

Landau Fermi Liquid and Heavy Fermion [Coleman]

Thus we have metals no longer justified to account states of.

"metals" do overview relevant about itinerant states atoms

robust Increasing localization: $5d \quad 4d \quad 3d \quad 5f \quad 4f$

methods of interest at the Fermi level (200-300 K) localization
Ce ... (A, Z) ... Yb

Interesting physics occur at the crossover between itinerant and localized system.

f-spins are always localized

{ high-T : local moment metal

{ low-T : spins "quench" to form heavy fermions.

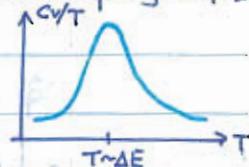
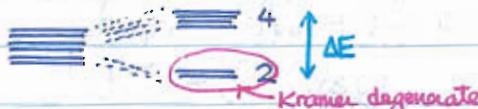
Spin = localized moment, when e^- lost all its charge degree of freedom

e.g. Ce^{3+} (4f¹) : $L=3, S=1/2, J=L-S=5/2$

At high-T, $\chi = \frac{n_e M^2}{3T}$, $M^2 = g_J^2 \mu_B^2 J(J+1)$.

$S_Q = k_B \ln(2J+1)$ [unquenched \equiv all states equally occupied]

Under crystal field,



At low-T, these materials form Fermi liquid:

$$E_F = \frac{p^2}{2m^*}; N^*(0) = \frac{m^* p_F}{\pi^2 \hbar^3}; \chi = \frac{\mu_B^2 N^*(0)}{1 + F_0^2}, \gamma = \lim_{T \rightarrow 0} \left(\frac{C_V}{T} \right) = \frac{\pi^2 k_B^2}{3} N^*(0)$$

$$\Rightarrow W = \frac{\chi}{\gamma} = 3 \left(\frac{\mu_B^2}{2\pi k_B} \right) \frac{1}{1 + F_0^2}$$

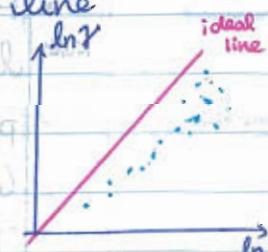
Although 4f/5f metals vary 3 orders of magnitudes in both

χ and γ , the ratio W seems to fit on a line

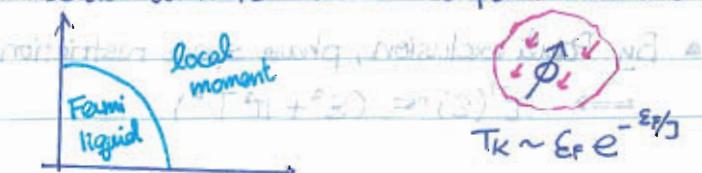
most effects are accounted for by $N^*(0)$

NOTE: actual material below the ideal line

because of correction by g_J and F_0^2



Local moment interact with itinerant e^- to form resonant states



[Caveat] remnant magnet has bipolar instead unipolar

- Possible Phases of heavy fermion system

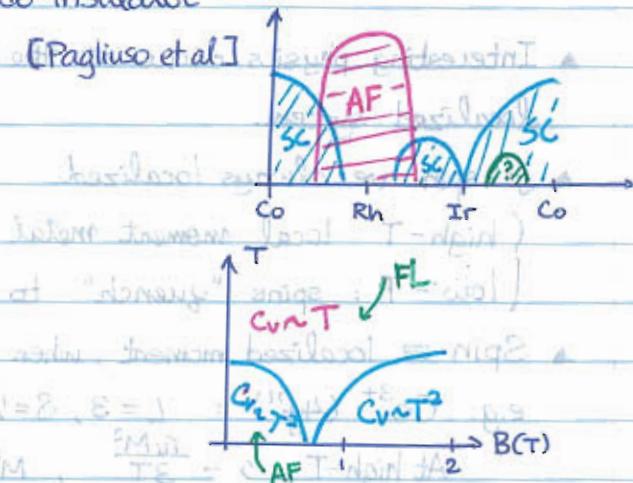
- metal, superconductor, Kondo insulator, quantum critical pt, "others"

- UBe_3 : local moment \rightarrow metal \rightarrow superconductor

But traditionally (\sim BCS) local moment is thought to destroy SC.

- $Ce_3Pt_4Bi_3$: Kondo insulator

- $Ce(Co, Rh, Ir)In$ [Pagliuso et al.]



- $YbRh_2Si_2$: QCP at "corner"

- UPd_2Al_3 : SC + AF coexistence

Landau Fermi Liquid

- Turn on interaction adiabatically; Pauli exclusion \rightarrow Fermi surface robust; excitation spectrum preserved.

Hence, $|e\rangle \xrightarrow{\text{same quantum number}} |qp\rangle = W \frac{m^*}{m} = \frac{N(O)^*}{N(O)} = 1 + \frac{F_s}{3}$

From 1950 \rightarrow 1960, it is realized that one can drop the long-ranged part of Coulomb and capture the essential physics by remnant short-ranged interaction (e.g. Hubbard model, Anderson model, etc.)

This is facilitated by discovery/experiments on 3He

- By Pauli exclusion, phase space restriction

$$\rightarrow \Gamma(\varepsilon) \propto (\varepsilon^2 + \pi^2 T^2)$$

▲ Thus we have Landau energy functional:

$$\mathcal{E}_l = \mathcal{E}_0 + \sum_{p,\sigma} (E_{p\sigma}^{(0)} - \mu) \delta n_{p\sigma} + \frac{1}{2} \sum f_{p\sigma p'\sigma'} \delta n_{p\sigma} \delta n_{p'\sigma'}$$

elements to utilize!



