Lecture I: Mesoscopic Fermi-Edge Singularity

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Boulder 2005 Summer School

(i) Fermi-Edge Singularity (FES) in electron transport

(ii) Review Feynman diagrams, Nozieres approach

(iii) New technique: functional determinants

(iv) FES out of equilibrium

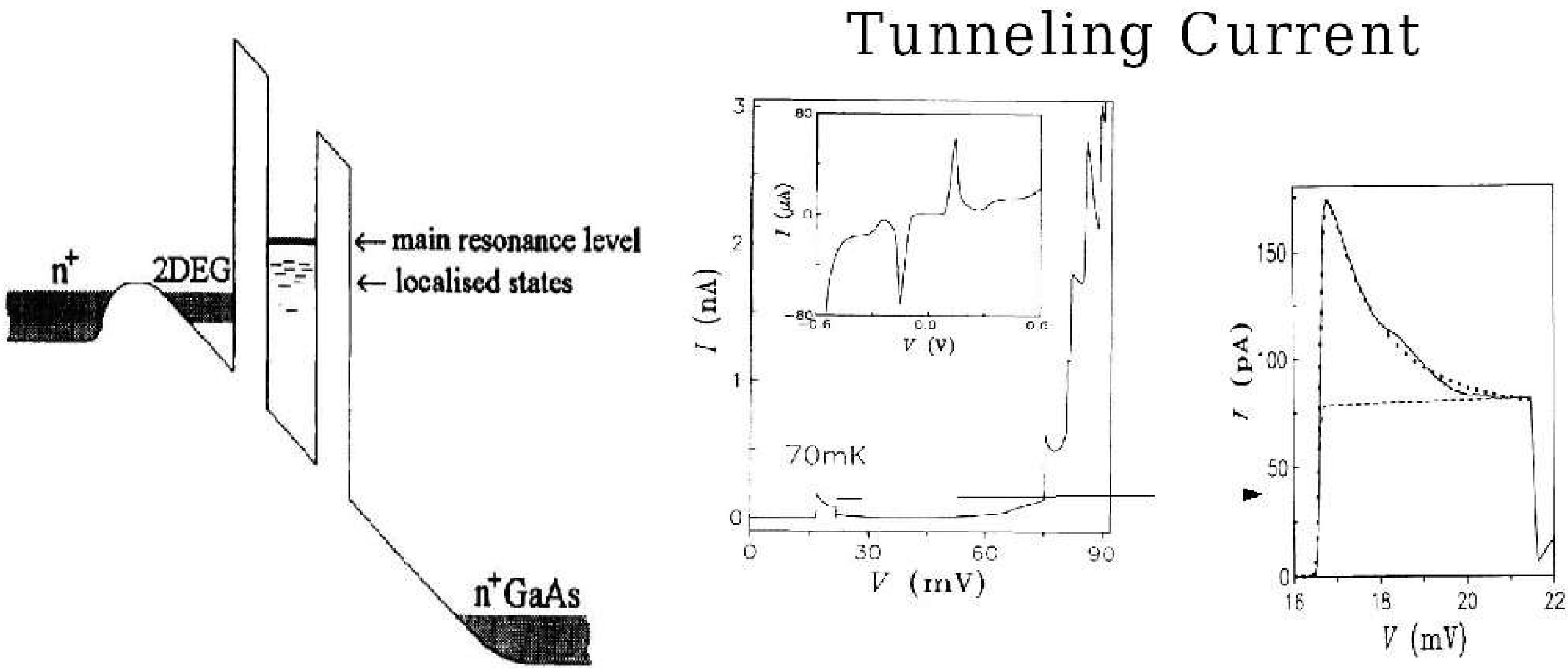
(v) Mesoscopic FES, chaotic scattering, quantum dots

D.A. Abanin & L.L., PRL 93, 126802 (2004), PRL 94, 186803 (2005)

Fermi-Edge Singularity in Resonant Tunneling

Many-body enhancement of transition rate in resonant tunneling

Matveev & Larkin '92



Tunneling from a localized level into 2DEG (Geim *et al.*, '93)

Resonant Tunneling Hamiltonian $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_T$

$$\mathcal{H}_0 = \sum_k \epsilon_k a_k^\dagger a_k + E_0 d^\dagger d + \sum_{kk'} U_{kk'} a_k^\dagger a_{k'} d d^\dagger,$$

(band electron a_k, $a_k^\dagger$, hole d, $d^\dagger$). Tunneling coupling:

$$H_T = \sum_k (T_k a_k^\dagger d e^{i\mu t} + T_k^* d^\dagger a_k e^{-i\mu t})$$

Current via Green's function: $I \propto \mathrm{Re}\,|T_k|^2 \int_{-\infty}^0 F(t) dt$

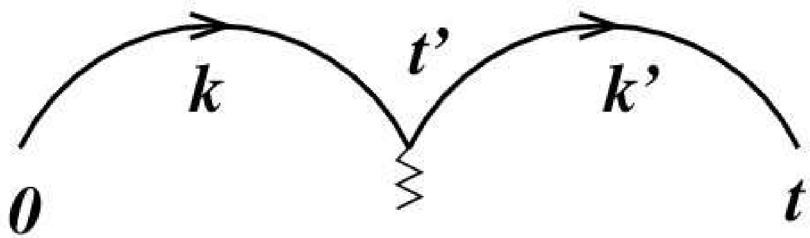
$$F(t) = \sum_{kk'} \langle 0 | a_k^\dagger(t) d(t) d^\dagger(0) a_{k'}(0) | 0 \rangle$$

No interaction: $U = 0$, $\langle a^\dagger d d^\dagger a \rangle = \langle a^\dagger a \rangle \langle d d^\dagger \rangle$, $F(t) \propto e^{i(\mu - E_0)t}/t$,
Obtain $I(\mu) \propto n_F(\mu - E_0)$: energy spectroscopy.

Enhancement of Tunneling Current

First-order perturbation in $U_{kk'}$:

$$\delta F(t) \propto \sum_{kk'} \int_t^0 dt' G_{k'}(-t') U_{k'k} G_k(t'-t)$$



$$\delta F(t) \propto \int_t^0 \frac{1}{(-t')(t'-t)} \propto \frac{1}{t} \ln|t| \quad - \quad \text{an essential divergence!}$$

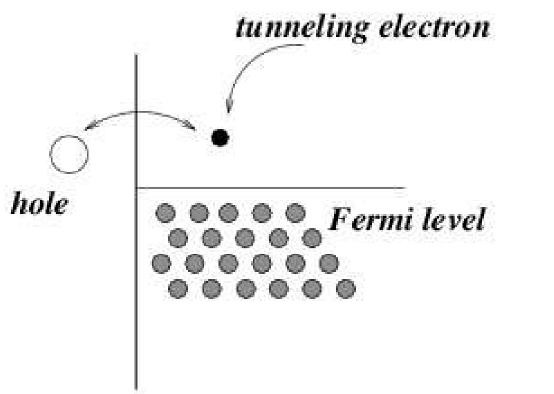
Higher orders of perturbation theory $\rightarrow$ more logs...

Mahan, Nozieres, DeDominicis, Anderson...

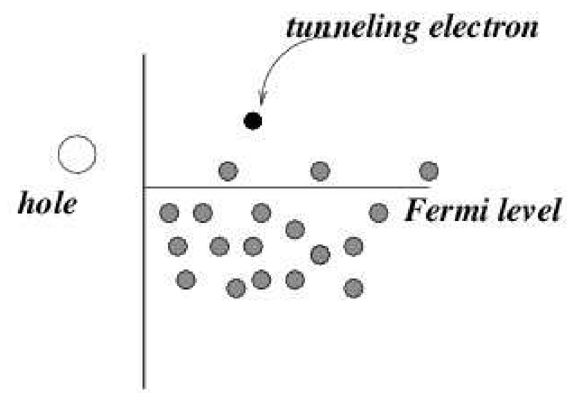
Two Competing Effects

1) Tunneling electron is attracted to the hole left behind: a quasiresonance, current enhancement.

