Plan:

Today: Properties of Luttinger Liquids

- Tunneling
- Single Impurity Problem
- Commensurate Potentials

Tuesday: Applications

- Quantum Hall Edge States
- Carbon Nanotubes
- Quantum Wires

Review:

Bosonization gives two equivalent representations of the non interacting Luttinger model:

$$L = \frac{1}{2\pi}\int dx \left\{ \frac{1}{\mathrm{v}_F}\left(\partial_t \theta\right)^2 - \mathrm{v}_F \left(\partial_x \theta\right)^2 \right\}$$

$$\frac{1}{2\pi}\int dx \left\{ \frac{1}{\mathrm{v}_F}\left(\partial_t \varphi\right)^2 - \mathrm{v}_F \left(\partial_x \varphi\right)^2 \right\}$$

where

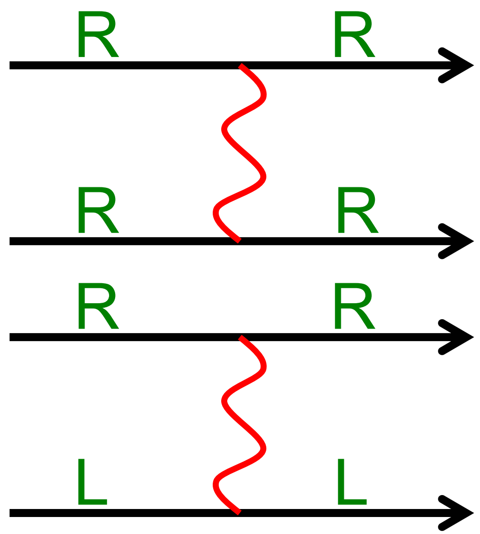
$$\left[\frac{1}{\pi}\partial_x \theta(x), \varphi(x') \right] = i\delta(x - x')$$

$$\psi^{\dagger}_{L,R} = \frac{\kappa}{\sqrt{2\pi x_c}} e^{i(\varphi \pm \theta)}$$

$$n(x) = \frac{1}{\pi}\partial_x \theta$$

Forward Scattering Interactions

(preserve chiral symmetry)



$$H_{\text{int}} = \frac{1}{2} V_0 \int dx \left(\psi_R^\dagger \psi_R + \psi_L^\dagger \psi_L \right)^2$$

$$= \frac{V_0}{2\pi^2} \int dx \left(\partial_x \theta \right)^2$$

$$L = \frac{1}{2\pi} \int dx \left\{ \frac{1}{v_F} \left(\partial_t \theta \right)^2 - \left[v_F + \frac{V_0}{\pi} \right] \left(\partial_x \theta \right)^2 \right\}$$

$$= \frac{1}{2\pi g} \int dx \left\{ \frac{1}{v_\rho} \left(\partial_t \theta \right)^2 - v_\rho \left(\partial_x \theta \right)^2 \right\} = \frac{g}{2\pi} \int dx \left\{ \frac{1}{v_\rho} \left(\partial_t \varphi \right)^2 - v_\rho \left(\partial_x \varphi \right)^2 \right\}$$

$$g = \left[1 + \frac{V_0}{\pi v_F} \right]^{-\frac{1}{2}} \quad ; \quad v_\rho = v_F / g$$

$g<1$: repulsive int. $V_0>0$

$g=1$: non interacting

$g>1$: attractive int. $V_0<0$

Properties of a Luttinger Liquid

1. $2k_F$ Density Correlations:

$$\left\langle \psi_L^\dagger \psi_R(x) \psi_R^\dagger \psi_L(0) \right\rangle \sim \left\langle e^{i(2\theta(x)-2\theta(0))} \right\rangle \sim x^{-2g}$$

"almost a crystal"

2. Pair Correlations:

$$\left\langle \psi_L \psi_R(x) \psi_R^\dagger \psi_L^\dagger(0) \right\rangle \sim \left\langle e^{i(2\varphi(x)-2\varphi(0))} \right\rangle \sim x^{-2/g}$$

"almost a superfluid"

