Dimensional Crossovers, Exotic SC, and Challenges in 2D

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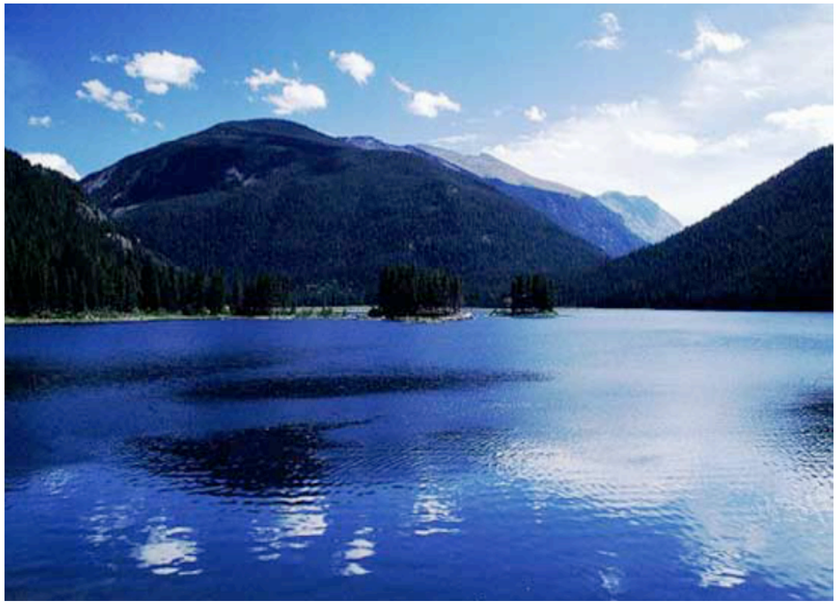
Boulder, Summer school 2008: Lecture II



Brief history of Superconductivity

1911: Kamerlingh Omnes

Hg becomes superconducting at 4K

1913: He won the Nobel price in physics

1933: Meissner effect

1941: Other superconducting metals

niobium-nitride, T_c=16K

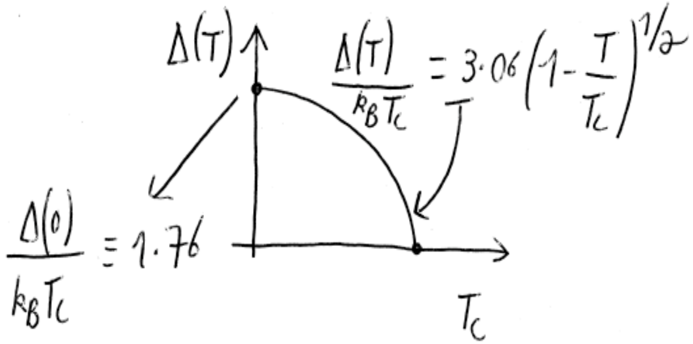
BCS Theory, 1957

BCS theory assumes that there is some attraction between electrons, which can overcome the Coulomb repulsion between electrons

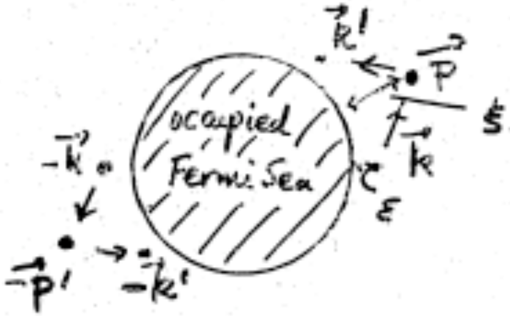
Coupling of electrons to the vibrating crystal lattice (phonons)

Fermi liquid to SC at: $k_B T_c = 1.1 E_D \exp -1/N(0)V$

Gap at T=0 grows with T_c
<u>Debye energy not high</u>: 100-500K



Landau Theory



Matrix Element connecting initial and final states

$$\Gamma(\varepsilon) = V + \frac{V^2}{\Omega} \sum_{\vec{p}}^{\xi < D} \frac{2}{2(\varepsilon - \xi)} + \cdots$$

D: Energy Cutoff

Scaling Hypothesis

$$\Gamma\left(\frac{\varepsilon}{D}, V\right) = \Gamma\left(\frac{\varepsilon}{D'}, V'\right)$$

works with $V \mapsto V' = \dfrac{V}{1 + \frac{m k_F}{2\pi^2} \cdot V \cdot \ln\left(\frac{D}{D'}\right)}$

- attractive Interactions

El-Phonon $D \sim \Theta_D$ scale to $\underline{\underline{-\infty}}$

$\longrightarrow$ BCS Theory based on Cooper Instability of an s-wave pair

Basic steps of BCS theory

Tinkham's book

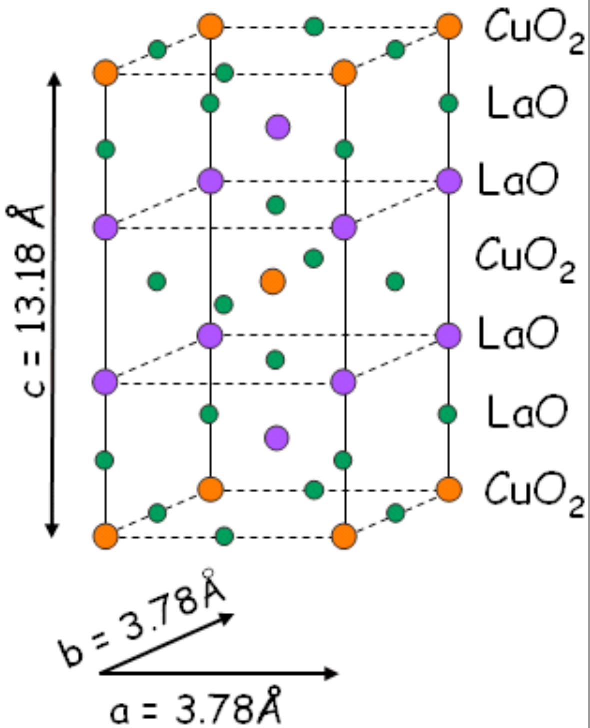
$$H_{BCS} = \sum_{\vec{k}\sigma} \xi_{\vec{k}} n_{\vec{k}\sigma} - \sum_{\vec{k}\vec{p}} V c^{\dagger}_{\vec{k}\uparrow} c^{\dagger}_{-\vec{k}\downarrow} c_{-\vec{p}\downarrow} c_{\vec{p}\uparrow}$$

$$\Delta = \sum_{\vec{k}} V \langle c_{-\vec{k}\downarrow} c_{\vec{k}\uparrow} \rangle$$

