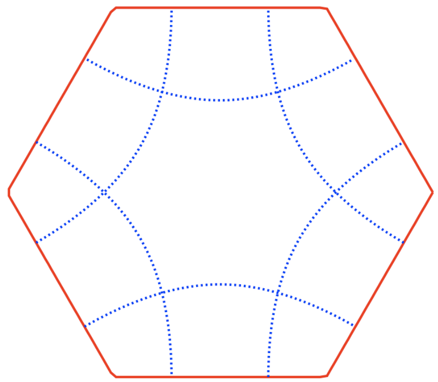
Gapless Spin Liquids in Two Dimensions

MPA Fisher (with O. Motrunich, Donna Sheng, Matt Block)

Boulder Summerschool
7/20/10

Interest – Quantum Phases of 2d electrons (spins) with emergent rather than broken symmetry



Focus on *gapless "Spin liquids"*

Especially - "Spin-Bose-Metals"

Spin liquids with "Bose" surfaces in momentum space

Access Quasi-1d descendent states on ladders

Useful references

Spin, Bose, and Non-Fermi Liquid Metals in Two Dimensions: Accessing via Multi-Leg Ladders; MPAF Fisher et al. **arXiv:0812.2955v1**

The introductions to the following 3 papers might be useful to look at:

d-wave correlated critical Bose liquids in two dimensions, O.Motrunich et al, PHYSICAL REVIEW B 75, 235116, 2007

Strong-coupling phases of frustrated bosons on a two-leg ladder with ring exchange, D. Sheng et al. PHYSICAL REVIEW B 78, 054520, 2008

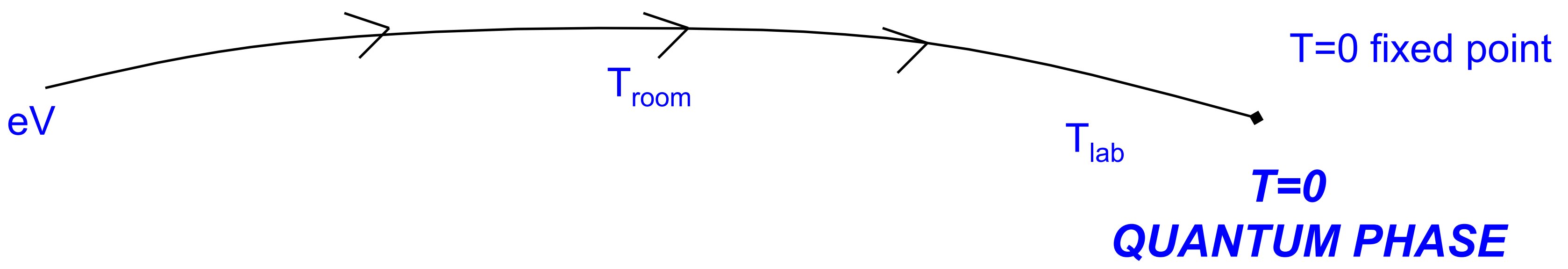
Spin Bose-metal phase in a spin-1/2 model with ring exchange on a two-leg triangular strip, D. Sheng et al. PHYSICAL REVIEW B 79, 205112, 2009

"Simplicity" of Electrons in solids

Separation of energy scales for the electrons;

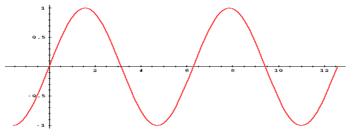
Kinetic energy
and Coulomb energy: E_{KE} , E_{coul} >> T_{room} >> T_{lab}

RG Flows

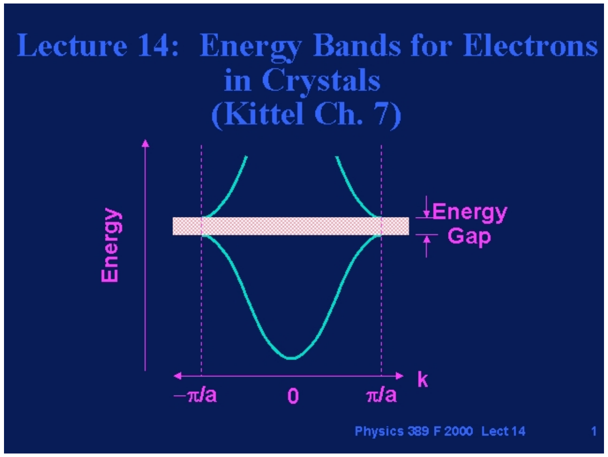


Band Theory: Metals versus insulators

$$H = \sum_j \frac{\mathbf{p}_j^2}{2m} + \sum_i V(\mathbf{r}_i)$$



- Energy Bands
- ***Band insulators***: Filled bands
- ***Metals***: Partially filled highest energy band



Even number of electrons/cell - (usually) a band insulator

Odd number per cell - always a metal

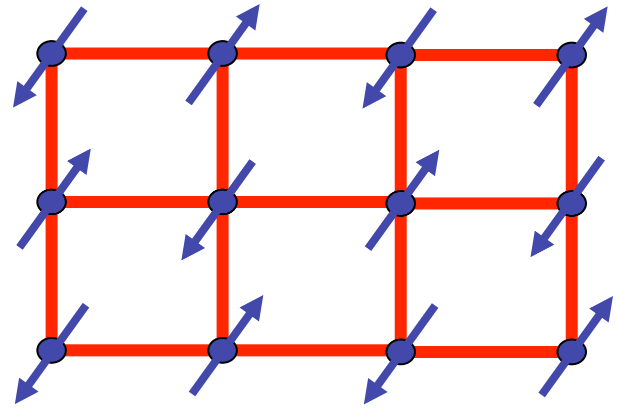
Quantum Theory of Solids: 2 dominant phases

Odd number of electrons/cell $\longrightarrow$ Metals

Even number of electrons/cell $\longrightarrow$ Band Insulators

But most d and f shell crystals with odd number of electrons are NOT metals

Due to Coulomb repulsion electrons gets stuck on atoms



“Mott Insulators”

Sir Neville Mott

Mott Insulators:
Insulating materials with odd number of electrons/unit cell

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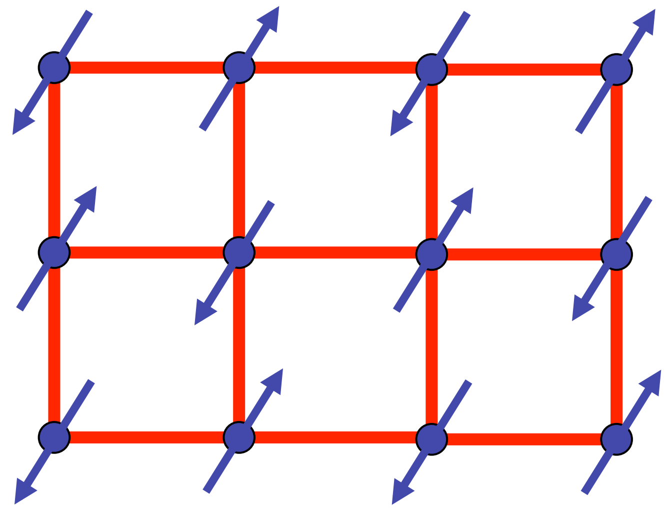
Insulating materials with an odd number of electrons/unit cell

Hubbard model with one electron per site on average:

$$\mathcal{H} = -t \sum_{\langle ij \rangle} [c^{\dagger}_{i\alpha} c_{j\alpha} + h.c.] + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$

For U>>t electron gets self-localized



Residual spin physics:

s=1/2 operators on each site

Heisenberg Hamiltonian:

$$H_{spin} = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j + \ldots$$

Quantum Phases of Electrons

Odd number of electrons/cell
(from atomic s or p orbitals) → Metal

Even number of electrons/cell → Band Insulator

Odd number of electrons/cell
(atomic d or f orbitals) → Mott insulator

Symmetry breaking in Mott insulators

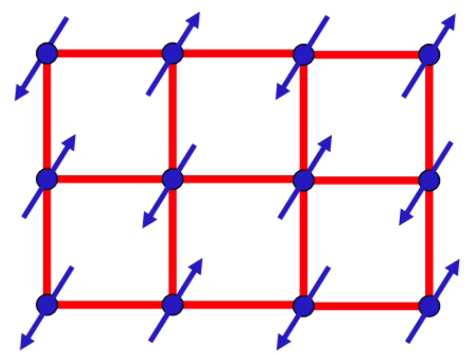
Mott Insulator → Unit cell doubling ("Band Insulator")

Symmetry breaking

2 electrons/cell

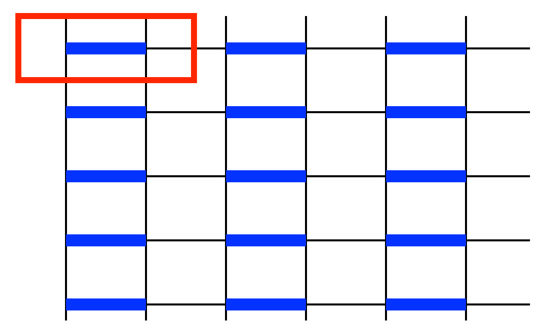
- Magnetic Long Ranged Order

Ex: 2d square Lattice AFM



- Spin Peierls

2 electrons/cell



Valence Bond

$$— \; = \; |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Quantum Phases of Electrons

Odd number of electrons/cell
(from atomic s or p orbitals) → Metal

Even number of electrons/cell → Band Insulator

Odd number of electrons/cell
(atomic d or f orbitals) → Mott insulator

Mott insulator → Symmetry breaking eg AFM

Mott insulator → ???

2d Spin liquids

Spin liquid – 2d Mott insulator with no broken symmetries

Theorem (Matt Hastings, 2005):
Mott insulators on an L by L torus have a low energy excitation with $(E_1 - E_0) < \ln(L)/L$

Implication: 2d Spin liquids are either Topological or Gapless

Quantum Phases of Electrons

Odd number of electrons/cell
(from atomic s or p orbitals) → Metal

Even number of electrons/cell → Band Insulator

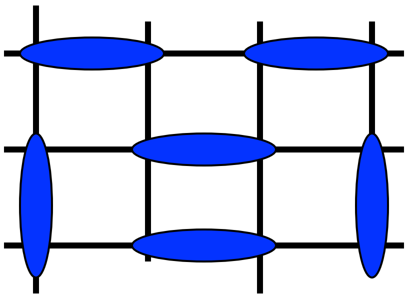
Odd number of electrons/cell
(atomic d or f orbitals) → Mott insulator

Mott insulator → Symmetry breaking
eg AFM

Mott insulator → $\boxed{\textbf{\textit{Spin Liquid}}}$

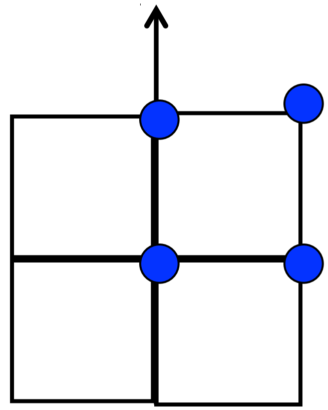
3 classes of 2d Spin liquids

Topological Spin Liquids



- ***Gap to all bulk excitations -*** (degeneracies on a torus)
- "Particle" excitations with fractional quantum numbers, eg spinon
- Simplest is short-ranged RVB, Z_2 Gauge structure

Algebraic Spin Liquids

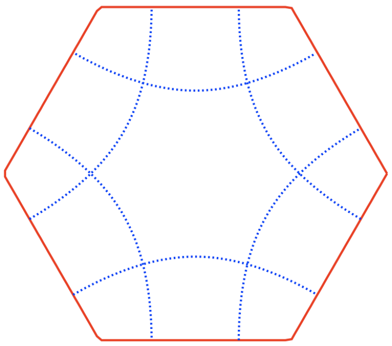


- ***Stable gapless phase*** with no broken symmetries
- no free particle description
- Power-law correlations at finite set of discrete momenta

"Spin Bose-Metals"

Gapless spin liquids with spin correlation functions singular along surfaces in momentum space

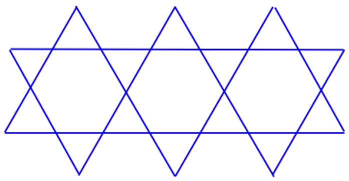
"Bose Surfaces"



2 Routes to gapless spin liquids

1.) Frustration, low spin, low coordination number

s=1/2 Kagome lattice AFM



Herbertsmithite $ZnCu_3(OH)_6Cl_2$

"Algebraic" spin liquids

2.) Quasi-itinerancy: "weak" Mott insulator with small charge gap

Charge gap comparable to exchange J

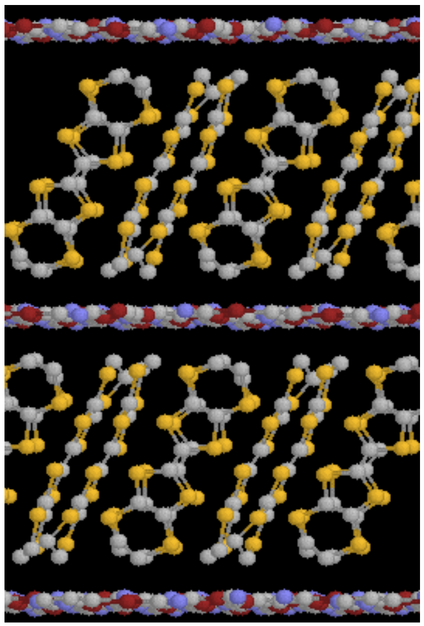
"Spin Bose-Metal" ?

