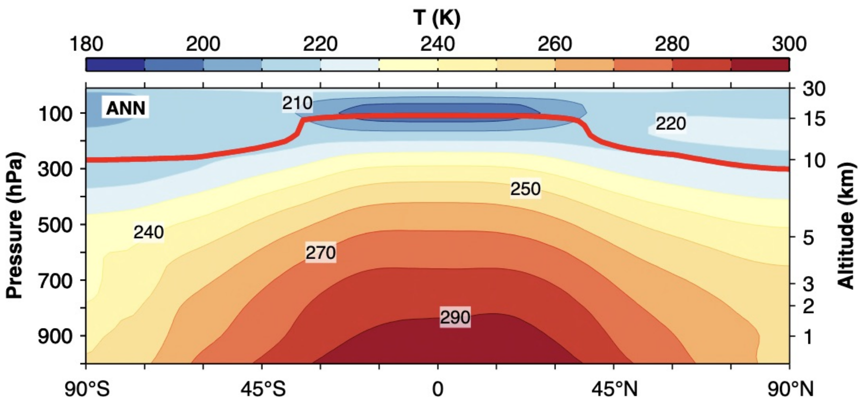
Boulder School 2022 Tiffany Shaw

Lecture 1 - Large scale circulation

Vallis (2006) Chapter 2

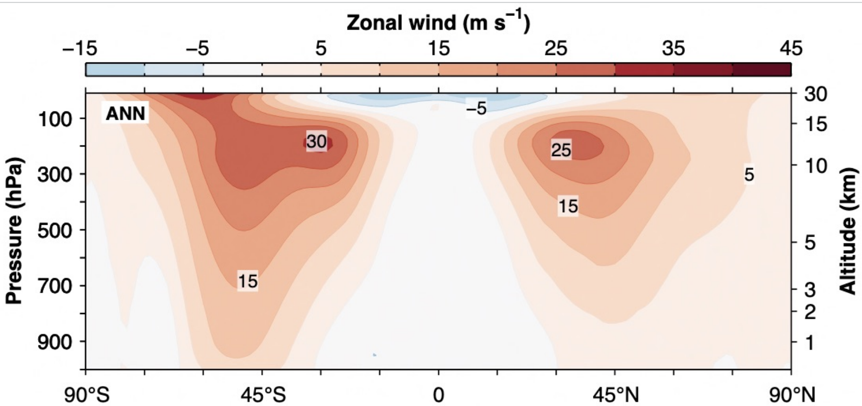
Observed large-scale features



Annual mean Reanalysis data

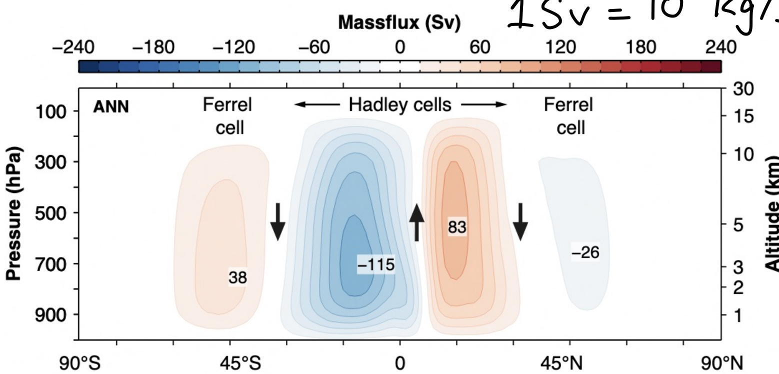
Figures: Physics of Earth's climate T. Schneider

Temperature decreases with height (mostly).



Upper level eastward flow, lower level westward – eastward

$1\,Sv = 10^9\,kg/s$

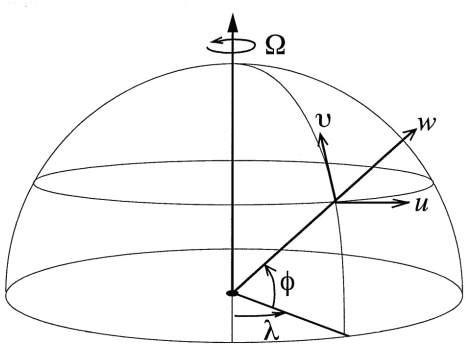


Air rises near the equator and sinks around 30° (two cell structure).

