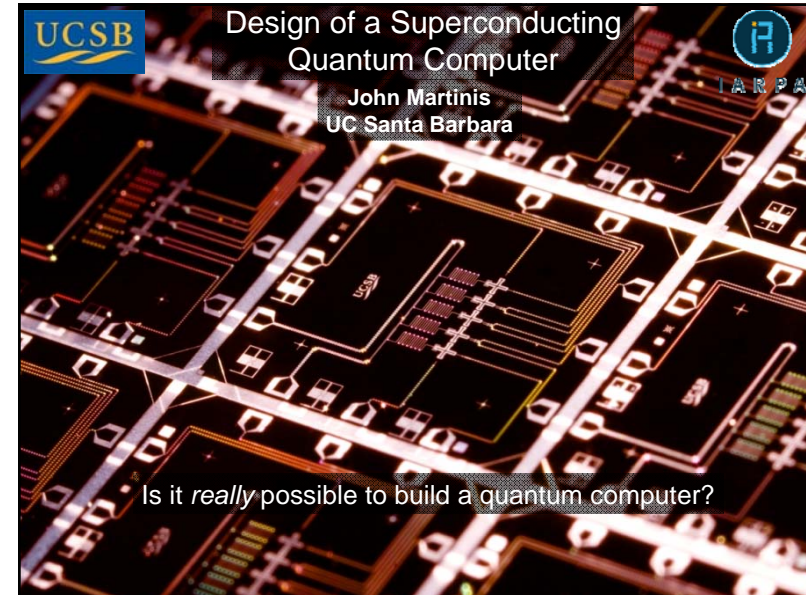


John Martinis : Suggested Reading

- 1) Overview of basic qubit physics
Implementing Qubits with Superconducting Integrated Circuits
Michel H. Devoret and John M. Martinis
 Quantum Information Processing vol. 3. (2004)
- 2) Recent review by Yale group
Superconducting Circuits for Quantum Information: An Outlook
M. H. Devoret and R. J. Schoelkopf
 Science **339**, 1169 (2013);
- 3) Architecture for a quantum computer
Surface codes: Towards practical large-scale quantum computation
Austin G. Fowler, Matteo Mariantoni, John M. Martinis, Andrew N. Cleland
 PRA **86**, 032324 (2012)



Outline

- 1) Exponential computing power
- 2) Hardware Requirements
- 3) Review of qubit physics: Bloch sphere
- 4) Need for fault-tolerant computation
- 5) Surface code theory
error-correction and architecture
- 6) Xmon superconducting qubits
integrated circuits for scaling
above fidelity threshold

Exponential Computation Power

- Classical computation power scales linearly:
 speed (GHz), size (RAM Mbytes), number processors

★ 1 GHz

★★ 2 GHz

★★★★ 2 GHz, Dual Processor

- Quantum computer scales exponentially!

qubit 1 qubit 2 qubit 3 **3 qubits :**
 $(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)$ **parallel processing of $2^3=8$ states**
 $= |0\rangle|0\rangle|0\rangle + |0\rangle|0\rangle|1\rangle + |0\rangle|1\rangle|0\rangle + |0\rangle|1\rangle|1\rangle + |1\rangle|0\rangle|0\rangle + |1\rangle|0\rangle|1\rangle + |1\rangle|1\rangle|0\rangle + |1\rangle|1\rangle|1\rangle$

Q-box 64 $\xrightarrow{2x}$ Q-box 65

200 bit quantum computer: More states than atoms in universe!

- **HOWEVER:** Only *measure* n qubits!
 Use only for certain algorithms (quantum simulation,
 factoring, optimization)

Classical Computing: Factoring 2048 bit number

Exponential scale up from 640 bit: 30 CPU years

10 year run time

\$10⁶ trillion

10⁶ terrawatt (world=10tw)

Consume all earth's energy in 1 day

"Ultimate Earth"
Made with "Virtual Nature" Studio 2
www.20thcentury.com
Copyright 2007 Chris Hanson

Quantum Computing: Factoring 2048 bit number

200M physical qubits
100k logical qubits

24 hour run time

(?) < \$few satellites

(?) < 10 M watt

Brute force design for wiring

1 m - 10 m
cells: 100 μm - 1 mm

Quantum Bits

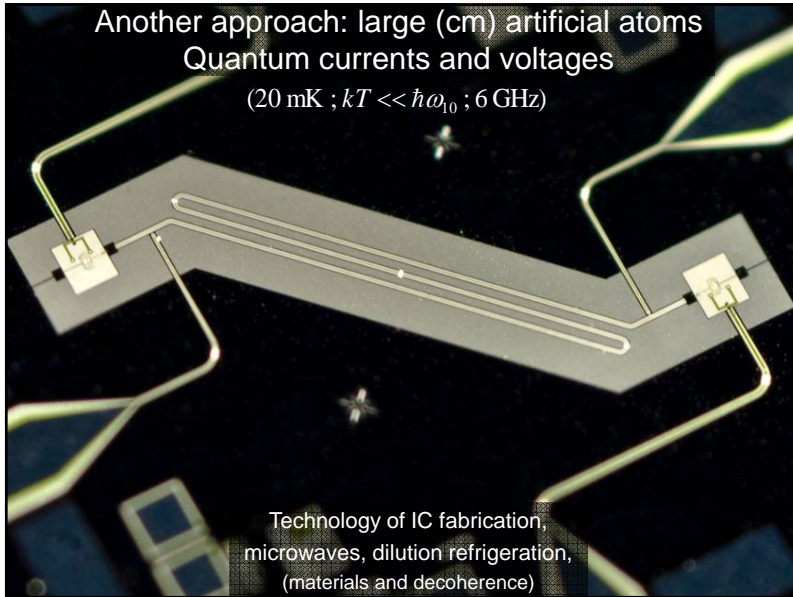
$|0\rangle + |1\rangle$

H atom wavefunctions:

Need Large "Molecules" for Control Signals

Manipulating atoms and electrons: μm length scale

For megascale quantum computer: need area for control



Macroscopic Variables

Single degree of freedom describing state of macroscopic number of atoms/electrons

Center of mass of ball
(single variable describes position)

Phase of superconductor
(Single phase for all Cooper pairs)

$$\Psi = \prod_k (u_k + v_k e^{i\phi} c_k^+ c_{-k}^+) |\Psi_0\rangle$$

$I \propto \nabla\phi$

Quantize macro-variables as for atomic variables

Qubit Physics: 2-level Quantum State

Spin in \vec{B} field
 B field is control!

$$H = -\frac{\mu_B}{2} \vec{\sigma} \cdot \vec{B}$$

$$= -\frac{\mu_B}{2} \left[\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} B_x + \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} B_y + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} B_z \right] \begin{pmatrix} |1\rangle \\ |0\rangle \end{pmatrix}$$

Example: strong B_z
 $E_{10} = \mu_B B_z$

Arbitrary amplitude and phase
 $\Psi = \cos\frac{\theta}{2} |0\rangle + \sin\frac{\theta}{2} e^{i\phi} e^{-iE_{10}t/\hbar} |1\rangle$
 → $|1\rangle'$ rotating frame

P_g : Measure prob. is projection z-axis

Qubits: B_z large; $B_x, B_y, \Delta B_z$ small controls

Control field ΔB_z is z-axis rotation

$$\Psi_f = \exp(-iHt/\hbar) \Psi$$

$$= \begin{pmatrix} e^{-i\Delta\phi/2} & 0 \\ 0 & e^{i\Delta\phi/2} \end{pmatrix} \Psi \quad \Delta\phi = -\frac{\mu_B}{\hbar} \int \Delta B_z(t) dt$$

$$= Z_{\Delta\phi} \Psi$$

Algebra of Z-gates:

$$\boxed{Z_\alpha} \boxed{Z_\beta} = \boxed{Z_{\alpha+\beta}}$$

$$Z_{2\pi} = -I$$

Qubits: B_z large; $B_x, B_y, \Delta B_z$ small controls

B_x, B_y at $\hbar\omega_{10} = \mu_B B_z$

- Rotating transverse control field B_t

$$Y_{\Delta\theta} = \begin{pmatrix} \cos \frac{\Delta\theta}{2} & -\sin \frac{\Delta\theta}{2} \\ \sin \frac{\Delta\theta}{2} & \cos \frac{\Delta\theta}{2} \end{pmatrix} \quad \Delta\theta = -\frac{\mu_B}{\hbar} \int B_t(t) dt$$

- Algebra of X- & Y-gates:

