# 2D Bose and Non-Fermi Liquid "Metals": Circumnavigating the Sign Problem

MPA Fisher, with O. Motrunich, D. Sheng, E. Gull and S. Trebst

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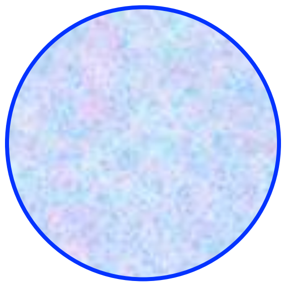
Interest: A class of exotic gapless 2D Many-Body States

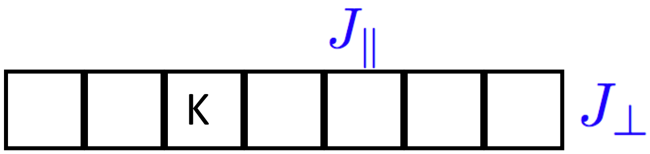
a) What are these "strange-metals"? Singular surfaces in momentum space (eg. ***Bose- surfaces***)

b) Variational wavefunctions?

c) "Strange-metal" phases as ground states for any Hamiltonians?



- 2D "Strange-Metals" have ***tractable quasi-1D descendents***
- Approach 2D via quasi-1D "ladders" with DMRG



# Useful references

Spin, Bose, and Non-Fermi Liquid Metals in Two Dimensions:
Accessing via Multi-Leg Ladders; MPAF Fisher et al. **arXiv:0812.2955v1**

The introductions to the following 3 papers might be useful to look at:

d-wave correlated critical Bose liquids in two dimensions, O.Motrunich et al, PHYSICAL REVIEW B 75, 235116, 2007

Strong-coupling phases of frustrated bosons on a two-leg ladder with ring exchange, D. Sheng et al. PHYSICAL REVIEW B 78, 054520, 2008

Spin Bose-metal phase in a spin-1/2 model with ring exchange on a two-leg triangular strip, D. Sheng et al. PHYSICAL REVIEW B 79, 205112, 2009

# What is a "Bose-Metal"?

First: Bose Condensate in Free Bose Gas

Superfluid in interacting Bose Gas

# 2D Free Bose Gas

Free particle Hamiltonian

$$H_0 = \sum_j \frac{\mathbf{p}_j^2}{2m}$$

Equal time Boson Green's function

$$G_b(\mathbf{r}) = \langle b^\dagger(\mathbf{r}) b(\mathbf{0}) \rangle$$

Momentum distribution fucntion

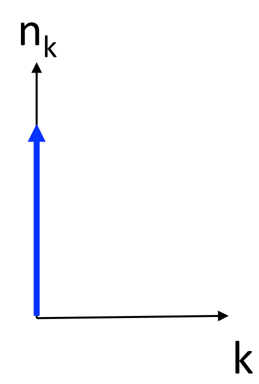
$$n_{\mathbf{k}}^b = G_b(\mathbf{k}) = \langle b_{\mathbf{k}}^\dagger b_{\mathbf{k}} \rangle$$

Off-diagonal long-ranged order

$$G_b(\mathbf{r} \to \infty) = \rho$$

BEC condensate

$$n_{\mathbf{k}}^{BEC} = N\delta_{\mathbf{k},\mathbf{0}}$$



# 2D Interacting Superfluid

Interacting Hamiltonian

$$H = \sum_j \frac{\mathbf{p}_j^2}{2m} + \sum_{i,j} V(\mathbf{r}_i - \mathbf{r}_j)$$

Green's function

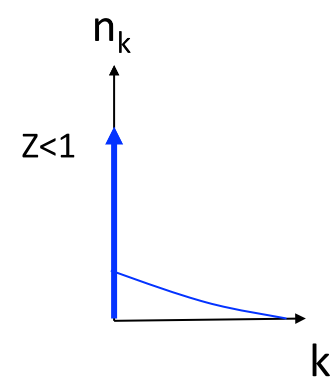
$$G_b(\mathbf{r}) = \langle b^\dagger(\mathbf{r}) b(\mathbf{0}) \rangle$$

Off-diagonal
long-ranged order

$$G_b(\mathbf{r} \to \infty) = \rho_c = Z\rho; \qquad Z < 1$$

Depleted Condensate
density in
Interacting Superfluid

$$n_{\mathbf{k}}^{SF} = ZN\delta_{\mathbf{k},0} + \delta n_{\mathbf{k}}^{SF}$$



# 2D Bose-Metal

$$G_b(\mathbf{r}) = \langle b^\dagger(\mathbf{r}) b(\mathbf{0}) \rangle$$

(Equal time Boson Green's function)

- A ***stable liquid phase*** of bosons that is not a superfluid

- Real space Green's function has oscillatory power law decay (***not*** a Bose condensate)

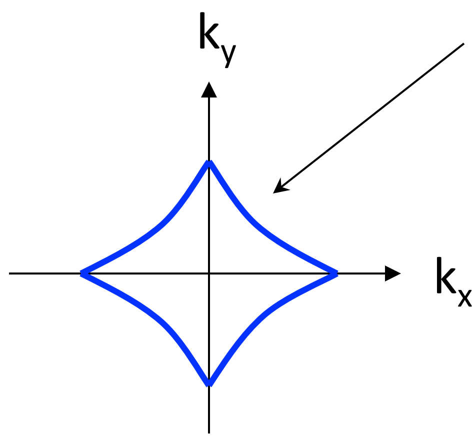
$$G_{BM}(\mathbf{r}) \sim \frac{\cos[\mathbf{k}_B(\hat{\mathbf{r}}) \cdot \mathbf{r}]}{|\mathbf{r}|^{\alpha(\hat{\mathbf{r}})}}$$

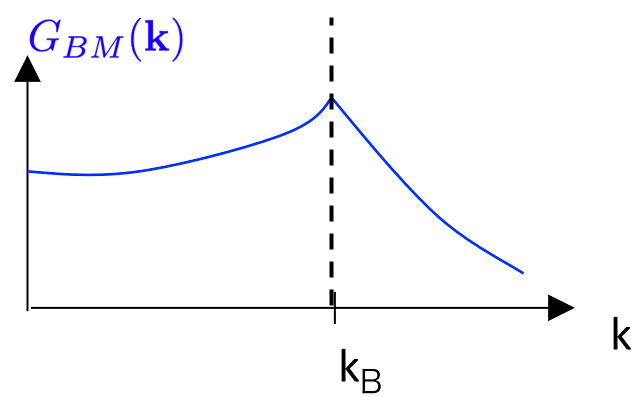
- ***Singularities*** in momentum distribution function

$$G_b(\mathbf{k}) = \langle b^\dagger_{\mathbf{k}} b_{\mathbf{k}} \rangle$$

- Singular momentum on a ***"Bose surface"***

$\mathbf{k}_B(\hat{\mathbf{r}})$




Angular dependent anomalous dimension $\alpha(\hat{\mathbf{r}})$

# What is a “Non-Fermi-liquid metal”?

First:  What is a Fermi Liquid Metal

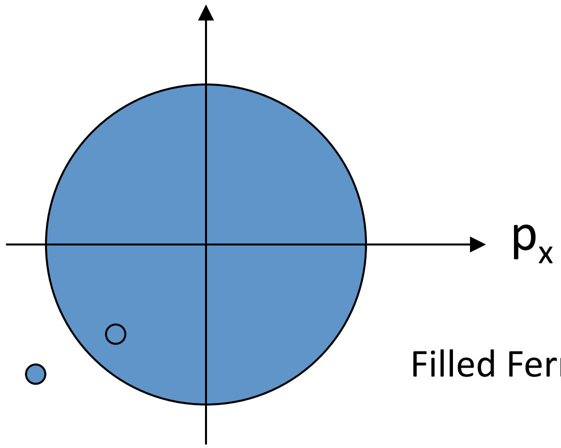
# 2D Free Fermi Gas

Free Fermions

$$H_0 = \sum_j \frac{\mathbf{p}_j^2}{2m}$$

$$\mathcal{H}_0 = \sum_k \epsilon_k c_k^\dagger c_k$$

$$\epsilon_k = \frac{k^2}{2m} - \mu$$

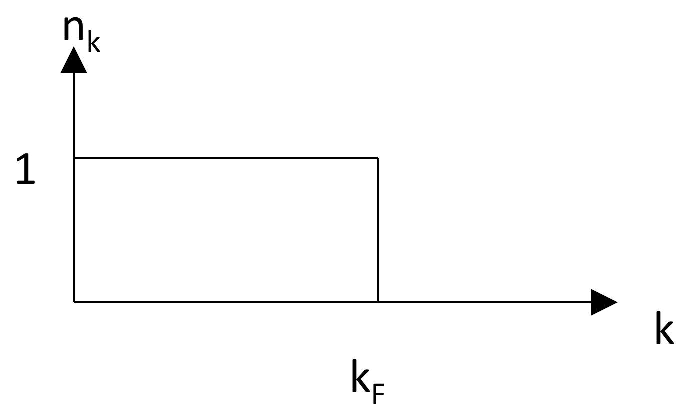



particle/hole excitations

Momentum Distribution Function:

$$n_k = \langle c_k^\dagger c_k \rangle$$

$$n_{\mathbf{k}}^{FF} = \Theta(k_F - |\mathbf{k}|)$$




Volume of Fermi sea determined by the density of particles $\rho = k_F^2/4\pi$

Fermion Spectral function:

Sharp quasiparticle excitations:

$$A_0(k,\omega) = ImG_0(k,\omega) = \delta(\omega - \epsilon_k)$$

# 2D Fermi-liquid Metal

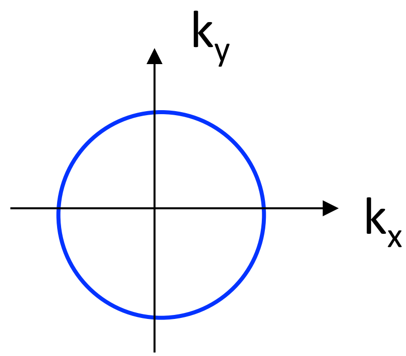
Equal time Green's function: $$G(\mathbf{r}-\mathbf{r}') = \langle c^\dagger(\mathbf{r}) c(\mathbf{r}') \rangle$$

Oscillatory decay $$G_{FL}(\mathbf{r}) \sim \frac{\cos(k_F|\mathbf{r}| - 3\pi/4)}{|\mathbf{r}|^{\alpha_{FL}}}; \quad \alpha_{FL} = 3/2$$

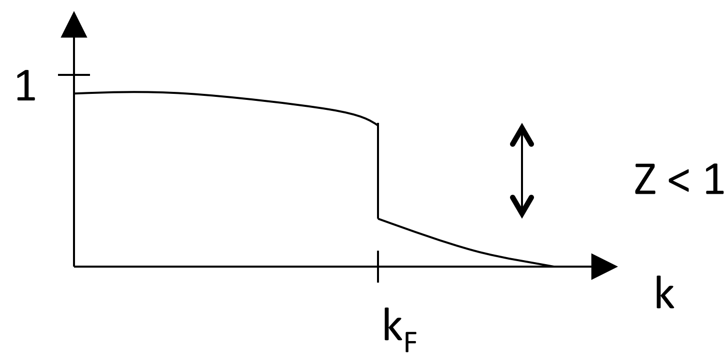
Momentum distribution function $$n_{\mathbf{k}}^{FL} = Z \cdot n_{\mathbf{k}}^{FF} + \delta n_{\mathbf{k}}^{FL} \qquad Z < 1$$

