Kondo effect

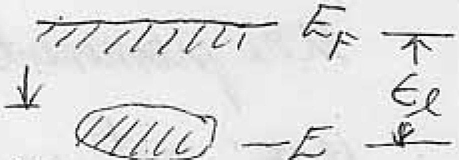
Dilute alloy $M_{1-x} I_x$

M - nonmagnetic metallic host

I - impurity ion with partially-filled d- or f-electron shell and spin $\underset{\sim}{S}$

$\mathcal{H}_{ex} = -2J \underset{\sim}{S} \cdot \underset{\sim}{s}$

$J \sim -\dfrac{\langle V_{kl}^2 \rangle}{\epsilon_l} < 0$ (AFM)



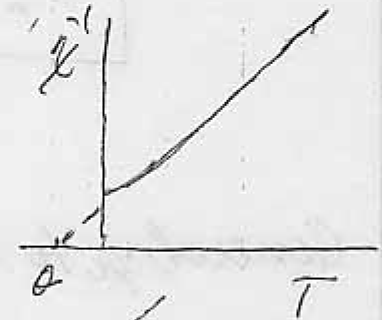
$\Delta = \pi \langle V_{kl}^2 \rangle N(E_F)$

$T_K \sim T_F \exp(-1/N(E_F)|J|)$

Gradual formation of many body singlet ground state as T decreases through T_K

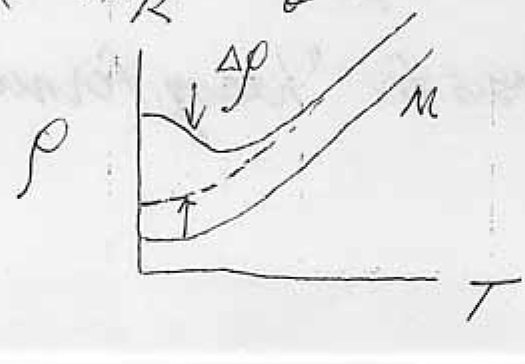
Physical properties scale with T_K

$T \gg T_K$ local moment behavior



$\chi(T) = \dfrac{N\mu_{eff}^2}{3k_B(T+\theta)}, \ |\theta| \sim (3-4)T_K$

$\Delta\rho(T) \sim -\ln(T/T_K)$



$T \ll T_K$ Nonmagnetic

Local Fermi liquid; effective Fermi temperature $T_F^* \simeq T_K$

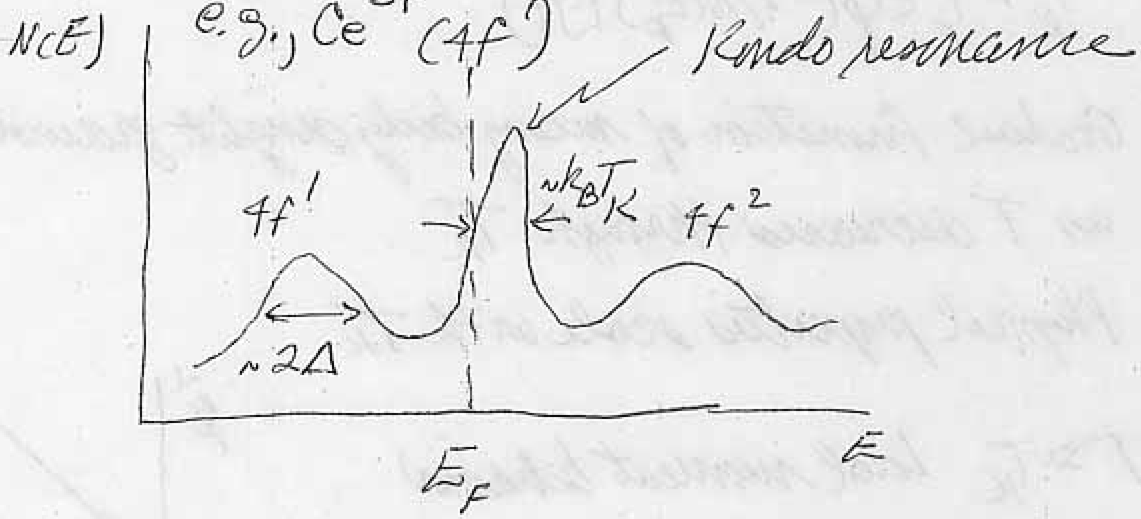
$\Delta\chi(T), \Delta C(T)/T = \gamma \xrightarrow[T \to 0]{} \text{Const.} \propto N(E_F) \rightarrow \propto m^* \propto \frac{1}{T_K}$

$\Delta\rho(T) = \Delta\rho(0)\left[1 - \left(\frac{T}{T_K}\right)^2\right]$

More prominent, the smaller T_K

NOTE: m^* diverges as $T_K \to 0$ ("heavy fermion")

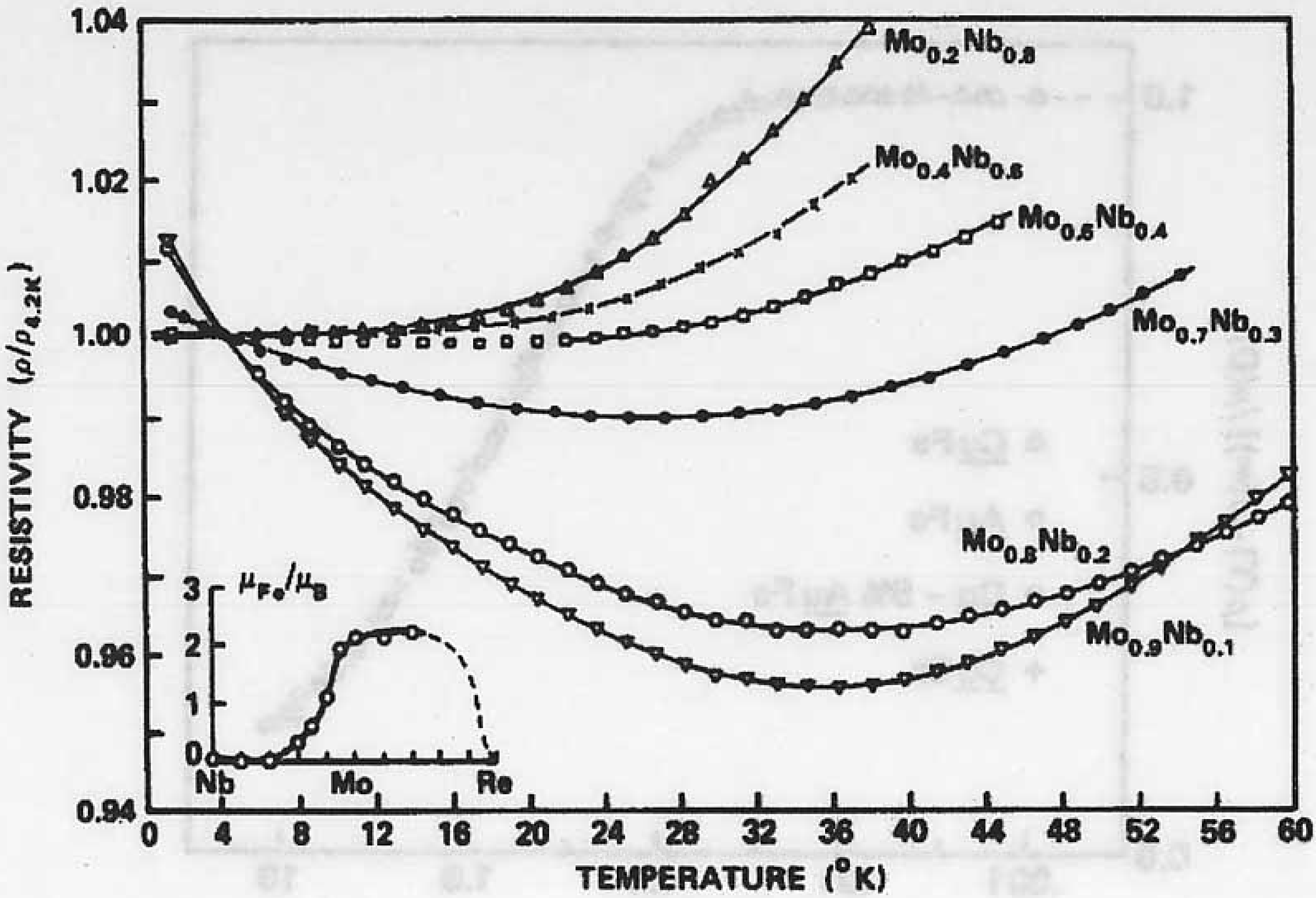
"Abrikosov-Suhl" or "Kondo" resonance



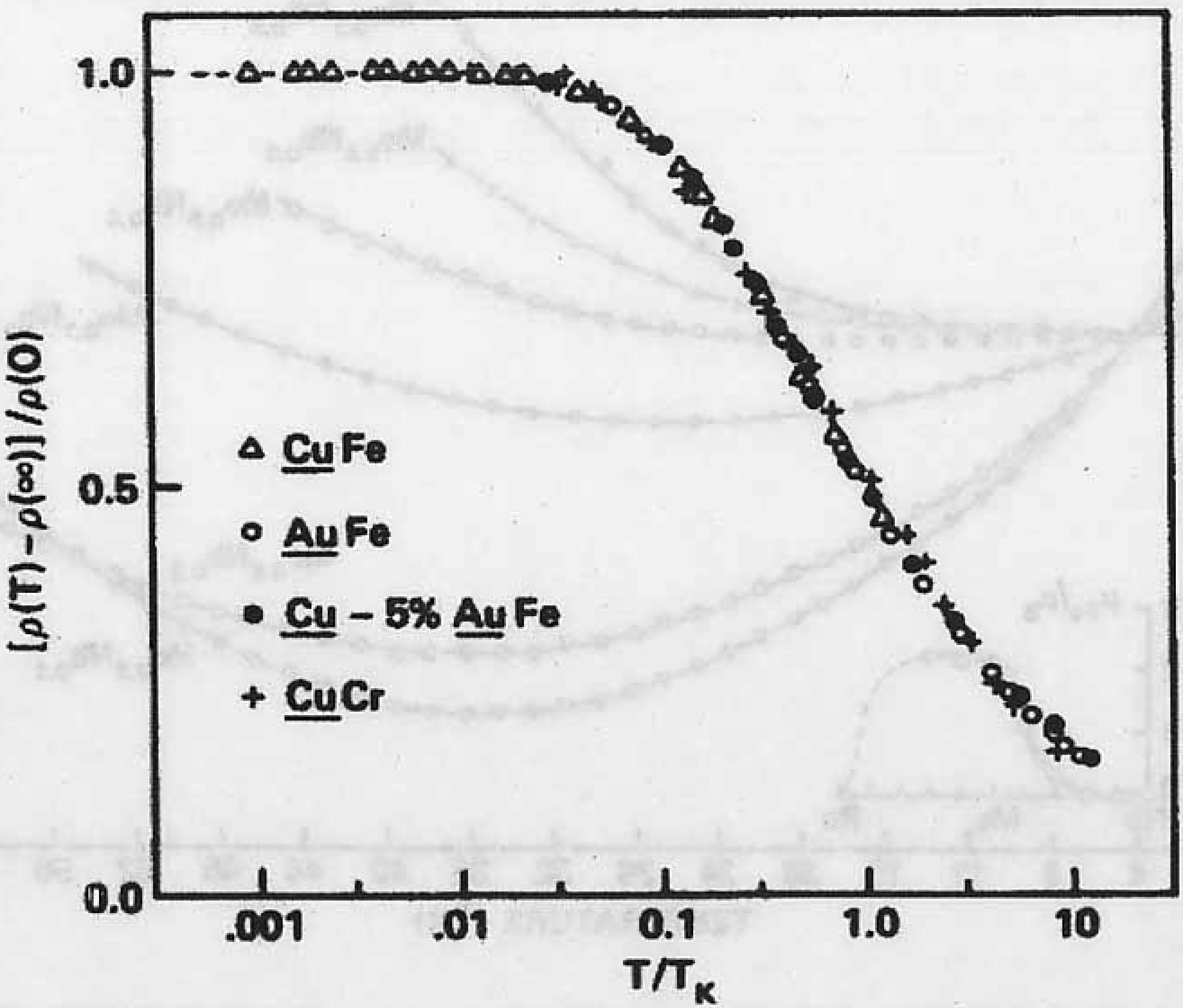
Generalize to lattice "Kondo lattice"