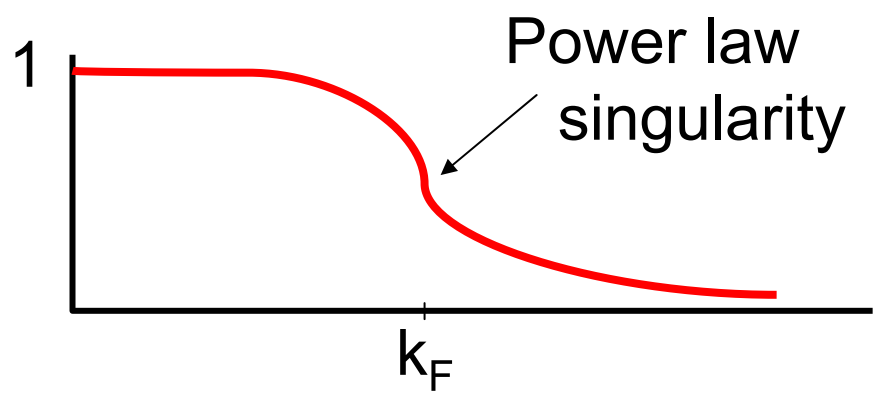
3. Single Particle :

$$\left\langle \psi_R^\dagger(x) \psi_R(0) \right\rangle \sim \left\langle e^{i(\theta(x)+\varphi(x)-\theta(0)-\varphi(0))} \right\rangle \sim x^{-\frac{1}{2}(g+1/g)}$$

4. Momentum Distribution

$$n(k) \sim \left\langle c_k^\dagger c_k \right\rangle = \int dx G(x) e^{-ikx}$$

$$\sim k^{\frac{1}{2}(g+1/g)-1}$$



Ballistic Conductance of a Luttinger Liquid

1. Kubo Formula :

$$G = \lim_{\omega \to 0} \frac{1}{i\omega} \int_{-\infty}^{0} dt \left\langle \left[J(t), J(0) \right] \right\rangle e^{i\omega t}$$

$$G = g \frac{e^2}{h}$$

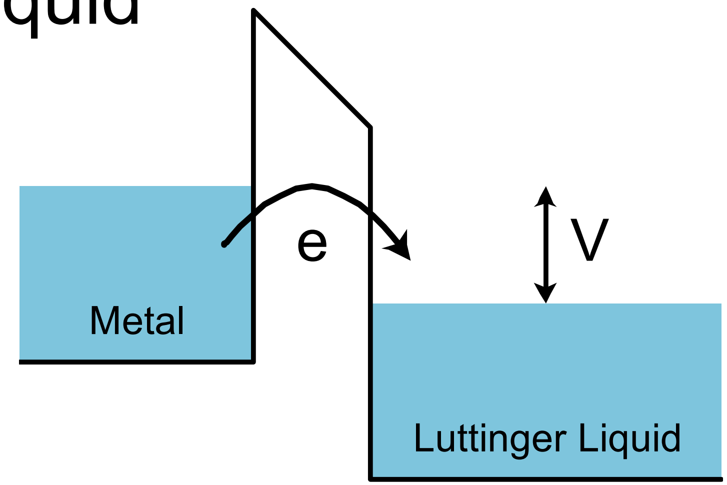
(NOT measured in DC transport expts!)

2. Fermi liquid Contacts

$$G = \frac{e^2}{h}$$

Landauer formula is valid because equilibration with the reservoirs is due to transfer of electrons

Tunneling into a Luttinger Liquid



Tunneling rate : (Golden Rule)

$$I = 2\pi t^2 \rho_{Metal}(E_F) \int_0^V dE \, \rho_{LL}(E)$$

Tunneling Density of States

$$\rho_{LL}(E) = \sum_N \left| \langle N | \psi_{LL}^\dagger | 0 \rangle \right|^2 \delta(E_N - E_0 - E)$$

$$= \int_{-\infty}^{\infty} dt e^{iEt} \langle \psi_{LL}(t) \psi_{LL}^\dagger(0) \rangle \sim E^{\alpha - 1}$$

$$\alpha = \frac{1}{2}\left(g + \frac{1}{g} \right) > 1$$

$$I(V) \sim V^\alpha$$

$$G(V) = \frac{dI}{dV} \sim V^{\alpha - 1}$$

Tunneling is completely suppressed at V (and T) = 0.