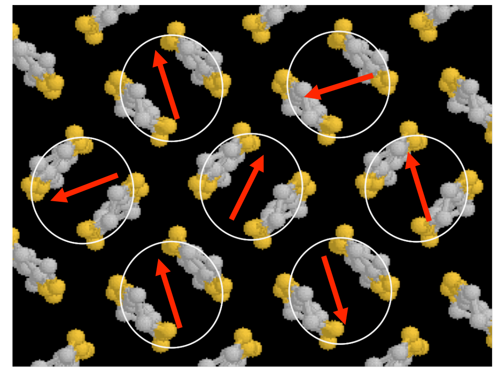
Organic Mott insulators; s=1/2 on a Triangular lattice

κ-(ET)$_2$X

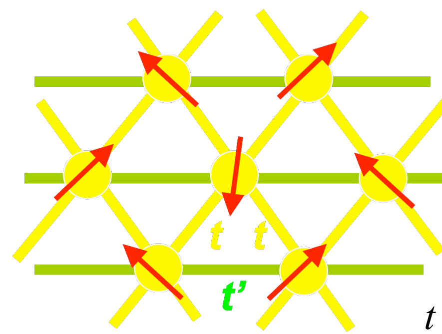
ET layer

X layer





Kino & Fukuyama dimer model



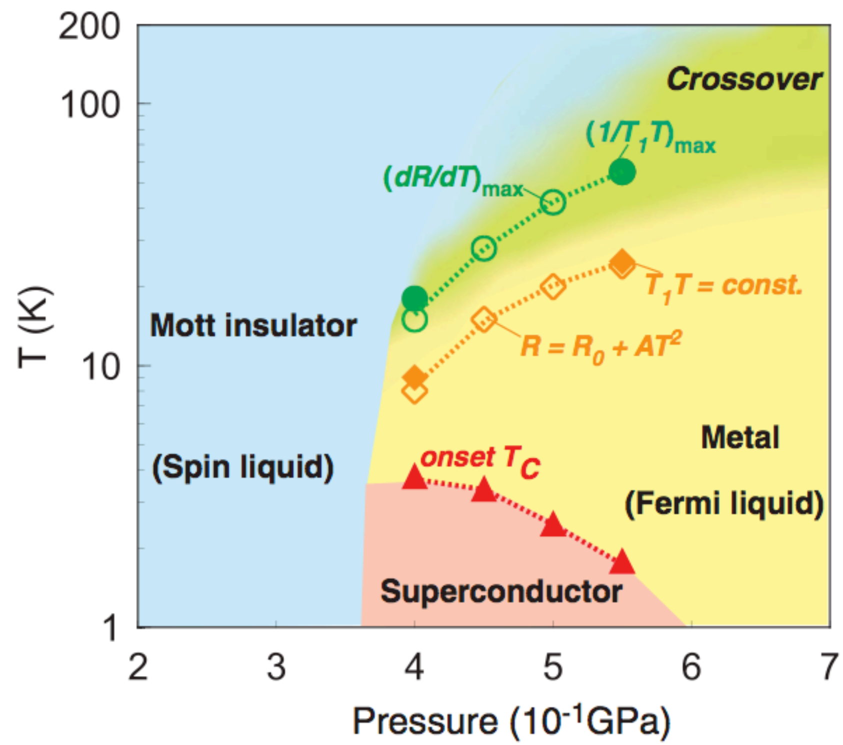
$t'/t = 0.5 \sim 1.1$

X = Cu(NCS)$_2$, Cu[N(CN)$_2$]Br, Cu$_2$(CN)$_3$.....

(Anisotropic) Triangular lattice
Half-filled Hubbard band

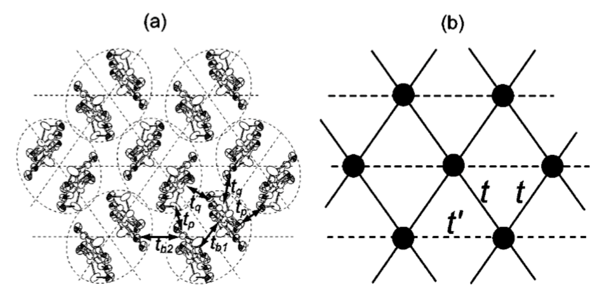
Candidate Spin Bose-Metal: k-$(ET)_2Cu_2(CN)_3$

Kanoda et. al. PRL 91, 177001 (2005)



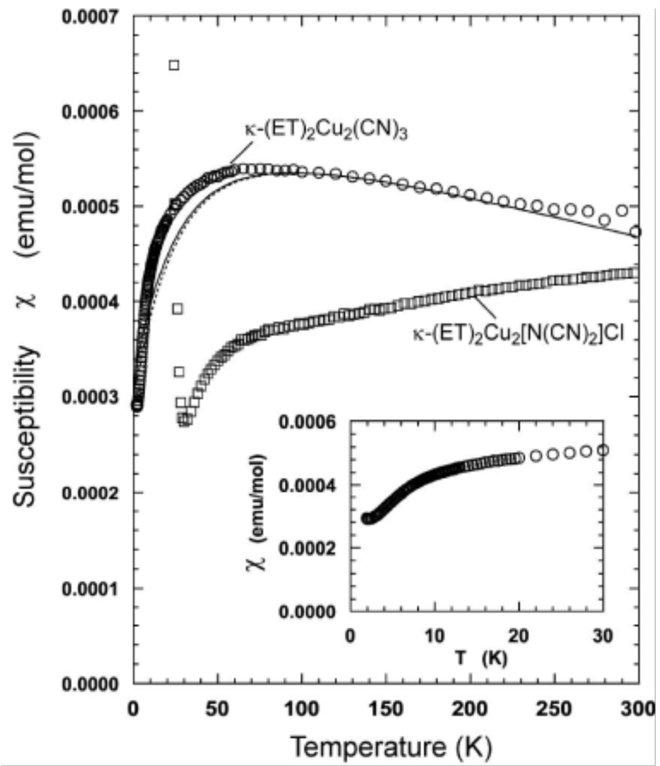
- Isotropic triangular Hubbard at half-filling
- Weak Mott insulator – metal under pressure
- No magnetic order down to 20mK ~ 10^{-4} J
- Pauli spin susceptibility as in a metal
- "Metallic" specific heat, C~T, Wilson ratio of order one

Motrunich (2005) , S. Lee and P.A. Lee (2005) suggested spin liquid with "spinon Fermi surface"



Spin and charge physics

Spin susceptibility J=250K



Shimizu et.al. 03

Charge Transport: small gap 200 K
"Weak Mott insulator"

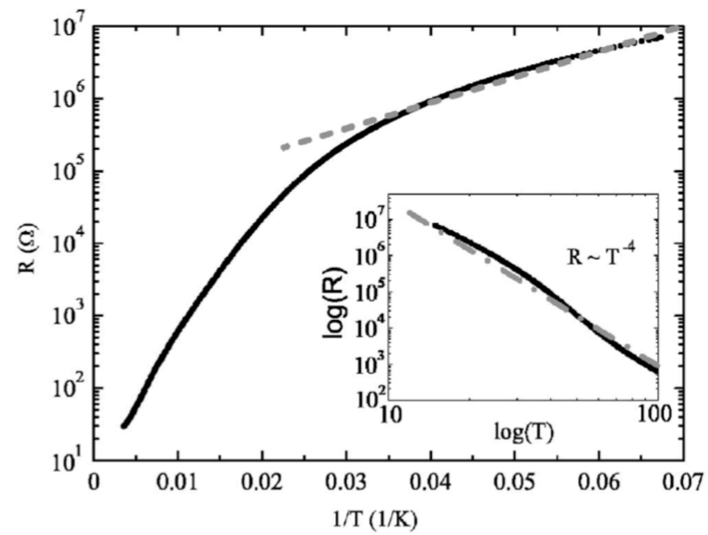


FIG. 1. Arrhenius plot of the resistance of a κ-$(ET)_2Cu_2(CN)_3$ single crystal. The gray dashed line at low temperature indicates the fitting of the gap value according to $R(T)=R_0\exp(\Delta/2k_BT)$. Inset: $\log(R)$ vs $\log(T)$ plot indicative of a power-law behavior of the resistance in the low-temperature region.

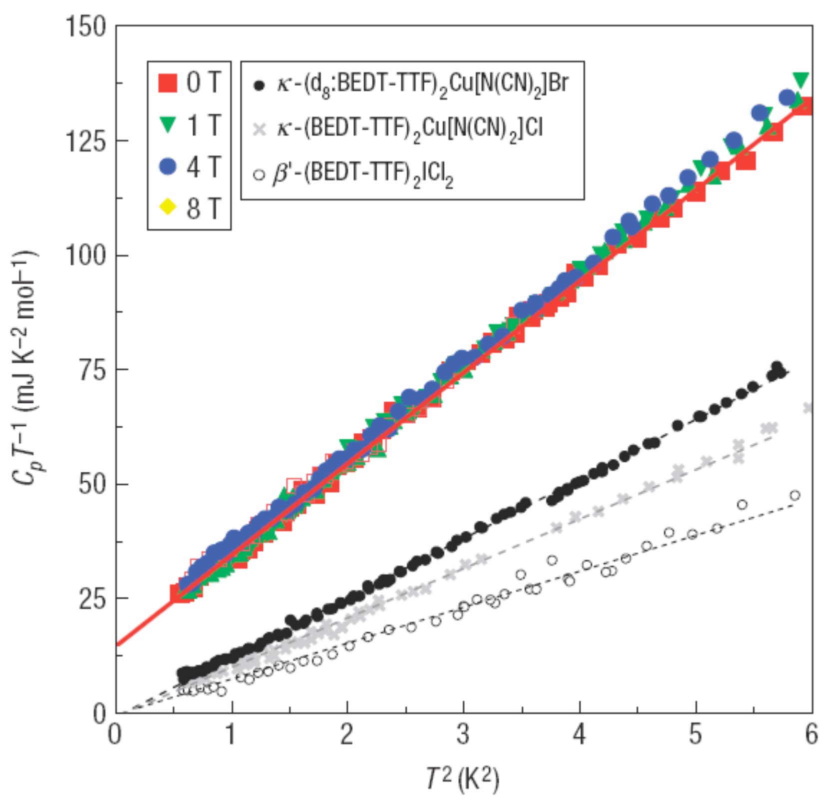
Kanoda et al PRB 74, 201101 (2006)

Heat capacity measurements

Thermodynamic properties of a spin-1/2 spin-liquid state in a κ-type organic salt

SATOSHI YAMASHITA[1], YASUHIRO NAKAZAWA[1,2*], MASAHARU OGUNI[3], YUGO OSHIMA[2,4], HIROYUKI NOJIRI[2,4], YASUHIRO SHIMIZU[5], KAZUYA MIYAGAWA[2,6] AND KAZUSHI KANODA[2,6]

S. Yamashita, et al., Nature Physics **4**, 459 - 462 (2008)



"Metallic" specific heat in a Mott insulator!!

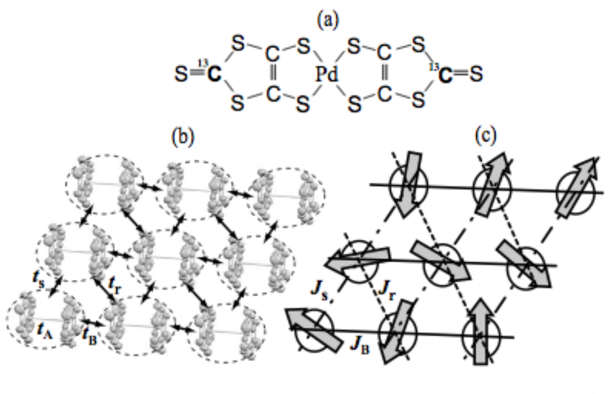
$$\gamma = 15 \,{}^{\mathrm{mJ}}\!/\!{}_{\mathrm{K^2 mol}}$$

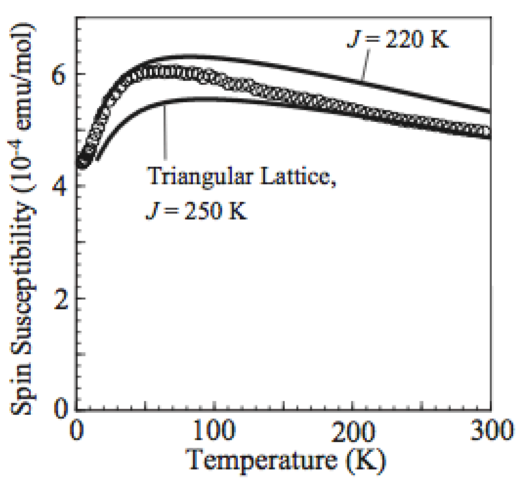
Wilson ratio between gamma and spin susceptibility of order one as in a metal

A new organic triangular lattice spin-liquid

$EtMe_3Sb[Pd(dmit)_{2]}2$

Itou, Kato et. al. PRB 77, 104413 (2008)