Basis for "heavy fermion" metals

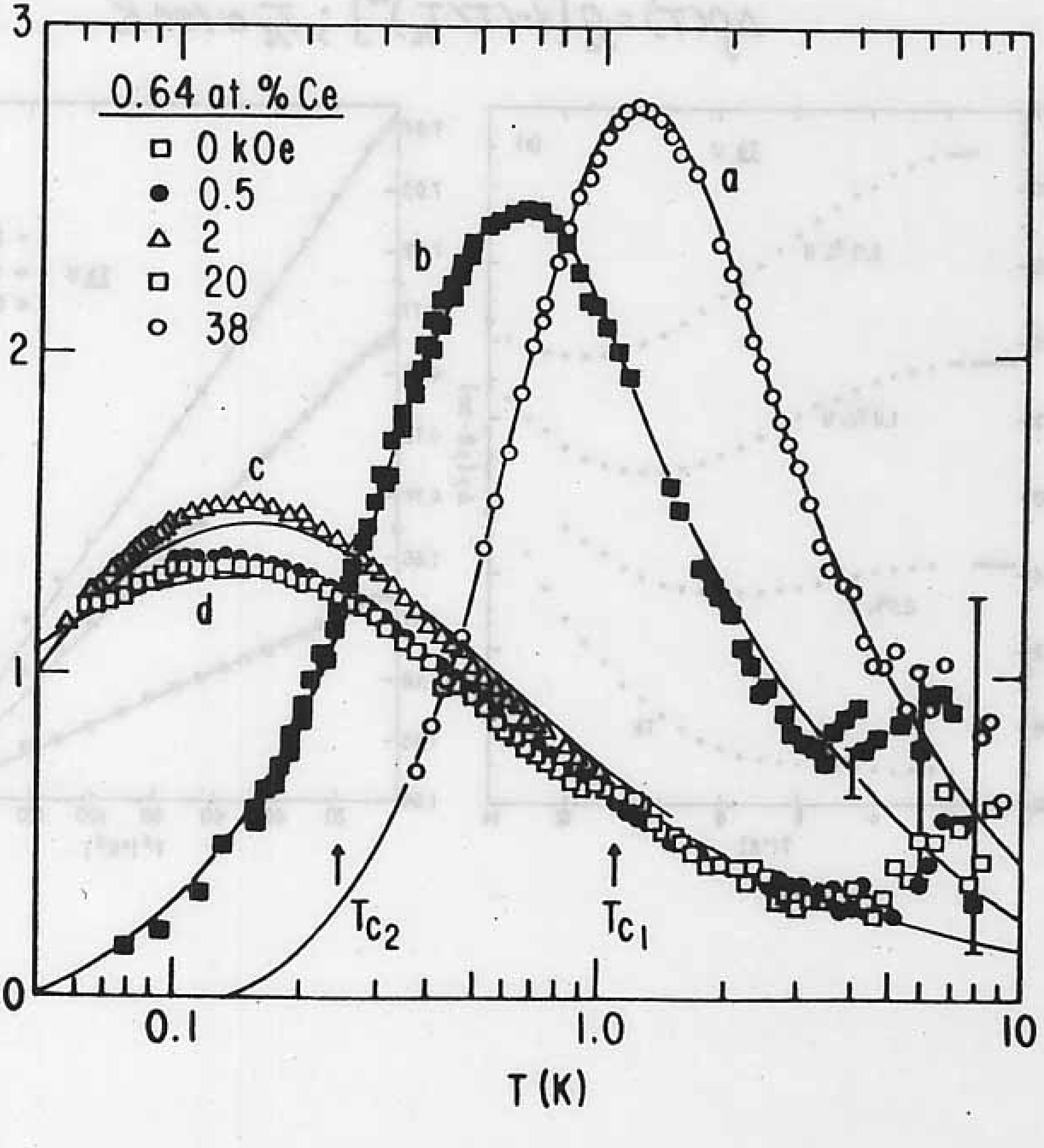
Characteristic Kondo resistance minimum associated with Fe dissolved in Mo-Nb alloys. After Sarachik *et al.* '64. Inset: μ_{eff} vs composition. After Clogston *et al.* '62.



Excess resistivity associated with scattering from the impurity spin. The resistivity saturates at low temperatures because all of the d-wave is being scattered. After White and Geballe '79.



$(\underline{La}\, Ce) Al_2$



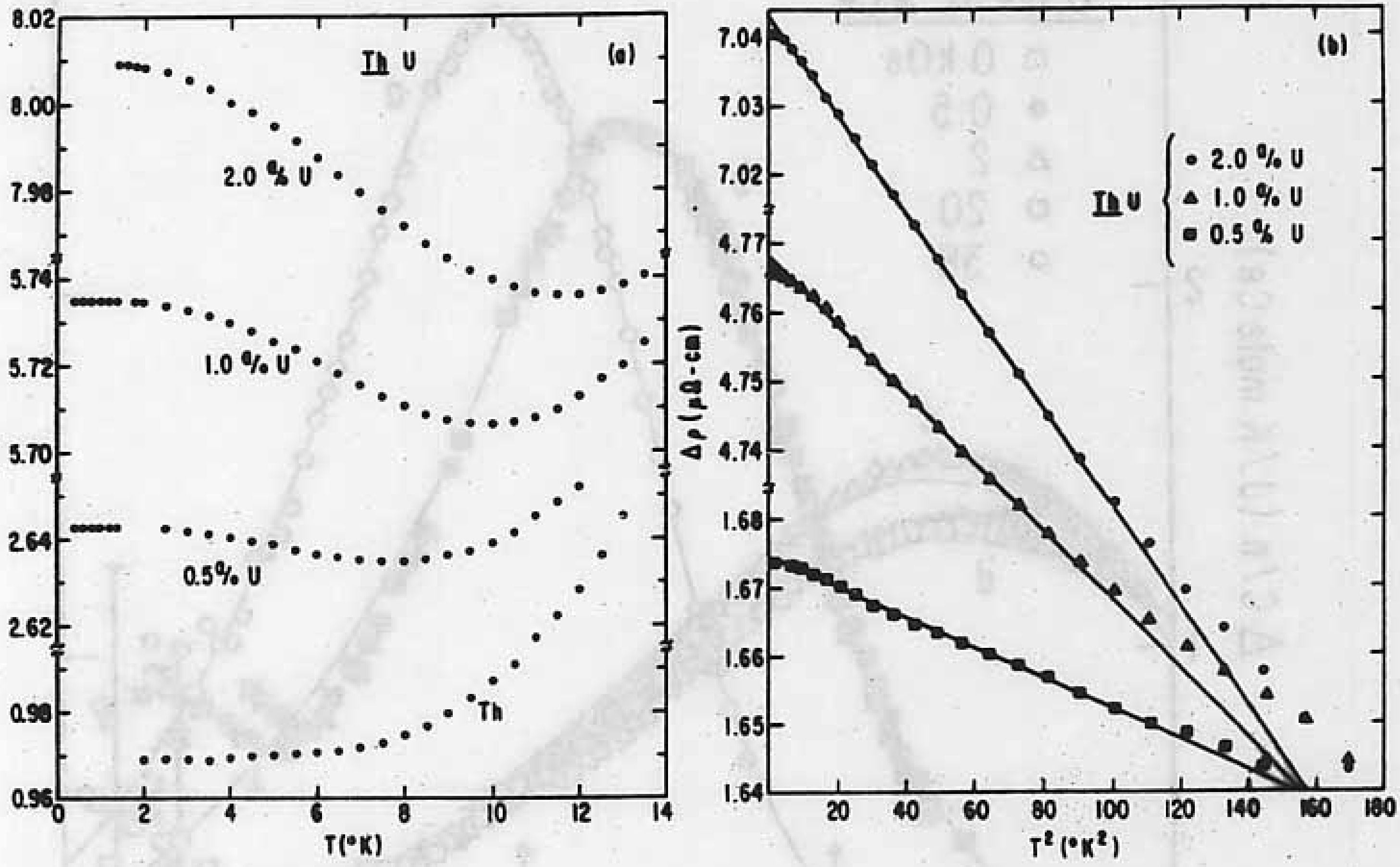
S.D. Bader, N.E. Phillips, M.B. Maple & C.A. Luengo (1975)

Curve d – Bloomfield-Hamann theory for $S = 1/2$, $T_K = 0.42K$
(also consistent with Bethe Ansatz solution)
Rajan et al. – 1983

$Th_{1-x}U_x$ Conventional Kondo effect (Fermi liquid - low T)