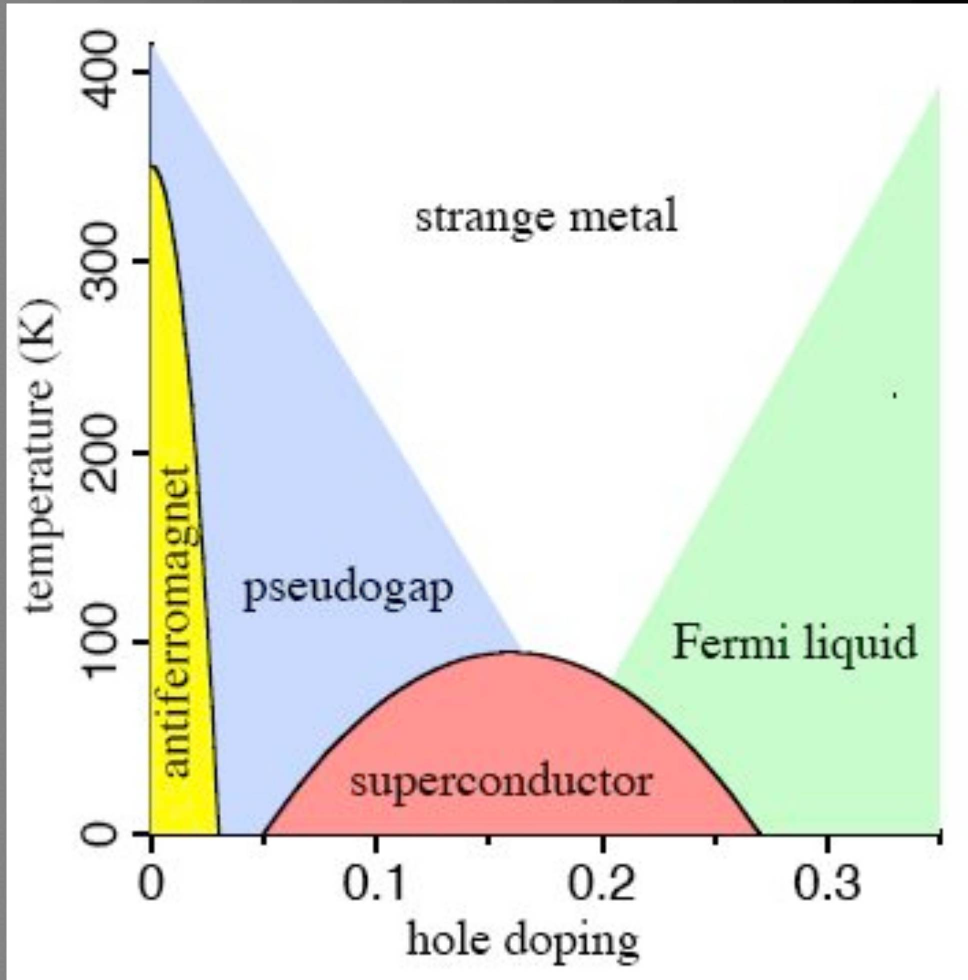
$$|\psi_G\rangle = \prod_{\vec{k}} \left(u_{\vec{k}} + v_{\vec{k}} c^{\dagger}_{\vec{k}\uparrow} c^{\dagger}_{-\vec{k}\downarrow} \right) |\phi_0\rangle$$

$$\frac{1}{V} = \frac{1}{2} \sum_{\vec{k}} \frac{\tanh(\beta E_{\vec{k}}/2)}{E_{\vec{k}}} \qquad E_{\vec{k}} = (\xi^2_{\vec{k}} + \Delta^2)^{1/2}$$

High Temperature Superconductors

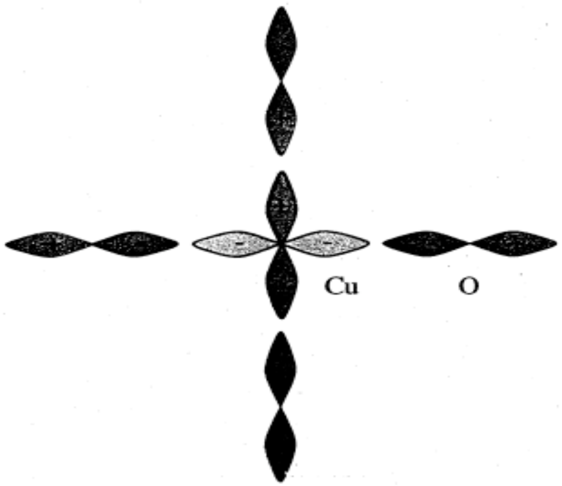


1986

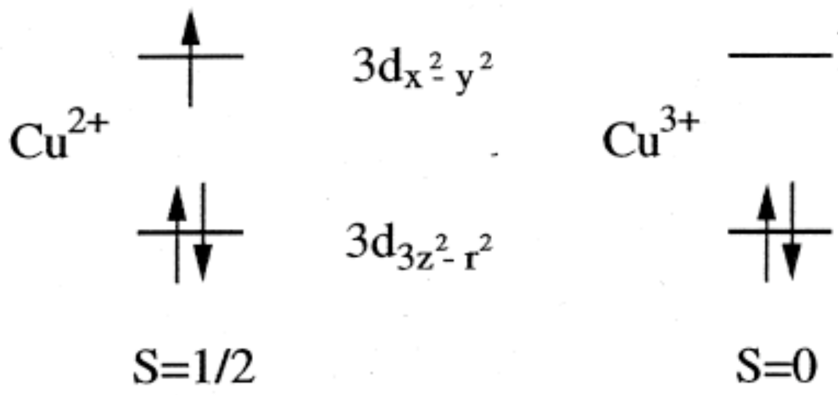


- Coupled CuO_2 layers
- Doping with holes leads to SC
- Nonmonotonic Tc versus doping
- Maximum Tc ~ 150 K
- Electronic SC without phonons?
- Normal phase is not a Fermi liquid at low doping

Planar cuprates



Oxidation States:

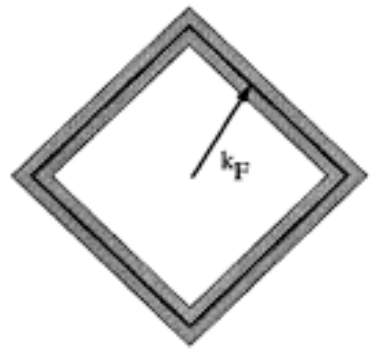


charge +|e|

Zhang-Rice singlets

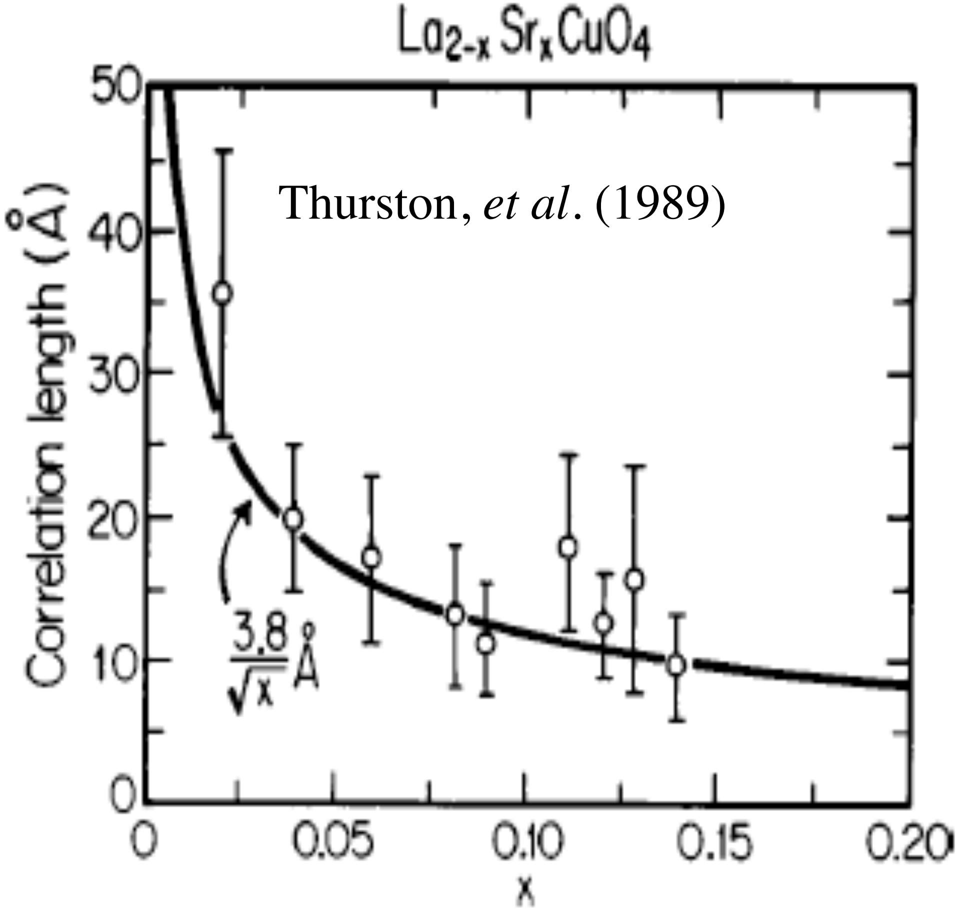
- Stochiometric Oxides:
  all $Cu^{2+} \rightarrow$ AF Mott Insulators.

