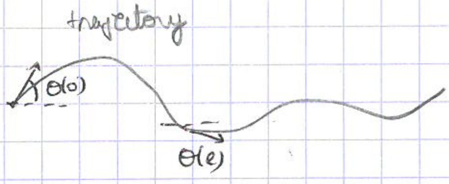
A Introduction on slides: Some quantitative evidence for jamming of cells.

B HOW DO YOU MODEL?

B1 interactions between cells:
- cells don't overlap
- cell adhere
- cells are polarized → may align.

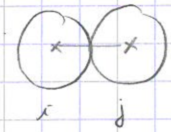
B2 activity:
- cells locomote
- they may have an active directionning → persistence length
- cell can change shape



$\langle \cos(\theta(\ell) - \theta(\ell+\ell')) \rangle \propto e^{-\ell/L}$ → persistence length.

C. SELF PROPELLED PARTICLE MODELS: ACTIVE MATTER CRASH COURSE

C1 interaction

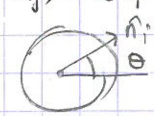


C2 activity

$$m \frac{d\vec{v_i}}{dt} = \vec{F}_{int} + \vec{F}_{propelled} + \vec{F}_{drag} + \vec{F}_{noise}$$

$$= \sum_j \nabla_r V(r_{ij}) + F_0 \hat{n}_i - b v_i + 0$$

→ overdamped limit $m \frac{d\vec{v_i}}{dt} \simeq 0 \Rightarrow \dot{r}_i = v_0 \hat{n}_i - \mu \sum_j \nabla_i V(r_{ij})$



preferred direction of particle i

$\hat{n}_i = \cos\theta_i \hat{x} + \sin\theta_i \hat{y}$

$\dot{\theta}_i = \eta_i \rightarrow \begin{cases} \langle \eta_i \rangle = 0 \\ \langle \eta_i(t) \eta_j(t') \rangle = 2 D_r \delta_{ij} \delta(t-t') \end{cases}$

rotational diffusion

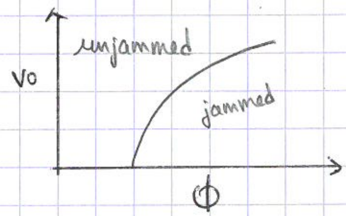
↳ relation $P_e = \frac{v_0}{2R D_r} \equiv \frac{L}{2R}$ "Peclet number".

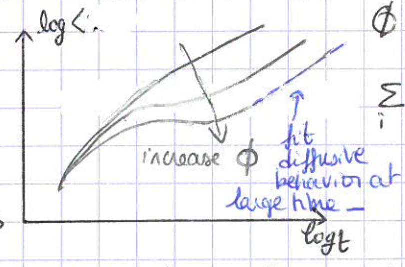
↳ model with very rich phenomenology, including "motility induced phase transition" MIPS.

Now what happens when going to very high densities? ⇒ GLASSY DYNAMICS?

↳ simulations in 2D
- Henkes et al PRE 84 2011
- Berthier et al PRL 112 2014

3 control parameters: $\{D_r, v_0, \phi\}$

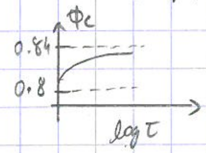


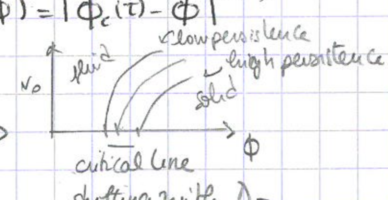


ϕ = packing fraction $= \sum_i \frac{\pi \sigma_i^2}{R^2}$ → particles, ↳ box

fit $\tau \equiv \frac{1}{D_r}$, $\langle \Delta r^2 \rangle \equiv 4 D_s(\tau, \phi) t$

Ansatz $D_s(\tau, \phi) = |\phi_c(\tau) - \phi|^{\gamma(\tau)}$

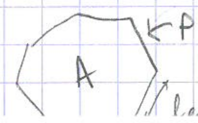




critical line shifting with D_r.

D VERTEX MODELS → the next simplest thing one can do after particles model.