$\Delta\gamma \approx 270$ mJ/mol U-K^2

$\Delta\rho(T) = \rho_0[1-(T/T_K)^2]$; $T_K \approx 100$ K



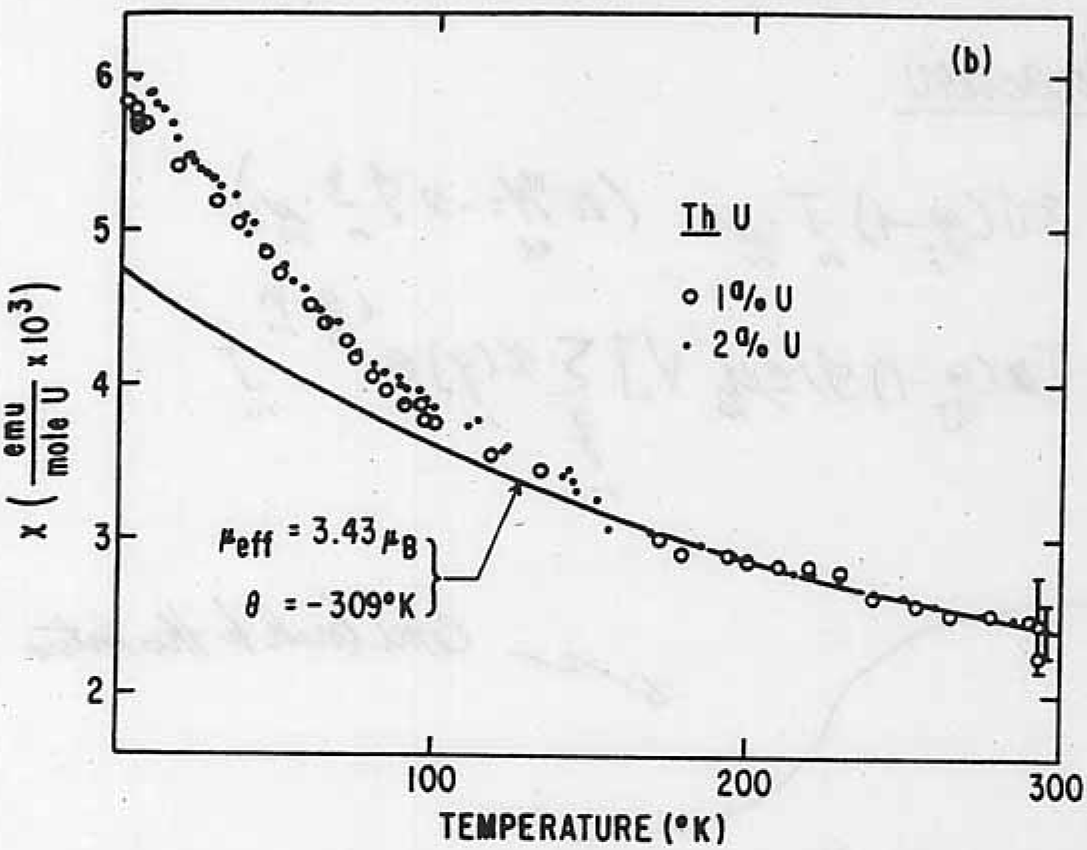
M.B. Maple et al. '70

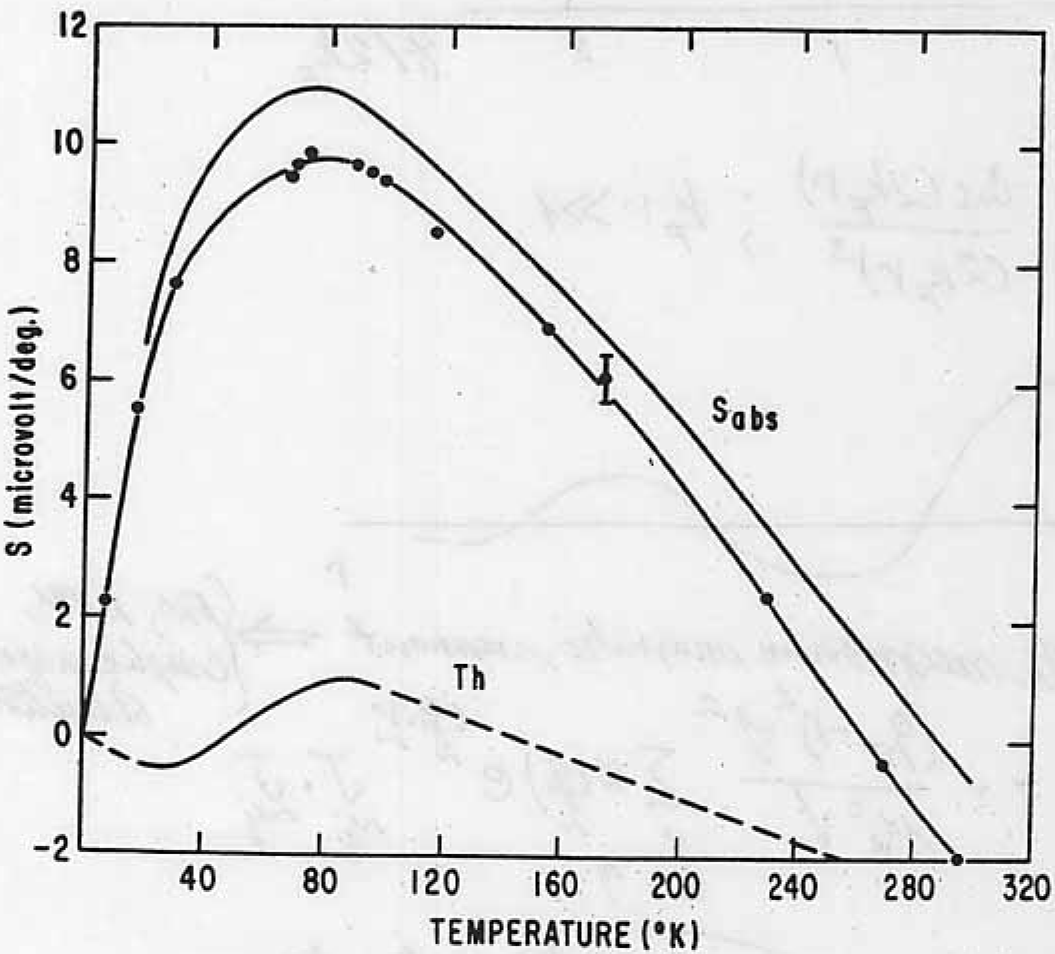
$Th_{1-x}U_x$

$$\chi(T) = C/(T-\theta)$$

$$\theta \approx -3T_K$$

$$C = N\mu_{eff}^2/3k_B$$



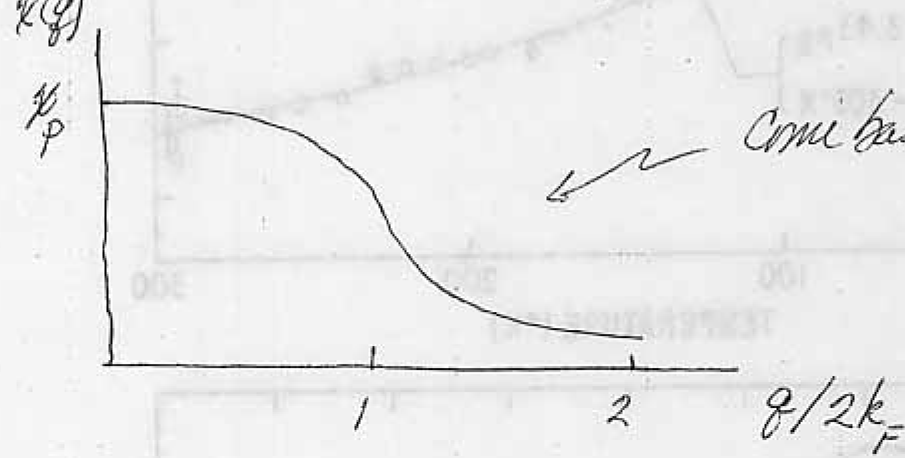


M.B. Maple et al. '70

RKKY interaction

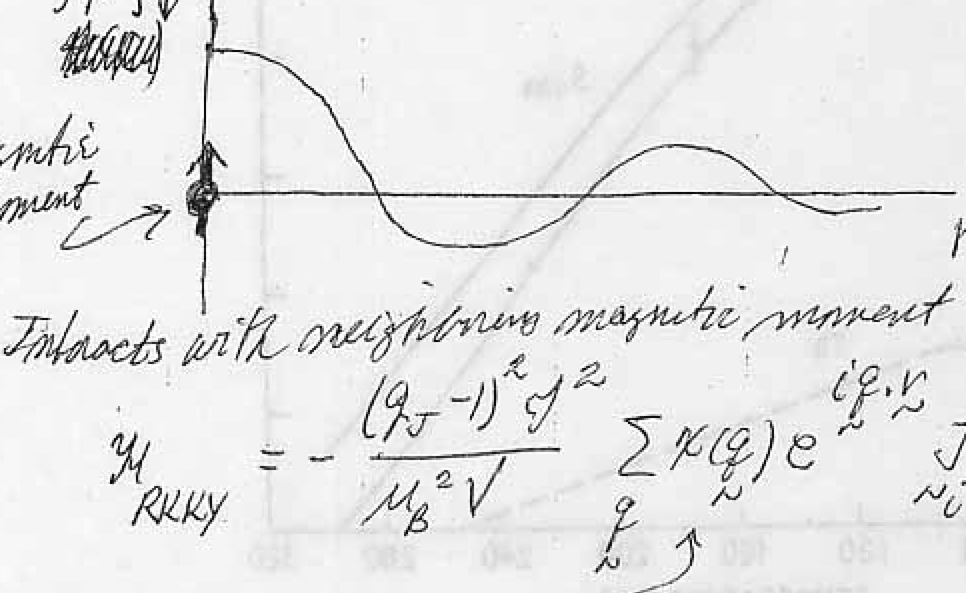
$$\mathcal{H}_{ex} = -2\mathcal{J}(g_J-1)\,\mathbf{J}\cdot\mathbf{s} \qquad (\text{or } \mathcal{H}_{ex} = -2\mathcal{J}\,\mathbf{S}\cdot\mathbf{s})$$

$$\mathbf{s}(\mathbf{r}) = [(g_J-1)\mathcal{J}/2\mu_B^2 V]\sum_{\mathbf{q}}\chi(\mathbf{q})\,e^{i\mathbf{q}\cdot\mathbf{r}}\,\mathbf{J}$$



Come back to this later

$$\mathbf{s}(\mathbf{r}) \sim \frac{\cos(2k_F r)}{(2k_F r)^3}\ ;\ k_F r \gg 1$$



Interacts with neighboring magnetic moment $\Longrightarrow$ {FM, AFM, complex magnetic structures

$$\mathcal{H}_{RKKY} = -\frac{(g_J-1)^2\mathcal{J}^2}{\mu_B^2 V}\sum_{\mathbf{q}}\chi(\mathbf{q})\,e^{i\mathbf{q}\cdot\mathbf{r}}\,\mathbf{J}_i\cdot\mathbf{J}_j$$

↑ electron band structure

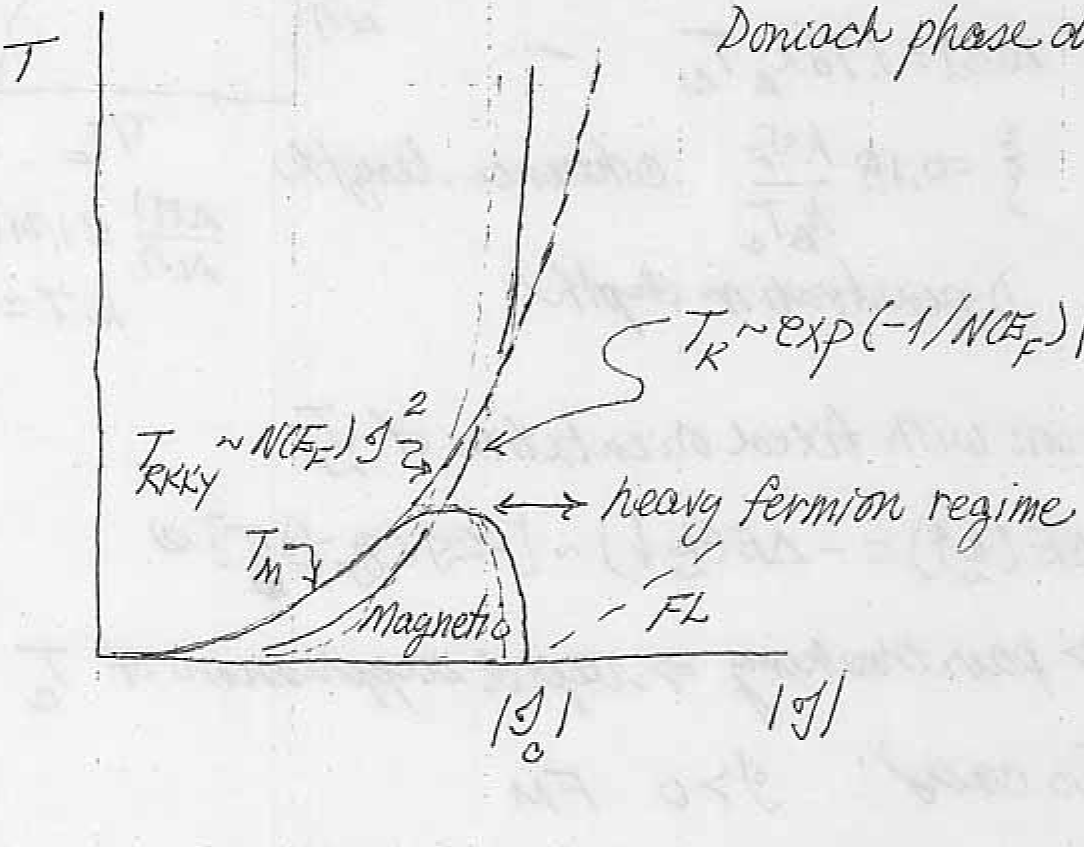
SC

Competition between magnetic and nonmagnetic ground states

Local singlet (Kondo): $T_K \sim \exp(-1/N(E_F)|J|)$

Magnetic order (RKKY): $T_{RKKY} \sim N(E_F) J^2$

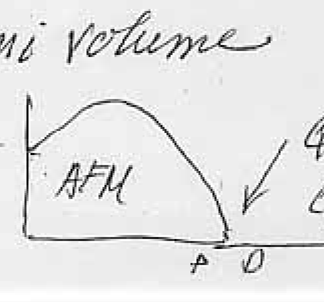
Doniach phase diagram



$0 \lesssim |J| << |J_c|$: Local moments - magnetic order
"small" Fermi volume

$|J| > |J_c|$: Local singlet - nonmagnetic
"Large" Fermi volume

NOTE: $P \Rightarrow$ increase $|J|$ for Ce



Paramagnetic impurities in conventional superconductors

SC: $(\underset{\sim}{k}\uparrow, -\underset{\sim}{k}\downarrow)$ Cooper pairs

$T_c \sim \theta_D \exp(-1/N(E_F)V)$ BCS

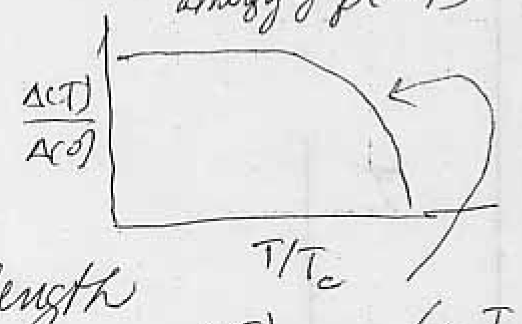
$F_n(0) - F_s(0) = \frac{1}{2} N(E_F)\Delta^2(0)$