J~4t^2/U



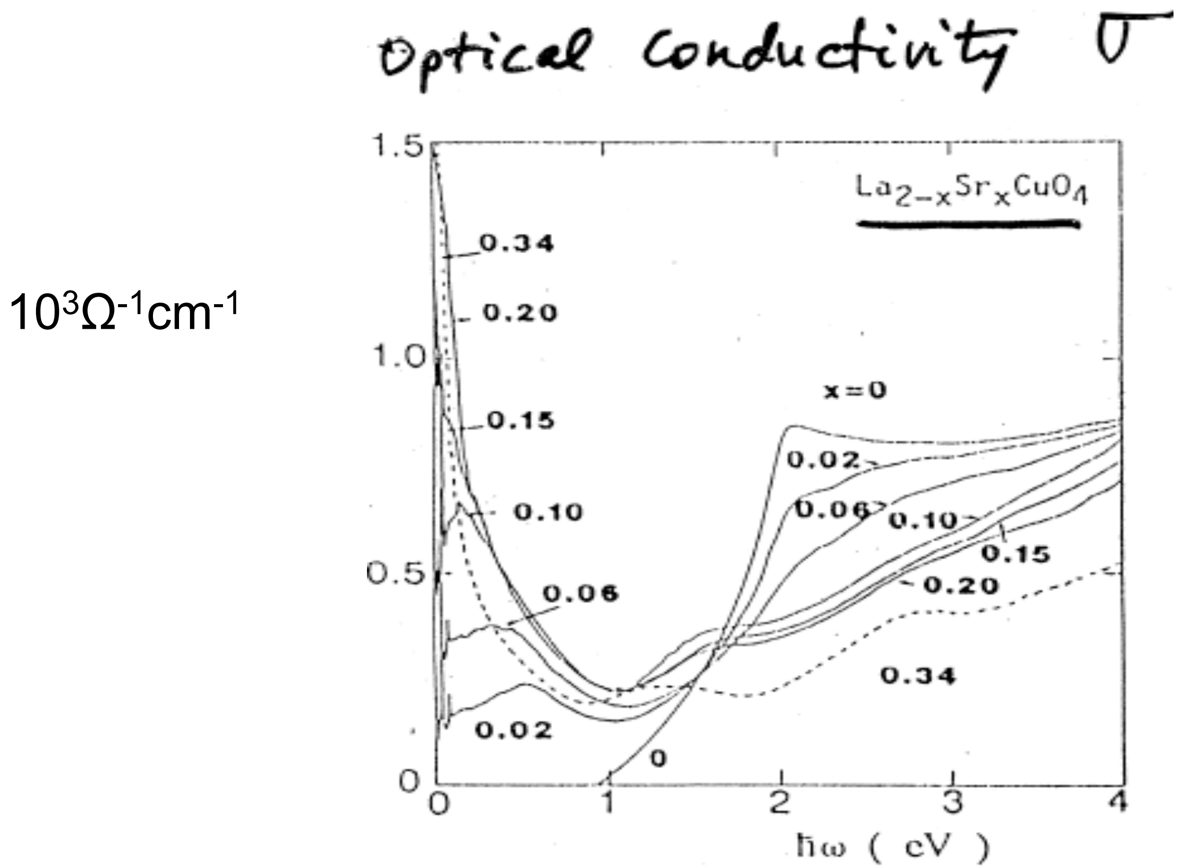
- Hole doped Oxides:
  $\sim 15\% Cu^{3+} \rightarrow$ High-T$_c$ Superconductors.

Short-ranged AF Correlations



- AF correlation length ξ equals mean distance between holes.
- AF correlations are short-ranged in optimally doped cuprates (2-3 lattice spacings).

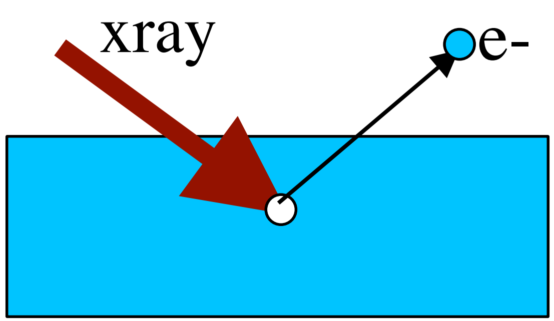
Optical conductivity: Mott versus Drude



Charge gap of 2 eV subsists in the pseudogap phase…

Strong Interaction U

Pseudogap



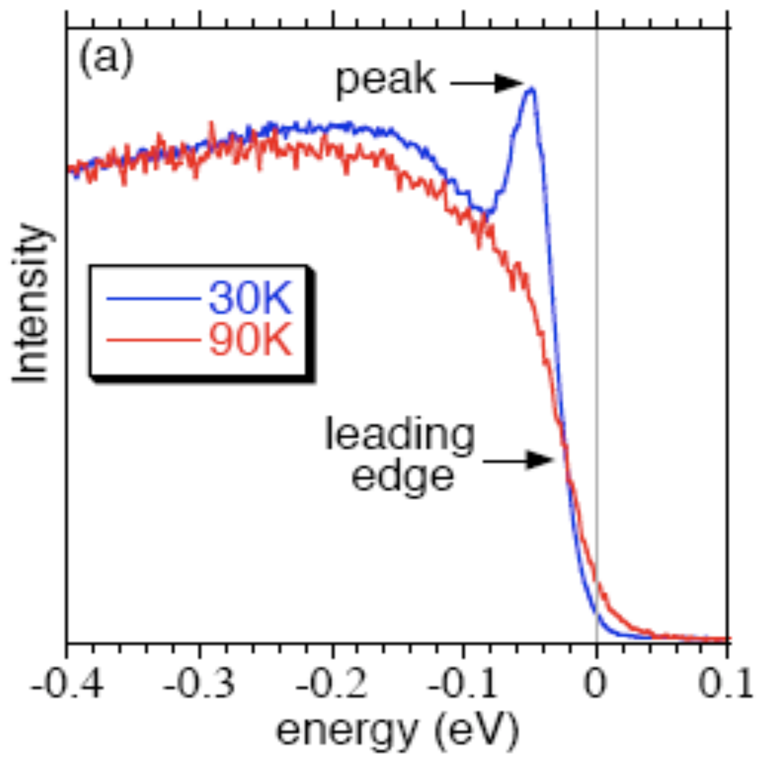
Photoemission

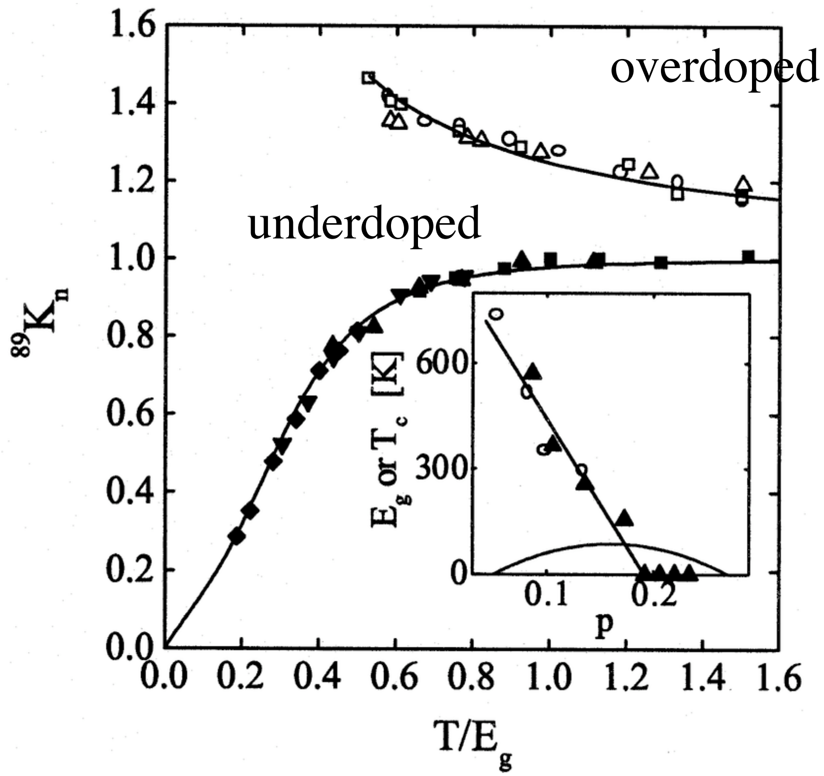
Kaminski-Campuzano

Spin Susceptibility
(Knight Shift)

J.L. Tallon JPCS (1998)