Luttingers Thm: Volume inside Fermi surface set by total density of fermions

$$\rho = k_F^2/4\pi$$




$$G(\mathbf{k}) = \langle c_{\mathbf{k}}^\dagger c_{\mathbf{k}} \rangle$$

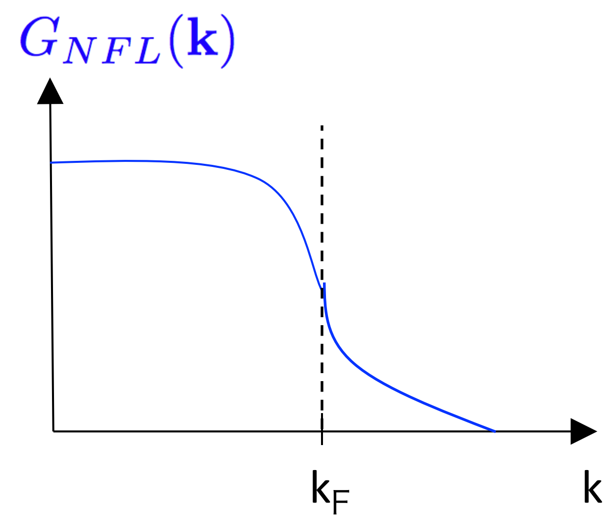



Quasi-particle excitation are (infinitely) long-lived on the Fermi surface

$$A(k,\omega) = Z\delta(\omega - \epsilon_k) + A_{inc}(k,\omega)$$

# 2D Non-Fermi Liquid Metal

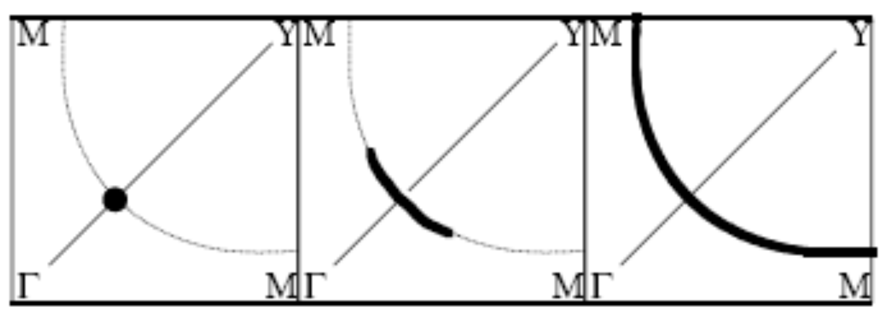
Various possibilities:




1) A singular "Fermi surface" that satisfies Luttinger's theorem but without a jump discontinuity in momentum distribution function

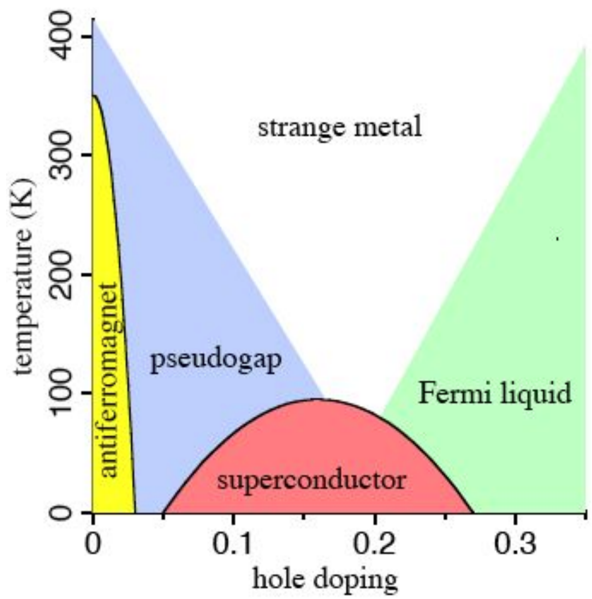
2) A singular Fermi surface that violates Luttinger's theorem (eg. volume "x" rather than "1-x")

3) A singular "Fermi surface" with ``arc"



# Motivation for Non-Fermi-Liquid Metal: "Abnormal" state of High $T_c$ Superconductors

Phase Diagram




**Strange metal**: "Fermi surface" but quasiparticles are not "sharp"

Spectral function measured with ARPES suggests Z=0

# Importance of the Strange Metal

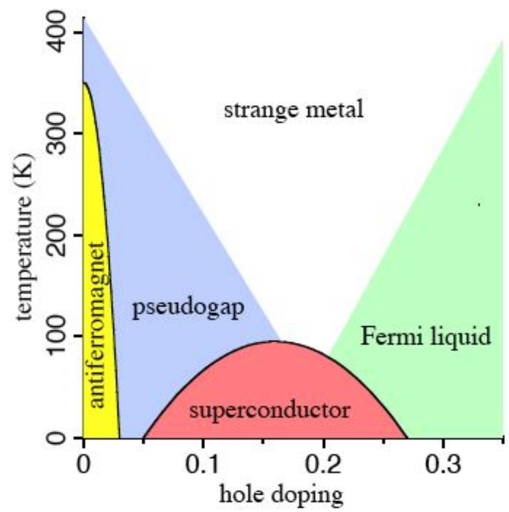
My bias:

- *The* theory of high Tc must **begin** with the Strange metal.
  (low energy physics emerges from high energy, not vice versa)

- Strange metal is a true (T=0) Non-Fermi liquid quantum phase or quantum phase transition
  (not just an “incoherent” finite T crossover)

- Pseudogap and d-wave SC should be understood as “instabilities” of the strange metal
  (akin to low Tc BCS sc emerging from Fermi liquid)

- Symmetry breaking “order” in the pseudogap regime is very beautiful, but (perhaps) a diversion




Strategy: Construct candidate Non-Fermi liquid quantum states as putative strange metals

# Wavefunction for 2D Bose-Metal?
# Wavefunction for 2D Non-Fermi liquid Metal?

First: Wavefunction for BEC and Superfluid phase of Bosons

Wavefunction for Free Fermions and a Fermi liquid

# Wavefunctions for Bose BEC and Superfluid

Bose Einstein Condensate (BEC)

$$\Psi_{BEC} = 1$$

Wavefunction is everywhere positive
ie. nodeless

Interacting Superfluid (SF)

Maintain the same nodeless structure,
put in a factor to keep the particles apart

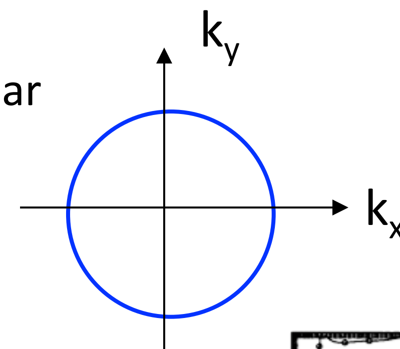
$$\Psi_{SF} = e^{-\sum_{i<j} u(\mathbf{r}_i - \mathbf{r}_j)} \geq 0$$

Jastrow form $u(\mathbf{r})$ is a variational parameter (function)

# Wavefunction for 2D Free Fermi gas

(N spinless fermions in 2D)

Free Fermion determinant: (eg with 2D circular Fermi surface)

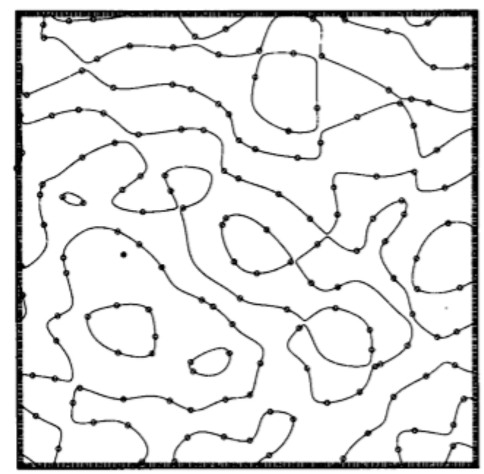



$$\Psi_{FF}(\{\mathbf{r}_i\}) = det[e^{i\mathbf{k}_i \cdot \mathbf{r}_j}]$$

Real space ***"nodal structure"***

Define a ``relative single particle function"

$$\Phi_{\mathbf{r}_2,\ldots,\mathbf{r}_N}(\mathbf{r}) \equiv \Psi(\mathbf{r}, \mathbf{r}_2, \ldots, \mathbf{r}_N)$$



Nodal lines:

Ultraviolet and infrared "locking"

# Wavefunction for interacting Fermi liquid?

Keep the sign (nodal) structure of free fermions, modifying the amplitude of the wavefunction, eg to keep the particles apart.

Common form: Multiply the free fermion wavefunction by a Jastrow factor,

$$\Psi_{Jastrow} = e^{-\sum_{i<j} u(\mathbf{r}_i - \mathbf{r}_j)} \geq 0$$

Proposed Fermi liquid wavefunction, with u(r) as a variational function

$$\Psi_{FL} = e^{-\sum_{i<j} u(\mathbf{r}_i - \mathbf{r}_j)} \psi_{FF}$$

Open question: Does the momentum distribution function that follows from this class of wavefunctions have a jump discontinuity on a Fermi surface with volume set by the density of particles?? Most probably yes!

$$G(\mathbf{r} - \mathbf{r}') = \int_{\mathbf{r}_2,..,\mathbf{r}_N} \Psi^*_{FL}(\mathbf{r}, \mathbf{r}_2, .., \mathbf{r}_N) \Psi_{FL}(\mathbf{r}', \mathbf{r}_2, .., \mathbf{r}_N) \quad \longrightarrow \quad \langle c^\dagger_{\mathbf{k}} c_{\mathbf{k}} \rangle = G(\mathbf{k})$$

# Wavefunction for a 2D (D-wave) Bose-Metal

O. Motrunich/ MPAF Phys. Rev. B (2007)

Wavefunctions:

N bosons moving in 2d: $\Psi(\mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_N)$

Define a ``relative single particle function" $\Phi_{\mathbf{r}_2,\ldots,\mathbf{r}_N}(\mathbf{r}) \equiv \Psi(\mathbf{r}, \mathbf{r}_2, \ldots, \mathbf{r}_N)$ .

