Outline of the series

American Football,
Barber Poles and Clouds

- Systems with more than one driven species
  "American football, Barber poles, and Clouds"
  - Variety of models with multiple species of particles
  - Surprises in "bare bones" NESM models with just two species
    - "Charged" particles driven in opposite directions
    - Phase transitions in the "ABC" model
- Summary and Outlook "Come and join in the fun!"

"Old" ones, from 90's!

Outline of ABC

- Motivations
  - Fundamental issues in Non-equilibrium Statistical Mechanics
  - Potential applications to physical/biological systems
- Surprises from a 2-D "Bare bones" model
- More surprises from 1-D: one *vs.* two lanes
  - Long range order … or ***not*** ?!?
  - Subtleties in coarsening
  - Effects of lane preference
- What else can we look forward to ?

Driven Diffusive Systems

- Diffusion of one or more particle species on a lattice
  - Relevant for many applications (biology, chemistry, …)
  - Simple local order parameter, satisfies conservation law
  - Well understood for systems in equilibrium ("model B")
- ... under non-equilibrium conditions
  - Driven by external forces
  - Many varieties … but only one (very simple) example here
  - Many surprises, even for one species (driven Ising lattice gas)
  - **Two species** of particles (Potts lattice gas)
  - Driven by external ("electric") field E - i.e., uniform bias
  - Periodic boundary conditions (so that **NESS** can be achieved)

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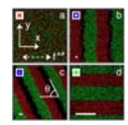
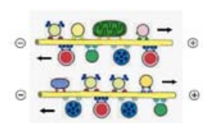
examples of Potential Applications

- Physical Systems
  - Pedestrian or vehicular traffic
  - Driven colloidal systems



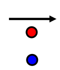
A. Wysocki & H. Löwen
PRE **79**, 041408 (2009)

- Biological Systems
  - Molecular motors on microtubules
  - Gel-electrophoresis (reptation model)



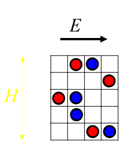
from a recent talk by R. Lipowsky

2-D bare bones model



Two "charged" particle species (+ ● , − ●)

with excluded volume, diffusing under an external, "electric" field E



all particle-hole exchanges with rate: 1
…except jumps against E : $exp(-E)$

all "charge" exchanges with rate: γ
…except jumps against E : γe^{-E}

…on an $H \times L$ lattice with PBC

control parameters: H, L, E, γ , and
$m \equiv (N_+ + N_-)/HL \quad q \equiv (N_+ - N_-)/HL$

here, mostly $E=\infty$, $m = 0.5$ and $q = 0$

One dimension ($1\times L$) … "one lane road"

exact solution ($E=\infty$, $\gamma > 0$) available

C. Godrèche and S. Sandow, 1998 unpublished
Y. Kafri, E. Levine, D. Mukamel, and M. Török, J. Phys. **A35**, L459 (2002).

- density homogeneous (i.e., no transition to jams)
- exponential distribution of (particle) cluster sizes:

$$\tilde{p}(s) \sim s^{-3/2} \exp(-s/\xi)$$

black (●) → ← gray (●)

2-D bare bones model

Two dimensions ($H \times L$) … "multilane road"

only MC and MFT (BS+RKPZ: 1991-now)

* anomalous, anisotropic, long-range correlations
* transitions to jammed states!!
* variety of ordered states
  - "American football" for $H \approx L$
  - "Barber poles" for $H \gg L$
  - drifting structures for $q \neq 0$
* both continuous; discontinuous transitions
* interesting coarsening phenomena ("clouds")
* Continuum Theory ***qualitatively*** adequate

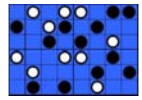
2-D bare bones model

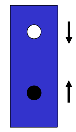
$E = \infty$, and turned vertical!

particle colors changed!

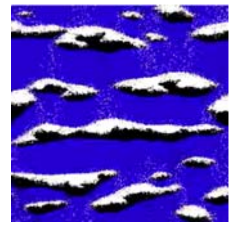
(just for these 2-d demos)

Barber poles



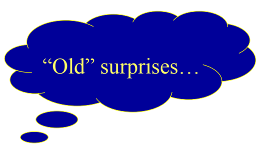


American football



Clouds

2-D bare bones model



- Disordered state: weird $\mathbf{S}(\boldsymbol{k})$'s
- Barber poles: winding number distributions?
- Clouds: dynamic scaling or not??