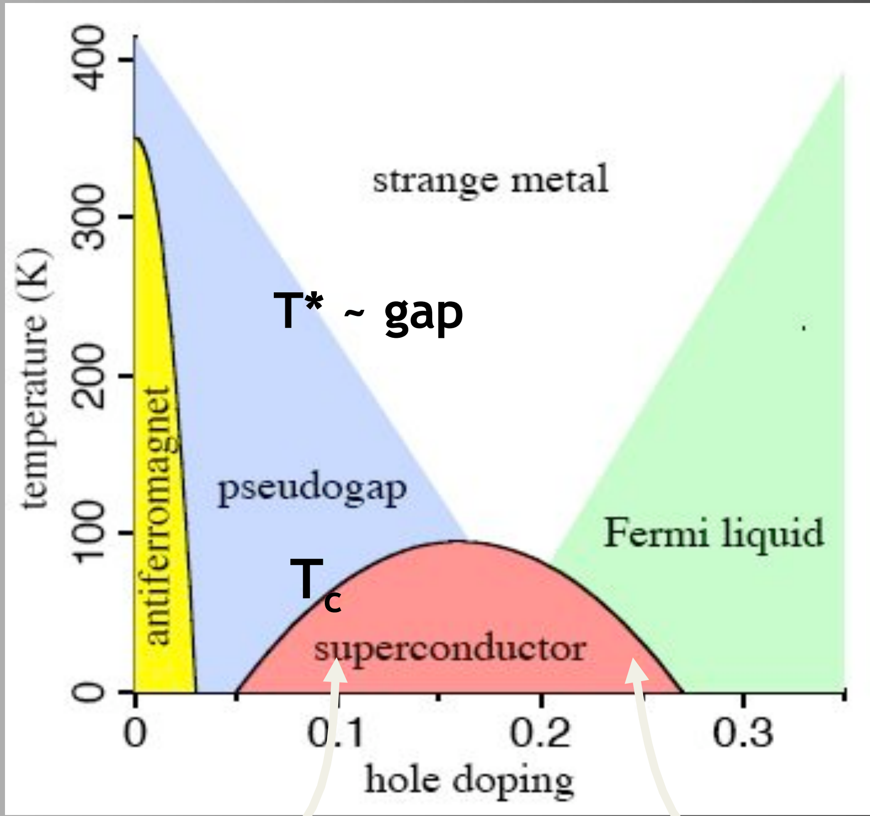
YBCO





Pseudogap in the antinodal directions

Is There a simple model that can explain this?



- Large gap
- Low Tc
- What is going on?
- Simple theory (BCS) would not give us two scales which go in opposite directions!
- Is there a simple model where this happens?

- Small gap
- Large pairs
- Low Tc
- Conventional BCS

…

Is |Neel> a good ground state?

$$H = \sum_{\langle i;j \rangle} J \vec{S}_i \cdot \vec{S}_j$$

The different components of $\vec{S}_i$ donot commute! More difficult than classical case

$\vec{S}(\vec{q}=\vec{0}) = \sum_i \vec{S}_i$ commutes with H: Eigenstates of H are also eigenstates of $\vec{S}(\vec{q}=\vec{0})^2$ and $S_z(0)$

2 sublattices: A and B

$$|Neel\rangle = \prod_{i \in A, j \in B} |S_{iz} = S, S_{jz} = -S\rangle$$

A $\uparrow$ $\downarrow$ $\uparrow$ $\downarrow$ B …

Not a good eigenstate of H and $\vec{S}(\vec{q}=\vec{0})^2$

But, |Neel>, ground state at half-filling in 2d: $\langle S_{iz} \rangle \sim \pm 0.3$

Physics of 2D doped Mott insulator

Doping 2D Mott insulator: Fermi surface

Challenging question: many channels

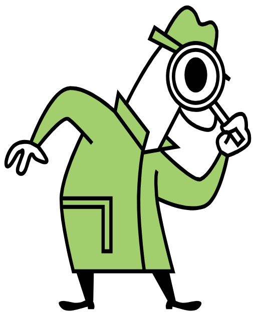
Antiferromagnetic fluctuations

Cooper and Umklapp channels

Difficult to build a 100% exact theory: Approximations

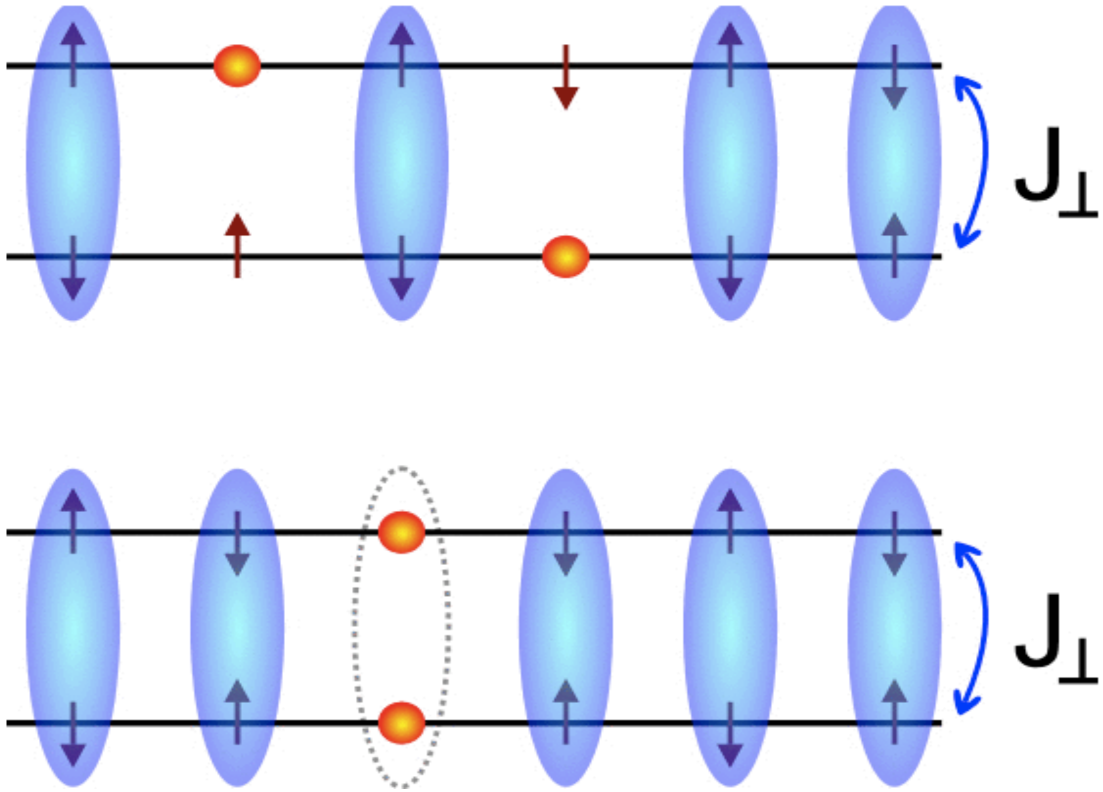
Quasi 1D: Crossover between weak and strong coupling…

We learn a lot from this (rigorous) approach



Pedestrian way: 2 chains…

Dagotto and Rice, Science **271**, 618 (1996)



Imagine a strong antiferromagnetic coupling between chains: Singlets

By doping, the holes pair to minimize magnetic energy $\longrightarrow$ SC

Mott versus SC: emergent in ladder

Undoped case: example of d-wave RVB system with Spin Gap (Anderson)

Dagotto and Rice, Science **271**, 618 (1996)

$$|RVB> = P_{D=0} \sum_{\langle i;j \rangle} F(i-j) c^{\dagger}_{1i\uparrow} c^{\dagger}_{2j\downarrow} |Vac>$$

$$P_{D=0} = \prod_i \prod_\alpha (1 - n^{\alpha}_{i\uparrow} n^{\alpha}_{i\downarrow})$$

Short-range function

Doping: Superconductivity emerges (quasi-long range order in 1D at T=0)