2) Fermi sea shakup by scatterer switching (orthogonality catastrophe): massive excitation of pairs, current suppression



Attraction of the tunneling electron to the hole



Shake-up of the Fermi sea (Orthogonality catastrophe)

Green's function factorization, $F(t) = e^{-i(E_0-\mu)t} L(t) D(t)$

Power law singularity (one scattering channel, phase shift δ):

$$I \propto \left(\frac{\xi_0}{E_0-\mu}\right)^{\alpha} \theta(E_i-\mu), \quad \text{with} \quad \alpha = 2\delta/\pi - (\delta/\pi)^2$$

Volume 18, Number 24 PHYSICAL REVIEW LETTERS 12 June 1967

INFRARED CATASTROPHE IN FERMI GASES WITH LOCAL SCATTERI

P. W. Anderson
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received 27 March 1967)

We prove that the ground state of a system of N fermions is orthogonal to
state in the presence of a finite range scattering potential, as $N\to\infty$. This
the response to application of such a potential involves only emission of exc
the continuum, and that certain processes in Fermi gases may be blocked t
ty in a low-T, low-energy limit.

PHYSICAL REVIEW VOLUME 163, NUMBER 3 15 NOVEMBER 1967

Excitons in Metals: Infinite Hole Mass

G. D. Mahan*
General Electric Research and Development Center, Schenectady, New York
(Received 5 May 1967)

The optical conductivity is evaluated for interband transitions between a flat valence band and a parabolic conduction band. The conduction band is filled with electrons to a Fermi energy μ_F. The conductivity is calculated assuming that the electron-hole interaction is attractive, static, and short range. The final-state interactions between the electron and hole cause a divergence in the conductivity at the interband threshold. This divergence appears to go as a power law. For this case of an infinite hole mass, the exciton binding energy vanishes, since the singularity in the scattering amplitude occurs just at threshold.

tion to the optical conductivity has the form

$$\sigma \sim \frac{1}{\omega}\left(\frac{\xi_0}{\omega-\omega_c}\right)^{2\Delta}\theta(\omega-\omega_c)\,,$$

where $\omega_c = E_G + \mu_F$ is the threshold frequency.

PHYSICAL REVIEW VOLUME 178, NUMBER 3 15 FEBRUARY 1969

Singularities in the X-Ray Absorption and Emission of Metals. III. One-Body Theory Exact Solution

P. Nozières*
University of California, San Diego, California

and

C. T. De Dominicis†
Lyman Laboratory, Harvard University, Cambridge, Massachusetts

The singularities of x-ray absorption or emission in metals are studied by a new "one-body" method, which describes the scattering of conduction electrons by the *transient* potential due to the deep hole. Using the linked-cluster theorem, the net transition rate in the time representation is expressed as the product of two factors: a one-electron transient Green's function L, and the deep-level Green's function $\mathcal{G}$. These factors obey simple Dyson equations, which can be solved asymptotically by using Muskhelishvili's method. The x-ray transition rate is found to behave as $1/\epsilon^\alpha$, where ϵ is the frequency measured from the threshold, and α an exponent involving the various phase shifts δ_l which describe scattering by the deep hole. α may be >0 (infinite threshold) or <0 (zero threshold). The experimental implications of these results and their relation to the Friedel sum rule are briefly discussed.

tering potential is explicitly time-dependent. Let $C(t-t')$ be the contribution of all single closed loops, $L_{\mathbf{kk'}}(t-t')$ being that of the open line in Fig. 3(a). The linked cluster theorem tells us that

$$\tilde{\mathcal{G}}(t) = e^{C(t)}\,,$$
$$\bar{F}_{\mathbf{kk'}}(t) = L_{\mathbf{kk'}}(t)e^{C(t)}\,. \qquad (15)$$

The net transition amplitude $\bar{F}_{\mathbf{kk'}}$ appears as the product of a one-body factor $L_{\mathbf{kk'}}$ multiplied by the deep-hole propagator $\tilde{\mathcal{G}}$.[10]

the various determinations of the integrand). The algebra is lengthy, but straightforward, so that we shall only quote the result.[15]

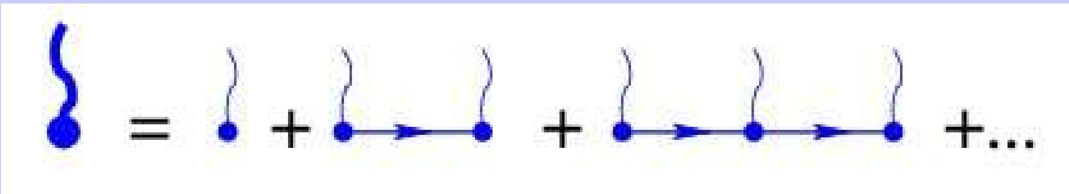
$$\varphi^a(\tau,\tau';t,t') = G^c(\tau-\tau')\left[\frac{(t'-\tau')(\tau-t)}{(t'-\tau)(\tau'-t)}\right]^{\delta/\pi}, \qquad (51)$$

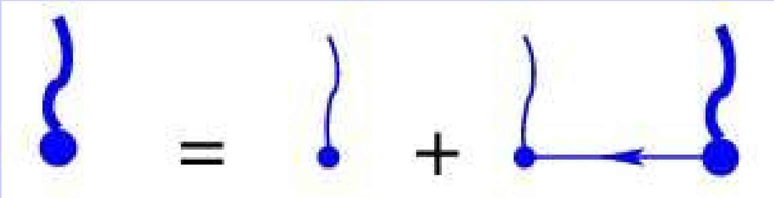
The quantities of physical interest are the spectral densities. Measured from their respective branch points, they behave as

$$\frac{1}{\epsilon}\left(\frac{\epsilon}{\xi_0}\right)^{(\delta/\pi)^2} \quad \text{for the Green's function } \mathcal{G},$$
$$\left(\frac{\xi_0}{\epsilon}\right)^{2\delta/\pi-(\delta/\pi)^2} \quad \text{for the response function } F. \qquad (66)$$

Elastic Scattering, Feynman Diagrams

One particle QM scattering amplitude, perturbation theory





$$F^{(1)}(\mathbf{k}_1,\mathbf{k}_2) = V(\mathbf{k}_2-\mathbf{k}_1)\,,$$

$$F^{(2)}(\mathbf{k}_1,\mathbf{k}_2) = \int \frac{V(\mathbf{k}_2-\mathbf{q})\,V(\mathbf{q}-\mathbf{k}_1)\,(d^3q)}{\varepsilon-\mathbf{q}^2/(2m)+i\delta}\,,$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$F^{(n)}(\mathbf{k}_1,\mathbf{k}_2) = \int\ldots\int \frac{V(\mathbf{k}_2-\mathbf{q}_{n-1})\ldots V(\mathbf{q}_1-\mathbf{k}_1)\,(d^3q_{n-1})\,\ldots\,(d^3q_1)}{\left(\varepsilon-\mathbf{q}_{n-1}^2/(2m)+i\delta\right)\ldots\left(\varepsilon-\mathbf{q}_1^2/(2m)+i\delta\right)}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\widehat{F} = \widehat{V} + \widehat{V}\widehat{G}_0\widehat{V} + \widehat{V}\widehat{G}_0\widehat{V}\widehat{G}_0\widehat{V} + \ldots = \widehat{V} + \widehat{V}\widehat{G}_0(\widehat{V} + \widehat{V}\widehat{G}_0\widehat{V} + \ldots) = \widehat{V} + \widehat{V}\widehat{G}_0\widehat{F}$$

Integral equation:

$$F(\mathbf{k}_1,\mathbf{k}_2) = V(\mathbf{k}_2-\mathbf{k}_1) + \int \frac{V(\mathbf{k}_2-\mathbf{q})\,F(\mathbf{k}_1,\mathbf{q})}{\varepsilon-\hbar^2\mathbf{q}^2/2m+i\delta}\,\frac{d^3q}{(2\pi)^3}\,.$$

Solution: $F(\epsilon) = (e^{2i\delta(\epsilon)}-1)/2\pi i\nu(\epsilon)$

with $\delta(\epsilon)$ the scattering phase, $\nu(\epsilon)=\int\delta(\epsilon-E(\mathbf{p}))d^3\mathbf{p}$ the density of states

