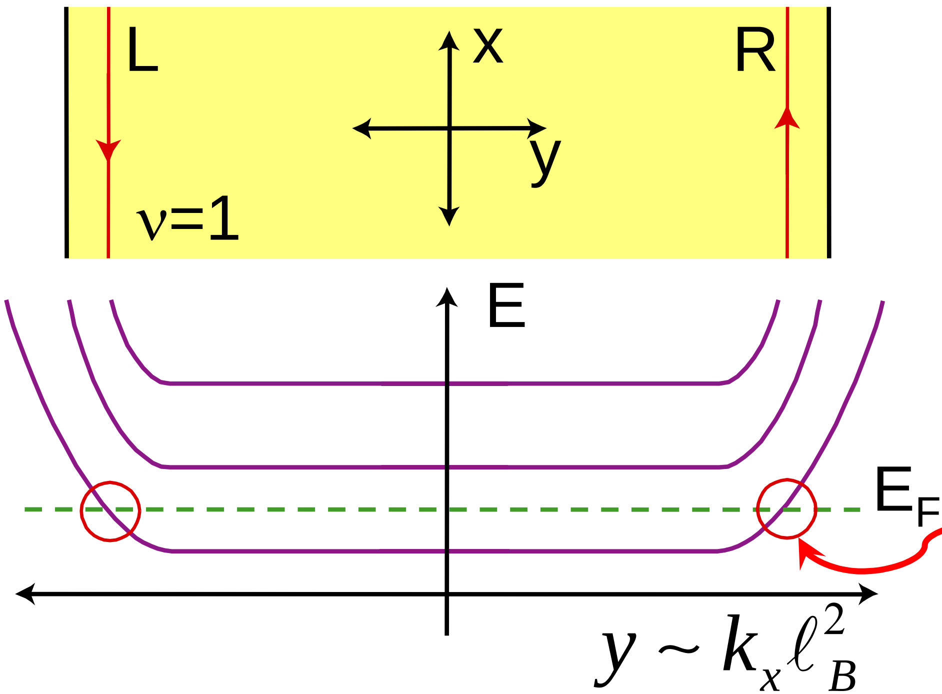
One Dimensional Electronic Systems:

- Quantum Hall Edge States:
  Chiral Luttinger Liquid

- Quantum Wires
  - Semiconductor wires
  - Polyacetalene
  - Carbon Nanotubes

Integer Quantum Hall Effect with edges (Halperin 1982)



Bulk Landau levels:

$$E_n(k_x) = \hbar\omega_c\left(n + 1/2\right)$$

$$\omega_c = eB/mc$$

$$\ell_B = \sqrt{\hbar c/eB}$$

Chiral Fermions ψ_R

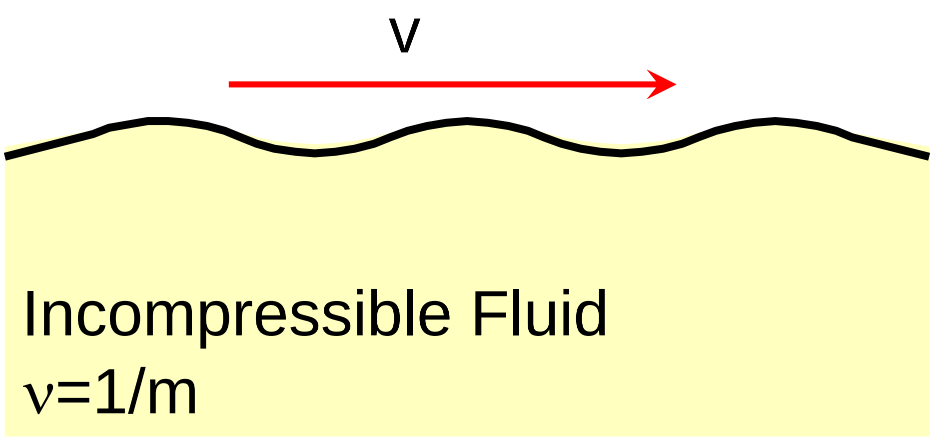
$$L = -\frac{1}{4\pi}\int dx\,\partial_x\phi_R\left(\partial_t\phi_R + \mathrm{v}\,\partial_x\phi_R\right) \quad ; \qquad \psi_R^\dagger \sim \frac{\kappa}{2\pi x_c} e^{i\phi_R} \quad ; \qquad n = \frac{\partial_x\phi_R}{2\pi}$$

Interactions: $V_0\left(\partial_x\phi_R\right)^2/\left(2\pi\right)^2$ only affect velocity v.

Fermi Liquid exponents (g=1) even with interactions

Fractional Quantum Hall Effect: Chiral Luttinger Liquid

Wen 1990



Assume a single edge mode:

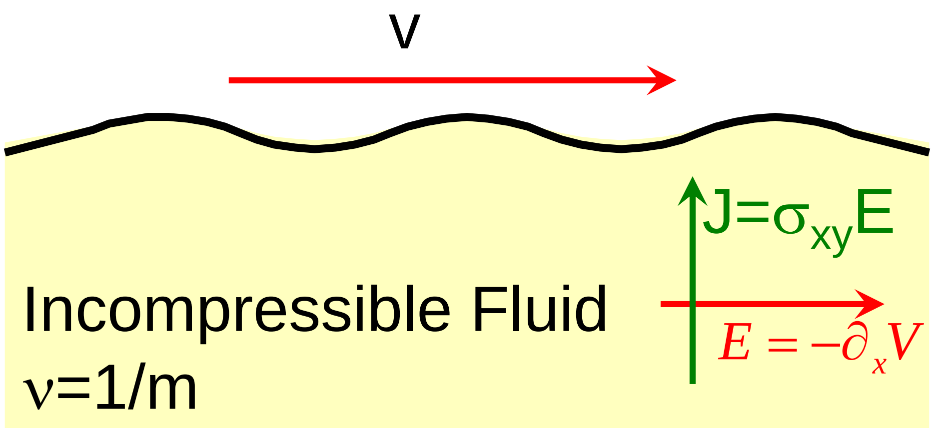
- Laughlin States ν=1/m
- “Sharp” Edge

$$L = -\frac{1}{4\pi g}\int dx\,\partial_x\phi_R\left(\partial_t\phi_R + \mathrm{v}\,\partial_x\phi_R\right)$$

g is determinied by the Chiral Anomaly

Fractional Quantum Hall Effect: Chiral Luttinger Liquid

Wen 1990



Single Mode approximation:
Valid for

- Laughlin States ν=1/m
- “Sharp” Edge

$$L = -\frac{1}{4\pi g}\int dx \partial_x \phi_R \left(\partial_t \phi_R + \mathrm{v}\partial_x \phi_R\right) + \mathrm{V(x)}\frac{\partial_x \phi_R}{2\pi}$$

Equation of motion :