X_α	X_β	=	$X_{\alpha+\beta}$
Y_α	Y_β	=	$Y_{\alpha+\beta}$
X_α	Y_π	=	iZ_α
A_α	B_β	=	C_γ

 Complicated as non-commuting
For arbitrary axis
- Quantum NOT gate:

$Y_\pi 0\rangle = 1\rangle$
$Y_\pi 1\rangle = 0\rangle$

 E level transitions

Rotating Wave Approximation

- Only control B_x or B_y
- Counterdriven $e^{i2\omega_0 t}$ (off resonance) amplitude small if drive slow (few ns pulses)
- B_x, B_y – not important
- Phase of gate (X or Y) set by phase δ of microwave drive

Probability Oscillations - Chevron Curve

Tune up sequence using measurement P_g

$|g\rangle$ X_π

Frequency Detuning δf

Rotation Phase (Pulse Amplitude x Time)

$\sqrt{\text{NOT}} = \pi/2$
NOT = π

Rabi Oscillations

Any gate (+ change μ_w phase)

Basis of all quantum logic

The Challenge: Qubit Errors

Physical qubits have T_1, T_2

Control Errors

Quantum gate errors: $\sim 10^{-2} - 10^{-4}$

Error rate required: $\sim 10^{-20}$

Can make up >16 orders of magnitude with 100-1000 qubits

Error Digitization with Measurement

Example: error from small X rotation:

$$|\Psi\rangle \rightarrow (\hat{I} + \epsilon\hat{X})|\Psi\rangle$$

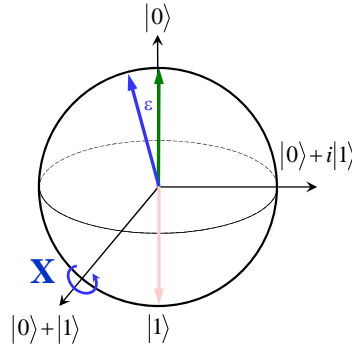
When measure Z, 2 outcomes:

1. Prob = $1-\epsilon^2$: erases error

$$(\hat{I} + \epsilon\hat{X})|\Psi\rangle \rightarrow |\Psi\rangle$$

2. Prob = ϵ^2 : X error

$$(\hat{I} + \epsilon\hat{X})|\Psi\rangle \rightarrow \hat{X}|\Psi\rangle$$



Understanding Qubit Errors

Classical bit:

Coin on table

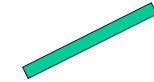
restoring force makes bit stable



Quantum bit (analogy):

Coin in space

rotation errors from any small force



Quantum bit (physics):

Qubit state $\psi = \cos\theta|0\rangle + \sin\theta e^{i\phi}|1\rangle$ described by amplitude and phase

$$\hat{X}\hat{Z} - \hat{Z}\hat{X} \neq 0$$

Order of amplitude/phase flips matter
Measuring amplitude randomizes phase

$$\hat{X}\hat{Z} + \hat{Z}\hat{X} = 0$$

Quantum anticommutation relation

Error Detection Basics

1) Consider 2 qubits, with parity-type measurement

simultaneous bit flip: $\hat{X}_{12} = \hat{X}_1\hat{X}_2$

simultaneous phase flip: $\hat{Z}_{12} = \hat{Z}_1\hat{Z}_2$

$$\begin{aligned} \text{new operators commute: } [\hat{X}_{12}, \hat{Z}_{12}] &= \hat{X}_1\hat{X}_2\hat{Z}_1\hat{Z}_2 - \hat{Z}_1\hat{Z}_2\hat{X}_1\hat{X}_2 \\ &= \hat{X}_1\hat{Z}_1\hat{X}_2\hat{Z}_2 - (-\hat{X}_1\hat{Z}_1)(-\hat{X}_2\hat{Z}_2) \\ &= 0 \end{aligned}$$

2) Behaves classically. Simultaneous eigenstates:
Stable outcomes when both \hat{X}_{12} and \hat{Z}_{12} measured

measurement:	\hat{Z}_{12}	\hat{X}_{12}	\hat{Z}_{12}	\hat{X}_{12}	\hat{Z}_{12}	\hat{X}_{12}	\hat{Z}_{12}	\hat{X}_{12}
outcome:	+1	-1	+1	-1	+1	-1	+1	-1

3) From detection to identification:
more qubits needed

\hat{Z}_{12} \hat{Z}_{23} id. single flips!

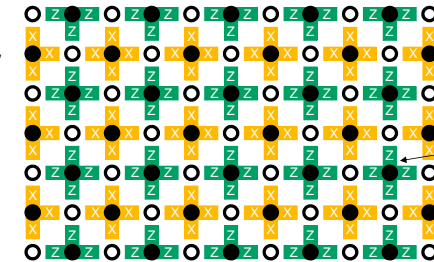
Surface Code Hardware

Toric code: Bravyi & Kitaev
arXiv:quant-ph/9811052 (1998)

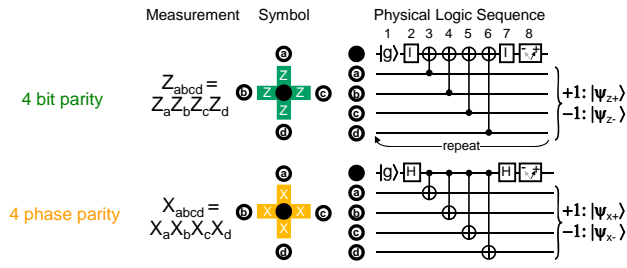
Logical CNOT: Raussendorf et. al.,
PRL 98, 190504 (2007)

Theory review: A. Fowler et. al.,
PRA 80, 052312 (2009)

Surface code for mortals: Fowler,
Mariantoni, Martinis, Cleland
PRA 86, 032324 (2012)

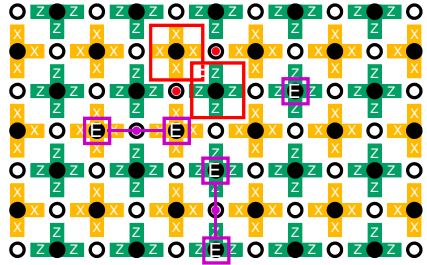


data qubits
measure qubits



\oplus
CNOT
= XOR

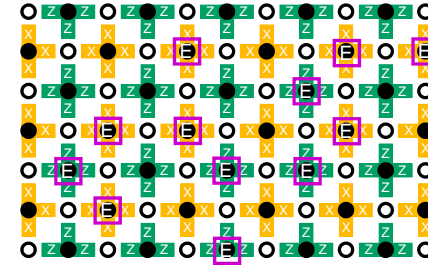
Stabilized State and Identifying Qubit Errors



All measurements XXXX and ZZZZ commute:
Measurement outcomes unchanging

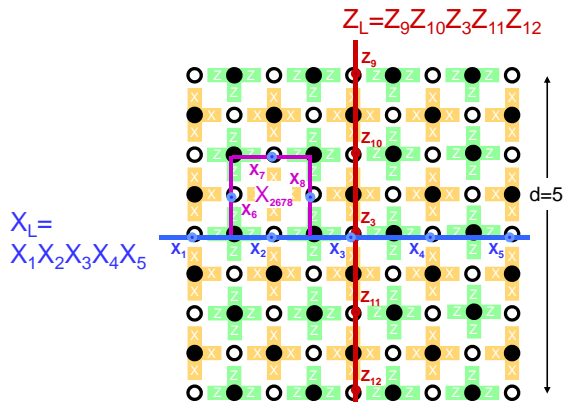
When errors:
Data qubit errors – pairs in space
Measure errors – pairs in time

Stabilized State and Identifying Qubit Errors



When large density -
Backing out errors may not be unique

Logical Qubit: 41 data qubits, 40 measure qubits

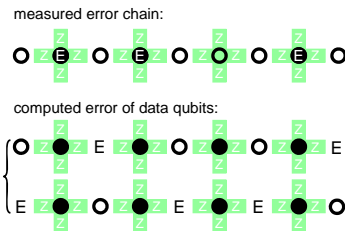
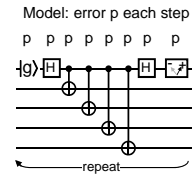


$$X_L = X_1 X_2 X_3 X_4 X_5$$

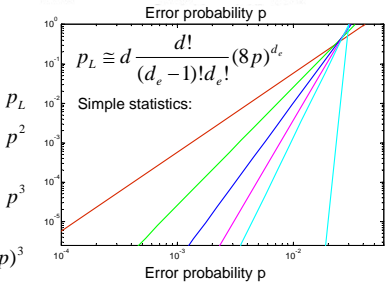
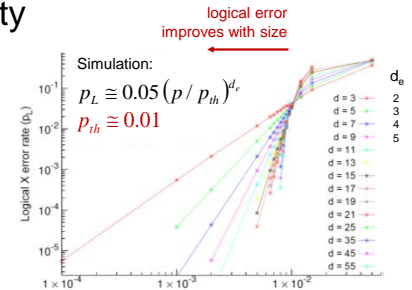
$$Z_L = Z_9 Z_{10} Z_3 Z_{11} Z_{12}$$