Many-Body Theory

Causal Green's function

$$G^c(x,x') = -i\langle \mathrm{T}\,\psi_\alpha(x)\psi_\beta^+(x')\rangle$$

Retarded and advanced Green's functions

$$G^c(t,t') = \begin{cases} G^R(t,t'), & t > t', \\ G^A(t,t'), & t < t'. \end{cases}$$

$$G^{R(A)}(\varepsilon,\mathbf{p}) = 1/(\varepsilon - \xi(\mathbf{p}) \pm i\delta)$$

$$G^c(\varepsilon,\mathbf{p}) = (1-n(\mathbf{p}))G^R(\varepsilon,\mathbf{p}) + n(\mathbf{p})G^A(\varepsilon,\mathbf{p}) = \frac{1-n(\mathbf{p})}{\varepsilon - \xi(\mathbf{p}) + i\delta} + \frac{n(\mathbf{p})}{\varepsilon - \xi(\mathbf{p}) - i\delta}$$

Summing the series for scattering on *V* obtain

$$F(\epsilon) = \begin{cases} (e^{2i\delta} - 1)/2\pi i\nu & \epsilon > \mu \\ -(e^{-2i\delta} - 1)/2\pi i\nu & \epsilon < \mu \end{cases}$$

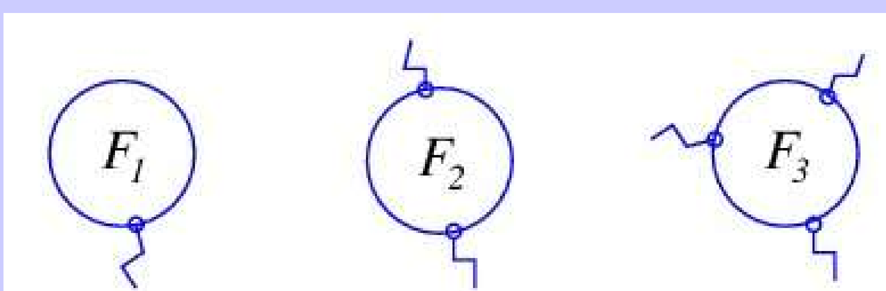
The orthogonality catastrophe

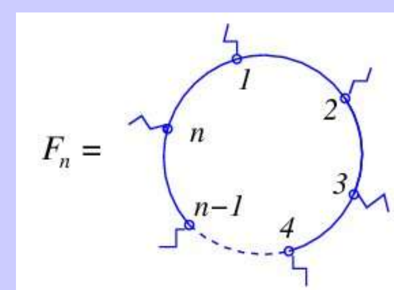
Dynamical overlap $K_\gamma = \left\langle \mathrm{T}\,\exp(-i\int_{-\infty}^{0} \widehat{\mathcal{H}}_{\rm int}(t)dt) \right\rangle,$

$$\sum_{n=0}^{\infty} \int_{-\infty}^{0} \int_{-\infty}^{t_1} \cdots \int_{-\infty}^{t_{n-1}} (-i)^n \left\langle \widehat{\mathcal{H}}_{\rm int}(t_1) \widehat{\mathcal{H}}_{\rm int}(t_2) \ldots \widehat{\mathcal{H}}_{\rm int}(t_n) \right\rangle dt_n \ldots dt_2 dt_1$$

Linked cluster expansion $K_\gamma = -\sum_{n=1}^{\infty} \frac{1}{n} F_n$

One diagram in each order of perturbation theory





Example: second order contribution

$$F_2 = \frac{(-i)^2}{2} \int_{-\infty}^{0} \int_{-\infty}^{0} \langle\langle \mathrm{T}\, \widehat{\mathcal{H}}_{\mathrm{int}}(t') \widehat{\mathcal{H}}_{\mathrm{int}}(t) \rangle\rangle \, dt'\, dt = - \int_{-\infty}^{0} \int_{-\infty}^{t} \langle\langle \widehat{\mathcal{H}}_{\mathrm{int}}(t) \widehat{\mathcal{H}}_{\mathrm{int}}(t') \rangle\rangle \, dt'\, dt$$

$$e^{\gamma t} e^{\gamma t'} \int\int U(\mathbf{r}) U(\mathbf{r}') \, \langle \psi^+(\mathbf{r}, t) \psi(\mathbf{r}', t') \rangle \, \langle \psi(\mathbf{r}, t) \psi^+(\mathbf{r}', t') \rangle \, d^3\mathbf{r}\, d^3\mathbf{r}'$$

$$U(\mathbf{r}) = \alpha \delta^{(3)}(\mathbf{r}),$$

(the exponent gamma regularizes contribution of long times)

$$F_2 = -(\nu_0 \alpha)^2 \int_{-\infty}^{0} \int_{-\infty}^{t} e^{\gamma t} e^{\gamma t'} \frac{dt'\, dt}{(\delta + i(t - t'))^2}$$

$$\mathrm{Re}\, F_2 = -\frac{1}{2} (\nu_0 \alpha)^2 \ln (E_F / \gamma) .$$

Power law for the overlap:

$$|K_\gamma| = (\gamma / E_F)^{\delta_0^2 / 2\pi^2} , \quad \delta_0 = \pi \alpha \nu_0$$

Ground state overlap in a finite system
a power law function of system size

$$\gamma \approx \Delta_0 = \hbar v_F / L , \qquad \langle 0 | 0' \rangle \approx (p_0 L)^{-\alpha} , \qquad \alpha = \delta_0^2 / \pi^2$$

(relate gamma to level spacing)

Good only for weak perturbation --> Nonperturbative methods?

Nozieres-deDominicis approach

(i) Resummation of Feynman diagrams:

$$G(t,t') = G_{fast}(t-t') + G_{slow}(t-t')$$

with G_{fast} unaffected by switching, $G_{slow}(t-t') = \frac{i}{\pi(t-t'+i0)}$

$$G(t,t') \to G_{slow}(t-t'), \quad V \to (e^{2i\delta}-1)/(2\pi i\nu)$$

(ii) Dyson equation for G gives a singular integral equation

(iii) Use special technique (Muschelishvili) to obtain solution

Scale decomposition of G good for clean Fermi liquid in equilibrium

Limitations of the conventional theory

Diagrammatic approach (Nozieres): (1) T-matrix by diagram resummation; (2) Dyson equation for $G(t)$; (3) Solve singular integral equation.

Too cumbersome, not easy to generalize, was replaced by bosonization (Schotte and Schotte)

Bosonization handles well many FES problems (e.g. Luttinger liquids, QH edge, nanotubes)

However, bosonization isn't always helpful:

(1) FES in a nonequilibrium electron gas;

(2) FES in a system with complex scattering (nonseparable potential, noncommuting scattering matrices, quantum dots)

Bosonization needs equilibrium Fermi sea, requires separable scattering channels, and thus is fairly fragile

Develop new technique (insight from work by Muzykantskii et al. on counting statistics and its relation to FES)

Generalized FES: A Three-Step Approach

Step I: A useful technical trick from linear algebra

Consider a many-body second quantized operator

$$\hat{A} = \sum_{\alpha,\beta} A_{\alpha\beta} c_\alpha^\dagger c_\beta$$

in many-particle Fock space. Restrict A to the one-particle subspace: $\hat{\mathbf{a}} = A_{\alpha\beta}$. Then

$$\mathrm{tr}\, e^{\hat{A}} = \sum_{n_1,..,n_N=0,1} \langle n_1, .., n_N | e^{\hat{A}} | n_1, .., n_N \rangle = \det(1 + e^{\hat{\mathbf{a}}})$$

(fermion partition function!)

Thus, a trace over many-particle Fock space can be expressed via the determinant over the one-particle subspace.