Geophysical Fluid Dynamics (Vallis 2006)

Geophysical Fluid Dynamics is the study of rotating, stratified fluids that describes large scale motions in the atmosphere and ocean on Earth and other planets

Equations of motion on a sphere



Vallis section 2.2.3

$\underline{r} = (\lambda, \phi, r)$

zonal, meridional, vertical

$\underline{v} = (u, v, w)$

Momentum Vallis (2.75)

$$\frac{Du}{Dt} - \left(\boxed{2\Omega\sin\phi} + \frac{u\tan\phi}{a}\right)v = -\frac{1}{\rho a\cos\phi}\frac{\partial p}{\partial \lambda}$$

$$\frac{Dv}{Dt} + \left(\boxed{2\Omega\sin\phi} + \frac{u\tan\phi}{a}\right)u = -\frac{1}{\rho a}\frac{\partial p}{\partial \phi}$$

Coriolis acceleration

$$\frac{Dw}{Dt} = \boxed{-\frac{1}{\rho}\frac{\partial p}{\partial z} - g}$$

Hydrostatic balance

N.B. $g \equiv g_{eff} = g_{grav} + \Omega^2 \underline{r}_{\perp}$

$\gamma \equiv \frac{H}{a} \ll 1 \qquad r = a + z$

3 equations, 5 unknowns (u, v, w, ρ, p)

Mass Vallis (2.52)

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \underline{v} = 0$$

Cartesian coordinates.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0$$

$$\nabla_p \cdot \underline{u} + \frac{\partial \omega}{\partial p} = 0$$

pressure coordinates
Vallis section 2.6

4 equations, 5 unknowns (u, v, w, ρ, p)

Equation of state

Ideal gas law $p = \rho R T \qquad R = \frac{R^*}{M}$

5 equations, 6 unknowns (u, v, w, p, ρ, T)

Energy

First law of thermodynamics for ideal gas

$$c_p \frac{DT}{Dt} - \frac{1}{\rho}\frac{Dp}{Dt} = \underbrace{\underset{\text{condensation}}{\boxed{LC}} + g\frac{\partial}{\partial p}\Big(\underset{\text{radiation}}{\boxed{F^R}} + \underset{\text{conduction temperature}}{\boxed{F^S}}\Big)}_{\text{heat added}}$$

$$L\frac{Dq}{Dt} = -\boxed{LC} + g\frac{\partial}{\partial p}\underset{\text{conduction moisture}}{\boxed{F_L}}$$

$$\frac{Dp}{Dt} = \omega = \frac{\partial p}{\partial t} + \underline{u}\cdot\nabla_p p + w\frac{\partial p}{\partial z}$$

$$= \frac{\partial p}{\partial t} + \underline{u}\cdot\nabla_p p - \rho g w$$

$$\omega \approx -\rho g \frac{Dz}{Dt} = -\rho\frac{D\Phi}{Dt}$$

N.B. Approximation neglects kinetic energy

Exercise: Use scale analysis to show kinetic energy is a small contribution

First law becomes:

$$c_p \frac{DT}{Dt} + \frac{D\Phi}{Dt} = LC + \frac{\partial}{\partial p}\left(F^R + F^S\right)$$

$$\Rightarrow \quad \frac{Ds}{Dt} = LC + \frac{\partial}{\partial p}\left(F^R + F^S\right)$$

$s = c_p T + gZ$ dry static energy

Combine with latent energy

$$\frac{Dm}{Dt} = g \frac{\partial}{\partial p}\left(F^R + F^S + F^L\right)$$

$m = c_p T + gZ + Lq$ moist static energy.

6 equations, 6 unknowns $(u, v, \omega, p, \rho, m)$

Equations of climate modeling

Dominant balances (scaling)

Vertical momentum: Hydrostatic balance

$$\frac{Dw}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g$$

$$\frac{UW}{L} \sim \left|\frac{1}{\rho}\frac{\partial p}{\partial z}\right| + g$$

$W \sim 10^{-2}\,ms^{-1}$ $\quad L \sim 10^{6}\,m$ $\quad g \sim 10\,ms^{-2}$

$U \sim 10\,ms^{-1}$ $\quad T = L/U$ $\quad H \sim 10^{4}\,m$

$$\frac{UW}{L} \sim \frac{10\,ms^{-1}\,10^{-2}\,ms^{-1}}{10^{6}\,m} \sim 10^{-7}\,\frac{m}{s^2} \ll g$$

$$\Rightarrow \quad \frac{\partial p}{\partial z} = -\rho g \qquad \delta = \frac{H}{L} \ll 1$$