$X_L Z_L = - Z_L X_L$ (so acts like qubit)
 X_L and Z_L commute with measures (so stabilized)
acts as extra degree of freedom

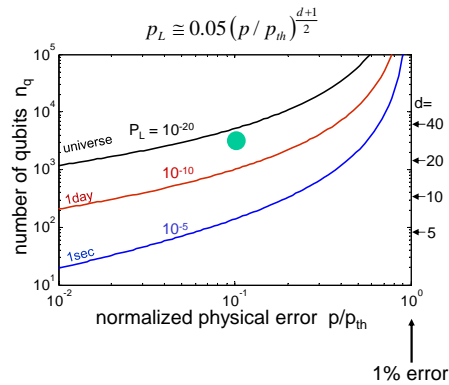
Logical Error Probability



$$P_L \approx \frac{5 \cdot 4 \cdot 3}{3 \cdot 2 \cdot 1} (8p)^3$$

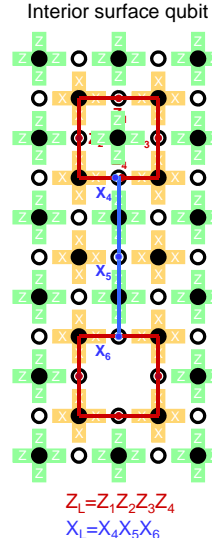


Size of Logical Qubits

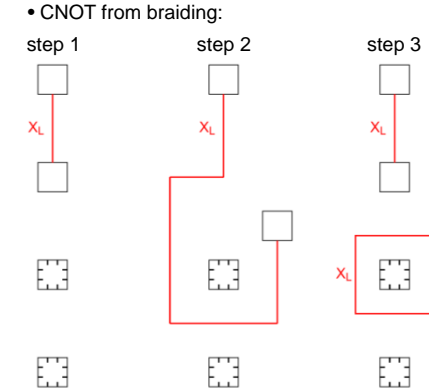


Need ~3000 physical qubits per logical qubit

Logical Qubits and Gates



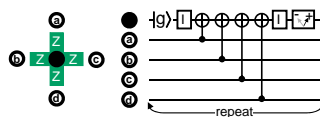
- Initialize Z_L with Z measurement
- Measure Z_L with Z measurement



• Single gates: $(X, Z), H, S = \sqrt{Z}, T = \sqrt{S}$

Review of Hardware & Requirements

Presently, surface code only realistic fault-tolerant architecture



- Unit cell requirements:
- Single gate
 - CNOT or CZ (nn)
 - Fast measurement
 - Nearest neighbor

Performance needed:
(99.3% threshold)

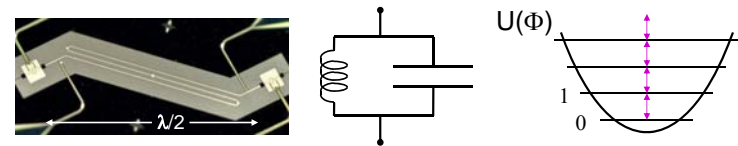
Target of 10-30x below threshold:
99.97% single gate
99.9% 2-qubit gate
99% measurement

Experiments (# qubits):
9 - basic test
20 - full test
200 - low error
1000 - logical op's
1 M - algorithm

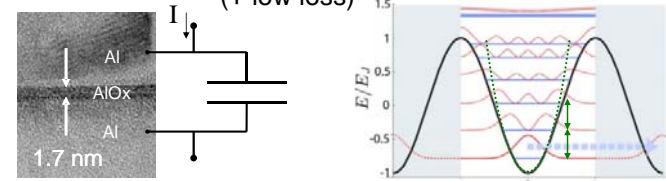
Scalability:
Qubits made reliably (IC fabrication)
Need room for control wiring
Eventually need control system

Superconducting Qubits

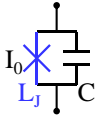
- Macroscopic "atom": quantize I and V, 5 GHz >> 20 mK
- LC oscillator (linear): memory and communication



- Josephson junction: non-linear inductance with 1 photon (+ low loss)



Qubit: Nonlinear LC resonator



$$I = I_0 \sin \delta$$

$$V = \frac{\Phi_0}{2\pi} \dot{\delta} \quad (V = \dot{\Phi})$$

δ is dimensionless flux

Junction response:
$$\dot{I}_j = I_0 \cos \delta \dot{\delta} \equiv (1/L_J)V$$

$L_J = \Phi_0 / 2\pi I_0 \cos \delta$
nonlinear inductor

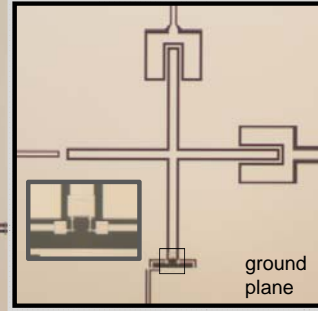
Junction energy:
$$U(\delta) = \int dt I V$$

$$= (I_0 \Phi_0 / 2\pi) \int dt \sin \delta d\delta / dt$$

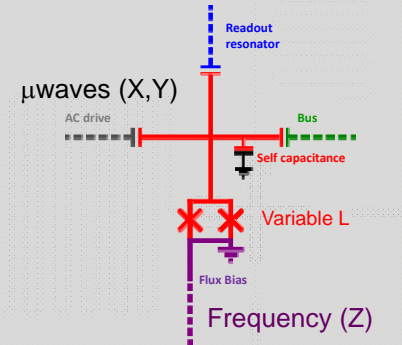
$$= -(I_0 \Phi_0 / 2\pi) \cos \delta$$

QM commutator: $[\hat{\Phi}, \hat{Q}] = i \hbar, \quad [\hat{\delta}, \hat{q}] = 2i$