"Known" example of
boson non-superfluid:
Laughlin nu=1/2 Bosons:

$$\Psi_{\nu=1/2}(z_1, z_2, \ldots, z_N) = \prod_{i<j}(z_i - z_j)^2 \ .$$

Point nodes in ``relative particle function"
Relative d+id 2-particle correlations

$$\Phi_{\nu=1/2}(z) \sim (z - z_i)^2$$

Goal: Construct time-reversal invariant analog of Laughlin,
(with relative $d_{xy}$ 2-particle correlations)

# Wavefunction for D-wave Bose-Metal (DBM)

Hint: nu=1/2 Laughlin is a determinant squared

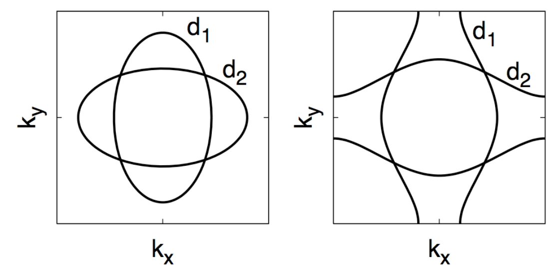
$$\Psi_{\nu=1/2} = [\Psi_{\nu=1}]^2 \qquad \Phi_{\nu=1}(z) \sim (z - z_i)$$

p+ip 2-body

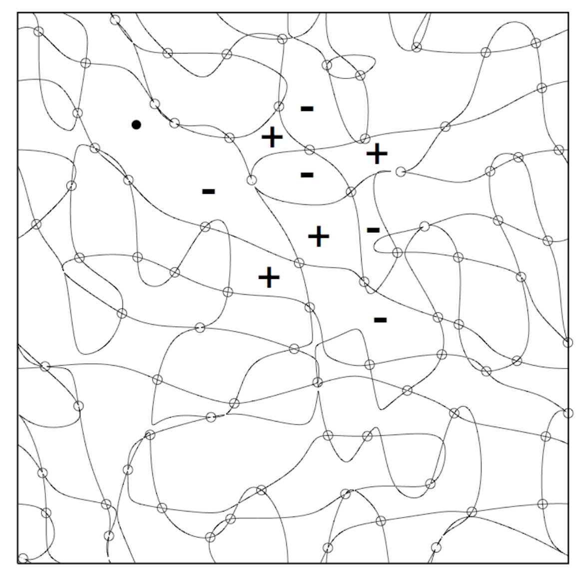
Try squaring Fermi sea wf:
No, “s-wave” with ODLRO

$$\Psi(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N) = (\det e^{i\mathbf{k}_i \cdot \mathbf{r}_j})^2 \ , \quad \text{(S-type).}$$

``D-wave” Bose-Metal:
Product of 2 different Fermi sea determinants, elongated in the x or y directions

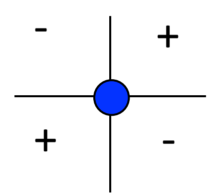



$$\Psi_{D_{xy}}(\mathbf{r}_1, \ldots, \mathbf{r}_N) = (det)_x \times (det)_y$$




Nodal structure of DBM wavefunction:

$$\Phi_{D_{xy}}(\mathbf{r}) \sim (x - x_i)(y - y_i)$$

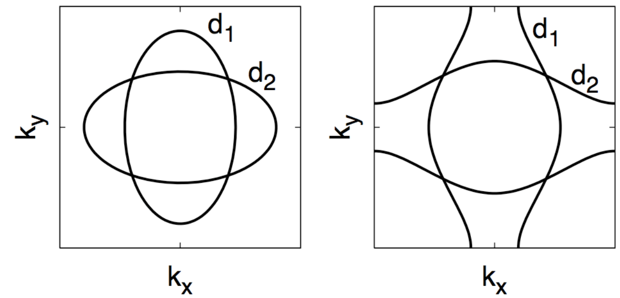



$D_{xy}$ relative
2-particle correlations

# Bose Surfaces in D-wave Bose-Metal

Mean Field Green's functions factorize:

$$G_b^{MF}(\mathbf{r},\tau) = G_{d_1}^{MF}(\mathbf{r},\tau) G_{d_2}^{MF}(\mathbf{r},\tau)/\bar{\rho}$$

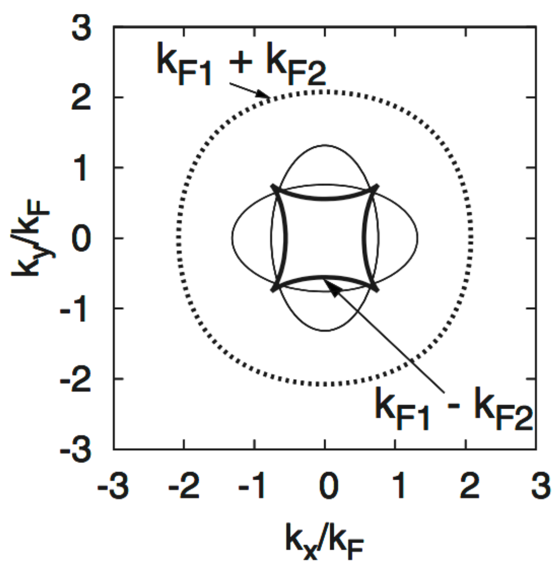



$$G_{d_\alpha}^{MF}(\mathbf{r}) \approx \frac{1}{2^{1/2}\pi^{3/2}} \frac{\cos(\mathbf{k}_{F_\alpha} \cdot \mathbf{r} - 3\pi/4)}{c_\alpha^{1/2}|\mathbf{r}|^{3/2}}$$

$$(\partial \epsilon_\alpha / \partial \mathbf{k})_{\mathbf{k}_{F_\alpha}(\hat{\mathbf{r}})} = (\hat{\mathbf{r}})$$

Momentum distribution function:

$$n_b(\mathbf{k}) = \int G_b(\mathbf{r}) e^{i\mathbf{k}\cdot\mathbf{r}} d\mathbf{r}$$




Two singular lines in momentum space, Bose surfaces:

$$\mathbf{k}_{F_1}(\hat{\mathbf{r}}) \pm \mathbf{k}_{F_2}(\hat{\mathbf{r}})$$

# Gutzwiller wavefunctions for Electron Non-Fermi-Liquids

Decompose the electron:
spinless charge e boson
and s=1/2 neutral fermionic spinon

$$c^{\dagger}_{\mathbf{r}\alpha} = b^{\dagger}_{\mathbf{r}} f^{\dagger}_{\mathbf{r}\alpha}$$

Mean Field Theory

Treat “Spinons” and Bosons as Independent: $\mathcal{H} = \mathcal{H}_f + \mathcal{H}_b$

Wavefunctions $\psi_f(\mathbf{x}_{i\uparrow}, \mathbf{x}_{i\downarrow})$ $\psi_b(\mathbf{r}_j)$

(enlarged Hilbert space - twice as many particles)

“Fix-up” Mean Field Theory

Gutzwiller projection: “glue” together Fermion and Boson “partons”

$$\Psi_G \equiv \psi_f(\mathbf{x}_{i\alpha}) \times \psi_b(\mathbf{r}_i \rightarrow \mathbf{x}_{i\alpha})$$

Project back into physical Hilbert space

# Fermi and Non-Fermi Liquids?

Put the Spinons in a filled Fermi sea

$$\psi_f = \det[e^{i\mathbf{k}_i\cdot\mathbf{x}_{j\uparrow}}] \times \det[e^{i\mathbf{k}_i\cdot\mathbf{x}_{j\downarrow}}]$$

***Fermi Liquid***: Put the Bosons into a superfluid

$$\psi_b^{SF} = e^{-\sum_{i<j} u(\mathbf{r}_i - \mathbf{r}_j)}$$

$$\Psi_{FL} = \mathcal{P}_G[\psi_f^{FF} \times \psi_b^{SF}]$$

***Non-Fermi Liquid***: Put Bosons into an *uncondensed* fluid - a “Bose metal”

$$\Psi_{NFL} = \mathcal{P}_G[\psi_f^{FF} \times \psi_b^{BoseMetal}]$$

***D-wave NFL Metal: Product of Fermi sea and D-wave Bose-Metal***

# ***Hamiltonians*** with Bose-Metal or NFL-Metal Ground states???

First: Hamiltonians for conventional FL metals?

$$H_0 = -\sum_{ij} t_{ij} c^\dagger_{i\sigma} c_{j\sigma} \qquad \Psi_{FF}(\{\mathbf{r}_i\}) = det[e^{i\mathbf{k}_i \cdot \mathbf{r}_j}]$$

$$H = -\sum_{ij} t_{ij} c^\dagger_i c_j + \sum_{ij} V_{ij} n_i n_j$$

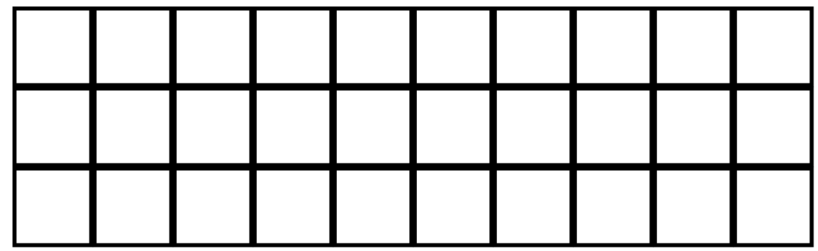
Can we demonstrate numerically that ***any*** interacting Fermion Hamiltonian has a Fermi-liquid ground state?? No!

Numerical challenge:

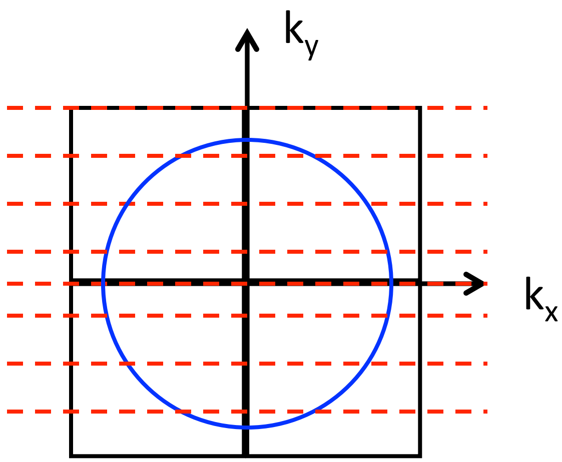
- Exact Diagonalization; too small
- QMC; sign problem
- Variational wavefunctions; biased
- DMFT; biased and uncontrolled

# Ladders to the Rescue

Transverse y-components of momentum become quantized

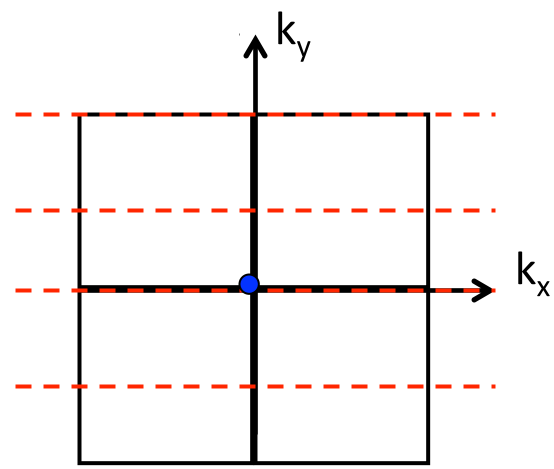


Put interacting Fermions on n-leg ladder
(assume in a FL state in 2D)




