Energy pumps with multi-tone driving

Anushya Chandran

Lecture 2 in “Non-Equilibrium Quantum Dynamics” summer school

Based on work with David Long, Phil Crowley, Martin Ritter + Alicia Kollar

Plan

1. Two tone driven qubit in the adiabatic regime
   - Quantized energy pump
   - Connections to topological invariants
   - Application: Cavity state boosting
   - Experiments with superconducting qubits
2. New driven phases of matter
   - Steady state energy pumps

Two-tone driven spin

$$H(t) = -\frac{1}{2}\overrightarrow{B}(t) \cdot \vec{\sigma}$$

$$\overrightarrow{B}(t) = \begin{pmatrix} \sin \omega_1 t \\ \sin \omega_2 t \\ B_z - \cos \omega_1 t - \cos \omega_2 t \end{pmatrix}$$

Observables of interest: energy currents

$$\underbrace{\partial_t \langle H \rangle}_{\text{Rate of change of energy in the system}} = \langle \partial_t H_1 \rangle + \langle \partial_t H_2 \rangle = \underbrace{P_1}_{\text{Power of drive 1}} + \underbrace{P_2}_{\text{Power of drive 2}}$$

$$\vec{B}(t) = \begin{pmatrix} \sin \omega_1 t \\ \sin \omega_2 t \\ B_z - \cos \omega_1 t - \cos \omega_2 t \end{pmatrix}$$

Observables of interest: energy currents

$$\overline{\partial_t \langle H \rangle} = \overline{\langle \partial_t H_1 \rangle} + \overline{\langle \partial_t H_2 \rangle} = \overline{P_1} + \overline{P_2} = 0$$

Rate of change of energy in the system

Power of drive 1

Power of drive 2

$$\vec{B}(t) = \begin{pmatrix} \sin \omega_1 t \\ \sin \omega_2 t \\ B_z - \cos \omega_1 t - \cos \omega_2 t \end{pmatrix}$$

Two regimes for dynamics

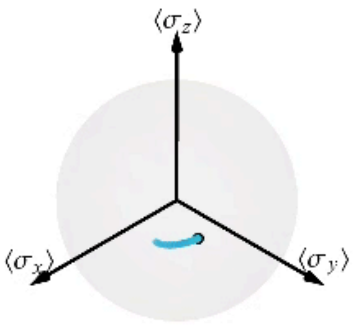
$$\overline{\partial_t \langle H \rangle} = \overline{\langle \partial_t H_1 \rangle} + \overline{\langle \partial_t H_2 \rangle} = \overline{P_1} + \overline{P_2} = 0$$

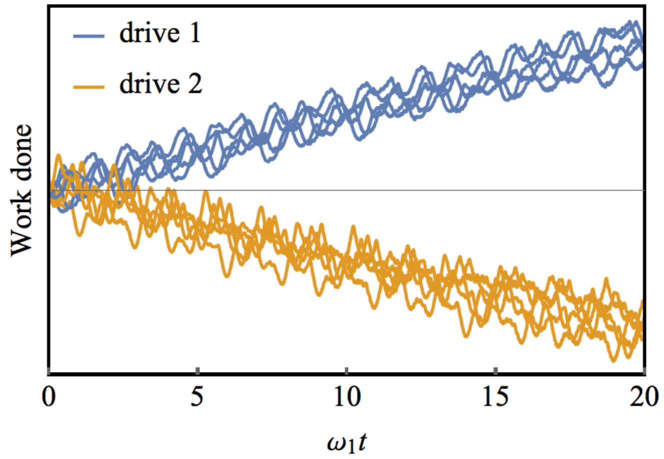
Rate of change of energy in the system

Power of drive 1

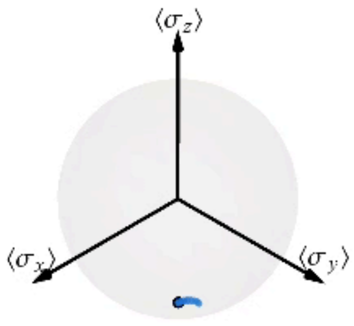
Power of drive 2

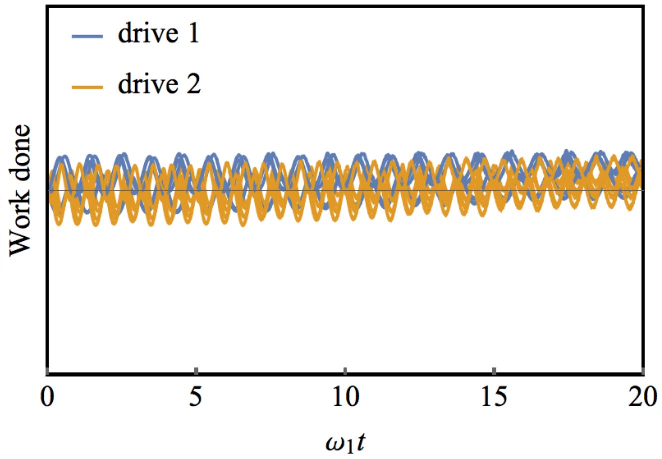
$B_z = 1$





$B_z = 3$





Martin, Refael, Halperin, 2017

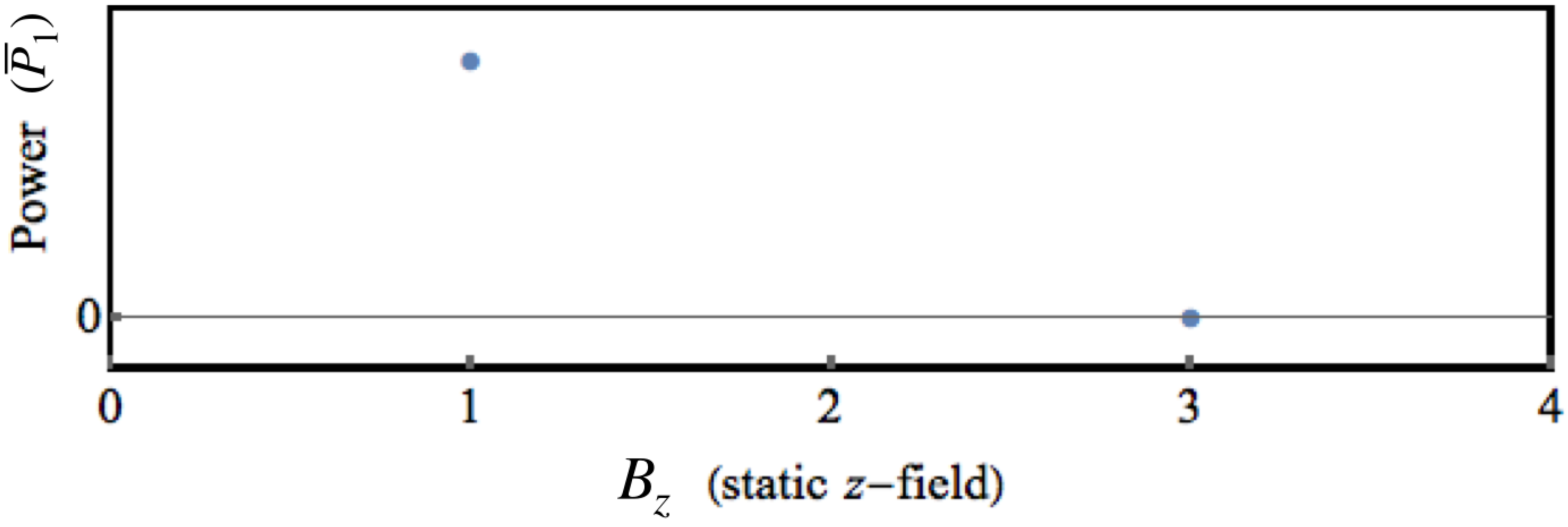
Quantum dynamics in driven systems

$$\overline{\partial_t \langle H \rangle} = \overline{\langle \partial_t H_1 \rangle} + \overline{\langle \partial_t H_2 \rangle} = \overline{P_1} + \overline{P_2} = 0$$

Rate of change of energy in the system

Power of drive 1

Power of drive 2



Martin, Refael, Halperin, 2017; **PC**, Martin, Chandran, PRB, 2018; **PC**, Martin, Chandran, PRL, 2020

Quantum dynamics in driven systems

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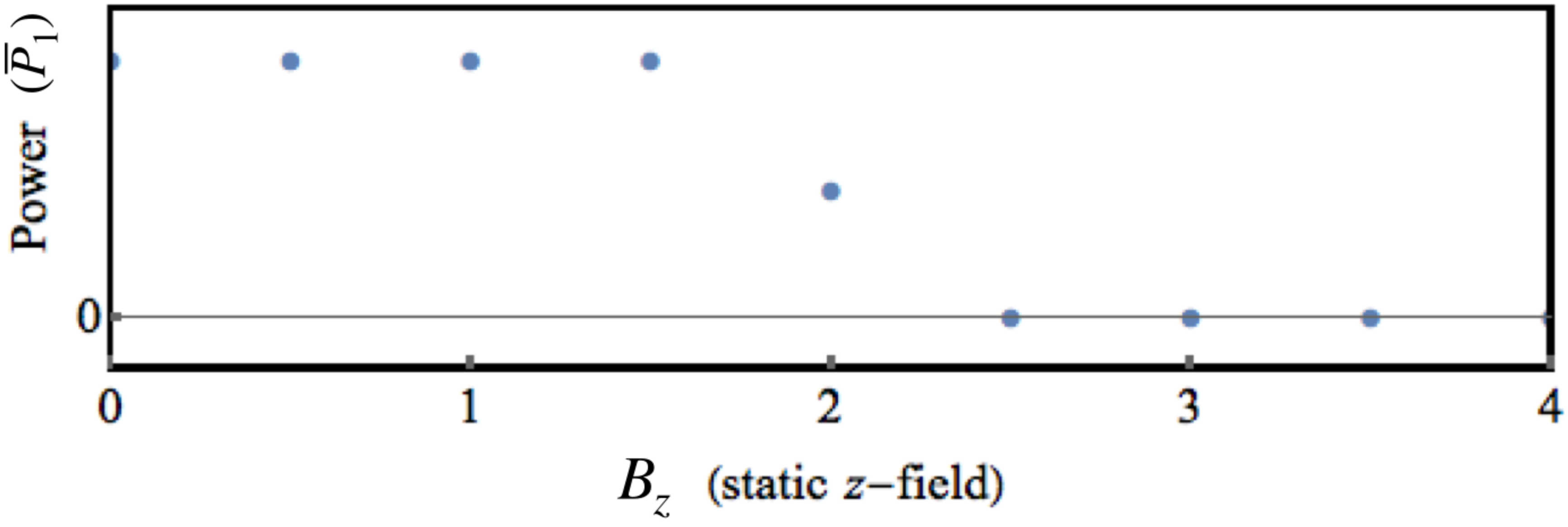
Rate of change of energy in the system

Power of drive 1

Power of drive 2



Martin, Refael, Halperin, 2017; Crowley, Martin, AC (2018, 2020)

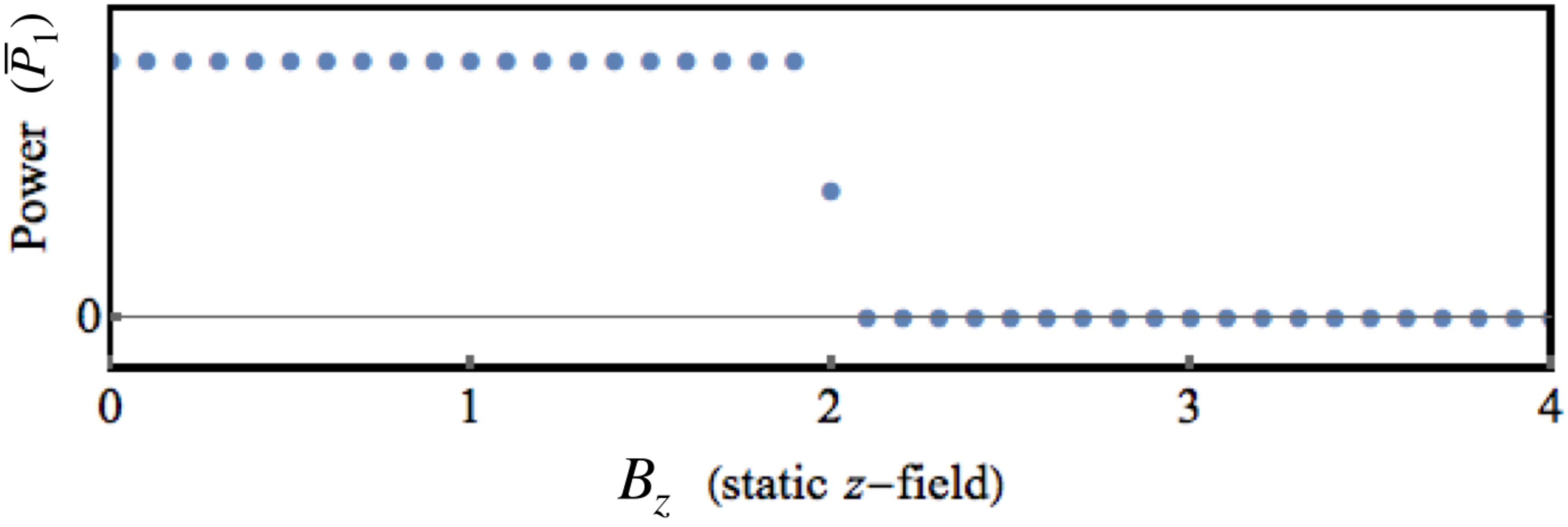
Quantum dynamics in driven systems

$$\overline{\partial_t \langle H \rangle} = \overline{\langle \partial_t H_1 \rangle} + \overline{\langle \partial_t H_2 \rangle} = \overline{P_1} + \overline{P_2} = 0$$

Rate of change of energy in the system

Power of drive 1

Power of drive 2



Martin, Refael, Halperin, 2017; Crowley, Martin, AC (2018, 2020)

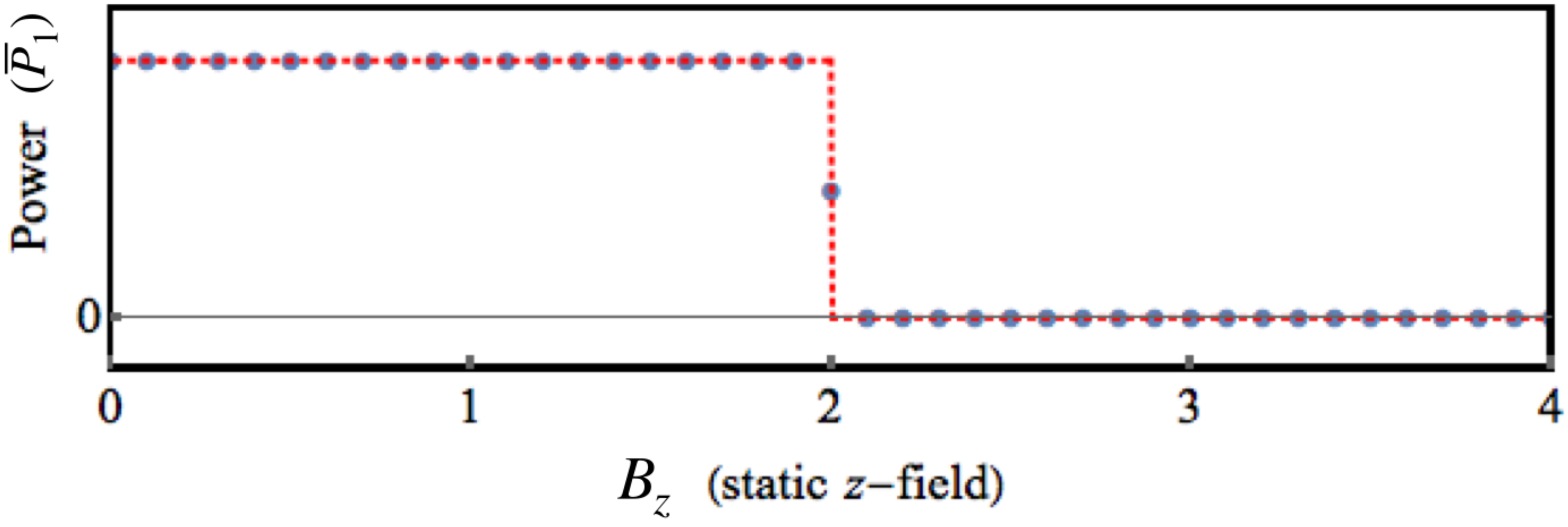
Quantum dynamics in driven systems

$$\overline{\partial_t \langle H \rangle} = \overline{\langle \partial_t H_1 \rangle} + \overline{\langle \partial_t H_2 \rangle} = \overline{P_1} + \overline{P_2} = 0$$

Rate of change of energy in the system

Power of drive 1

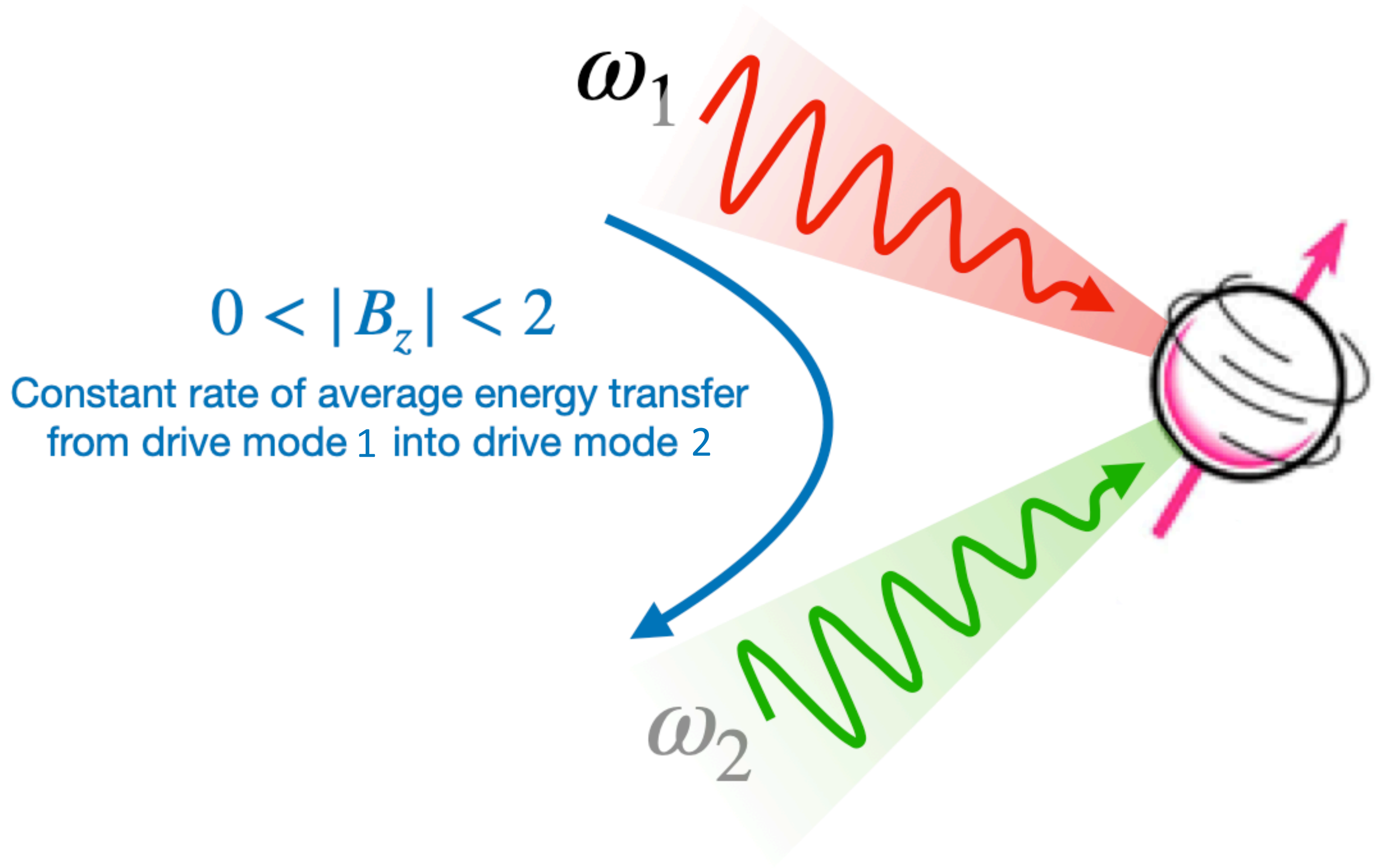
Power of drive 2



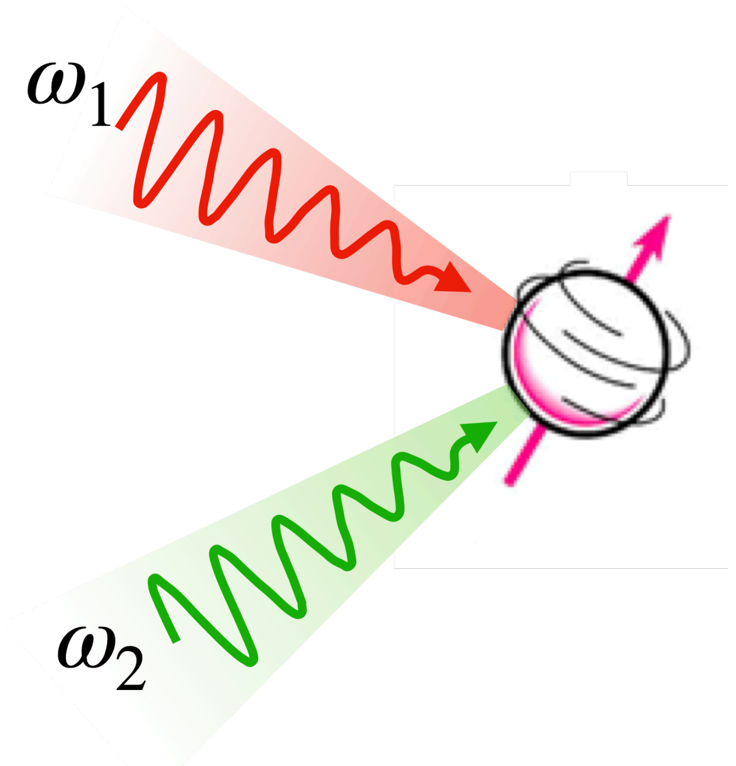
Martin, Refael, Halperin, 2017; Crowley, Martin, AC (2018, 2020)

Quantum dynamics in driven systems

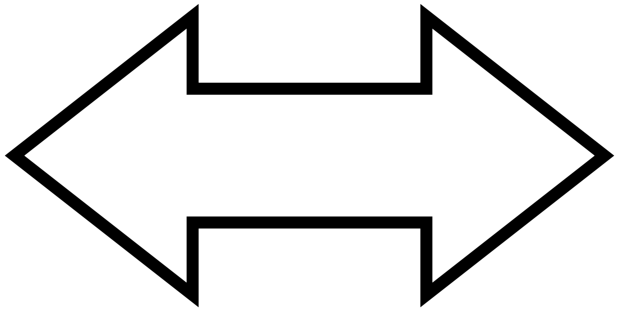
Why would a single qubit exhibit this quantised response?