Xmon Circuit



ground plane

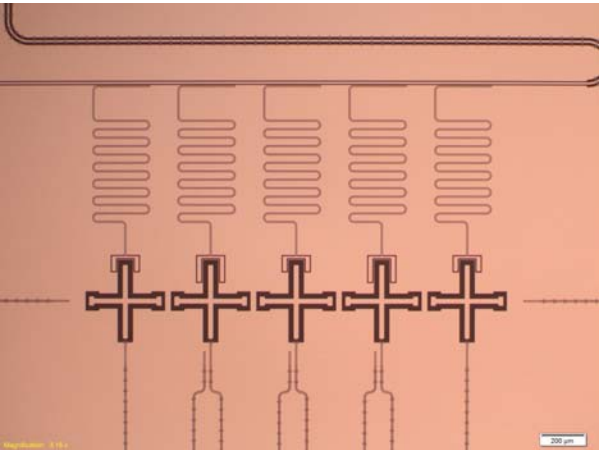


Transmon is non-linear LC oscillator

Xmon: Superconducting Qubit Fabrication

Al metal & Josephson junctions
Sapphire substrate
IC fabrication technology

10+ years
50+ researchers

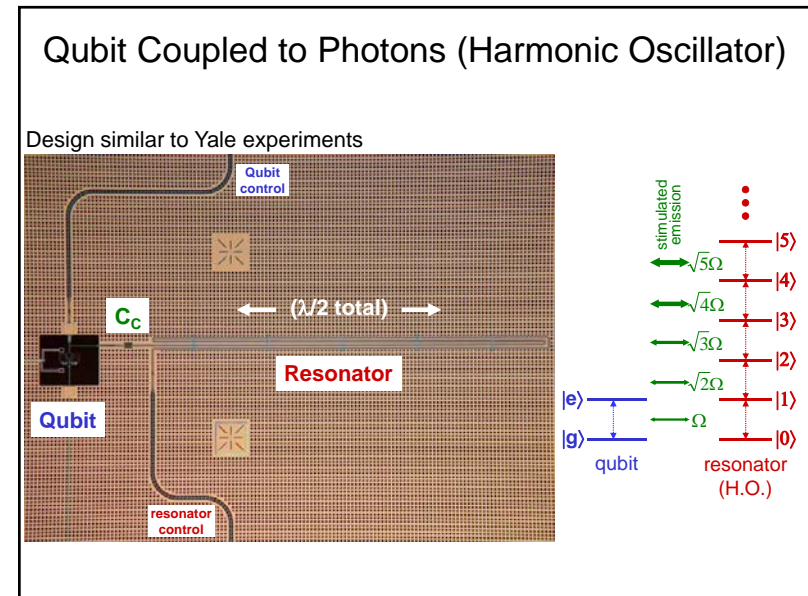
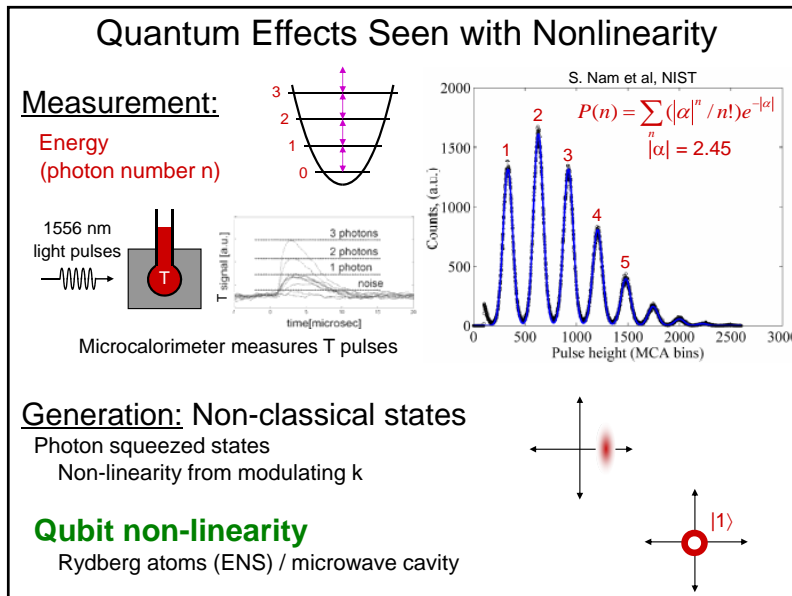
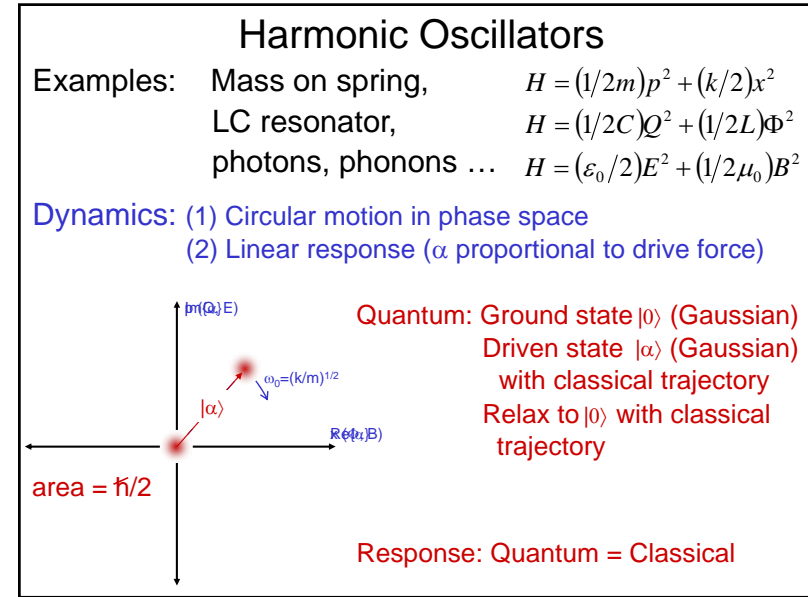
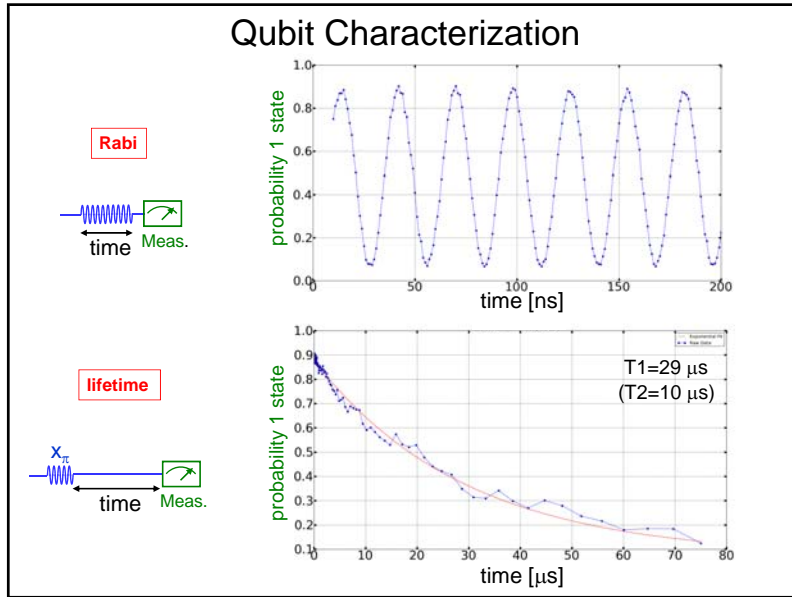


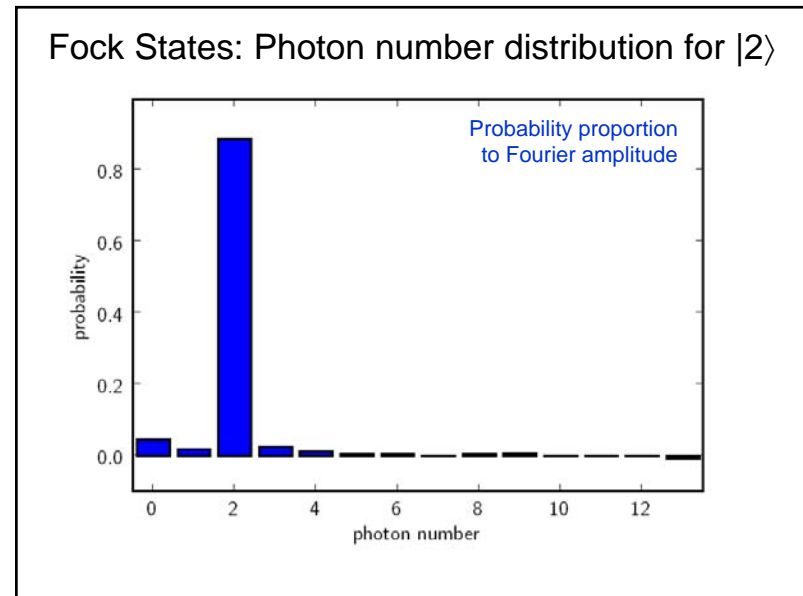
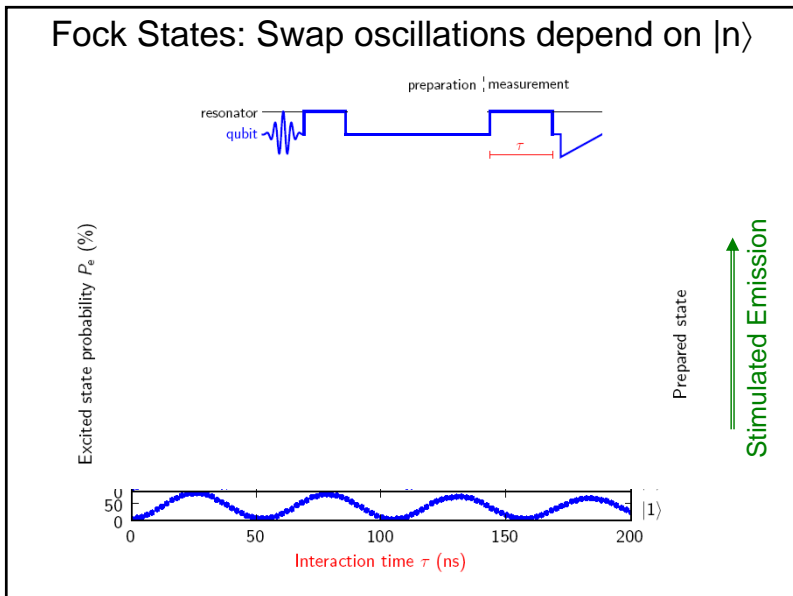
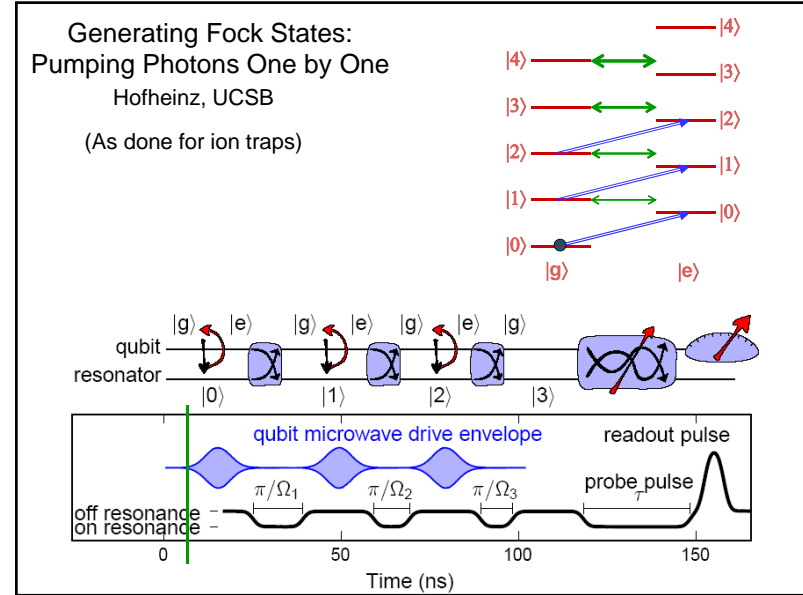
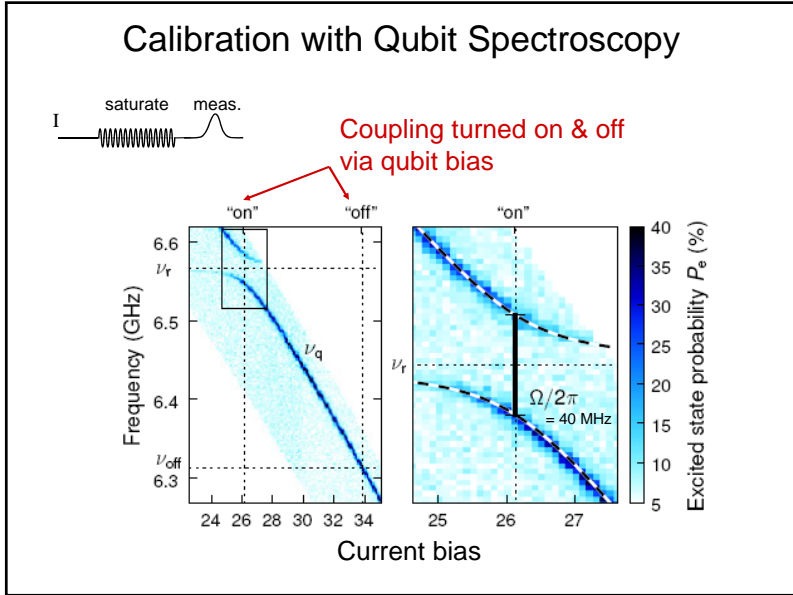
qubit readout (freq. mux'd)