Many gapless 1d modes, one for each Fermi point

Put hard core Bosons hopping on n-leg ladder
(assume 2D Hamiltonian in superfluid phase)




Single gapless 1d mode

***"Fingerprint" of 2D Fermi surface already present on 2-leg ladder***

(for N=1 Fermions and hard core Bosons are the "same")

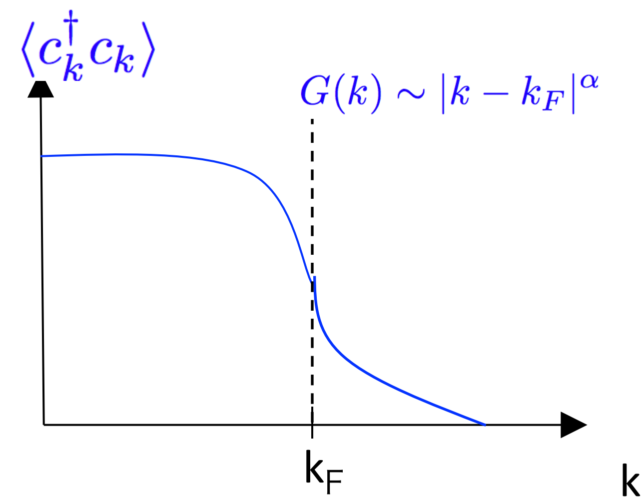
# Access fingerprint of 2D FL with DMRG on Ladders

## Interacting Fermions on 1-leg ladder (1D)

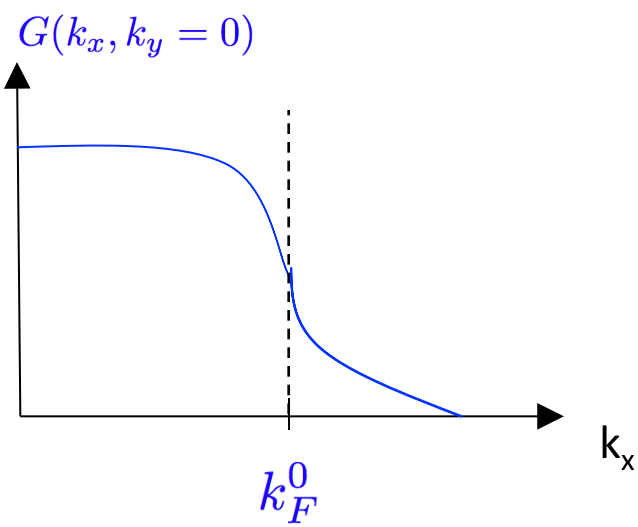
$$G(x) \sim \sin(k_F x)/x^{1+\alpha}$$

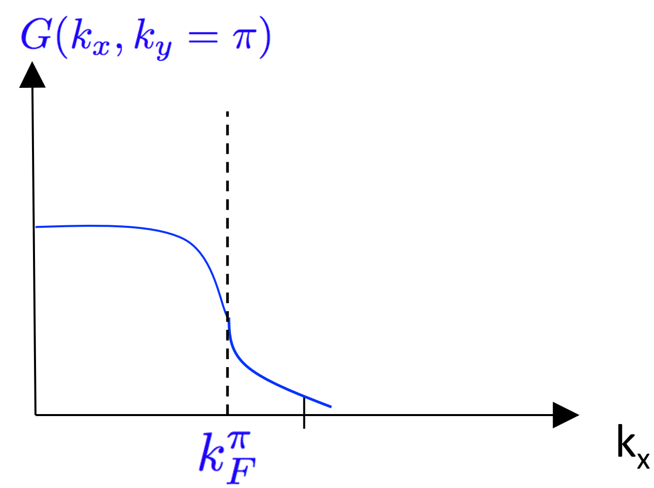
Luttinger liquid exponent: $\alpha$

Momentum distribution function




## Interacting Fermions on 2-leg ladder $\alpha_0, \alpha_\pi$



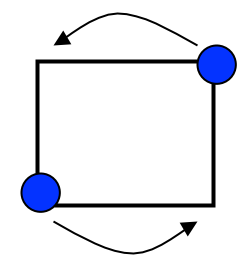




***Luttinger "volume" sum rule:*** $$k_F^0 + k_F^\pi = \pi n_{1d}$$

# Hamiltonian for D-wave Bose-Metal?

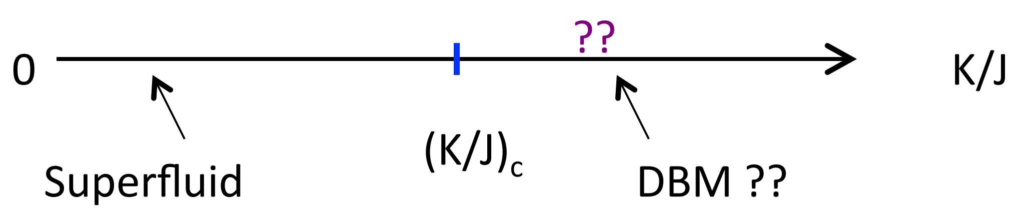
(Strong coupling limit of gauge theory)

***"Ring exchange"***



$$
\begin{aligned}
H &= H_J + H_4 \ , \\
H_J &= -J \sum_{\mathbf{r};\, \hat{\mu}=\hat{\mathbf{x}},\hat{\mathbf{y}}} (b^\dagger_{\mathbf{r}} b_{\mathbf{r}+\hat{\mu}} + h.c.) \ , \\
H_4 &= K_4 \sum_{\mathbf{r}} (b^\dagger_{\mathbf{r}} b_{\mathbf{r}+\hat{\mathbf{x}}} b^\dagger_{\mathbf{r}+\hat{\mathbf{x}}+\hat{\mathbf{y}}} b_{\mathbf{r}+\hat{\mathbf{y}}} + h.c.) \ ,
\end{aligned}
$$

Phase diagram: K/J and density of bosons



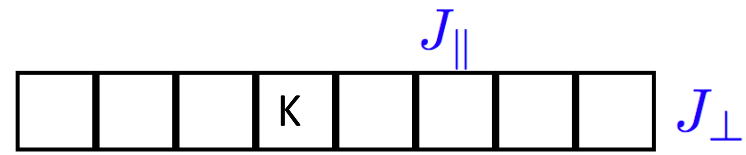

J-K Model has a sign problem - completely intractable

# Ladders to the rescue:
# Boson ring model on the 2-Leg Ladder

- Exact Diagonalization (2 x 18)
- Variational Monte Carlo
- DMRG  (2 x 50)

E. Gull, D. Sheng, S. Trebst,
O. Motrunich and MPAF,
Phys. Rev. B 78, 54520 (2008) )

$$
\begin{aligned}
H &= H_J + H_4 \ , \\
H_J &= -J \sum_{\mathbf{r};\, \hat{\mu}=\hat{\mathbf{x}},\hat{\mathbf{y}}} (b^\dagger_{\mathbf{r}} b_{\mathbf{r}+\hat{\mu}} + h.c.) \ , \\
H_4 &= K_4 \sum_{\mathbf{r}} (b^\dagger_{\mathbf{r}} b_{\mathbf{r}+\hat{\mathbf{x}}} b^\dagger_{\mathbf{r}+\hat{\mathbf{x}}+\hat{\mathbf{y}}} b_{\mathbf{r}+\hat{\mathbf{y}}} + h.c.) \ ,
\end{aligned}
$$



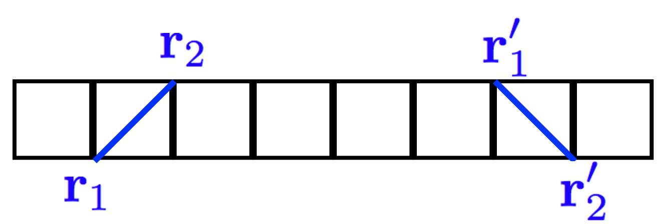

Correlation Functions:

1) Momentum Distribution function $n(k_x, k_y) = \langle b^\dagger_{\mathbf{k}} b_{\mathbf{k}} \rangle; \quad k_y = 0, \pi$

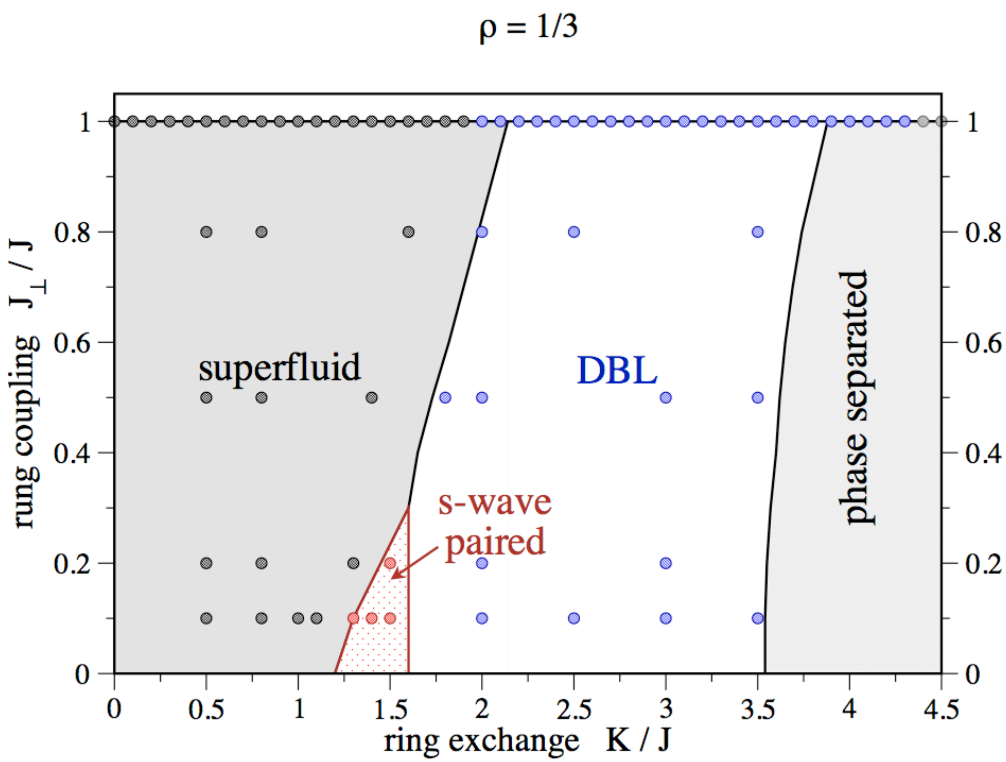
2) Density-density structure factor $\mathcal{D}(\mathbf{k}) = \sum_{\mathbf{r}} e^{i\mathbf{k}\cdot\mathbf{r}} \langle n_{\mathbf{r}} n_{\mathbf{0}} \rangle \qquad n_{\mathbf{r}} = b^\dagger_{\mathbf{r}} b_{\mathbf{r}}$