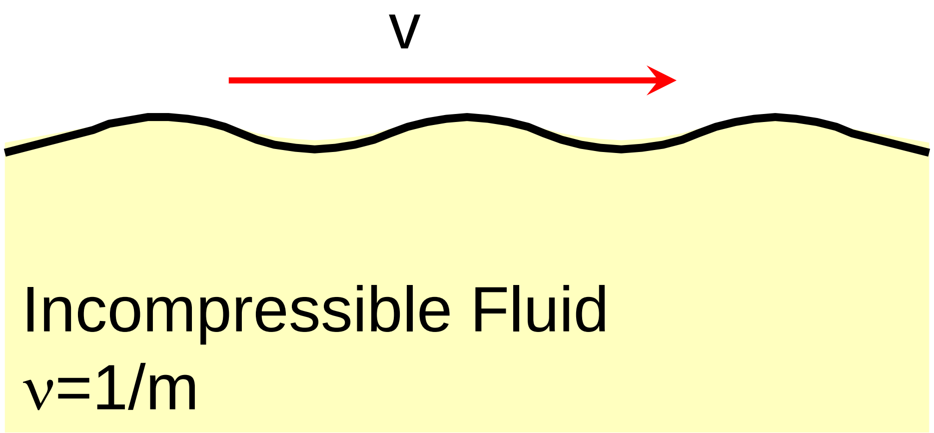
$$\partial_t \frac{\partial_x \phi_R}{2\pi} + \partial_x \frac{\mathrm{v}\partial_x \phi_R}{2\pi} = -\frac{g}{2\pi}\partial_x V$$

$$\partial_t \rho + \partial_x j = g\frac{e^2}{h}E = \sigma_{xy} E$$

$$g = \nu = \frac{1}{m}$$

Fractional Quantum Hall Effect: Chiral Luttinger Liquid

Wen 1990



Assume a single edge mode:

- Laughlin States ν=1/m
- “Sharp” Edge

$$L = -\frac{m}{4\pi}\int dx\,\partial_x\phi_R\left(\partial_t\phi_R + \mathrm{v}\,\partial_x\phi_R\right)$$

$$\left[\frac{\partial_x\phi(x)}{2\pi},\phi(x')\right] = \frac{1}{m}\delta(x-x')$$

Electron op. (charge e)

$$\psi_e^\dagger \sim e^{im\phi}$$

Laughlin Q.P. op. (charge e/m)

$$\psi_{QP}^\dagger \sim e^{i\phi}$$

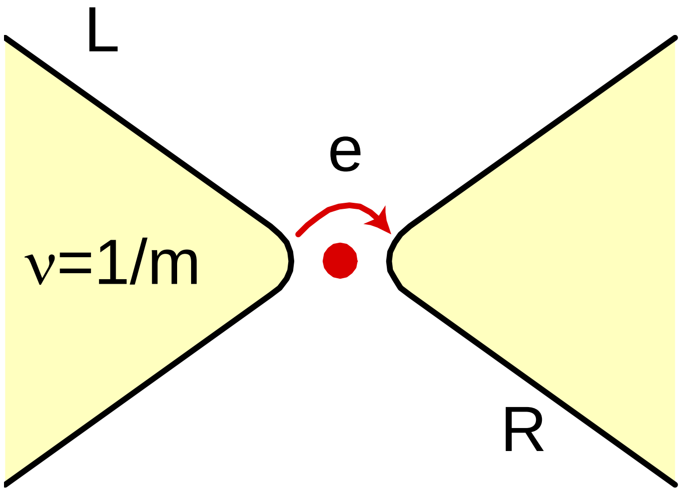
Quantum Point Contact: Analog of single barrier in L.L.

Weak (electron) Tunneling

$$H_T = t\psi_R^\dagger \psi_L \sim te^{im(\phi_R - \phi_L)}$$

$$G(T) \sim t^2 T^{2m-2}$$

$$\sim t^2 T^4 \text{ for } \nu = 1/3$$

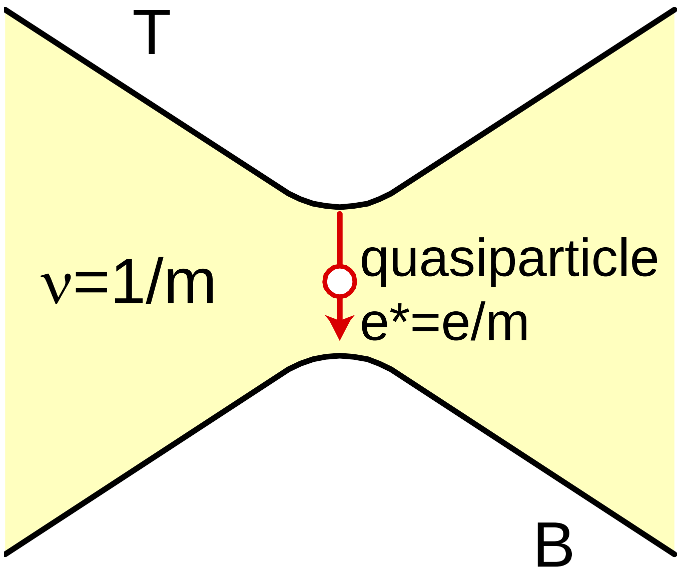


Weak (quasiparticle) Backscattering

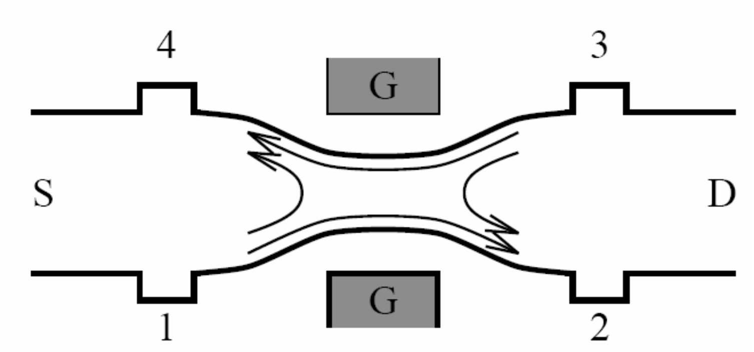
$$H_T = \mathrm{v}\,\psi_{Q.P.,B}^\dagger \psi_{Q.P.,T} \sim te^{i(\phi_B - \phi_T)}$$