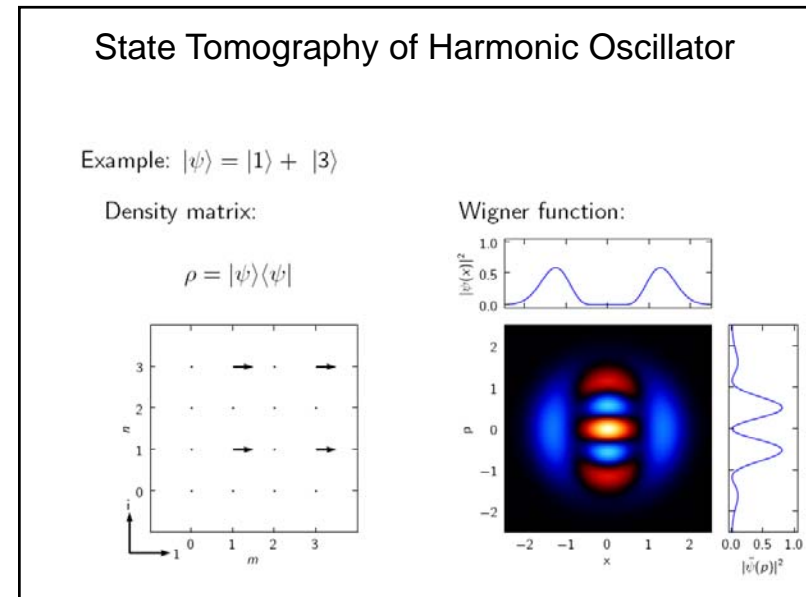
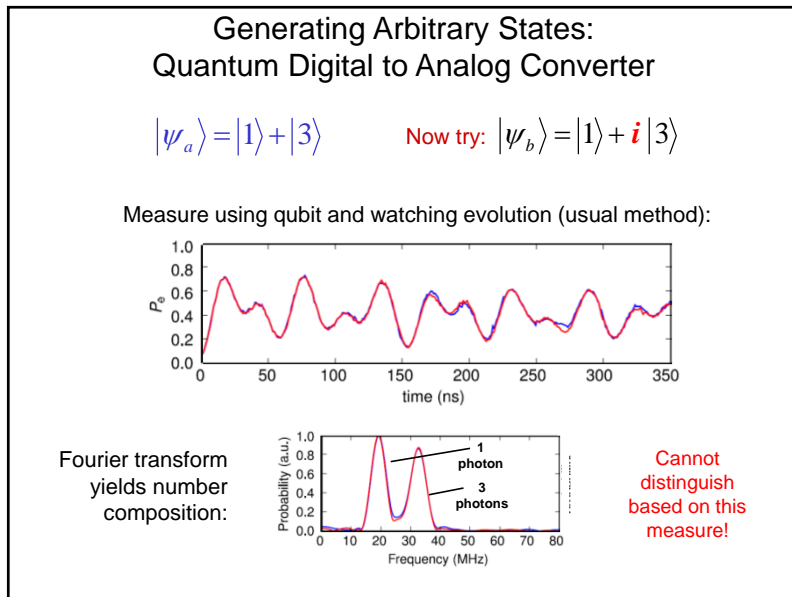
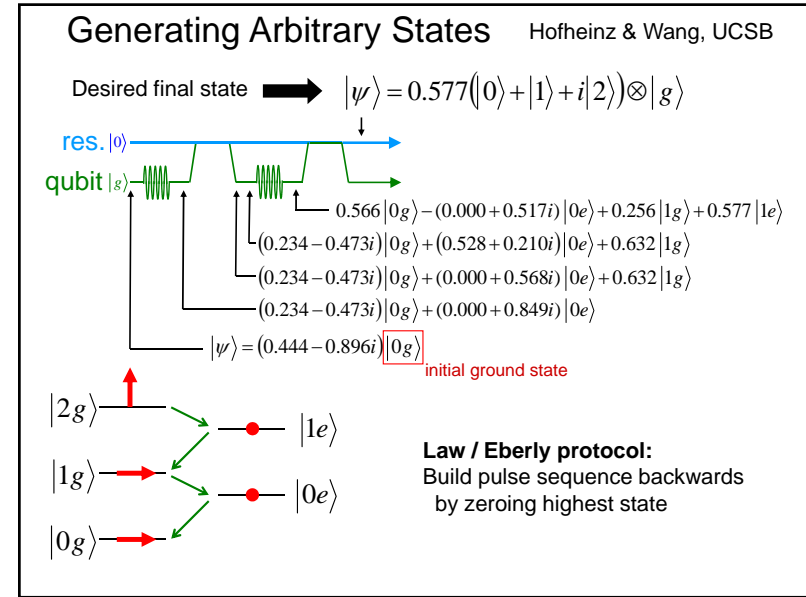
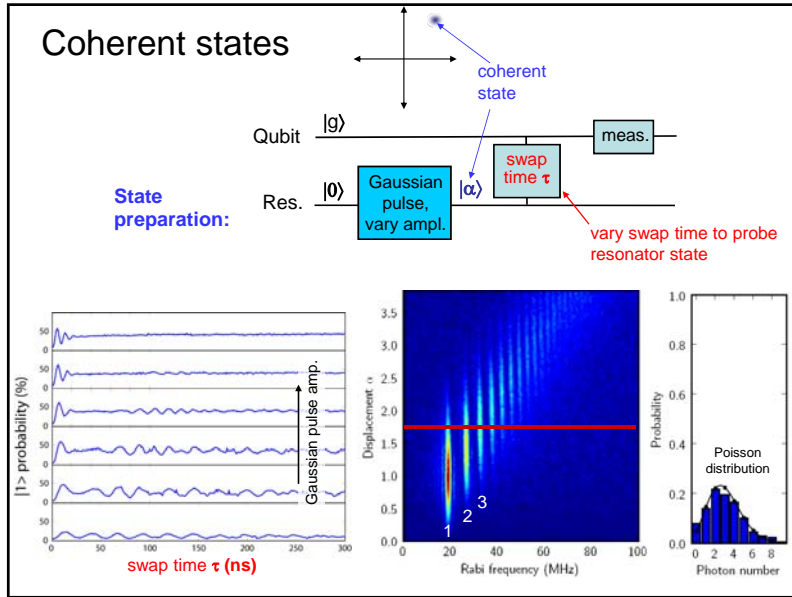
5 Xmon qubits, C coupling