$\Delta(0) = 1.76\, k_B T_c$

$\xi = 0.18 \frac{\hbar v_F}{k_B T_c}$ coherence length

λ penetration depth

Energy gap (OP)



$\frac{\Delta(T)}{\Delta(0)} \simeq 1.74\left(1-\frac{T}{T_c}\right)^{1/2}$ for $T \simeq T_c$

Ln ion with fixed orientation of $\underset{\sim}{J}$

$\Delta E(\underset{\sim}{k}\uparrow) = -\Delta E(\underset{\sim}{k}\downarrow) \sim [-2\mathcal{J}(g_J-1)\underset{\sim}{J}]\, s$

$\Rightarrow$ pair breaking $\Rightarrow$ rapid suppression of T_c

Two cases: $\mathcal{J} > 0$ FM

$\mathcal{J} < 0$ AFM (Kondo effect)

$\mathcal{J} > 0$ FM

$T_c/T_{c_0} = U_n(\alpha/\alpha_{cr})$ Abrikosov & Gor'kov (AG) '60

α pair breaking parameter

$\alpha = \tau^{-1} = n\hbar N(E_F)\mathcal{J}^2(g_J-1)^2 J(J+1)$ $\tau \to$ lifetime of Cooper pair

$\alpha_{cr} = k_B T_{c_0}/4\hbar\gamma$ (ln γ Euler's constant)

$\alpha \to 0$

$T_c/T_{c_0} = 1-0.691(\alpha/\alpha_{cr}) = 1-0.691(n/n_{cr})$

Linear region $n/n_{cr} \ll 1$

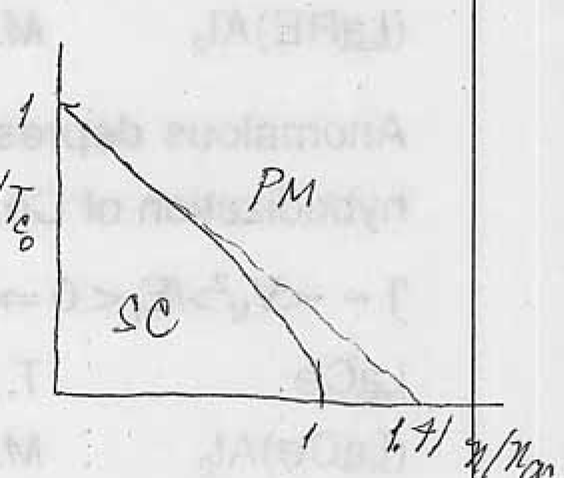
$$\left.\frac{dT_c}{dn}\right|_{n=0} = -(\pi^2/2)k_B^{-1}N(E_F)\mathcal{J}^2\underbrace{(g_J-1)^2J(J+1)}_{\text{de Gennes' factor}}$$