Horizontal momentum: Geostrophic balance

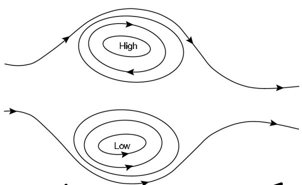
$$\frac{Du}{Dt} - \left(2\Omega\sin\phi + \frac{u\tan\phi}{a}\right)v = \frac{-1}{a\cos\phi}\frac{\partial \Phi}{\partial \lambda}$$

$$\frac{U^2}{L} \qquad fU \qquad \frac{U^2}{a} \qquad \left|\frac{1}{a\cos\phi}\frac{\partial \Phi}{\partial \lambda}\right|$$

$$Ro \equiv \frac{U}{fL} \qquad \text{Rossby number}$$

If $Ro \ll 1 \Rightarrow -2\Omega \sin\phi\, v = -fv = \frac{-1}{a\cos\phi}\frac{\partial \Phi}{\partial \lambda}$

$$2\Omega \sin\phi\, u = fu = \frac{-1}{a}\frac{\partial \Phi}{\partial \phi}$$



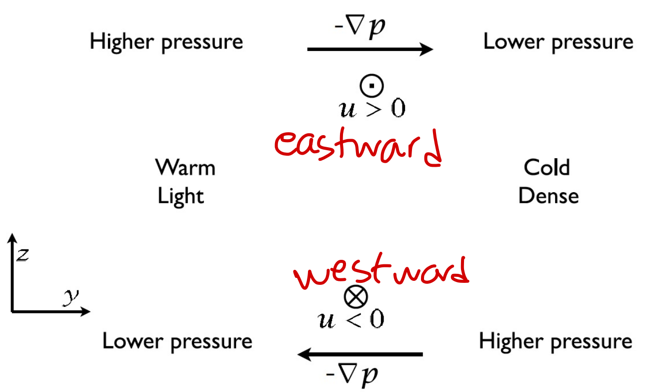
Vallis Fig 2.5

Thermal wind balance

Combine hydrostatic and geostrophic balance

$$-f\frac{\partial v}{\partial p} = \frac{-1}{a\cos\phi}\frac{\partial^2 \Phi}{\partial p\,\partial \lambda} \overset{\text{hydrostatic balance}}{=} -\frac{1}{a}\frac{\partial}{\partial x}(-\alpha) \overset{\text{ideal gas law}}{=} \frac{1}{a}\frac{R}{p}\frac{\partial T}{\partial x}$$

$$f\frac{\partial u}{\partial p} = -\frac{1}{a}\frac{\partial^2 \Phi}{\partial p\,\partial \phi} = -\frac{1}{a}\frac{\partial}{\partial \phi}(-\alpha) = \frac{1}{a}\frac{R}{p}\frac{\partial T}{\partial \phi}$$



Vallis Fig 2.6

Zonal momentum balance regimes

(Schneider 2006)

Exploit symmetry about the axis of rotation (connected to angular momentum conservation

$$\frac{Du}{Dt} - 2\Omega \sin\phi\, v - \frac{uv}{a}\tan\phi = \frac{-1}{a\cos\phi}\frac{\partial \Phi}{\partial \lambda}$$

$$\frac{\partial u}{\partial t} + \underbrace{\underline{v}\cdot\nabla u}_{= \nabla\cdot(\underline{v}u)} - 2\Omega\sin\phi\, v - \frac{uv}{a}\tan\phi = \frac{-1}{a\cos\phi}\frac{\partial \Phi}{\partial \lambda}$$

Consider steady state

Let $\overline{\cdot} \equiv \frac{1}{2\pi}\int_0^{2\pi} (\cdot)\, d\lambda$ zonal average

$$u = \bar{u} + u' \quad v = \bar{v} + v' \quad \omega = \bar{\omega} + \omega'$$

Exercise: Show zonal momentum budget away from surface becomes

$$(f + \bar{\zeta})\bar{v} - \bar{\omega}\frac{\partial \bar{u}}{\partial p} = \frac{1}{a\cos^2\phi}\frac{\partial}{\partial \phi}\left(\cos^2\phi\, \overline{u'v'}\right) + \frac{\partial}{\partial p}\left(\overline{u'\omega'}\right)$$