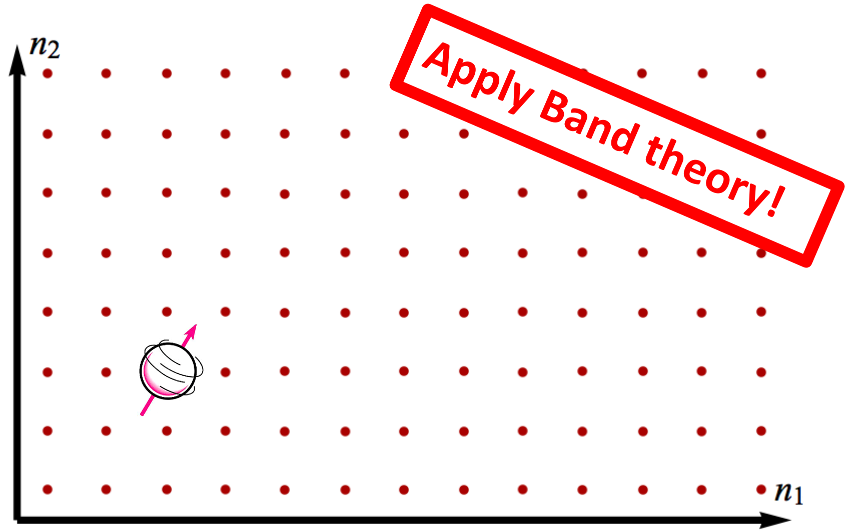


Synthetic dimensions for a driven qubit



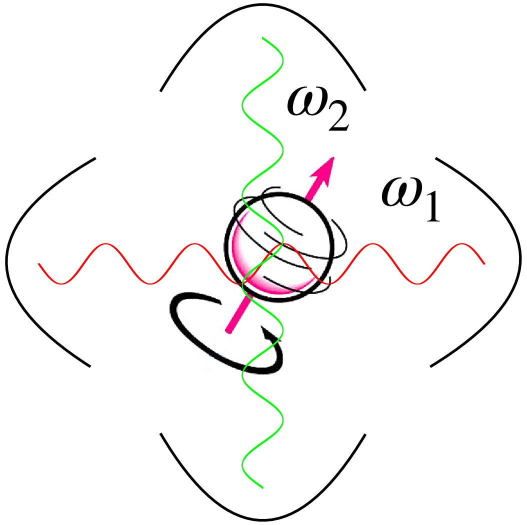
Driven two-level system

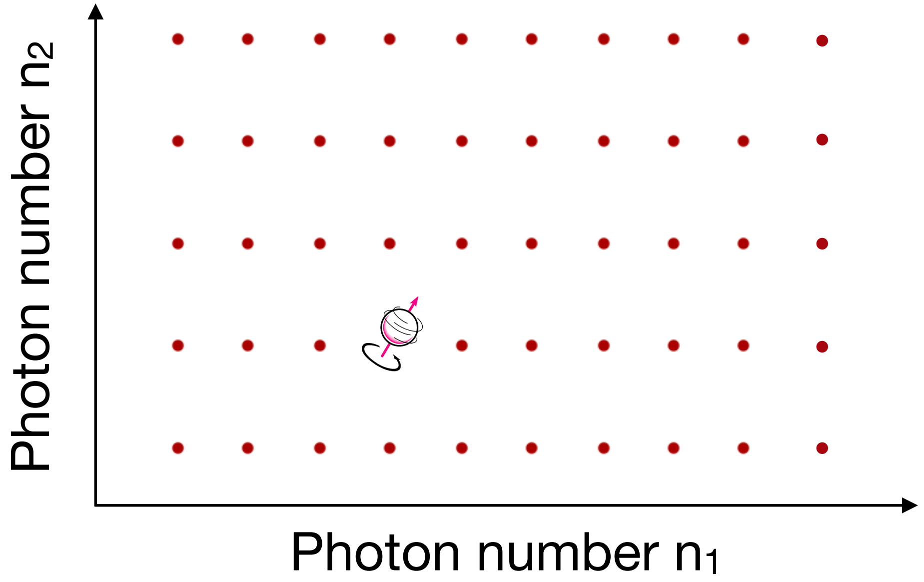




Two-band lattice system

Synthetic lattice model

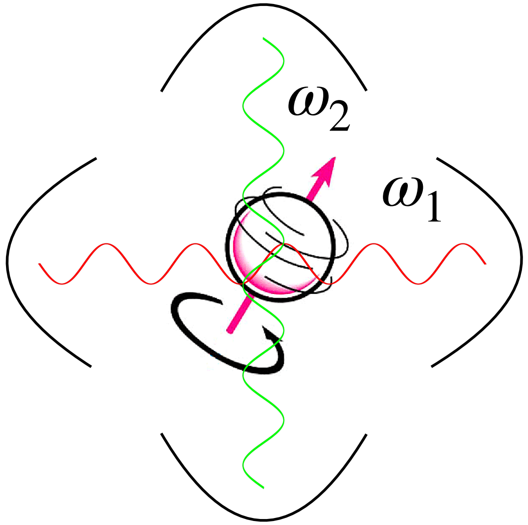


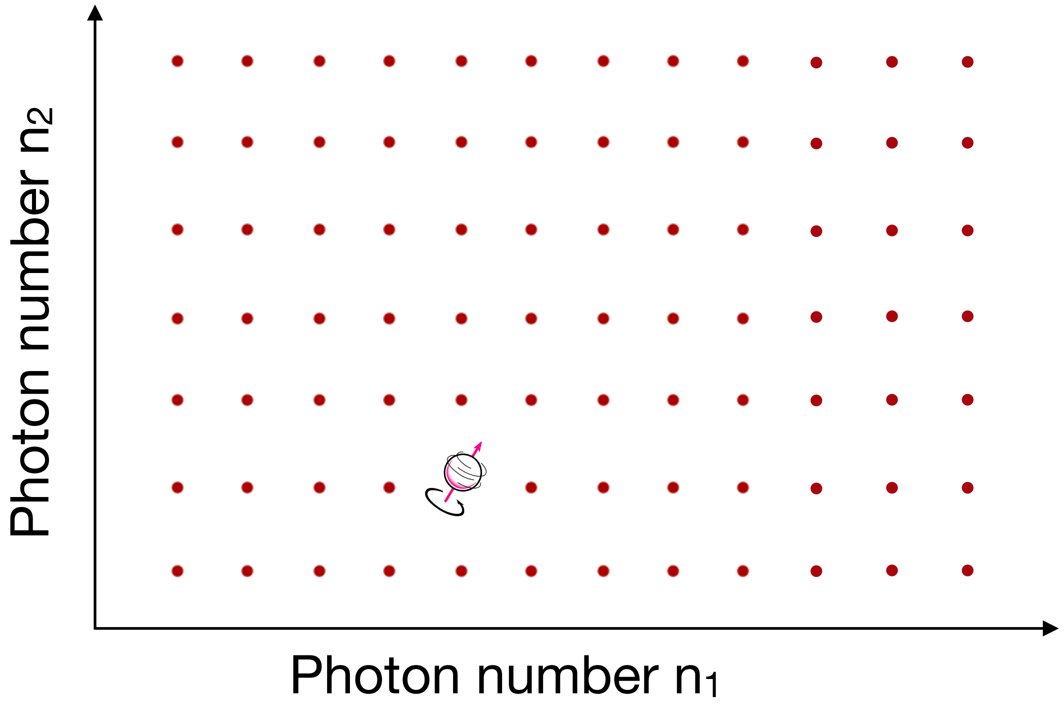


2d lattice with two levels
per lattice site

Synthetic lattice model

$$H = B_m S_x + B_0(a^\dagger S^+ + \text{h.c.}) + B_0(b^\dagger S^- + \text{h.c.}) + \dots$$
$$+\omega_1 a^\dagger a + \omega_2 b^\dagger b$$





Synthetic lattice model

$$H = B_m S_x + B_0(a^\dagger S^+ + \text{h.c.}) + B_0(b^\dagger S^- + \text{h.c.}) + \ldots$$
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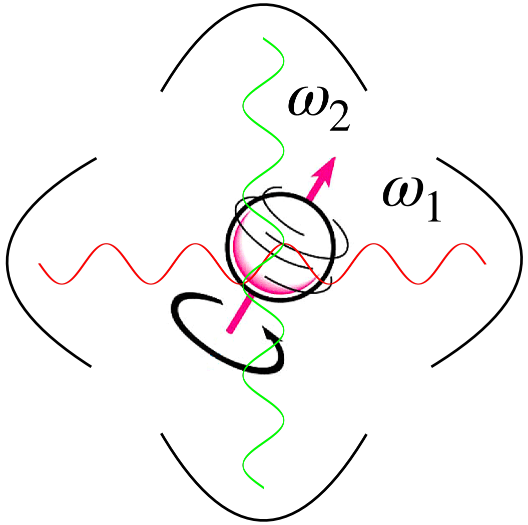

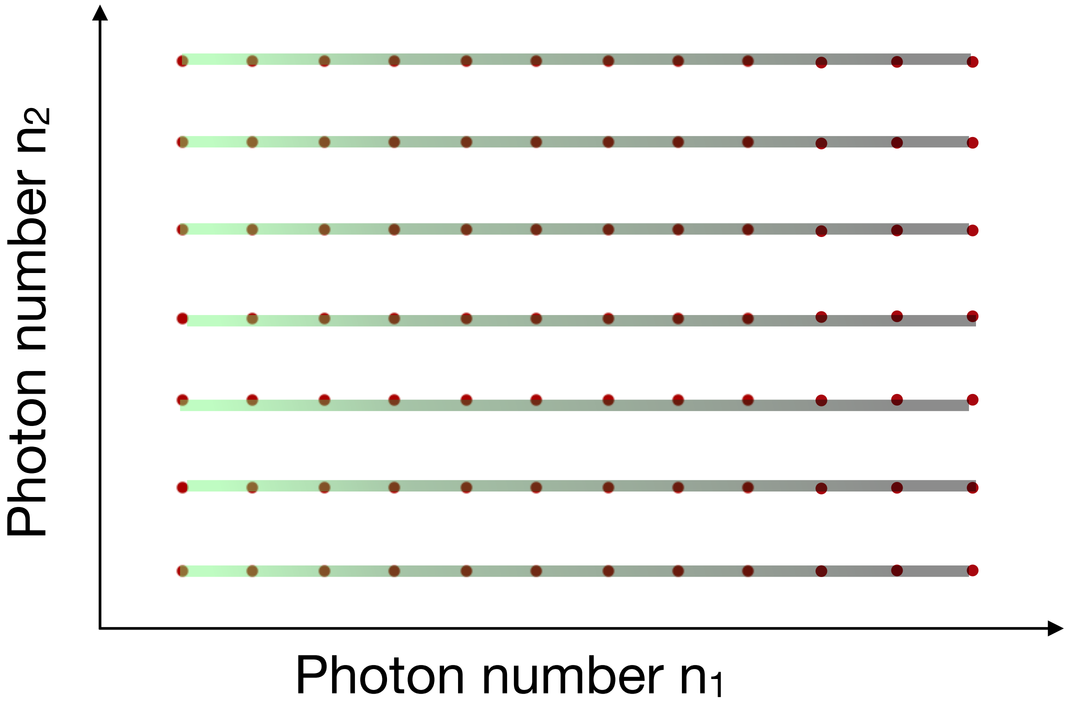

Synthetic lattice model

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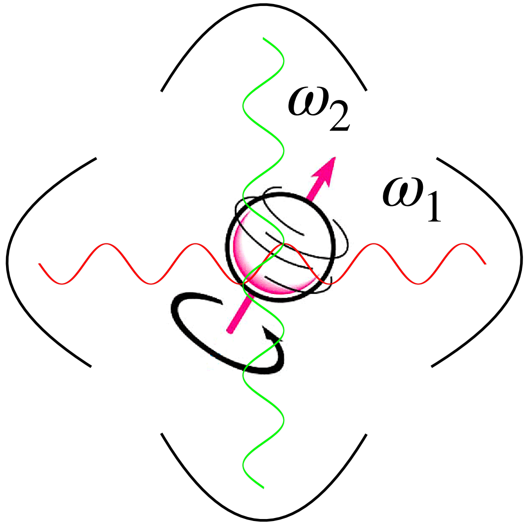