A pedagogical example: With $\hat{A} = 0$ have

$$\mathrm{tr}(e^{\hat{A}}) = 2^N, \quad \det(1 + e^{\mathbf{a}}) = 2 \times \ldots \times 2 = 2^N$$

This can be generalized to ANY number of operators*

Klich '02

$$\mathrm{tr}(e^{\hat{A}} e^{\hat{B}}) = \det(1 + e^{\hat{\mathbf{a}}} e^{\hat{\mathbf{b}}}),$$

$$\mathrm{tr}(e^{\hat{A}} e^{\hat{B}} e^{\hat{C}}) = \det(1 + e^{\hat{\mathbf{a}}} e^{\hat{\mathbf{b}}} e^{\hat{\mathbf{c}}})$$

$$\cdots$$

$(*)$ Holds for quadratic operators only.

Apply the determinant formula to FES

Link the orthogonality catastrophe overlap factor

$$D(t) = \langle 0|0'\rangle = \mathrm{tr}\left(e^{-iH_1 t} e^{iH_0 t} \hat{\rho}_e\right)$$

to one-particle evolution operators $e^{-\hat{h}_1\tau}$, $e^{-i\hat{h}_0\tau}$. Note that

$$\hat{\rho}_e = \prod_p (n_p a_p^+ a_p + (1-n_p) a_p a_p^+) = \frac{1}{Z}\exp\left(-\sum_p \lambda_p \hat{a}_p^+ \hat{a}_p\right)$$

with $e^{-\lambda_p} = \frac{n(\epsilon_p)}{1-n(\epsilon_p)}$. From the determinant formula obtain

$$D(t) = \det\left(1 - \hat{n}(\epsilon) + e^{-\hat{h}_1\tau} e^{i\hat{h}_0\tau} \hat{n}(\epsilon)\right)$$

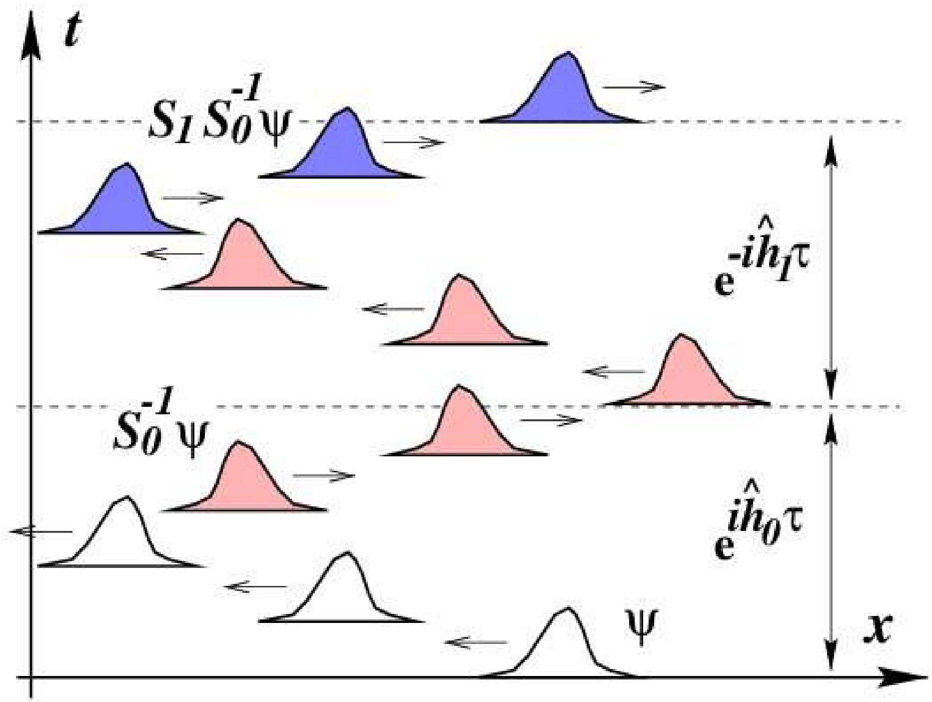
Holds for generic evolution operators (quantum dots, billiards, scattering, etc.)

Step II: Scattering Matrix

Interpret forward-backward evolution operator

$$\hat{S} = e^{-\hat{h}_1 \tau} e^{i\hat{h}_0 \tau} = \delta_{t,t'} \times \begin{cases} S_1^{-1} S_0, & 0 < t, t' < \tau \\ 1, & \text{else} \end{cases}$$

as the single-particle scattering operator!



Single channel: $S_{0,1} = e^{2i\delta_{0,1}}$, $S_1^{-1} S_0 = e^{2i(\delta_0 - \delta_1)}$

Step III: Solving an Integral Equation

Evaluate $D = \det\left(1 - \hat{n} + \hat{S}\hat{n})\right)$ with noncommuting $\hat{n}$, $\hat{S}$:

$$\delta \ln D = \mathrm{tr}\left[\left(1 - \hat{n} + \hat{S}\hat{n}\right)^{-1} (\hat{S} - \hat{1})\delta\hat{n}\right]$$

Need to invert integral operator $\Delta = 1 - \hat{n} + \hat{S}\hat{n}$

(linked by Muzykantskii & Adamov to generalized Riemann-Hilbert problem)

(1) Nonequilibrium FES: used time-energy duality;

(2) Nonseparable FES at $T = 0$: $n_F(\epsilon)$ generates Cauchy kernel $\hat{n}(t, t') \propto 1/(t - t' + i0)$, same equation as in Nozieres

Example: Orthogonality Catastrophe

Wanted: the determinant $e^C = \det(1+(\hat{S}-1)\hat{n})$, with the scattering operator S diagonal in the time domain, $\hat{n}$ diagonal in the energy domain:

$$\hat{S} \equiv e^{-i\hat{h}_1\tau} e^{i\hat{h}_0\tau} = \delta_{t,t'} \times \begin{cases} S_1 S_0^{-1}, & 0<t<\tau \\ 1, & \text{else} \end{cases} \quad , \quad \hat{n}_{t,t'} = \frac{i}{2\pi(t-t'+i0)} = \int e^{-i\epsilon(t-t')} n_F(\epsilon)\frac{d\epsilon}{2\pi}$$

$\hat{S}$, $\hat{n}$ are operators in the Hilbert space of functions of t (or, alternatively, of ϵ).

Using the identity $d\ln\det X = \mathrm{tr}\,(X^{-1}dX)$, obtain

$$d_\delta C = d\ln\det(1+(\hat{S}-1)\hat{n}) = \mathrm{tr}\left[(1+(\hat{S}-1)\hat{n})^{-1} d_\delta \hat{S}\hat{n}\right], \quad d_\delta \hat{S} = 2ie^{2i\delta}\begin{cases} 1, & 0<t<\tau \\ 0, & \text{else} \end{cases}$$

To invert $1+(\hat{S}-1)\hat{n}$ represent $S(t)$ as a product of $Y_+(t)$, $Y_-^{-1}(t)$, analytic in the upper and in the lower halfplane, respectively:

$$S(t) = Y_+(t)Y_-^{-1}(t), \quad \ln Y_\pm(t) = -\frac{1}{2\pi i}\int \frac{\ln S(t')}{t-t'\pm i0}\,dt' \qquad (1)$$

which gives $Y_\pm(t) = (t-\tau\pm i0)^{\delta/\pi}/(t\pm i0)^{\delta/\pi}$. In terms of $Y_\pm$ the variation dF reads

$$dC = \mathrm{tr}\left[\left(Y_-^{-1}\hat{n} + Y_+^{-1}(1-\hat{n})\right) Y_-\, dS\, \hat{n}\right]$$

Note analytic properties of $Y_\pm$ and $\hat{n}$: $\hat{n}Y_-\hat{n} = Y_-\hat{n}$, $(1-\hat{n})Y_+(1-\hat{n}) = Y_+(1-\hat{n})$.
Simplify: $dC = \mathrm{tr}\left[Y_-^{-1}\hat{n}Y_- d_\delta S\right]$ and, taking into account the form of $Y_\pm$, obtain

$$e^C = (-i\tau\xi_0)^{-\delta^2/\pi^2}$$

PRL 94, 186803 (2005)

FES OUT OF EQUILIBRIUM

Generic energy distribution $n(\epsilon)$, e.g. a split Fermi step

$$n(\epsilon) = (1 - x)n_F(\epsilon - \mu_1) + xn_F(\epsilon - \mu_2)$$

with $\mu_{1,2}$ potentials in the leads (Pothier *et al.* '97)