3) Pair-boson correlator

$$P(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}'_1, \mathbf{r}'_2) = \langle b^\dagger_{\mathbf{r}_1} b^\dagger_{\mathbf{r}_2} b_{\mathbf{r}'_1} b_{\mathbf{r}'_2} \rangle$$



# Phase Diagram for 2-leg ladder



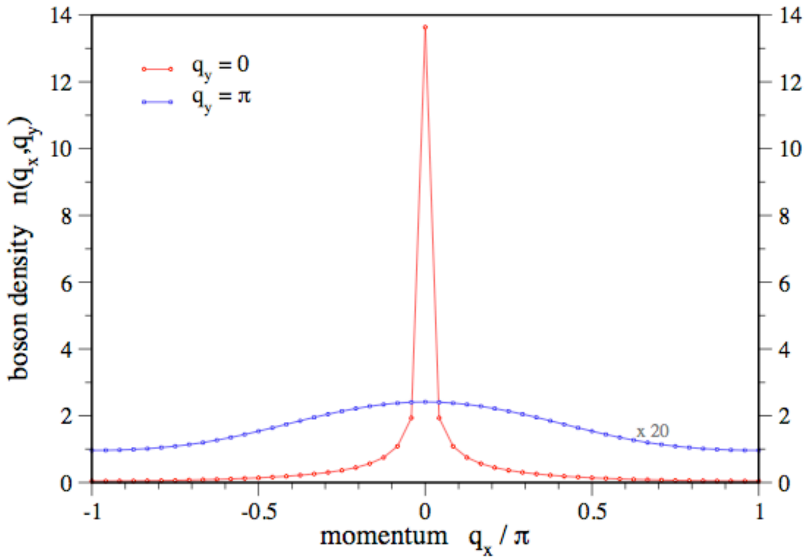

Phases:

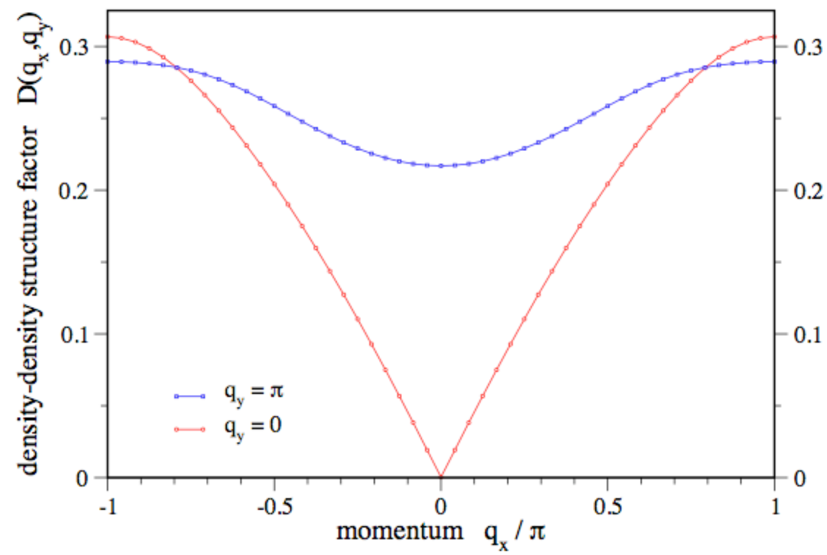
1) Superfluid – "Bose condensate"
2) D-Wave Bose Metal - DBL
3) s-wave Pair-Boson "condensate"

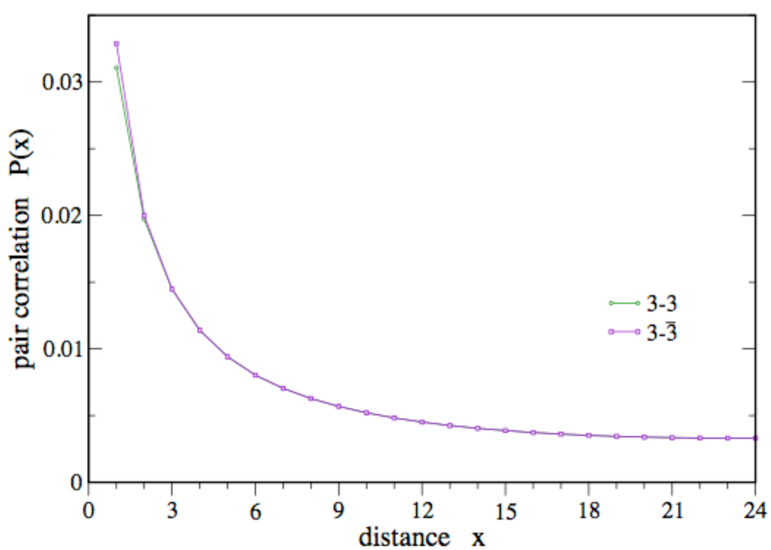
D-wave Bose-Metal occupies large region of phase diagram