9

In the upper troposphere $\omega \approx 0$

$$(f + \bar{\zeta})\bar{v} \approx \frac{1}{a\cos^2\phi}\frac{\partial}{\partial\phi}\left(\cos^2\phi\, \overline{u'v'}\right)$$

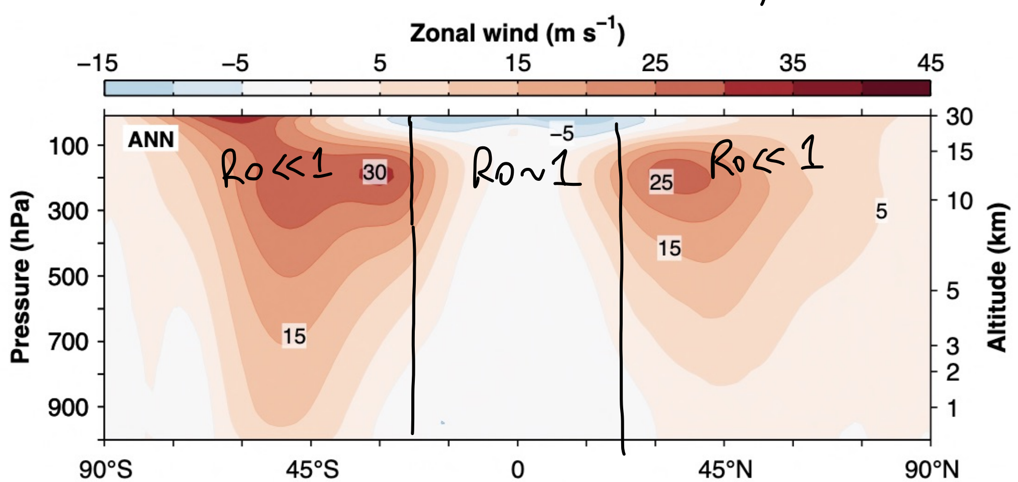
Let $\quad Ro \equiv -\frac{\bar{\zeta}}{f} \quad$ where $\bar{\zeta} = \frac{-1}{a\cos\phi}\frac{\partial}{\partial\phi}\left(\cos\phi\, \bar{u}\right)$

$$f(1 - Ro)\bar{v} \approx \frac{1}{a\cos^2\phi}\frac{\partial}{\partial\phi}\left(\cos^2\phi\, \overline{u'v'}\right)$$

Two regimes:

$Ro \sim 1 \qquad f(1-Ro)\bar{v} \approx 0$

$Ro \ll 1 \qquad f\bar{v} \approx \frac{1}{a\cos^2\phi}\frac{\partial}{\partial\phi}\left(\cos^2\phi\, \overline{u'v'}\right)$



Energy balance regimes

Miyawaki et al (2022)

Consider steady state, zonal average, vertically integrated moist static energy budget

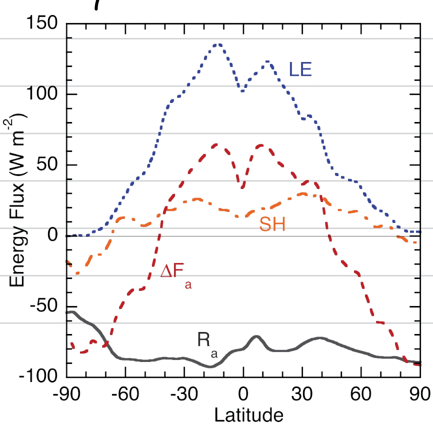
vertical integral $\langle \cdot \rangle = \frac{1}{g} \int \cdot \, dp$

$$\frac{Dm}{Dt} = g \frac{\partial}{\partial p} \left(F^R + F^S + F^L \right)$$

$$\frac{\partial m}{\partial t} + \nabla \cdot (\underline{v} m) = g \frac{\partial}{\partial p} \left(F^R + F^S + F^L \right)$$

$$\frac{1}{a\cos\phi} \frac{\partial}{\partial \phi} \left(\cos\phi \langle \overline{vm} \rangle \right) = \boxed{F^R_T - F^R_B} \boxed{- F^S - F^L}$$

$$\frac{\partial}{\partial y} \left(\langle \overline{vm} \rangle \right) = Ra + TF$$



$$\Delta F_a \equiv \frac{\partial}{\partial y} \left(\langle \overline{vm} \rangle \right)$$

$$TF = LE + SH$$

Hartmann (2016)

Divide by Ra radiative cooling

$$\frac{\frac{\partial}{\partial y}(\langle \overline{vm} \rangle)}{Ra} = 1 + \frac{TF}{Ra}$$

Let $R_1 \equiv \frac{\frac{\partial}{\partial y}(\langle \overline{vm} \rangle)}{Ra}$

What balances radiative cooling?