$$G(T) \sim \mathrm{v}^2 T^{2/m-2}$$

$$\sim \mathrm{v}^2 T^{-4/3} \text{ for } \nu = 1/3$$

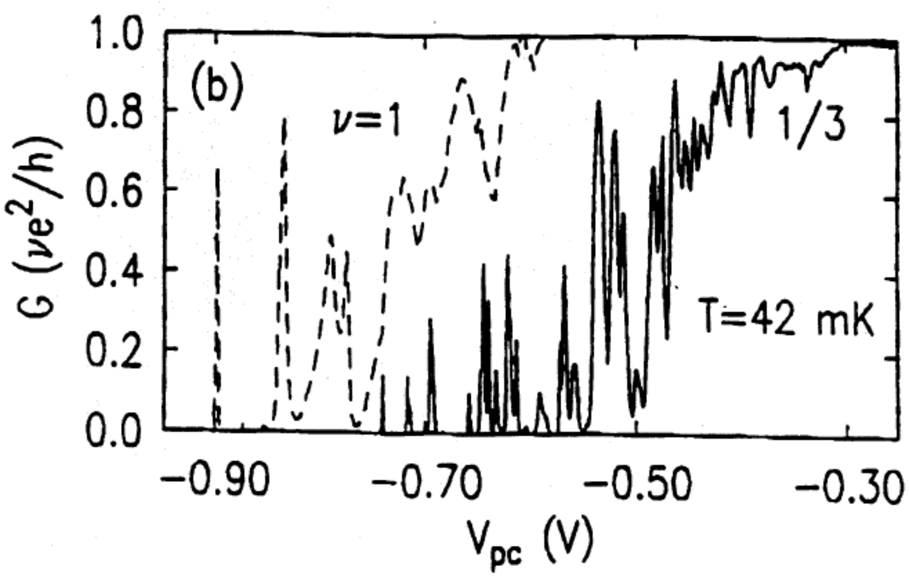


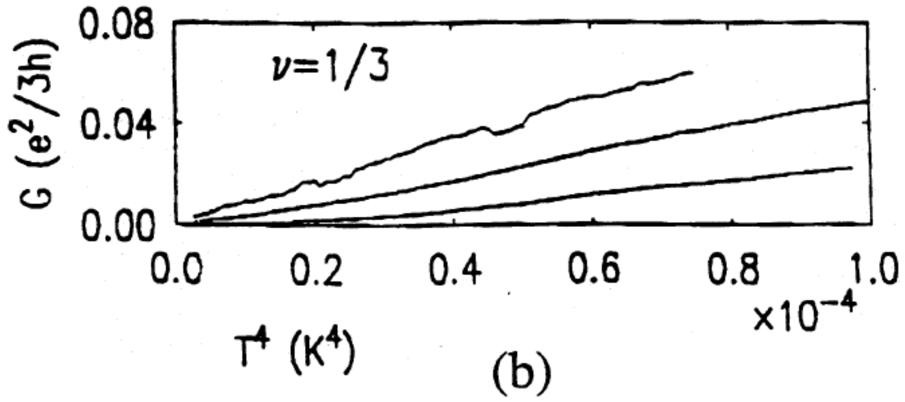
Early point contact experiments (Milliken, Umbach and Webb 1994)



For weak tunneling theory predicts

$G \sim T^4$

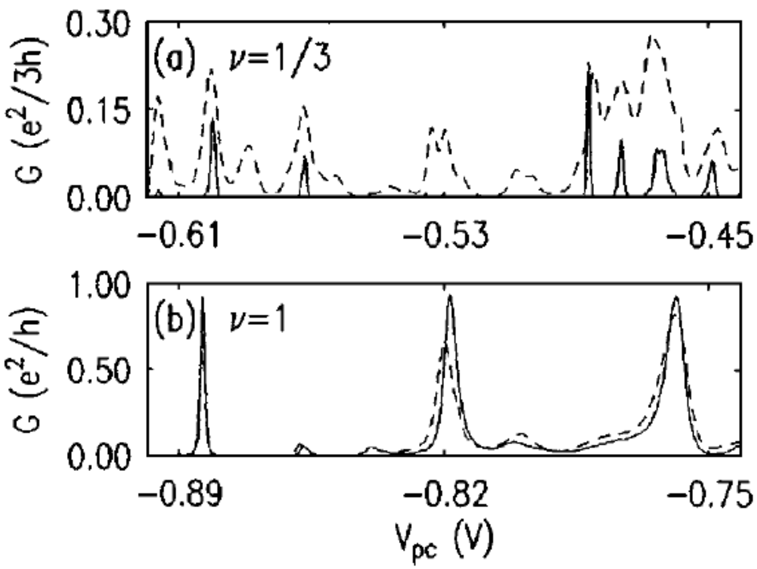


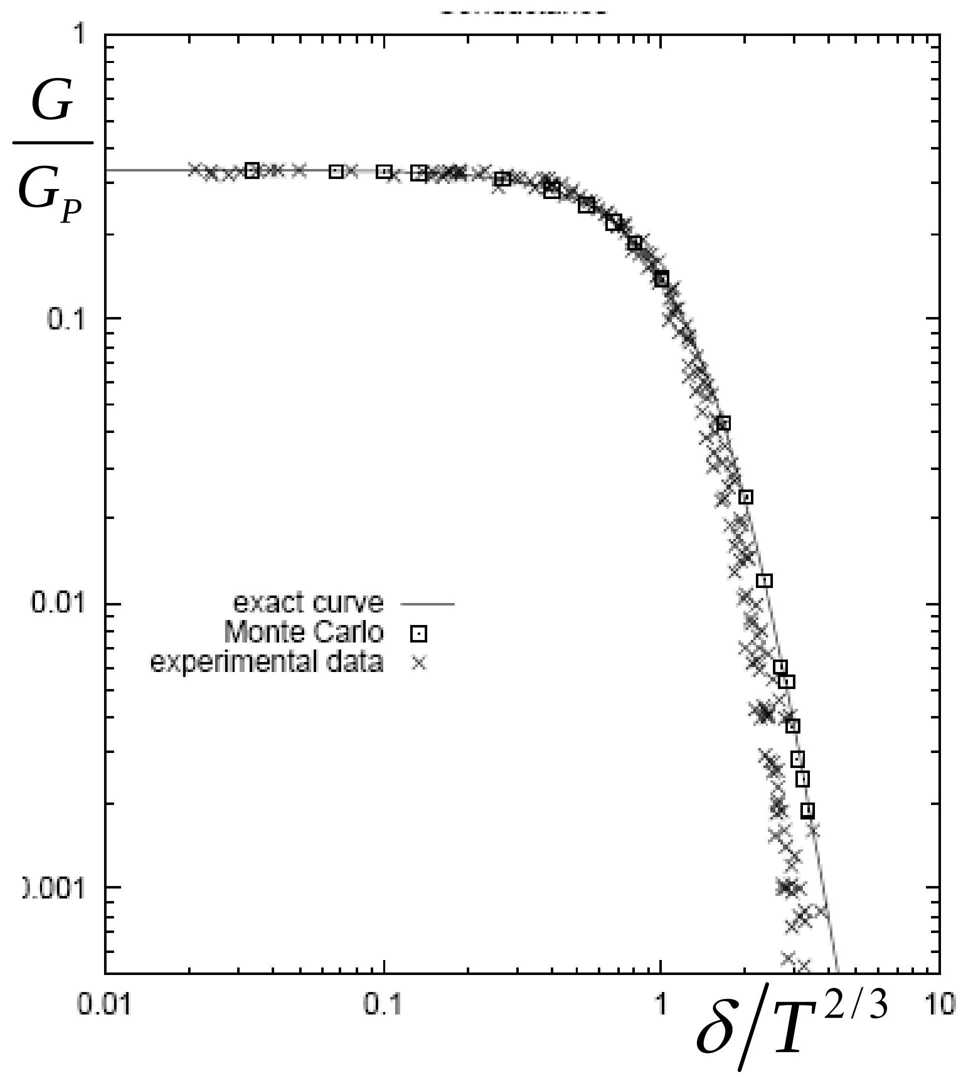


Resonance Lineshapes (Milliken, Umbach and Webb 1994)

