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Chetan Nayak - Lecture 2

Outline:

- More motivation, QC.
- Non Abelian statistics

From last time; $H = J_1 \sum A_i - J_2 \sum B_p$

"Dichotomy" of "soluble" v.s. "realistic" models
Aim/need to connect soluble & realistic models.

Effective Hamiltonians.

eg, Hubbard model, $H = -t\sum_{\langle ij\rangle} c^{\dagger}_{i\sigma} c_{j\sigma} + U\sum_i n_{i\uparrow} n_{i\downarrow}$

For $U \to \infty$, $n_i = 1$.

$$\begin{array}{cccc} \uparrow & \downarrow & \uparrow & \uparrow \\ \downarrow & \uparrow & \uparrow & \downarrow \\ \downarrow & \downarrow & \downarrow & \uparrow \end{array}$$

One electron per site, random spins.

But, for finite t, virtual hopping causes splitting of singlet/triplet state, $\Delta E_{s,t} = t^2/U$

For energy scales much less than U.

$$H = +J\sum_{\langle ij\rangle} \underset{\sim}{S}_i \cdot \underset{\sim}{S}_j \quad ; \quad J = t^2/U$$

effective model contains nearest neighbour (& to order t^4/U^3, ring exchange processes, analogy to Plaquette.

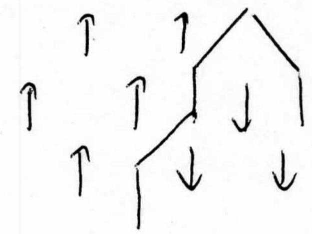
For square lattice problem; $B_p = \sigma_x \sigma_x \sigma_x \sigma_x$

centering $= \sigma_+ \sigma_- \sigma_+ \sigma_- + \dots$

(Terms can arise in effective Hamiltonians that may not seem realistic.) ring exchange.

Fluctuating loops in other systems:

Domain walls



However, cannot have termination, so dynamics differ from previous case.

Dimer models

— dimer

---- reference configuration

Aim for spin singlet pairs, but strong onsite repulsion, so ground state is filled with dimers

To form loops, compare to reference pattern.
Dynamics of dimers leads to dynamics of loops,

If $H = -J \sum \left(| = X / / | + \text{equivalent pictures (by symmetry)} \right)$

$+ V \sum \left(| = X = | + \dots \right)$

For $V \lesssim J$, equivalent phase to previous model.

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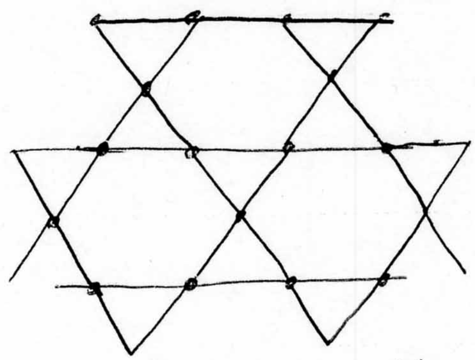
For lattice of links $\Rightarrow$ sites:



Hard core bosons living on vertices. (or possibly spin $\frac{1}{2}$ particles).

More motivation – possible relation to Quantum Computing.

Consider spin system $|s_1, s_2, \ldots, s_n\rangle$

$$|\uparrow\rangle = 0, \quad |\downarrow\rangle = 1.$$

Evolve $U|s_1, s_2, \ldots s_n\rangle$, & measure some spin(s) to answer some question.

At a more abstract level:

- Hilbert space of states with similar energy, $\mathcal{H}$
- Control of initial state.
- Apply any unitary transform
- Read out state

Traditionally $\mathcal{H} = \bigotimes_{i=1,\ldots N} \mathcal{H}_i$ single particle (qubit) Hilbert spaces.

Problem: Want to create superposition of inputs, but such states are delicate. Need to isolate from source of errors & to be able to correct errors without measuring state.

Solution: Use redundancy (as is also true for classical communication & computation).

e.g. $|0\rangle, |1\rangle \Rightarrow |000\rangle$ & $|111\rangle$

three "spins" per qubit.

Can check for errors with

$$P = |100\rangle\langle 100| + |011\rangle\langle 011|$$

So, if initial state $\alpha|000\rangle + \beta|111\rangle$

$$\Rightarrow \alpha|100\rangle + \beta|011\rangle$$

Can diagnose & then correct with a $\pi/2$ pulse

To correct also for phase errors, diagnose in σ_x basis.

With 5 "qubits", reduce error rate $\varepsilon \to \varepsilon^2$ (2 spin flips).

Idea: Local degrees of freedom encode topology in a redundant way.

e.g. topological equivalence of doughnut v.s. coffee cup.

Regard change of shape as "error", but genus of object remains unchanged.

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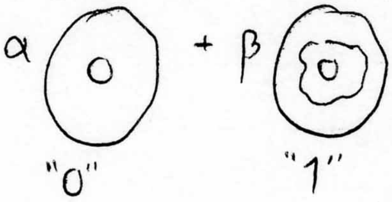
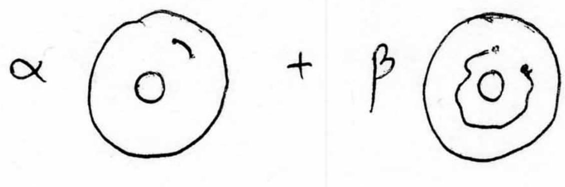
From last time, states



(analog with loop configurations)

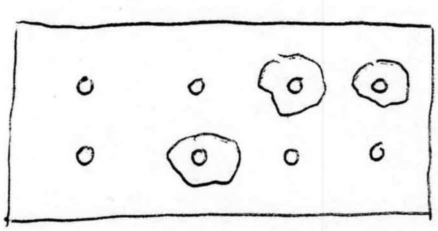
Consider a spin flip; Introduce excitation.