Other predictions

$\Delta C/\Delta C_0 = V_n(T_c/T_{c_0})$

Depression relative to BCS law of corresponding states

$(\Delta C/\Delta C_0 = T_c/T_{c_0})$

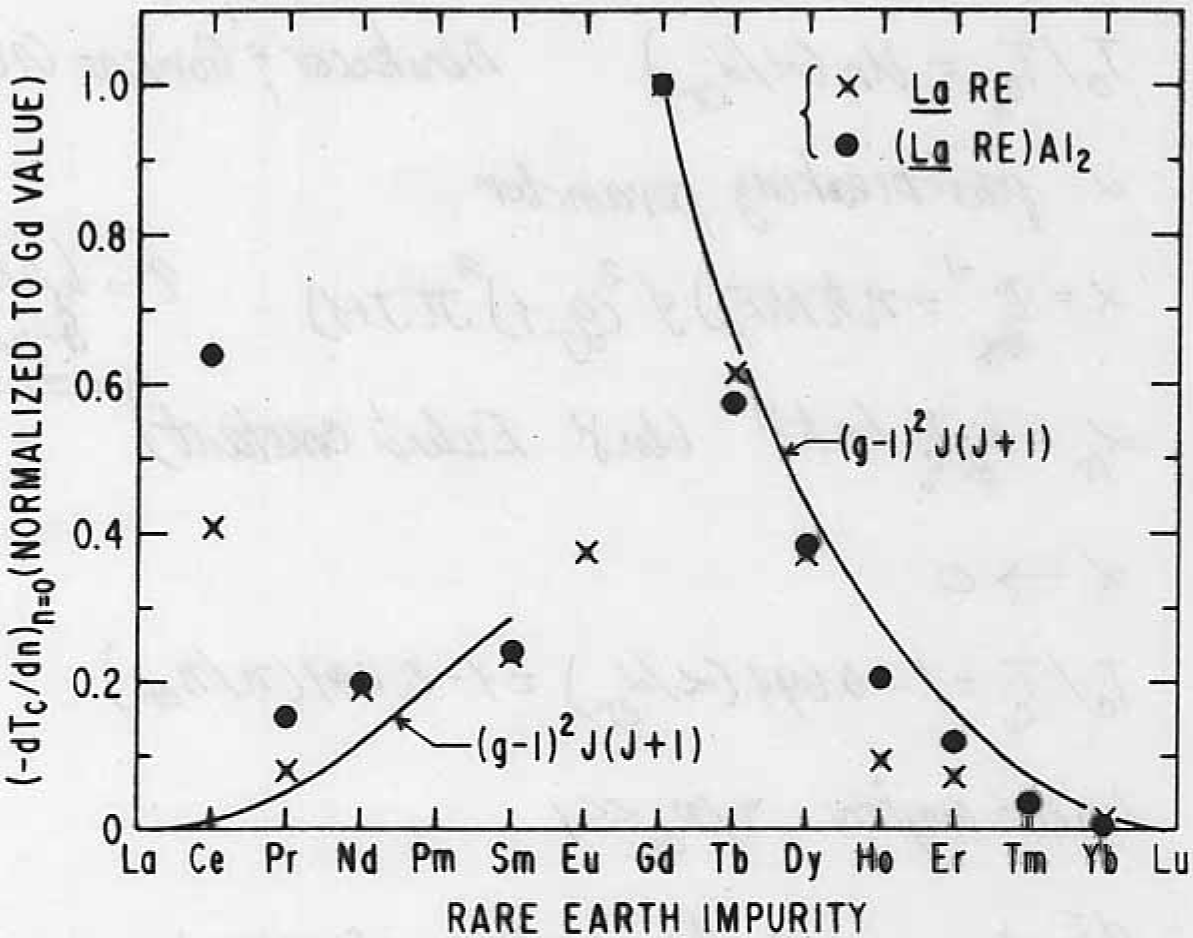


Gapless SC

$\Delta \not\leftrightarrow \Omega$

$\Omega \to 0$ faster than $\Delta \to 0$ with α

Paramagnetic impurities in superconductors



La: T_∞ = 6 K, $LaAl_2$: T_∞ = 3.3 K, $\mathcal{J}$ ~ 0.1 eV

LaRE *B. T. Matthias, H. Suhl, E. Corenzwit '58*

$(LaRE)Al_2$ *M. B. Maple '70*

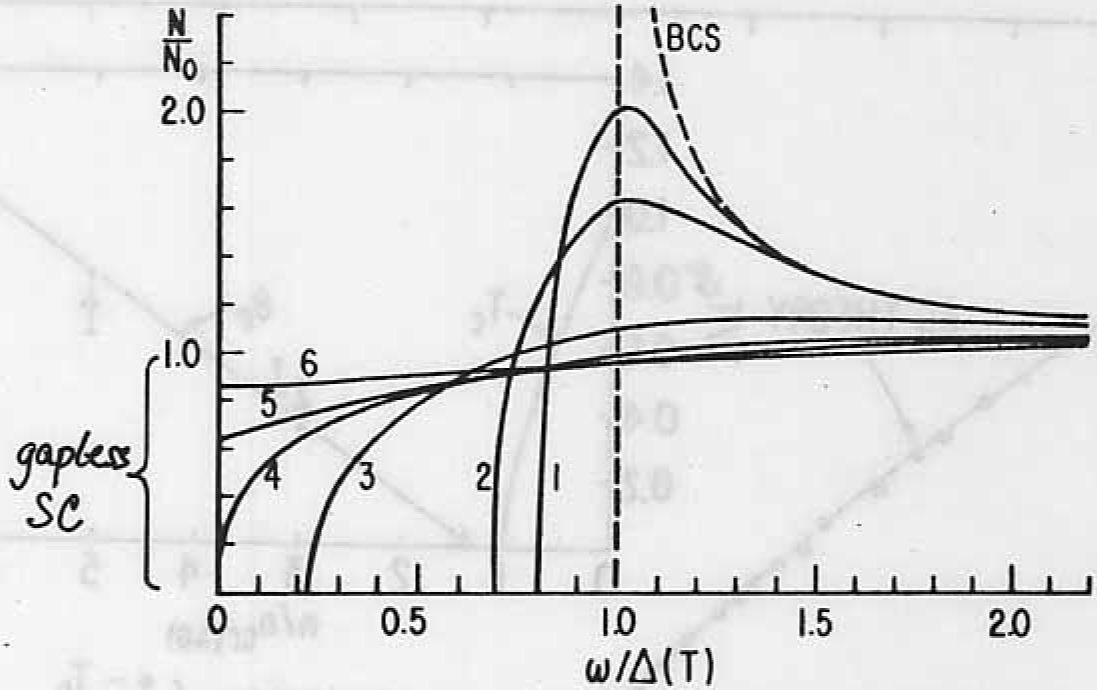
Anomalous depression of T_c for Ce ⇒

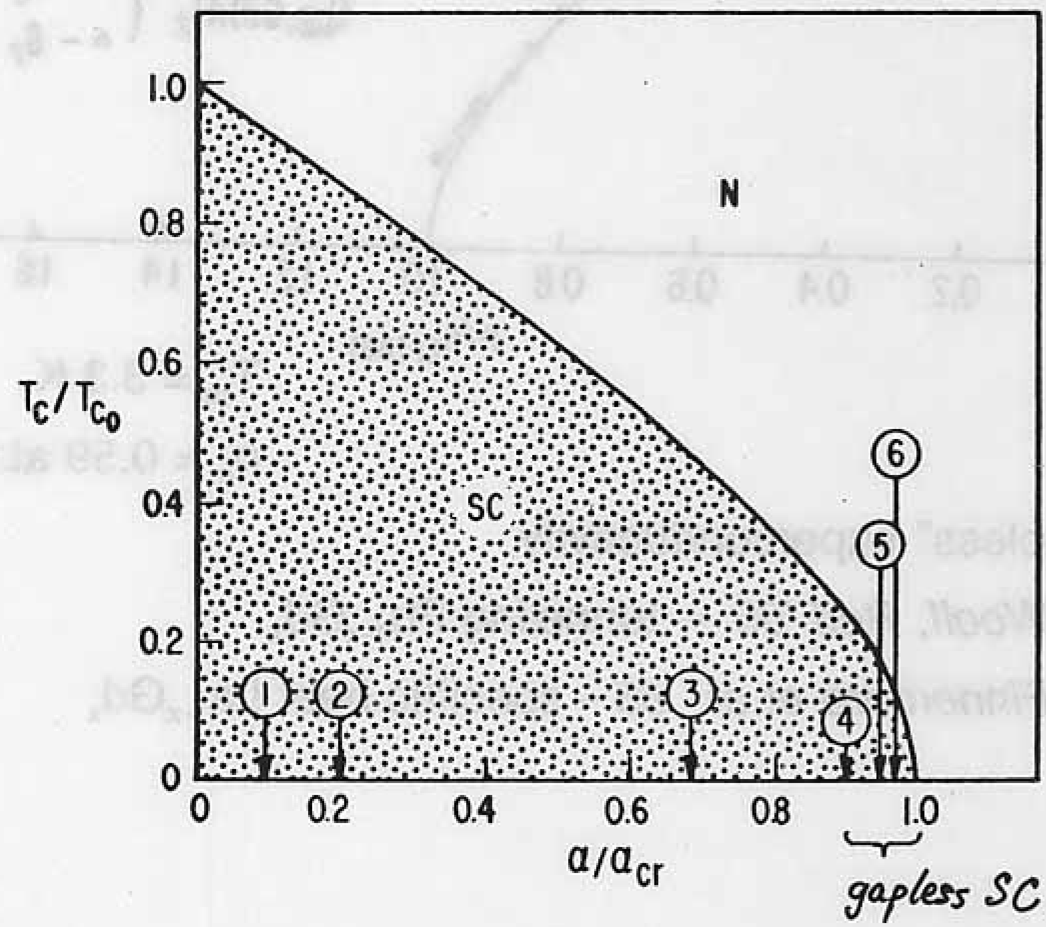
hybridization of Ce 4f and conduction electron states ⇒

$\mathcal{J} \sim -\langle V_{kf}^2 \rangle / E_f < 0$ ⇒ Kondo effect

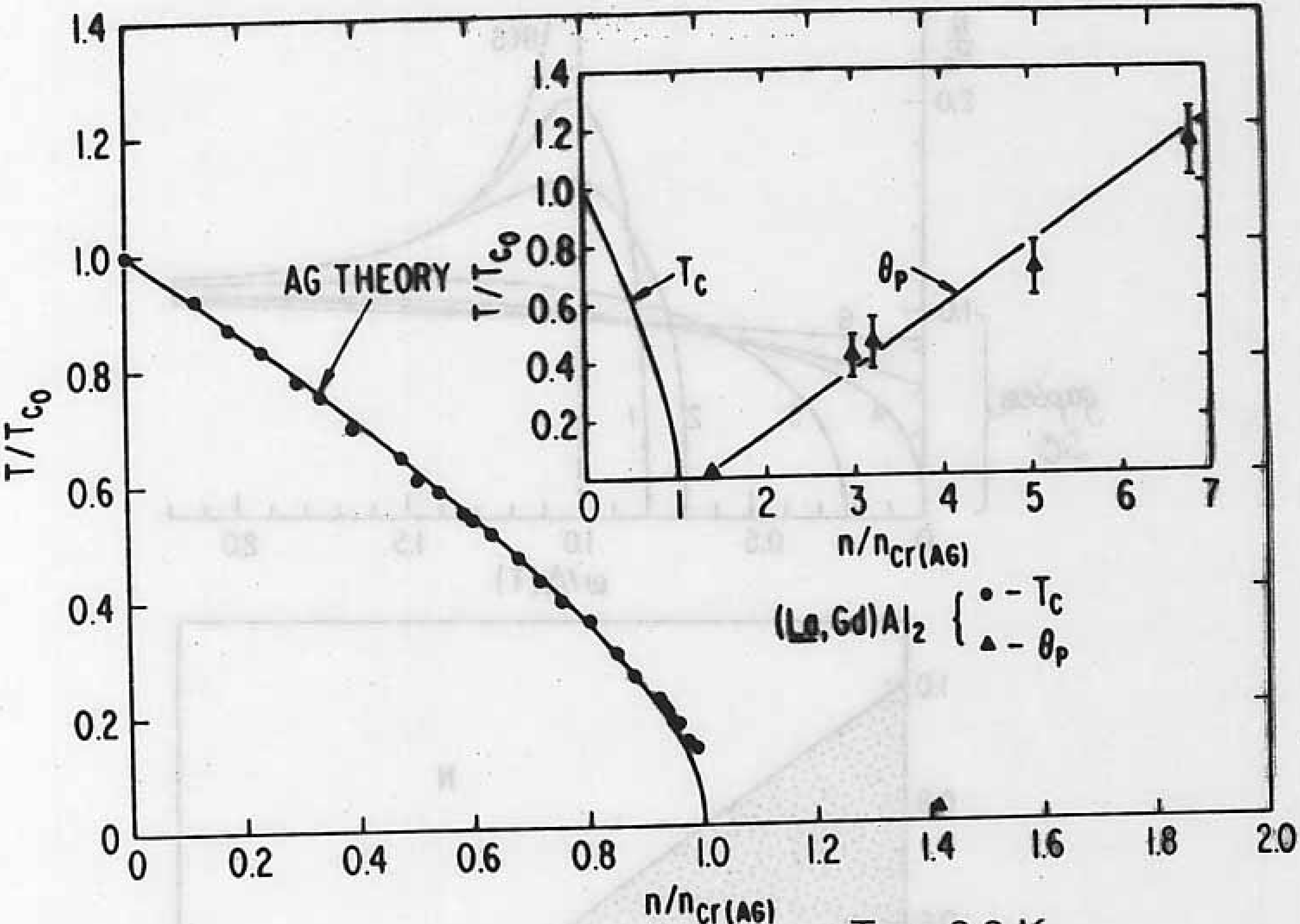
LaCe *T. Sugawara, H. Eguchi '66*

$(LaCe)Al_2$ *M. B. Maple, Z. Fisk '68*


N/N0
BCS
2.0
1.0
6
5
4
3
2
1
gapless SC
0
0.5
1.0
1.5
2.0
ω/Δ(T)


1.0
0.8
0.6
0.4
0.2
0
N
SC
Tc/Tc0
1
2
3
4
5
6
0
0.2
0.4
0.6
0.8
1.0
α/αcr
gapless SC

Paramagnetic impurities in superconductors



$T_{\infty} = 3.3$ K

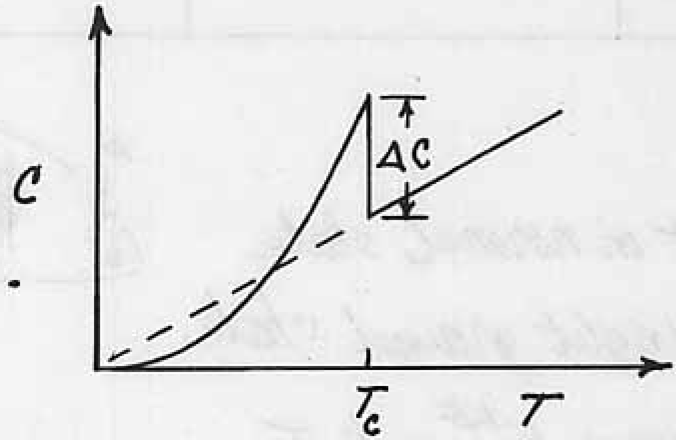
$n_{\sigma} = 0.59$ at.% Gd

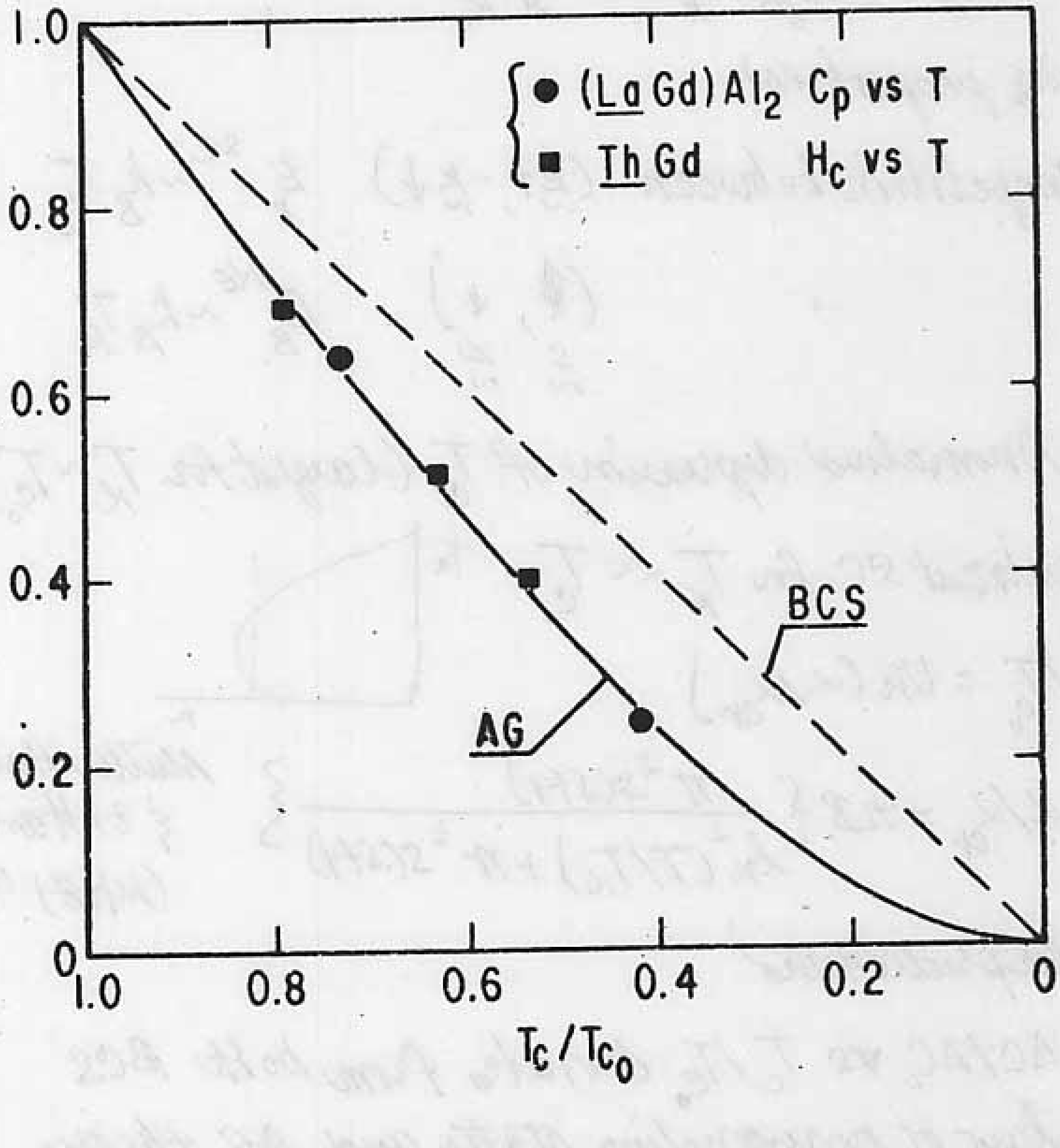
"Gapless" superconductivity

Woolf, Reif '65 — tunneling $Pb_{1-x}Gd_x$

Finnemore et al. '65 – specifric heat $La_{1-x}Gd_x$

M. B. Maple '68





W. R. Decker & D. K. Finnemore (1968)
C. A. Luengo & M. B. Maple (1973)

$\mathcal{J} < 0$ AFM

Kondo effect in normal state

Many body singlet ground state

Binding energy $E_B^{KE} \sim k_B T_K$

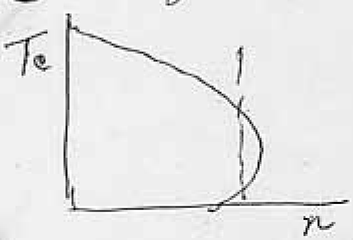
SCing properties

Competition between $(\underset{\sim}{k}\uparrow, -\underset{\sim}{k}\downarrow)$ $\quad E_B^{SC} \sim k_B T_{c_0}$