For “perfect” resonances theory predicts:

$$G(T,\delta) = \frac{e^2}{3h}\tilde{G}\left(\delta/T^{2/3}\right)$$

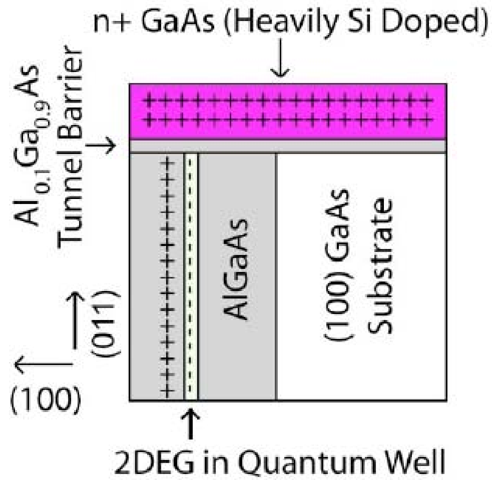


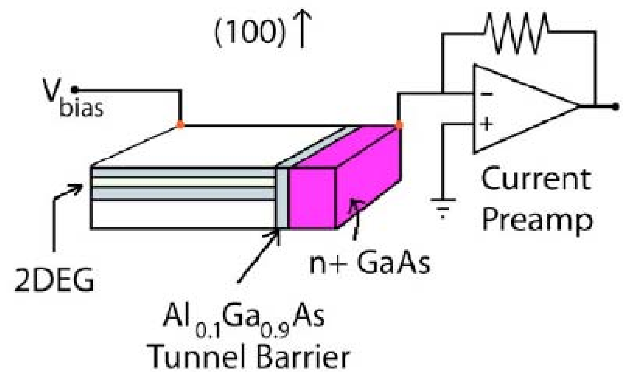


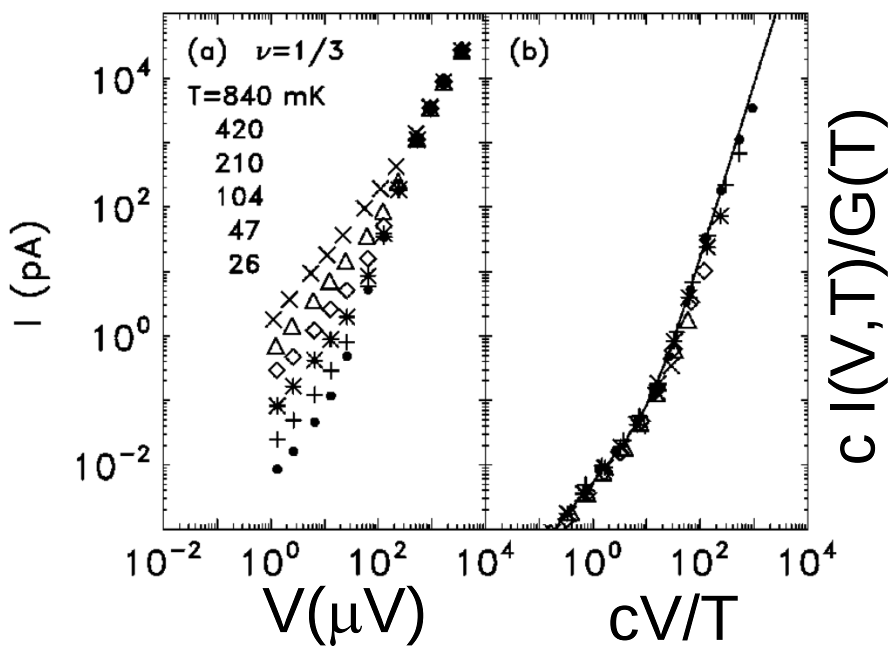
Difficulty for interpretation: Gates produce a smooth confining potential

Cleaved Edge Experiments Chang, Pfeiffer and West, 1996

Tunneling from metal to ν=1/3 at an atomically sharp edge







Predicted Scaling Form:

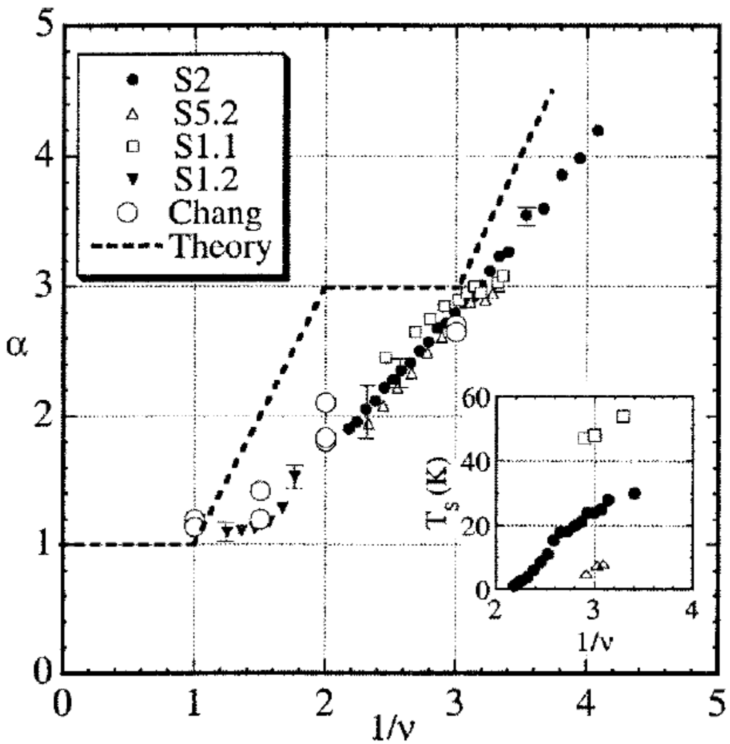
$$I(V,T) = cT^{\alpha} F_{\alpha}(V/T); \quad \alpha = 3$$

$$F_3(x) = x + x^3$$

Fit from Experiment:

α ~ 2.7

Dependence of Tunneling Exponent on Filling Factor



Theory: Kane, Fisher 95
Shytov, Levitov, Halperin 98

Exponent should depend on "neutral modes" predicted to be present for $\nu \neq 1/m$. Leads to plateau structure in $\alpha(\nu)$

Experiment: Grayson, et al, 1998

Exponent seems to depend only on the "charge mode": $\alpha \sim 1/\nu$.

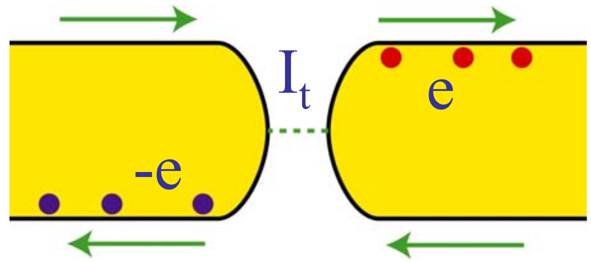
Experiments must not be in the low energy, long wavelength limit described by theory.