G. Korniss, B. Schmittmann and R.K.P. Zia, Physica **A239**, 111 (1997).
K. Bassler, B. Schmittmann and R.K.P. Zia, Europhys. Lett. **24**, 115 (1993)
D. Adams, B. Schmittmann and R.K.P. Zia, Phys. Rev. **E75**, 041123 (2007)

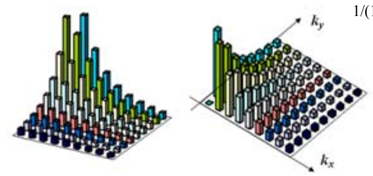
"Old" surprises

$S(k)$ = FT of correlation of density fluctuations $\langle \rho(0)\rho(x) \rangle$

Structure Factors $S(k)$

in the homogeneous phase of an ordinary system (e.g., gas)

Ornstein-Zernike $S(k_x,k_y)$



Isotropic

Lorentzian: $1/(1+k^2\xi^2)$

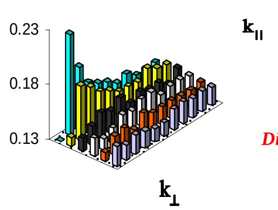
FT $\Rightarrow$ $exp(-r/\xi)$

"Old" surprises

Structure Factors $S(k)$

in the homogeneous phase of this driven system ("snowmist")

$\mathbf{S_{++}(k)}$ FT of $\langle \rho_+(0)\rho_+(x) \rangle$



Positive (necessary)

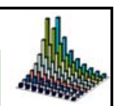
Anisotropic (no surprise)

Discontinuity at origin!

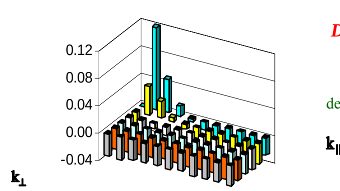
generic for DDS
due to ***violation*** of FDT

"Old" surprises

Imaginary parts are always positive!
WHY ?!?



Re $\mathbf{S_{+-}(k)}$ FT of $\langle \rho_+(0)\rho_-(x) \rangle$



Complex (typically)

Anisotropic (no surprise)

Discontinuity (like before)

Some change signs !
comes out of Langevin description + FDT ***violation***

"Old" surprises

Distributions of Structure Factors

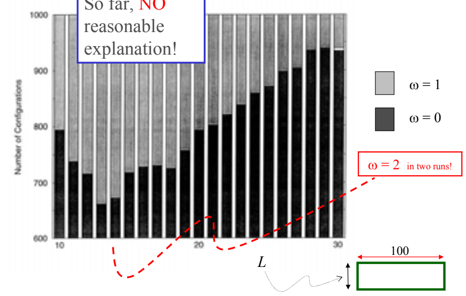
- How do the averages come about? (especially for the *negative* SF's)
- SF for each snapshot is a mess (speckle patterns from, e.g., laser scattering)
- What is the whole distribution of SF's?

Asymmetries in Structure Factor Histograms
G. Korniss, B. Schmittmann and R.K.P. Zia
J. Phys. **A30**, 3837 (1997).

"Old" surprises

Histogram of Barber Poles

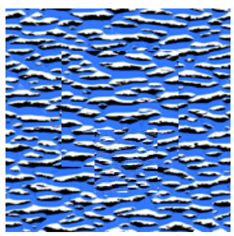
So far, NO reasonable explanation!



"Old" surprises

Coarsening (dynamic scaling)

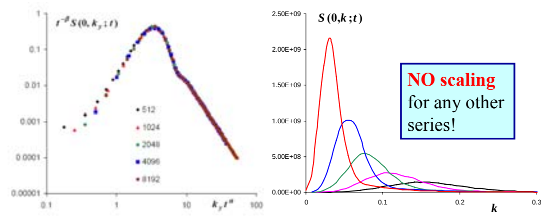
1000 x 1000
$m = 1/2 \quad q = 0$
$E = 50$



"Old" surprises

NO explanation, except for large ***k***

Coarsening (dynamic scaling)



NO scaling for any other series!