Global mean $\frac{\partial}{\partial y}\langle \overline{vm} \rangle \to 0$

$R_1 \lesssim \varepsilon$ Radiative convective equilibrium

$$Ra \sim -TF$$

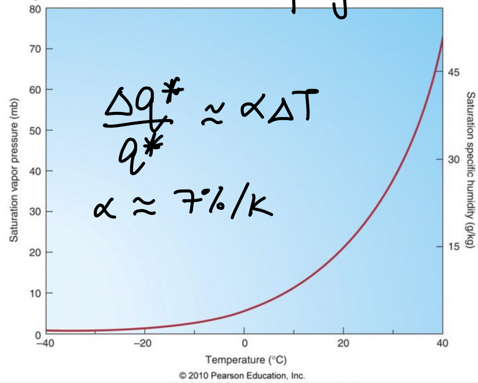
Thermal stratification follows moist adiabat

$$\Gamma = -\frac{\partial T}{\partial z} = \Gamma_m = \Gamma_d \left[\frac{1 + \frac{L q^*(T,p)}{R_d T}}{1 + \frac{L^2 q^*(T,p)}{c_p R_v T^2}} \right]$$

$$\Gamma_d = \frac{g}{c_p} \qquad R_d = R \qquad R_v = \frac{R^*}{M_v}$$

$$\Gamma_m < \Gamma_d$$

Clausius-Clapeyron relation

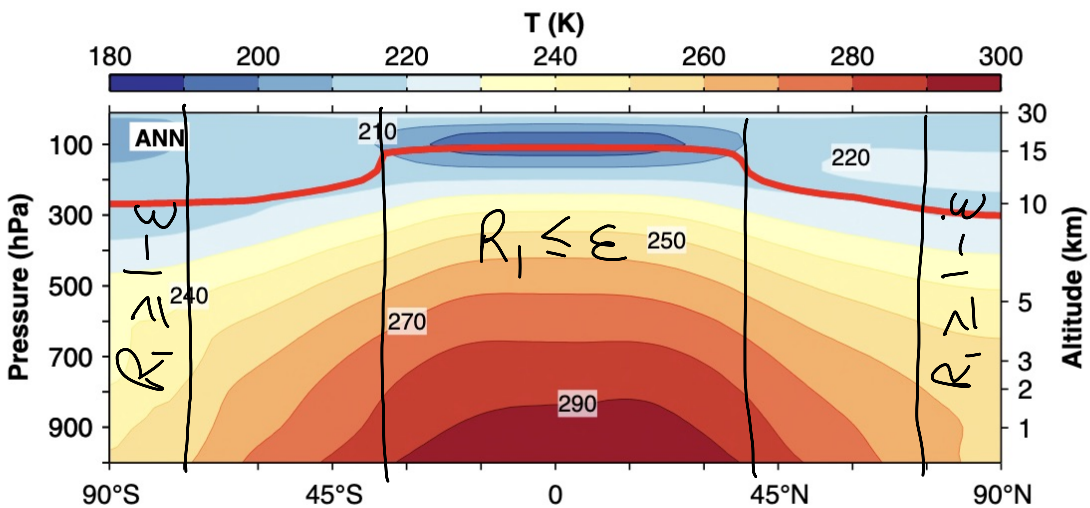


$$\frac{\Delta q^*}{q^*} \simeq \alpha \Delta T$$

$$\alpha \simeq 7\%/K$$

$R_1 \geq 1-\varepsilon$ Radiative advective equilibrium

Advective heating balances radiative cooling



$R_1 \geq 1-\varepsilon$ $\quad R_1 \leq \varepsilon$ $\quad R_1 \geq 1-\varepsilon$

$\varepsilon = 0.1$