# BOULDER 2017 II

Correction in PT in $\Delta_0$ → $\eta \sim L^{z-2}$

no corr. $c=1$

$\Delta(u)$ flows yields $\zeta$

$\delta f = -f_c$

$$\eta\, \partial_t u(x,t) = c\nabla_x^2 u(x,t) + F(x,u(x,t)) + \cancel{f}$$

$$+\, m^2 (w(t) - u(x,t))$$

$\uparrow\, t$ 

$m^2 w$

$$\overline{F(x,u)F(x',u')} = \delta^d(x-x')\Delta_0(u-u')$$

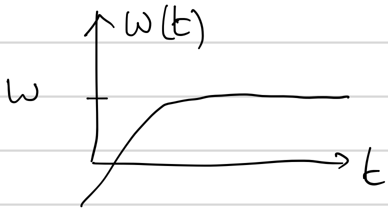
$$\Delta_0 = -R_0''$$

$$\delta H_V[u] = \int d^dx\ \tfrac{1}{2} m^2 (w-u)^2$$

$L_m \sim 1/m$ flat

Middleton $\dot{w}(t) \geq 0 \ \forall t \geq t_0$, $\dot{u}(x,t) \geq 0 \ \forall x$ $\Rightarrow$ $\dot{u}(x,t) \geq 0 \ \forall x \ \forall t \geq t_0$

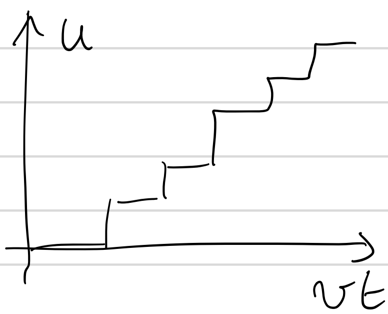
partial order, memory loss, convergence to Middleton attractor




$$\Rightarrow u(x,t) \xrightarrow[t\to\infty]{} u(x;w)$$

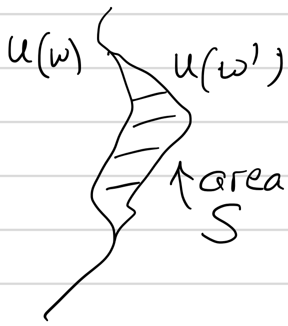
unique leftmost met. state $H_V[u]$

$$u(w) = \frac{1}{L^d}\int dx\, u(x;w)$$




$v = 0^+$ quasistatic

Avalanche def




motion from one Middleton state to next

Smooth disorder $w_i$

$$S = L^d\left(u(w_i^+) - u(w_i^-)\right)$$

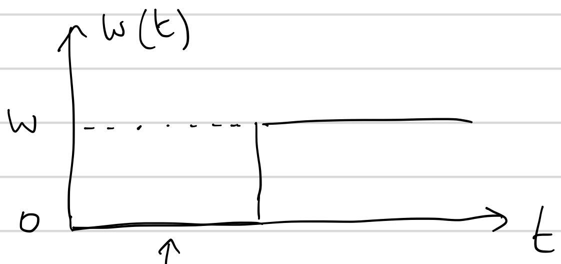
# Protocols / stat.