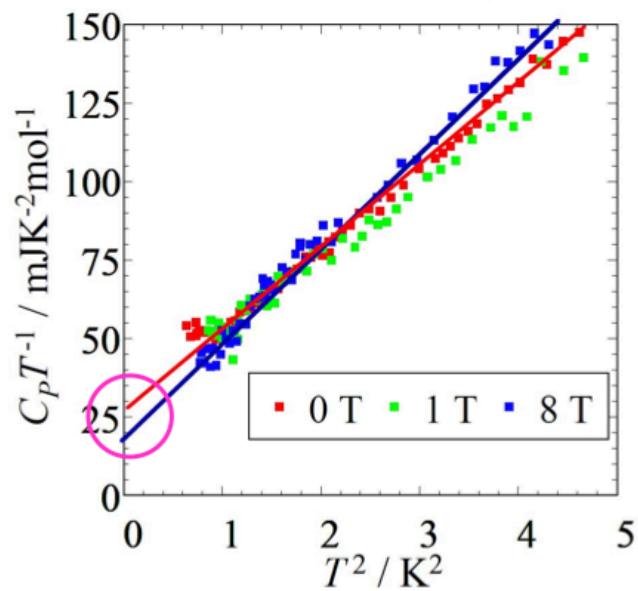




Wilson ratio of order one as in a metal

β'-$EtMe_3Sb[Pd(dmit)_2]_2$:

Heat Capacity

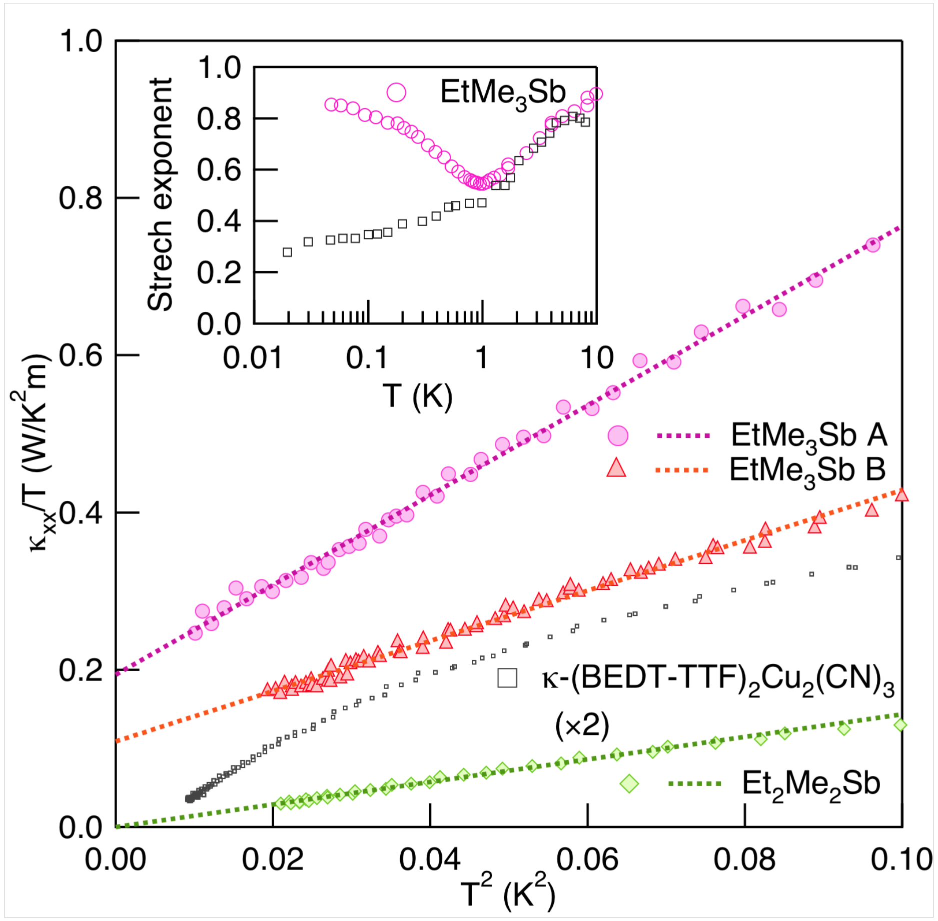


$$C_P T^{-1} = AT^{-3} + \gamma + \beta T^2$$

S. Yamashita, Y. Nakazawa et al. (Osaka Univ.), Annual Meeting of Japan Society for Molecular Science (2008), Fukuoka

Thermal Conductivity: K_{xx} ~T

Yamashita et al unpublished

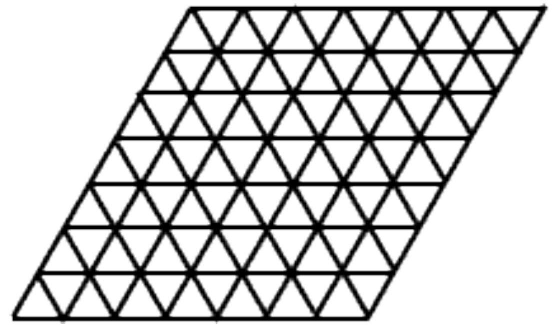


Thermal conductivity K ~ T at low temperatures, just as in a metal!!

Et_2Me_2Sb has a spin gap below 70K, so just have phonon contribution

Hubbard model on triangular lattice

$$\mathcal{H} = -t \sum_{\langle ij \rangle} [c^\dagger_{i\alpha} c_{j\alpha} + h.c.] + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



Phase diagram at Half filling?

Weak coupling

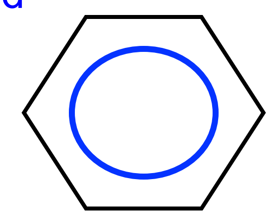
Strong coupling

metal ????? insulator

0 U/t

Free Fermi gas,
un-nested

Fermi liquid

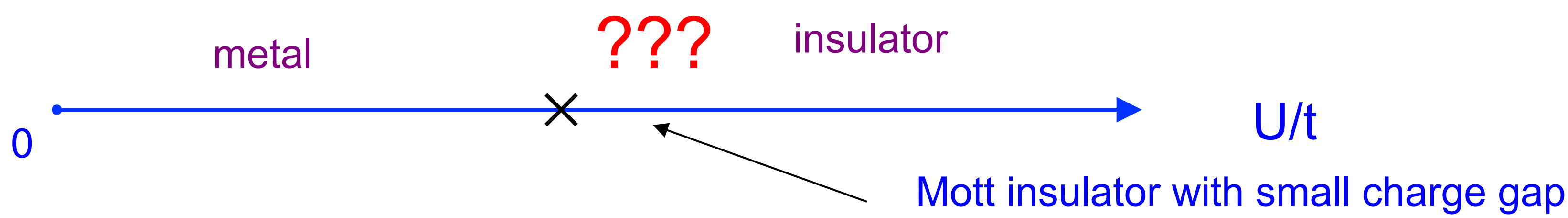


$$H_{spin} = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

Heisenberg n.n. AFM
3-sublattice Neel state

$J=t^2/U$

“Weak” Mott insulator - Ring exchange



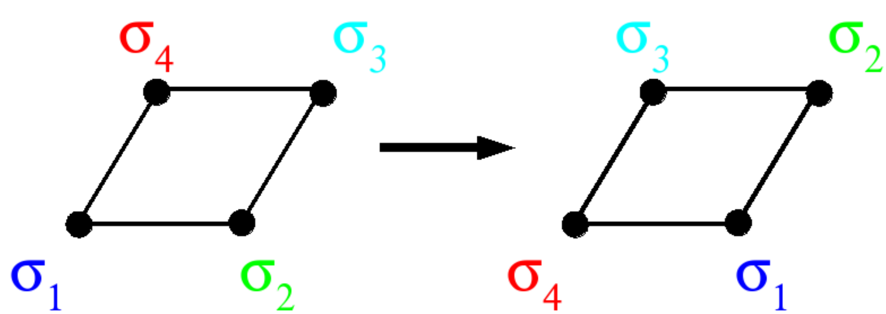
$$\hat{H}_{\text{Hubbard}} = -t\sum_{ij} c^{\dagger}_{i\alpha}c_{j\alpha} + U\sum_{i} n_{i\uparrow}n_{i\downarrow}$$

Insulator --> effective spin model

$$\hat{H}_{\text{eff}} = \frac{2t^2}{U}\sum_{ij}\mathbf{S}_i\cdot\mathbf{S}_j + \frac{20t^4}{U^3}\sum_{\text{rhombi}}(P_{ijkl} + \text{h.c.}) + \ldots$$

Ring exchange:
(mimics charge
fluctuations)

P_{1234} :



Slave-fermions

Fermionic representation of spin-1/2

$$\mathbf{S}_i = f_i^\dagger \frac{\boldsymbol{\sigma}}{2} f_i; \qquad f_{i\alpha}^\dagger f_{i\alpha} = 1;$$

General "Hartree-Fock" in the singlet channel

$$\mathcal{H}_{\text{trial}} = -\sum_{ij} t_{ij} f_{i\alpha}^\dagger f_{j\alpha}$$

$$\longrightarrow \quad |\Psi_0\rangle \quad \longrightarrow \quad |\Psi_{\text{spin}}\rangle = P_G(|\Psi_0\rangle)$$

free fermions spins Gutzwiller projection

- easy to work with numerically – VMC (Ceperley 77, Gros 89)

Gutzwiller-projected Fermi Sea

$$P_G \left(|\text{Fermi Sea}\rangle = \prod_{k<k_F} c^\dagger_{k\uparrow} c^\dagger_{k\downarrow} |vac\rangle \right)$$

$$= a|(\uparrow)\ (\uparrow)\ (\downarrow)\ (\uparrow)\ (\downarrow)\ (\uparrow)\rangle + \cancel{b|(\uparrow\downarrow)\ (0)\ (\downarrow)\ (\uparrow)\ (\uparrow)\ (\downarrow)\rangle}$$
$$\cancel{+c|(\uparrow)\ (\downarrow)\ (\downarrow)\ (\uparrow\downarrow)\ (\uparrow)\ (0)\rangle} + d|(\downarrow)\ (\uparrow)\ (\uparrow)\ (\downarrow)\ (\uparrow)\ (\downarrow)\rangle + \ldots$$

real-space configurations

-- insulator wave function (Brinkman-Rice picture of Mott transition)

$$\Psi_{\text{spin}}(\{R\uparrow\},\{R'\downarrow\}) = \det[R\uparrow]\det[R'\downarrow](-1)^{p(\{R\uparrow\},\{R'\downarrow\})}$$

Gauge structure

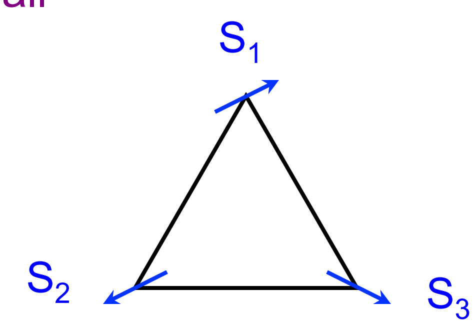
$$\mathcal{H}_{\rm trial} = -\sum_{ij} t_{ij} f^\dagger_{i\alpha} f_{j\alpha} = -\sum_{ij} \boxed{|t_{ij}|e^{ia_{ij}}} f^\dagger_{i\alpha} f_{j\alpha}$$

variational parameter

Slow spatial variation of the phases a_{ij} produces only small trial energy change ~ (curl a)^2