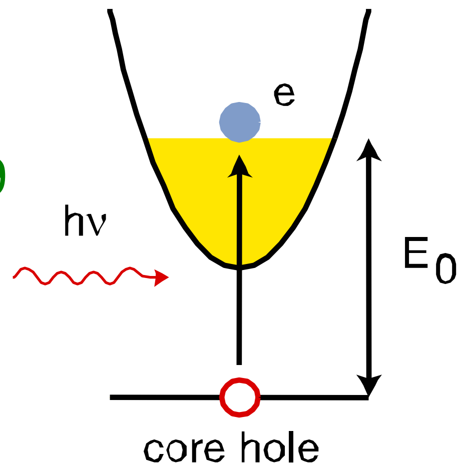
Orthogonality Catastrophe (Anderson, 1967)

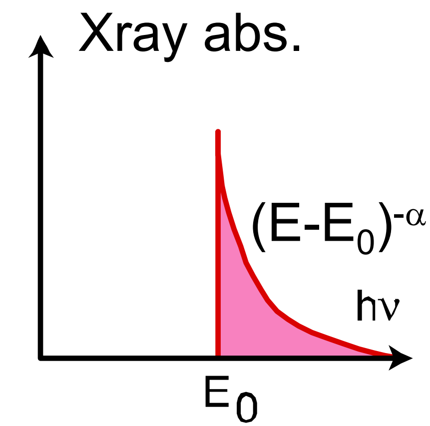
$$\left\langle \Psi^0_{N+1} \middle| \psi^\dagger \middle| \Psi^0_N \right\rangle = 0$$

due to rearrangement of all the other electrons

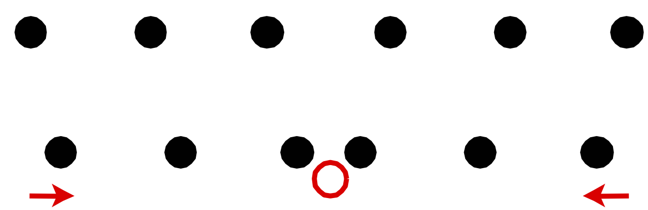
X ray edge problem:
Nozieres and DeDominicis, 1969

Formulation using bosonization:
Schotte and Schotte 1969





$$\left\langle \Psi^0_{\text{before}} \middle| \Psi^0_{\text{after}} \right\rangle \propto L^{-(\delta/\pi)^2}$$



Power law behavior describes the “shake up” spectrum of excitations created by the sudden potential change.

Related Problems with orthogonality catastrophes:

- Kondo Problem (see Doniach and Sondheimer's book)

2. Suppression of Coulomb Blockade due to Coupling to electromagnetic environment

Girvin, et al., PRL 1990
Devoret, et al., PRL 1990

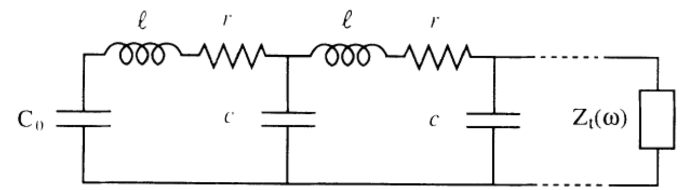


FIG. 1. A schematic model of a tunnel junction C_0 connected to an RLC transmission line terminated by an impedance $Z_t(\omega)$ after a length d.