This is an excited state, but can radiate energy.

However, any local excitation cannot distinguish topological states, so global phase, but no phase difference between states.
[If we allow the end-points to move, there is a possibility for intermediate excitation to transform β to α]

Scaling up to multiple "qubits".



Need however to • Prepare initial state • Manipulate state • Read final state

No trivial way to overcome problems here ⇒ Non Abelian statistics.

Non Abelian Statistics:

Particles: $\quad \underset{x_1}{\times} \quad \underset{x_2}{\times} \quad \underset{x_3}{\times} \quad \underset{x_4}{\times} \quad \cdots \quad \underset{x_n}{\times}$

Consider degenerate ground states, $\psi_\alpha \quad \alpha = 1, \dots, g$.

Where previously, on braiding/exchange $\psi \to \psi e^{i\theta_{12}}$, since there was an unique ground state.

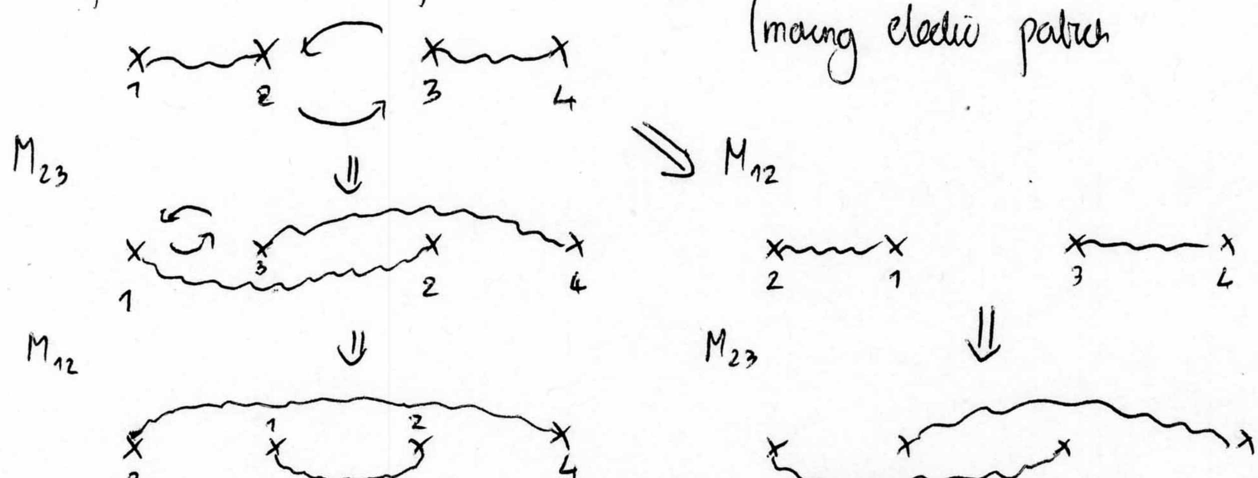
Now, $\psi_\alpha \to M^{(12)}_{\alpha\beta} \psi_\beta$ $\quad$ (ψ_α is many body wavefunction $\psi_\alpha(x_1, x_2, \dots, x_n)$.)

or $\quad \psi_\alpha \to M^{(23)}_{\alpha\beta} \psi_\beta \quad$ (swap 2 & 3)

In general, $[\underset{\approx}{M}^{12}, \underset{\approx}{M}^{23}] \neq 0$, thus non Abelian.

Example, "broken loop" excitation

(moving electric particle



In previous model, by "surgery" rules, all such states equivalent, but if non-equivalent, then order would matter.

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For many cases, such exchange matrices provide a way of building any unitary operation.

i.e, Can access entire degenerate manifold by exchanges.

Fusion: Consider various species of particles:

$a = 0, 1, 2, \ldots$

In previous case, 0 = vacuum
1 = electric
2 = magnetic
3 = electric & magnetic

Given two particles, 

Can consider as a single particle, but may be a superposition of species.

$$\left.\begin{array}{l} \text{e.g.} \quad \text{electric} + \text{electric} = \text{vacuum} \\ \qquad\ \ \text{electric} + \text{magnetic} = \text{electric \& magnetic} \end{array}\right\} \text{fusion}$$

In Non Abelian systems, fusion causes a superposition.

So, Hilbert space: $2n$-quasi particles at fixed positions, $x_1, \ldots, x_{2n}$ (of fixed type). Ψ_α, $\alpha = 1, \ldots, g$.

Initial state; create pcl/anti-pcl pairs from the vacuum, e.g. flip spin $\Rightarrow$ two electric particles.

(2n particles ⇒ n particles & n anti particles)

Unitary transformation:

Arbitrary braiding procedure, with no local interaction.

[An advantage for QC here is that operation is discreet, so operation is always accurate].

Read out:

Combine & annihilate/fuse particles.

For abelian model, can perform surgery & reduce to vacuum. In general, states are different, & can then be distinguished by testing whether the state is or is not the vacuum.