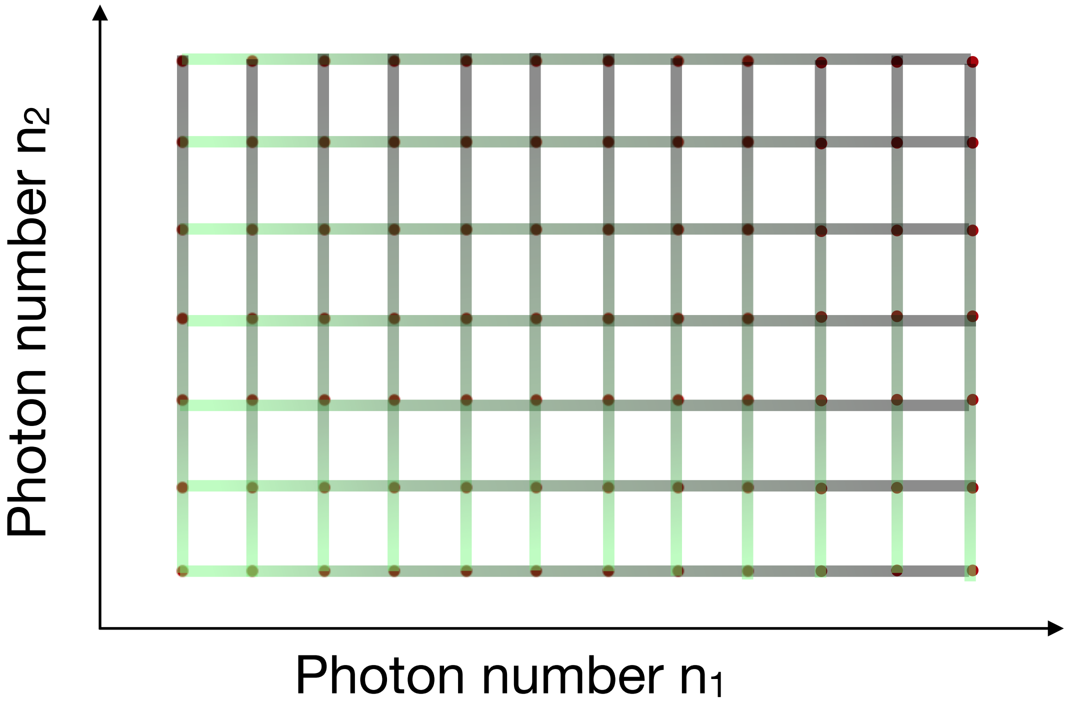

Synthetic lattice model

$$H = B_m S_x + B_0(a^\dagger S^+ + \text{h.c.}) + B_0(b^\dagger S^- + \text{h.c.}) + \dots$$
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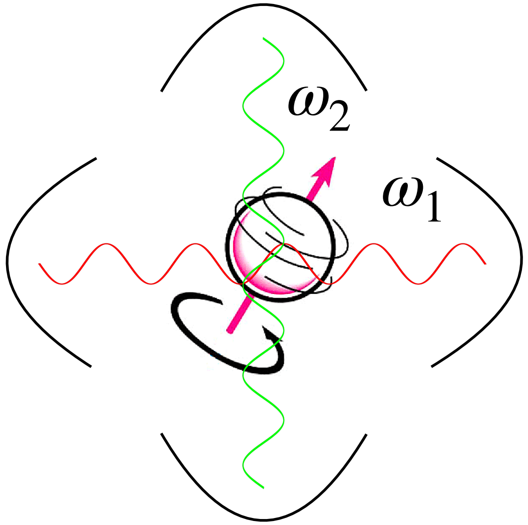
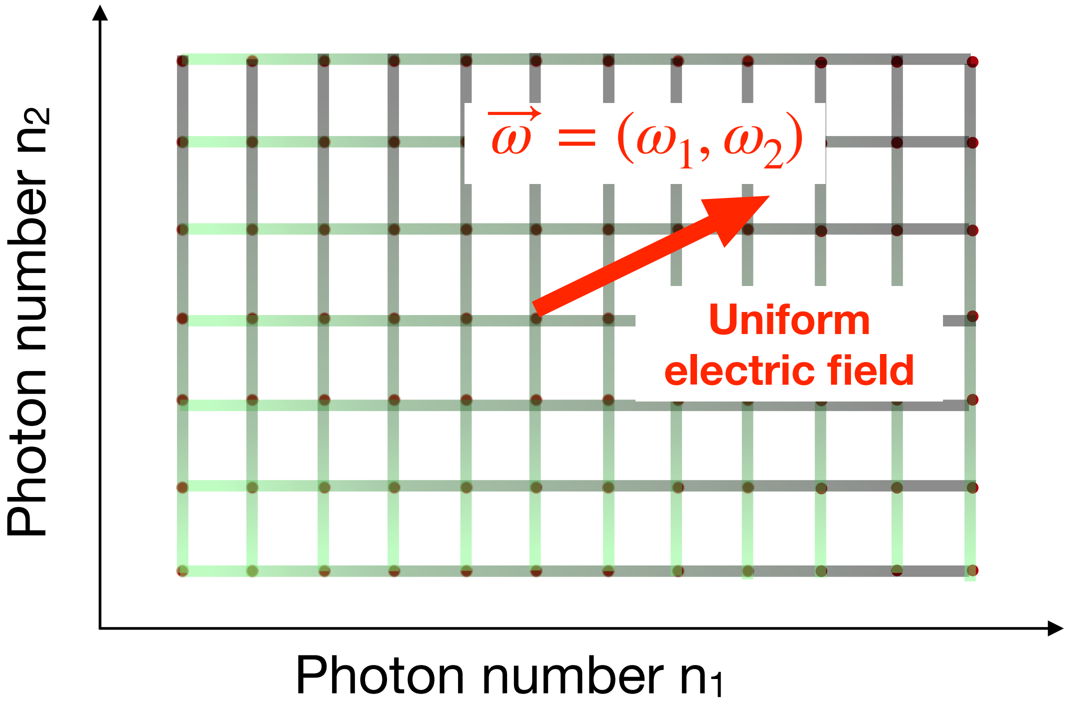
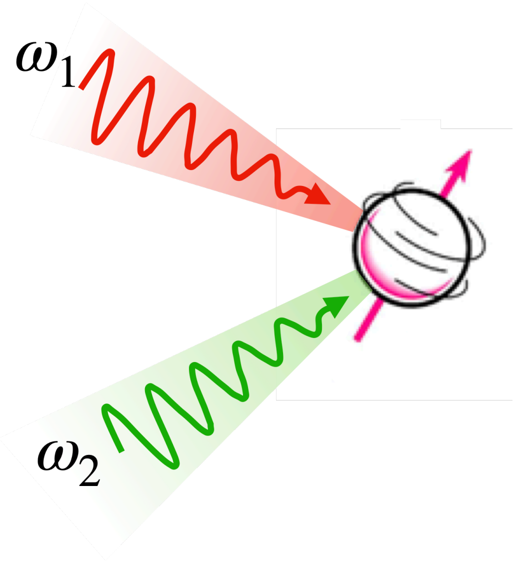
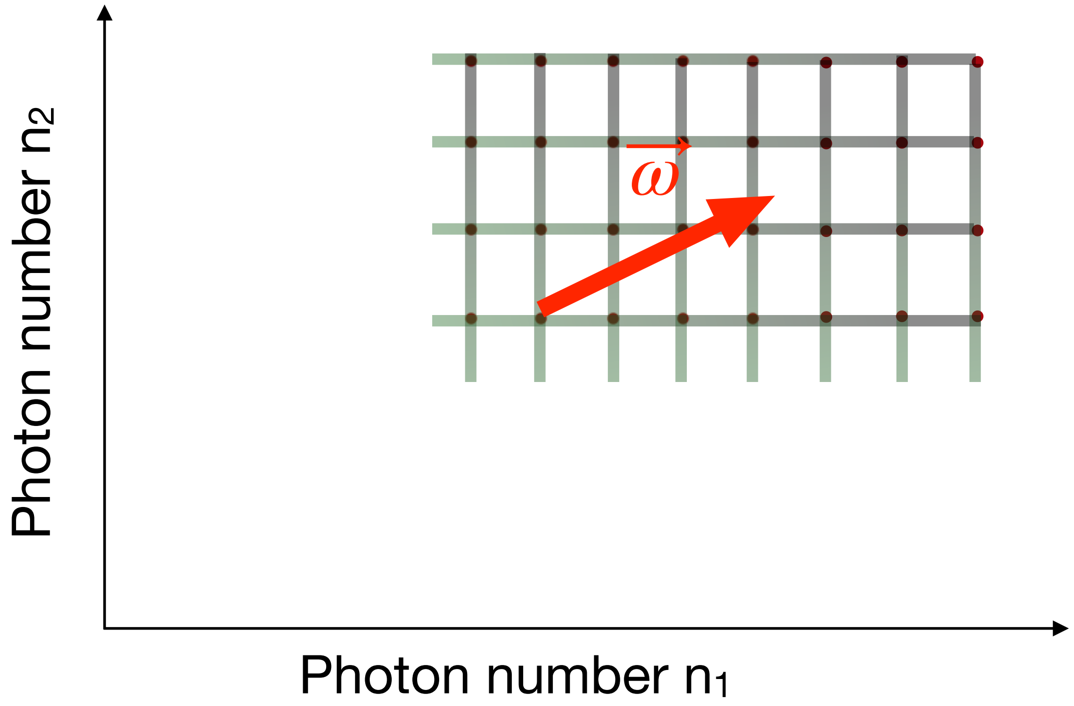




Classical limit: large n1, n2

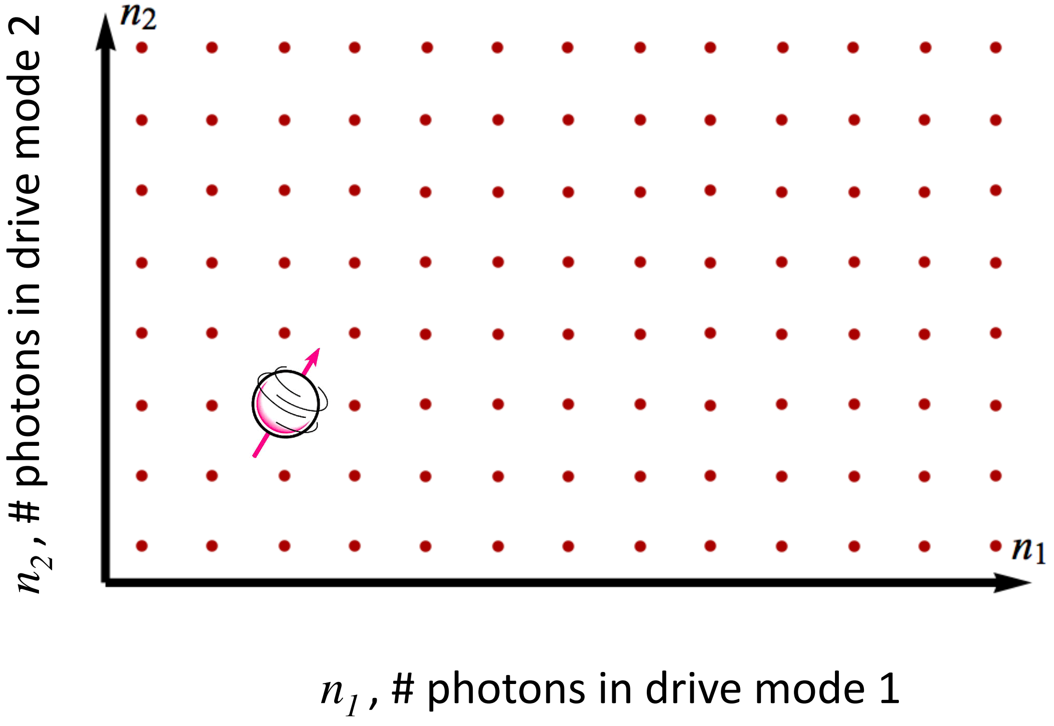
1. translationally invariant tight-binding model
2. with 2-orbitals per site (m levels, m orbitals per site)
3. uniform electric field in non-lattice direction





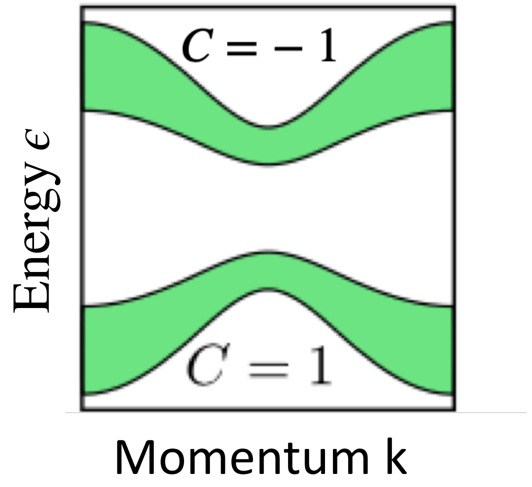
Synthetic band structure

For a moment, ignore the electric field. Then we have a translationally invariant 2d tight-binding model with Bloch bands.

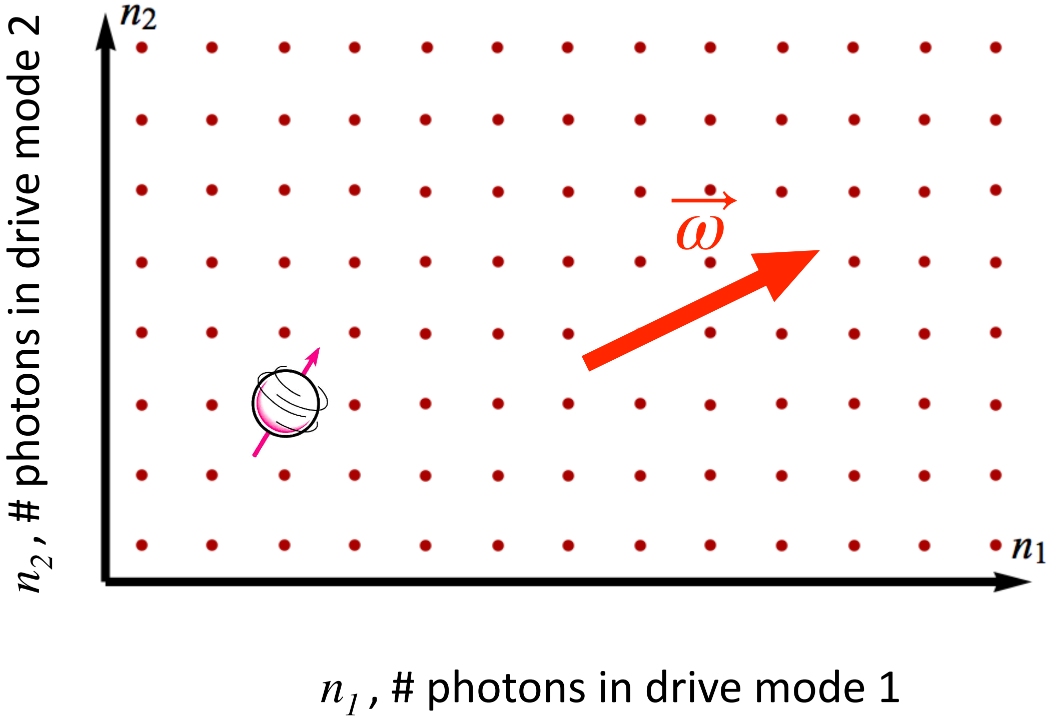


Synthetic bands can have topological invariants

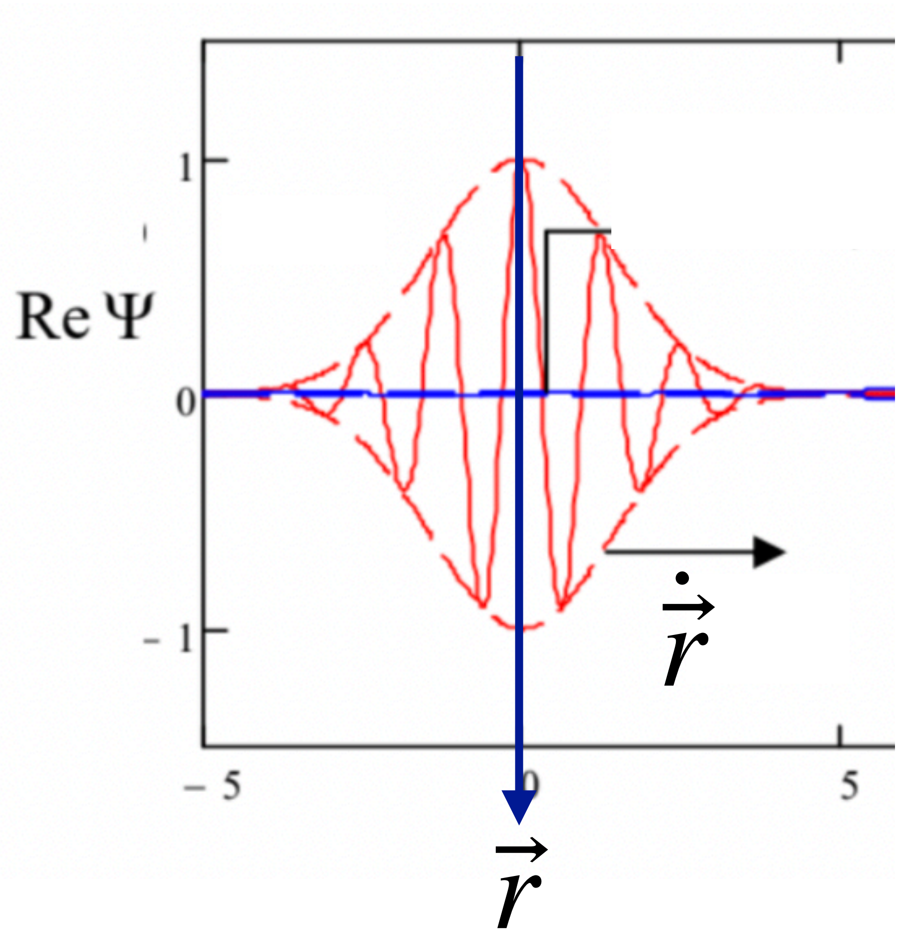
Chern numbers have physical consequences for wave packet motion in the presence of a weak electric field = low frequency driving

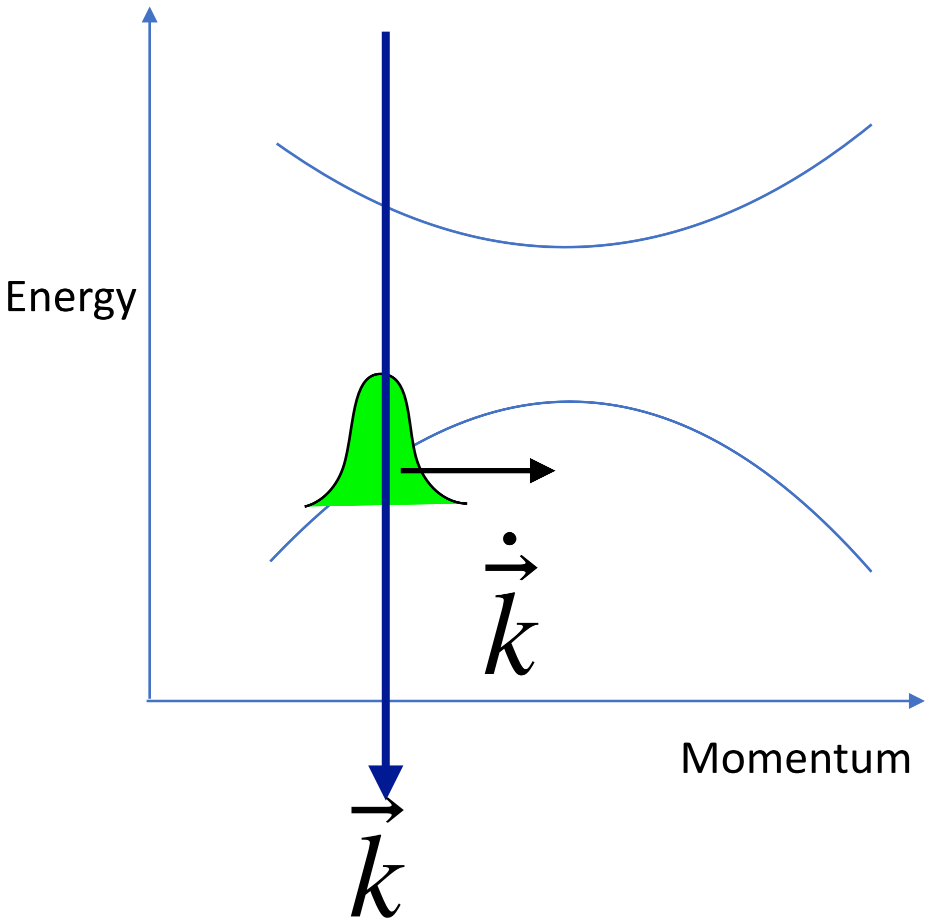


Bands can have Chern numbers!



Wave-packet dynamics





Wave-packet dynamics

$$\hbar \dot{\boldsymbol{k}} = -e\boldsymbol{E} - e\dot{\boldsymbol{r}} \times \boldsymbol{B}$$

Force due to external electric field

Lorentz force due to the external magnetic field

Wave-packet dynamics

Group velocity due to the dispersion of the band

Anomalous velocity due to Berry curvature $\vec{\Omega} = \vec{\nabla} \times \vec{A}_{\vec{k}}$

$$\dot{\boldsymbol{r}} = \frac{\partial \varepsilon_n}{\hbar \partial \boldsymbol{k}} + \Omega \times \dot{\boldsymbol{k}}$$

$$\hbar \dot{\boldsymbol{k}} = -e\boldsymbol{E} - e\dot{\boldsymbol{r}} \times \boldsymbol{B}$$

Force due to external electric field

Lorentz force due to the external magnetic field

$$\vec{A}_{\vec{k}} = \langle u_{n\vec{k}} | i \vec{\nabla}_{\vec{k}} | u_{n\vec{k}} \rangle$$

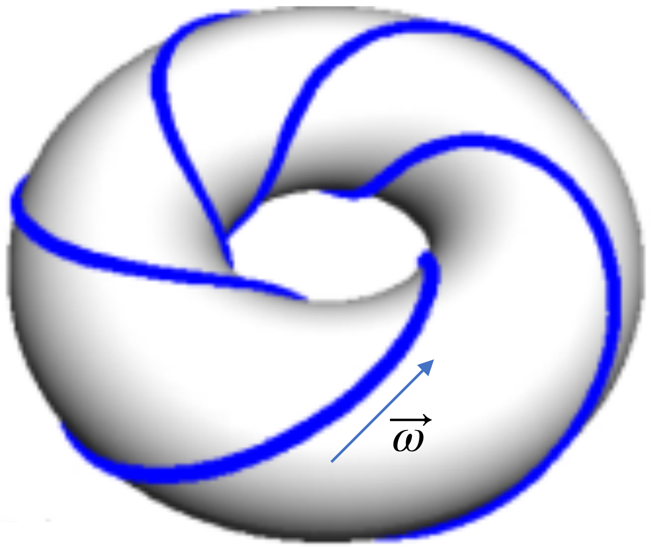
Wave-packet dynamics for the synthetic model

$$\hbar\dot{\boldsymbol{k}} = -e\boldsymbol{E}$$

The electric field $\vec{E} = (\omega_1, \omega_2)$

$$\hbar(k_x, k_y) = (\omega_1 t, \omega_2 t)$$

Brillouin zone



Wave-packet dynamics for the synthetic model

$$\int d^2k \Rightarrow \int dt$$

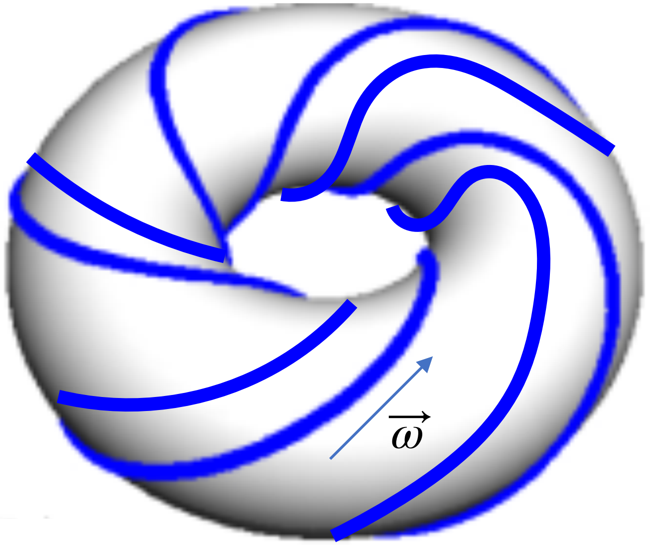
$$\hbar \dot{\boldsymbol{k}} = -e\boldsymbol{E}$$