qubit control







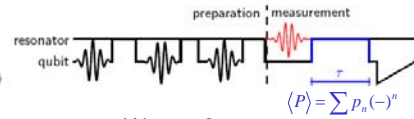
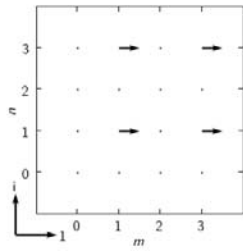


State Tomography of Harmonic Oscillator

Example: $|\psi\rangle = |1\rangle + |3\rangle$

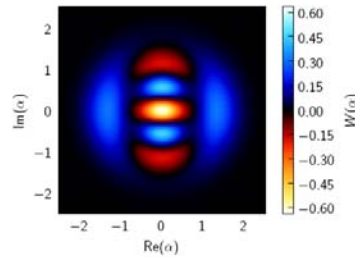
Density matrix:

$$\rho = |\psi\rangle\langle\psi|$$



Wigner function:

$$W(\alpha) = \frac{2}{\pi} \langle\psi|D(\alpha)PD(-\alpha)|\psi\rangle$$

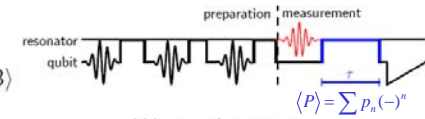
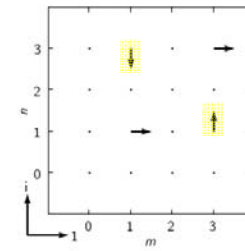


State Tomography of Harmonic Oscillator

Example: $|\psi\rangle = |1\rangle + i|3\rangle$

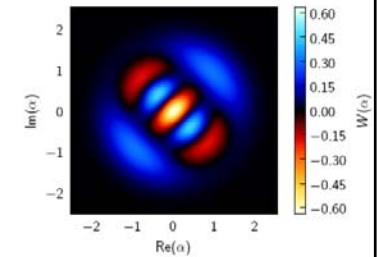
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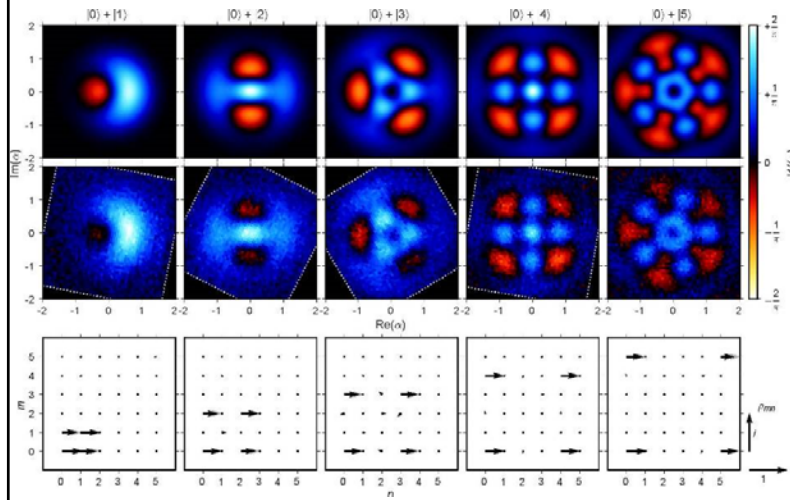


Wigner function:

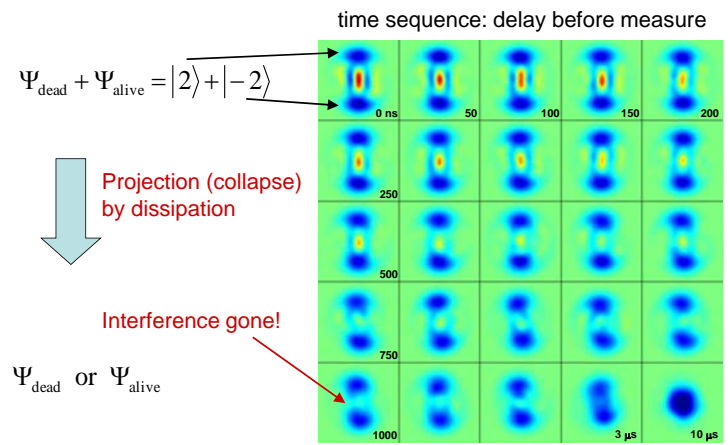
$$W(\alpha) = \frac{2}{\pi} \langle\psi|D(\alpha)PD(-\alpha)|\psi\rangle$$



Wigner Tomography, $|0\rangle + |N\rangle$ states:



Collapse of Schrodinger Cat State

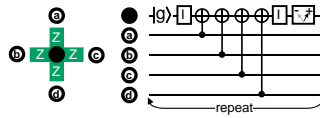


Collapse rate scales with distance between Ψ_{dead} & Ψ_{alive}

See "Exploring the Quantum", Haroche & Raimond

Review of Hardware & Requirements

Presently, surface code only realistic fault-tolerant architecture



Unit cell requirements:
 Single gate
 CNOT or CZ (nn)
 Fast measurement
 Nearest neighbor

Performance needed:
 (99.3% threshold)

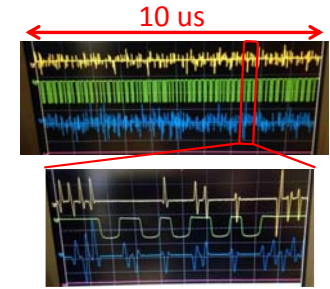
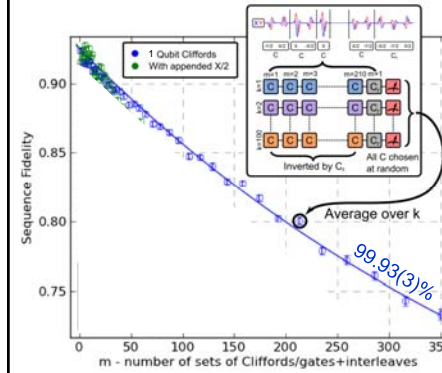
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Experiments (# qubits):
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 20 - full test
 200 - low error
 1000 - logical op's
 1 M - algorithm

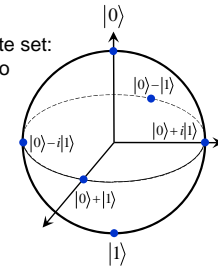
Scalability:
 Qubits made reliably (IC fabrication)
 Need room for control wiring
 Eventually need control system

Randomized Benchmarking*

Realistic multi-qubit test of *long* algorithm (1000+ gates)



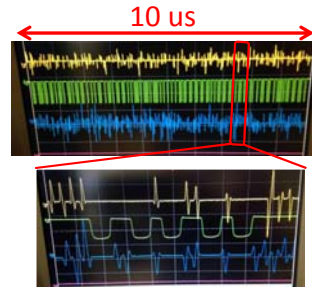
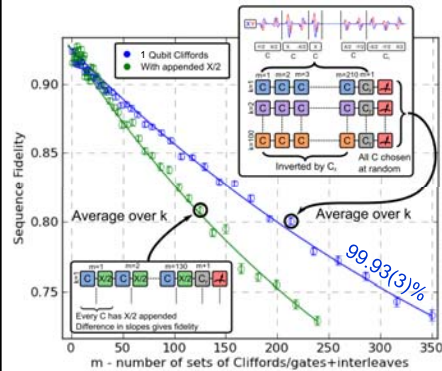
Clifford gate set: "rotation" to 6 states