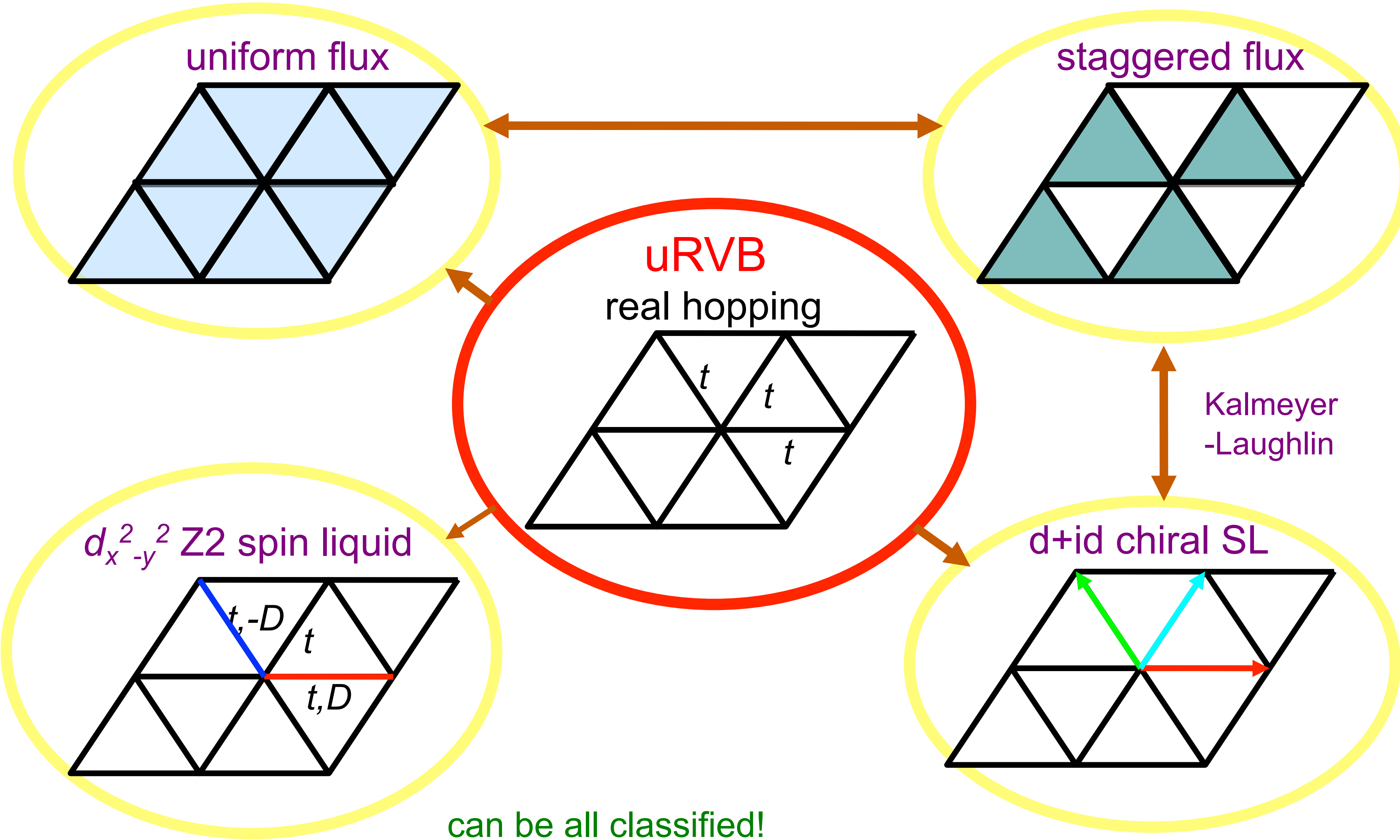
Physics of gauge flux: Spin chirality

$$\nabla \times a \sim \vec{S}_1 \cdot (\vec{S}_2 \times \vec{S}_3)$$



$\Rightarrow$ $\boxed{\text{need to include } a_{ij} \text{ as dynamical variables}}$

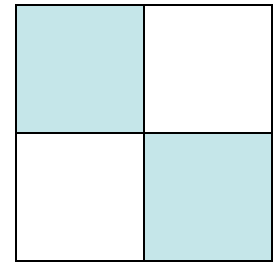
Examples of “fermionic” spin liquids



can be all classified!

Wen 2001; Zhou and Wen 2002

Algebraic Spin Liquid (example)

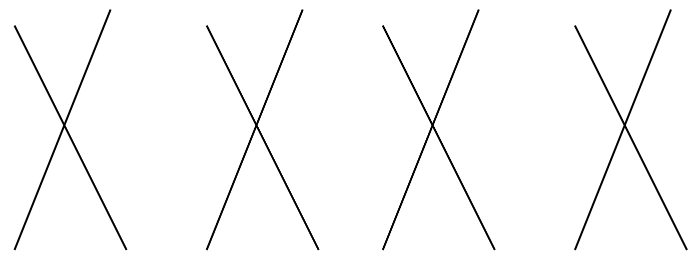


Staggered flux state on 2d square lattice

Mean field Hamiltonian:

$$\mathcal{H}_{\mathrm{sF}}^{0} = -\sum_{\boldsymbol{r}\in A}\ \sum_{\boldsymbol{r}'\ \mathrm{NN}\ \boldsymbol{r}} \{[it + (-1)^{(r_y - r_y')}\Delta] f_{\boldsymbol{r}\alpha}^{\dagger} f_{\boldsymbol{r}'\alpha} + \mathrm{H.c.}\},$$

Band structure has relativistic dispersion
with four 2-component Dirac fermions



Effective field theory is non-compact QED3

$$\mathcal{L}_E = \bar{\Psi}[-i\gamma^{\mu}(\partial_{\mu} + ia_{\mu})]\Psi + \frac{1}{2e^2}\sum_{\mu}(\epsilon_{\mu\nu\lambda}\partial_{\nu}a_{\lambda})^2 + \cdots,$$

Note: can argue that the monopoles are irrelevant due to massless Fermions,
(cf Polyakov confinement argument for pure compact U(1) gauge theory)

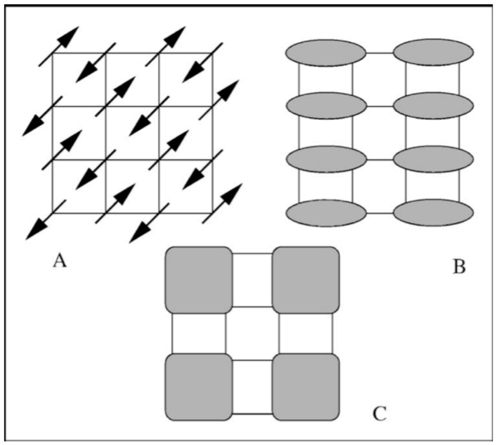
Emergent symmetry in Algebraic spin liquid

Spin Hamiltonian has ***global SU(2)*** spin symmetry

$$\mathcal{H} = J \sum_{\langle rr' \rangle} \boldsymbol{S}_r \cdot \boldsymbol{S}_{r'} + \cdots$$

Low energy effective field theory is non-compact QED3 with ***SU(4) flavor symmetry and U(1) flux conservation symmetry***

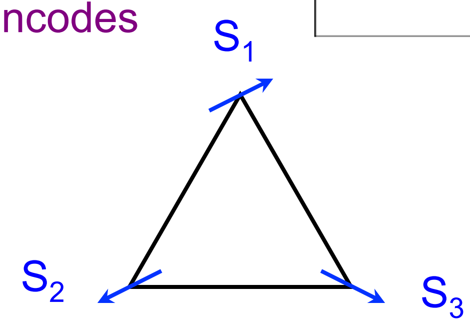
$$\mathcal{L}_E = \bar{\Psi}[-i\gamma^\mu(\partial_\mu + ia_\mu)]\Psi + \frac{1}{2e^2}\sum_\mu (\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda)^2 + \cdots ,$$



The SU(4) symmetry encodes slowly varying competing order parameters

The U(1) flux conservation symmetry encodes a conserved spin chirality

$$\nabla \times a \sim \vec{S}_1 \cdot (\vec{S}_2 \times \vec{S}_3)$$

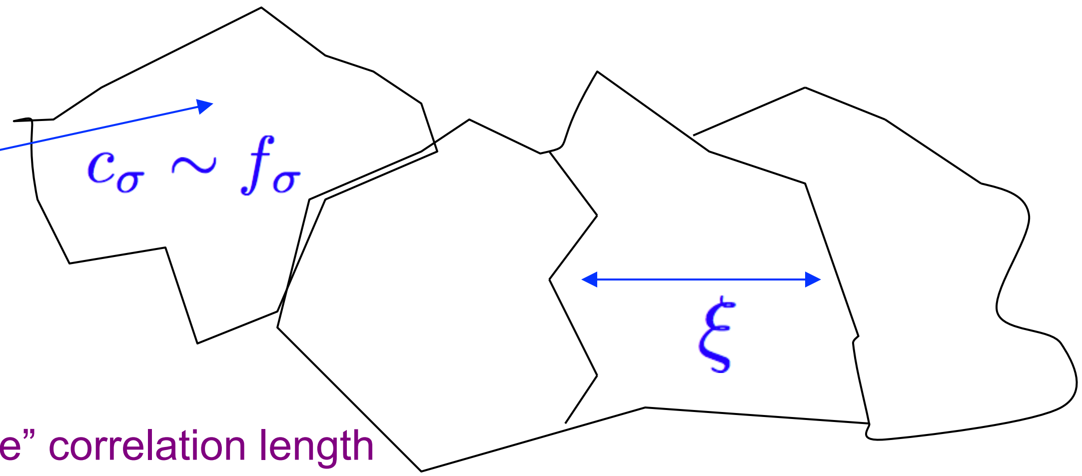


Weak Mott insulator: Which spin liquid?

Motrunich (2005)

Long charge correlation length, $\langle c_\sigma(x) c_\sigma^\dagger(0) \rangle \sim e^{-x/\xi}$ $\xi >> a$

Inside correlation region electrons do not "know" they are insulating

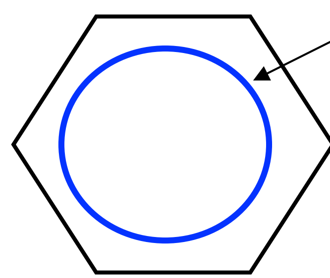


Guess: ***Spin*** correlations "inside" correlation length "resemble" spin correlations of free fermion metal. oscillating at $2k_F$

$$\langle \vec{S}(x) \cdot \vec{S}(0) \rangle \sim \cos(2k_F x)/x^{3/2}$$

Appropriate spin liquid:

Gutzwiller projected Filled Fermi sea (a "Spin Bose-Metal")

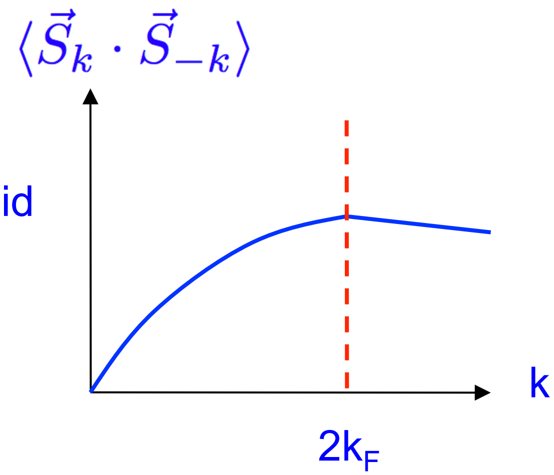


Spinon fermi surface is not physical in the spin model

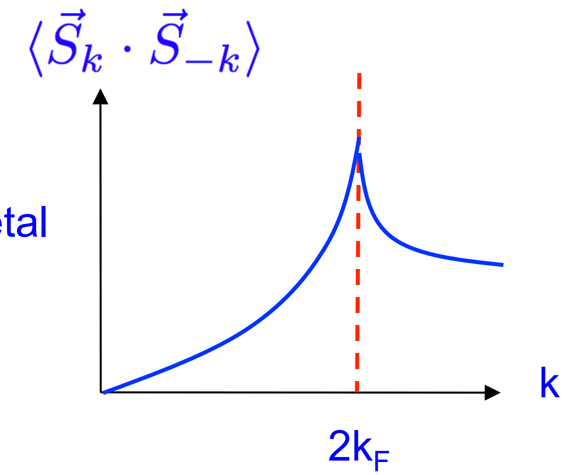
Phenomenology of Spin Bose-Metal
(from wf and Gauge theory)

Singular spin structure factor at “$2k_F$” in Spin Bose-Metal
(more singular than in Fermi liquid metal)

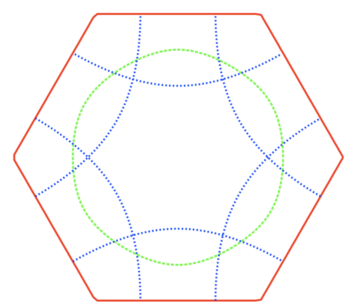
Fermi liquid



Spin Bose-metal



$2k_F$ “Bose surface” in
triangular lattice Spin Bose-Metal



Is projected Fermi sea a good caricature of the Triangular ring model ground state?

$$\hat{H}_{\mathrm{ring}} = J_2 \sum_{ij} \mathbf{S}_i \cdot \mathbf{S}_j + J_4 \sum_{\mathrm{rhombi}} (P_{ijkl} + \mathrm{h.c.})$$

Variational Monte Carlo analysis suggests it might be for $J_4/J_2 > 0.3$
(O. Motrunich - 2005)

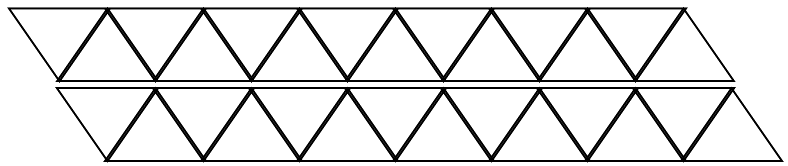
A theoretical quandary: Triangular ring model is intractable

- Exact diagonalization: so small,
- QMC - sign problem
- Variational Monte Carlo - biased
- DMRG - problematic in 2d

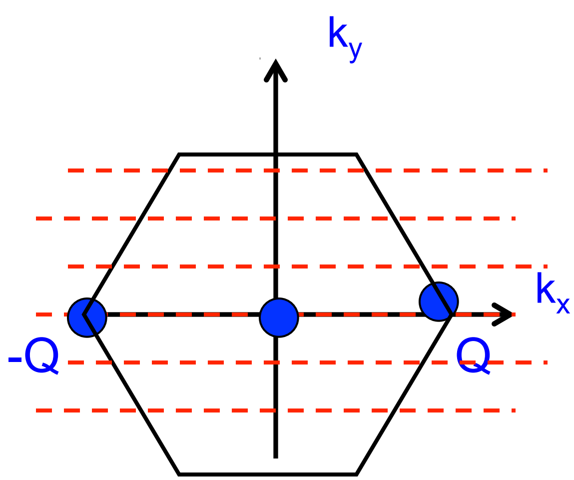
?????