$(\Uparrow, \downarrow)$ with $\underset{\sim}{S}$, $\underset{\sim}{s}$ $\quad E_B^{KE} \sim k_B T_K$

$\Rightarrow$ Anomalous depression of T_c (largest for $T_K \sim T_{c_0}$)

Reentrant SC for $T_K \ll T_{c_0}$



$T_c / T_{c_0} = U_n(\alpha/\alpha_{cr})$

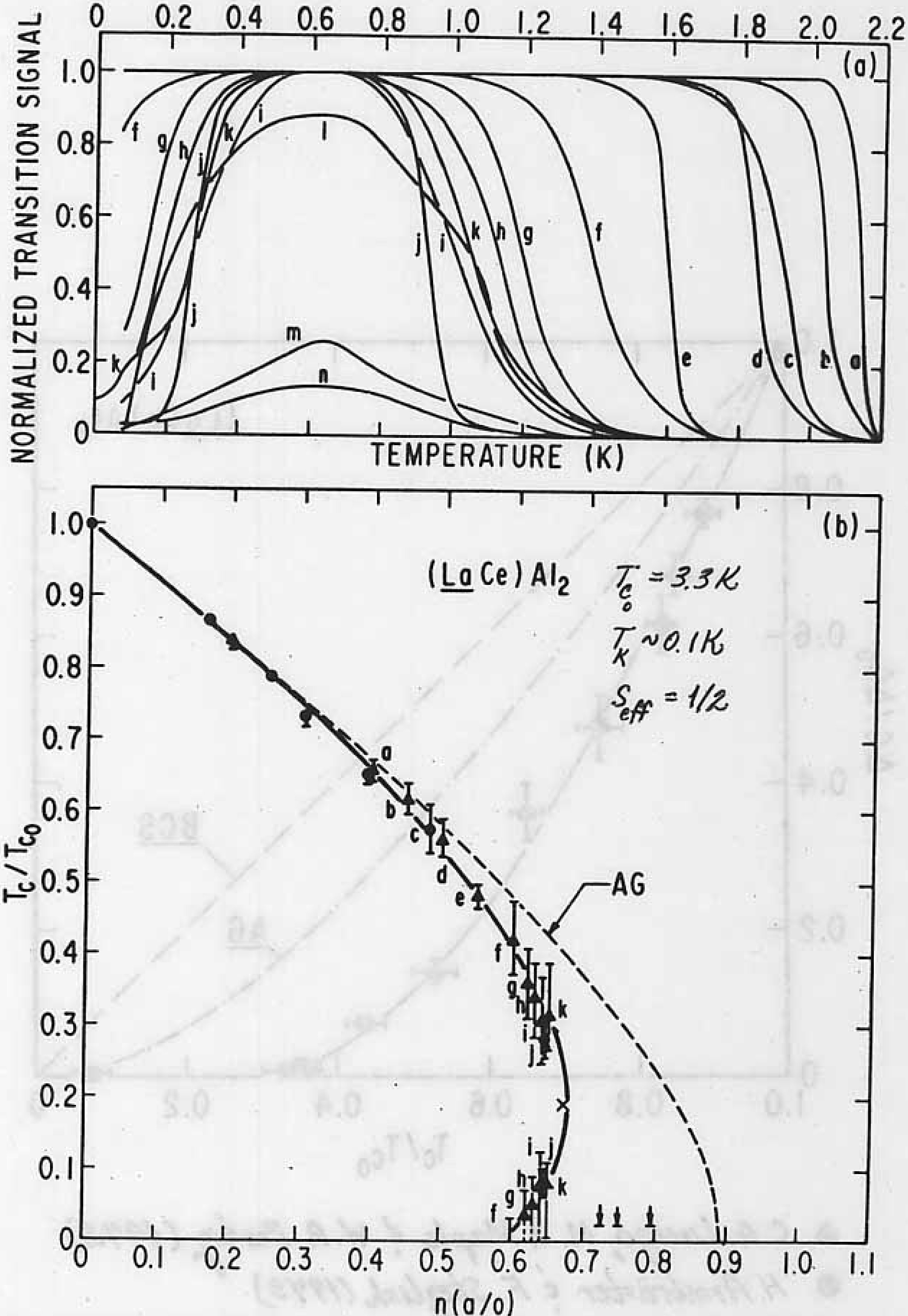
$$\alpha/\alpha_{cr} = nB\left\{\frac{\pi^2 S(S+1)}{\ln^2(T/T_K) + \pi^2 S(S+1)}\right\}$$

Müller Hartmann & Zittartz (MHZ) '71

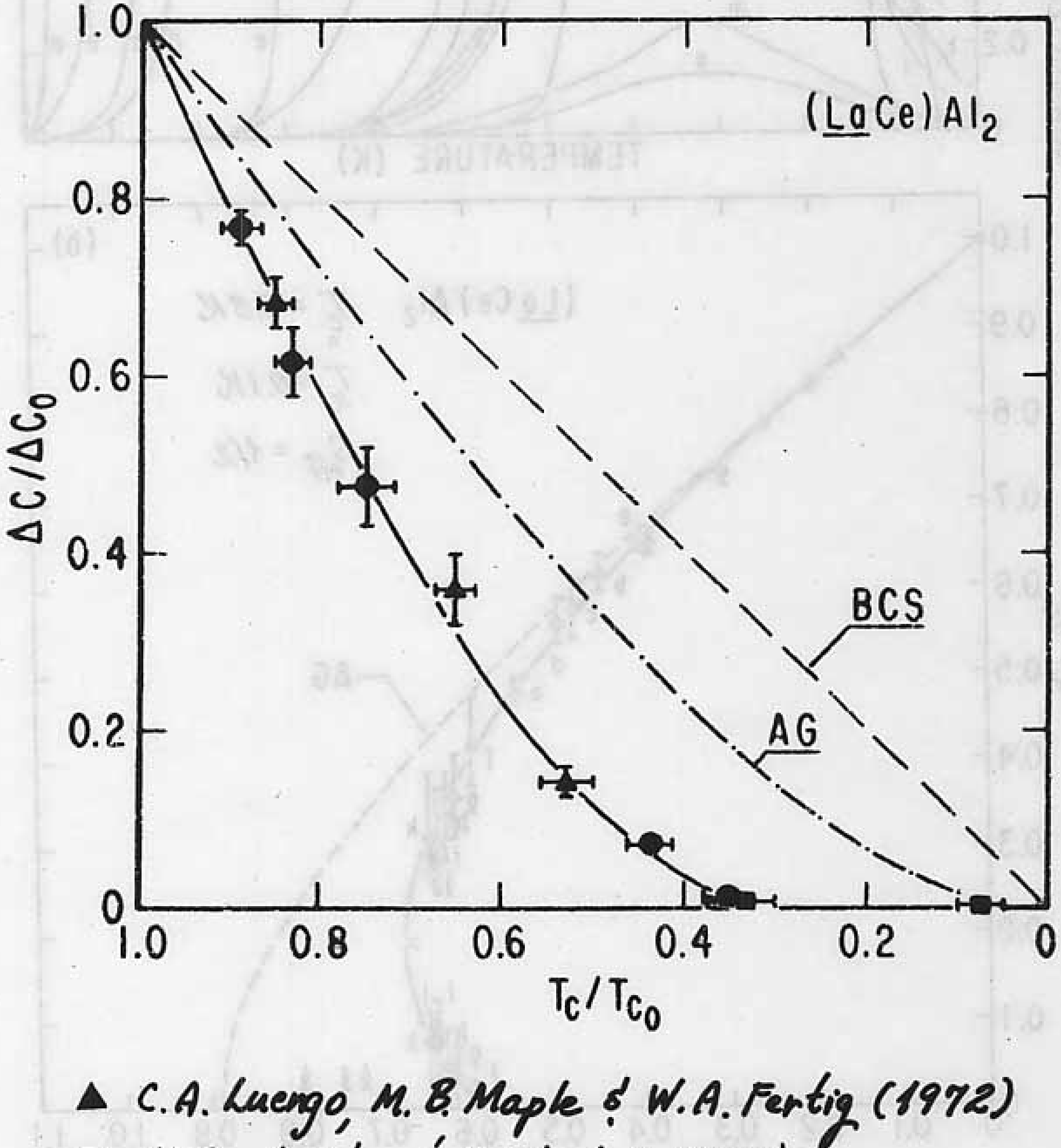
Other predictions

$\Delta C/\Delta C_0$ vs T_c/T_{c_0} deviates from both BCS law of corresponding states and AG theory

Bound state in gap



M. B. Maple, W. A. Fertig, A. C. Mota, L. E. DeLong, D. Wohlleben & R. Fitzgerald (1972)



▲ C.A. Luengo, M.B. Maple & W.A. Fertig (1972)

● H. Armbrüster & F. Steglish (1973)

■ S.D. Bader, N.E. Phillips, M.B. Maple & C.A. Luengo (1972)

$T_K \gg T_{c_0}$

$T_c = T_{c_0} \exp[-An/(1-Dn)]$ A.B. Kaiser '7

Based on nonmagnetic resonant state at E_F

① Reduction of $N(E_F)$ – one body dilution effect

② Reduction of V by Coulomb repulsion between electrons that scatter into resonant state – two body effect