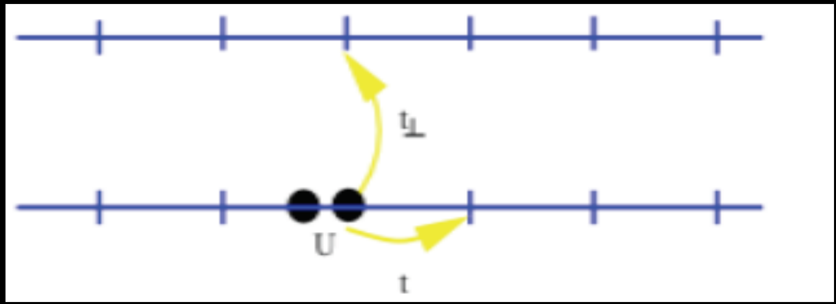
$$< \Delta(x) \Delta^{\dagger}(0) > \propto x^{-1/2}$$

See also Gogolin/Nersesyan/Tsvelik book

Universal exponent for a few holes

Weak coupling regime

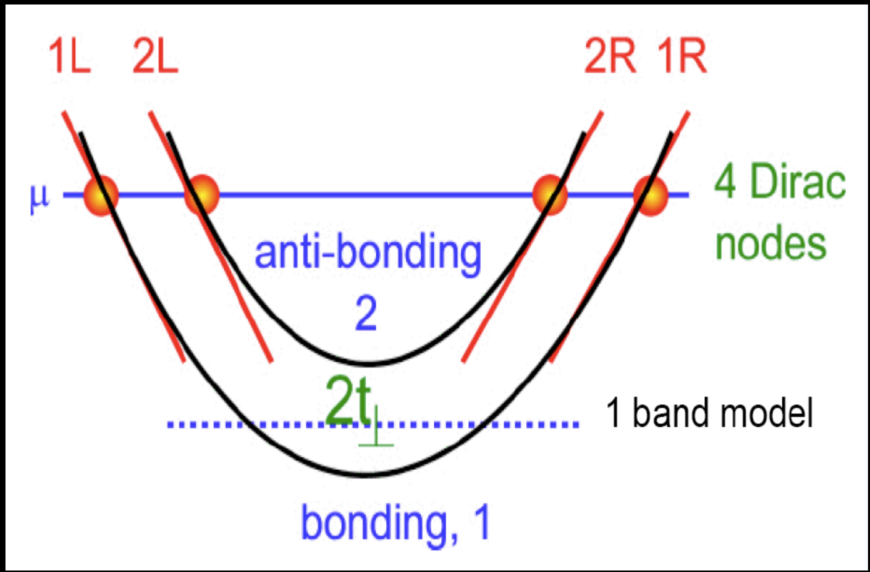
First, diagonalize the spectrum



$$\epsilon_j(k) = \mp t_\perp - 2t\cos(k)$$

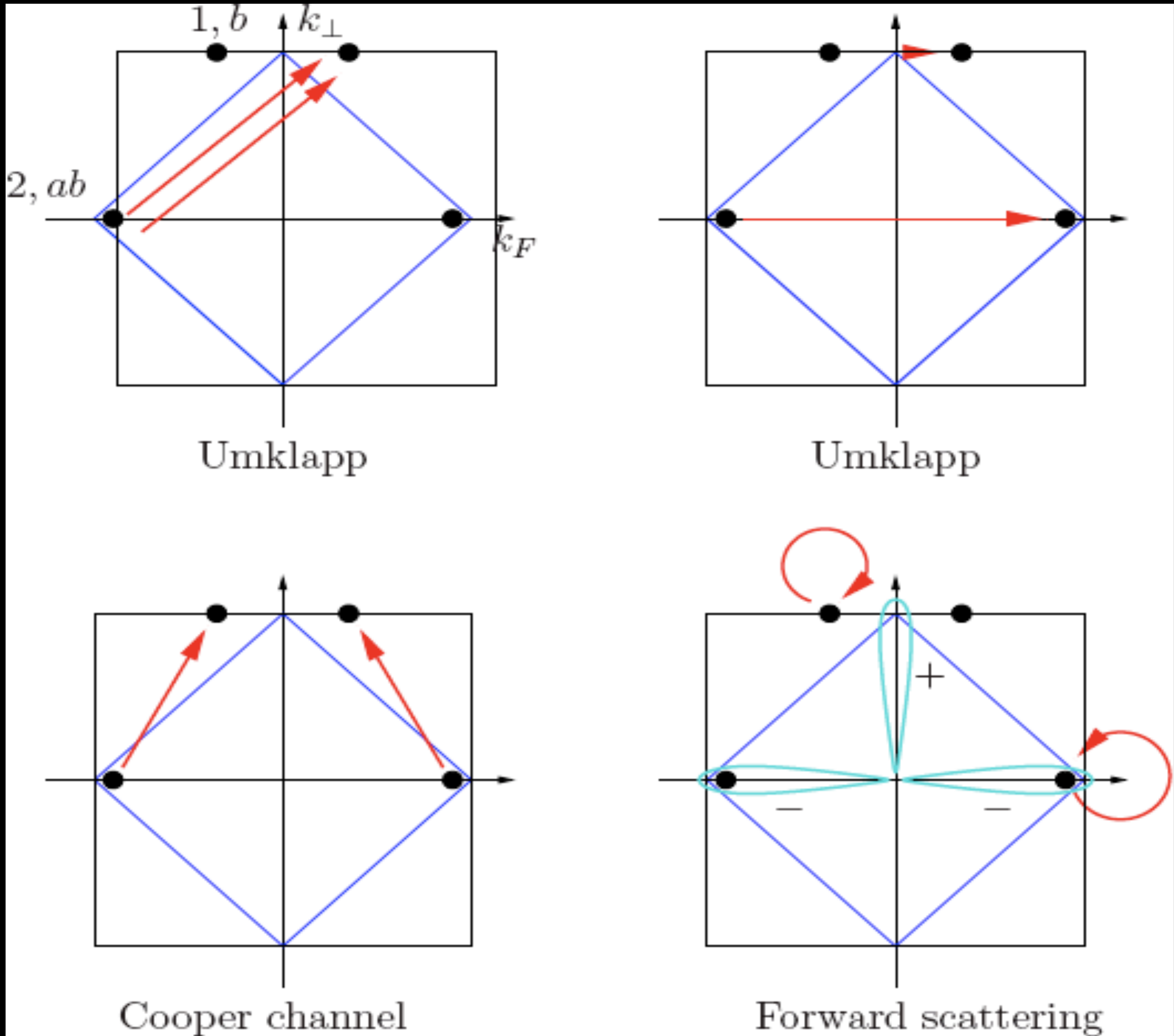
Half-filling: $\mathbf{v_1} = \mathbf{v_2}$
and $k_{F1} + k_{F2} = \pi$

Large Doping: $\mathbf{v_2} \ll \mathbf{v_1}$



Urs Ledermann & K. Le Hur, PRB **61**, 2497 (2000)
M. P. A. Fisher, Les Houches Notes, 1998

2D Interpretation of couplings



Example: RG for spinless fermions

Urs Ledermann & K. Le Hur, PRB **61**, 2497 (2000)

Away from Half-filling

$$H_{Int} = \int dk_1 dk_2 dk_3 dk_4 \delta(k_1 + k_3 - k_2 - k_4)$$
$$\times [c_1 \Psi^\dagger_{R1}(k_1)\Psi_{R1}(k_2)\Psi^\dagger_{L1}(k_3)\Psi_{L1}(k_4) + c_2(1 \leftrightarrow 2)$$
$$+ f_{12}(\Psi^\dagger_{R1}(k_1)\Psi_{R1}(k_2)\Psi^\dagger_{L2}(k_3)\Psi_{L2}(k_4) + 1 \leftrightarrow 2)$$
$$+ c_{12}(\Psi^\dagger_{R1}(k_1)\Psi_{R2}(k_2)\Psi^\dagger_{L1}(k_3)\Psi_{L2}(k_4) + 1 \leftrightarrow 2)].$$

$$\frac{dc_1}{dl} = -\frac{1}{2\pi v_2} c_{12}^2$$
$$\frac{dc_2}{dl} = -\frac{1}{2\pi v_1} c_{12}^2$$
$$\frac{df_{12}}{dl} = \frac{1}{\pi(v_1 + v_2)} c_{12}^2$$
$$\frac{dc_{12}}{dl} = \frac{c_{12}}{\pi}\left(\frac{2 f_{12}}{v_1 + v_2} - \frac{c_1}{2v_1} - \frac{c_2}{2v_2}\right)$$