Rescale both ***S*** and ***k***

Horrible comparison with continuum theory!
unlike the case for models A,B

2-D bare bones model

Natural Questions:

No transitions in 1-d *vs.* ordered states in 2-d reminiscent of Ising model …

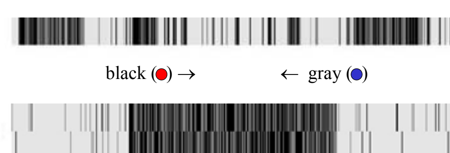
- How does cross-over occur here?
- What happens if we "gradually" progress from one to the other?
- Study "two lane" system, $2 \times L$;
- …*then* to "multi-lane" cases.

More surprises from 1-D: one *vs.* two lanes

Snapshots of $H = 1,2$ systems

(both in steady state)

$E = \infty$, $\gamma = 0.1$, $L = 1000$



…a few details of this two-lane system

- $m = 0.5$, $E = \infty$, $\gamma = 0.1$, $L \le 10{,}000$
- *length* of "jam" ~ $0.47L$

⇔ 94 % of particles are in the "jam"

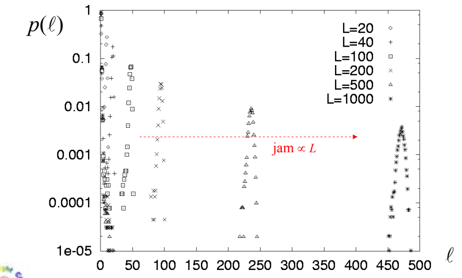
⇔ 6 % of particles are "travelers"

~ $\gamma/(2-\gamma)$

…describe clusters via the residence distribution, $p(\ell)$: probability for particle to be in cluster of length ℓ…

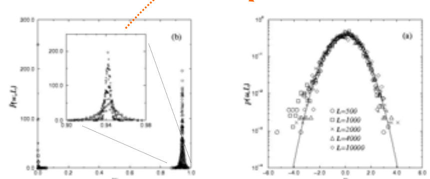
Residence distribution has *two* components:

exponential + Gaussian (center ∝ L)



…replot against ℓ/L

center + scale



"travelers" "jam"

s/L ~ $2\ell/L$

particles in a cluster *two* lanes

$(\ell - \bar{\ell})/\delta\ell$

with $\bar{\ell} \sim 0.47L$, $\delta\ell \sim \sqrt{L}$

G. Korniss, B. Schmittmann, and R. K. P. Zia
Europhys. Lett. **45**, 431 (1999).

…yet another twist – a conjecture

- The two lane system, just like the $1{\times}L$ case, is homogeneous in "the thermodynamic limit."
- The "jam" will not go with L for "***large enough L***," with cross-over length beyond those in MC (may be as large as 10^{10} or even 10^{70}).
- …based on MC + exact solution of similar model

N. Rajewsky, T. Sasamoto, and E.R. Speer, Physica **A279**, 123 (2000).
T. Sasamoto and D. Zagier, J. Phys. **A34**, 5033 (2001).

- …and criterion associated with asymptotic properties of currents of finite clusters

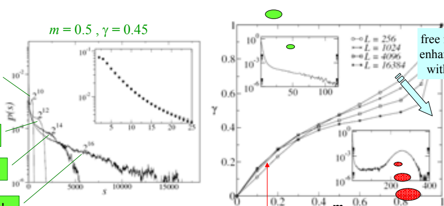
Y. Kafri, E. Levine, D. Mukamel, G.M. Schütz, and J. Török, Phys. Rev. Lett. **89**, 035702 (2002).

…yet another twist – MC results

here, free flowing

Extensive MC, with L up to 2^{20} and various γ's, is *consistent* with it...

$m = 0.5$, $\gamma = 0.45$



macro-cluster

going…

going…

gone!

free flow enhanced with L

But, jamming enhanced here as L increases!!

jammed in this corner!

I.T. Georgiev, B. Schmittmann, and R.K.P. Zia,
Phys. Rev. Lett. **94**, 115701 (2005)

Regardless of the issues of $L \to \infty$, we may ask:

How do the $1{\times}L$ and $2{\times}L$ systems evolve toward these very different steady states (for *presently accessible* L's) ?

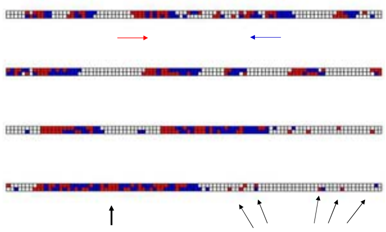
- Investigate the t-dependent residence distribution:

$p(s,t)$ *or* $p(\ell,t)$.