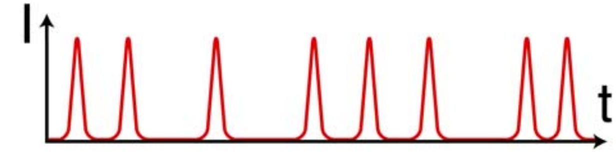
Shot Noise in a point contact: DePicciotto et. al.; Saminadayer et. al. 97

Direct measurement of the charge of the Laughlin Quasiparticle

Strong Pinch Off

e tunneling

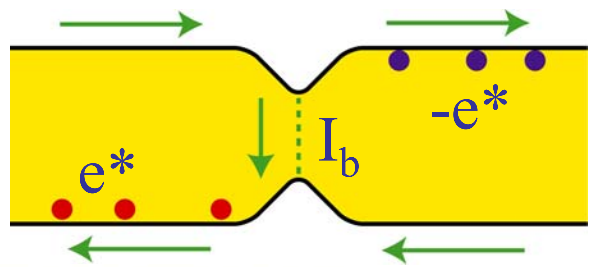


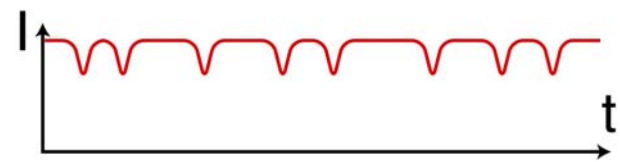


$S = 2e\,I_t$

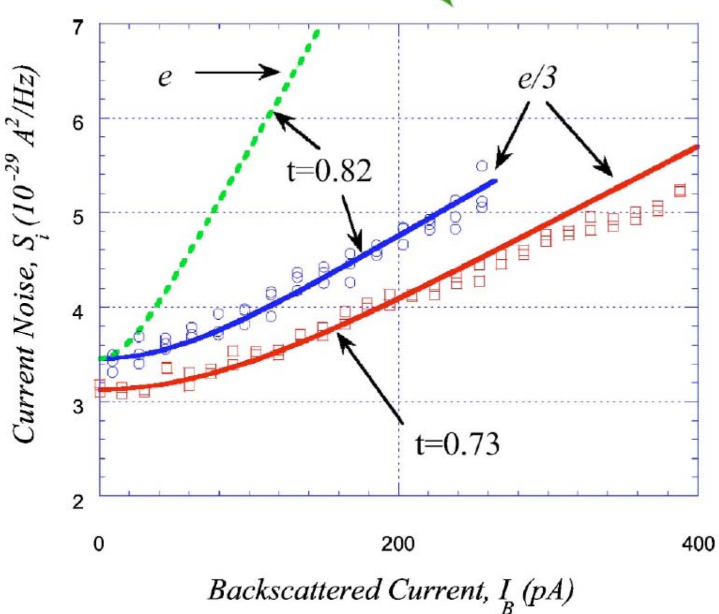
Weak Pinch Off

Q.P. tunneling





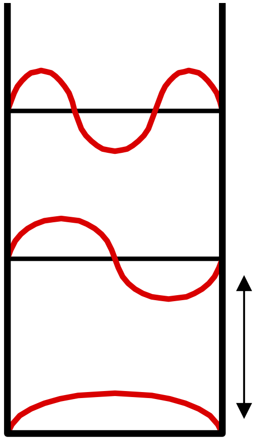
$S = 2e^*\,I_b$



$e^* = e/3$

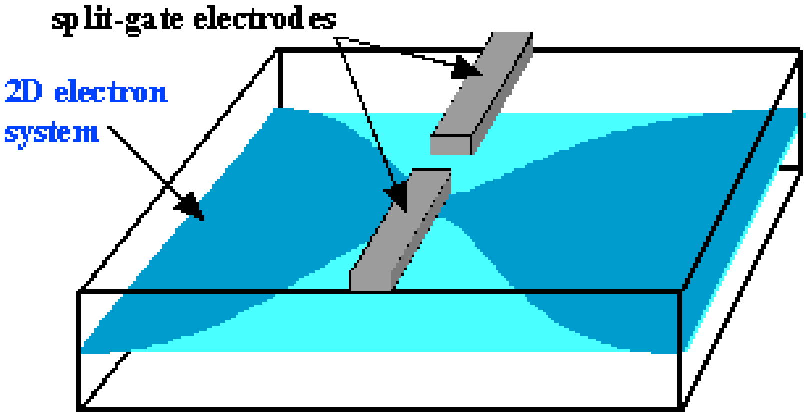
Quantum Wires:

1D for E < ΔE: the subband spacing



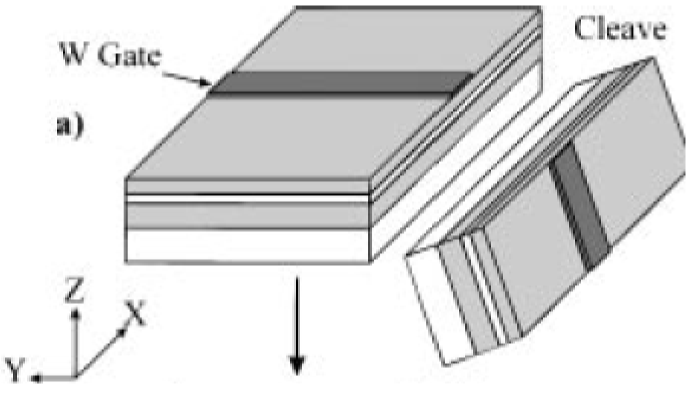
$$\Delta E \sim \frac{\hbar^2}{2m^* L^2}$$

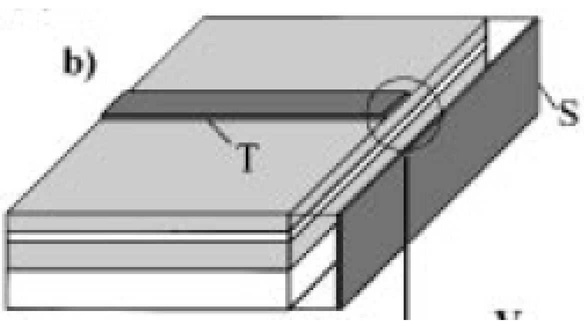
1. Split Gate Devices: (Tarucha et al. 95)