▲ Landau energy functional \sim "fixed point" Hamiltonian
(Shankar, RMP, 94')

▲ Warning: $E_{p\sigma} = E_{p\sigma}^{(0)} + \sum_{p'\sigma'} f_{p\sigma p'\sigma'} \delta n_{p'\sigma'} [E_{p'\sigma'}] \neq E_{p\sigma}^{(0)}$

This is a feedback (aka self-consistent) condition & lead to renormalization of excitation energy.

▲ Entropy: $S = -k_B \sum_{p,\sigma} (n_{p\sigma} \ln n_{p\sigma} + (1-n_{p\sigma}) \ln (1-n_{p\sigma}))$
 $\implies n_{p\sigma} = \frac{1}{e^{\beta E_{p\sigma}} + 1} = f(E_{p\sigma})$

As $T \rightarrow 0$, $\delta n_p \rightarrow 0$, $n_{p\sigma} = f(E_{p\sigma}^{(0)})$
 $V_F = \frac{\partial E_{p\sigma}}{\partial p} = \frac{p_F}{m^*}$

▲ This give rise to linear heat capacity.

▲ By rotation invariance, $\begin{cases} f_{p\sigma p'\sigma'} = f_{pp'}^s + f_{pp'}^a \sigma \sigma' \\ f^{sa} = f^{sa}(\cos \theta) = \hat{p} \cdot \hat{p}' \end{cases}$
 $\implies f^{sa}(\cos \theta) = \frac{1}{N^*(0)} \sum_{l=0}^{\infty} (2l+1) F_l^{s,a} P_l(\cos \theta)$

Landau parameters

▲ Let $\delta E_{p\sigma}^{(0)} = b_l P_l(\cos \theta)$

$$\delta E_{p\sigma} = a_l P_l(\cos \theta)$$

large in heavy fermion

▲ From this we find $\begin{cases} \chi_s = \frac{\mu_B N^*(0)}{1 + F_0^s} = \mu_B^2 N^*(0) (1 - A_0^s) \\ \chi_c = \frac{N^*(0)}{1 + F_0^s} = N^*(0) (1 - A_0^s) \end{cases}$

$$\text{where } A_0^{s,a} = \frac{F_0^{s,a}}{1 + F_0^{s,a}}$$

= N(0) in heavy fermion

(The A 's can be interpret as T-matrix amplitude for s-wave scatter

▲ Since χ_s large while χ_c unrenormalized, $1 - A_0^s \approx 0$.

• Heavy Fermion \approx Local Landau Fermi Liquid

$$\Delta A_{\sigma\sigma'\sigma'} = \frac{1}{N^*(0)} (A_0^s + A_0^a \sigma\sigma')$$

$$A_0^{(0)} = A_0^s + A_0^a \approx 0 \quad \text{and} \quad \chi_c \approx 0 \quad \text{locality of moments}$$

$$\Rightarrow A_0^s = -A_0^a \approx 1$$

$$\Delta \begin{cases} p(T) = p_0 + AT^2 \\ C_V(T) = \gamma T \end{cases}; \quad \frac{A}{\gamma} = \alpha_{\text{kw}} \text{ is approx. const., \& is universal}$$

$$\text{at low } T: \log \beta = \log \beta_0 - \frac{\mu_0}{kT} + \frac{AT^2}{2} = \log \beta_0 - \frac{\mu_0}{kT} + \frac{A}{2}$$

at low T : moments (functions of β) $\propto \beta^{-1}$ \propto $\exp(-\mu/kT)$

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$$(6, 7, -1) \beta_0, (6, 7, -1) \beta_0 \sqrt{3} \beta_0^2 = \beta_0; \text{ partial}$$

$$\beta_0^2 = \frac{\beta_0^2}{\beta_0^2} = \beta_0^2 \leftarrow \text{approx. localization}$$

$$\frac{\beta_0^2}{\beta_0^2} \beta_0^2 = \beta_0^2 \propto \beta_0^2 \propto T^2 \propto A$$

$$\frac{\beta_0^2}{\beta_0^2} = \frac{\beta_0^2}{\beta_0^2} = \beta_0^2$$

higher term $\propto \beta_0^2$ $\propto \beta_0^2 \propto T^2$

$$7\beta_0^2 \beta_0^2 + \beta_0^2 = \beta_0^2 \leftarrow \text{commutator relation}$$

$$\hat{q} \cdot \hat{q} = (\hat{q}_x \hat{q}_y)^2 = \beta_0^2$$

$$(0, 2, 0) \beta_0^2 \beta_0^2 (1, 2, 1) \beta_0^2 (0, 1, 1) = (0, 2, 0) \beta_0^2 \beta_0^2 \leftarrow \text{anticommuting relation}$$

$$(0, 2, 0) \beta_0^2 \beta_0^2 = \beta_0^2 \beta_0^2$$

moment moment no sign!

$$(0, 2, 0) \beta_0^2 \beta_0^2 = \beta_0^2 \beta_0^2$$

$$(7A-1)(0)M_0^2 = \frac{(0)M_0^2}{\beta_0^2 + 1} = 0 \quad \text{but we write out!}$$

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$$\frac{0}{\beta_0^2 + 1} = 0 \quad \text{but we write out!}$$

(principle reason of abelian gauge theory \propto temperature of mass eA unit)

$$0 \neq \beta_0^2 A - 1 \quad \text{where } \beta_0^2 \text{ is sign of } eA^2$$