$E \sim t e^{-l}$

Forward scattering favors exotic SC…

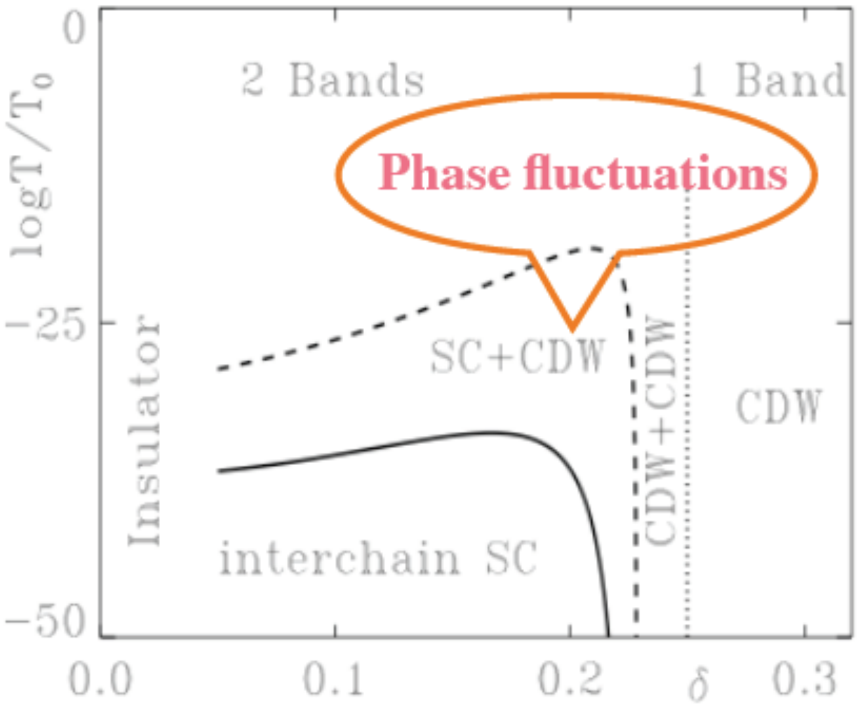
Solvable set of differential equations; strong coupling analysis: bosonization

Half-filling: 7 Couplings diverge with a fixed ratio

"SO(8) Gross-Neveu Model"

Spin- & Charge Gaps $\propto \exp -\pi v_1/U$

Short-Range RVB insulating system with preformed Cooper pairs



U. Ledermann and K. Le Hur, PRB 61, 2497 (2000)

Doping: Cooper pairs liberated
Superconductivity
SO(6) symmetry

Long-range Interactions:
SO(5) symmetry

Lin-Balents-Fisher and others…

SC remains for spinless fermions:
p-wave type SC

Emergent symmetries in IR

SO(5) theory in two dimensions

AF order parameter (S_x,S_y,S_z) and SC order parameter (Re Δ,ImΔ)

<u>2 sites</u>: 2 fermion operators

$$N^\alpha = \frac{1}{2}\left(c^\dagger \tau^\alpha c - d^\dagger \tau^\alpha d\right)$$

$$\Delta^\dagger = -\frac{i}{2} c^\dagger \tau^y d^\dagger$$

10 generators: $\pi^\dagger_\alpha = -\frac{1}{2} c^\dagger \tau^\alpha \tau^y d^\dagger$ + Total charge and spin

E. Demler and S.C. Zhang; 2D non-Fermi liquid with SO(5) theory: LeClair, 2008

Ladders: route to 2 dimensions

2-leg ladder, nice prototype:
Doping (D-Mott) RVB material, d-wave superconductivity

Rice et al., Schulz, Balents and MPA Fisher, Emery-Kivelson,…

3-leg ladder: still better
One can focus both at nodal and antinodal directions

Urs Ledermann, Karyn Le Hur, T. Maurice Rice, PRB **62**, 16383 (2000)
John Hopkinson and Karyn Le Hur, PRB **69**, 245105 (2004)

Extension to large N & comparison to 2 dimensions (in progress)

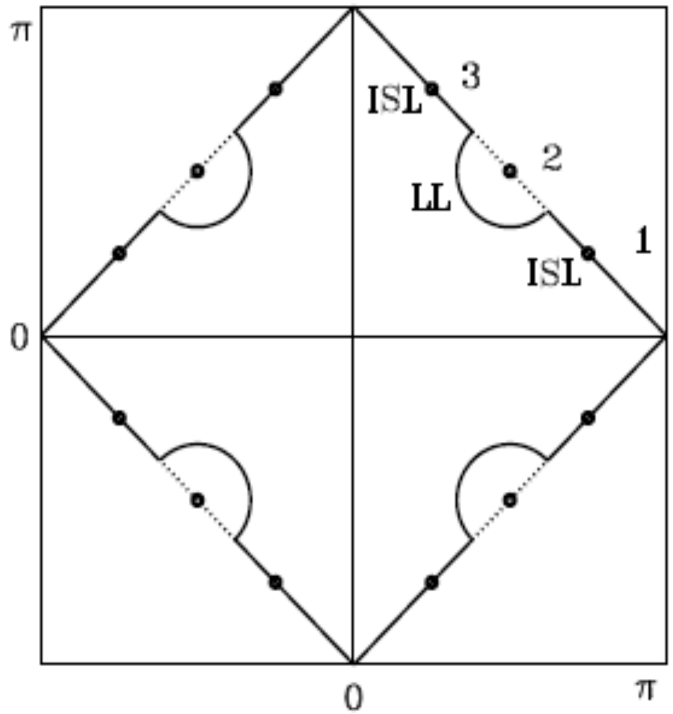
3 band model: summary

$$\boxed{\epsilon_j(k) = -2t\cos(k) - 2t_\perp \cos(k^y_{Fj})}$$

$$\mathbf{k}_{Fj} = \pi - \arccos\left[\frac{t_\perp}{t}\cos\left(\frac{\pi j}{N+1}\right)\right]$$

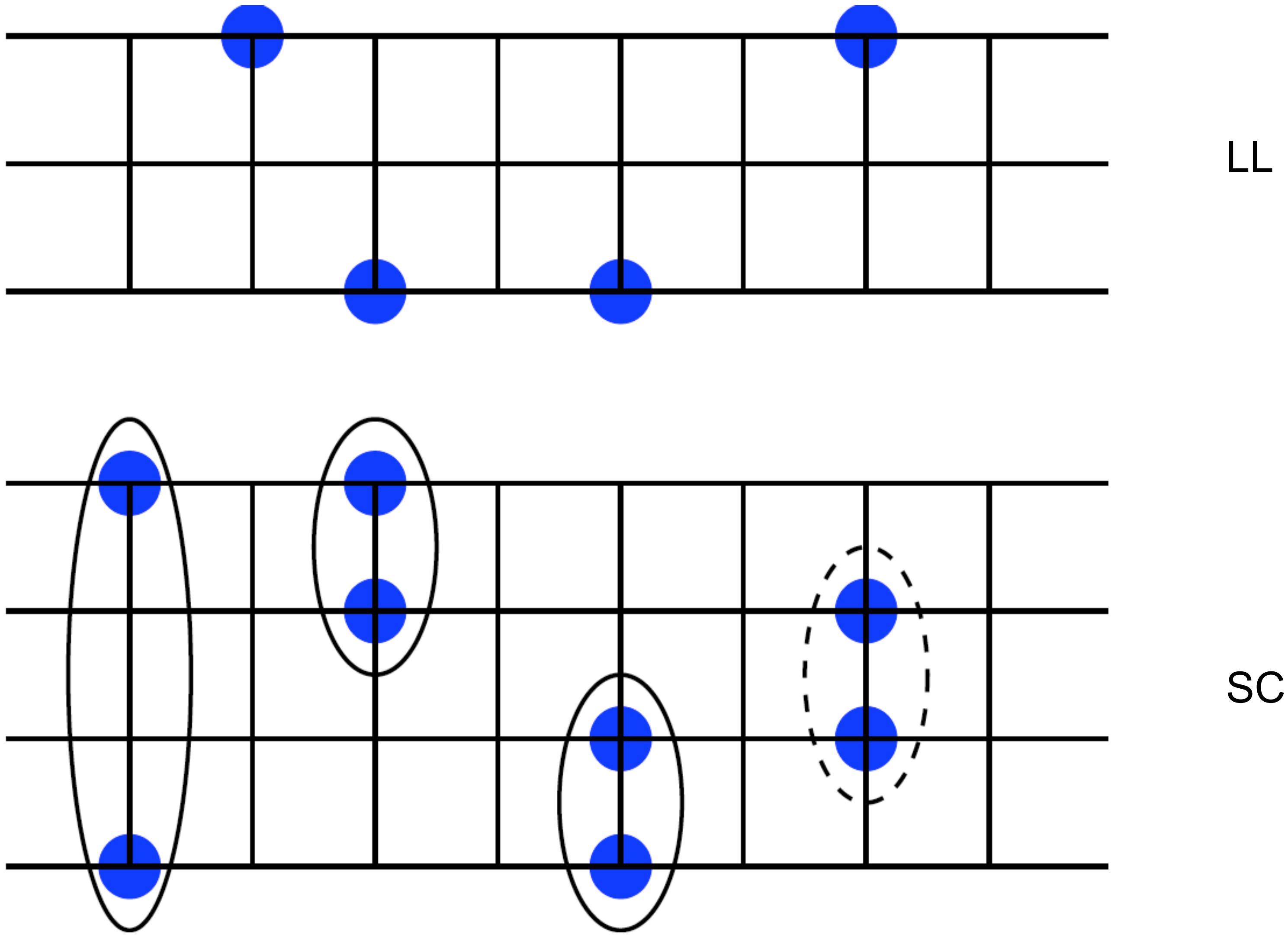
$$\mathbf{k}^y_{Fj} = \pm\frac{\pi j}{N+1}$$