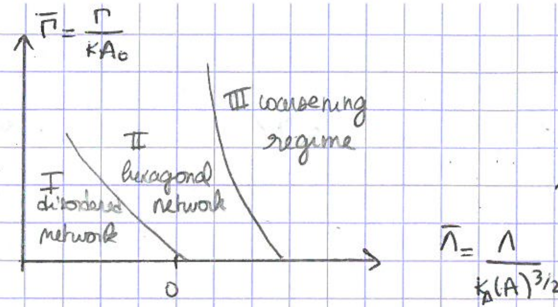
D1 zero motility limit (no activity).



perimeter

M_1 $E = \sum \frac{K_A}{2}(A-A_0)^2 + \sum \Lambda \ell_{ij} + \Gamma P^2$

phase diagram for the available ground states :



Growing : bulk stiffness

$$E_{cell} = K_A (A - A_0)^2 + K_p (P - P_0)^2 \qquad P_0 = \frac{\Lambda}{2\Gamma} \quad K_A = \frac{\bar{K_A}}{2} \quad K_p = \frac{\Gamma}{2}$$

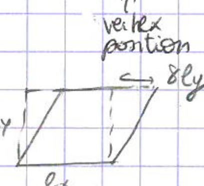
$$\mathcal{E}_{tot} = \frac{1}{K_A A_0^2} \sum_i E_i = \sum_{i=1}^{N} \left[(a_i - 1)^2 + \frac{1}{r} (p_i - p_0)^2 \right] \qquad r = \frac{K_A A_0}{K_p} \quad p_0 = \frac{P_0}{\sqrt{A_0}}$$

HOMEWORK : Calculate the algebraic expressions of the critical lines of the phase diagram.

Beyond ground states → need to generate an ensemble

↳ infinite temp quench : find nearest minimum, gradient ascent $\mu \dot{r}_i = -\nabla_i \mathcal{E}$.
uniformly
vertex position

D1b linear response



→ calculate shear modulus g at a minimum energy state

→ $\mathcal{E} = \mathcal{E}(\{r_i\}, \gamma)$ potential energy landscape

↳ $\mathcal{E}_{min}(\gamma) = \min_{\{r_i\}} \mathcal{E}(\{r_i\}, \gamma) \rightarrow g = \frac{1}{V} \frac{d^2 \mathcal{E}_{min}(0)}{d^2 \gamma}$

rewrite in terms of dynamical matrix or hessian $D_{j\alpha, k\beta} = \frac{\partial^2 \mathcal{E}}{\partial r_i^\alpha \partial r_i^\beta}$

$$= \sum_m \omega_m^2 \, U_{j\alpha}^m U_{k\beta}^m \qquad \text{eigen decomp.}$$

extended dynamical matrix : $\bar{D}_{pq} = \frac{\partial^2 \mathcal{E}}{\partial z_p \partial z_q}$ $\quad z = (\vec{r}_1, \dots \vec{r}_N, \gamma)$
vertex positions

$$= \sum_m \bar{\omega}_m^2 \, \bar{U}_p^m \bar{U}_q^m \qquad \dot{r}_{min} \equiv \frac{dr^{min}}{d\gamma}$$

$$\dot{z}_{min} = (\dot{r}_1^{min}, \dots \dot{r}_{Nd}^{min})$$

$$V g \, \bar{u}_\gamma^m = \bar{\omega}_m^2 \sum_q \bar{u}_q^m \, \dot{z}_q^{min} \quad \forall m$$

→ comment 1 : if $\exists$ a zero mode with $U_\gamma^m \neq 0$ then $g = 0$.

↳ comment 2 : otherwise all zero modes have vanishing overlap with shear dgree of freedom.

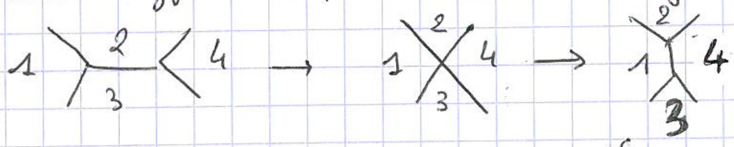
$$g = \frac{1}{V} \left[\sum_{\tilde{m}} \frac{(\bar{u}_\gamma^{\tilde{m}})^2}{\bar{\omega}_{\tilde{m}}^2} \right]^{-1}$$ weighted sum of overlaps of eigenvectors with shear DOF

$\tilde{m} \,/\, \omega_{\tilde{m}} \neq 0$.

↳ $g = \begin{cases} 0 & \exists \text{ non trivial modes} \\ \text{weighted sums.} \end{cases}$

Staple EPJE 2010 :
for ordered system → linearly stable below $p_0 = p_{hex} \simeq 3.72$
For disordered states, numerically calculate # non zero modes

Calculate energy barriers for localized rearrangement



T_1 transition

↳ execute T_1, minimize global energy