# Superfluid (DMRG)

DMRG 2 x 48



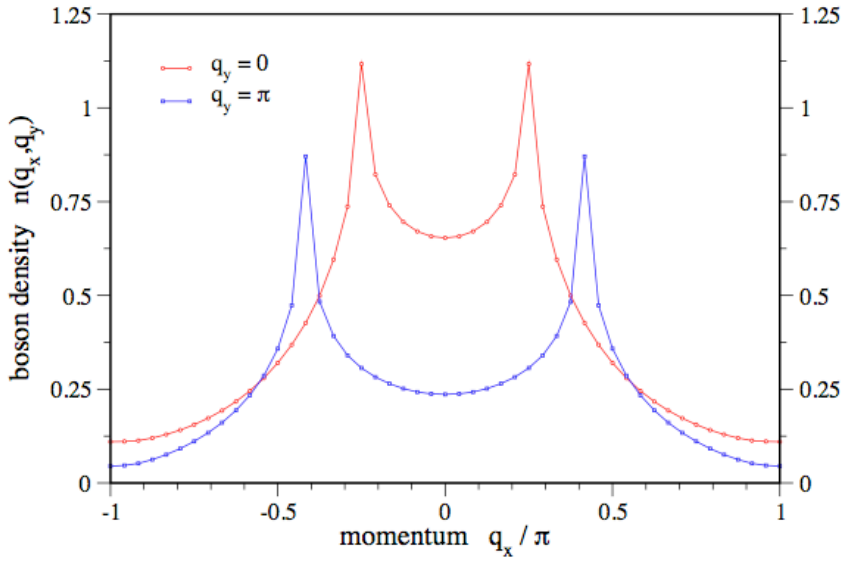

$$\rho = 1/3$$

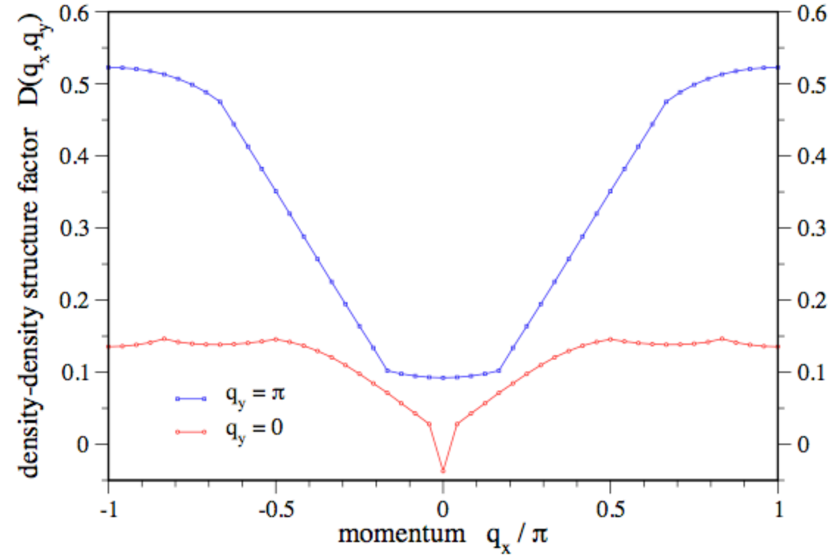
$$K/J = 1$$

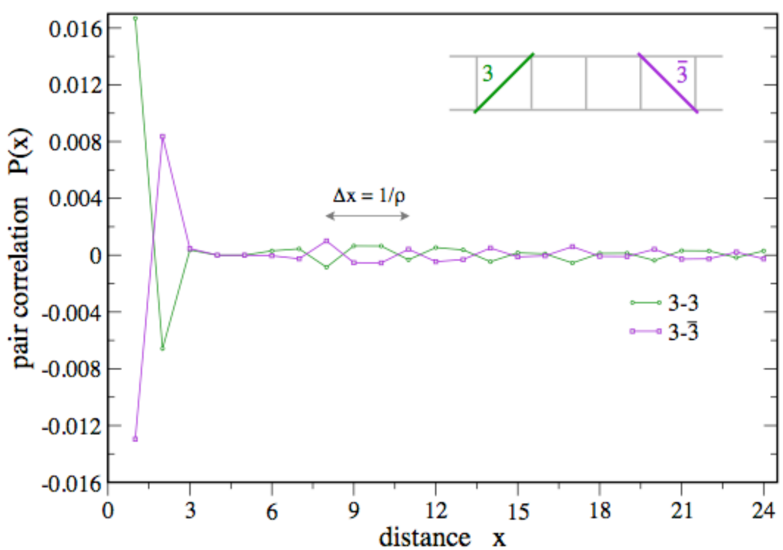
$$J_\perp/J = 1$$

# D-Wave Bose Metal (from DMRG)

DMRG 2 x 48



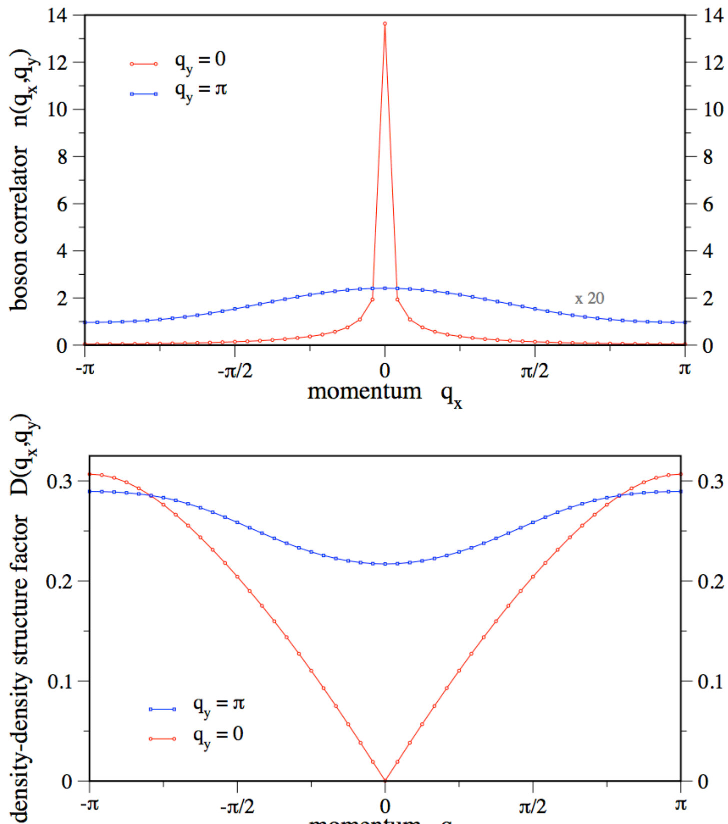



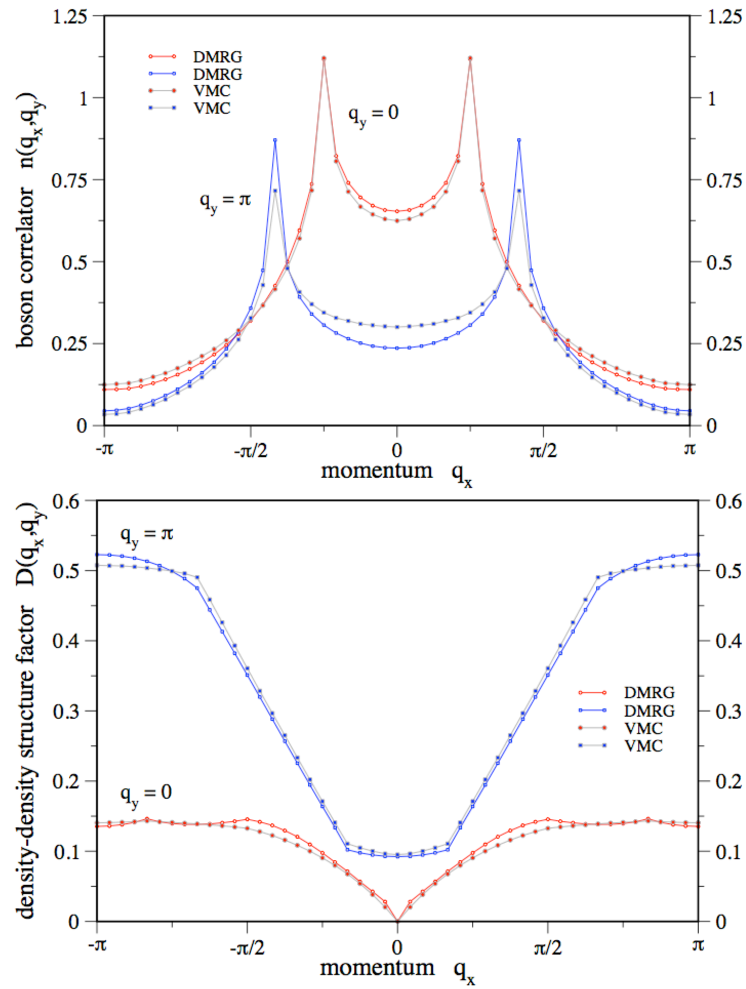

$$\rho = 1/3$$

$$K/J = 3$$

$$J_{\perp}/J = 1$$

# Superfluid versus D-wave Bose-Metal

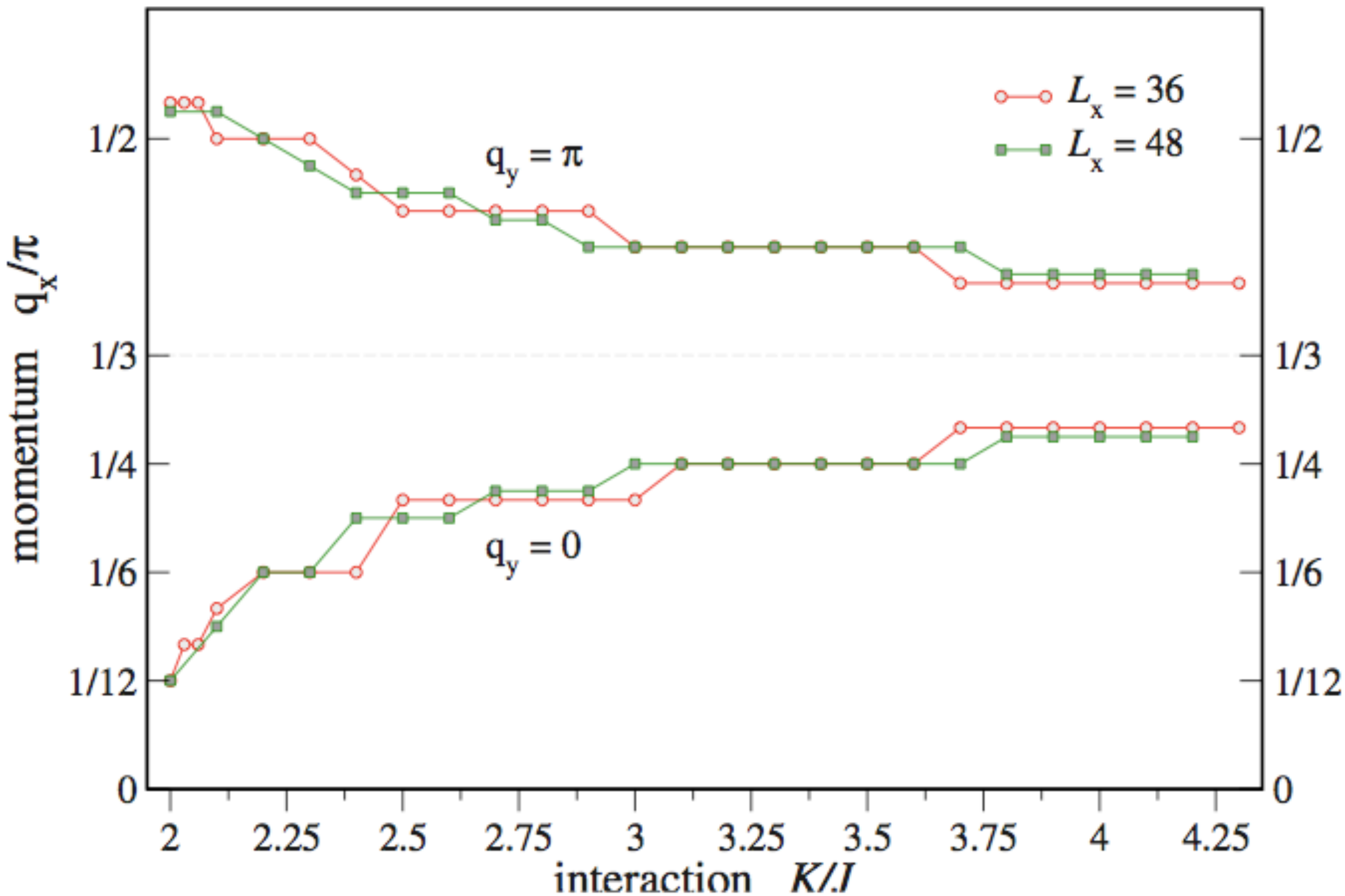


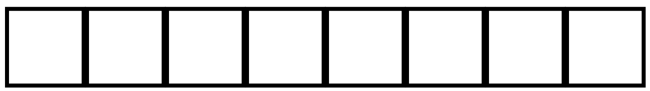

Superfluid - "condensed" at zero momentum

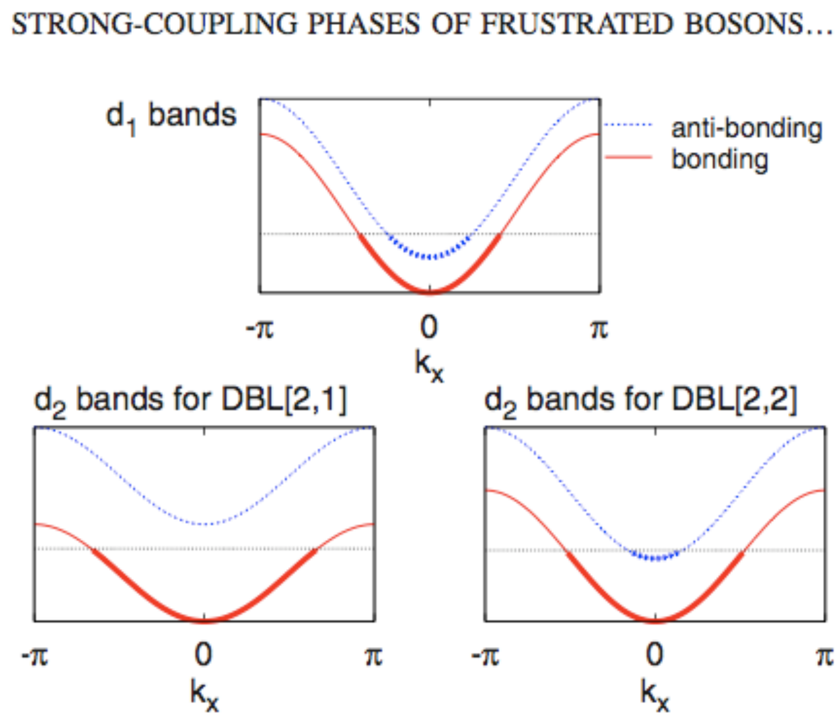
D-wave Bose-Metal; Singular "Bose points" at $q_y = 0, \pi$

# Singular Momentum in D-wave Bose-Metal (Bose "surface")



# Variational Wavefunction for ladder






In DBM:

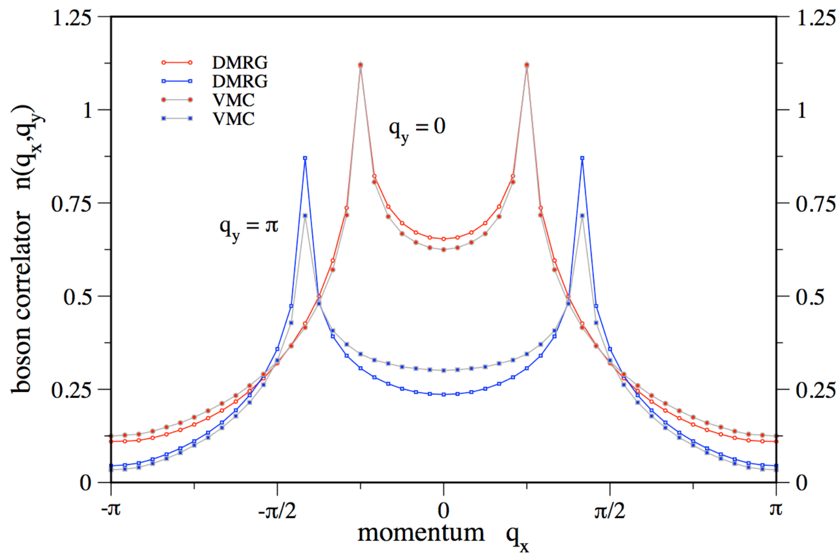
Bonding/Antibonding occupied
For $d_1$ Fermion