The electric field $\vec{E} = (\omega_1, \omega_2)$

$$\hbar(k_x, k_y) = (\omega_1 t, \omega_2 t)$$

Later time

Brillouin zone



Wave-packet dynamics for the synthetic model

$$\dot{\boldsymbol{r}} = \frac{\partial \varepsilon_n}{\hbar \partial \boldsymbol{k}} + \Omega \times \dot{\boldsymbol{k}}$$

$$\propto \Omega \times \overrightarrow{E}$$

$$\hbar \dot{\boldsymbol{k}} = -e\boldsymbol{E} -$$

Motion of the wave packet transverse to the electric field

As Ω changes with $\boldsymbol{k}$, this transverse velocity changes in time

Wave-packet dynamics for the synthetic model

$$\dot{\boldsymbol{r}} = \qquad + \Omega \times \dot{\boldsymbol{k}}$$

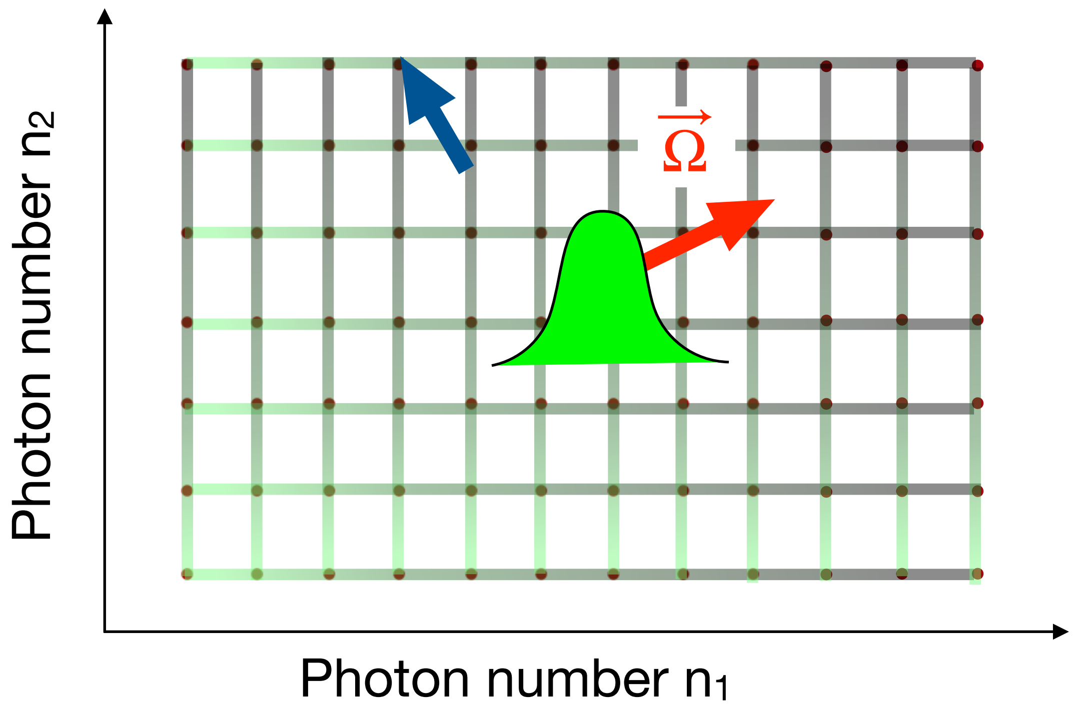
$$\propto \Omega \times \vec{E}$$

Motion of the wave packet
transverse to the electric field

Chern number => $\int d^2k\Omega$ is quantized

=> $\frac{1}{T}\int_0^T dt$ (transverse velocity) is quantized

Energy pump from the synthetic quantum Hall effect

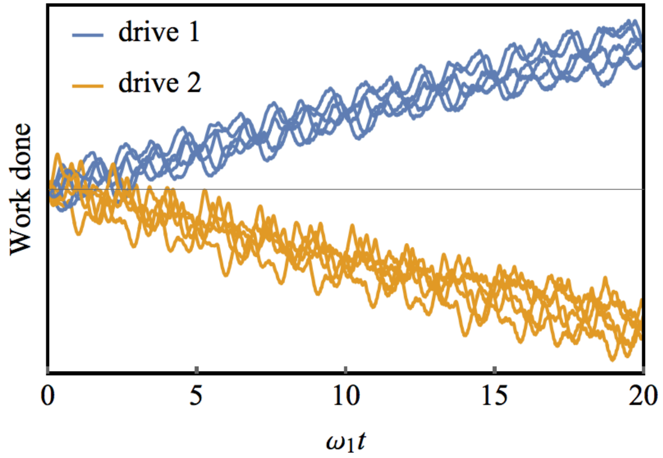


$$\overline{\dot{n}_2} = C/T_1$$

"Topological frequency conversion"

Chern number

$$\overline{P_2} = -\,\omega_2 \overline{\dot{n}_2}$$



TKNN, 1982

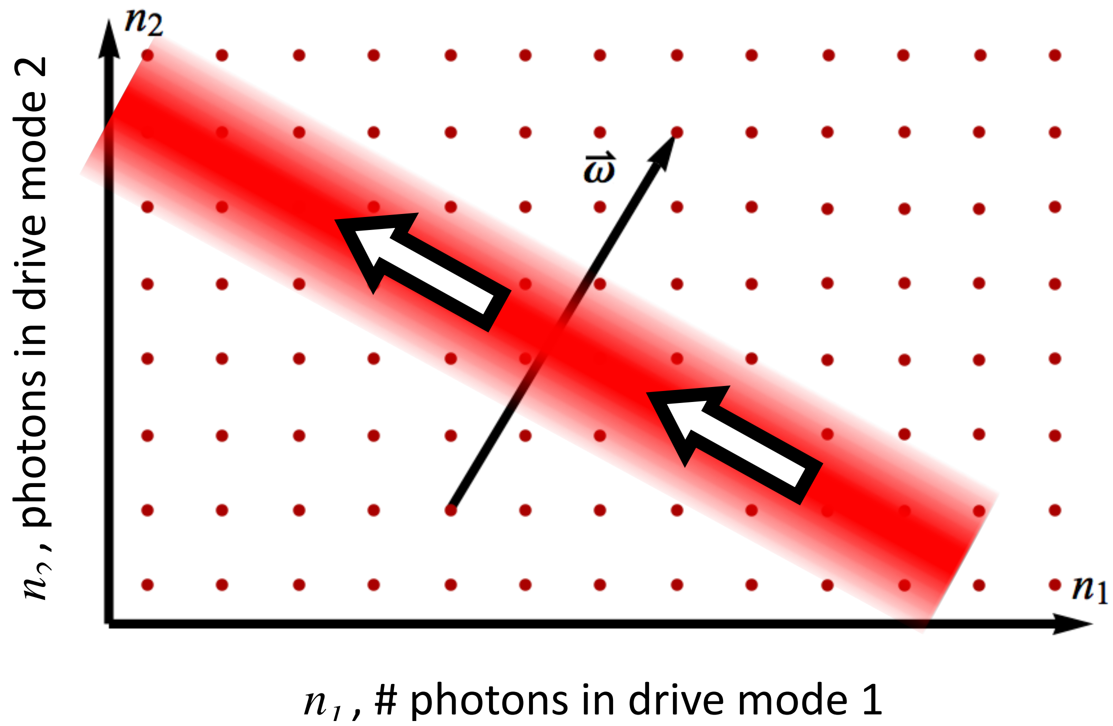
Martin, Refael, Halperin (2017)

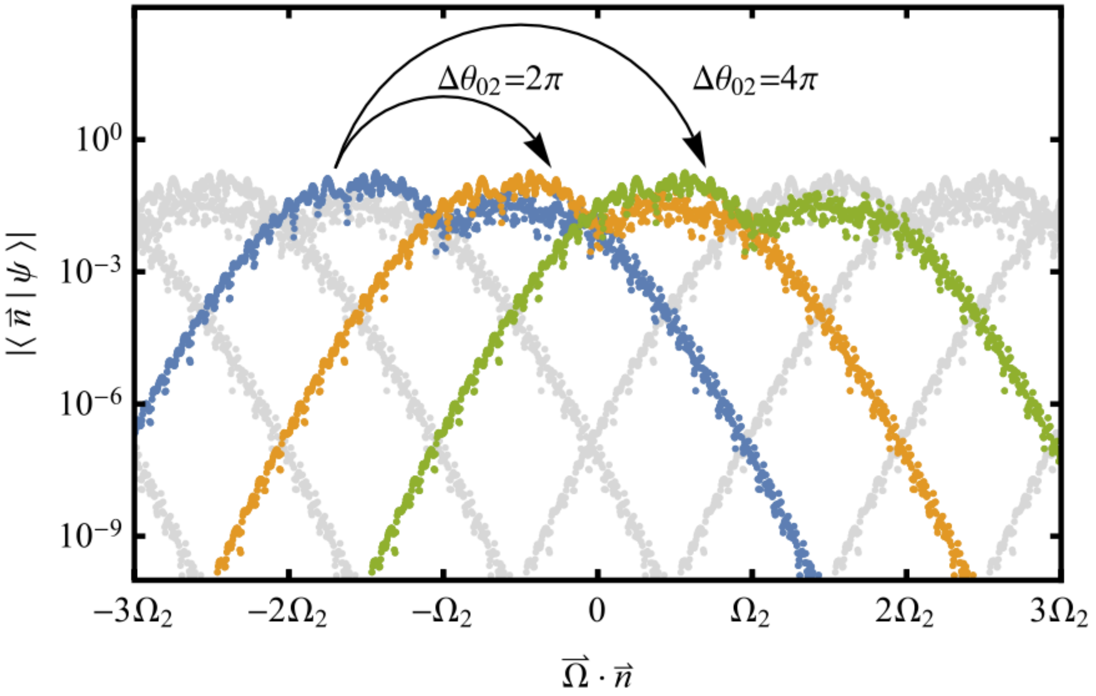
Crowley, Martin, AC (2018)

Invariants with a finite electric field

By flux threading

Flux threading of 2π moves the quasi-energy state up or down the Stark ladder by C units





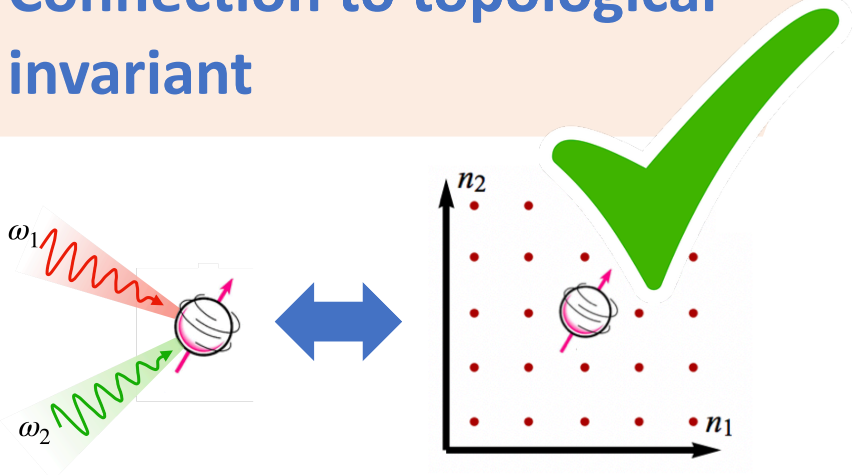
Crowley, Martin, AC (2018)

Questions?

①

Quantized energy pump

Connection to topological invariant



Interesting application: Cavity quantum state Boosting

- Method to produce non classical states of light
  - Fock states
  - Cat states
  - …

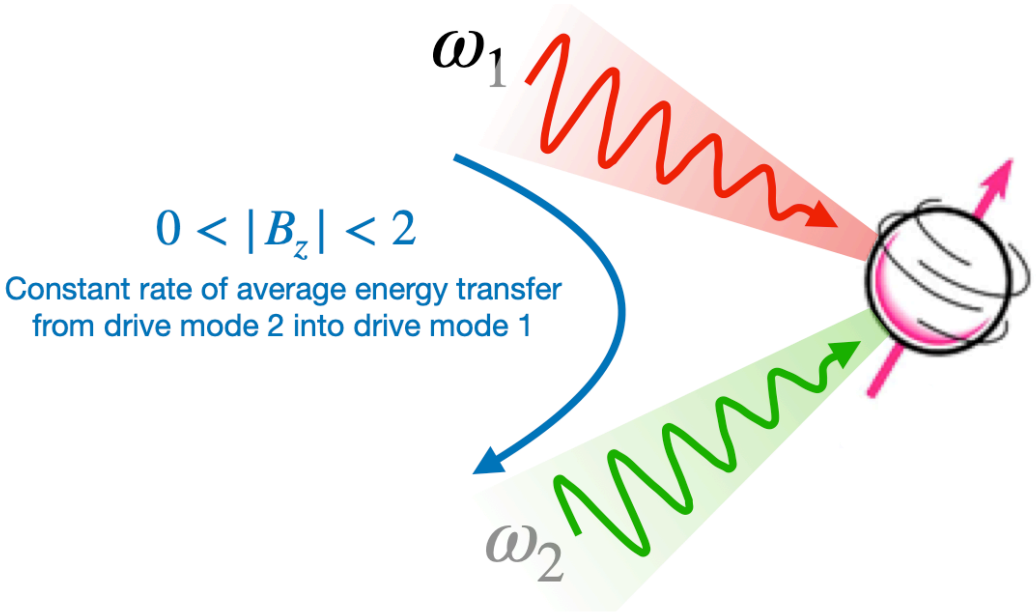
$$|0\rangle \to |n\rangle$$

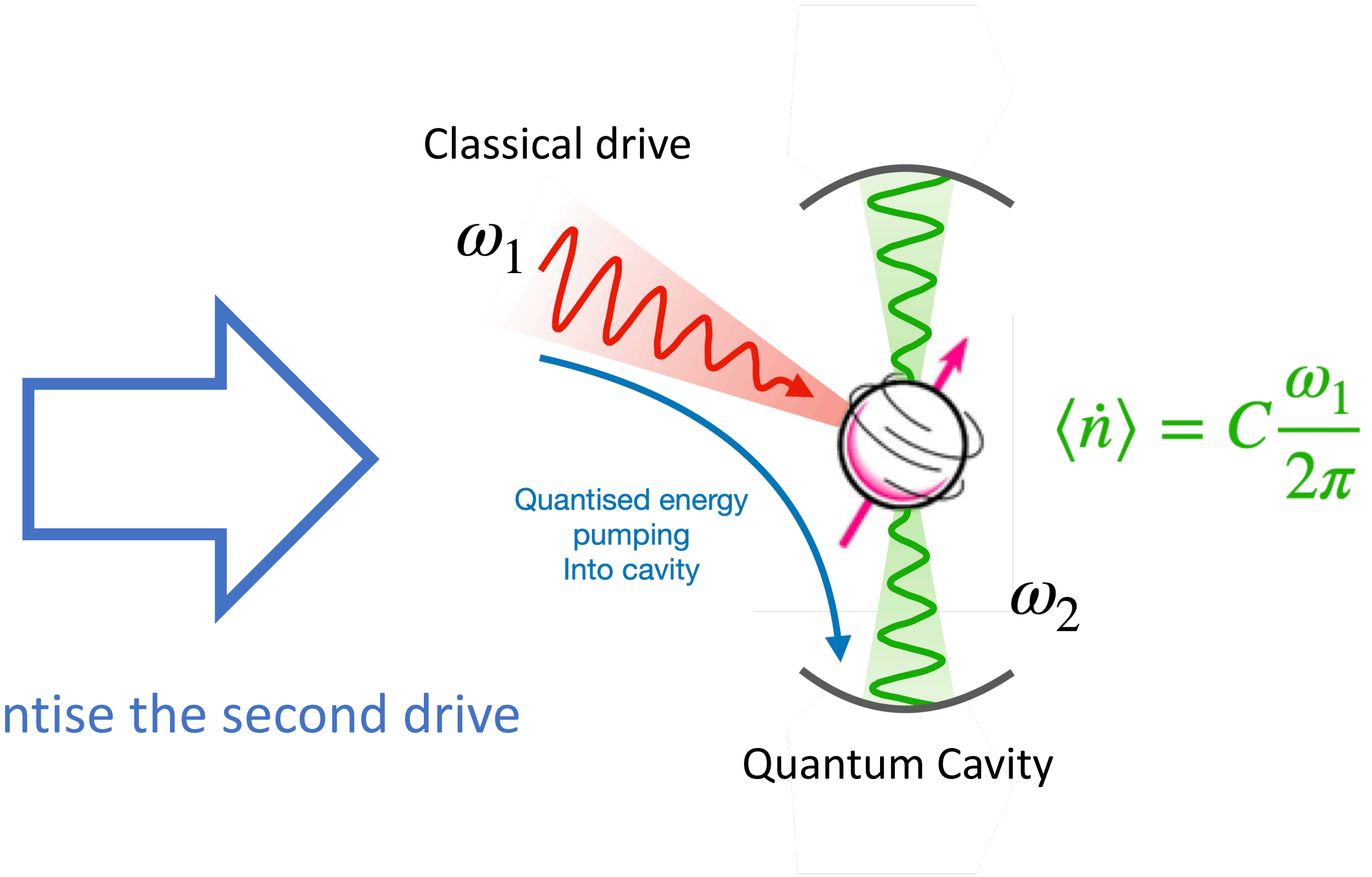
- Non classical states of light are a quantum resource

  Review: Gilchrist et al (J. Opt. B 2004)

  - Quantum metrology
  - Universal photonic quantum computation

Cavity quantum state Boosting





Quantise the second drive

Nathan, Martin, Refael, PRB, 2018, Long, Crowley, Kollar, Chandran, PRL, 2022

Cavity quantum state Boosting

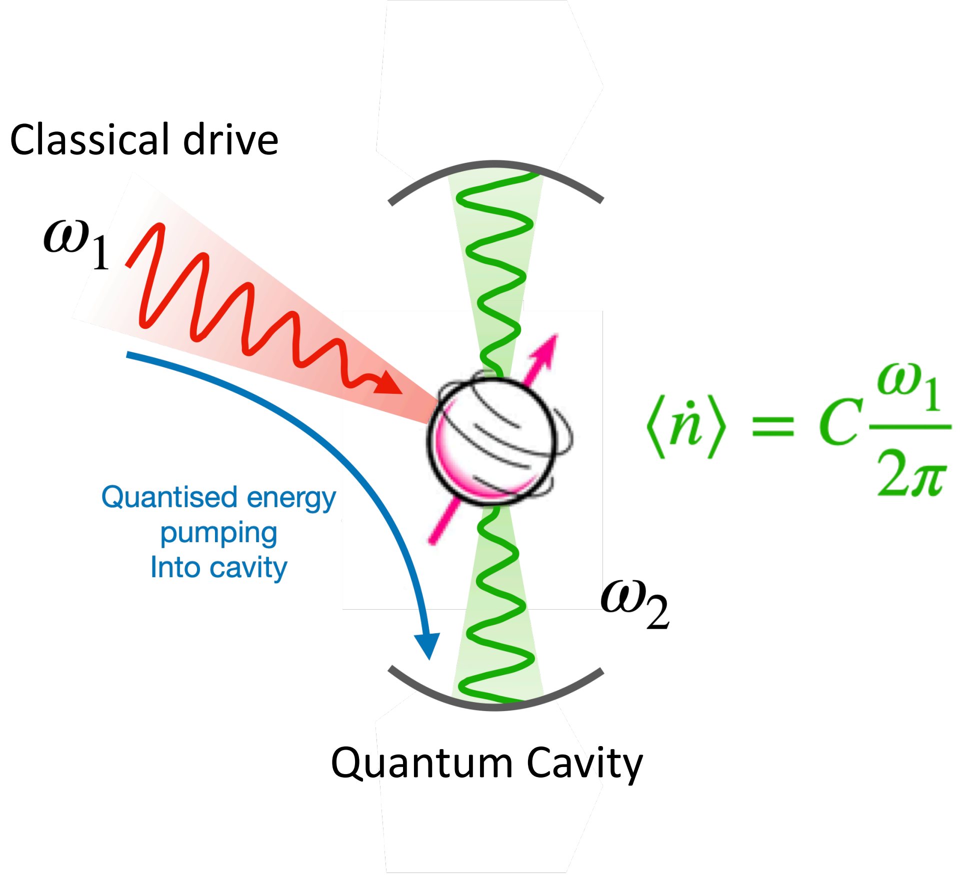
Initial state:

$$|\psi(0)\rangle = |\rightarrow\rangle \sum_n c_n |n\rangle$$

Special times T_m

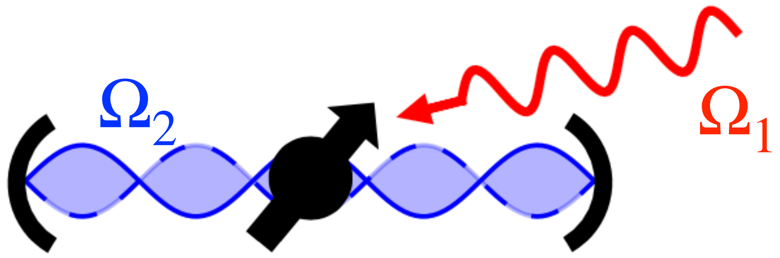
$$|\psi(T_m)\rangle \approx |\rightarrow\rangle \sum_n c_n |n + \langle \dot{n} \rangle T_m \rangle$$

Cavity state is coherently boosted!