Quasi-1d route to "Spin Bose-Metals"

Triangular strips:

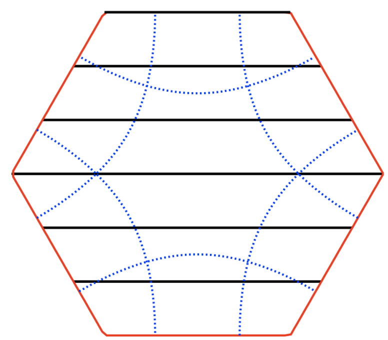


Neel state or Algebraic Spin liquid



Few gapless 1d modes

Spin Bose-Metal

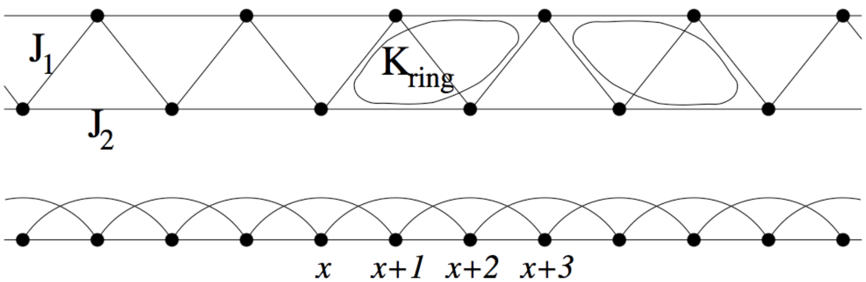


Fingerprint of 2d singular surface -
many gapless 1d modes, of order N

New spin liquid phases on quasi-1d strips, each a descendent of a 2d Spin Bose-Metal

2-leg zigzag strip

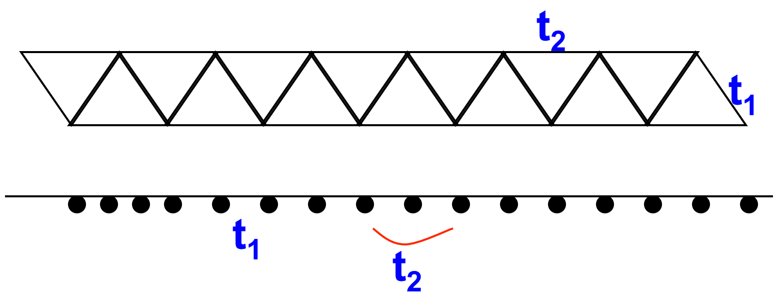
$$\mathcal{H}_\Delta = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + K \sum_{\langle 1234 \rangle} [\mathcal{P}_{1234} + \mathcal{P}_{1234}^{-1}]$$



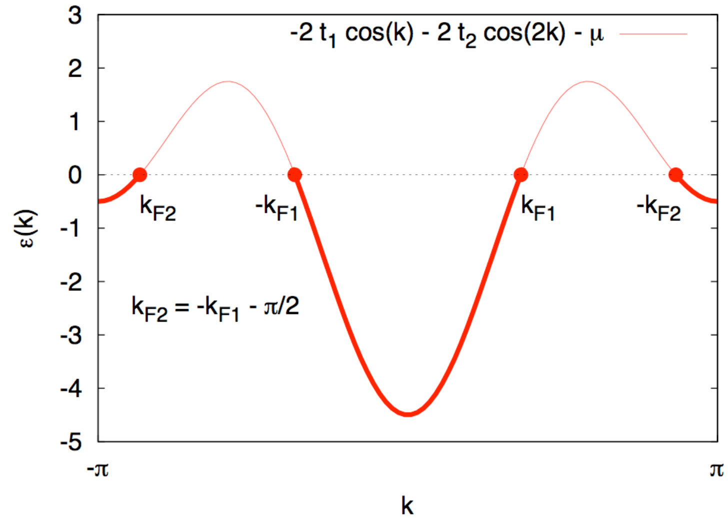
Analysis of J_1-J_2-K model on zigzag strip

Exact diagonalization
Variational Monte Carlo of Gutzwiller wavefunctions
Bosonization of gauge theory and Hubbard model
DMRG

Gutzwiller Wavefunction on zigzag



$$\mathcal{H}_{\rm trial} = -\sum_{ij} t_{ij} f^{\dagger}_{i\alpha} f_{j\alpha}$$



$|\Psi_0\rangle \quad\quad |\Psi_{\rm spin}\rangle = P_G(|\Psi_0\rangle)$

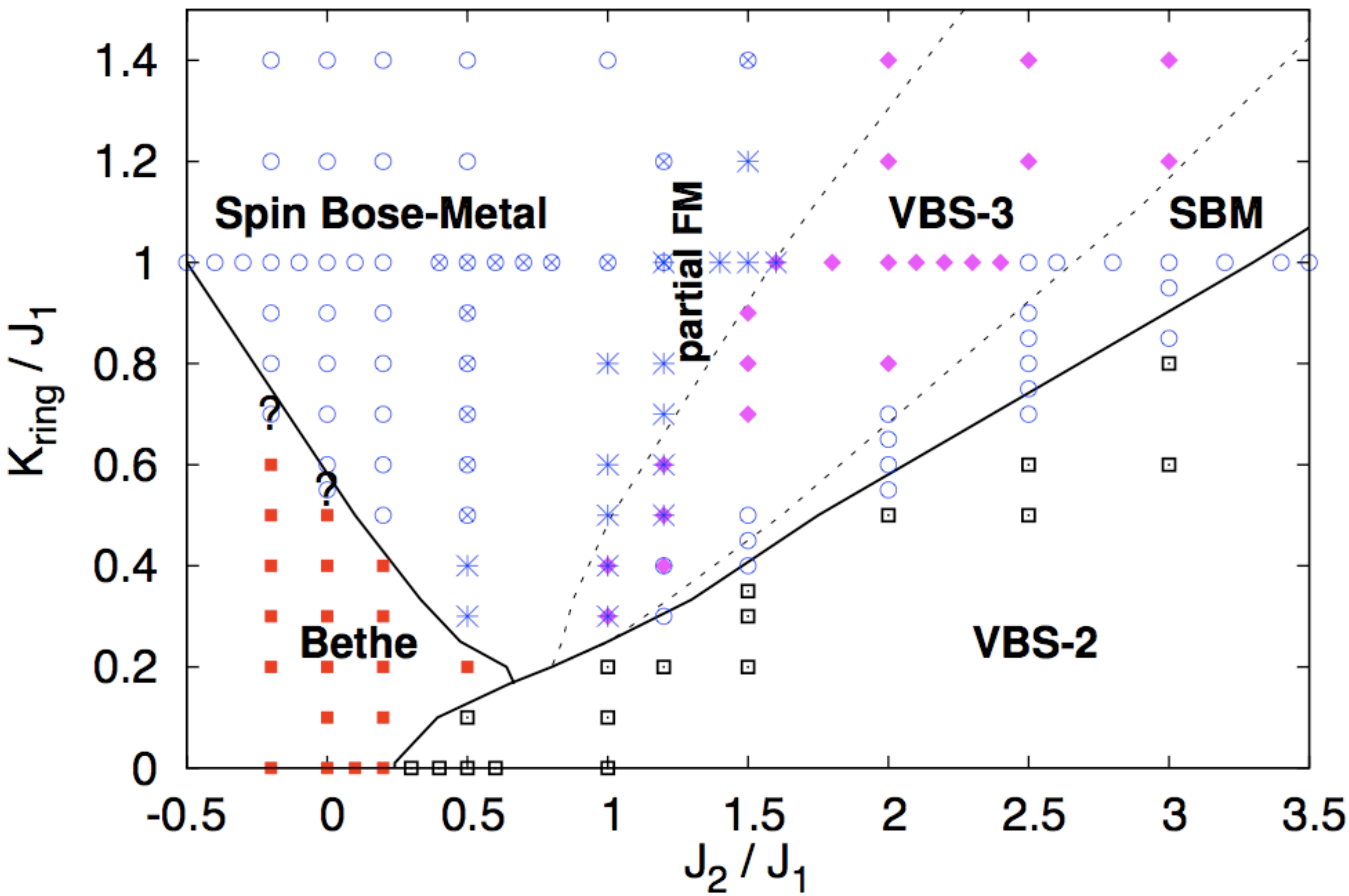
free fermions — spins — Gutzwiller projection

Spinon band structure

Single Variational parameter: t_2/t_1 or k_{F2}

$(k_{F1}+k_{F2} = \mathrm{pi}/2)$

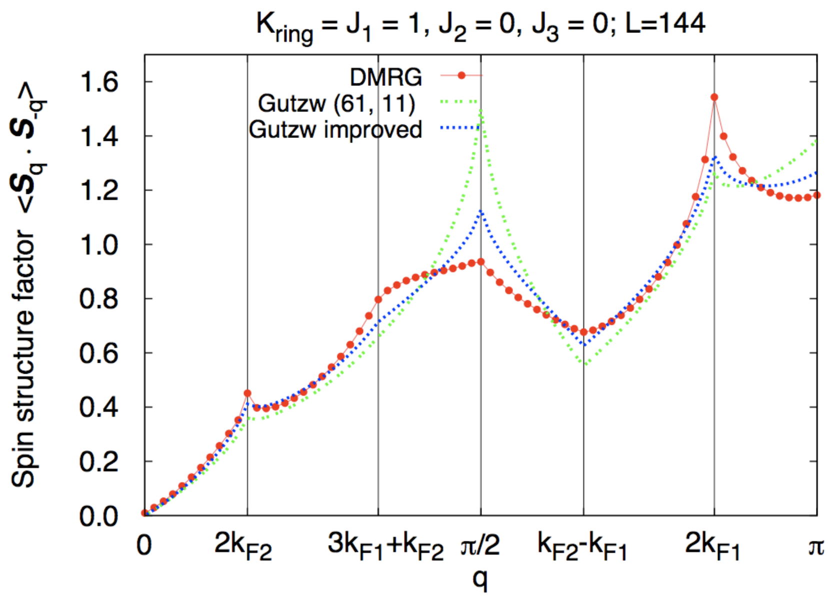
Phase diagram of zigzag ring model



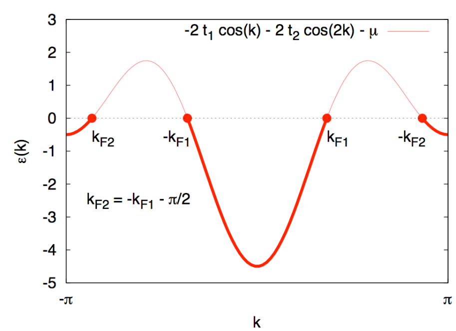
Spin Structure Factor in Spin Bose-Metal

Singularities in momentum space locate the "Bose" surface (points in 1d)

$$\langle \vec{S}_k \cdot \vec{S}_{-k} \rangle$$



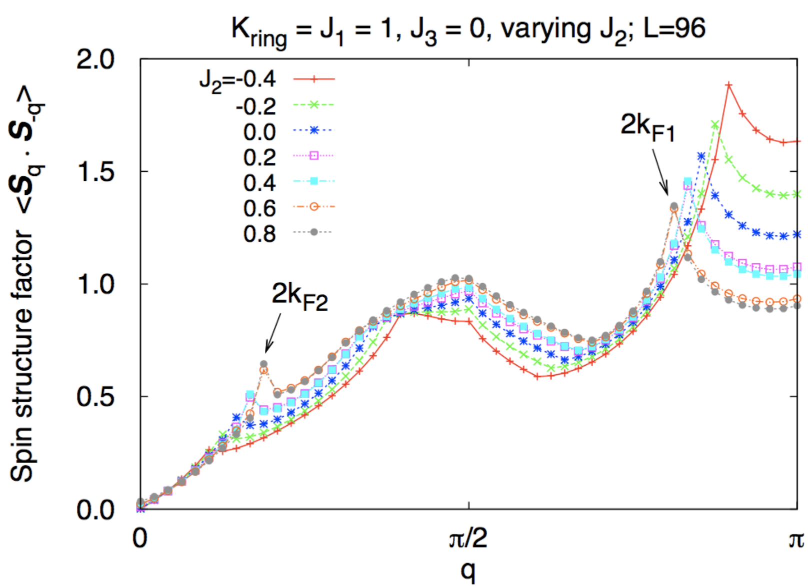
(Gutzwiller improved has 2 variational parameters)

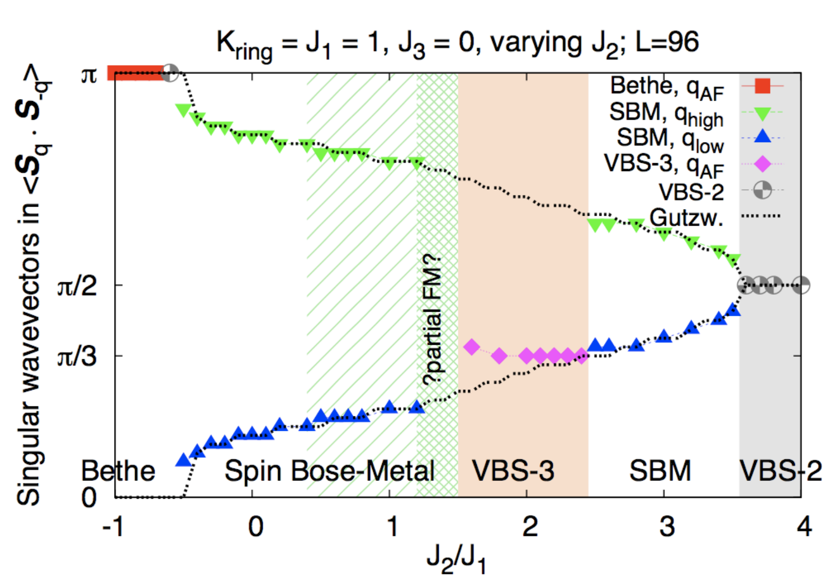


Singular momenta can be identified with $2k_{F1}$, $2k_{F2}$ which enter into Gutzwiller wavefunction!

