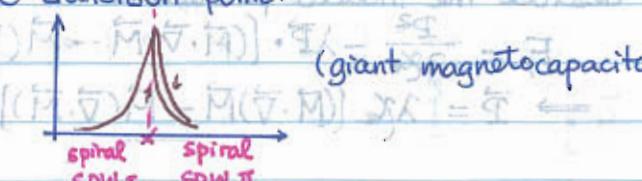


Multiferroics and Magnetoelectrics [Mostovoy]

- The Maxwell Eqns. has duality properties $\{E\} \leftrightarrow \{B\}$, $\{B\} \leftrightarrow \{-E\}$.
 - However, ferromagnets (FM) breaks time-reversal, while ferroelectric (FE) breaks inversion symmetry. e.g. BaTiO_3
 - From quantum chemistry, FM wants partial d-shell, FE wants empty d-shell $\text{Ti}^{+4} \overset{\delta}{\text{O}}^-$, $\text{Ti}^{+3} \overset{\delta}{\text{O}}^-$ \Rightarrow small overlap for multiferroics
 - First linear magnetoelectric effect: Cr_2O_3 .
 $P = \chi_e E + \alpha H$, $M = \alpha E + \chi_m H$.
 - Around the same time multiferroics is also observed.
 - In these material, electric polarization changes dramatically around magnetic transition point.
 - e.g. DyMnO_3

(giant magnetocapacitance)
 - Phenomenology of linear magnetoelectric effect
- $$P_i = \alpha_{ij} H_j \quad \Leftrightarrow \quad \Phi_{ME} = -\alpha_{ij} E_i H_j$$
- $$M_i = \alpha_{ji} E_j$$
- These material breaks time-reversal & space-inversion symmetry. But respects the combination of both.
- Practically, one just need to check that the observed symmetry of a material satisfies the desired symmetry to find magnetoelectric candidate.
 - Given symmetry, write down all possible terms respecting the symmetry to obtain a theory.

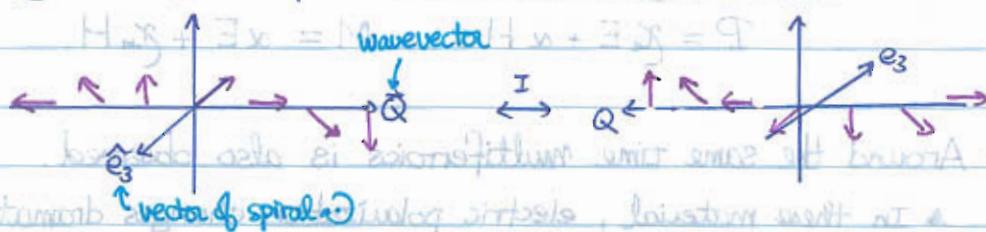
[unstable] ferroelectricity in 2D Mott insulator

- More theoretically, one can write down all the Ginzburg-Landau term consistent with symmetry both odd and even powers of M .

Ferroelectrics space-inversion breaking mechanism materials

proper	covalent bond	BaTiO ₃
	atom polarizability (geometric FE)	BiMnO ₃
improper	structural transition (electronic FE)	K ₂ SeO ₄
	charge ordering (magnetic FE)	LuFe ₂ O ₄
	magnetic ordering	CoCr ₂ O ₄

- Magnetic FE — spiral SDW breaks inversion



- To describe the induced polarization:

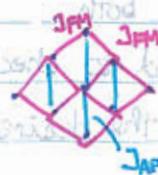
$$\begin{aligned} F_p &= \frac{P^2}{2\chi e} - \lambda \vec{P} \cdot [(\vec{M} \cdot \vec{\nabla}) \vec{M} - \vec{M} (\vec{\nabla} \cdot \vec{M})] \\ \rightarrow \vec{P} &= \lambda \chi_e [(\vec{M} \cdot \vec{\nabla}) \vec{M} - \vec{M} (\vec{\nabla} \cdot \vec{M})] \end{aligned}$$

- In comparison, sinusoidal SDW has no polarization; while for spiral SDW,

$$\begin{array}{ccc} \uparrow \vec{E} & & H_{\perp} \propto \vec{E} \\ \vec{e}_3 \leftarrow & \rightarrow Q & \Rightarrow \\ & & \vec{E} \propto \vec{M} \end{array}$$

- To get multiferroic, one way is to consider frustration (geometric or by further bonds).

e.g. RMnO_3 (not stable at low temperature)



[Section 2] (III) explain the spin-orbit coupling

- $T_{FE} < T_M$ because there is a sinusoidal-helicoidal transition

$$\Phi = \alpha_x M_x^2 + \alpha_y M_y^2 + \alpha_z M_z^2 + \frac{b}{2} M^4 + c \vec{M} \left(\frac{\partial^2}{\partial x^2} + Q^2 \right) \vec{M}$$

$$\Rightarrow \vec{M} = M_x \hat{x} \cos(Qx) \quad \rightarrow \quad \vec{M} = M_x \hat{x} \cos(Qx) + M_y \hat{y} \sin(Qx)$$

- Microscopically, Dzyaloshinskii-Moriya causes multiferroic effect

$$E_{DM} = \vec{D}_{12} \cdot (\vec{S}_1 \times \vec{S}_2)$$
$$\vec{D}_{12} \propto \lambda \vec{x} \times \vec{F}_{12}$$

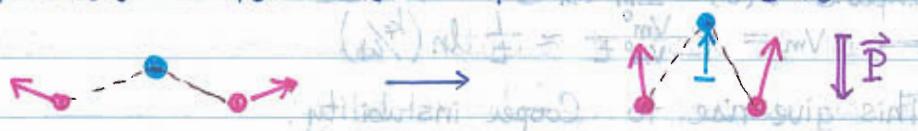
- DM is originated from 1st-order correction to super-exchange by spin-orbit coupling.

- Another microscopic mechanism is magnetostriction.



- A third mechanism is exchange striction

recall $\theta \sim 180^\circ \Rightarrow J > 0$ & $\theta \sim 90^\circ \Rightarrow J < 0$



- Another mechanism is higher order term in effective spin Hamiltonian from Hubbard model

- Finally, polarization can come from bi-hopping of e- on O²⁻