Long, Crowley, Kollar, AC, PRL, 2022

Hamiltonian



$$H = \Omega_2 \hat{b}^\dagger \hat{b} + \quad B_0(\hat{b} S^+ + \mathrm{h.c.}) - \quad \overrightarrow{B}_c(t) \cdot \vec{S}$$

Cavity energy — J-C interaction — Slow classical drive

$$\overrightarrow{B}_{\mathrm{c}}(t) = (B_m + B_d \cos \Omega_1 t)\hat{x} + B_d \sin(\Omega_1 t)\hat{z}$$

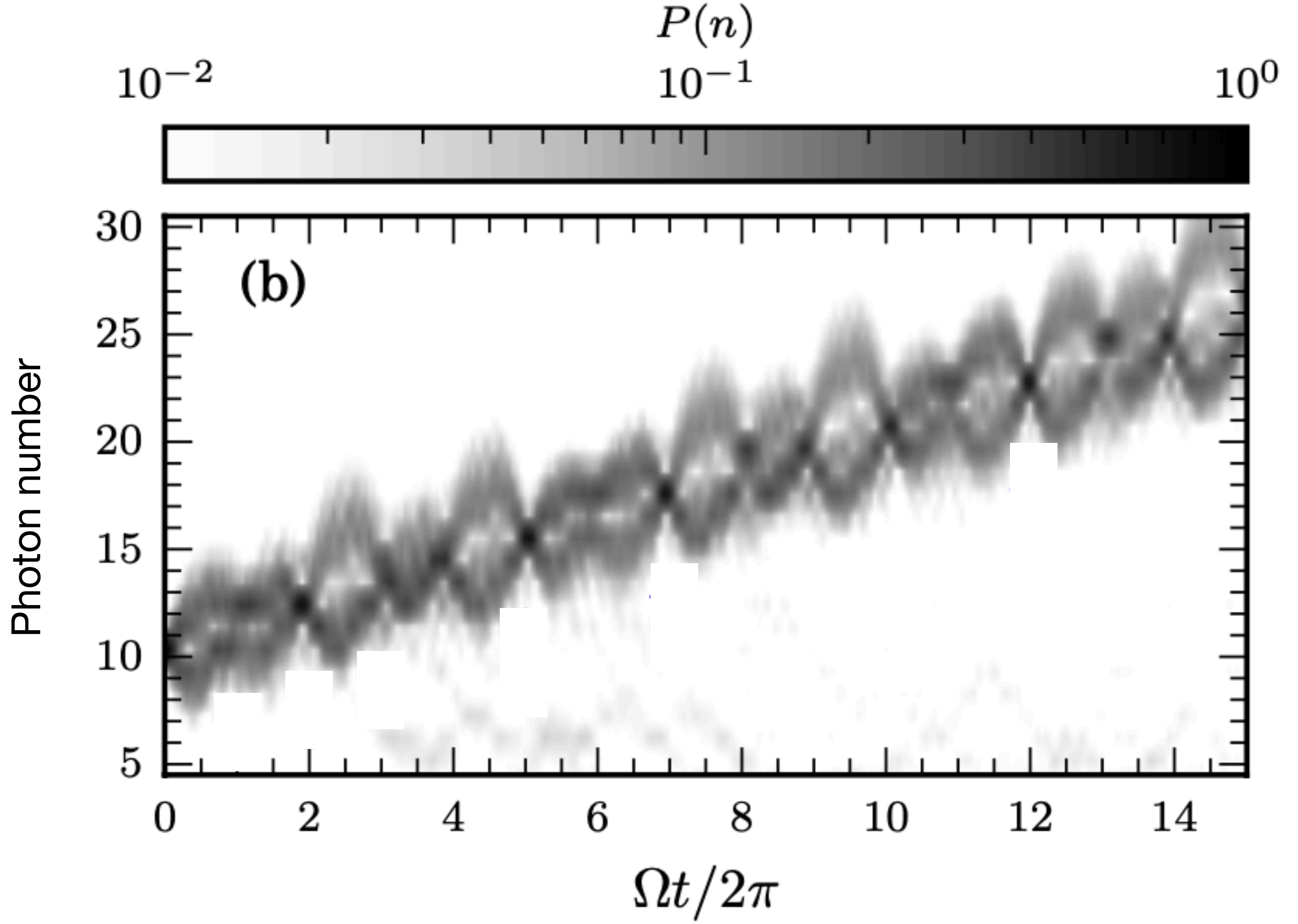
Adiabatic limit: $\Omega_{1,2} \ll B_0\sqrt{n}, B_m, B_d$

Incommensurate: $\Omega_1/\Omega_2 \notin \mathbb{Q}$

Long, Crowley, Kollar, AC (2021)

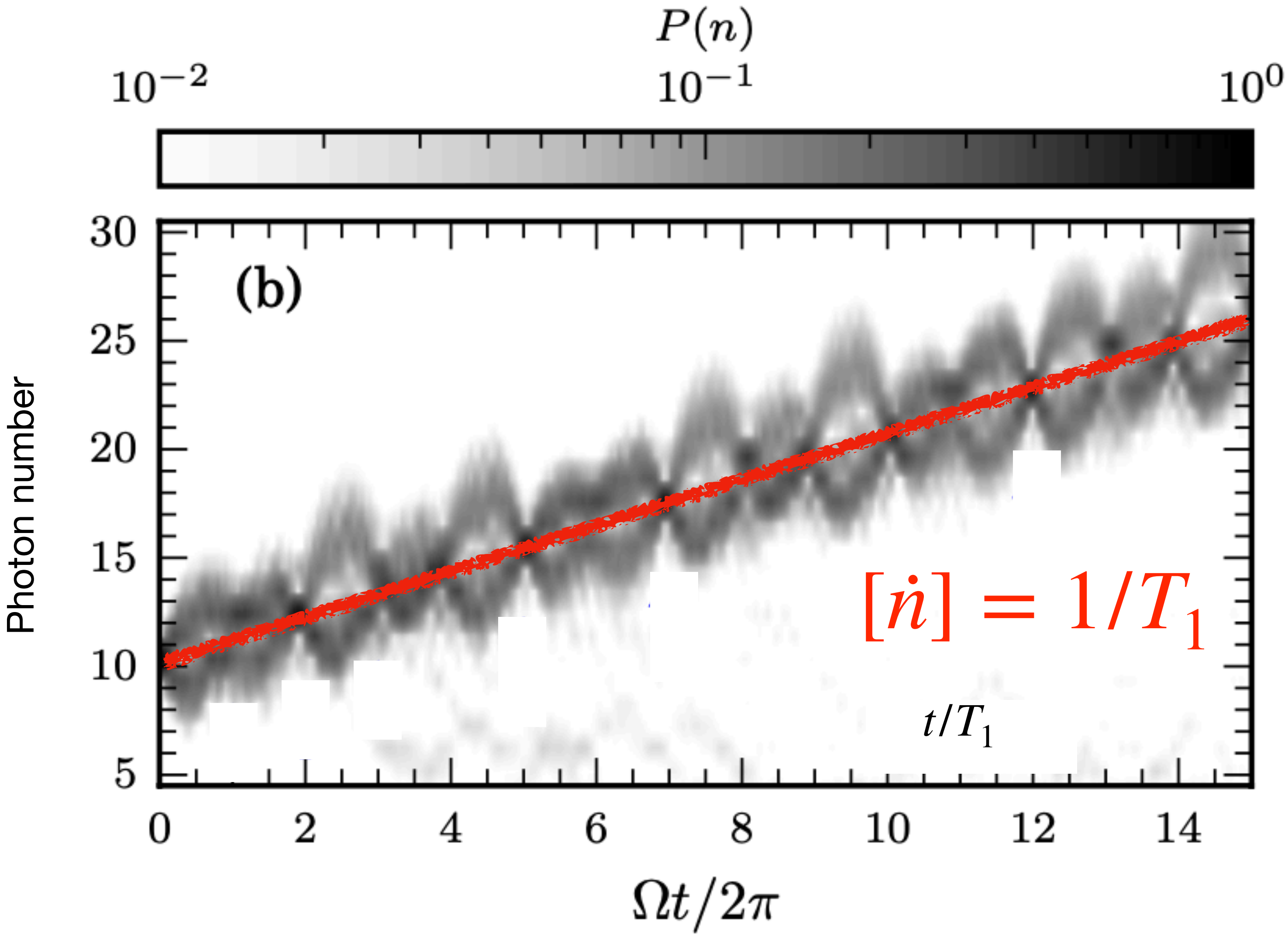
See also: Nathan et al (2021)

A numerical simulation showing boosting



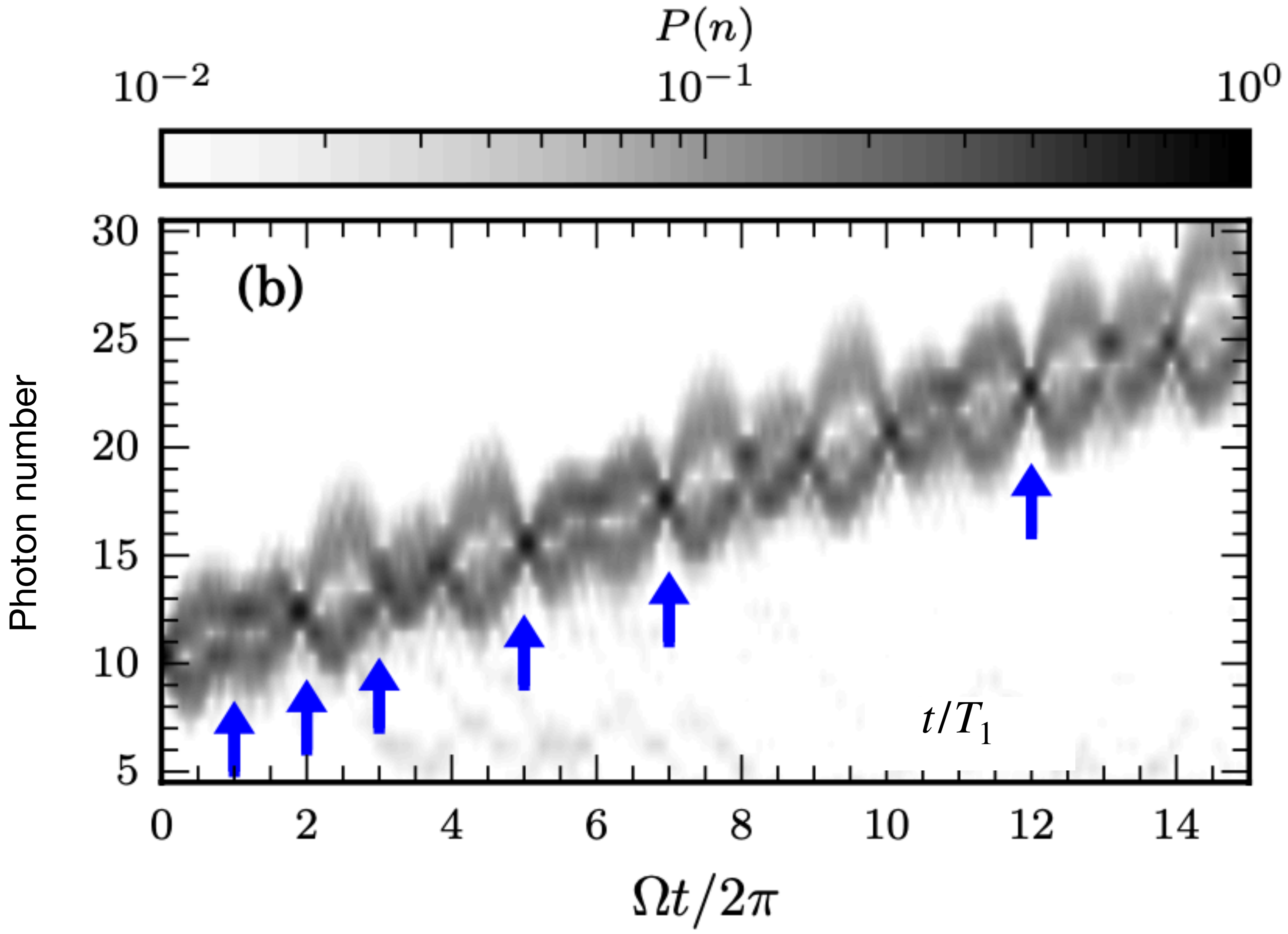
Initialize $|\psi(t=0)\rangle = |\rightarrow\rangle|10\rangle$

Energy pump in the cavity limit



Nathan et al (2021)

Cavity state boosting



Blue arrows: theoretically predicted rephasing times

Nathan et al (2021)

Why cavity state boosting

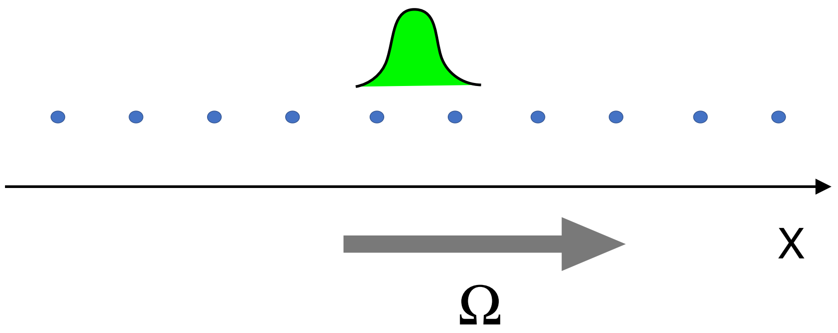
- Because of Bloch oscillations of the wave packet along the electric field

Causes oscillatory motion along the direction of the electric field in a tight-binding model

$$\dot{\boldsymbol{r}} = \frac{\partial \varepsilon_n}{\hbar \partial \boldsymbol{k}} + \Omega \times \dot{\boldsymbol{k}}$$

$$\hbar \dot{\boldsymbol{k}} = -e\boldsymbol{E} -$$

Reminder: Bloch oscillations in d=1



$$\dot{\boldsymbol{r}} = \frac{\partial \varepsilon_n}{\hbar \partial \boldsymbol{k}}.$$

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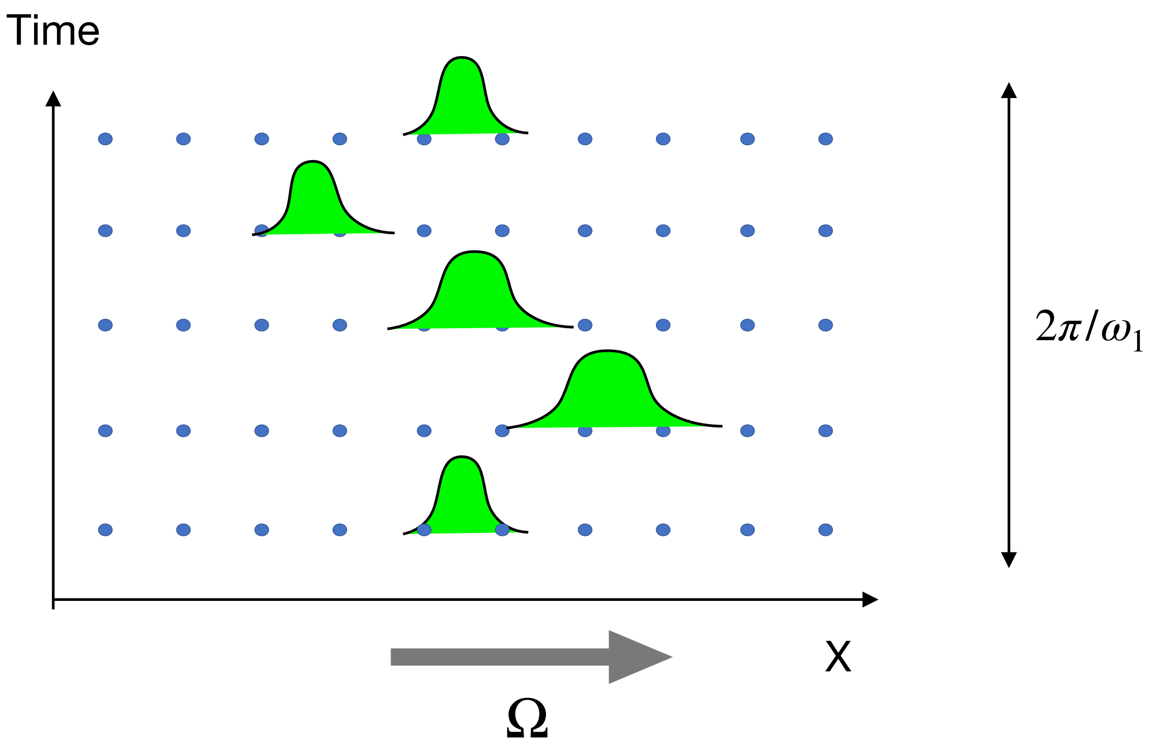
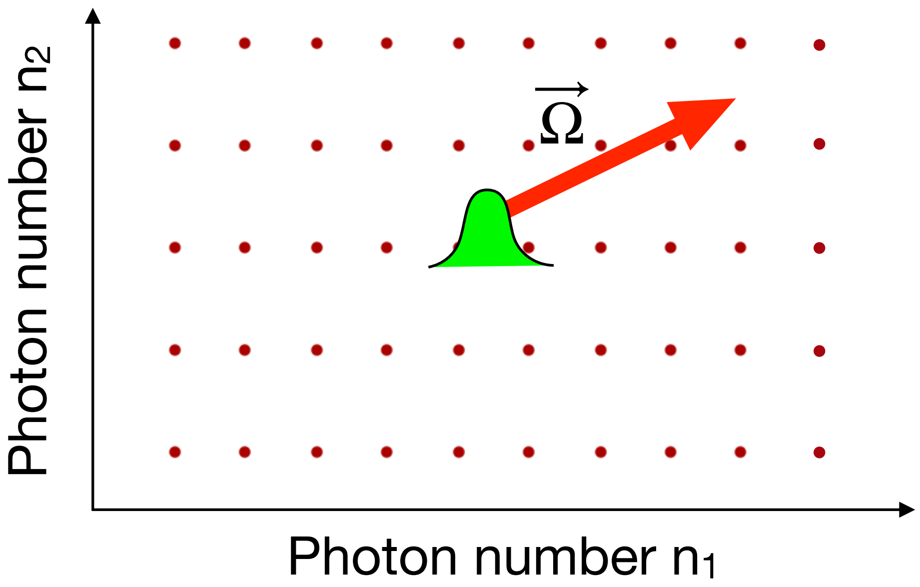
Reminder: Bloch oscillations in d=1