Found: splitting of FES resonance $N(\epsilon) = \mathrm{Im}\, G(\epsilon)$,

$$G(\epsilon) \propto \int \frac{1 - n(\epsilon')}{(\epsilon' - \mu_1)^{\alpha_1}(\epsilon' - \mu_2)^{\alpha_2}} \times D(\epsilon - \epsilon')d\epsilon'$$

with *complex exponents* $\alpha_1 = 2(\delta - \tilde{\delta})/\pi$, $\alpha_2 = 2\tilde{\delta}/\pi$ and

$$\tilde{\delta} = \frac{1}{2i} \ln\left(1 - x + e^{2i\delta}x\right)$$

At small $\delta \ll 1$, the exponents are expressed through partial density of states: $\alpha_1 = (1 - x) \times 2\delta/\pi$, $\alpha_2 = x \times 2\delta/\pi$

Broadening of FES: Orthogonality catastrophe

The broadening function $D(\epsilon) = \int e^{i\epsilon\tau} D(\tau) d\tau$ with

$$D(\tau) = \frac{(1 - i\mu\tau)^{\delta\tilde{\delta}/\pi^2}}{(1 + \mu^2\tau^2)^{\tilde{\delta}^2/2\pi^2}} (-i\tau\xi_0)^{-\delta^2/2\pi^2} \exp(-\gamma\tau)$$

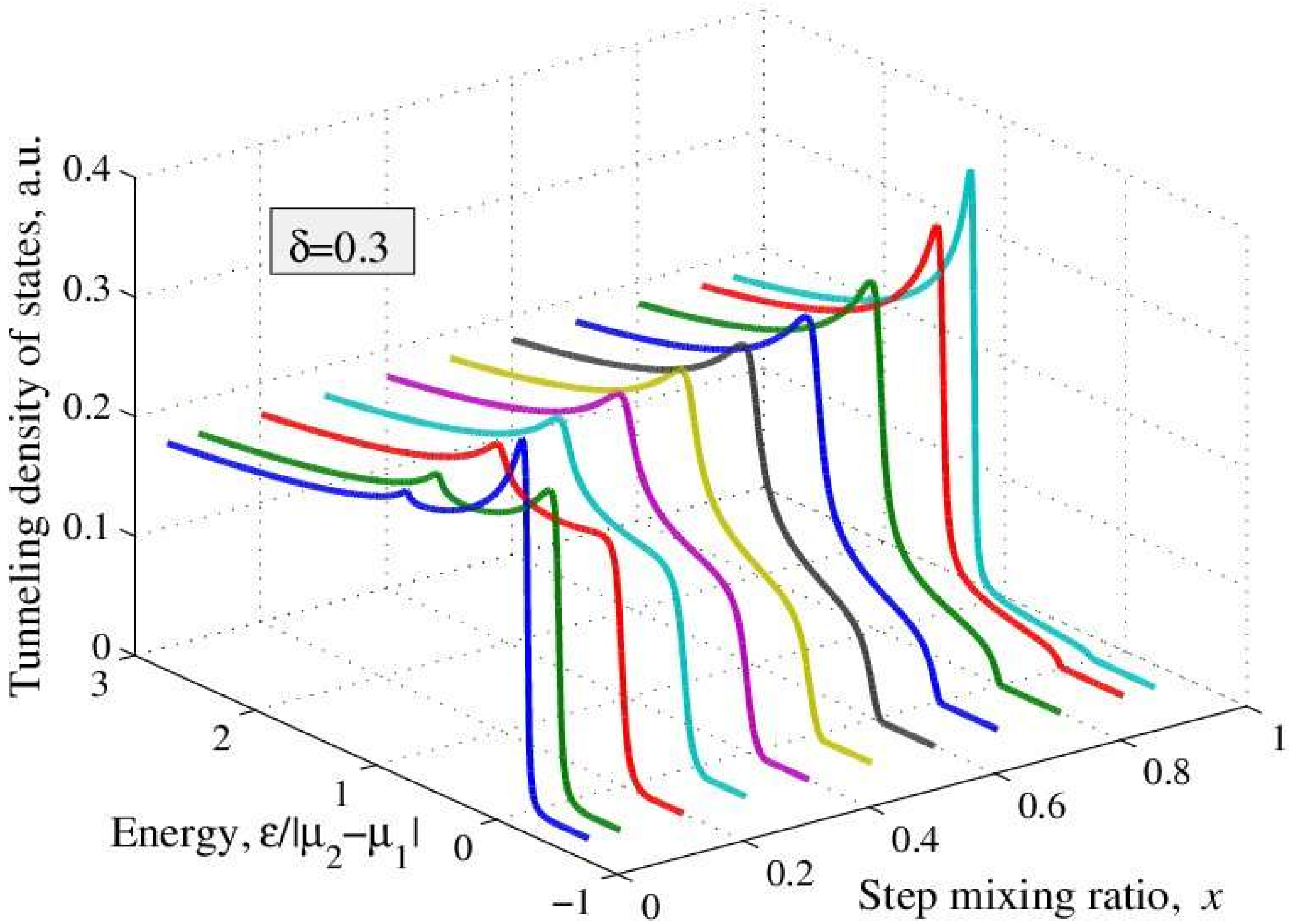
The energy width parameter *(Effective temperature)*

$$\gamma = -\frac{|\mu|}{4\pi\hbar} \ln\left(1 - 4x(1-x)\sin^2\delta\right), \quad \mu \equiv \mu_2 - \mu_1$$

Width scales with the step separation: $\gamma \propto \mu \times x(1-x)\delta^2$ (small δ)

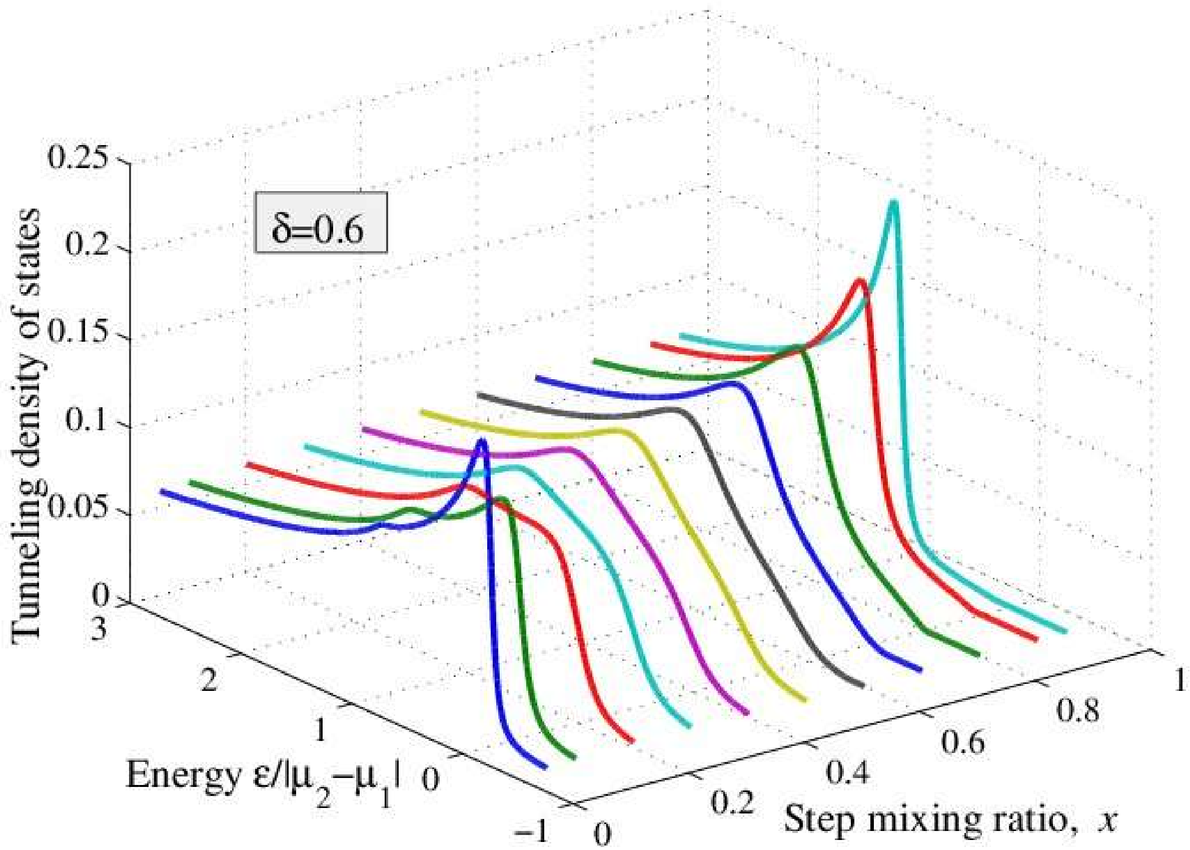
Note divergence at $x = 1/2$, $\delta = \pi/2$; A nonperturbative effect!