1) $w(t) = vt$  steady state

fixed velocity driving

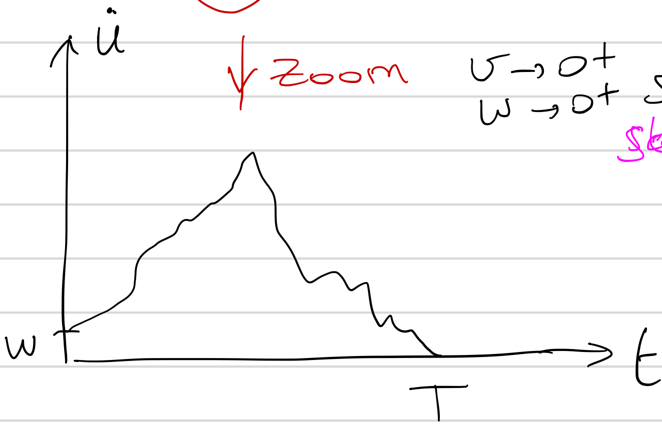

$v \to 0^+$
well separated

2) following a kick




Middleton $u(0)$
prepared in .. state

↓ zoom




$v \to 0^+$
$w \to 0^+$ same ~~stat~~ stat

$$S = \int_0^\infty dt\, \dot{u}(t)$$

$$P_w(S) \underset{w \to 0^+}{\sim} w \rho(S)$$

avalanche density

$$\rho(S) = \overline{\sum_i \delta(S - S_i)\, \delta(w - w_i)}$$

$$\rho(S) \sim \frac{1}{S^\tau}$$

$$\rho(T) \sim \frac{1}{T^\alpha}$$

$$\rho(\dot{u}) \sim \frac{1}{\dot{u}^a}$$

avalanche $\quad \overline{\dot{u}^p} \sim \langle S^p \rangle v$ $\qquad$ smooth $\quad \overline{\dot{u}^p} \sim v^p$

# Mean field theory of avalanches

1) $u(t) \to u(x,t)$

velocity theory

$$\eta \partial_t \dot u(t) = m^2(\dot w(t) - \dot u(t)) + \partial_t F(u(t))$$

2) exact for

$\dot u(t) \equiv \int d^dx\, \dot u(x,t)$

$\dot w(t) = L^d \dot w(t)$

total velocity, driving

$$\int D\dot u(t) \prod_t \delta(\partial_t \dot u - \dots) = \int D\dot u D\tilde u\, e^{\int_t i\tilde u_t(\partial_t \dot u_t - \dots)}$$

$e^{-S}$

$$S_0 = \int_t \tilde u_t\left((\partial_t + m^2)\dot u_t - m^2 \dot w_t\right)$$

$$S_{dis} = -\frac{1}{2}\int_{tt'} \tilde u_t \tilde u_{t'} \partial_t \partial_{t'} \Delta(u_t - u_{t'})$$

$$e^{\int_t \tilde u_t \partial_t F(u_t)}$$

Middleton!

$$\dot u_t \partial_{t'} \Delta'(u_t - u_{t'})$$

$\mathrm{sgn}(t-t')$

$\Delta(u) = -\sigma|u| + O(u^2)$

$\sigma = -\Delta'(0^+)$

$$\dot u_t \Delta'(0^+) \partial_{t'} \mathrm{sgn}(u_t - u_{t'}) \to \delta(t-t')$$

$$= -\sigma \int_t \tilde u_t \tilde u_t \dot u_t + O(\epsilon)$$

$\epsilon = d_c - d$

theory linear in $\dot u_t$

$$e^{\int dt\, \lambda(t)\dot u(t)} = \int D\tilde u D\dot u\, e^{m^2\int_t \tilde u_t \dot w_t}\, e^{\int_t \dot u_t(\partial_t \tilde u_t - m^2 \tilde u_t + \sigma \tilde u_t^2 + \lambda(t))}$$

$\to \delta(\dots)$

theorem

$$\| \quad e^{m^2 \int_t \tilde u_t \dot w(t)}$$

where $\tilde u_t$ solu $\quad \partial_t \tilde u_t - m^2 \tilde u_t + \sigma \tilde u_t^2 = -\lambda(t)$

$\tilde u_\infty = 0 \quad \lambda_\infty = 0$

Brownian force model

mean field $\quad F(u)$ is BM

ABBM model 1 particle

$d \geq d_{uc}$ BM landscape

$m \to 0$ Bessel process

$\dot u \equiv r \quad$ BM $d$ dim



$\dot u(u) \qquad W(t) = vt$

$$\eta \frac{d\dot u}{du} = m^2\left(\frac{v}{\dot u} - 1\right) + \xi(u)$$

$$P(\dot u) \sim \frac{1}{\dot u^a} e^{-\dot u \frac{m^2}{\sigma}}$$

$$a = 1 - \frac{m^2 v}{\sigma} = 2 - \frac{d}{2}$$

$\to$ Total size distrib $\quad S = \int_0^\infty \dot{u}(t)\,dt \longrightarrow \int d^dx\, \dot{u}(x,t)$

$$\overline{e^{\lambda S}} = e^{\lambda \int_0^\infty \dot{u}(t)\,dt} \qquad \lambda(t) = \lambda \Rightarrow \hat{u}_t = \tilde{u}$$

$$-m^2 \tilde{u} + \sigma \tilde{u}^2 = -\lambda \qquad S_m = \sigma/m^4 = \frac{|\Delta'(0^+)|}{m^4} = \frac{\langle S^2\rangle}{2\langle S\rangle}$$

$$\tilde{u} = \frac{1}{2}\left(1 - \sqrt{1-4\lambda}\right) \qquad \tau_m = \eta/m^2$$

$$\int_0^\infty e^{\lambda S} P_w(S)\,dS = e^{\frac{w}{2}\left(1-\sqrt{1-4\lambda}\right)}$$

$$LT^{-1} \Longrightarrow P_w(S) = \frac{w}{2\sqrt{\pi}\,S^{3/2}} e^{-\frac{(S-w)^2}{4S}} \qquad \langle S\rangle = w$$