- $1{\times}L$ Small clusters form at early t and dissolve.
- $2{\times}L$ ***Coarsening***, like quenches below T_c…

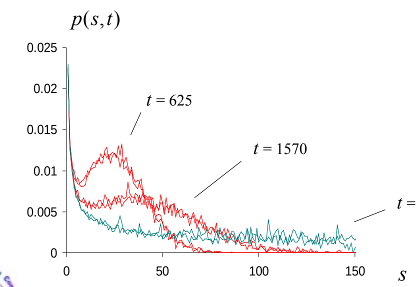
…except much faster!

Snap shots of a small 2×L system



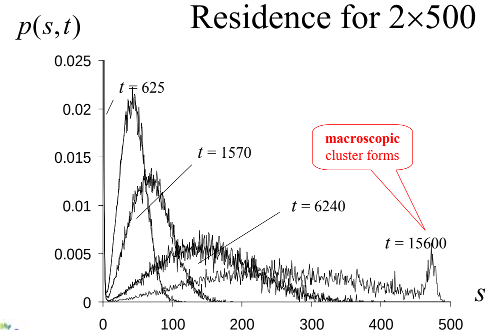
…so, it’s easy to identify a cluster and its size (s) and/or length (ℓ)

Residence for 1×500 and 1×1000

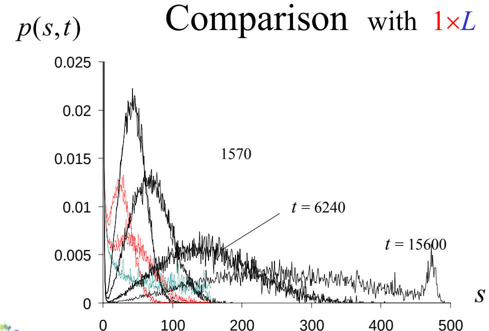


…already essentially same as theoretical prediction for SS $p(s)$

Residence for 2×500



Comparison with 1×L



- From residence distribution $p(\ell,t)$
- Find growth rate of average cluster size :

$$\bar{\ell}(t) \equiv \sum \ell \, p(\ell,t)$$

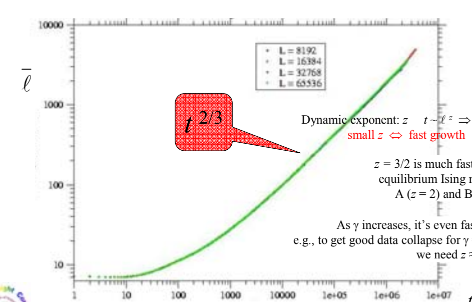
- See if dynamic scaling exists :

$$\bar{\ell}(t)\, p(\ell,t) \overset{?}{=} f(x) \qquad x \equiv \ell / \bar{\ell}(t)$$

- Check L independence (during growth regime)

J.T. Mettetal, B. Schmittmann, and R.K.P. Zia, Europhys. Lett. **58**, 653 (2002)
I.T. Georgiev, B. Schmittmann, and R.K.P. Zia, J. Phys. **A 39**, 3495 (2006)

growth of cluster size $\gamma = 0.1 \quad m = 0.5$



Dynamic exponent: z $\quad t \sim \ell^z \Rightarrow z = 3/2$
small z ⇔ fast growth

z = 3/2 is much faster than equilibrium Ising models: A (z = 2) and B (z = 3)

As γ increases, it’s even faster… e.g., to get good data collapse for γ = 0.3, we need $z \approx 1.45$

"New" surprises: Fast coarsening

…brief summary

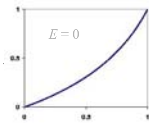
of published material on fast coarsening in $2\times L$

- MC: $m = 1/2$, $q = 0$, $E = \infty$, $\gamma = 0.1$, $L \leq 10{,}000$
- coarse grained clusters grow: $\ell \sim t^{2/3}$
- dynamic scaling ok for $p(\ell, t)$
  …with scaling function $\sim$ theory for 1 species
  but wrong exponent
- *improved* "theory" gives reasonably good fit to both exponent and $p(\ell, t)$
  but no analytic understanding