Split FES (I)



Tunneling density of states for different step mixing ratios x at $\delta = 0.3$

Split FES (II)



Tunneling density of states for different step mixing ratios x at $\delta = 0.6$

Note broadening nonmonotonic in x, increasing with δ

Nonequilibrium FES features:

(1) Factorization into open line and closed loop contributions: $G(t) = L(t)D(t)$

(2) In equilibrium, $\mu = 0$, agree with the textbook result;

(3) Golden Rule interpretation of broadening: tunnel below Fermi sea with de-excitation of pairs; effective temperature $T_* = \int n(\epsilon)(1 - n(\epsilon))d\epsilon$

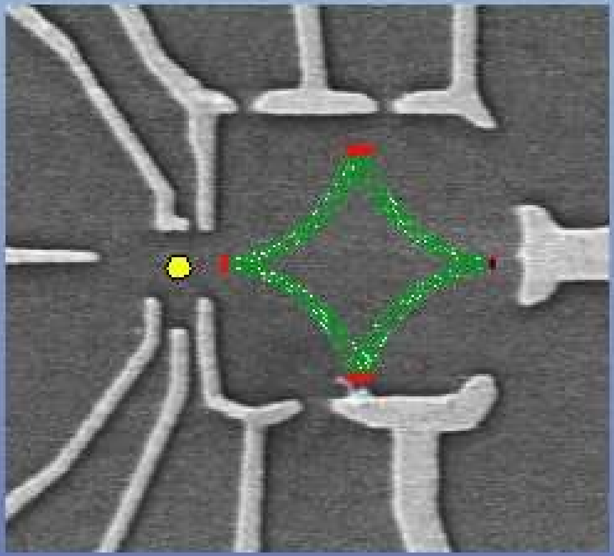
(4) True (split) singularity in the open line contribution $L(\epsilon)$ broadened after convolution with $D(\epsilon)$

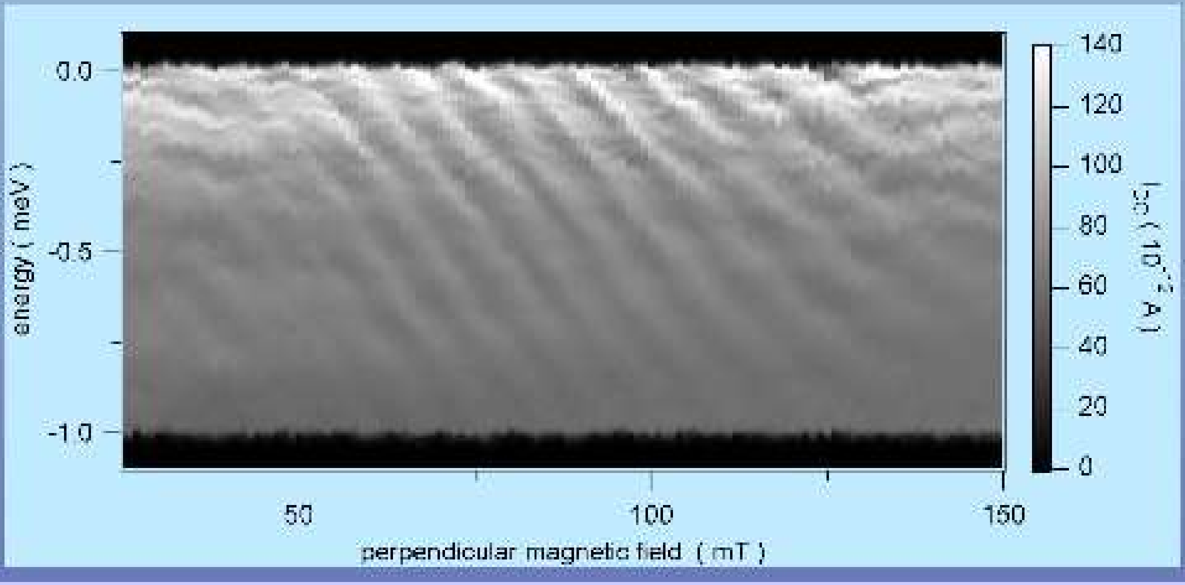
(5) Relation to Kondo problem? (Recent controversy about singlarities at finite bias)

Tunneling in an Open Quantum Dot: Mesoscopic FES

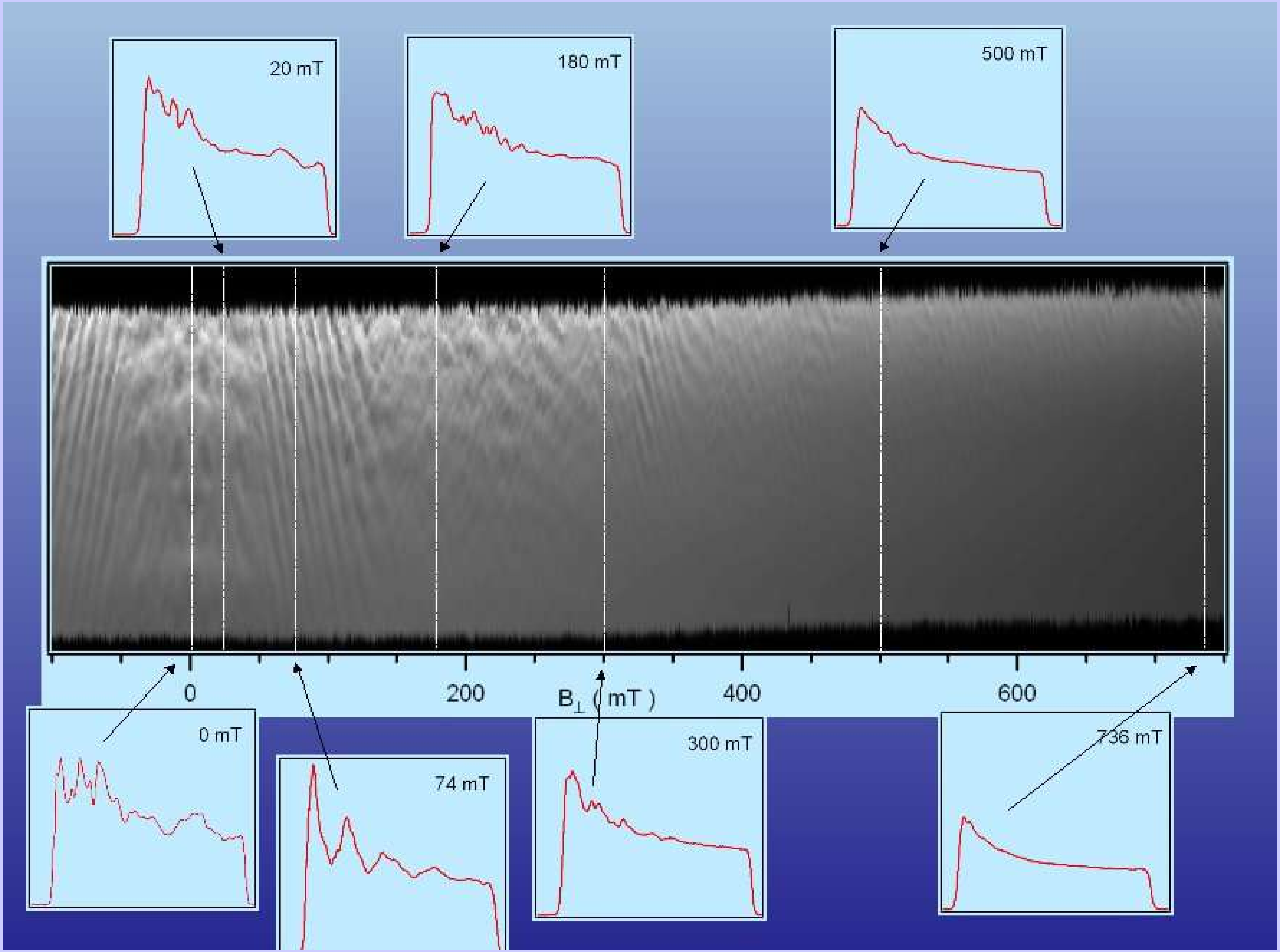
(i) Scattering after tunneling, multiple returns, enhanced interaction
(ii) FES depends on full S-matrix --- tunability
(iii) Mesoscopic fluctuations over FES