$$v_1 = v_N < v_2 = v_{N-1} < \ldots.$$



Hierarchy of energy scales allow for the truncation of Fermi surface without breaking any symmetry: difficult many couplings (3-leg ladder: 21 RG equations)

Real space picture...



In agreement with White-Scalapino: DMRG

Even-Odd Effect at small N

- **N even: Spin gap**

"disordered d-wave superconductor": 2-chain

- **N odd: No spin gap**

"deconfined spinons": Insulating Single chain

Even-Odd effect like for spin ladders

$SrCu_2O_3$ Spin gap / $Sr_2Cu_3O_5$ No spin gap

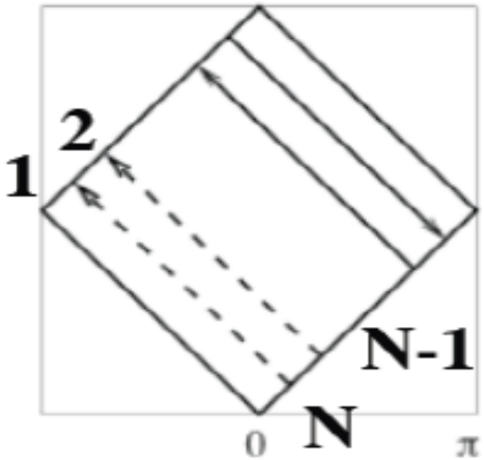
Quasi-1D approach makes sense as long as the energy difference between 2 neighboring bands is larger than the largest energy scale of the system

$$t_\perp/N > te^{-t/U}$$

$$\boxed{U \ll t/\ln N, t_\perp/\ln N}$$

Large N limit at half-filling

AFM channels



Umklapps on the whole Fermi surface favor a uniform Mott Gap…

$$v_1 = v_n \sim t/N \qquad (van\ Hove\ singularity)$$
$$\Delta \sim t e^{-\lambda t/U}$$

$$\langle \vec{S}_i(x) \cdot \vec{S}_j(0) \rangle \propto (-1)^{i+j} \cos(\pi x)/x^{1/N}$$

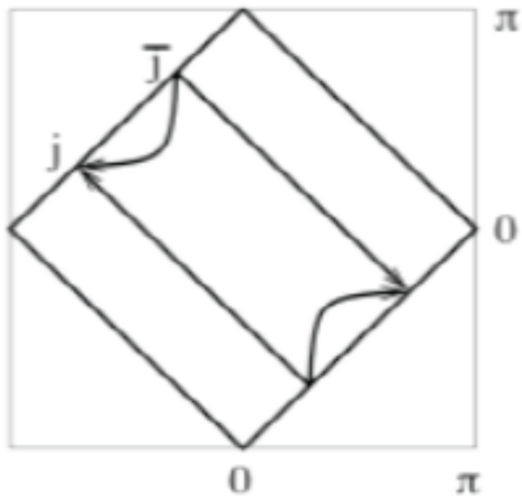
Instability at E_c towards a spin liquid at (π;0)

$$Small\ U \rightarrow E_c \sim t \exp(-a \exp(bN))$$
$$Large\ U \rightarrow J \exp(-0.68N)$$

$$and\ J = 4t^2/U$$

For the small U case, the spin gap now decreases as a double exponential…

D-wave Cooper channel



Unconventional SC when doping

All 4-band AFM channels are cutoff by the chemical potential for $\delta > (t_\perp/t)/N$

But lead to Cooper instabilities between bands $(j, \bar{j})$ and $(j, \bar{j} \pm 1)$

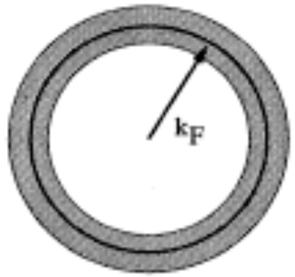
D-wave Superconductivity

$$H = H_{Kin} + \sum_{i,j} \int dx V_{ij} \Delta_i^\dagger \Delta_j$$

where $V_{ij}<0$ for $(i,j)\leq N/2$ and $(i,j)>N/2$ and $V_{ij}>0$ in all other cases

Fermi liquid…

occurs when (4-band) AFM fluctuations and umklapp disappear (no nesting)



Forward scattering gives a contribution of order 1

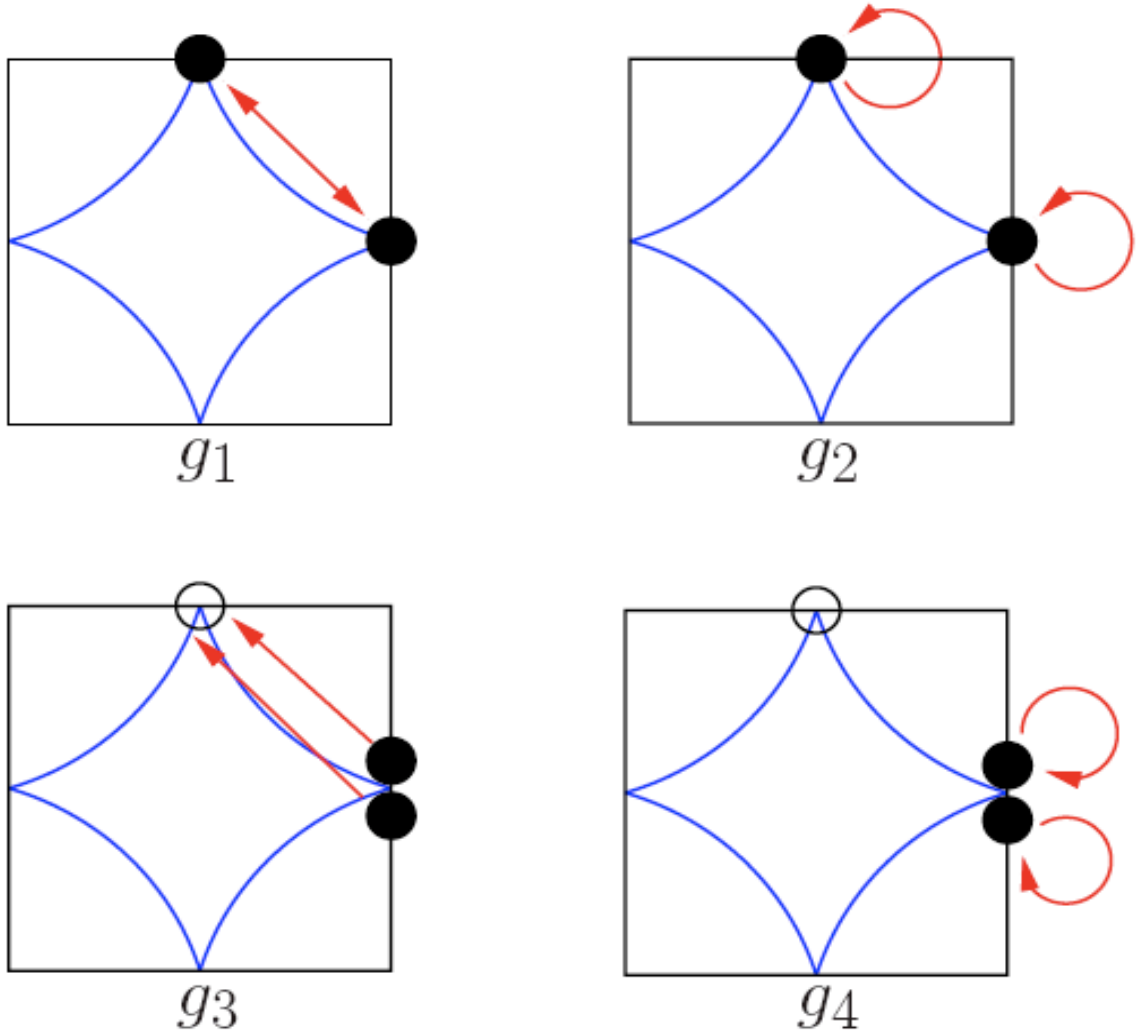
Cooper processes, that favor the Fermi liquid, have a weight ~ N