1) Wavepacket oscillates

2) Wavepacket breathes

Bloch oscillations in 2d

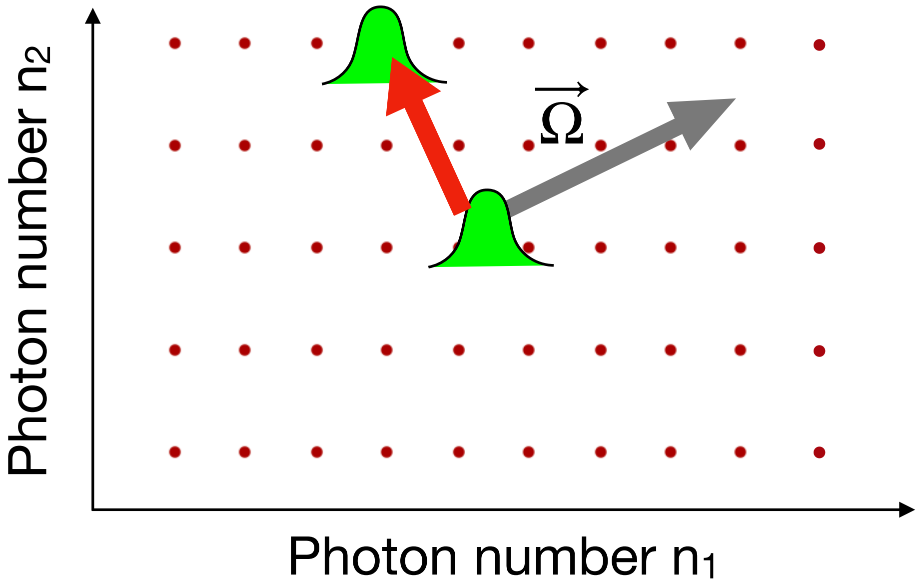


Almost periods T_n

$$\omega_1 T_N \approx 2\pi, \omega_2 T_N \approx 2\pi$$

Wavepacket rephases

Bloch oscillations in 2d



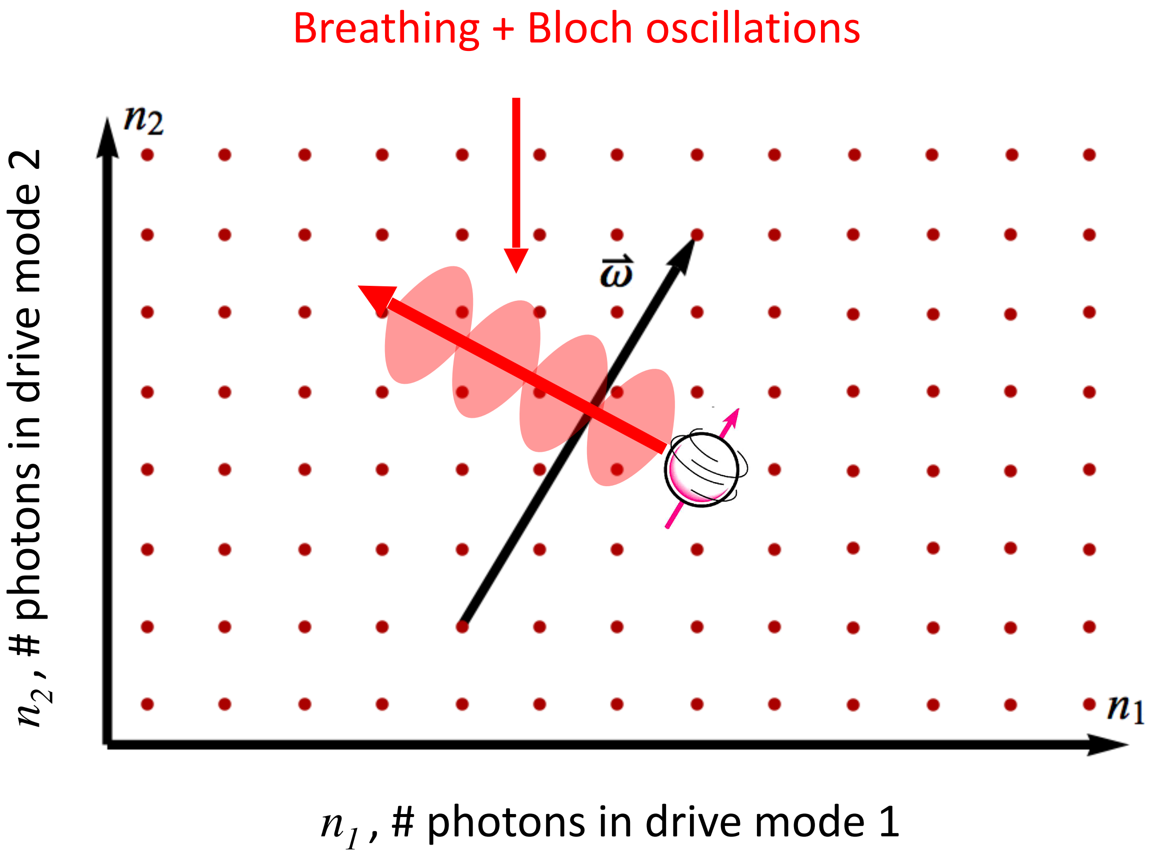
Almost periods T_n

$$\omega_1 T_N \approx 2\pi, \omega_2 T_N \approx 2\pi$$

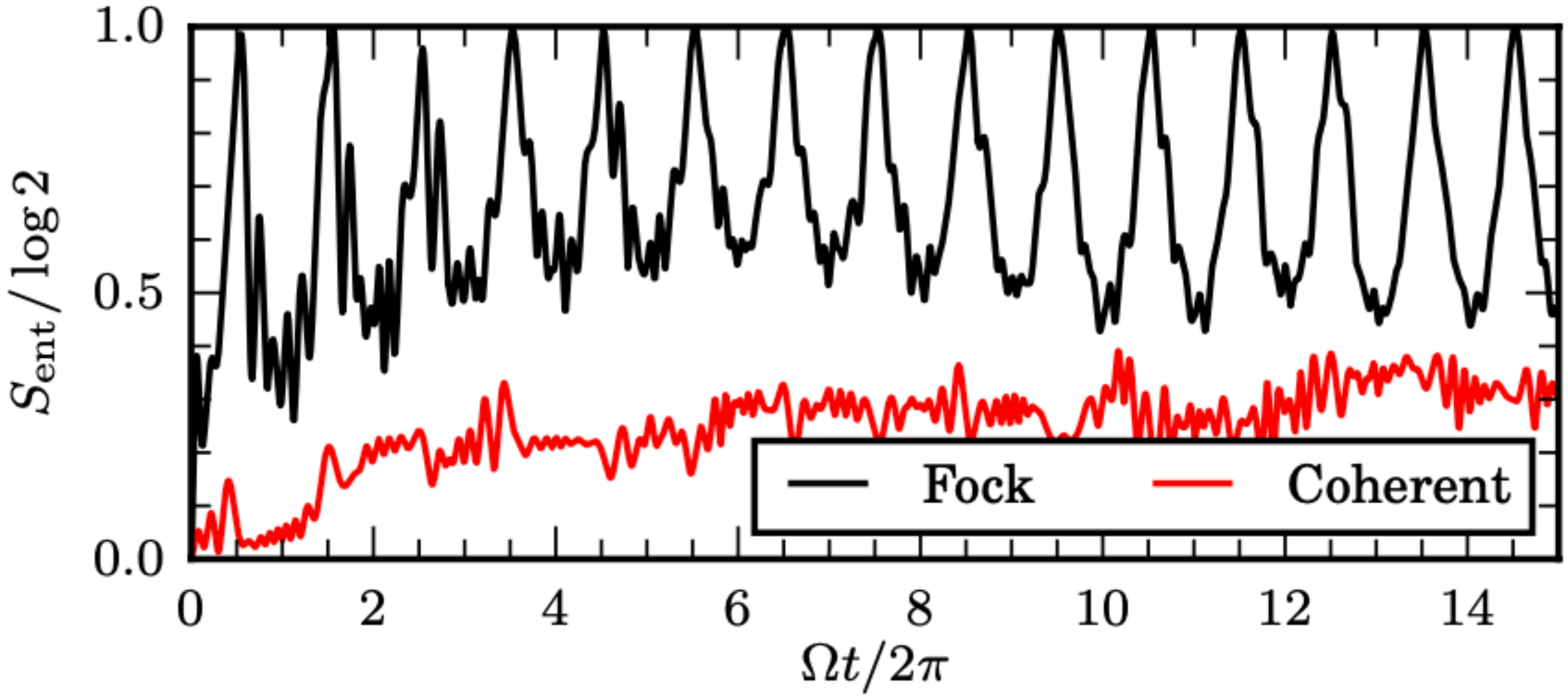
Wavepacket rephases

Anomalous velocity ⇒ wave packet is boosted!

Cavity quantum state Boosting



An aside: entanglement between qubit and cavity

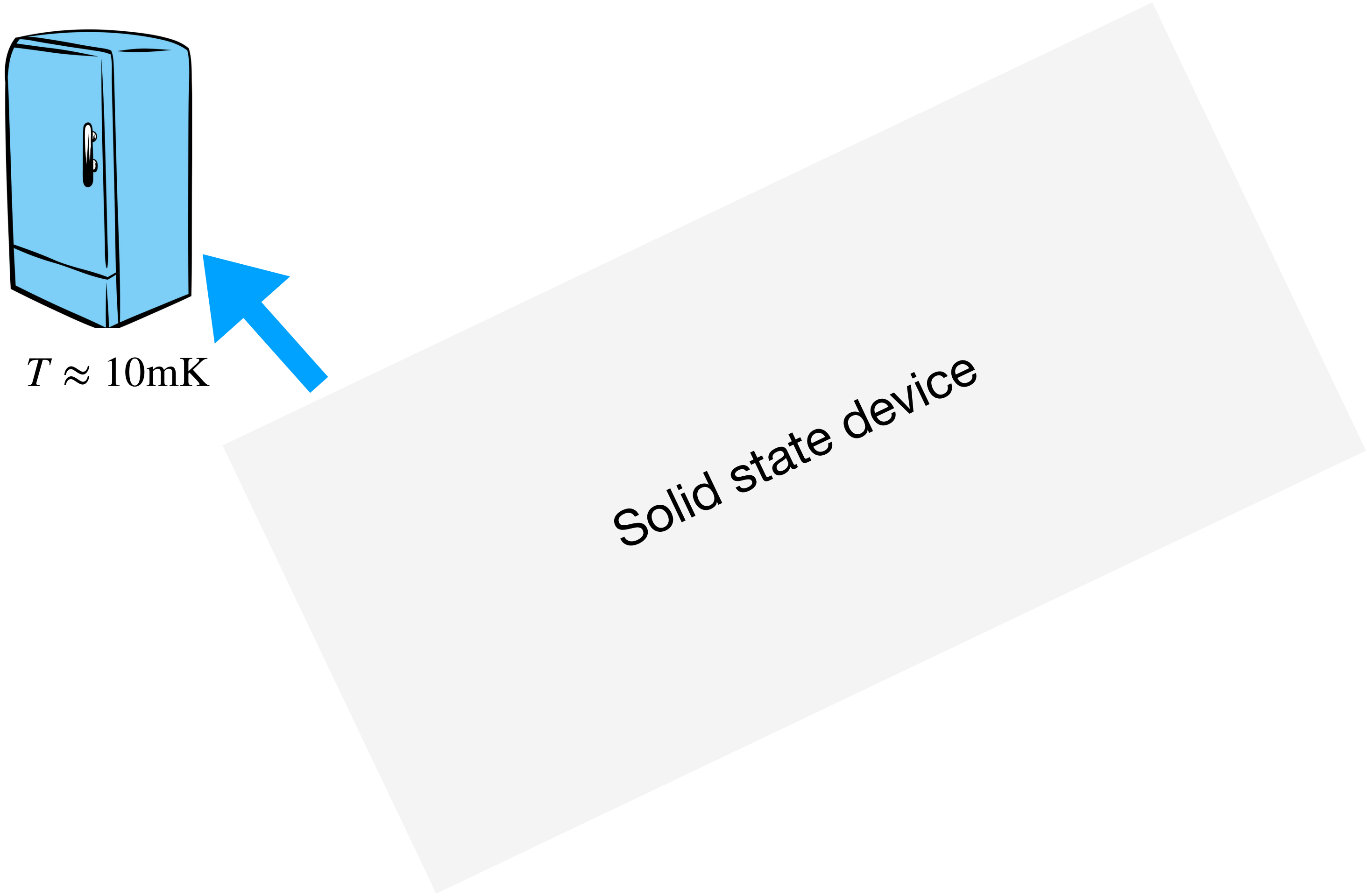


Observing cavity state boosting

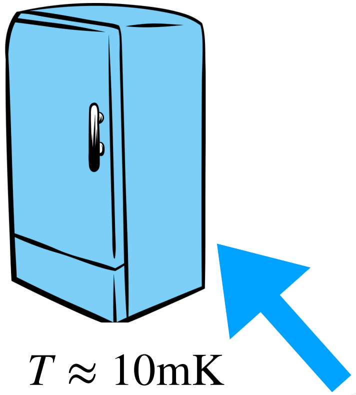
Ongoing: Martin Ritter + Kollar group, D. Long

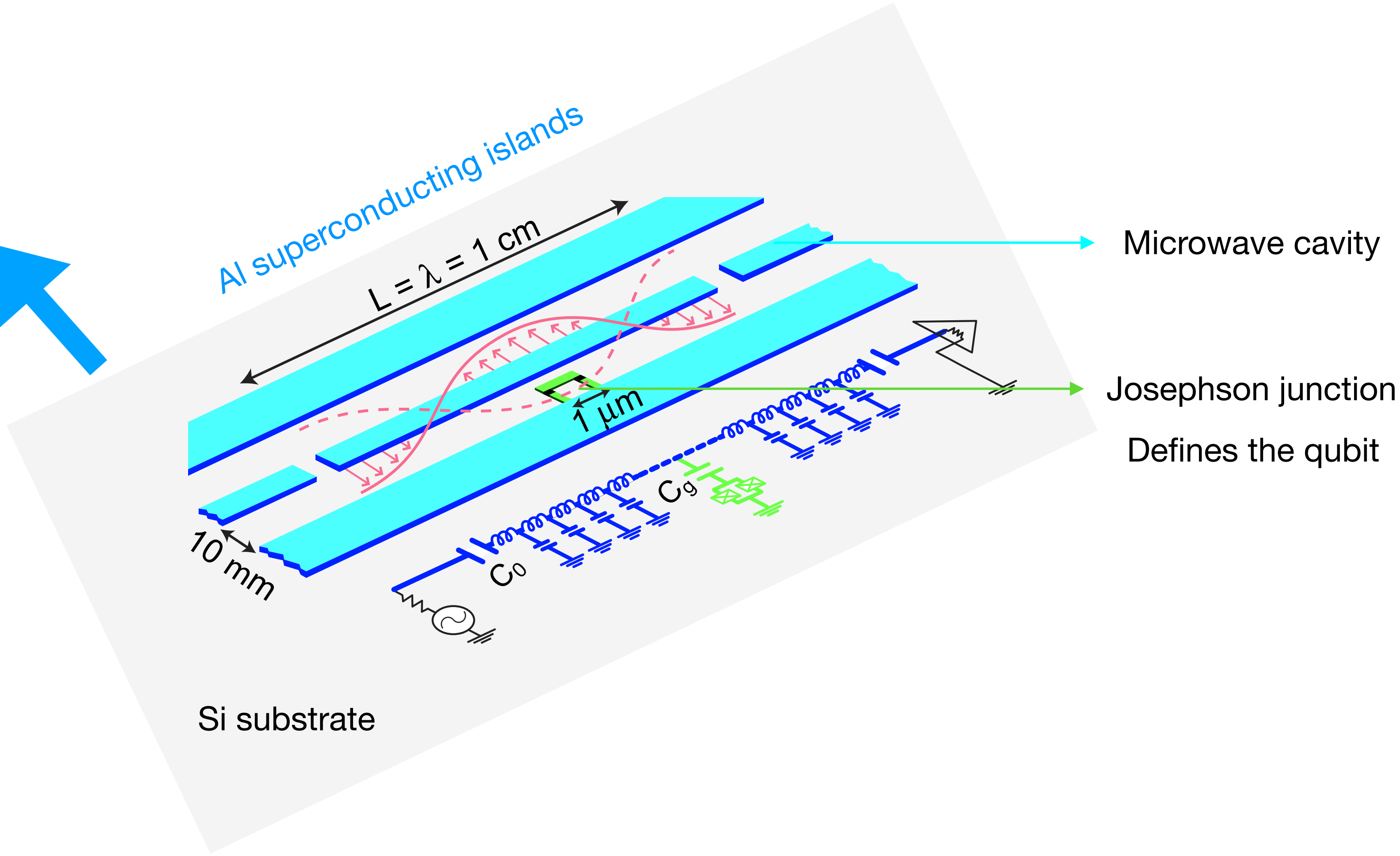
Superconducting circuit-QED architecture

Basics of the architecture



Basics of the architecture





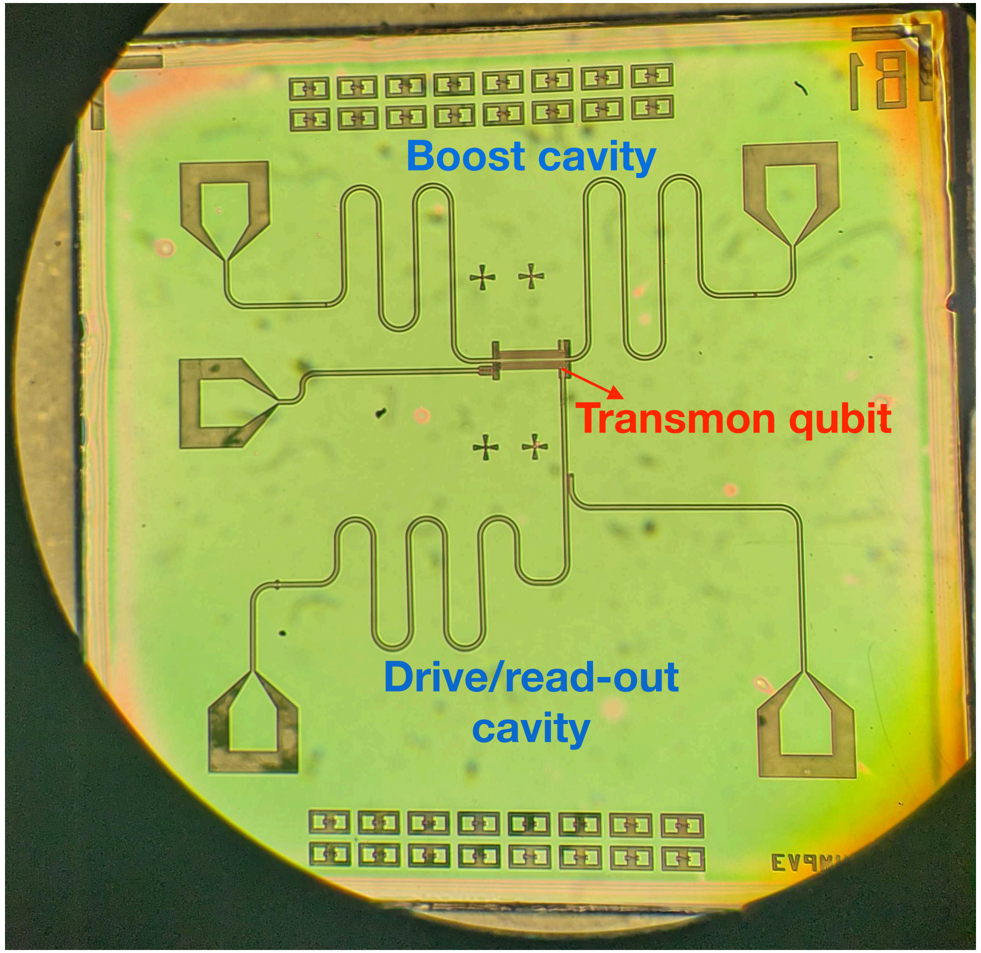
Blais *et al.*, PRA **69**, 062320 (2004)

Chip for cavity state boosting

- Boost cavity: 4.8 GHz
    - Quality factor ~ 100,000
- Readout cavity: 7.4 GHz
- Qubit: max frequency 6.5 GHz
- Rabi coupling: ~50 MHz

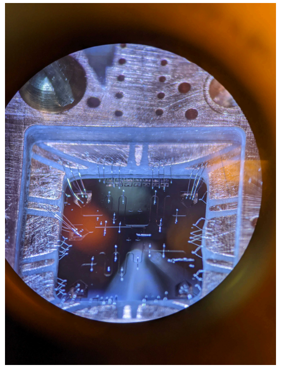
$$H = \Omega_2 \hat{b}^\dagger \hat{b} + \quad B_0(\hat{b}S^+ + \mathrm{h.c.}) - \quad \overrightarrow{B}_c(t) \cdot \vec{S}$$

Realize in a rotating frame



The Kollar group

Observing cavity state boosting

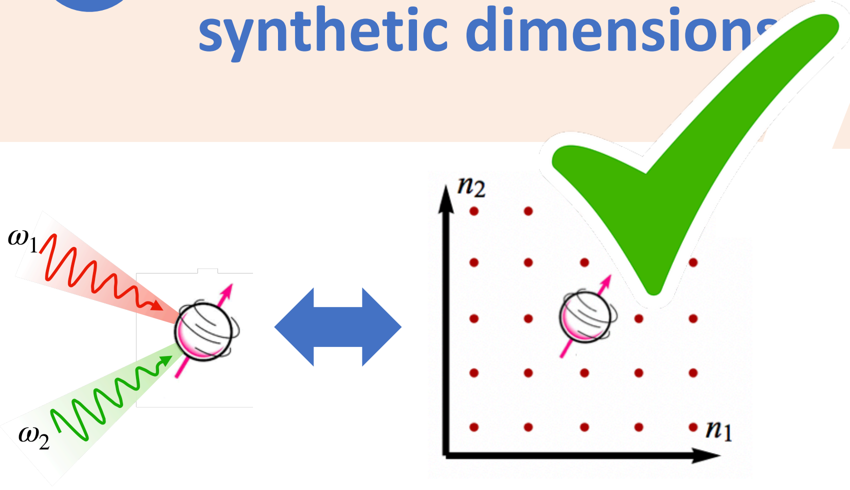


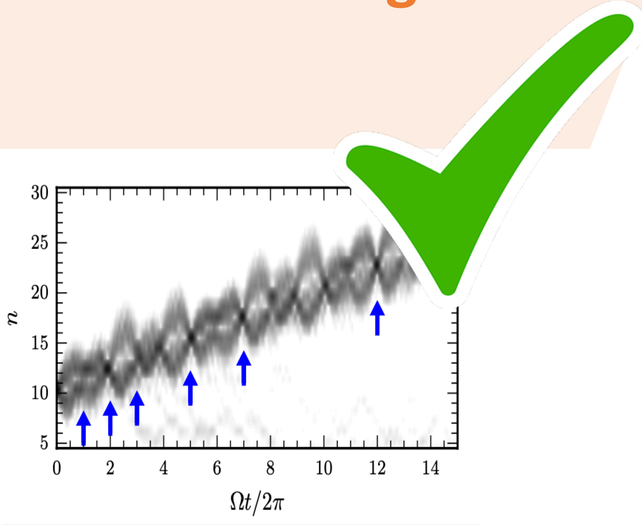
- ☑ High Q boost-cavity (~100,000)
- ☑ Low direct cross-talk between cavities
- ☑ Slow drives on qubit

Questions?