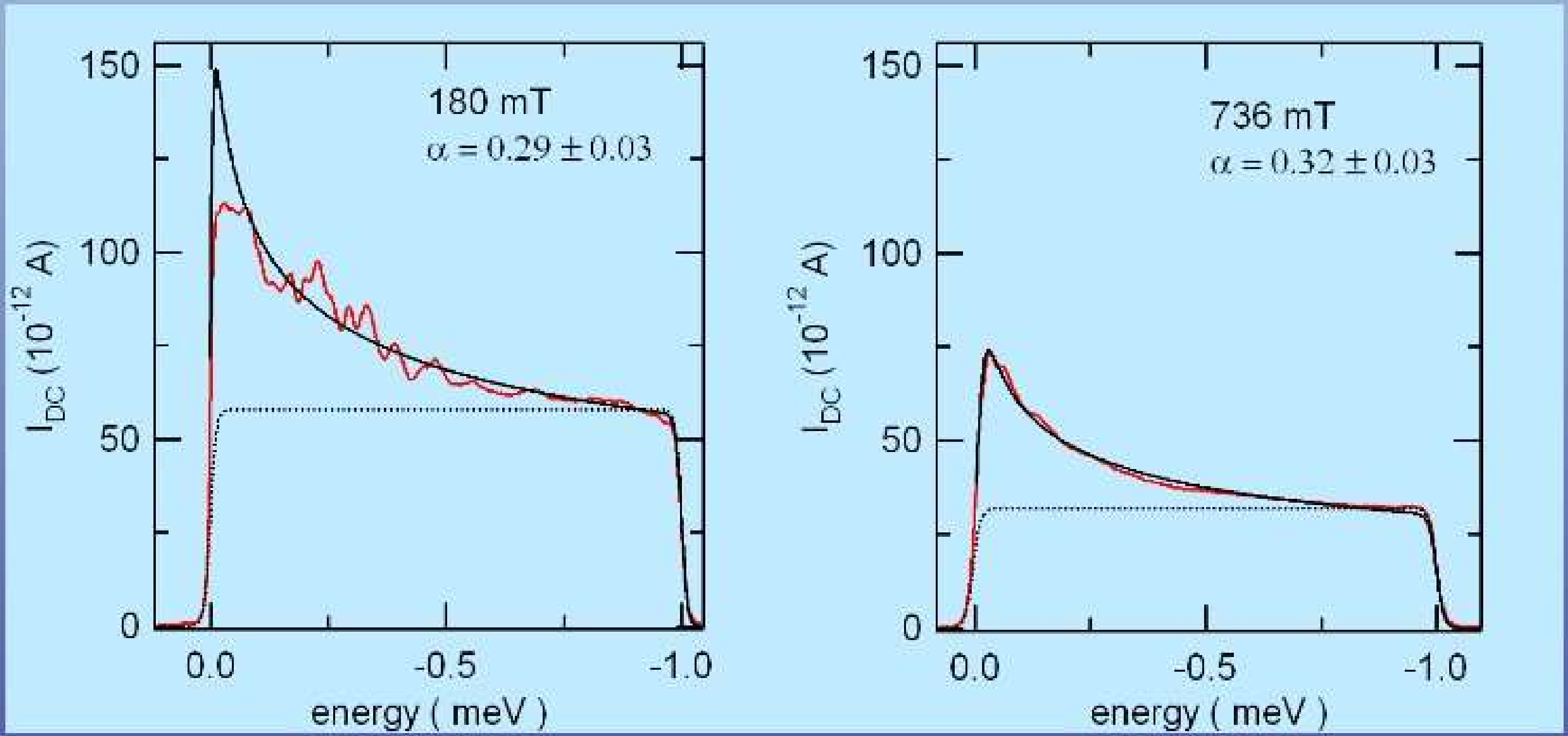
Resonant tunneling spectroscopy, Marcus, Zumbuhl (unpublished)