$$I \propto V^{1+R/R_Q} \qquad R_Q = \frac{h}{e^2}$$

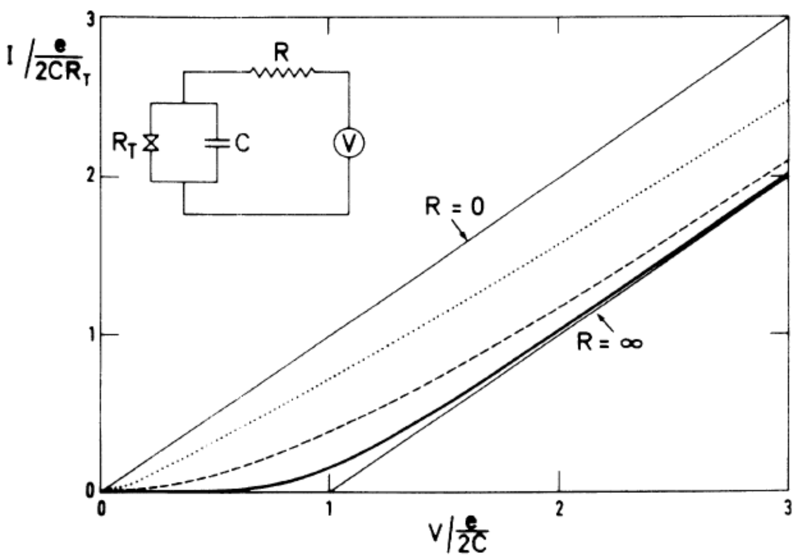


FIG. 2. The I-V characteristic of a tunnel junction coupled to an environment characterized by a resistance R (see inset) for R/R_Q =0, 0.1, 1, 10, and ∞.

Also related to dissipative quantum tunneling

Caldeira and Leggett Ann. Phys. 1983

Tunneling into a Luttinger Liquid at finite Temperature

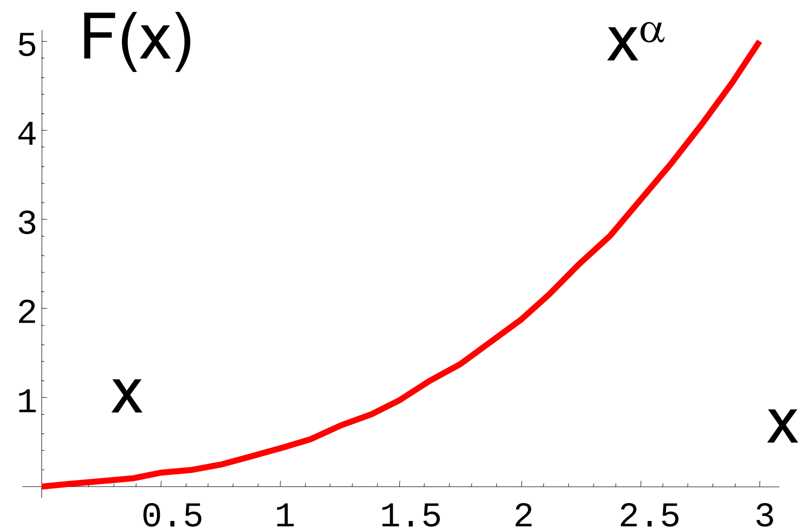
$$I(T,V) \propto \begin{cases} V^{\alpha} & \text{for } T \ll V \\ T^{\alpha-1}V & \text{for } T \gg V \end{cases}$$

Scaling Behavior for V ~ T

$$I(T,V) \propto T^{\alpha} F\left(\frac{V}{2\pi T}\right)$$

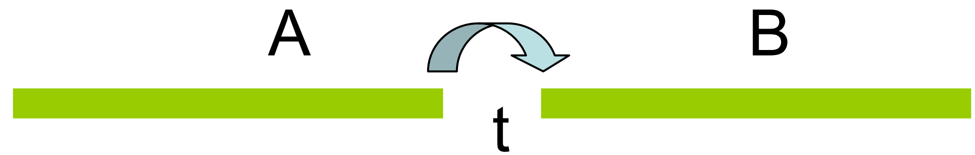
Universal Scaling Function

$$F(x) = \frac{1}{\pi} \frac{\left|\Gamma\left(\frac{1+\alpha}{2} + ix\right)\right|^2}{\Gamma(1+\alpha)} \sinh \pi x$$



Single Barrier in a Luttinger Liquid Kane,Fisher 92

1. Large Barrier Limit



$$L = \frac{g}{2\pi}\int_{-\infty}^{0} dx\left(\partial_\mu \varphi_A\right)^2 + \frac{g}{2\pi}\int_{0}^{\infty} dx\left(\partial_\mu \varphi_B\right)^2 + \frac{t}{\pi x_c}\cos\left[\varphi_A(0) - \varphi_B(0)\right]$$

$$\psi_A^\dagger \psi_B + \text{h.c.} \quad \left(\theta_{A,B}(0) = 0\right)$$

Perturbation theory : $G(T) \propto t^2 T^{(2/g-2)}$

2/g-2>0, so the orthogonality catastrophe is similar to tunneling from a metal, but the exponent is different:

- Tunneling from LL to LL.
- Tunneling into the end of the LL.