*Magesan et. al., PRL 106.18 (2011): 180504

Randomized Benchmarking*

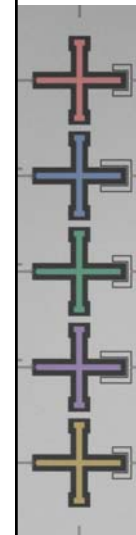
Realistic multi-qubit test of *long* algorithm (1000+ gates)



Gate	Fidelity (%)	Gate Time
X	99.95	20ns
Y	99.95	20ns
X/2	99.93	20ns
Y/2	99.93	20ns
-X	99.92	20ns
-Y	99.90	20ns
-X/2	99.93	20ns
-Y/2	99.93	20ns
H	99.91	40ns
Z	99.97	10ns
Z/2	99.98	10ns

*Magesan et. al., PRL 106.18 (2011): 180504

Single qubit gate fidelities

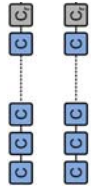


gates	Q ₀	Q ₁	Q ₂	Q ₃	Q ₄
I	0.9990	0.9996	0.9995	0.9994	0.9991
X	0.9992	0.9996	0.9992	0.9991	0.9991
Y	0.9991	0.9995	0.9993	0.9992	0.9991
X/2	0.9992	0.9993	0.9993	0.9994	0.9993
Y/2	0.9991	0.9993	0.9995	0.9994	0.9994
-X	0.9991	0.9995	0.9992	0.9989	0.9991
-Y	0.9991	0.9995	0.9991	0.9987	0.9991
-X/2	0.9991	0.9992	0.9993	0.9990	0.9995
-Y/2	0.9991	0.9992	0.9995	0.9990	0.9994
H	0.9986	0.9986	0.9991	0.9981	0.9988
Z	0.9995	0.9988	0.9994	0.9991	0.9993
Z/2	0.9998	0.9991	0.9998	0.9995	0.9996
2T ^a		0.9989	0.9994	0.9989	0.9990
average over gates	0.9992	0.9992	0.9994	0.9991	0.9992
average over qubits			0.9992		

Simultaneous Benchmarking

Frequency (GHz):

5.50 4.60



(Detuning turns off interaction)

Gate Fidelity (%):

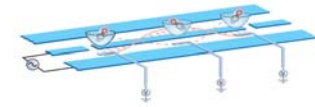
Q0	Q1
99.94	
	99.95
99.91	

Operation of Q0 & Q1 simultaneously has same errors as Q0+Q1 →negligible crosstalk

Coupled Qubits: Thinking Outside the Cavity

Conventional thinking:

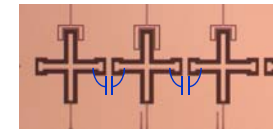
- (1) Operate qubits at fixed frequency (gives longest memory)
- (2) Couple qubits through "quantum bus" (long distance communication)
- (3) Use complex photon drive/control (need tuning for proper gate)



SC qubits: 200ns, 90-98% fidelity

New coupler design:

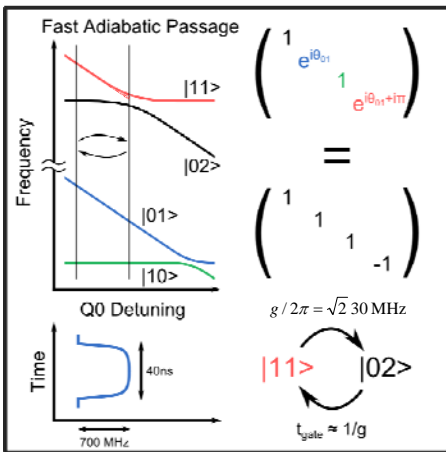
- (1) Adjustable frequency qubit (use to turn on/off interaction)
- (2) Direct qubit-qubit coupling (no extra mode giving decoherence)
- (3) DC drive for qubit frequency (need accurate, but 1 param. tuning)



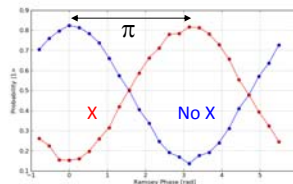
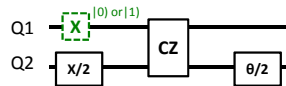
Theory: 40ns, 99.99% intrinsic fidelity (no decoherence)

Tune up of Controlled-Z

truth table $\begin{cases} |00\rangle \rightarrow |00\rangle \\ |01\rangle \rightarrow |01\rangle \\ |10\rangle \rightarrow |10\rangle \\ |11\rangle \rightarrow -|11\rangle \end{cases}$



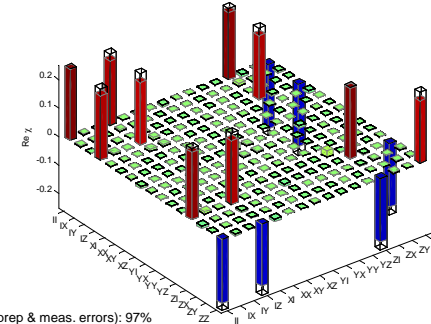
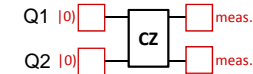
Functionality Check (truth table)



*Similar to Strauch; DiCarlo; Yamamoto

Controlled-Z QPT Fidelity


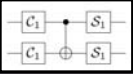
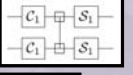

Quantum Process Tomography



total fidelity (with state prep & meas. errors): 97%


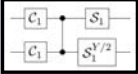
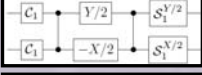
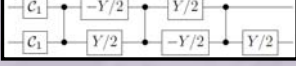
Two qubit Clifford group C_2

Classes of C_2

-  single qubit class: 576
-  CNOT-like class: 5184
-  iSWAP-like class: 5184
-  SWAP-like class: 576

