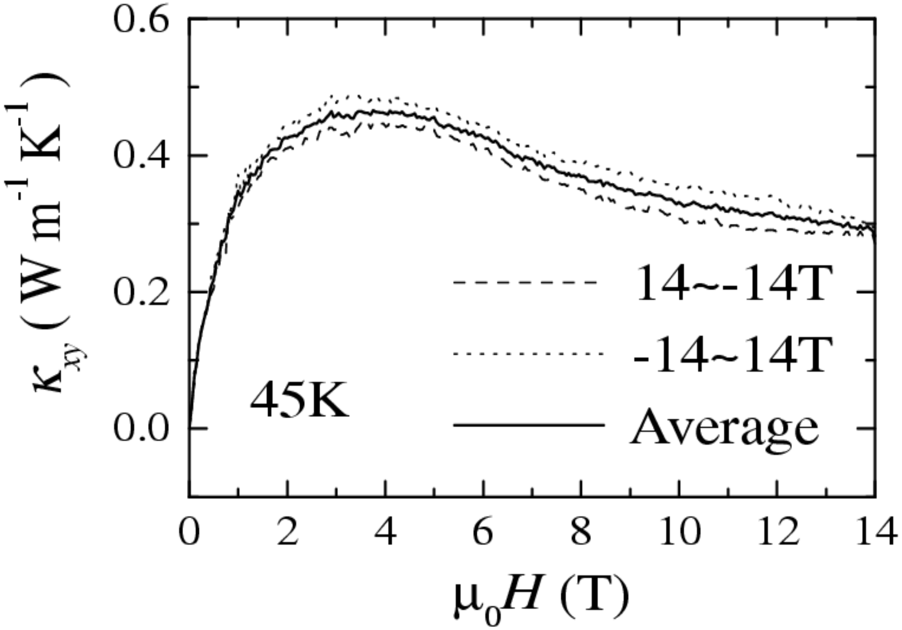
Is *H** determined by
close-packing of fat vortices?



$\kappa_a(H)/\kappa_a(0)$
1.00
0.95
0.90
0.85
0.80
0.75
0.70
0.65
0.60
0.55
0
2
4
6
8
10
12
14
$\mu_0 H$ (T)
15K
20
30
40
50
60
70
80



0.3
0.2
0.1
0.0
κ_{xy} (W m^{-1} K^{-1})
0
1
2
3
4
$\mu_0 H$ (T)
22.5K
4~-4T
-4~4T
Average



0.6
0.4
0.2
0.0
κ_{xy} (W m^{-1} K^{-1})
0
2
4
6
8
10
12
14
$\mu_0 H$ (T)
45K
14~-14T
-14~14T
Average