① **Theory: quantised energy pumping & synthetic dimensions**

② **Applications: Cavity state boosting**



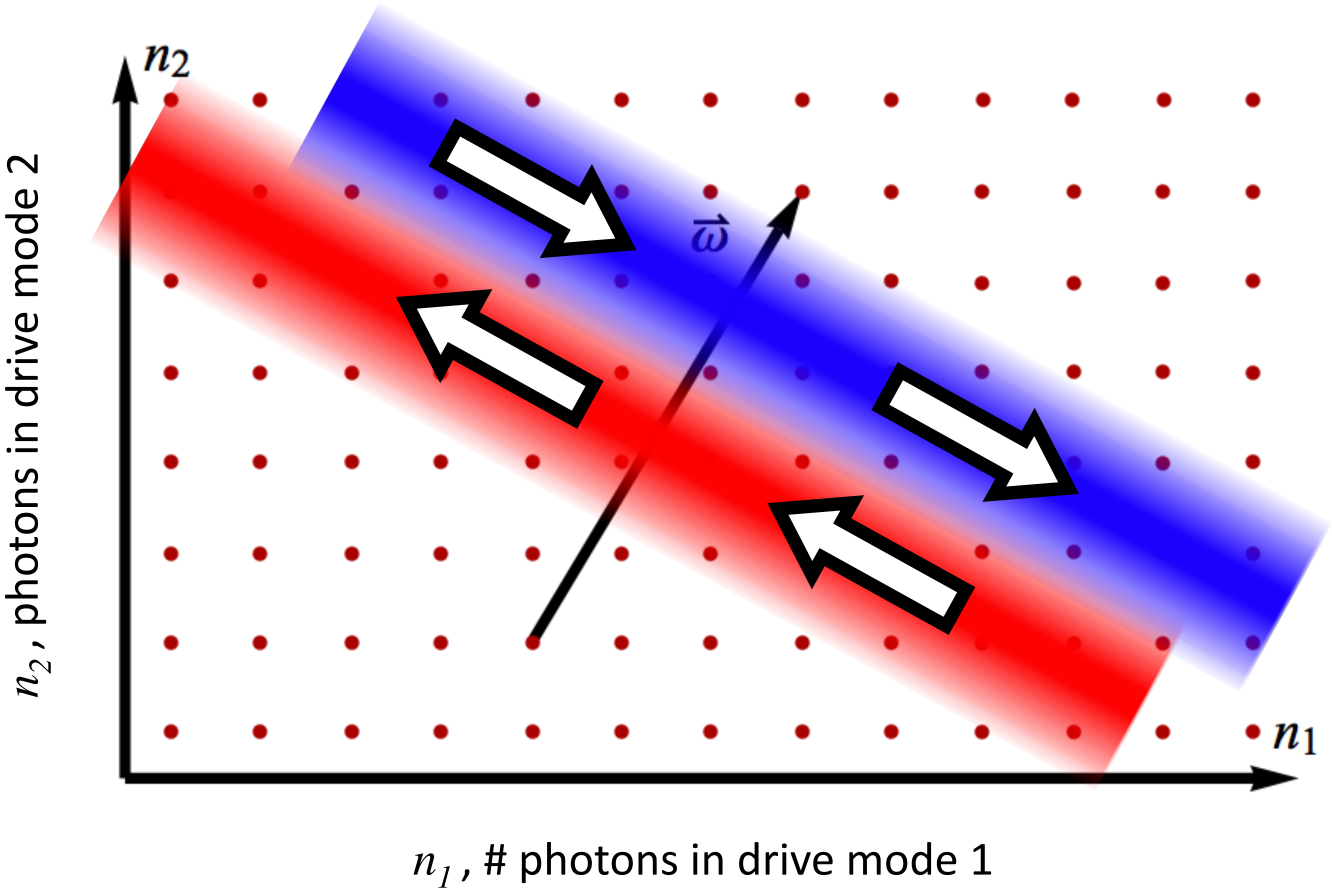


New driven phases of matter

Topological pumping effect + Spatially extended systems = Robust pumping away from the low frequency limit

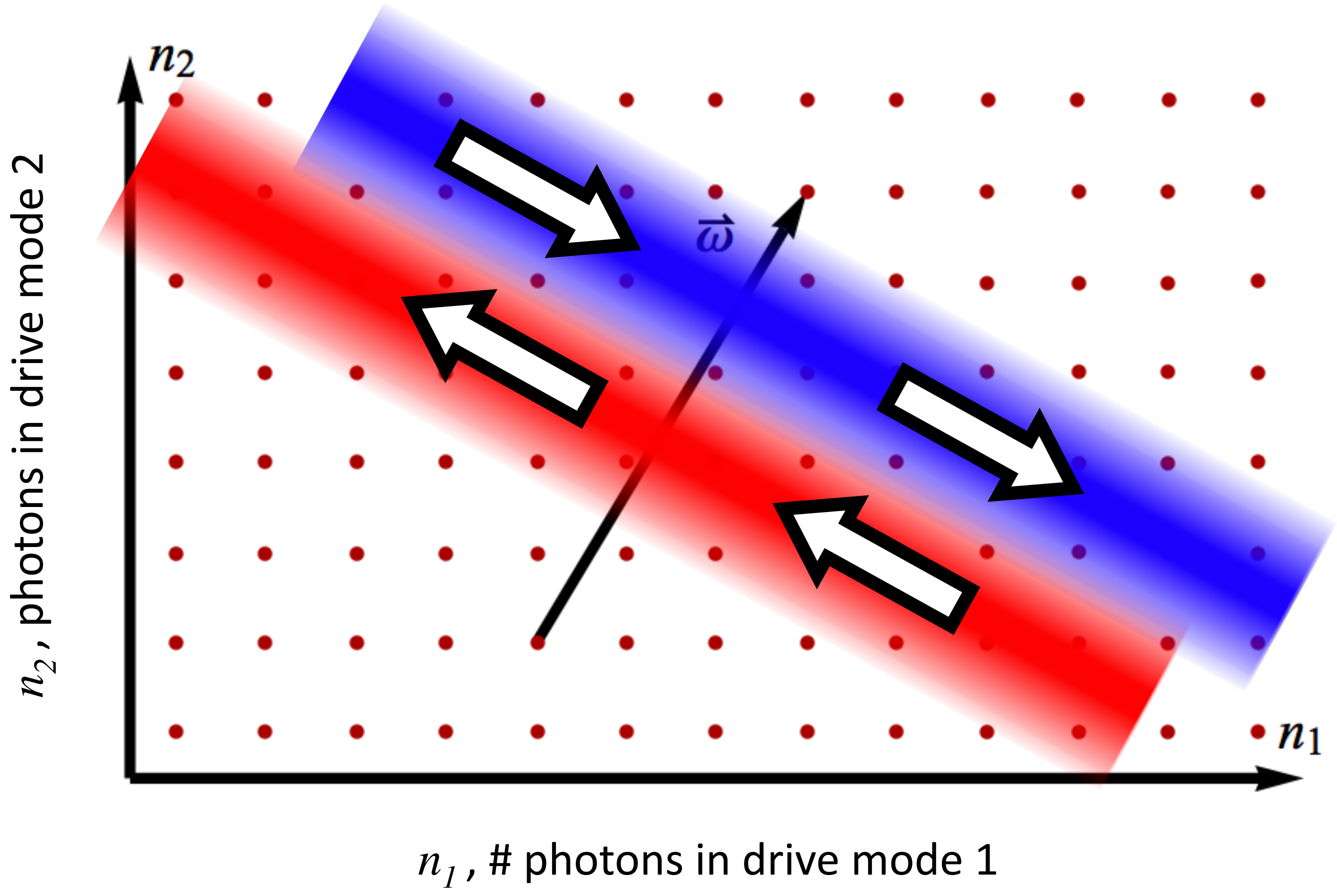
Quasi-energy states

In order that there be energy pumping between the drives, the quasi-energy states have to be delocalized and chiral on the synthetic lattice.



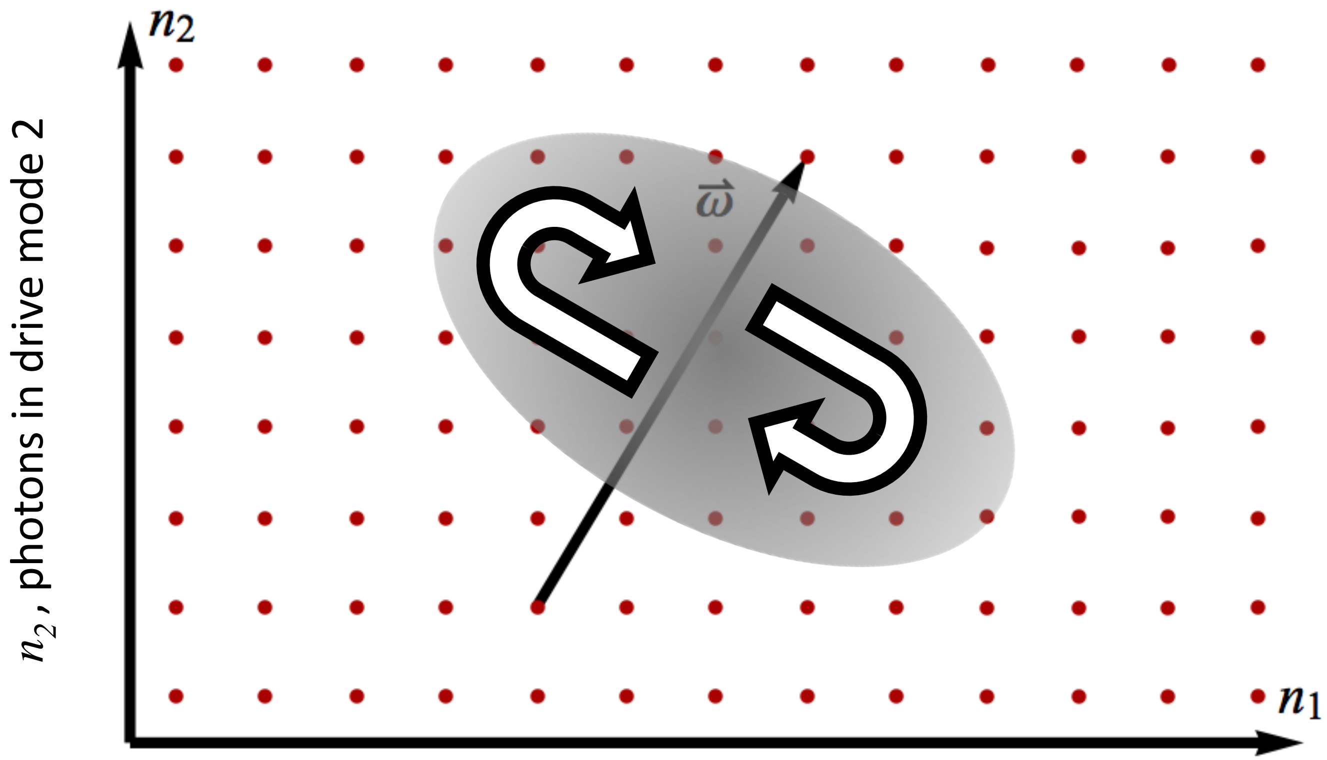
Quasi-energy states

But this situation is unstable because any perturbation would couple the two states.



Localized quasi-energy states generically

Localization => Energy pumping cannot proceed indefinitely

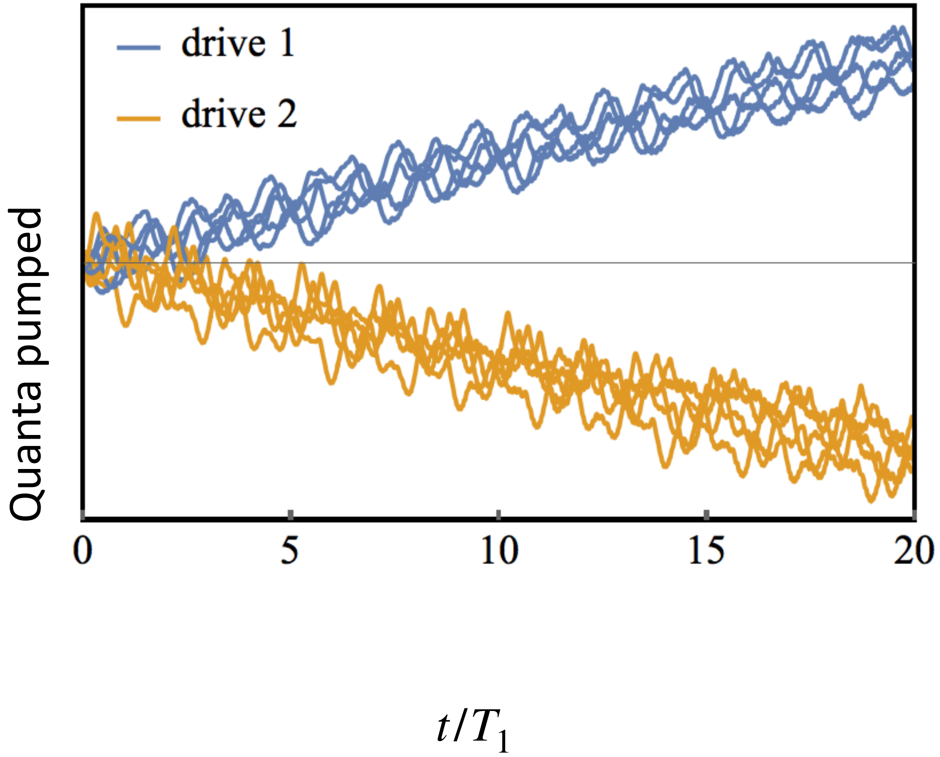


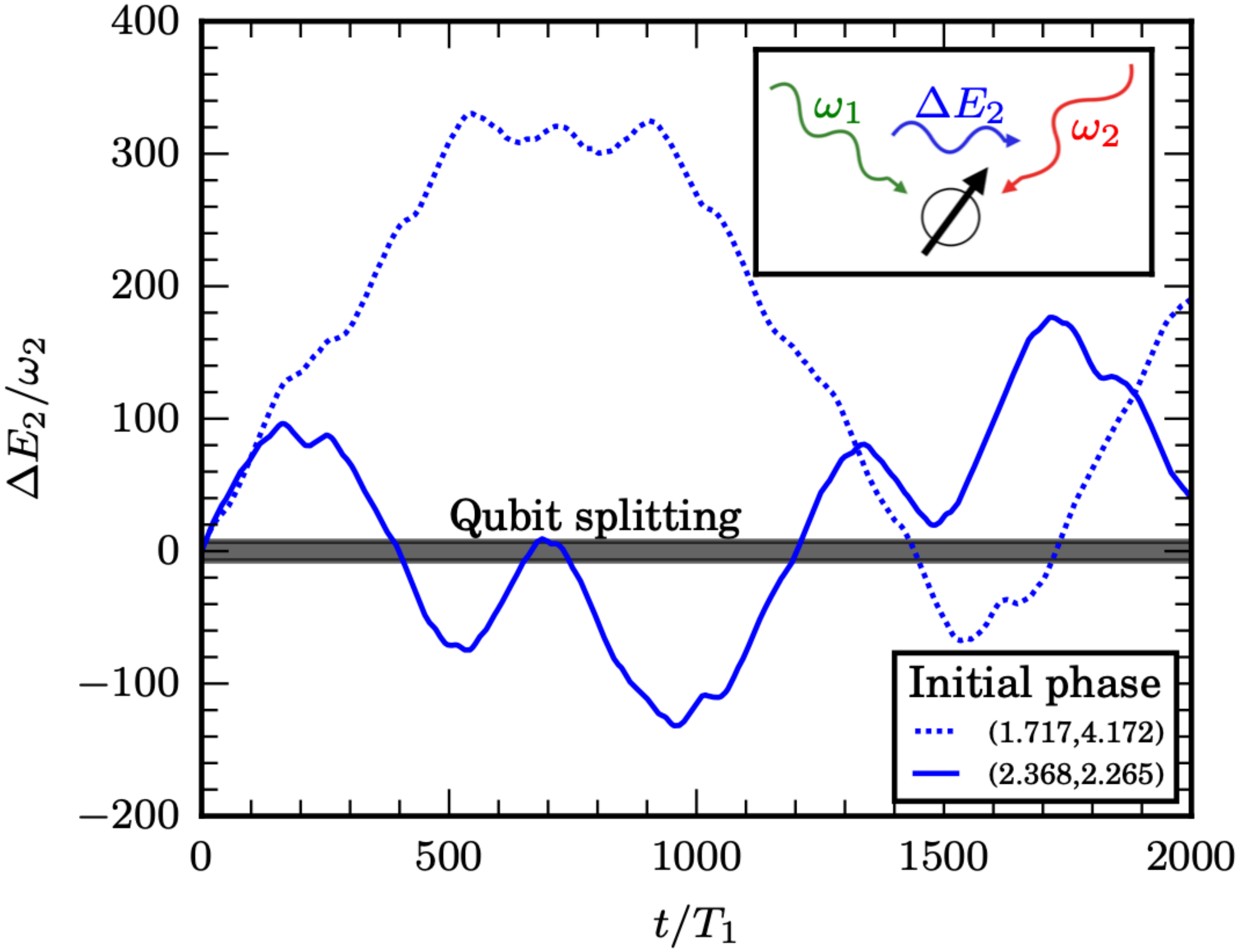
n_1, # photons in drive mode 1

$$\tau_{\text{pump}} \sim e^{B/\omega}$$

Pumping is pre-thermal

Giant energy oscillations

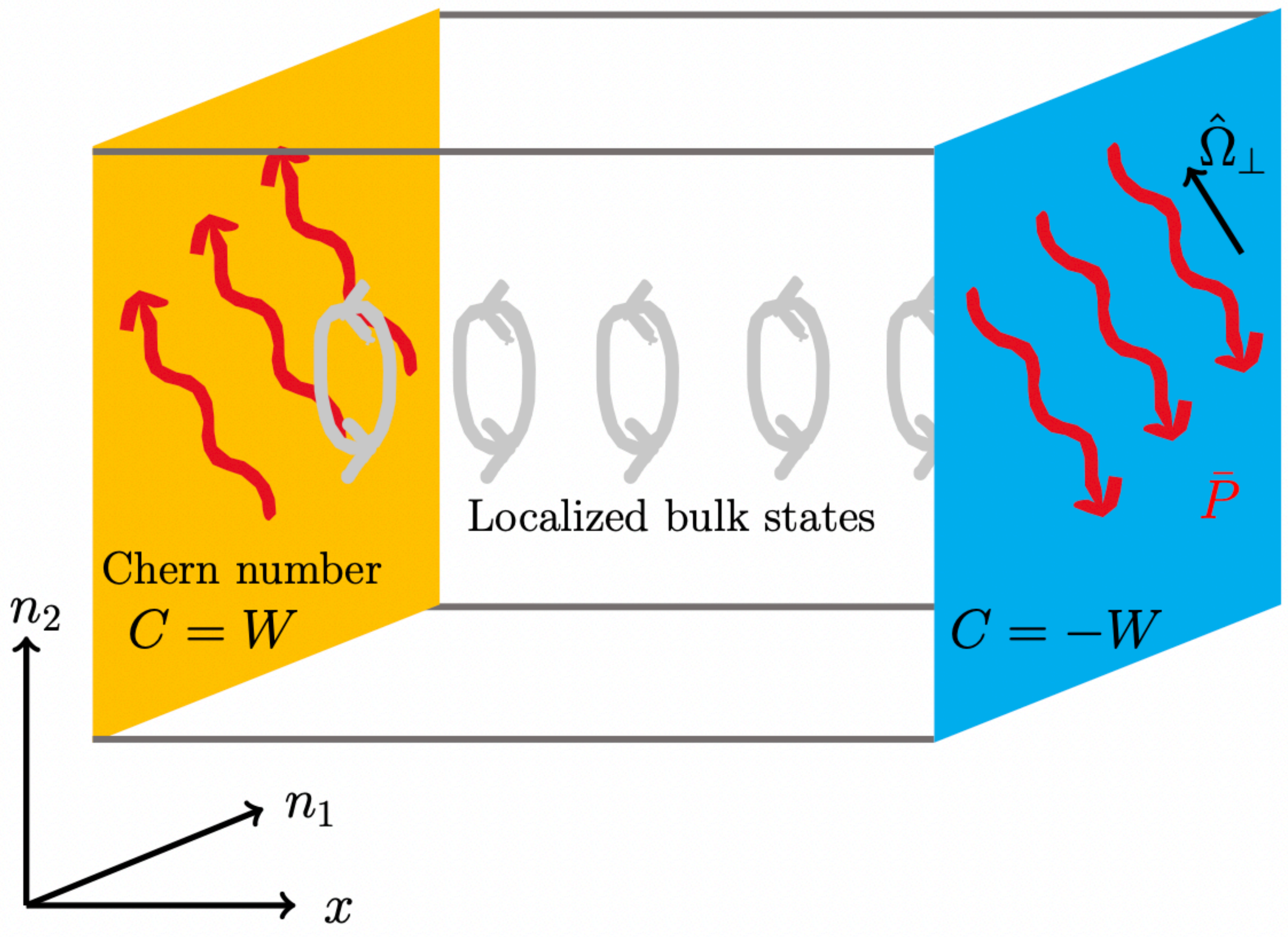




Vuina, Long, Crowley, AC (2023)

Can we create a steady state energy pump?