Evolution of singular momentum ("Bose" surface)

DMRG



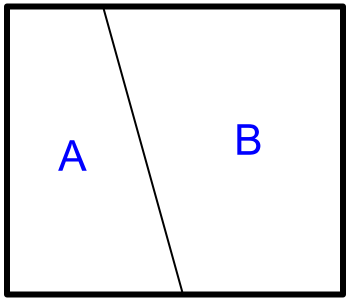


Entanglement Entropy

Density matrix for system $$\rho = |\psi\rangle\langle\psi|$$

Reduced density matrix for Sub-system A $$\rho_A = Tr_B[\rho]$$

Entanglement entropy $$S_A = -Tr_A[\rho_A \ln \rho_A]$$

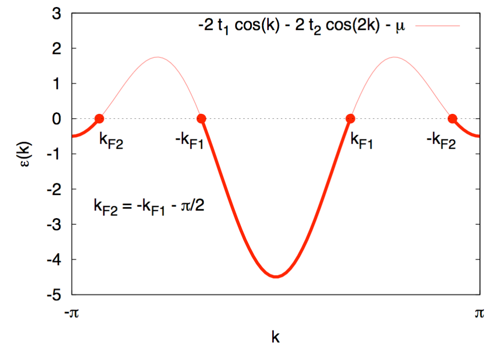


$$|\psi\rangle = |\psi\rangle_A |\psi\rangle_B$$ Product state has S_A=0

1D Gapless system (Conformal field theory, CFT)

Length X subsystem has $$S(X) = \frac{c}{3}\ln(X)$$ c = central charge

Entanglement in SBM?
Quasi-1d Gauge Theory



Linearize about two Fermi points, Bosonize and integrate out gauge field

$$f_\alpha(x) = \sum_{a,P} e^{iPk_{Fa}x} f_{Pa\alpha}$$

$$f_{Pa\alpha} \sim e^{i(\varphi_{a\alpha} + P\theta_{a\alpha})}$$

"Fixed-point" theory of zigzag Spin Bose-Metal

$$\mathcal{L}_{s\ell} = \mathcal{L}_\sigma + \mathcal{L}_\chi$$

Two gapless spin modes

$$\mathcal{L}_\sigma = \frac{1}{2\pi} \sum_{a=1,2} \left[\frac{1}{v_a} (\partial_\tau \theta_{a\sigma})^2 + v_a (\partial_x \theta_{a\sigma})^2 \right]$$

Gapless spin-chirality mode

$$\mathcal{L}_\chi = \frac{1}{2\pi g} \left[\frac{1}{v} (\partial_\tau \theta_\chi)^2 + v (\partial_x \theta_\chi)^2 \right]$$

$$\chi = \vec{S}_{x-1} \cdot [\vec{S}_x \times \vec{S}_{x+1}] \qquad \chi \sim \partial_x \varphi_\chi$$

Emergent global symmetries: SU(2)xSU(2) and U(1) Spin chirality

3 Gapless Boson modes – central charge c=3

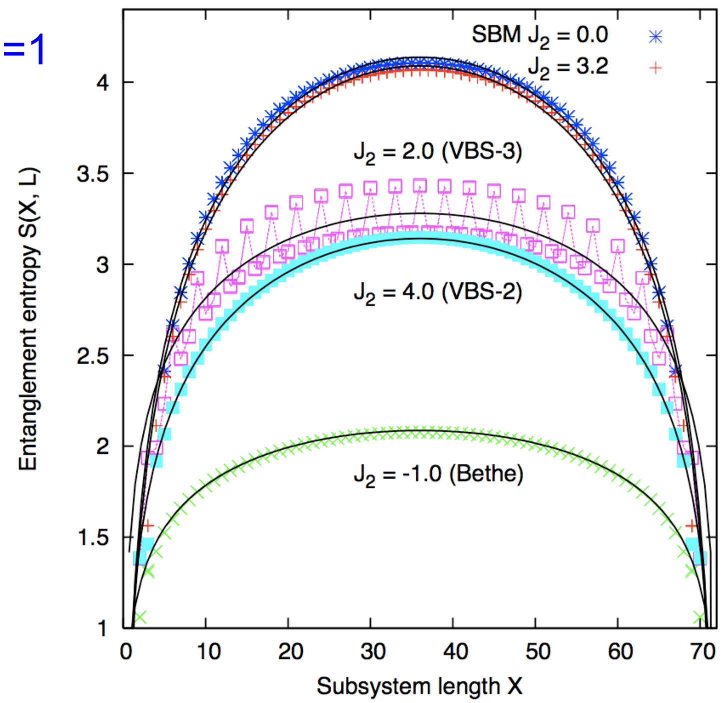
Measure c=3 with DMRG? Entanglement Entropy

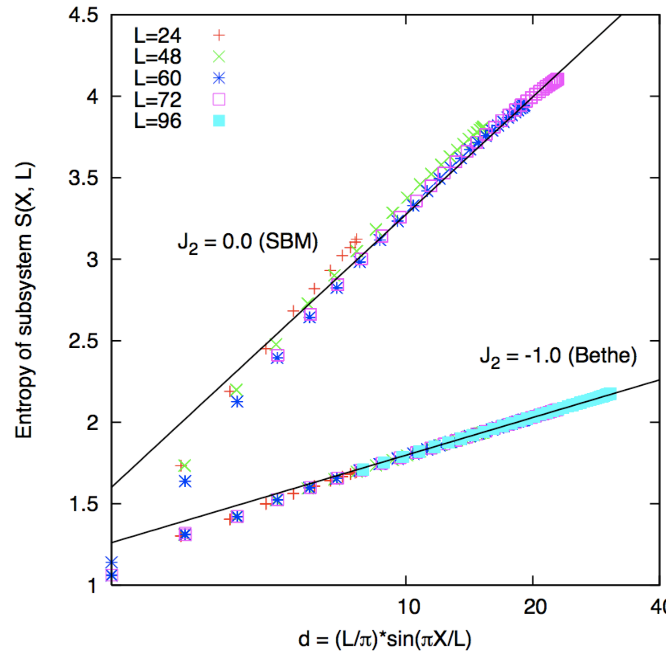
$$S(X,L) = \frac{c}{3}\log\left(\frac{L}{\pi}\sin\frac{\pi X}{L}\right) + A$$

Bethe c=1
Spin Bose-metal c=3.1

(VBS-2 c~2; VBS-3 c~1.5)

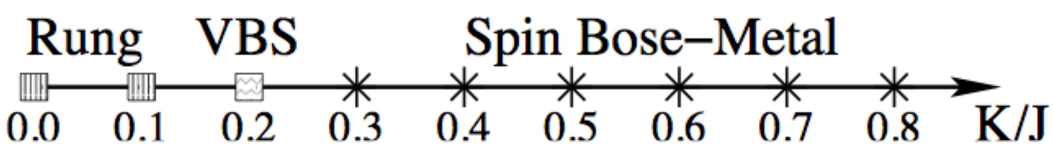
$K/J_1=1$

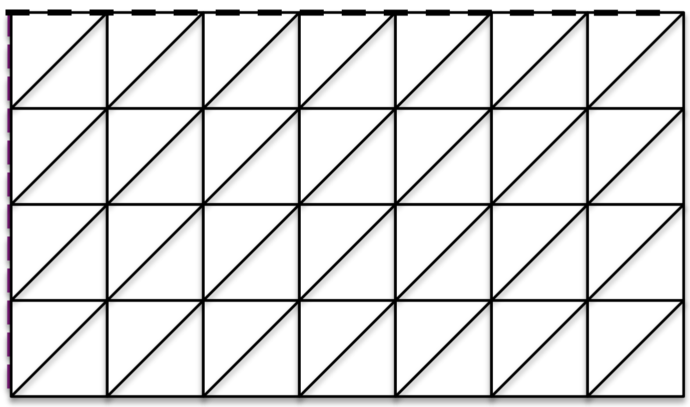


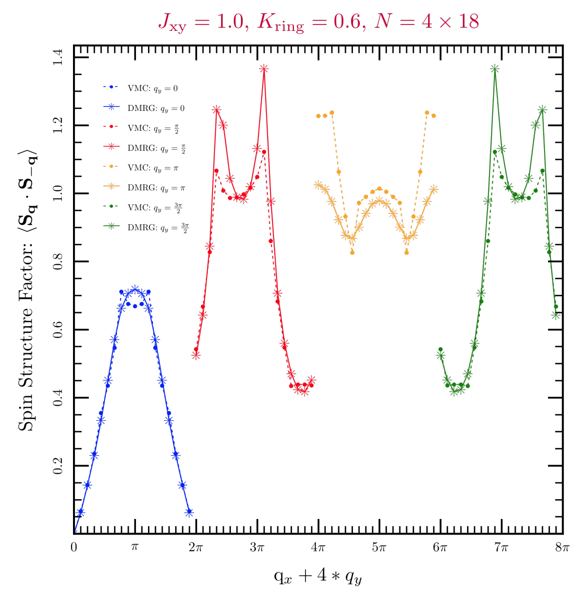


Entanglement entropy in SBM consistent with c=3 for 3 gapless Boson modes!

Phase Diagram; 4-leg Triangular Ladder







Singlets along the “rungs”
for K=0

Spin structure factor shows singularities
consistent with a Spin-Bose-Metal,
(ie. 3-band Spinon-Fermi-Sea wavefunction)
with 5 gapless modes, ie. c=5

Entanglement Entropy for SBM on N-leg ladder

For length L segment on N-leg ladder expect

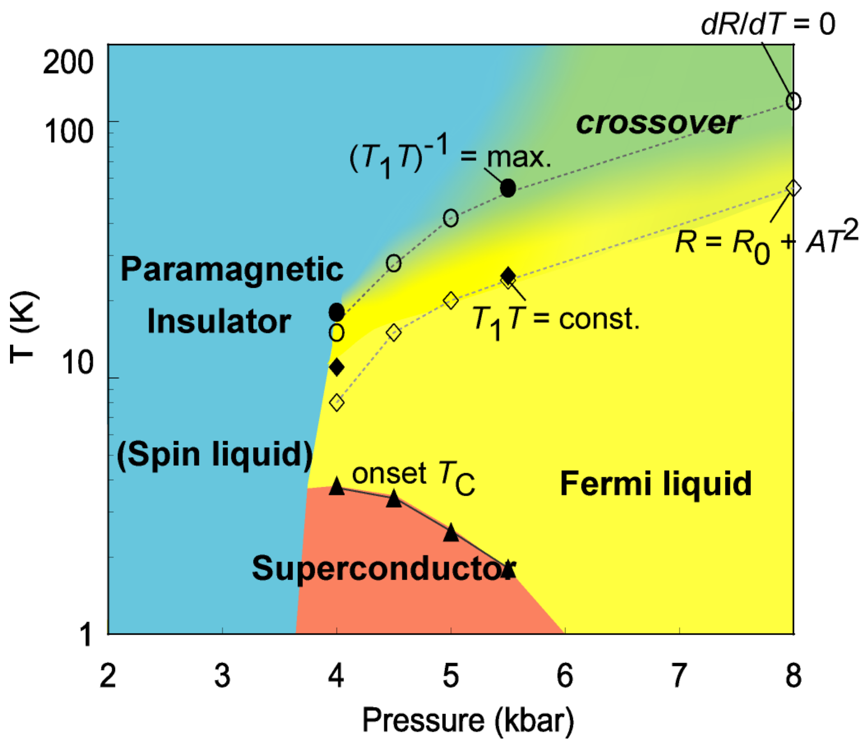
$$S_N = \frac{c_N}{3}\log(L/a) + A \qquad\qquad c_N \sim N$$

For 2d Spin Bose-Metal expectation is that L by L region has entanglement entropy

$$S_{2d}(L) \sim L\log(L/a)$$

2d Spin Bose-Metal as entangled as a 2d Fermi liquid

“Triangular Spin-Liquid” and “Square AFM”

