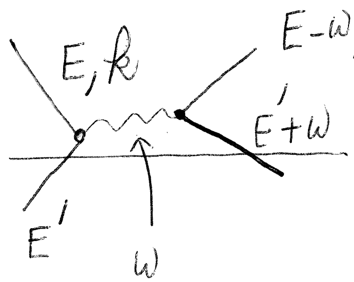
Breakdown of FL in one dimension

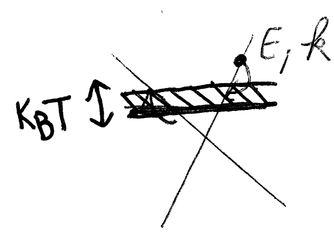
D. Maslov (2005); K. LE HUR (2006)

Fermi liquid assumption $G(k,t) = Z\, e^{i(k-k_F)v_F t}\, e^{-t/\tau_F}$

$$\tau_F \gg \frac{\hbar}{\xi_k} \qquad \tau_F \sim \frac{1}{T^2}$$

can we make a simple argument to estimate the electron lifetime in 1D





FINAL STATE for the excited electron

electrons in one dimension : $(E-\omega) = v_F(k-q)$

correspondence between energy & momentum

If other electron, a left moving electron $\omega = -v_F q$

Thus: $$\frac{\hbar}{\tau_e(E)} \simeq |U|^2 \int_0^E d\omega \int_{-\omega}^{0} dE' \int dq\, \delta(\omega + v_F q)\ \delta\left[(E-\omega) - v_F(k-q)\right]$$

let us consider electrons close to the Fermi surface, $k \to 0$

$v_F q = -\omega$

$\delta(E - \omega + v_F q)$

$\delta(E - 2\omega)$

$$\frac{\hbar}{\tau_e(E)} \simeq |U|^2 E$$

This argument requires screened Coulomb interactions

$U\delta(x)$

Conclusion : In 1D, $\boxed{\tau_F \simeq \frac{\hbar}{E}}$ (electron à priori badly defined at low E)

Jordan-Wigner transformation (Simple view of bosonization)

$$c_j = \exp\left(i\pi \sum_{j'<j} n_{j'}\right) \underbrace{b_j}_{\text{hard-core bosons}}$$

$$[b_i, b_j] = [b_i^+, b_j^+] = [b_i^+, b_j] = 0 \quad i \neq j$$

$$\{b_i^+, b_i\} = 1$$

$$c_j^+ = b_j^+ \exp\left(-i\pi \sum_{j'<j} n_{j'}\right)$$

$$n_j = c_j^+ c_j = b_j^+ \exp\left(-i\pi \sum_{k<j} n_k\right)\left(\exp i\pi \sum_{k<j} n_k\right) b_j$$

kink operator is unitary

$$c_j^+ c_j = b_j^+ b_j$$

* check that $c_j c_k = - c_k c_j \quad k \neq j$ fermionic statistics

$k > j$
$$c_j c_k = \exp i\pi \sum_{k<j} n_k \; b_j \; \exp i\pi \sum_{k'<k} n_{k'} \; b_k$$

$$b_j \exp\left(i\pi \sum_{k'<k} n_{k'}\right) = \exp i\pi \sum_{\substack{k'<k \\ k' \neq j}} \exp i\pi n_{k'} \; b_j \; \exp(i\pi n_j)$$

hard-core bosons $\equiv$ SPINS

$$b_i^+ = S_i^+$$
$$b_i = S_i^-$$
$$n_i \equiv b_i^+ b_i = \frac{1}{2} + S_i^z$$
$$b_i b_i^+ = \frac{1}{2} - S_i^z$$

$$b_j \exp(i\pi n_j) = S_j^- \underbrace{\exp\left(i\pi\left[\frac{1}{2} + S_j^z\right]\right)}_{}$$

$S_j^z = \pm\frac{1}{2}$ $\quad 1 - 2\left[\frac{1}{2} + S_j^z\right]$

$$\left[\begin{array}{l}\exp i\pi = -1 \\ \exp 0 = 1\end{array}\right.$$

$$b_j \exp(i\pi n_j) = -S_j^-(2S_j^z)$$

now, we can use that $\{S_j^-, S_j^z\} = 0$

(anticommutation relations at the same site)

$$S_j^- S_j^z = -S_j^z S_j^-$$

thus: $$b_j \exp(i\pi n_j) = \underline{+2S_j^z}\, S_j^-$$

$$-\left[1 - 2\left[\frac{1}{2} + S_j^z\right]\right] = -\exp\left(i\pi\left(\frac{1}{2} + S_j^z\right)\right)$$

$$= -\exp(i\pi n_j)\, b_j$$

FINALLY $$b_j \exp\left(i\pi \sum_{k'<k} n_{k'}\right) = -\exp i\pi \sum_{k'<k} n_{k'}\, b_j$$