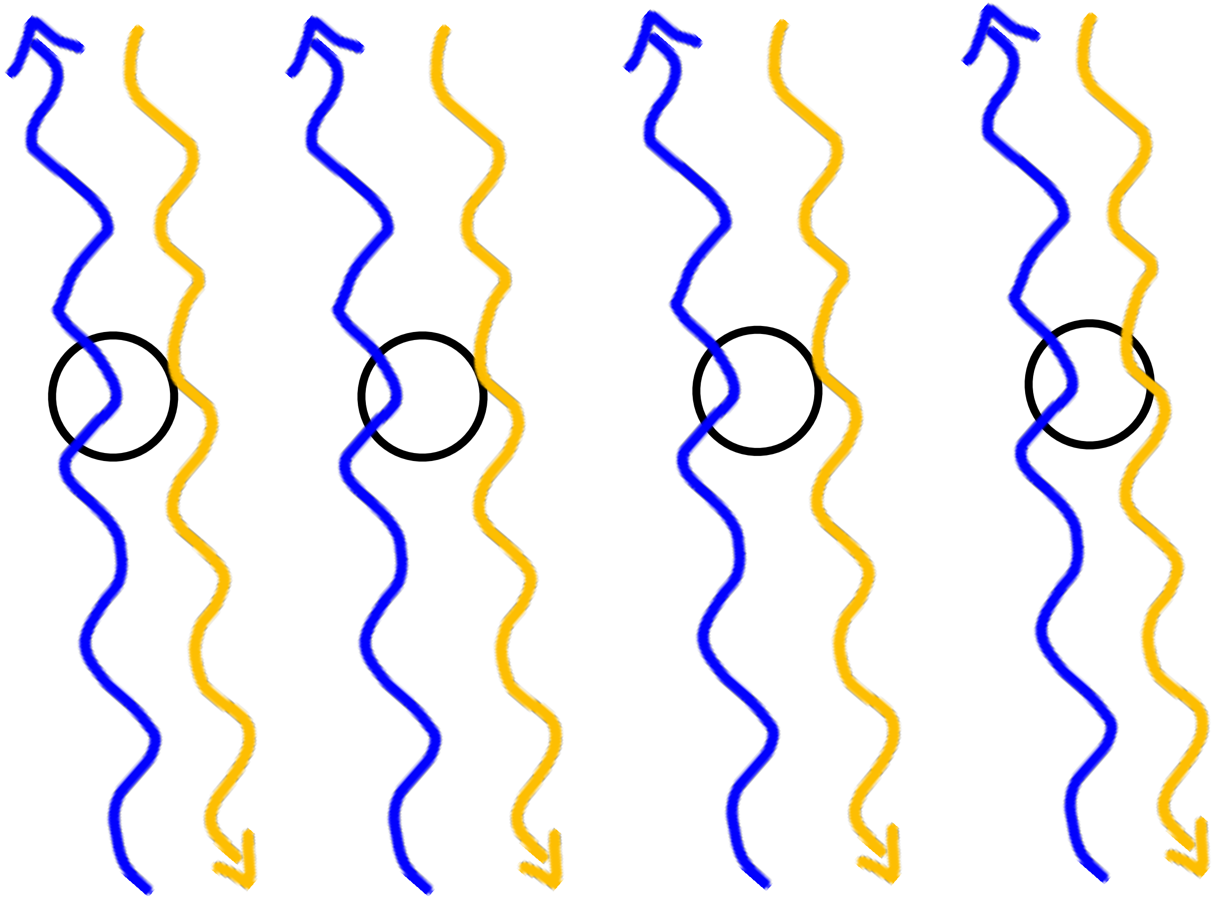
Yes! Separate the delocalized states on the frequency lattice



Coupled layer model

[Long, Crowley, AC, PRL **126**, 106805 (2021)]
[Nathan, *et al.*, PRL **127**, 166804 (2021)]
[Long, Crowley, AC, PRB **106**, 144203 (2022)]

Start with a chain of decoupled spinful fermionic sites. Fine-tune the on-site Hamiltonian to have pumping modes.



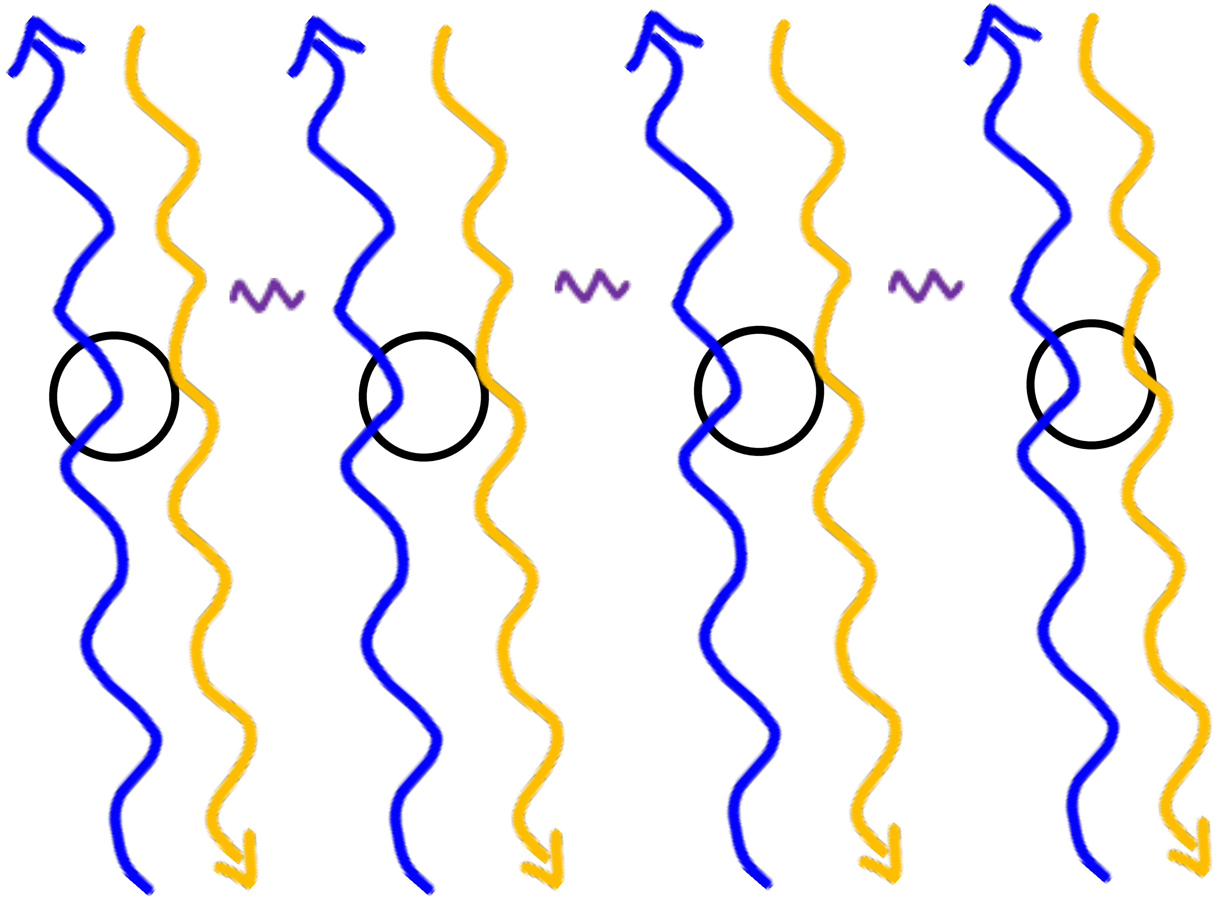
$c^{\dagger}_{j,+}(t)$ – "up" pumping mode

$c^{\dagger}_{j,-}(t)$ – "down" pumping mode

Coupled layer model

[Long, Crowley, AC, PRL **126**, 106805 (2021)]
[Nathan, *et al.*, PRL **127**, 166804 (2021)]
[Long, Crowley, AC, PRB **106**, 144203 (2022)]

Now couple the down modes on site j to the up modes on the site j+1



$$H_{hop}(t) =$$

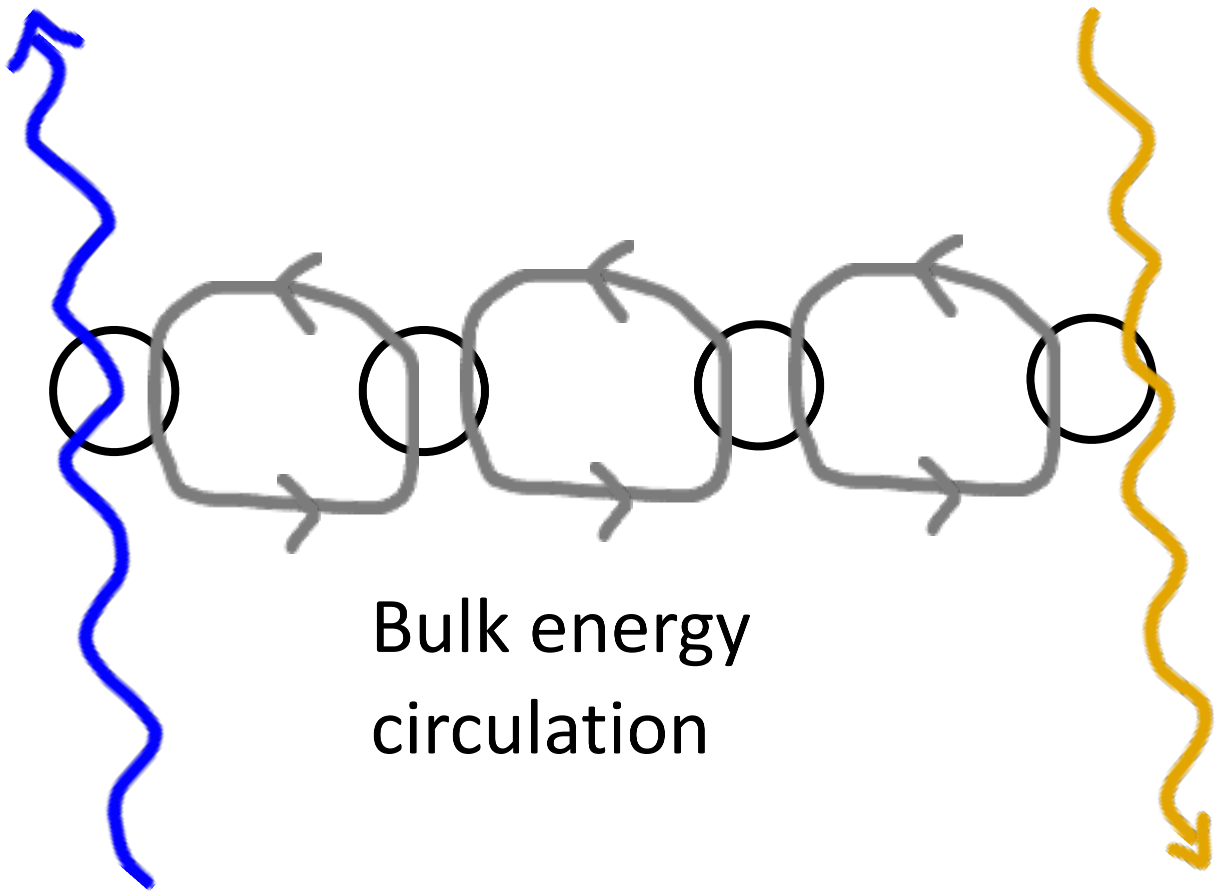
$$+\epsilon c^{\dagger}_{j,-}(t)\, c_{j+1,+}(t)]$$

$c^{\dagger}_{j,+}(t)$ – "up" pumping mode

$c^{\dagger}_{j,-}(t)$ – "down" pumping mode

[Long, Crowley, AC, PRL **126**, 106805 (2021)]
[Nathan, *et al.*, PRL **127**, 166804 (2021)]
[Long, Crowley, AC, PRB **106**, 144203 (2022)]

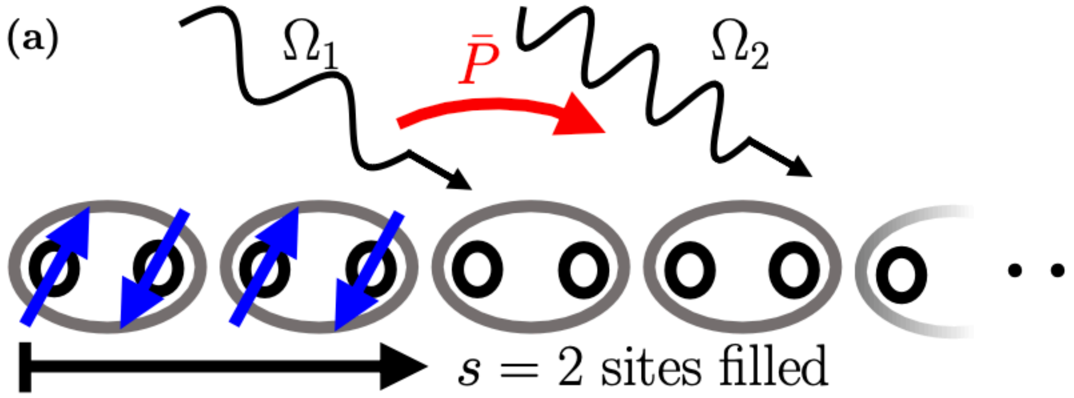
Coupled Layer Model



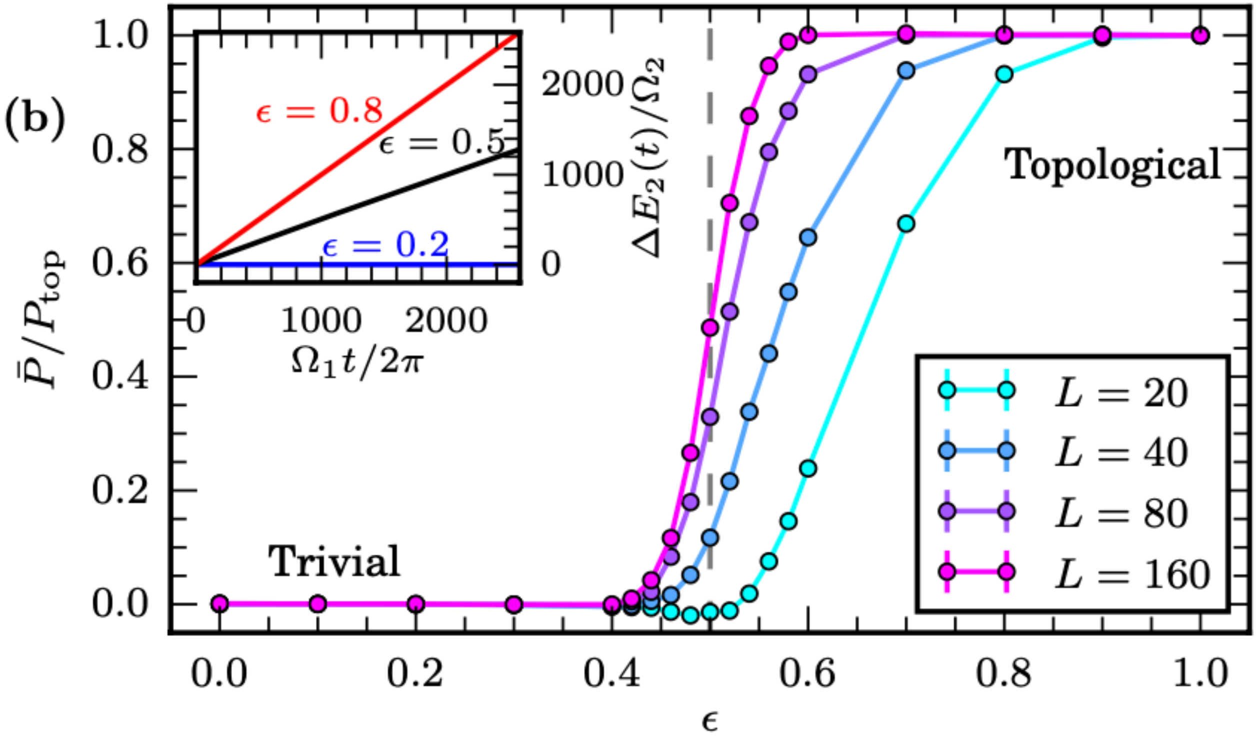
$$H_{\text{hop}}(t) =$$

$$+\epsilon c^{\dagger}_{j,-}(t)\, c_{j+1,+}(t)]$$

A non-adiabatic energy pump in (1+2)



($\Omega_{1,2}$ are the drive frequencies)



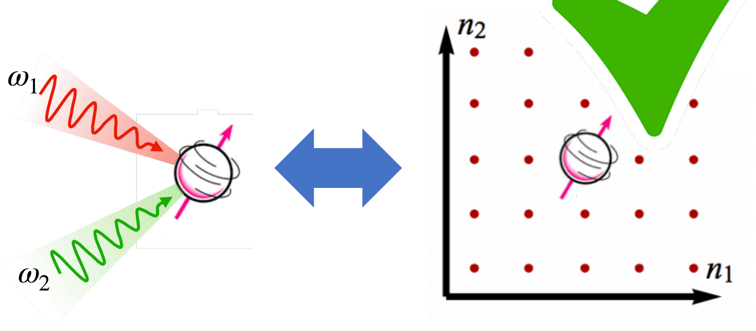
Long, Crowley, AC, 2021, 2022, Nathan et al, 2021

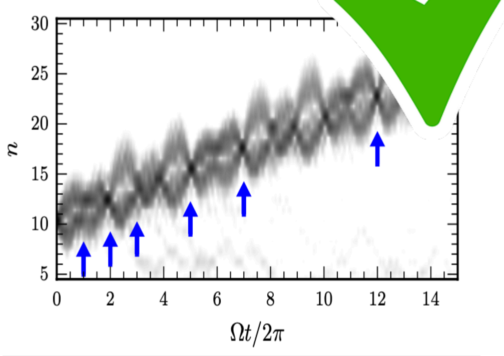
Questions?

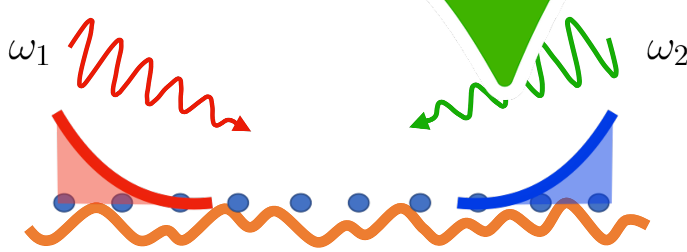
1 **Theory: quantised energy pumping & synthetic dimensions**

2 **Applications: Cavity state boosting**

3 **Beyond: new phases of driven quantum matter**







Last lecture

1. Localization/delocalization on the frequency lattice
    1. Connections to chaos
    2. Floquet's theorem
2. Topological classification of multi-tone driven hopping models
    - Non-trivial classification: quantized non-adiabatic pumps
3. MBL with two tone driving
    1. Interacting, disordered topological pumps