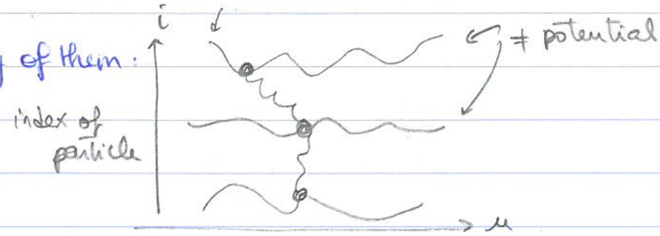
26/07/2017

DISORDERED ELASTIC SYSTEMS

→ Disordered elastic systems are a bit more simple than spin glasses:
- polynomial time algorithms to find ground states
- amenable to analytic methods RG, MF

Equation for a particle in 1d u. $H = V(u) \rightarrow$ $\cancel{m\ddot{u}} + \eta \dot{u} = -V'(u) + f$. (overdamped)

↓ couple elasticly many of them:



↳ random potential: $\overline{V_i(u) V_j(u')} = \delta_{ij} R_0(u-u')$

→ Hamiltonian for all the particles: $H = \sum_i \frac{c}{2}(u_{i+1} - u_i)^2 + V_i(u_i)$

↳ eq. motion: $\eta \partial_t u_i = -\frac{\delta H}{\delta u_i} = c(u_{i+1} + u_{i-1} - 2u_i) - V'(u_i) + f$

⇒ continuous space limit $u_i \rightarrow u(x)$: $H_V[u] = \frac{1}{2}\int d^dx \, c(\nabla_x u)^2 + V(x, u(x))$

$\overline{V(x,u)V(x',u')} = \delta^d(x-x') R_0(u-u')$ → breaking of translational invariance

properties / remarks: 1) $u(x) \rightarrow \vec{u}(\vec{x}) \in \mathbb{R}^N$ wit $\vec{x} \in \mathbb{R}^d$ $N=1$ interface, $d=1$ line

2) Elastique energy: $\frac{1}{2}\int_q c q^2 u_q u_{-q} \rightarrow |q|^\alpha u_q u_{-q}$ LR elasticity

DISORDER:

3) $R_0(u)$ smooth → short range: random bond RB

long range: random field RF

$R_0(u)$ periodic for lattices

↳ we want look at the zero temperature $\min_u H = E_{min} \rightarrow u_{min}(x)$

with the equation of motion $\eta \partial_t u(x,t) = c\nabla_x^2 u(x,t) + F(x, u(x,t))$

↳ $F(x,u) = -\partial_x V(x,u)$ depinning force

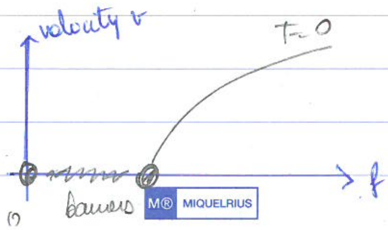
$\Delta(u) = R''(u)$

two T=0 critical points

FRG fixed point:

$\overline{(u(x)u(0))^2} \sim x^{2\zeta_{eq}}$ T=0 f=0

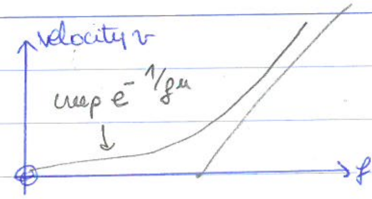
$\sim x^{2\zeta_{dep}}$



→ critical exponent $\zeta(d, N, \alpha, \text{RB}, \text{RF}, \text{RP})$ $\zeta_{eq}\left(\begin{smallmatrix} d=1 \\ N=1 \\ SR \end{smallmatrix}\right) = 2/3$

Activation at finite temperature:

$\hookrightarrow$ prediction $\mu = \dfrac{d-2+2\zeta_{eq}}{2-\zeta_{eq}} \overset{d=1}{=} 1/4$



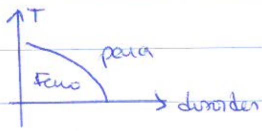
$\tau \sim e^{E_b/T}$

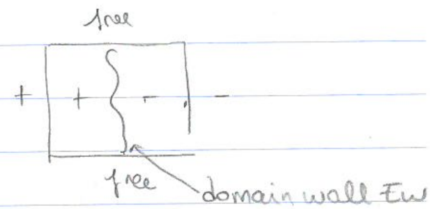
1. $v \sim f\, e^{-E_b/T}$

2. $E_b(f) \sim v_c \left(\frac{f_c}{f}\right)^{\mu}$ $\quad f \to 0$

$v \sim v_0\, e^{-\frac{v_c}{T}\left(\frac{f_c}{f}\right)^{\mu}}$

$H = -\sum_{\langle ij \rangle} (J + \delta J_{ij}) s_i s_j - h_i s_i$
RB $\qquad$ RF

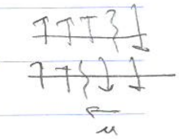




energy of domain wall $E_{dw} \sim 2J \,\text{length} \sim 2J \int \frac{dx}{a} \sqrt{1+\left(\frac{du}{dx}\right)^2} \sim cste + \frac{J}{a} \int dx\, (\nabla_x u)^2$

RB $\to$ short range disorder $V(x,u) = 2/a\, \delta J(x,u)$

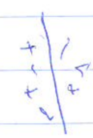
RF $\to$ long range disorder $V(u) - V(0) = 2 \sum_i h_i \quad \sim \overline{(V(u)-V(0))^2} = u$



PINNING, DEPINNING PHENOMENOLOGY.

1) periodic potential $V(x,u) = g \sin u \quad \eta \dot{u} = -g \cos u + f \quad f_c = g$

2) $V(x,u)$ random for $c = \infty \to$ straight line, $F_{dis} \sim \int dx\, \partial_u V(x,0) \sim L^{1/2}$
$\qquad \doteq -fL$
can adapt $\quad \{ f_c = 0 \quad$ sum of gaussian random numbers



Larkin model $\quad V(x,u) = -f(x) u$
$\hookrightarrow$ random $\overline{f(x) f(x')} = W \delta^d (x-x') \quad W = -R_0''(0) = \Delta_0(0)$