Haldane (1981) Long distance properties

let's forget the hard core constraint

$$b_j \mapsto \sqrt{n_j}\, \exp(i\theta_j)$$

superfluid phase

$$\theta_j \mapsto \theta(x)$$

$n_j \mapsto \rho(x)$ is the 1d electron density

The density can be decomposed as $\rho = \rho_0 + \tilde{\rho}$

measures fluctuations of the electron density

electron wave function:

$$\exp i\pi \sum_{j<i} n_j \mapsto \exp i\pi \int^x \rho$$

$$= \exp i(k_F x + \phi)$$

CHARGE MODE

$$\rho_0 = \frac{k_F}{\pi}$$

As usual, the density and phase are canonically conjugate quantum variables taken to satisfy

$$[\theta(x), \tilde{\rho}(x')] = i\,\delta(x-x') \quad (*)$$

One dimension: PHONON-like displacement field

$$\tilde{\rho}(x) = \frac{\partial_x \phi}{\pi}$$

The factor π has been chosen so that the full density takes the simple form

$$\rho(x) = (k_F + \partial_x \phi)/\pi$$

$$(*) \mapsto \quad [\theta(x), \phi(x')] = \frac{i\pi}{2} \operatorname{sgn}(x - x')$$

Notice that $\partial_x \theta$ is the momentum conjugate to ϕ

IMPORTANT OBSERVABLES: $\rho_0 = \frac{N}{L}$ $\quad k_F = \frac{\pi N}{L}$

$$\hat{N} = N + \hat{Q} = N + \frac{1}{\pi} \int_0^L \nabla\phi(x)\, dx$$

fixed by number of particles or chemical potential

$$\hat{Q} = N_R + N_L$$

$$\hat{J} = -(N_R - N_L) = -\frac{1}{\pi} \int_0^L \nabla\theta(x)\, dx$$

(with those chosen conventions)

1d density wave takes the form (Gaussian theory)

density Hamilto: $H = \frac{v}{2\pi} \left[g (\partial_x \theta)^2 + g^{-1} (\partial_x \phi)^2 \right]$

EXAMPLE

kinetic term for bosons

$$H_{kin} = \int dx \, \frac{1}{2m} (\nabla b^\dagger(x))(\nabla b(x))$$

$$b(x) \simeq \sqrt{\rho_0}\, e^{i\theta(x)}$$

NOT only restricted to fermions

$$H_{kin} = \int dx \, \frac{\rho_0}{2m} (\nabla \theta)^2$$

(good)

Momentum representation

$$H = \sum_q \frac{v}{2\pi L} \cdot \left[g\, q^2 \phi(q)^* \phi(q) + g^{-1} |\varrho(q)|^2 \right]$$

$$\begin{cases} \phi(x) = \phi_0 + \frac{\pi}{L} \hat{Q} x + \frac{1}{\sqrt{L}} \sum_{q \neq 0} \phi_q e^{iqx} \\ \theta(x) = \theta_0 + \frac{\pi}{L} \hat{J} x + \frac{1}{\sqrt{L}} \sum_{q \neq 0} \theta_q e^{iqx} \end{cases}$$

wave propagating at velocity v

$$\text{equations of motion} \quad \partial_t^2 \theta = v^2 \partial_x^2 \theta$$

$$\partial_t \theta = i [H, \theta] = \left(\frac{iv}{2\pi}\right) g^{-1} \left[(\partial_x \phi)^2, \theta \right]$$

$$-\left[\theta(x'), \frac{\partial_x \phi(x)}{\pi} \right] = -i \delta(x - x')$$

$$\left[\partial_x \phi, \theta(x') \right] = +i\pi \, \delta(x - x')$$

$$= -\frac{v}{2} 2 g^{-1} \partial_x \phi$$

equation of continuity

$$\nabla \vec{j} + \frac{\partial \tilde{\rho}}{\partial t} = 0$$

$$\partial_t (\partial_t \theta) = -v g^{-1} \partial_t \partial_x \phi$$

$$+ \partial_x v_F \partial_x \theta + \frac{\partial_t \partial_x \phi}{\pi}$$

$$\partial_x \theta \Rightarrow i [H, \phi] = \partial_t \phi$$

$$= \frac{ivg}{2\pi} \left[(\partial_x \theta)^2_{(x')}, \phi(x) \right]$$

$$= -vg \, \partial_x \theta \qquad \partial_x \theta \equiv \frac{-1}{vg} \partial_t \phi$$

$$\partial_x (\partial_x \theta) = -\frac{1}{vg} \partial_x \partial_t \phi \qquad \partial_t^2 \theta = v^2 \partial_x^2 \theta$$