ΔE ~ 5 meV

2. Cleaved Edge Overgrowth (Yacoby et al. 96)

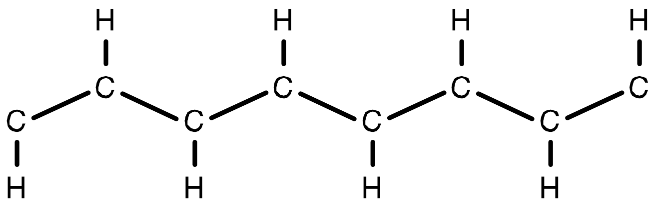


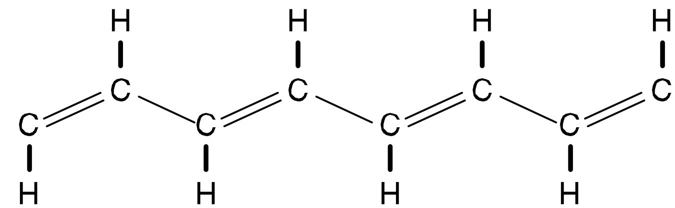


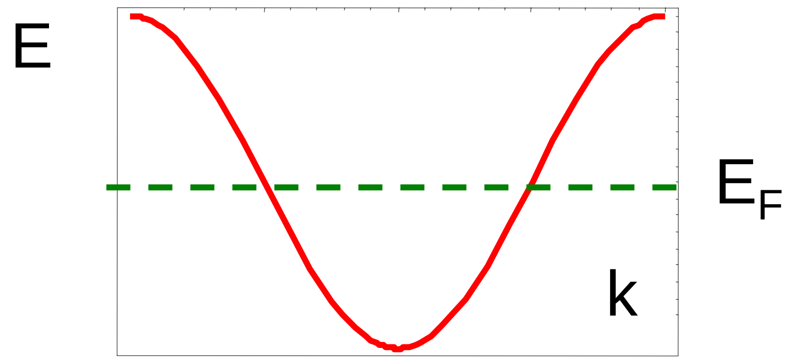
ΔE ~ 20 meV

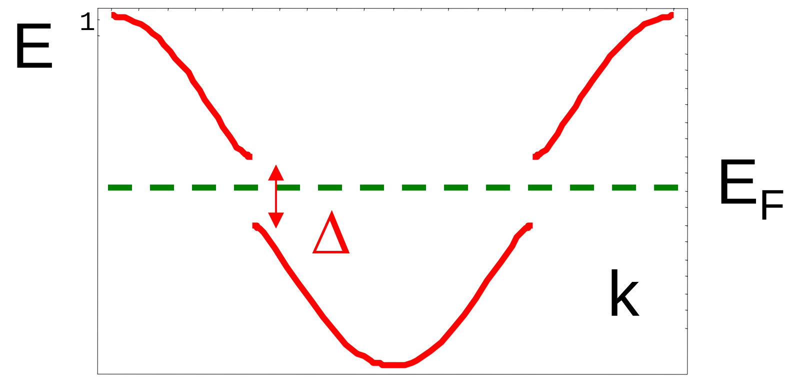
Molecular Wire: Polyacetalene

(See Heeger et al. Rev. Mod. Phys. 1988)









Truly 1D .... but Peierl's Instability:

Any commensurate 1D metal is unstable

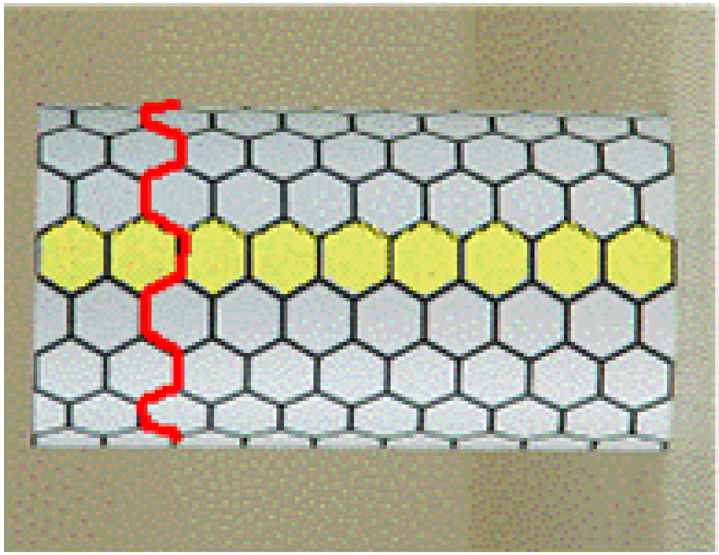
Elastic energy cost : $\sim \Delta^2$

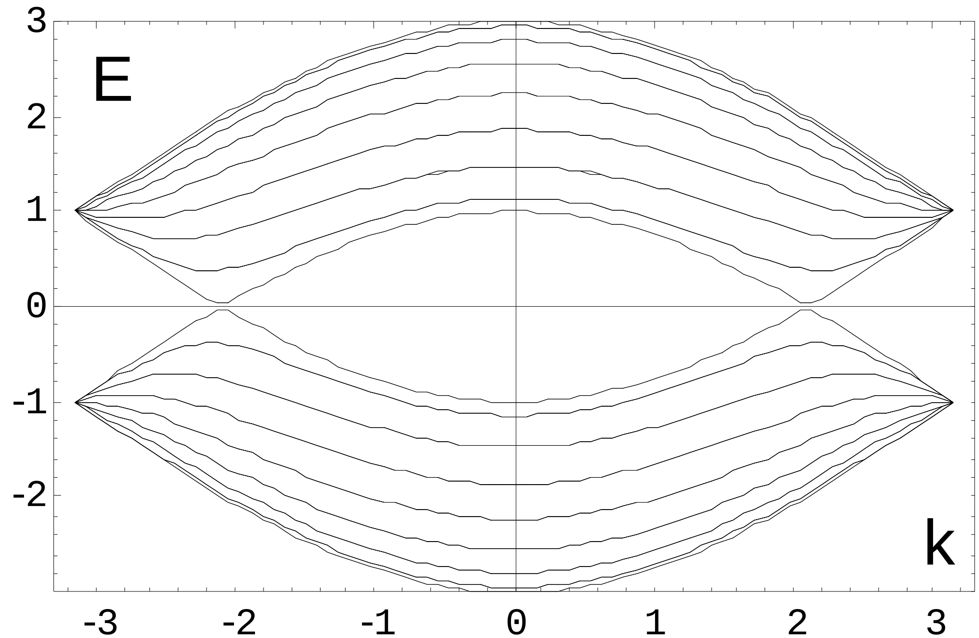
Electronic energy gain: $\sim \lambda\Delta^2 \log \omega_0/\Delta$

Peirels Energy Gap:

$$\Delta \sim \omega_0\, e^{-1/\lambda} \sim 1.8 \text{ eV}$$

"Armchair" [N,N] Carbon Nanotubes





- One Dimensional for $E < \hbar v_F / R \sim 1\ \mathrm{eV}$

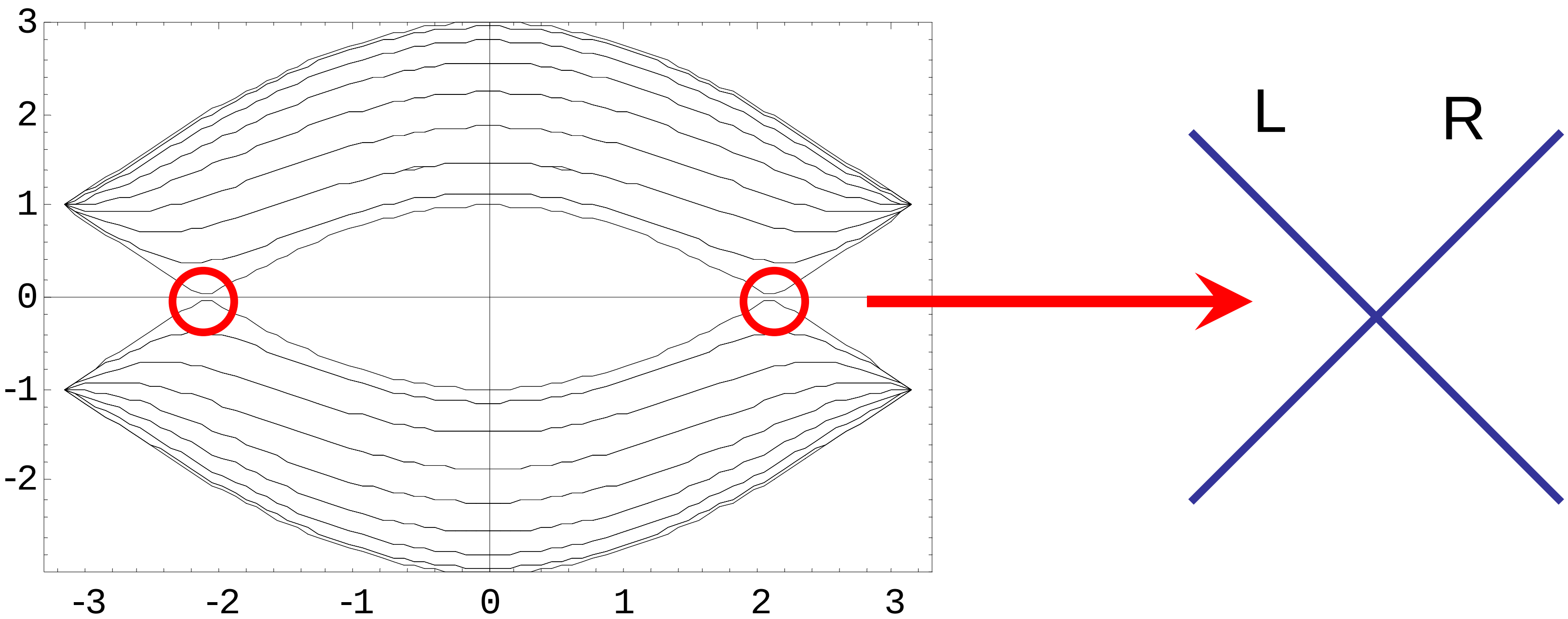
- Peierls Instability suppressed for large N ~ R/a:

Elastic energy cost : $\sim N\, \Delta^2$

Electronic energy gain: $\sim 1\, \lambda\Delta^2 \log \omega_0/\Delta$

$$\Delta \sim \omega_0\, e^{-N/\lambda}$$

Low Energy Theory



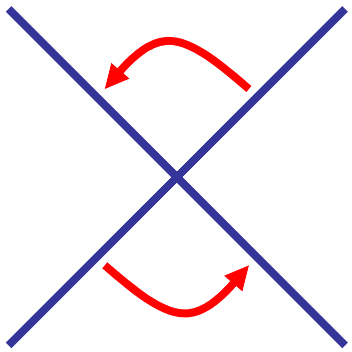
4 Channels:

$$H = \sum_{a=K,K'} \sum_{s=\uparrow,\downarrow} -i\mathrm{v}_{\mathrm{F}} \left\{ \psi^{\dagger}_{asR} \partial_x \psi_{asR} - \psi^{\dagger}_{asL} \partial_x \psi_{asL} \right\}$$

$$L = \frac{1}{2\pi} \sum_{a,s} \left\{ \frac{1}{\mathrm{v}_F} \left(\partial_t \theta_{a,s} \right)^2 - \mathrm{v}_F \left(\partial_x \theta_{a,s} \right)^2 \right\}$$

Electron Interactions $V_{\alpha\beta\gamma\delta}\psi^\dagger_\alpha\psi^\dagger_\beta\psi_\gamma\psi_\delta$

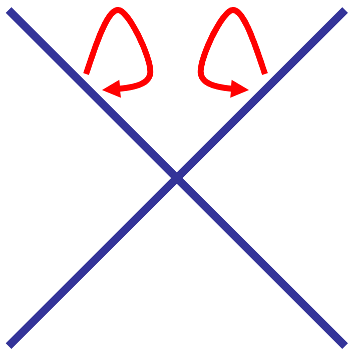
1. "Backscattering Interactions"



- Violate Chiral symmetry
- Have form $\lambda \cos(\phi_\alpha+\phi_\beta-\phi_\chi-\phi_\delta)$
- Can lead to electronic Instabilities

- Depend on the short range part of V(r) (q ~ 1/a)
- Suppressed by 1/N

2. "Forward Scattering Interactions"



- Preserve Chiral symmetry
- Have form $n_\alpha n_\beta \propto \partial_x\phi_\alpha\partial_x\phi_\beta$
- Lead to Luttinger Liquid

- Depend on Long range part of V(r)
- NOT suppressed by 1/N

Screened Coulomb Interaction : $H_{\text{int}} = e^2 \log\left(R_s / R\right) n_{tot}^2$

Luttinger Liquid Theory

Charge/Spin/Flavor variables: $\theta_{\rho,\sigma;\pm} = \pm\theta_{K\uparrow} \pm \theta_{K\downarrow} \pm \theta_{K'\uparrow} \pm \theta_{K'\downarrow}$

1 Charge Mode

$$L_{\rho+} = \frac{1}{2\pi g}\sum_{a,s}\left\{\frac{1}{v_\rho}\left(\partial_t\theta_{\rho+}\right)^2 - v_\rho\left(\partial_x\theta_{\rho+}\right)^2\right\}$$

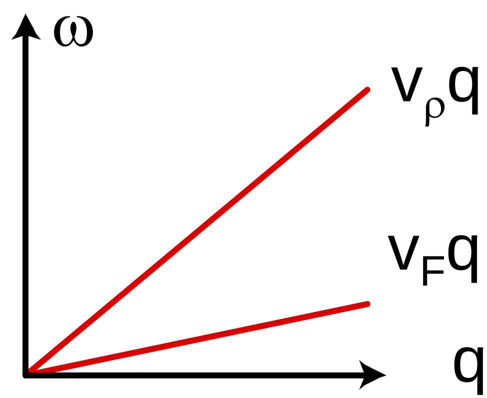
3 Neutral Modes

$$L_a = \frac{1}{2\pi}\sum_{a,s}\left\{\frac{1}{v_F}\left(\partial_t\theta_a\right)^2 - v_F\left(\partial_x\theta_a\right)^2\right\}$$

$$g = \left(1 + \frac{8}{\pi}\frac{e^2}{\hbar v_F}\log\frac{R_s}{R}\right)^{-\frac{1}{2}} \sim .2$$

$$v_\rho = v_F / g$$

Spin Charge Separation:

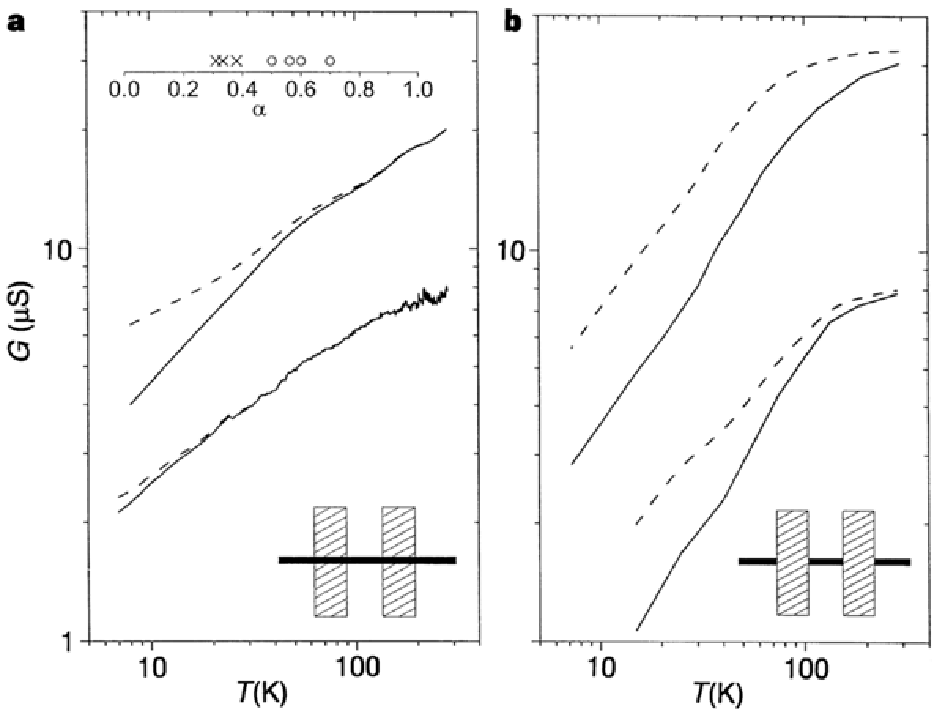


Tunneling Experiments

Metal contact to rope of nanotubes

Bockrath et al. 1999

$$G(T) \sim T^{\alpha}$$



Bulk contacted

End contacted

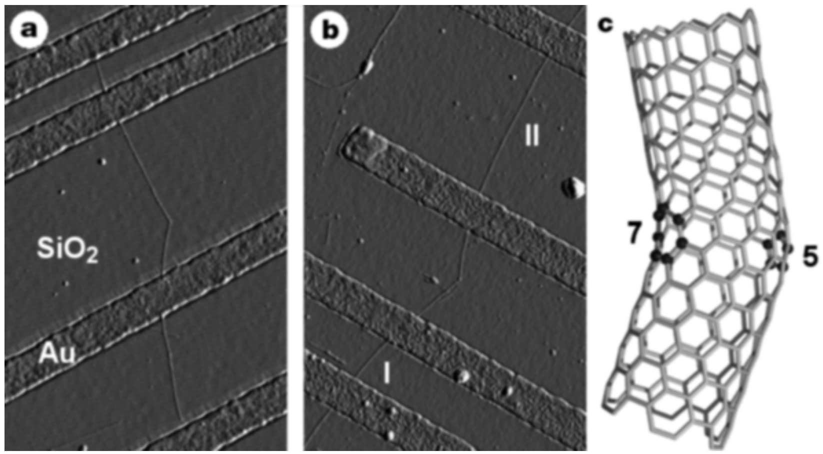
$$\alpha_{bulk} = 0.3 \qquad \alpha_{end} = 0.6$$

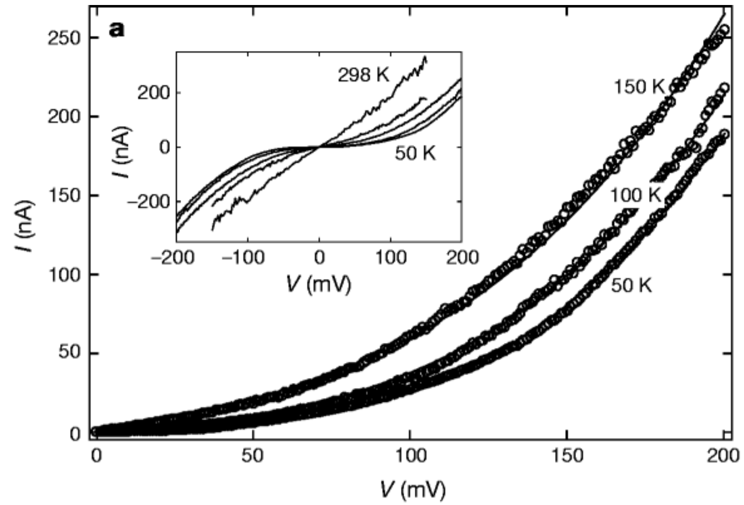
Theory:

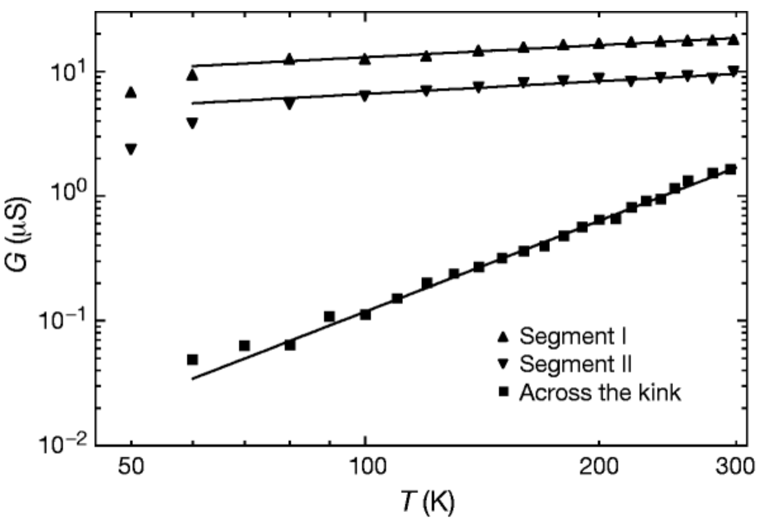
$\alpha_{bulk} = (g+1/g-2)/8 \sim .25$

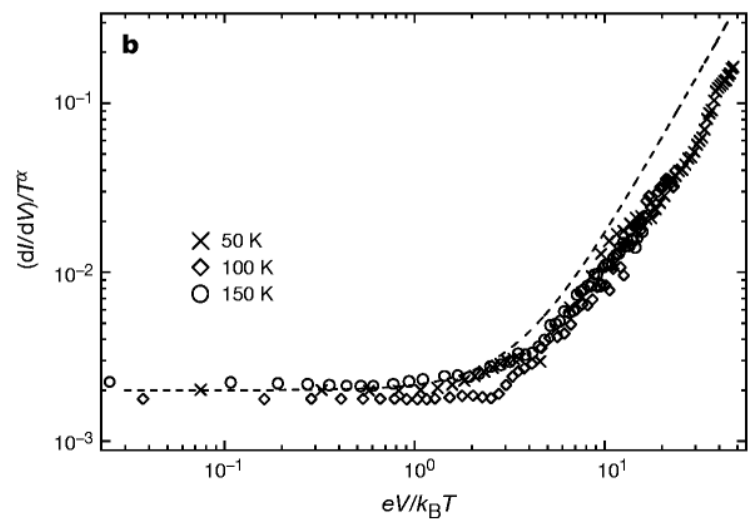
$\alpha_{end} = (1/g-1)/4 \sim .65$

Tunneling across a barrier in a single nanotube Yao et al. 99









Scaling Plot $T^{\alpha}F(V/T)$