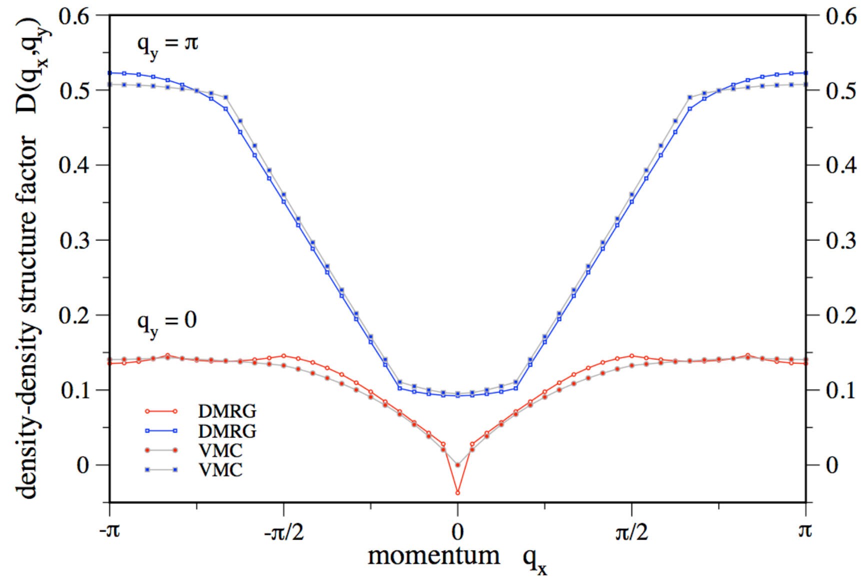
Just Bonding occupied
For $d_2$ Fermion

Variational parameter:
Fermi wavevectors in $d_1$ bands

$$\Psi_{\mathrm{bos}}(r_1, r_2, \ldots) = \Psi_{d_1}(r_1, r_2, \ldots) \cdot \Psi_{d_2}(r_1, r_2, \ldots).$$

# DBM: How good is ladder variational wavefunction?



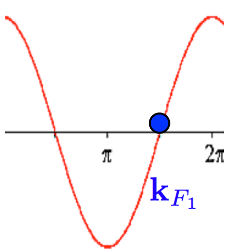




Gauge mean field theory predicts singularities in momentum distribution function at: $\mathbf{k}_{F_1} \pm \mathbf{k}_{F_2}$

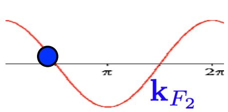
Both DMRG and $det_1$ x $det_2$ Wavefunction show singular cusps *only* at: $\mathbf{k}_{F_1} - \mathbf{k}_{F_2}$

Why? Ampere's Law - Parallel currents attract

$d_1$ and $d_2$ Fermions have opposite gauge charge, so right moving $d_1$ attracts left moving $d_2$ to form boson at momentum: $\mathbf{k}_{F_1} - \mathbf{k}_{F_2}$




$d_1$




$d_2$

# "D-Wave Metal"

Itinerant non-Fermi liquid phase of 2d electrons?

Gauge theory
(parton) construction

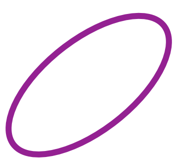
$$c^\dagger_\alpha(\mathbf{r}) = b^\dagger(\mathbf{r}) f^\dagger_\alpha(\mathbf{r}) = d^\dagger_x(\mathbf{r}) d^\dagger_y(\mathbf{r}) f^\dagger_\alpha(\mathbf{r})$$

Wavefunction; Product of determinants

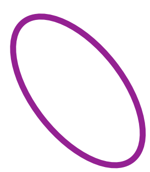
$$\{\vec{R}_i\} = \{\vec{r}_{i\uparrow}, \vec{r}_{j\downarrow}\}$$

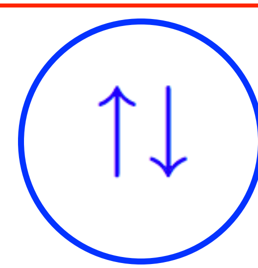
$$\Psi^{Metal}_{d_{x^2-y^2}} = \det_{x+y}[e^{i\vec{K}_i\cdot\vec{R}_j}] \cdot \det_{x-y}[e^{i\vec{K}_i\cdot\vec{R}_j}] \times \det[e^{i\vec{k}_i\cdot\vec{r}_{j\uparrow}}] \cdot \det[e^{i\vec{k}_i\cdot\vec{r}_{j\downarrow}}]$$

$x+y$



$x-y$





Filled Fermi sea

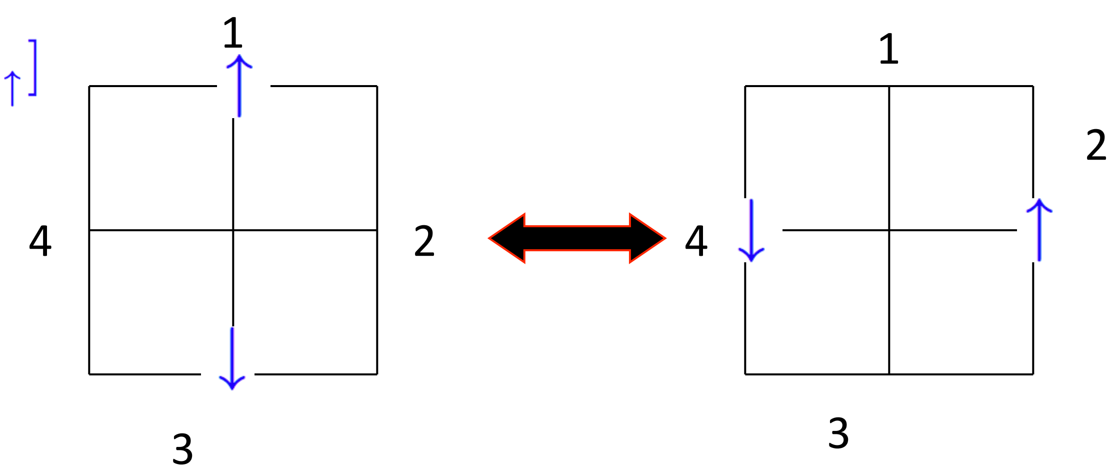
# Hamiltonian for D-wave Metal?
# (from gauge theory)

## t-K "Ring" Hamiltonian (no double occupancy constraint)

$$\mathcal{H}_{tK} = -t\sum_{\langle ij\rangle}[c^\dagger_{i\alpha}c_{j\alpha} + h.c.] + K\sum_{\langle 1234\rangle}[\mathcal{S}^\dagger_{13}\mathcal{S}_{24} + h.c.]$$

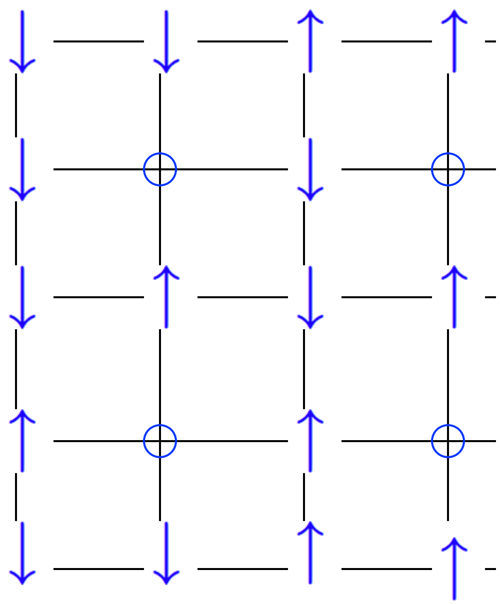
$$\mathcal{S}^\dagger_{ij} = \frac{1}{\sqrt{2}}[c^\dagger_{i\uparrow}c^\dagger_{j\downarrow} - c^\dagger_{i\downarrow}c^\dagger_{j\uparrow}]$$

Electron singlet pair
"rotation" term

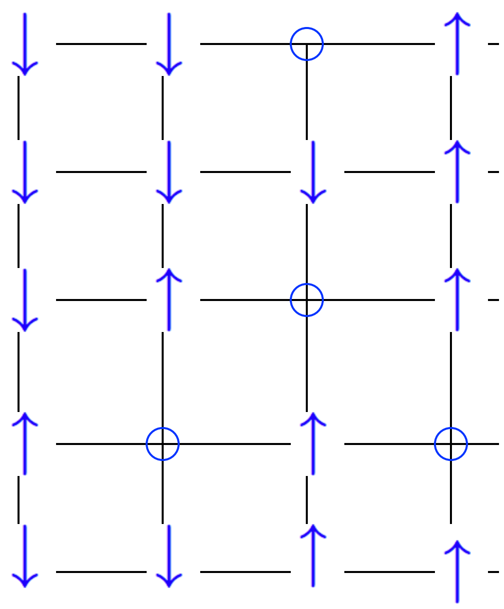



- Ring term will be generated when projecting into a single band model
- Ring term operates when two doped holes are nearby
- Ring term induces 2-particle singlet d-wave correlations (for K>0)

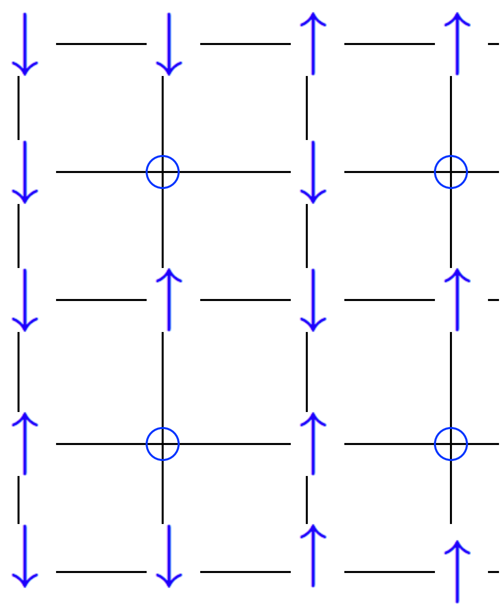
# Doping near optimal



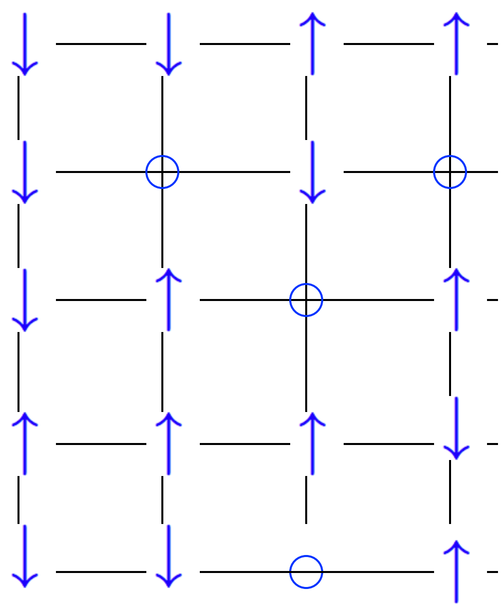
# Doping near optimal



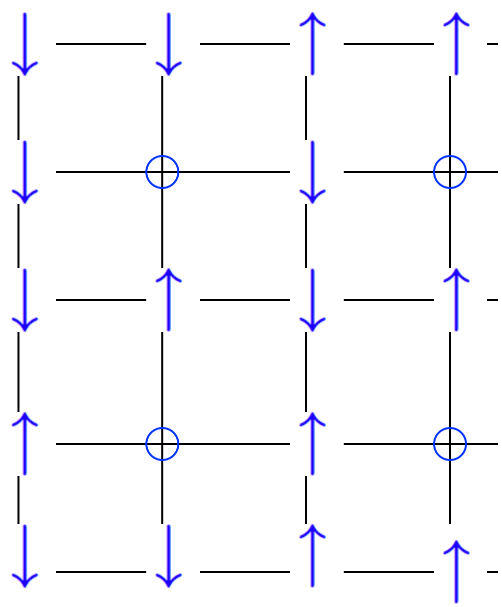
# Doping near optimal



# Doping near optimal



# Doping near optimal



# Phase diagram of electron t-K Hamiltonian?

$$\mathcal{H}_{tK} = -t \sum_{\langle ij \rangle} [c_{i\alpha}^\dagger c_{j\alpha} + h.c.] + K \sum_{\langle 1234 \rangle} [\mathcal{S}_{13}^\dagger \mathcal{S}_{24} + h.c.]$$