Small Barrier Limit

Weakly scattering impurity

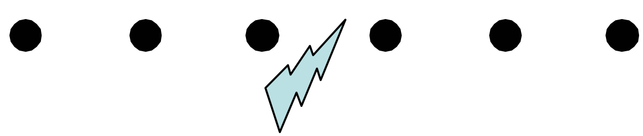
$$L = \frac{1}{2\pi g}\int_{-\infty}^{\infty} dx\left(\partial_\mu \theta\right)^2 + \frac{\mathrm{v}}{\pi x_c}\cos\left[2\theta(0)\right] \qquad \psi_R^\dagger \psi_L + \mathrm{h.c.}$$

Perturbation Theory:

$$G(T) = \frac{e^2}{h} - c\mathrm{v}^2 T^{(2g-2)}$$

2g-2<0, so perturbation theory diverges for T→0.

Interpretation: Impurity "pins" the Wigner crystal



$$V(\theta) \sim \cos 2\theta$$

Renormalization Group Analysis

RG Transformation:

- "Integrate out" short wavelength degrees of freedom with $\Lambda/b < q < \Lambda$.
- Rescale space and time $(x,\tau) \rightarrow b\,(x',\tau')$

This leads to a "coarse grained" action, in which the parameters have "flowed".

The Free Action is unchanged:

$$S = \frac{1}{2\pi g}\int_{-\infty}^{\infty} dx\,d\tau\left(\partial_\mu \theta\right)^2 \rightarrow \frac{1}{2\pi g}\int_{-\infty}^{\infty} dx'\,d\tau'\left(\partial'_\mu \theta\right)^2$$

The Luttinger liquid is a "fixed point" of the RG.

Renormalization of the Impurity Scattering Term

$$S_{imp} = \mathrm{v}\int d\tau \cos 2\theta \qquad \theta(x,\tau) = \theta^{<}(x,\tau) + \theta^{>}(x,\tau)$$

slow ($q<\Lambda/b$) fast ($\Lambda/b<q<\Lambda$)

$$\left\langle e^{2i\theta} \right\rangle_{>} = e^{2i\theta_<} \left\langle e^{2i\theta_>} \right\rangle_{>} = e^{2i\theta_<} e^{-2\left\langle \theta_>^2 \right\rangle} = b^{-\Delta} e^{2i\theta_<}$$

$$S'_{imp} = \mathrm{v} b^{1-\Delta} \int d\tau' \cos 2\theta_< = \mathrm{v}' \int d\tau' \cos 2\theta_< \qquad \left(\mathrm{v}' = \mathrm{v} b^{1-\Delta} \right)$$

For $b = e^{\ell}$, $\quad \dfrac{d\mathrm{v}}{d\ell} = (1-\Delta)\mathrm{v}$

$\Delta<1$: v is "relevant"
$\Delta>1$: v is "irrelevant"
$\Delta=1$: v is "marginal"