total : 11520


Classes of C_2 : CZ

- 
- 
- 
- 

$S_i = \{ [I], [Y/2, X/2], [-X/2, -Y/2] \}$

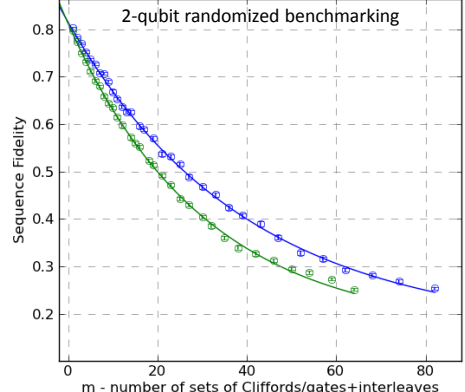
A. Corcoles et al., PRA 2013

Performance of CZ gate




Fast (40ns)
Accurate $F_{CZ} = 99.45(5)\%$

At best in world



2-qubit randomized benchmarking

CZ fidelity for all pairs

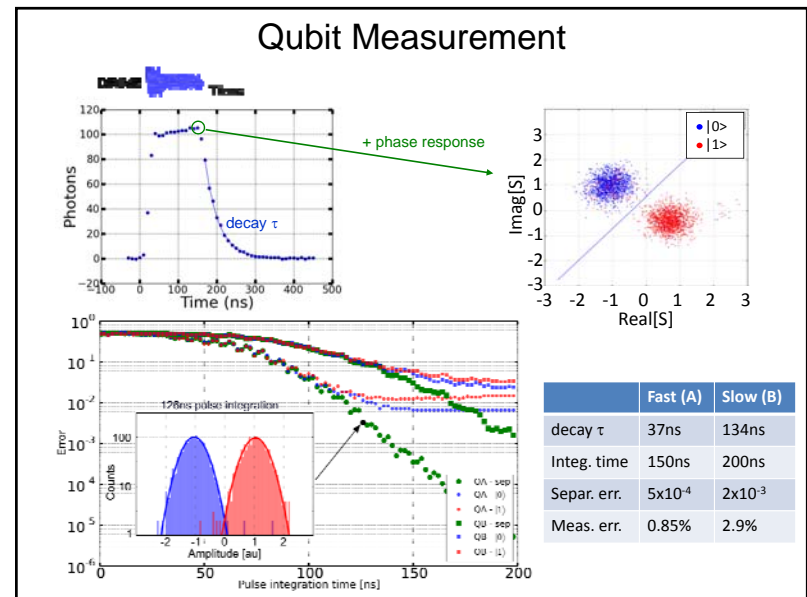
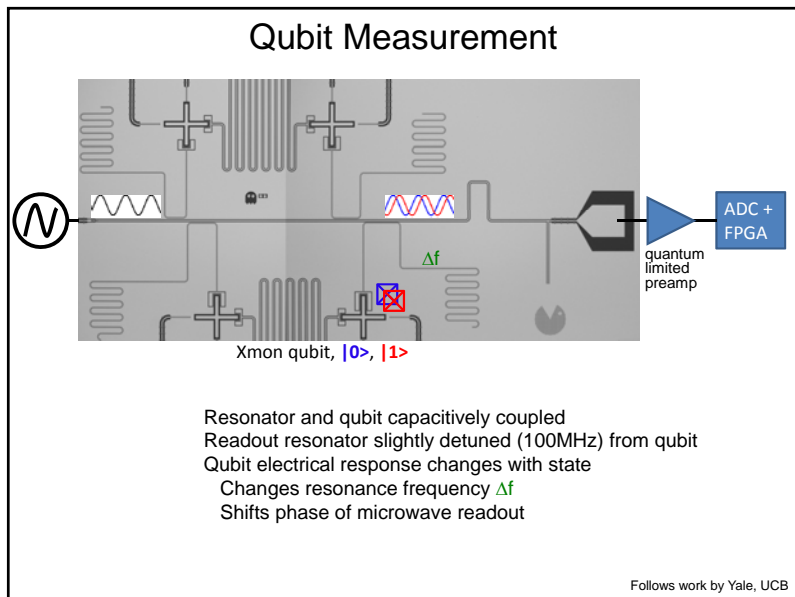
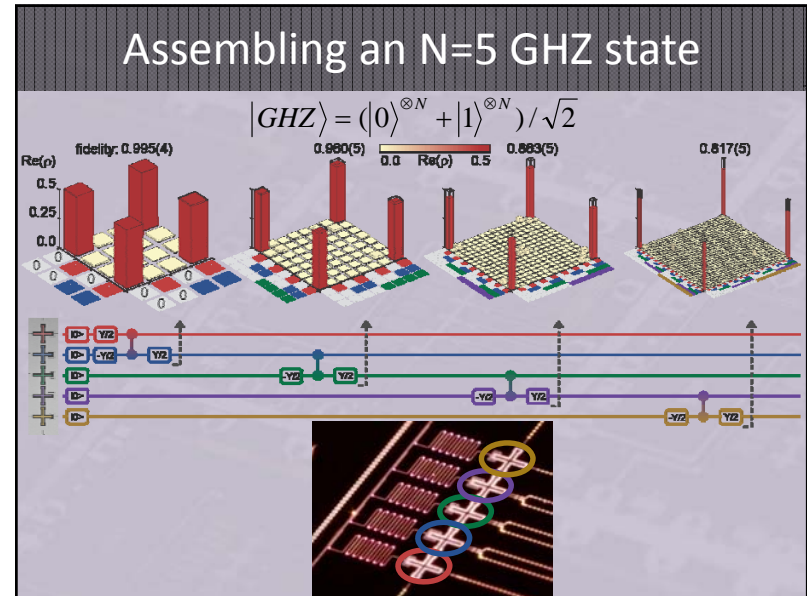
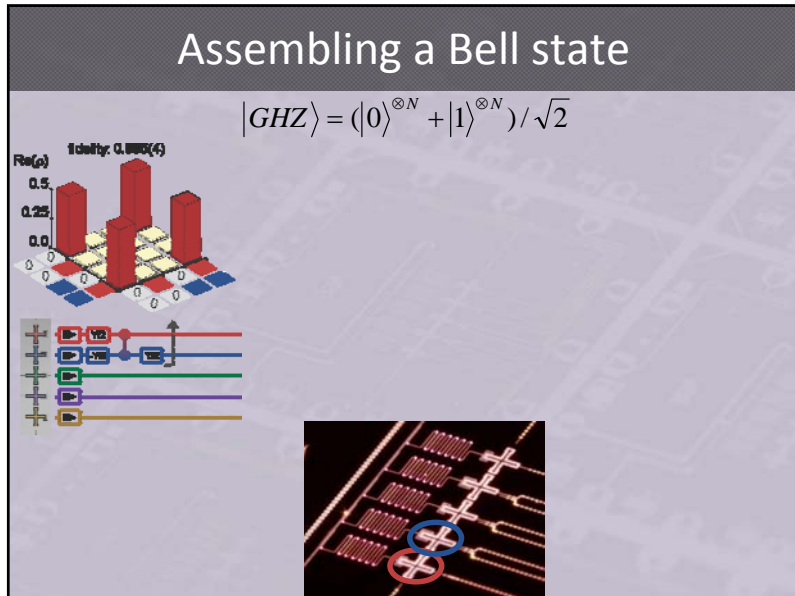


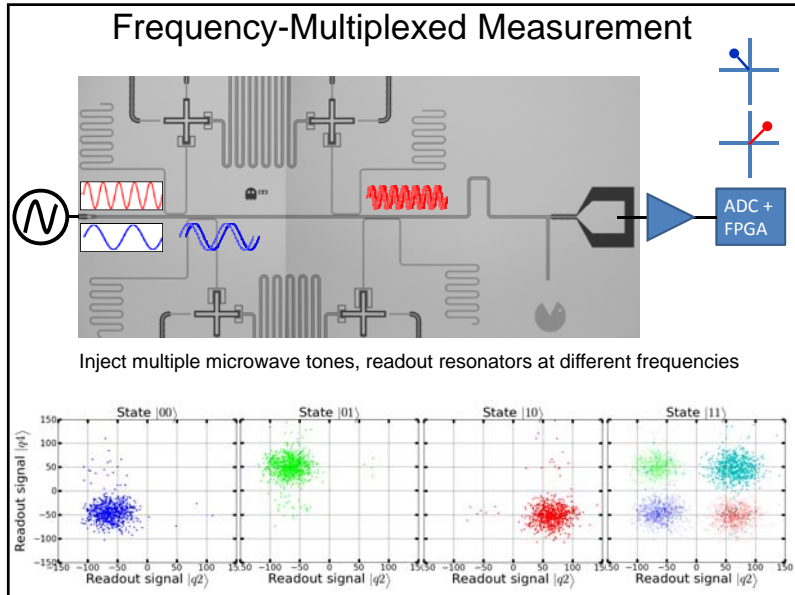
qubits	Q_0	Q_1	Q_2	Q_3	Q_4
$CZ_{Q_0-Q_1}$	0.9924 ± 0.0005				
$CZ_{Q_1-Q_2}$	0.9936 ± 0.0004				
$CZ_{Q_2-Q_3}$	0.9944 ± 0.0005				
$CZ_{Q_3-Q_4}$	0.9900 ± 0.0006				

Comparison to other technologies

system	# qubits	entangling gate fidelity	single qubit gate fidelity
liquid NMR [1]	3,1	0.995	0.9999
UCSB Xmon	5	0.994	0.9992
ion traps [2]	1		0.99998
ion traps [3] (QPT, 1/2 CNOT)	2	0.993	
ion traps [4]	5	0.95	
sup MIT LL, planar [5]	1		0.998
sup IBM, planar [6]	2	0.98	
sup IBM, planar [7]	2	0.93	0.998
sup IBM, planar [8]	3	0.96	0.997

1. Ryan et al., New J. Phys 2009
2. Brown et al., PRA 2011
3. Benhelm et al., Nat Phys 2008
4. Choi et al., arxiv 2014
5. Gustavsson et al., PRL 2013
6. Chow et al., PRL 2012
7. Corcoles et al., PRA 2013
8. Chow et al., arxiv 2013





Summary & Outlook

Xmon qubits are high-fidelity technology

- 1 qubit: 99.93(3)%
- 2 qubit: 99.45(5)%
- Measure: 99-99.9%
- Improvements likely

Now understand path to fault-tolerant computation

- Surface code: at needed fidelity of 99%
- SC qubits are IC scalable

Surface code project to test science of error correction:

- 100-1000 qubit, surface code architecture
- Logical error rate 10^{-15}
- Logical operations (H, CNOT, ...) at 10^{-6} errors



Market: Solve optimization problems (spin glass)

Conjecture: Build QC without much coherence

Technology: Use standard Josephson fabrication

Machine has superb engineering

Physicists: No exponential computing power

What does Nature have to say?

Preliminary Results:
No faster than classical code

Typical (median) execution times

Time [μs] vs \sqrt{N}

- Belief Propagation (exact)
- Simulated Quantum Annealing
- Simulated Annealing generic
- optimized
- parallelized
- GPU
- D-Wave

Matthias Troyer (ETH) and collaborators