BCS law of corresponding states

$\Delta C / \Delta C_0 = T_c / T_{c_0}$

Magnetic – nonmagnetic transition

$|J| \sim - \frac{\langle V_{kf}^2 \rangle}{\epsilon_f}$

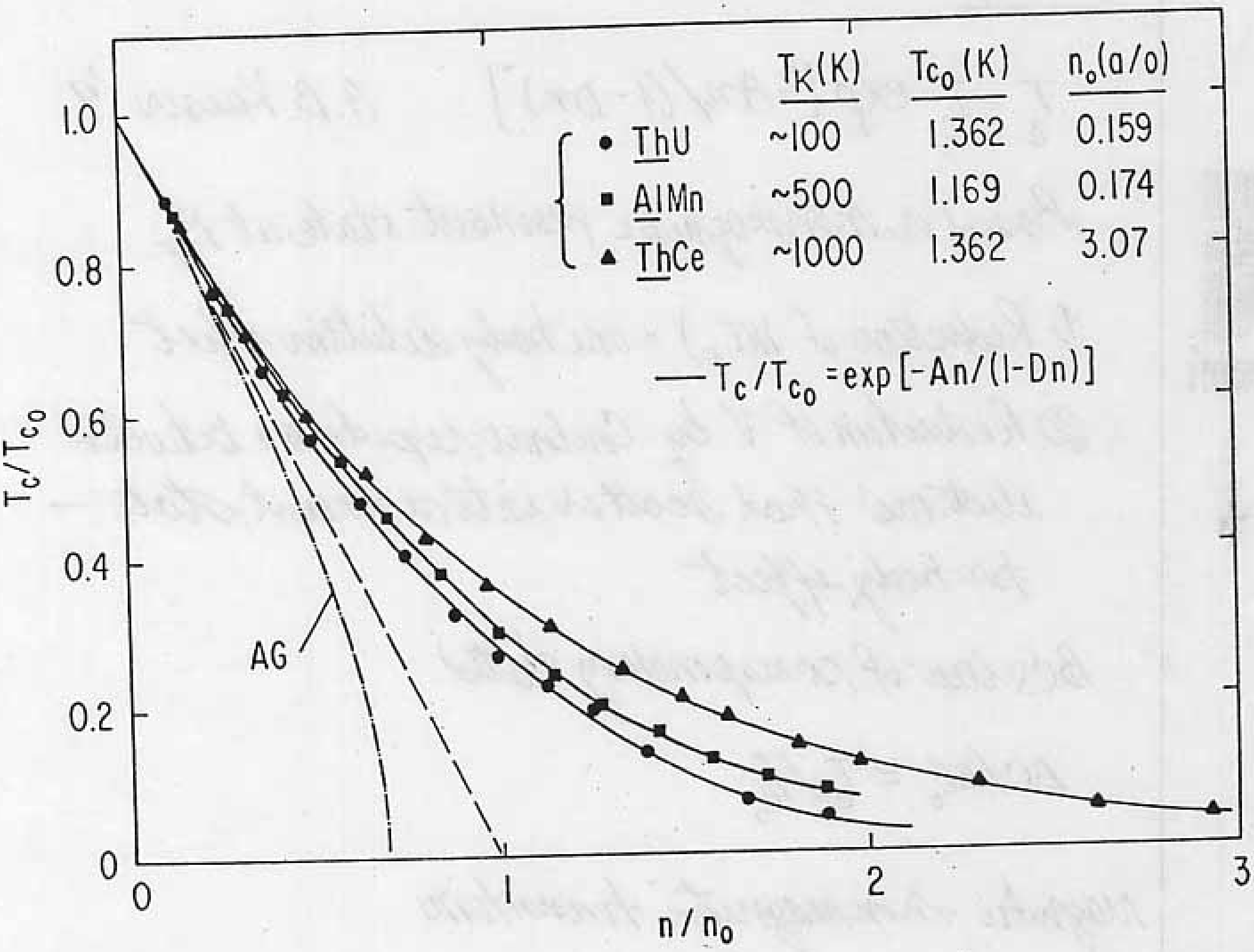
$P \Rightarrow$ increase of $\langle V_{kf}^2 \rangle$ and/or decrease of ϵ_f

$\Rightarrow$ increase of T_K

$(dT_c/dn)_{n=0}$ passes through maximum at $T_K \sim T_{c_0}$

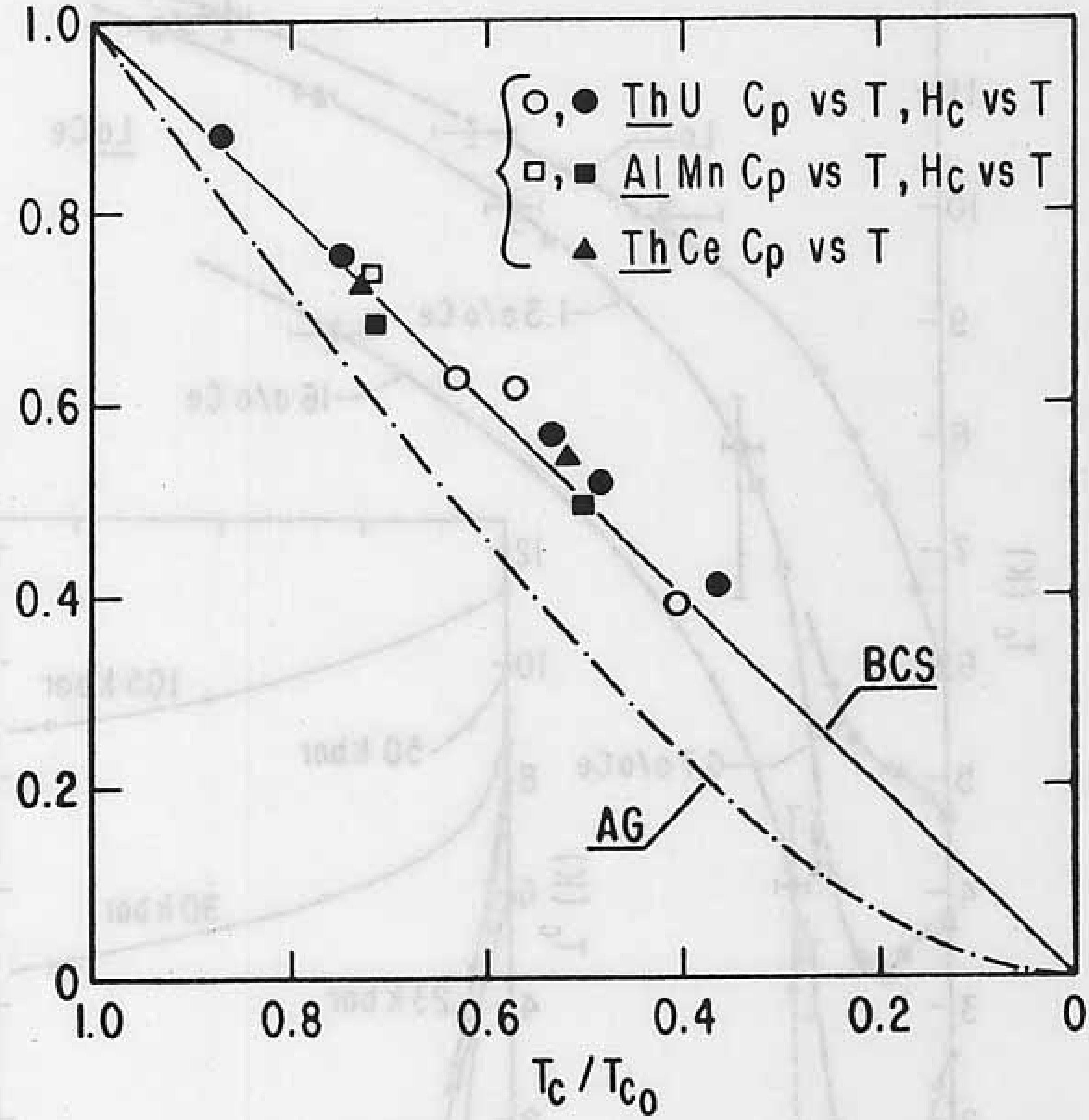
Paramagnetic impurities in superconductors

Kondo effect: $T_K \gg T_{c_0}$



M.B. Maple, J.G. Huber, B.R. Coles, A.C. Lawson '70

J.G. Huber & M.B. Maple '70



○ C.A. Luengo, J.M. Cotignola, J. Sereni, A.R. Sweedler, M.B. Maple & J.G. Huber (1972)

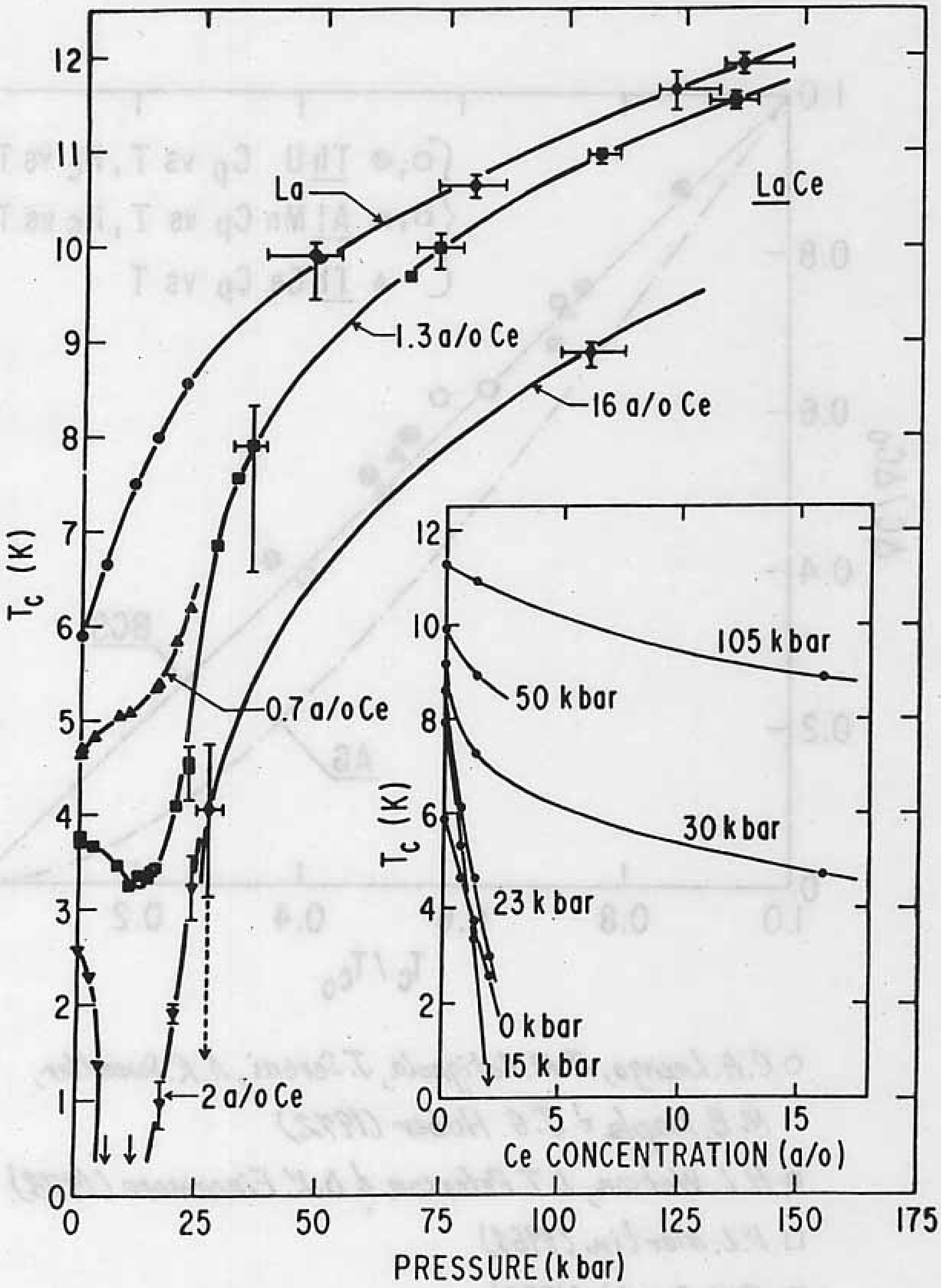
● H.L. Watson, D.T. Peterson & D.K. Finnemore (1973)

□ D.L. Martin (1961)

■ F.W. Smith (1972)

▲ C.W. Dempesy (1970)

Pressure-induced demagnetization of Ce impurities in La



Analogue of γ-α transition in Ce

M.B. Maple, J. Wittig, K.S. Kim '69

▸ Magnetic–nonmagnetic transition of Ce impurities in (La, Th) alloys –

(La, Th) Ce   note La Ce $\begin{cases} T_{c_0} = 6K \\ T_K \sim 0.1K \end{cases}$