BM rough disorder
avalanche all scales

$$\sim w\,\rho(S) \qquad \rho(S) \sim \frac{1}{S^{3/2}} e^{-S}$$

most $S \sim w^2$

rare large $\rho(S)$

not norm

$$\tau_{MF} = 3/2$$

$$\lambda(t) = \delta(t-t_0) \Longrightarrow \tilde{u}(t) = \frac{\lambda\,\theta(t<t_0)}{\lambda + (1-\lambda)e^{-(t-t_0)}}$$

duration

$\Rightarrow P(T)$

$$LT^{-1} \hookrightarrow P(\dot{u}) = \underset{\text{piece}}{\delta(\dot{u})} + \text{smooth} \qquad \rho(\dot{u}) \underset{\dot{u}\to 0}{\sim} 1/\dot{u}$$

$$a_{MF} = 1$$

$$\overline{e^{\lambda \dot{u}(t_0)}} = e^{\int dt\, \dot{w}(t)\,\tilde{u}(t)} = e^{w\tilde{u}(0)}$$

kick $t=0$

$\delta(t)$

$$= e^{w \frac{\lambda}{\lambda + (1-\lambda)e^{t_0}}}$$

$$\lim_{\lambda\to\infty} = P(T \le t_0) = e^{-w/(e^{t_0}-1)}$$

$$P(\dot{u}(t_0) = 0)$$

$$P(T) = \frac{w e^{-w/(e^T-1)}}{(2\,\mathrm{sh}\,T/2)^2} \underset{w\to 0}{\sim} w/(2\,\mathrm{sh}\,T/2)^2 \sim \frac{1}{T^2}$$

$$\alpha_{MF} = 2$$

$$P_w(S) = \frac{wL^d}{2\sqrt{\pi}\, S^{3/2}} e^{-\frac{(S-wL^d)^2}{4S}}$$

$w \sim L^{-d}$
single avalanche regime
$L \to \infty$ gaussian

$$P_w(T) = \frac{wL^d \, e^{-wL^d/(e^T-1)}}{4\,\mathrm{sh}^2(T/2)}$$

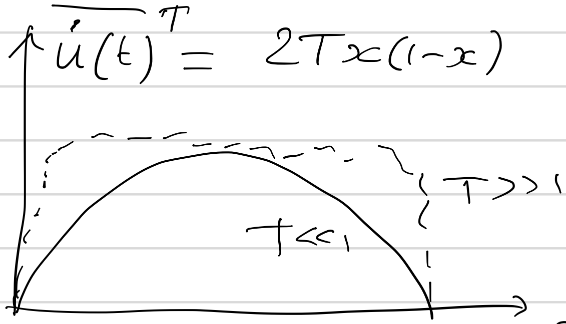
$L \to \infty$
$w$ fixed
gumbel
max $wL^d$ indep. events

MFT avalanches uncorrelated

$P(S,T)$ $\quad \overline{S}_T = 2T\coth T/2 - 4 \sim T^2 \quad T \ll 1$

$\sim T \quad T \gg 1$

"shape" at fixed $T$ $\quad \overline{\dot u(t)}^T = 2Tx(1-x)$ $\quad$ parabola $T \ll 1$




$x = t/T$

at fixed $S$ $\quad \overline{\dot u(t)}^S = 2t \cdot e^{-t^2/S}$ $\quad \int dt\, \dot u = S$

<u>beyond MFT</u> $d = d_{uc} - \epsilon$ $\quad S_m \sim m^{-(d+\zeta)} \quad t_m \sim m^{-z}$

$$\tau = 2 - \frac{2}{d+\zeta} \qquad \alpha = 1 + \frac{d-2+\zeta}{z}$$ scaling

NF Conj $\quad a = 2 - \frac{2}{d+\zeta-z}$ $\quad$ gen. NF checked numerically

local exponents $S_0$

universal "shapes"