$$\frac{dV(\theta_1,\theta_2)}{dl} = -\frac{1}{v_F}\int d\theta V(\theta_1,\theta)V(\theta,\theta_2)$$

Θ is the angle parametrizing the Fermi surface

See Lin-Balents-Fisher, 1997

2D: 2-patch model



$$\epsilon(\mathbf{k}) = -2t(\cos k_x + \cos k_y) - 4t' \cos k_x \cos k_y$$

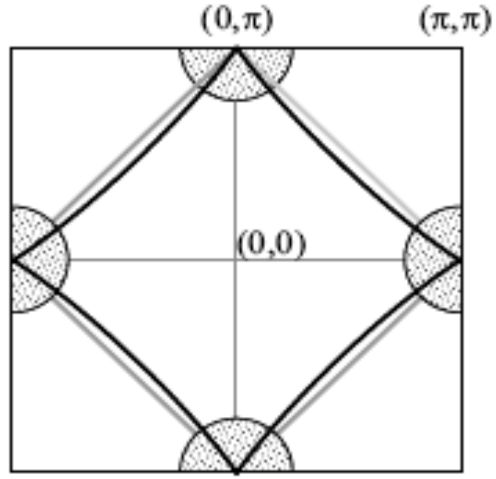
(1-loop) RG equations

P. Lederer, G. Montambaux, D. Poilblanc, J. Physique 48, 1613 (1987)
H.J. Schulz, Europhysics. Lett. 4, 609 (1987)
I. Dzyaloshinskii, Sov. JETP 66, 847 (1987)
N. Furukawa and T.M. Rice, 1999, C. Honerkamp et al.,…

•Pure Hubbard model, t'=0, half-filling
The fixed point is a Mott insulator with long range AF order
$\chi_s(\pi;\pi)$ diverges more strongly
Doping: d-wave SC

•t'=t: Dzyaloshinskii weak-coupling fixed point in 2D

•t'/t<<1 close to half-filling, ~ half-filled 2-leg ladder:
D-Mott state for $U>U_c = F(t'/t)$
T.M. Rice et al.: numerical strong coupling analysis (A. Lauchli)



(a)

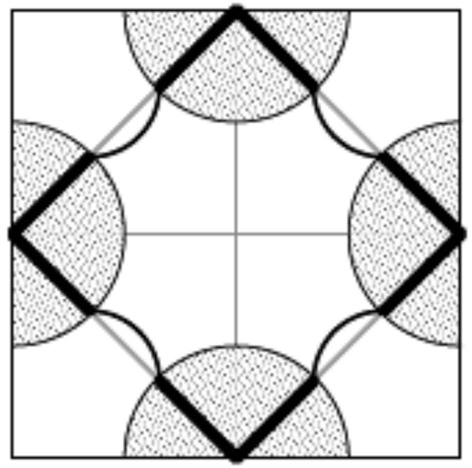
Picture of the pseudogap & SC phase

Insulating antinodal directions:

Spin and Charge gap

At weak U, a unique energy scale

(in agreement with ladder systems)



Nodal directions: Fermi arcs

The remaining (hole-like) Fermi surface consists of 4 arcs: Fermi liquid

Consistent with ARPES experiments, for example, on BSCCO

SC: proximity effects of the Fermi arcs with the RVB region
Andreev scattering

Strong coupling Analysis

t-J model (C. Gros, R. Joynt, and T.M. Rice; J.E. Hirsch (1985))

$$H = -t \sum_{\langle i;j \rangle;\alpha} P_{D=0} \left(c^{\dagger}_{i\alpha} c_{j\alpha} + H.c. \right) P_{D=0} + J \sum_{\langle i;j \rangle} \left(\mathbf{S}_i \cdot \mathbf{S}_j - n_i n_j / 4 \right)$$

•Half-filling: Antiferromagnetic ordering, $J=4t^2/U$ and $\langle S_i^z \rangle = \pm 0.3$

•Away from half-filling: Gutzwiller projection cumbersome to implement

Gutzwiller approximation: Renormalization of bare parameters

t becomes $2xt/(1+x)$ and x is the number of holes

F.C. Zhang and T.M. Rice, 1988
Anderson

Connection with D-wave SC

$$H = -t\delta \sum_{\langle i;j \rangle} \left(c^{\dagger}_{i\alpha} c_{j\alpha} + H.c. \right) - J \sum_{\langle i;j \rangle} b^{\dagger}_{ij} b_{ij}$$

$$b^{\dagger}_{ij} = \frac{1}{\sqrt{2}} \left(c^{\dagger}_{i\uparrow} c^{\dagger}_{j\downarrow} - c^{\dagger}_{i\downarrow} c^{\dagger}_{j\uparrow} \right)$$

Antiferromagnetic fluctuations favor d-wave SC

$$b_{ij} = \Delta = |\Delta| e^{i\theta}$$

Baskaran, Zou, Anderson, 1987: mean-field treatment
Also, slave theories + Gauge theories: Lee, Wen, Nagaosa

BUT: Many decoupling possibilities

At short distance, no difference between AF and D-wave pairing ansatz

However, other decoupling possibilities:

Spin density wave at $Q=(\pi,\pi)$

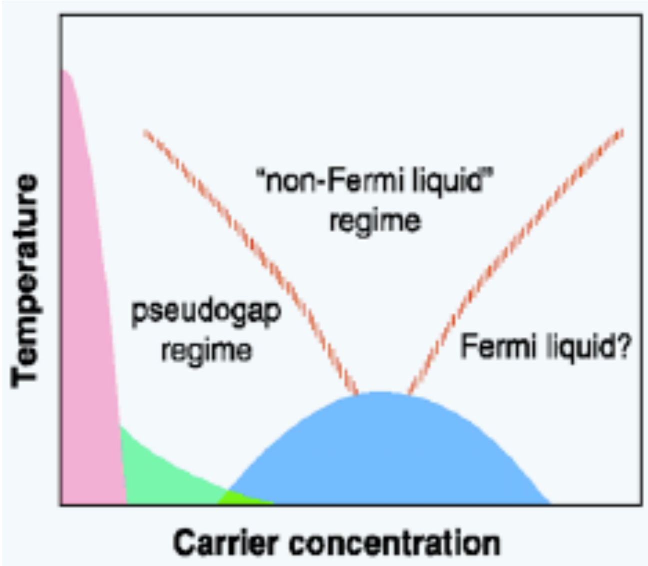
U(1) staggered flux or DDW state (orbital currents),…

See, e.g., E. Zhao & A. Paramekanti, Phys. Rev. B 76, 195101 (2007)

Systematic approach to find the good ground state?

Quasi-1D approach: Bosonization! In 2D?

Conclusion of Lecture II:



A fascinating phase diagram

Rich physics of doped Mott insulator in 2D: truncated Fermi surface, pseudogap

Unconventional SC

Intuition can be gained from Ladder systems

2008: New Superconductors FeAs… Tc ~50K