Bosonization: SIMPLE view

$$c(x) = \exp i\pi \int_{-\infty}^{x} dx' \, \rho(x') \quad \exp(i\theta)$$

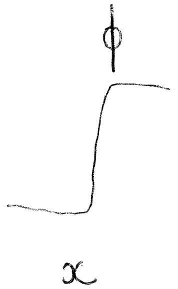
$$\rho(x') \rightarrow \rho_0 + \tilde{\rho}$$

statistics

$$\exp(i\theta(x)) = \exp i\pi \int_{-\infty}^{x} dx' \, P(x')$$

$$P(x') = \frac{\partial_x \theta}{\pi} \quad \text{"momentum" conjugate to } \phi$$

momentum : generator of translations (in ϕ), this creates a kink of height π in ϕ centered at x



This corresponds to a localized unit charge

$$\tilde{\rho} = \frac{\partial_x \phi}{\pi}$$

Following Haldane, the most general charge e fermion operator is

$$\psi(x) = \sum_{m \text{ odd}} e^{im(k_F x + \phi(x))} e^{i\theta(x)}$$

statistical argument!

electron field operator can be expanded into a right & left moving piece

($\pm k_F$ Fermi points)

$$\psi(x) = \psi_R + \psi_L \simeq e^{ik_F x} e^{i\Phi_R} + e^{-ik_F x} e^{i\Phi_L}$$

$$m = \pm 1 \qquad \Phi_{R/L} = (\theta \pm \phi)$$

$\Phi_{R/L}$ describe the slowly varying piece of the electron field.

One can check:
$$\begin{aligned}[\Phi_R(x), \Phi_R(x')] &= [\phi(x) + \theta(x), \phi(x') + \theta(x')] \\ &= [\phi(x), \theta(x')] + [\theta(x), \phi(x')] \\ &= \frac{i\pi}{2}\,\mathrm{sgn}(x-x') + \frac{i\pi}{2}\,\mathrm{sgn}(x-x') \\ &= i\pi\,\mathrm{sgn}(x-x')\end{aligned}$$

$$[\Phi_L(x), \Phi_L(x')] = -i\pi\,\mathrm{sgn}(x-x')$$

These 2 fields commute with one another.

These fields are simply related to the right & left moving electron densities

$$N_{R/L} = \pm \frac{1}{2\pi}\partial_x \Phi_{R/L}$$

$$(N_R + N_L) = \frac{1}{\pi}\partial_x \phi = \tilde{\rho} \qquad N_R - N_L = \frac{1}{\pi}\partial_x \theta$$

Idea: Rewrite the wave theory in terms of an interacting electron theory

$$H = \frac{v}{2\pi}\int dx\, [g(\partial_x\theta)^2 + g^{-1}(\partial_x\phi)^2]$$

$$H = \frac{v}{2\pi}\int dx \left(g\,\pi^2[N_R - N_L]^2 + g^{-1}\pi^2[N_R+N_L]^2\right)$$

$$H = \frac{v\pi}{2}\int dx \left[(g+g^{-1})(N_R^2 + N_L^2) - 2N_R N_L [g - g^{-1}]\right]$$

$$H = \pi v_0 \int dx\, [N_R^2 + N_L^2 + 2\lambda N_R N_L]$$

$$v_0 = v(g+g^{-1})/2$$

$$\lambda = (1-g^2)/(1+g^2)$$

interaction between the 2 species

free electron $g \equiv 1$ $\quad v = v_0 = v_F$

repulsive interactions $g < 1$

definition of the Luttinger parameter g? (See later)

Field theory techniques. link with Dirac Hamiltonian $\quad H = i v_F \psi_R^\dagger \partial_x \psi_R$

normal ordering $\quad \psi_R^\dagger(x+\epsilon)\psi_R(x-\epsilon) = :\quad: + \langle 0|\psi_R^\dagger(x+\epsilon)\psi_R(x-\epsilon)|0\rangle$

$$\langle 0|\psi_R^\dagger(x+\epsilon)\psi_R(x-\epsilon)|0\rangle = \int_{-\infty}^{0} \frac{dk}{2\pi} e^{-2ik\epsilon} = \frac{i}{4\pi\epsilon}$$

$$J_R(x+\epsilon)J_R(x-\epsilon) \longmapsto \frac{i}{\pi} :\psi_R^\dagger \partial_x \psi_R:$$