Tunneling Spectroscopy

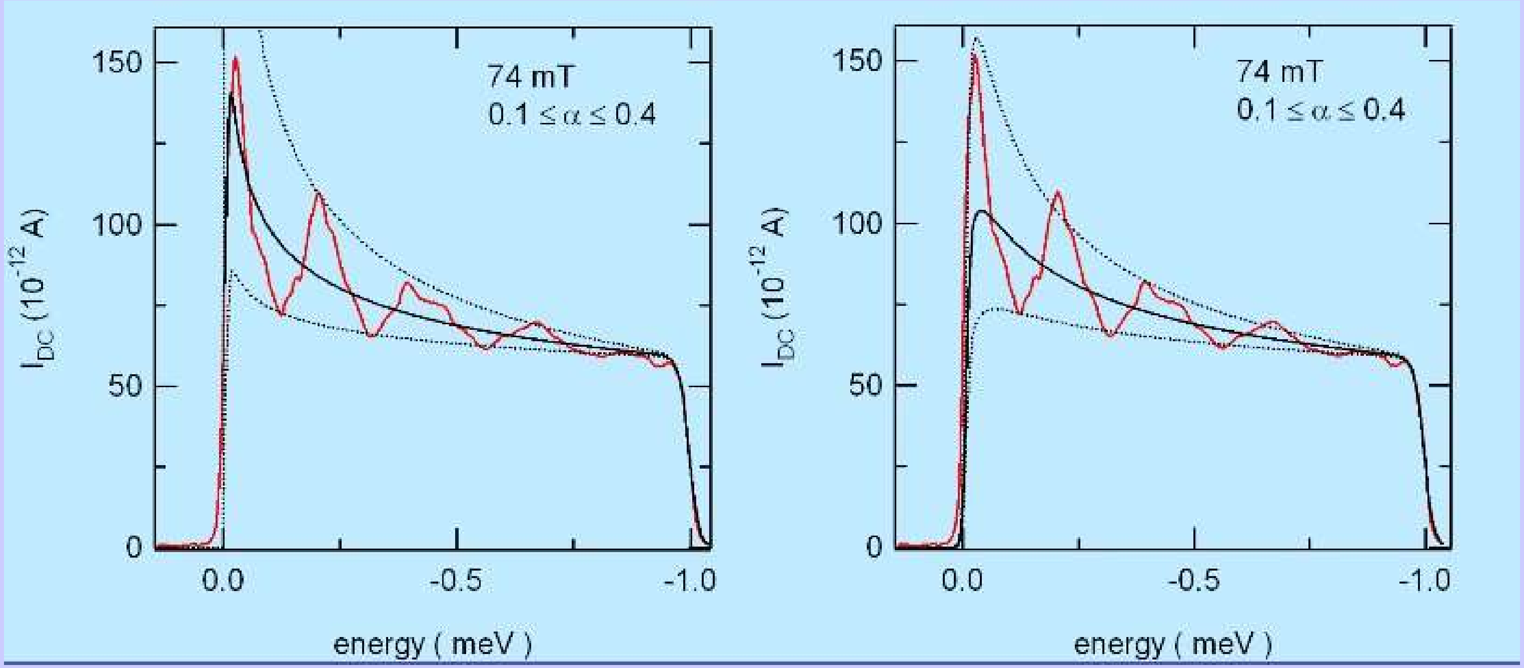


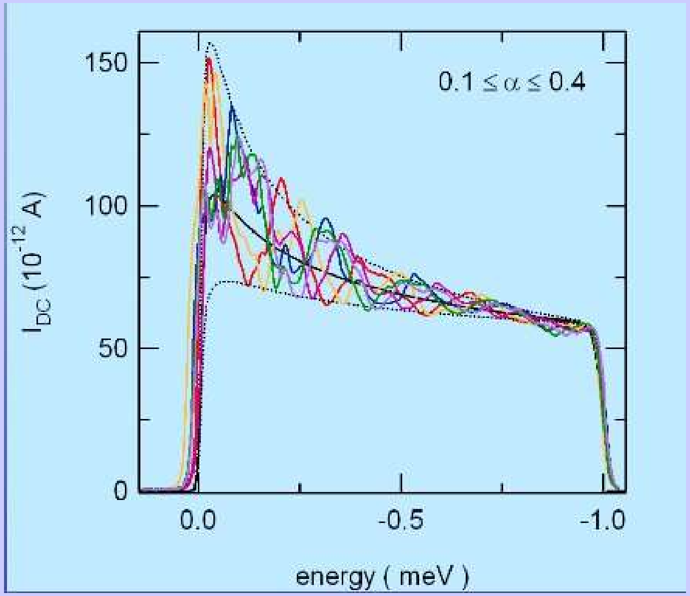


$$I(\epsilon) \propto (E_F - \epsilon)^{-\alpha}$$

Fermi Edge Resonance?

Mesoscopic fluctuations





Effect of backscattering

Power law dependence?

Average? Variance?

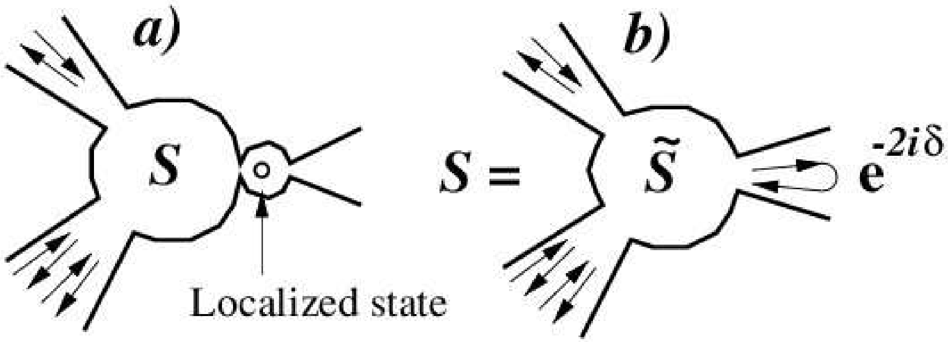
Quantum dot as a compound scatterer

Difficulty for theory: non-commuting matrices (Yamada & Yosida)

Microscopic parameter: the backscattering phase δ

Open up the small dot, define an extended scattering matrix $\tilde{S}$ of size $(N+1)\times(N+1)$.

Relate S and $\tilde{S}$?



$$S_{ij} = \tilde{S}_{ij} + \frac{\tilde{S}_{i(N+1)}\tilde{S}_{(N+1)j}}{e^{2i\delta} - r}, \quad i,j = 1...N,$$

with the backscattering amplitude $r = \tilde{S}_{(N+1)(N+1)}$

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Matrix R

FES for non-commuting scattering matrices is described by

$$R = S(\delta)S^{-1}(\delta')$$

Energy-independent below Thouless energy

Note: in the simplest case $N = 1$ have $S = e^{2i\delta}$ and $R = e^{2i\delta - 2i\delta'}$ as in Mahan and Nozieres *et al.*

For open quantum dot, using extended S-matrix, obtain

$$R_{ij} = \delta_{ij} + \left(\frac{U(\delta)}{U(\delta')} - 1\right) u_i^* u_j$$

where

$$U(\delta) = \frac{e^{2i\delta} - r}{e^{2i\delta} r^* - 1}$$

Note: $R = \hat{1} +$ (a rank one matrix)

The effect of scattering on FES exponent

The matrix R has just one nontrivial eigenvalue. Thus obtain

$$\alpha = 2\beta - \beta^2, \quad \beta = \frac{1}{2\pi i} \ln\left(\frac{U(\delta)}{U(\delta')}\right)$$

Scattering affects the FES exponent through a single parameter, the backscattering amplitude r.

The amplitude r is part of a random matrix. Thus,

1) Changing r, one can tune α in the range $0 < \alpha < 1$;

2) Statistics of α sensitive to magnetic field;

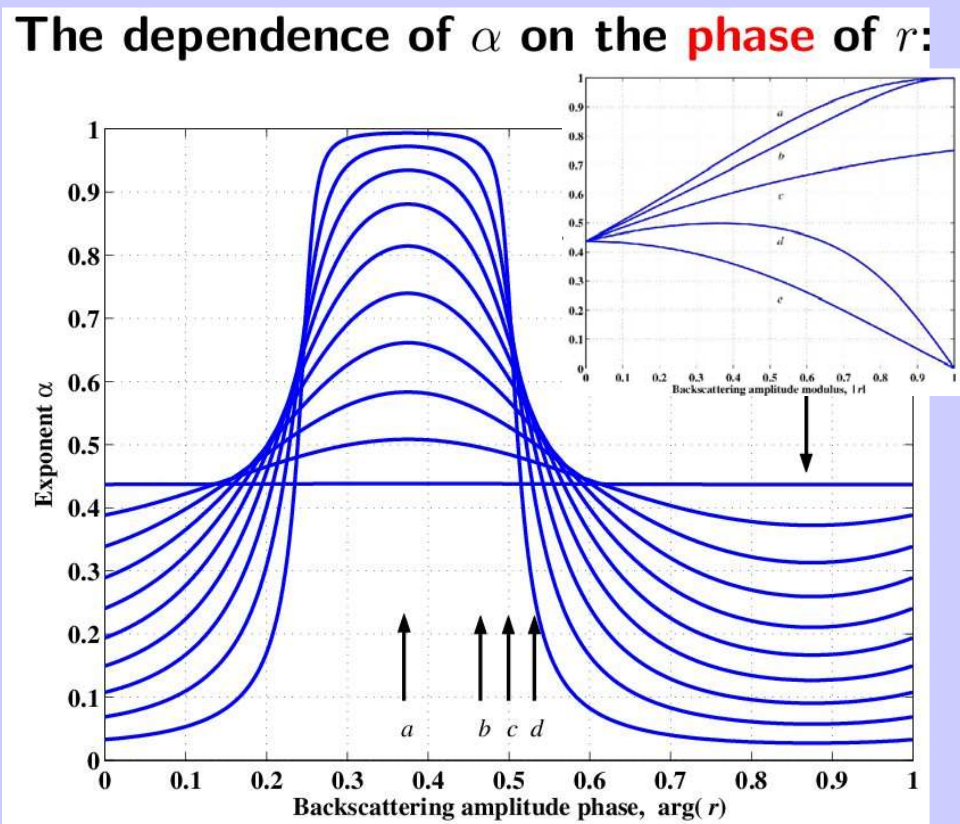
3) Small r limit:

$$\langle\beta\rangle = \beta_0, \quad \langle\alpha\rangle = \alpha_0 - \frac{\sin^2(\delta - \delta')}{\pi^2}\langle|r|^2\rangle + O(r^2)$$

— disorder-enhanced orthogonality.

Mesoscopic FES summary

- FES depends on backscattering, full S-matrix
- FES exponent tunable
- Different power law for fluctuations (in progress)



<u>More broadly:</u>

(i) FES for generic mesoscopic scatterer;

(ii) Generalization to disordered metals:
resonant tunneling, two-level systems

Conclusions

- Efficient technique for FES outside conventional approach
- One particle S-matrices appear early in the calculation
- Applicable to general energy distribution and noncommuting scattering matrices: nonequilibrium transport, mesoscopic systems