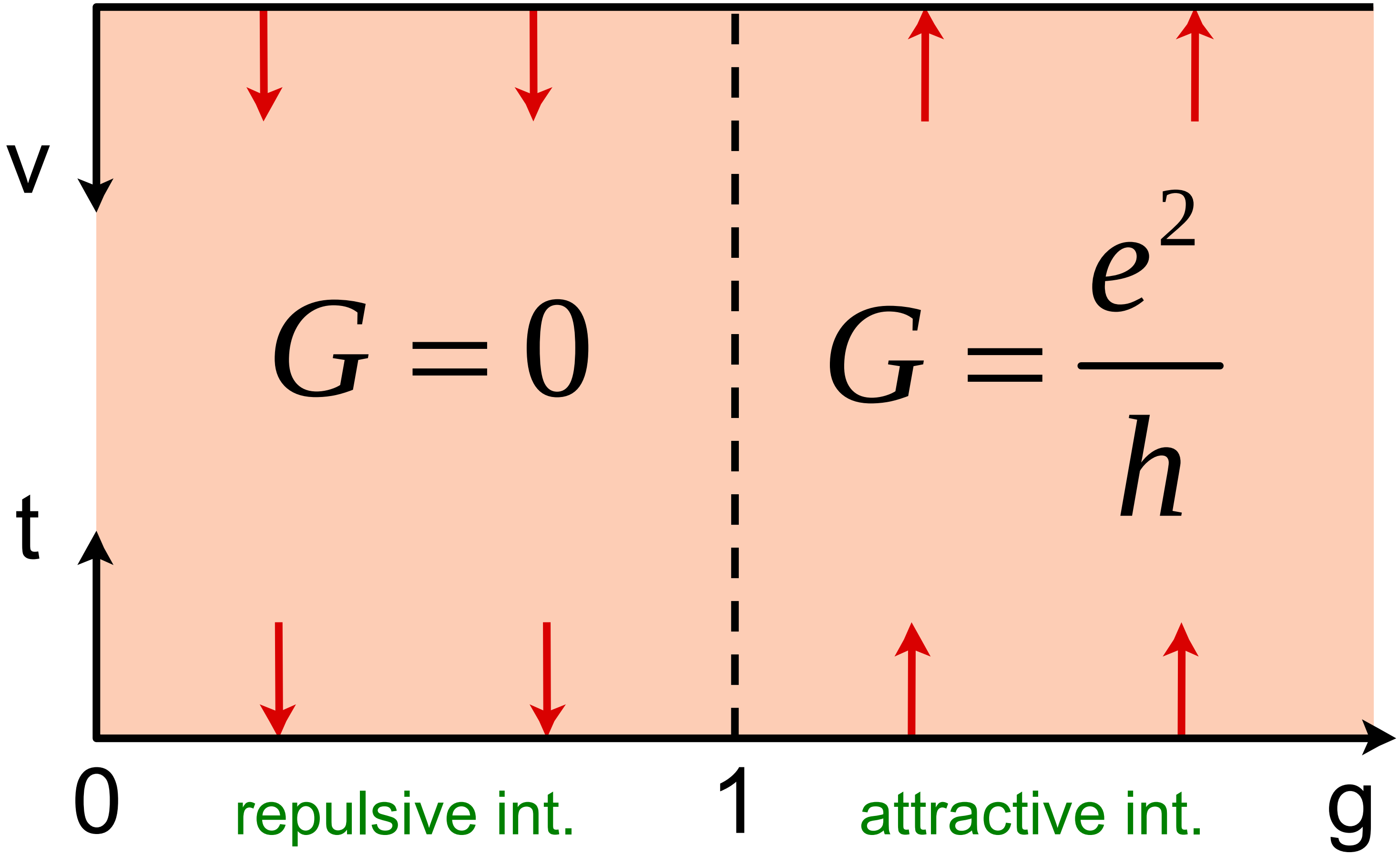
The scaling dimension Δ can be determined by computing : $\left\langle e^{2i\theta(x)} e^{-2i\theta(0)} \right\rangle \propto x^{-2\Delta}$

We already calculated this: $\Delta = g$

Phase Diagram

$$\frac{d\mathrm{v}}{d\ell} = (1-g)\mathrm{v}$$

$$\frac{d\mathrm{t}}{d\ell} = (1-\frac{1}{g})\mathrm{t}$$

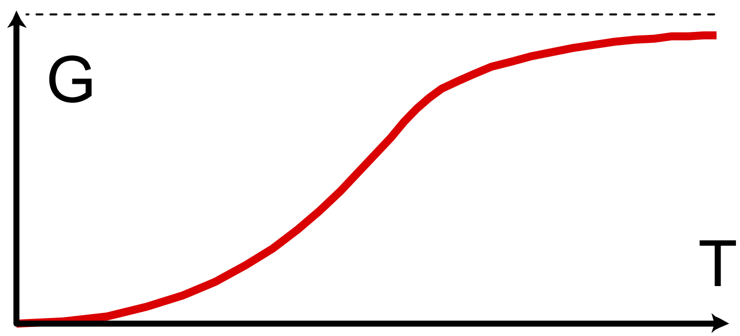


An arbitrarily weak impurity “flows” to the large barrier limit at low energy, so $G(T) \sim T^{2/g-2}$ for $T \rightarrow 0$.