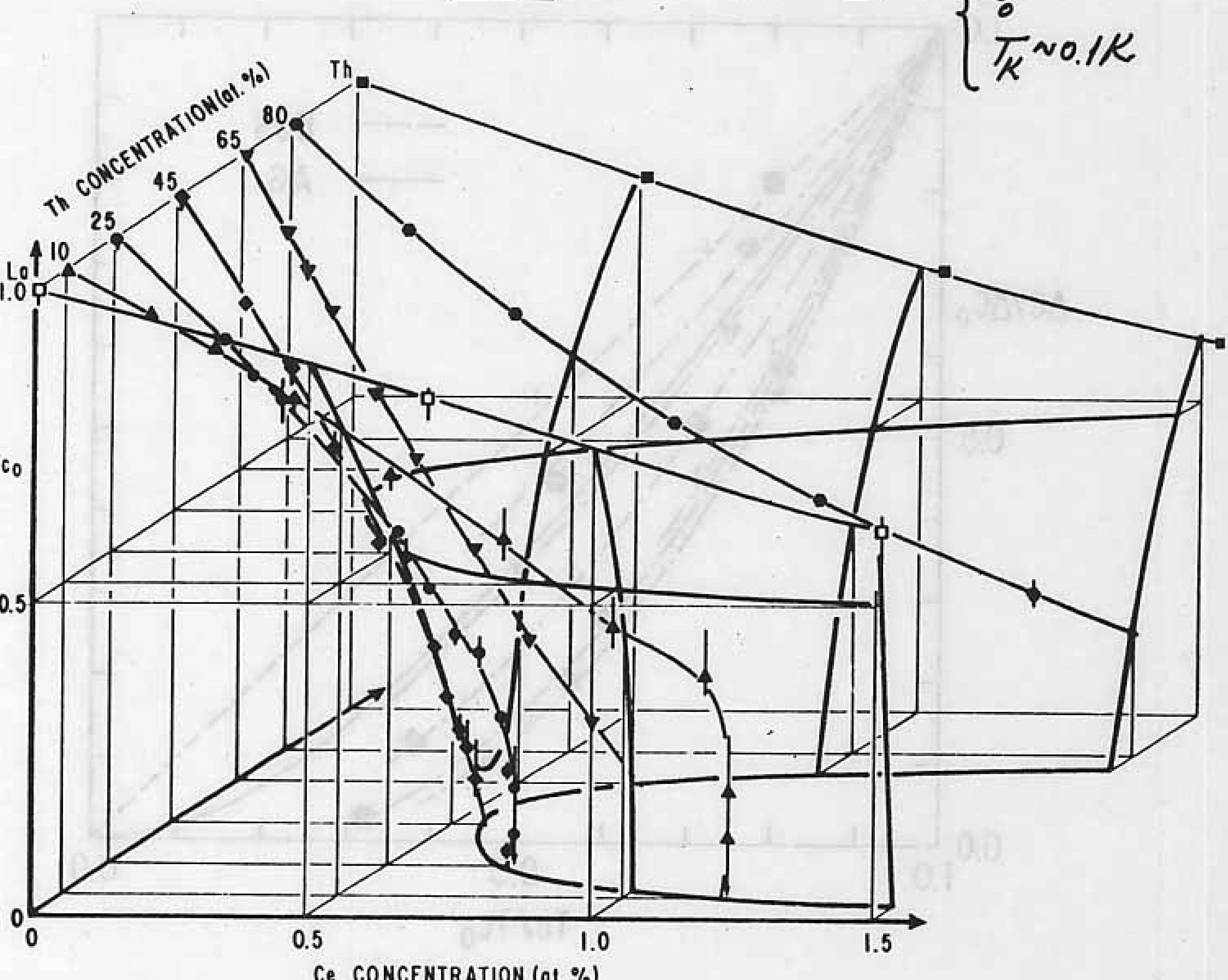
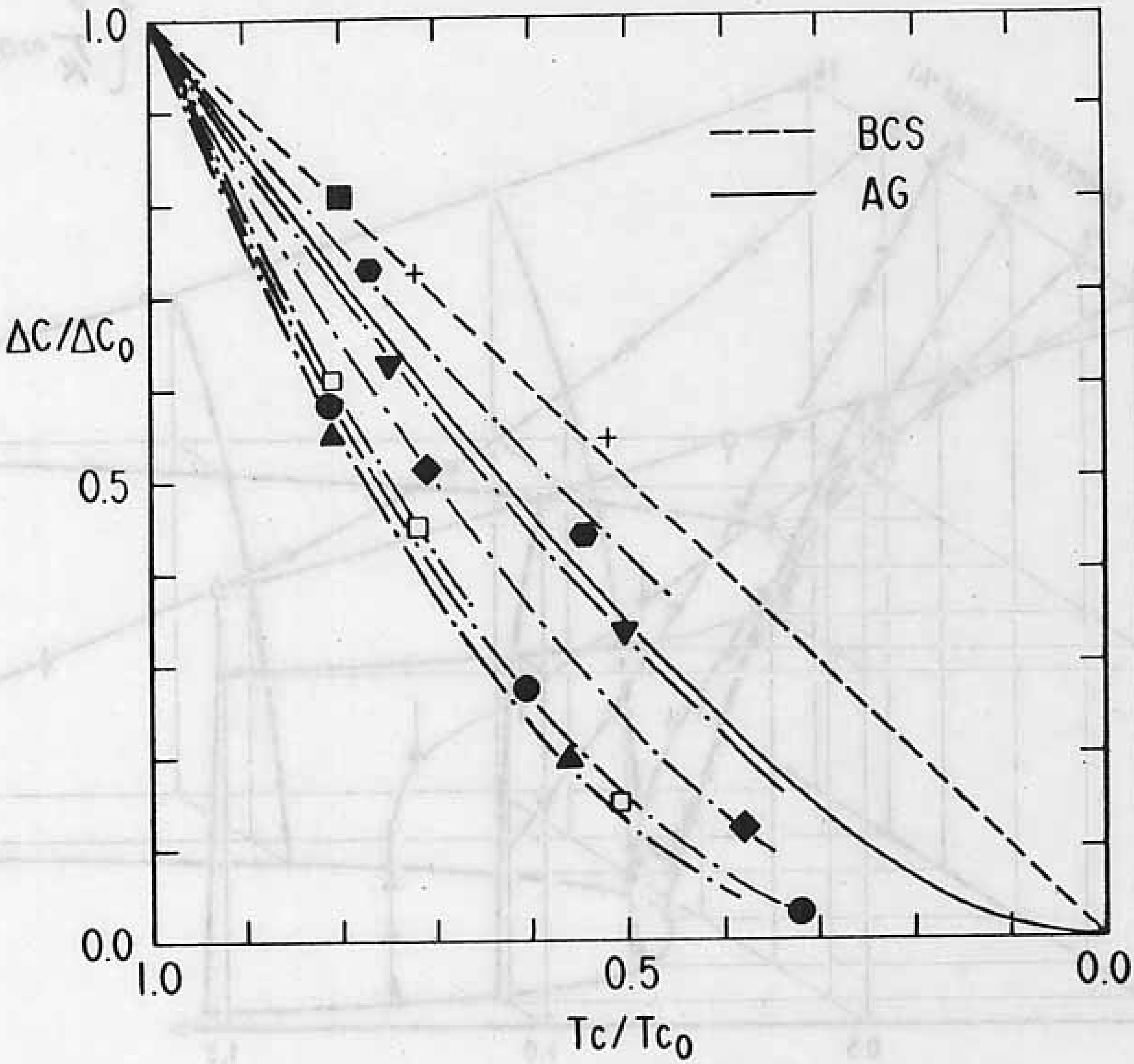


Fig. 18

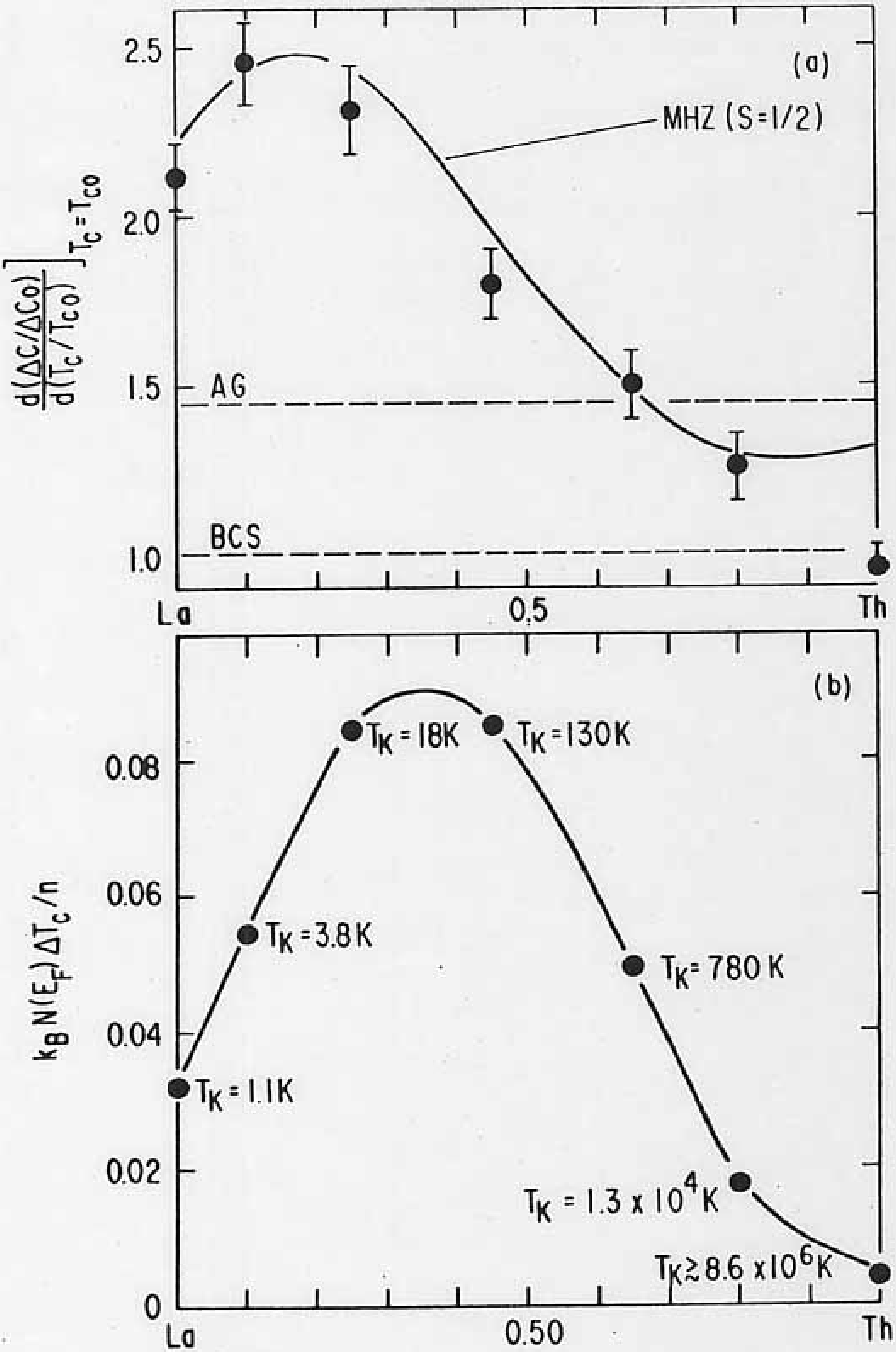
J.G. Huber, W.A. Fertig & M.B. Maple (1974)

$T_K \sim T_F \exp(-1/N(E_F)|\mathcal{J}|)$; $\mathcal{J} \sim -\frac{\langle V_{kf}^2 \rangle}{E_f}$

$P \Rightarrow$ increase $|\mathcal{J}| \Rightarrow$ increase $\langle V_{kf}^2 \rangle$ and/or decrease E_f



C.A. Luengo, J.G. Huber, M.B. Maple & M. Roth (1974)



C.A. Luengo, J. G. Huber, M. B. Maple ē M. Roth (1974)