"New" surprises: Lane preference

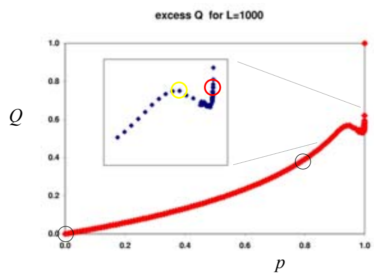
Lane preference

- "cars/trucks" (sometimes) tend to stay in "fast/slow" lane
- p probability for choosing "preferred" lane
- $p = 0, 1$ cases are clear:
  - jam as before *vs* free flow
  - equal mix (on the average) *vs* pure cars/trucks
    i.e., $Q \equiv$ "excess" $= 0$ *vs* $\equiv 1$
- *expect* $Q(p)$ to be
  monotonically increasing, e.g.,…



"New" surprises: Lane preference

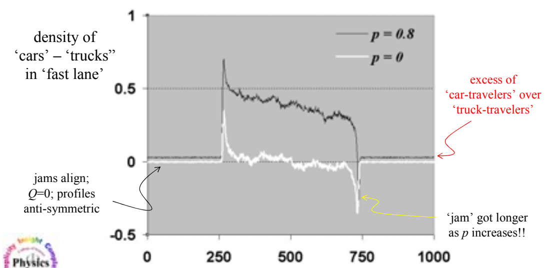
…instead, there is a twist :



"New" surprises: Lane preference

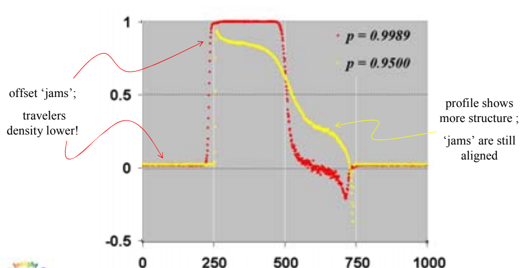
Profiles at 'small' p

(CM of entire 'jam' centered at 500, then averaged)



"New" surprises: Lane preference

Profiles at 'large' p



"New" surprises: Lane preference

brief remarks

- Simple model seems to show effects we might see (should expect?) on two-lane roads
- Numerical integration of MFT display qualitatively same behavior
- Need a better understanding of '***negative*** *response*'
- Need better theories for quantitative predictions

B. Schmittmann, J. Krometis, and R.K.P. Zia, Europhys. Lett. **70**, 299 (2005)

What else can we look forward to?

- Despite its ***simplicity***, this model continues to present many ***interesting*** & ***challenging*** issues:
  - Many already raised and …
  - What about 3 lanes? and … 13 lanes?
  - Exclusion at larger distances (big trucks); interactions…
  - Inhomogeneous jump rates (gravel patches, road works…)
- Other "ABC" models: driven in parallel, overtaking cyclically
- Multispecies models: widely differing driver preferences

⋮

Outline of the series

- Overview/Review "Equilibrium SM *vs.* Nonequilibrium SM"
- An Ising-like model in DDS "Shattered expectations"
- DDS in one-dimension "Bare bones NESM"
- Systems with more than one driven species "American football, Barber poles, and Clouds"
- ***Summary and Outlook…***

Summary and Outlook

- Adding a "simple" drive to the equilibrium Ising model adds dimensions ***far beyond expectations***.
- Even a "bare bones" system (1-D, "non-interacting") provided many ***amazing phenomena***, while adding other species leads to further surprises.
- Potential applications to wide range of systems in nature exist.
- Some ***new insights*** have been garnered, but the goal of an ***overarching framework for NESM*** is far from being just "around the corner".

Summary and Outlook

- Equilibrium SM is not an easy subject, but "full" Nonequilibrium SM is really ***far out*** !!
- Nonequilibrium SM topics here form a tiny corner, in which
- Driven Diffusive Systems occupy a minute part, in which
- Models presented in these lecture are a small fraction, with…

Lots of open questions ⇒

- Lots of work to be done
- Lots of ideas to pursue
- Lots of interesting phenomena, *waiting to be discovered*!
- Lots of ways/levels to participate:
  - Computer simulations
  - Numerical/analytical approaches to ODEs,PDEs,SDEs
  - Field theory (QFT,SFT)
  - Rigorous mathematical methods