Crossover from Weak to Strong Barrier

For g>1/4, v is the only relevant term. Then, the crossover between the two limits is described by a universal scaling function

$$G(T,\mathrm{v}) = \frac{e^2}{h}\tilde{G}\left(\frac{c\mathrm{v}}{T^{1-g}}\right)$$



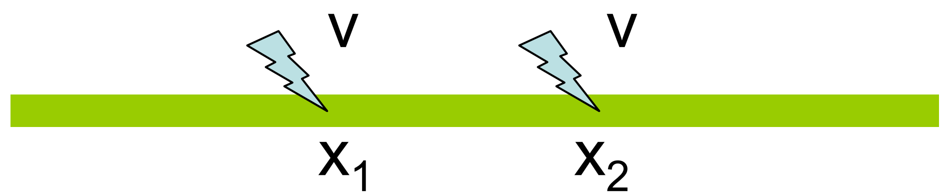
This crossover scaling function has been computed exactly:

g=1/2 Kane and Fisher 1992 (Using “fermionization”)

All g : Fendley, Ludwig, Saleur, 1995 (Thermodyanamic Bethe Ansatz).

Resonant Tunneling

2 Symmetric Impurities :



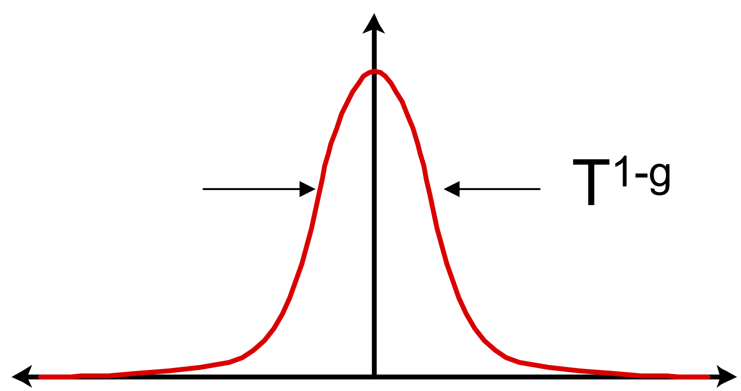
$$\mathrm{v}\left(\cos 2\theta_1 + \cos 2\theta_2\right) = 2\mathrm{v}\cos\left(\theta_1 - \theta_2\right)\cos\left(\theta_1 + \theta_2\right)$$

$$= 2\mathrm{v}\cos\left(\pi n\right)\cos 2\bar{\theta} \equiv \mathrm{v}_{\mathrm{eff}}\cos 2\bar{\theta}$$

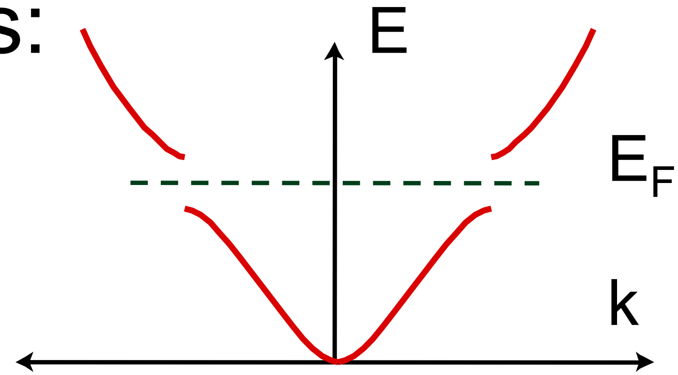
By tuning n (with a gate) a “perfect resonance” can be found, where $\mathrm{v}_{\mathrm{eff}}=0$. In general, 2 parameters must be tuned to find a perfect resonance.

$$G(T,\mathrm{v}_{\mathrm{eff}}) = \frac{e^2}{h}\tilde{G}\left(\frac{c\mathrm{v}_{\mathrm{eff}}}{T^{1-g}}\right)$$

Universal Resonance Lineshape



Commensurate Potentials:



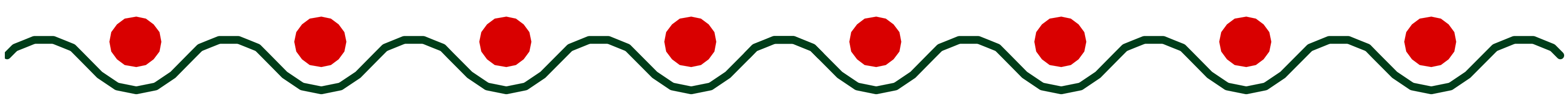
1. Band Insulator: $V(2k_F) \neq 0$

$$S = \int_{-\infty}^{\infty} dx d\tau \left\{ \frac{1}{2\pi g} \left(\partial_\mu \theta \right)^2 + \frac{\mathrm{v}_{2\mathrm{k_F}}}{\pi x_c} \cos 2\theta \right\}$$

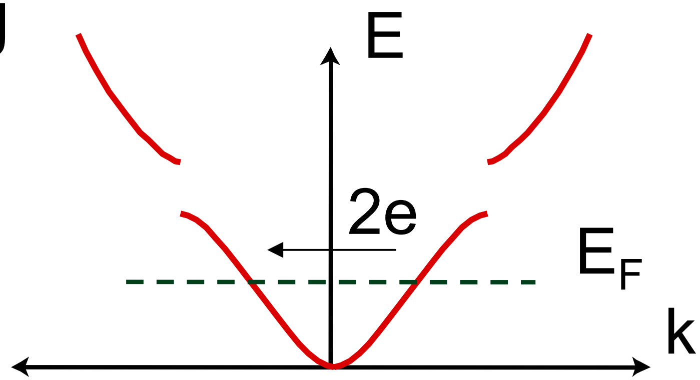
$$\frac{d\mathrm{v}_{2k_F}}{d\ell} = \left(2 - g\right) \mathrm{v}_{2k_F}$$

Strongly Relevant:

$$S_{eff} = \int_{-\infty}^{\infty} dx d\tau \left\{ \frac{1}{2\pi g} \left(\partial_\mu \theta \right)^2 + \frac{1}{2} k \theta^2 \right\}$$



2. Mott Insulator at half filling



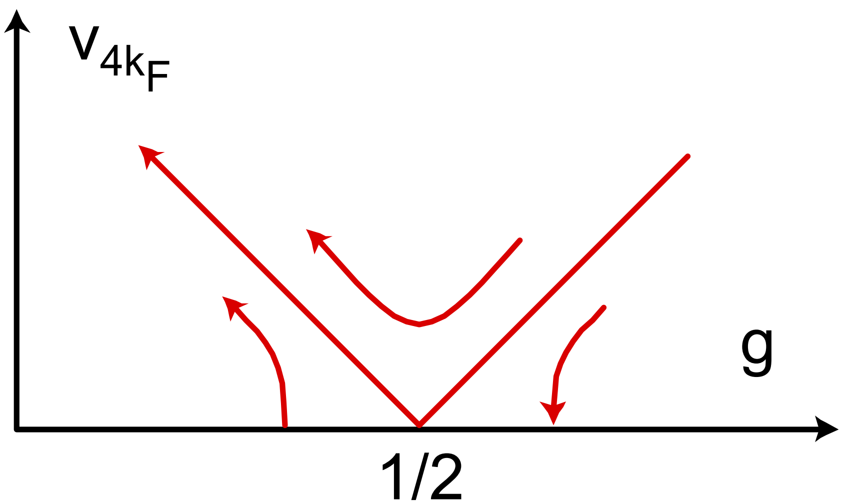
Umklapp Scattering:

$$v_{4k_F}\left[\psi_L^\dagger\partial_x\psi_L^\dagger\psi_R\partial_x\psi_R\right]$$

$$S=\int_{-\infty}^{\infty}dx\,d\tau\left\{\frac{1}{2\pi g}\left(\partial_\mu\theta\right)^2+\frac{v_{4k_F}}{\pi x_c}\cos 4\theta\right\}$$

$$\frac{dv_{4k_F}}{d\ell}=\left(2-4g\right)v_{4k_F}$$

$$\frac{dg}{d\ell}=-c\,v_{4k_F}^2$$



Kosterlitz Thouless Flow Diagram

g>1/2: Luttinger Liquid

g<1/2: Mott Insulator