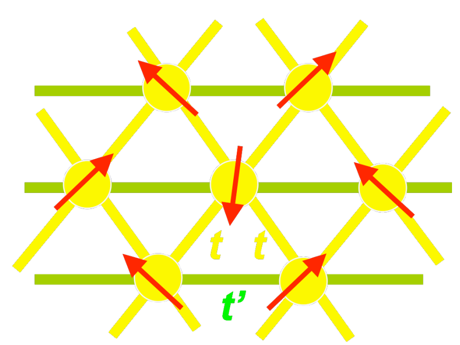


2D spin liquid

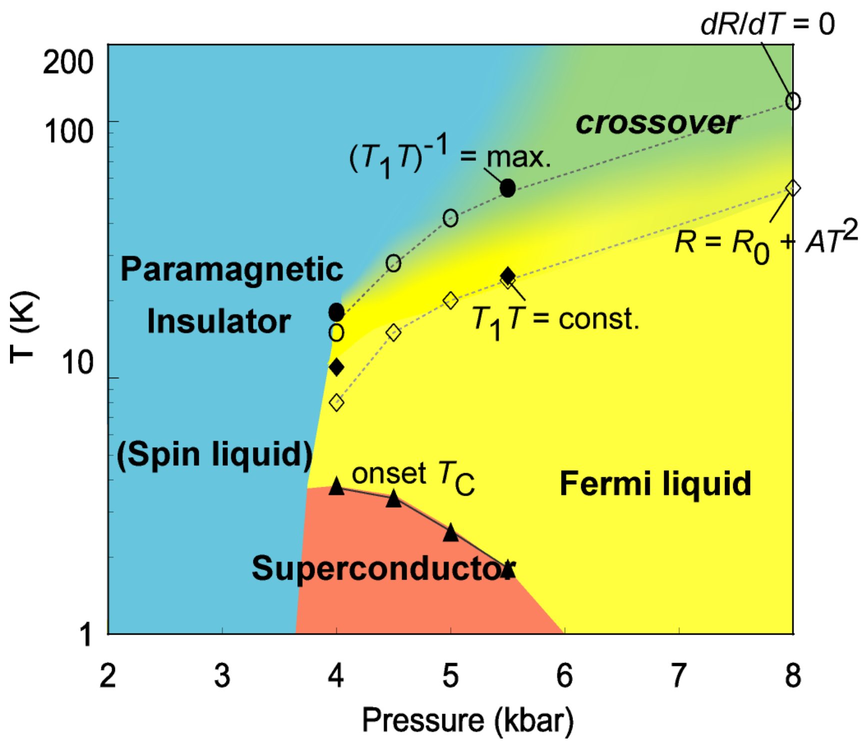
κ-$Cu_2(CN)_3$

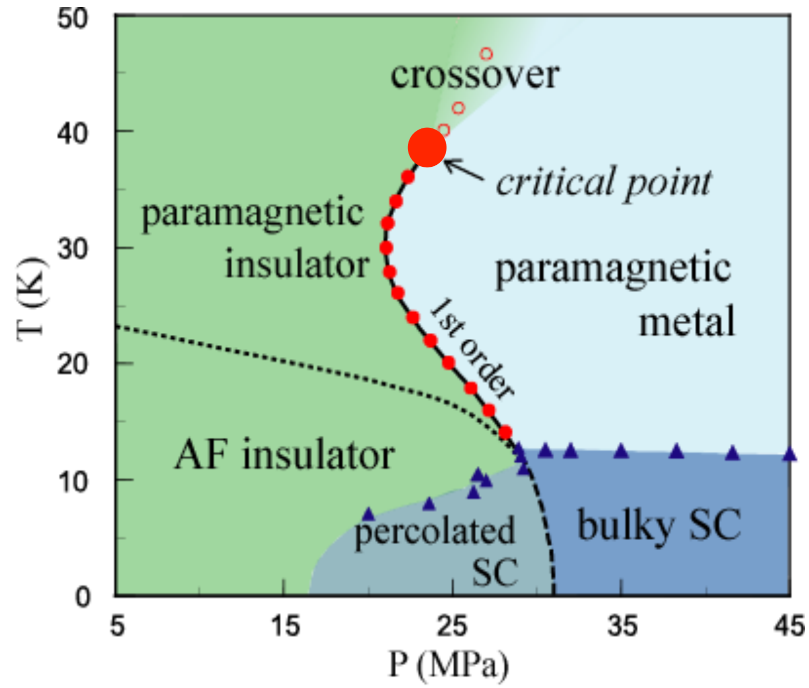
t’/t=1.06

Isotropic triangular lattice



"Triangular Spin-Liquid" and "Square AFM"



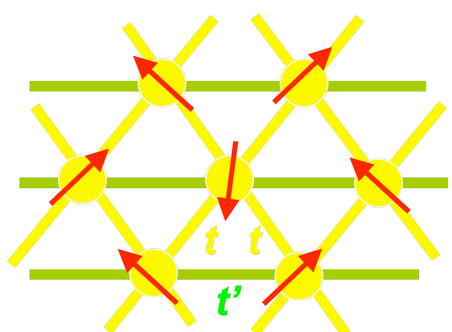


2D spin liquid

κ-$Cu_2(CN)_3$

t'/t=1.06

Isotropic triangular lattice



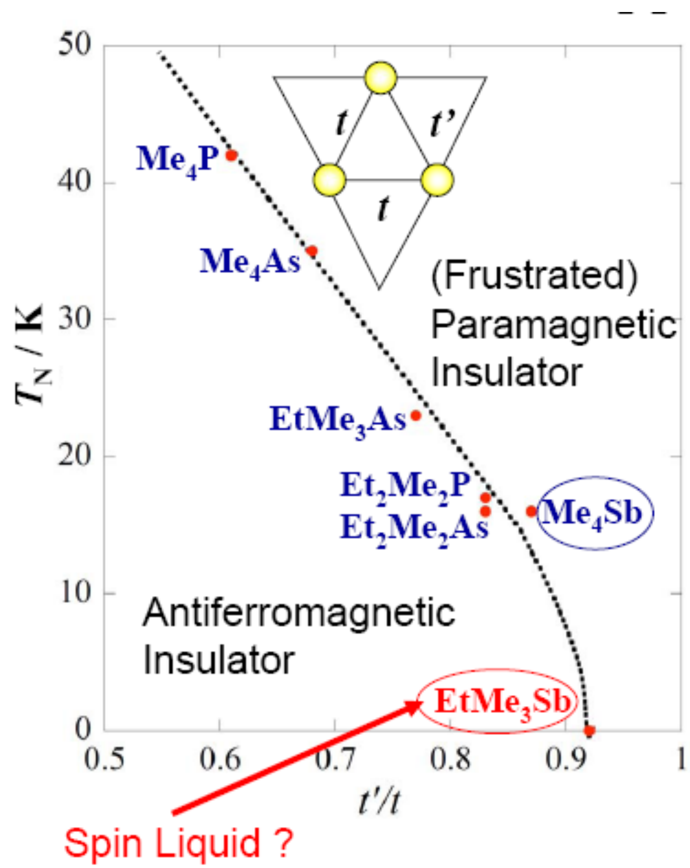
2D antiferromagnet

κ-$Cu[N(CN)_2]Cl$

t'/t=0.75

"Anisotropic" triangular lattice

New class of organics; AFM versus spin-liquid



T. Itou et al., *PRB*, **77** (2008) 104413

$EtMe_3Sb[Pd(dmit)_{2]}2$

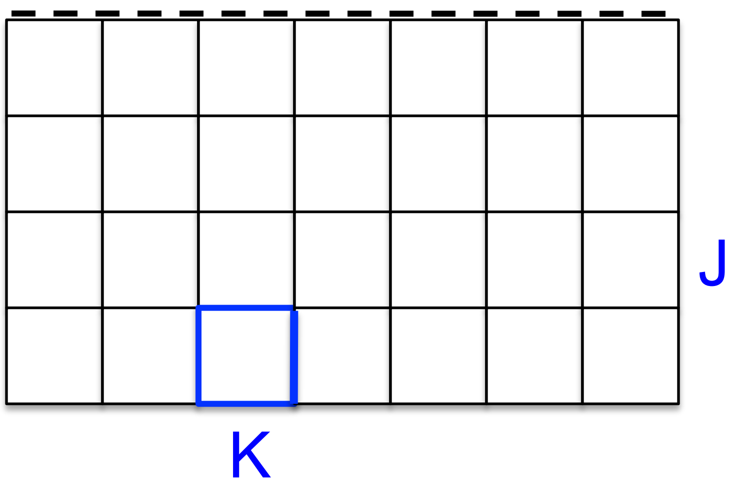
Kato et. al.

“Square lattice” - AFM,

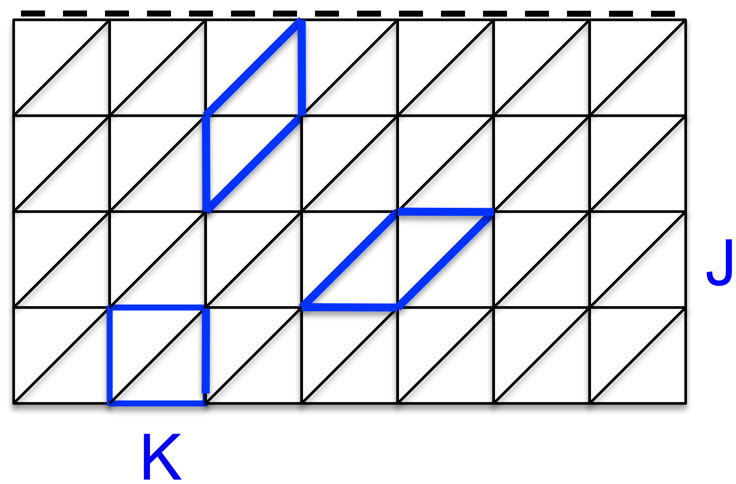
“Triangular lattice” – Spin-liquid

4-leg Ladders; Square-vs-triangular (preliminary)

Square J-K model



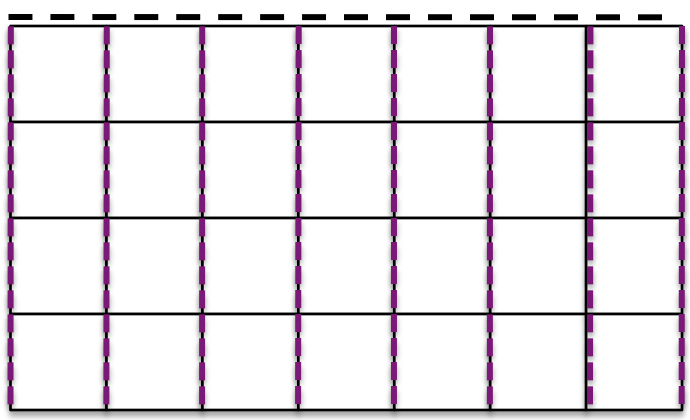
Triangular J-K model



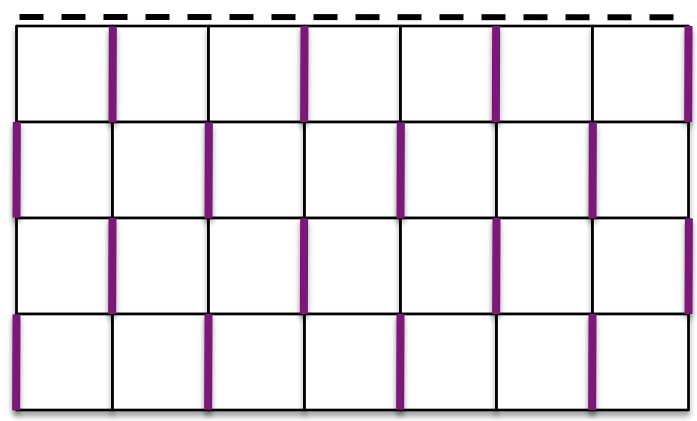
(Periodic b.c. in “rectangular” geometry)

DMRG, ED, VMC of Gutzwiller wavefunctions, Bosonization,…

Phase Diagram; 4-leg Square Ladder

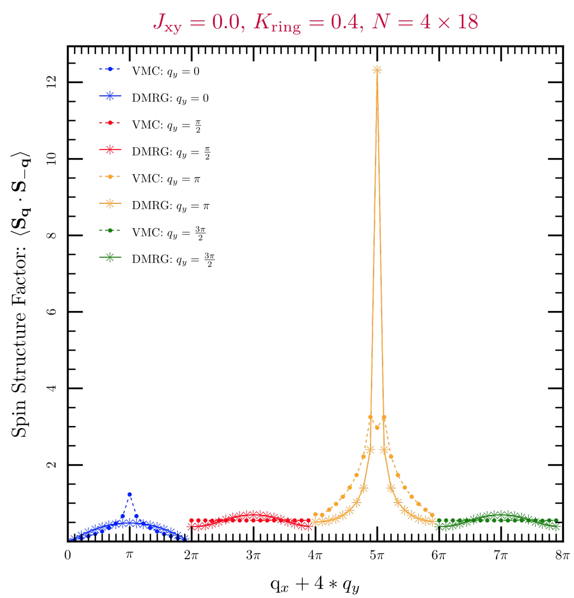


Singlets along the “rungs”