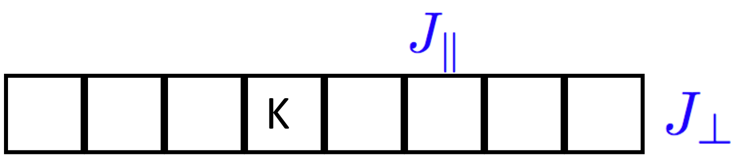
Doping and K/t ??

Fermi liquid for K<<t ?
D-wave metal for K ~ t ?
D-wave superconductor ?

*Future: Put t-K Hamiltonian on a*
*2-leg ladder and attack with DMRG,…*

# Summary & Outlook

- Bose-Metals are 2D gapless liquids with singular “Bose” surfaces
- Every 2D Bose-Metal has a distinct set of quasi-1D descendents states which should be numerically accessible via DMRG
- Hard core bosons with 4-site ring term on the 2-leg ladder has a quasi-1D descendant Bose-Metal ground state over a large part of phase diagram




Future generalizations (DMRG, VMC, gauge theory):

- Boson Ring exchange models on 3-leg, 4-leg ladders
- Quasi-1D descendents of 2D non-Fermi liquids of itinerant electrons?
  (D-Wave Metal on the n-leg ladder?)
- Other Hamiltonians with Bose-Metal or non-Fermi-liquid states???

# Wavefunctions for a Bose-metal and non-Fermi-liquid metal?

One attempt - Towards a “D-wave Metal”

Underlying is a “D-wave ***Bose***-Metal”

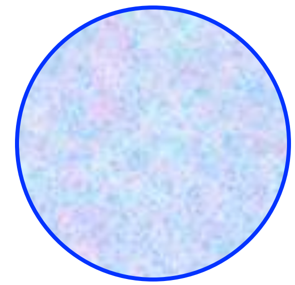
Informed/Motivated by the FQHE:  Why?

- Worked for Laughlin!
- Have Boson fluid states which are not-superfluids
- Have Non-Fermi liquids in QHE (Composite Fermi liquids)

# Half-filled Landau band for Bosons/Fermions

nu=1/2 for Bosons,
Laughlin wf, a non-superfluid

$$\Phi^{\nu=1/2}_{Laughlin} = \prod_{ij}(z_i - z_j)^2 \prod_j e^{-|z_j|^2/4}$$



nu=1/2 for electrons;
a “Composite Fermi-liquid”

$$\Psi_{CFL} = \prod_{i<j}(z_i - z_j)^2 \prod_j e^{-|z_j|^2/4} \cdot \det[e^{i\vec{k}_i \cdot \vec{r}_j}]$$

$$\Psi_{Fermi-sea} = \det[e^{i\mathbf{k}_i \cdot \mathbf{r}_j}]$$

Laughlin times a filled Fermi-sea state

$$\Psi_{CFL} = \Phi^{\nu=1/2}_{Laughlin} \times \Psi_{Fermi-sea}$$

Problem: QHE wavefunction breaks T-reversal invariance

Goal: Construct time-reversal invariant analogs of QHE states

# Gauge Theory for D-wave Bose Metal phase

Slave Fermion decomposition for lattice bosons: $b^\dagger(\mathbf{r}) = d_1^\dagger(\mathbf{r}) d_2^\dagger(\mathbf{r})$

Gauge Theory Hamiltonian: $H_{U(1)} = H_t + H_a$

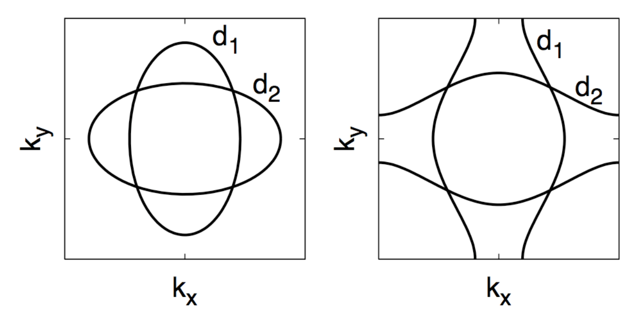
$$H_t = -\sum_{\mathbf{r}} \left[ t_\parallel e^{i a_x(\mathbf{r})} d_1^\dagger(\mathbf{r}) d_1(\mathbf{r}+\hat{\mathbf{x}}) + t_\perp e^{i a_y(\mathbf{r})} d_1^\dagger(\mathbf{r}) d_1(\mathbf{r}+\hat{\mathbf{y}}) + h.c. \right] - \sum_{\mathbf{r}} \left[ t_\perp e^{-i a_x(\mathbf{r})} d_2^\dagger(\mathbf{r}) d_2(\mathbf{r}+\hat{\mathbf{x}}) + t_\parallel e^{-i a_y(\mathbf{r})} d_2^\dagger(\mathbf{r}) d_2(\mathbf{r}+\hat{\mathbf{y}}) + h.c. \right]$$

$$H_a = h \sum_{\mathbf{r}} \sum_{\mu=x,y} e_\mu^2(\mathbf{r}) - K \sum_{\mathbf{r}} \cos[(\nabla \times a)_{\mathbf{r}}]$$

$$(\nabla \cdot e)_{\mathbf{r}} = d_1^\dagger(\mathbf{r}) d_1(\mathbf{r}) - d_2^\dagger(\mathbf{r}) d_2(\mathbf{r})$$

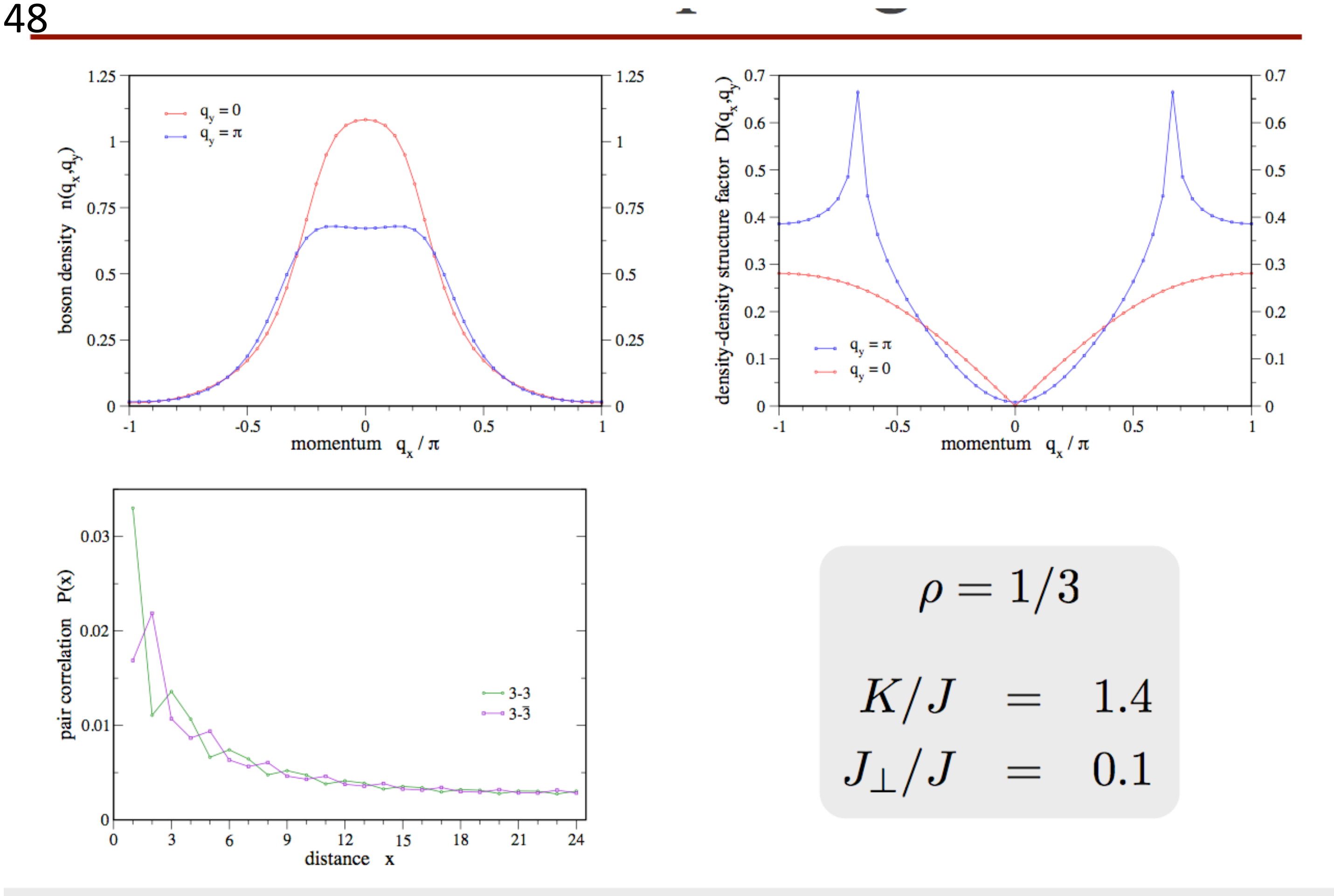
Strong coupling: h>>K,t integrate out gauge field gives Boson Hamiltonian: $\mathcal{H}(\hat{b}, \hat{b}^\dagger)$

Weak Coupling: K>> h,t Anisotropic Fermi surfaces of $d_1$ and $d_2$ minimally coupled to a (non-compact) U(1) gauge field



# S-wave Pair Boson “Condensate” (DMRG)

DMRG 2 x 48




DBM[2,2] VMC wavefuncton is favored in this region of the phase diagram – but unstable to s-wave pairing with gauge fluctuations present