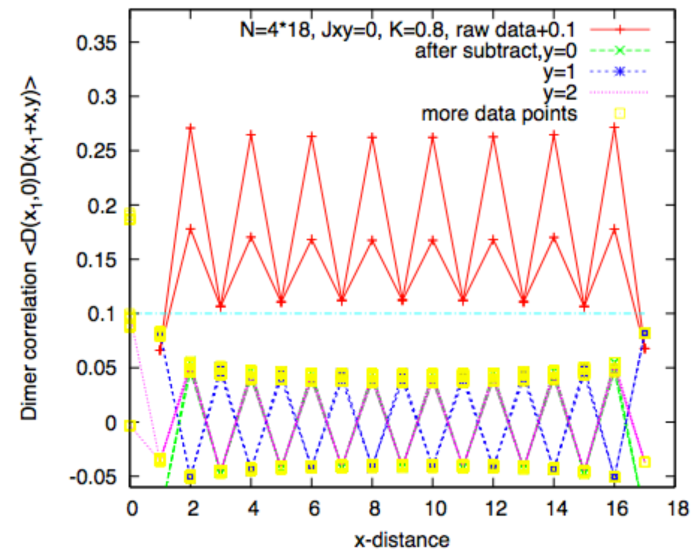


Valence bond crystal

Square ladder correlators

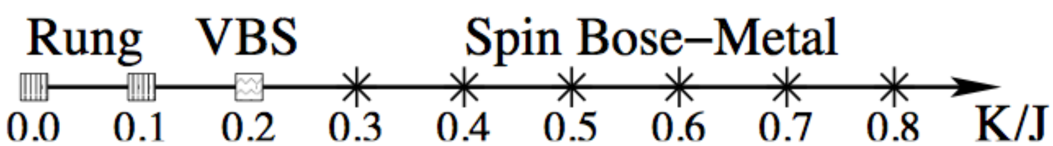


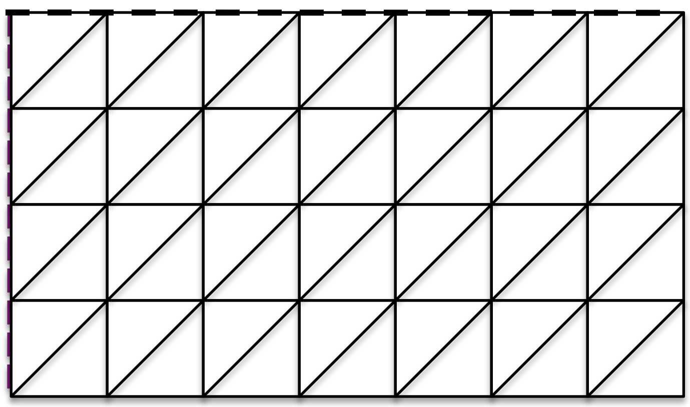
Spin structure factor in "rung" phase
for K/J=0.4 (DMRG/VMC)
(large peak at pi-pi like in AFM)

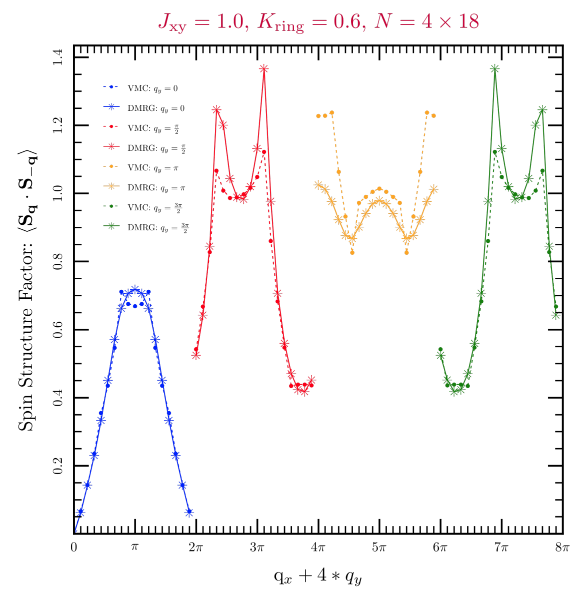


"Frozen" Dimer correlations
in the staggered-dimer phase
(K/J = 0.8)

Phase Diagram; 4-leg Triangular Ladder



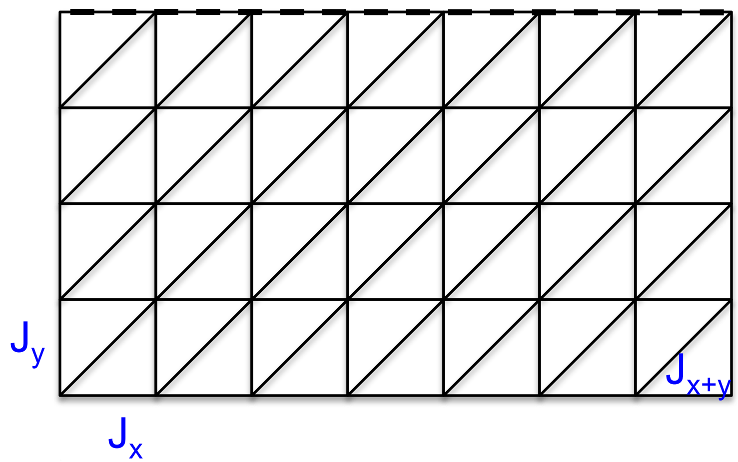




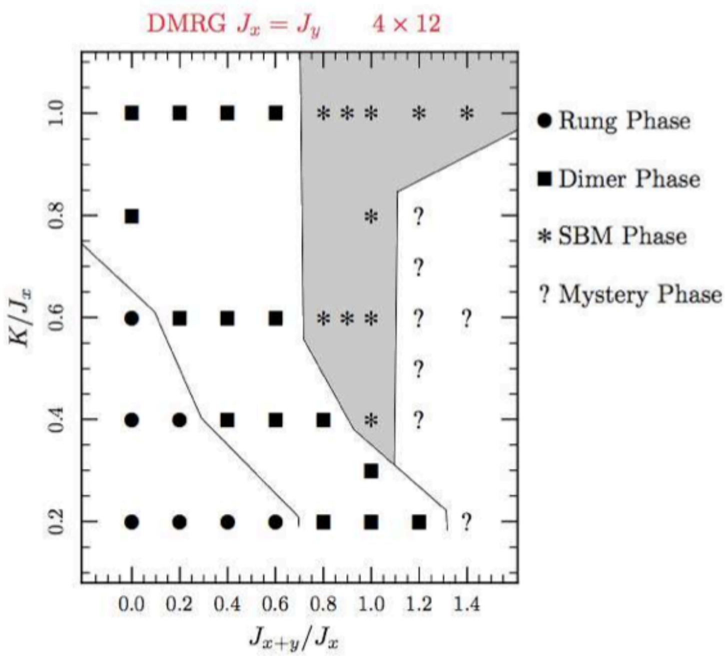
Singlets along the “rungs” for K=0

Spin structure factor shows singularities consistent with a Spin-Bose-Metal, (ie. 3-band Spinon-Fermi-Sea wavefunction) with 5 gapless modes, ie. c=5

Goal: Evolution from Square to Triangular via DMRG



4-leg Ladder
Towards 2d

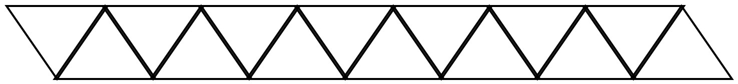


K/J

Summary

- Spin Bose-Metals - 2d spin liquids with singular "Bose" surfaces
  - quasi-1d descendents are numerically accessible
- Heisenberg + ring-exchange on zigzag strip exhibits quasi-1d descendent of the triangular lattice Spin Bose-Metal

$$\mathcal{H}_\Delta = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + K \sum_{\langle 1234 \rangle} [\mathcal{P}_{1234} + \mathcal{P}_{1234}^{-1}]$$



Future?

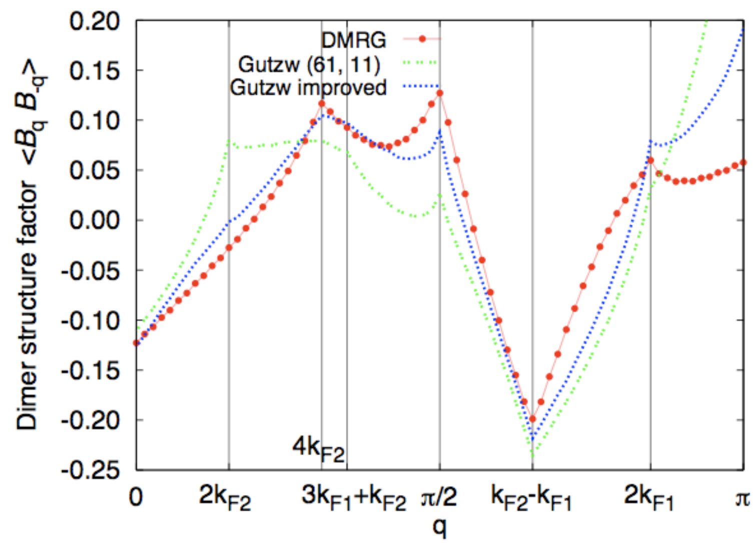
DMRG/VMC/gauge theory for

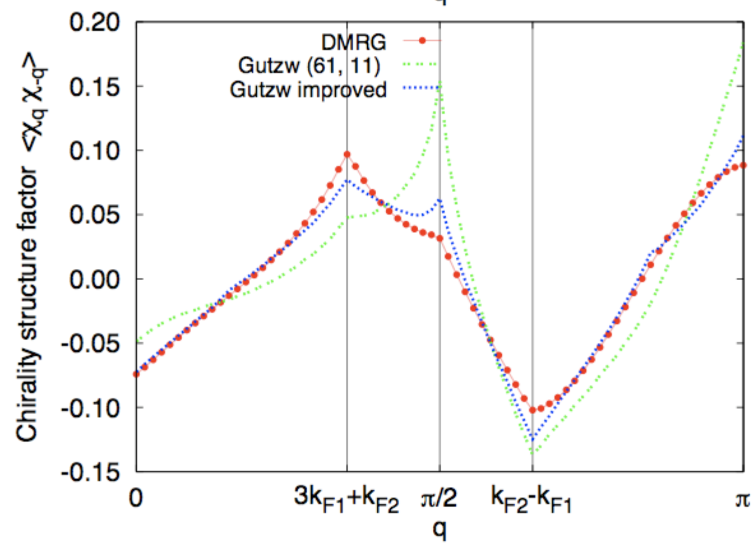
- Hubbard on the zigzag strip
- Ring exchange model on 6-leg square/triangular strips?
- Descendents of 2d non-Fermi liquids (D-wave metal) on the 2-leg ladder?

Dimer and chirality correlators in Spin Bose-Metal on 2-leg zigzag strip

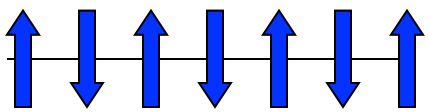
$$
\begin{aligned}
\mathcal{B}(x) &= \vec{S}(x) \cdot \vec{S}(x+1)\,, \\
\chi(x) &= \vec{S}(x-1) \cdot [\vec{S}(x) \times \vec{S}(x+1)]
\end{aligned}
$$

$$
\begin{aligned}
D(x, x') &= \langle \mathcal{B}(x)\mathcal{B}(x') \rangle - \langle \mathcal{B} \rangle \\
X(x, x') &= \langle \chi(x)\chi(x') \rangle\,.
\end{aligned}
$$





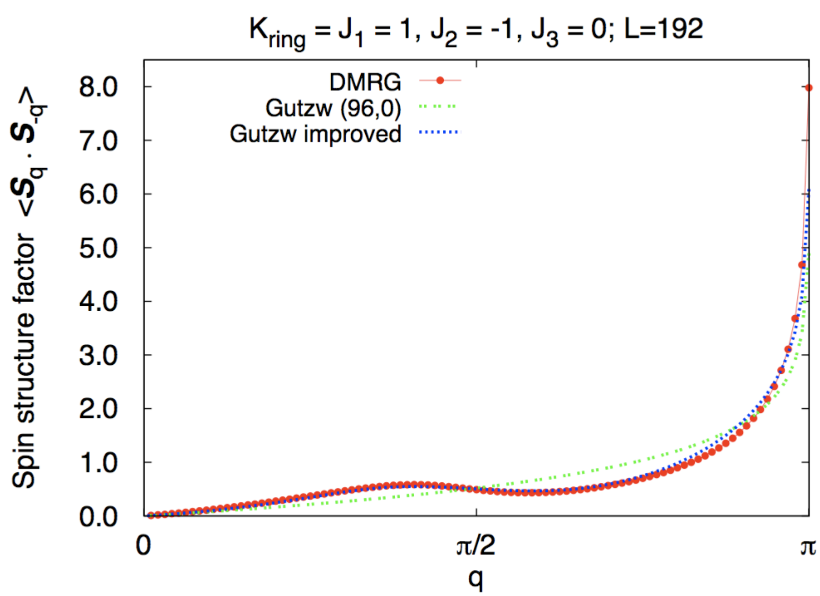
Bethe chain and VBS-2 States

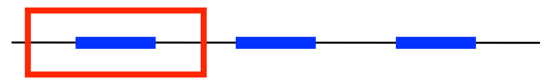


Bethe chain state; “1d analog of Neel state”

$$\langle \vec{S}_x \cdot \vec{S}_0 \rangle \sim (-1)^x / x$$

Spin structure factor

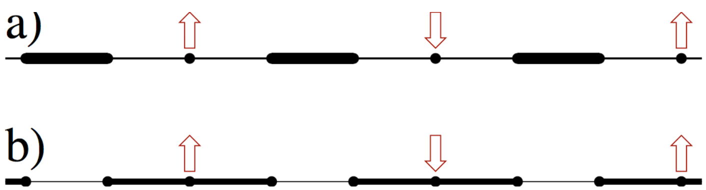




 $= \ |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

Valence Bond solid (VBS-2)

“VBS-3” Phase



Period 3 dimer long-range order
Period 6 spin correlations;

$2k_{F1}$ = 2 pi/3 instability in gauge theory, gaps out the first spinon band, leaving second band gapless like a Bethe chain