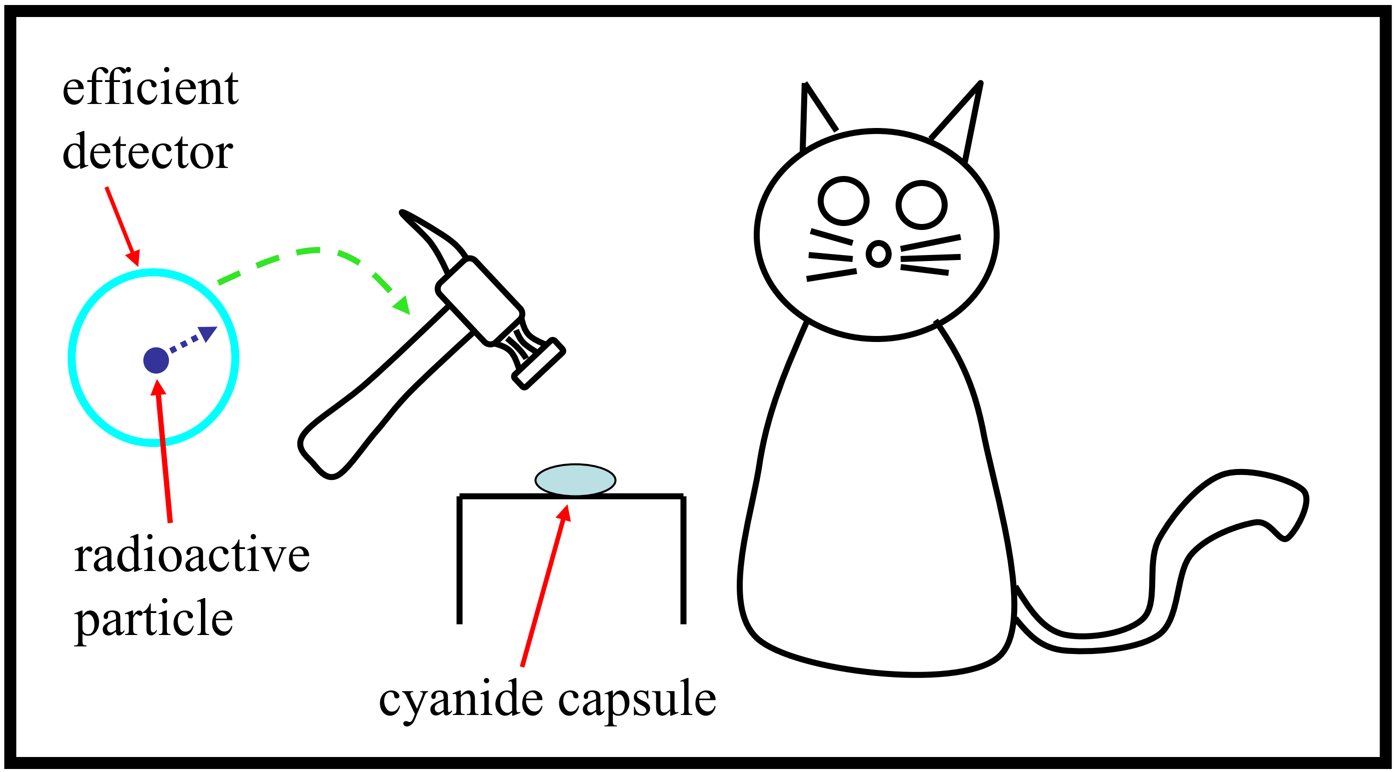
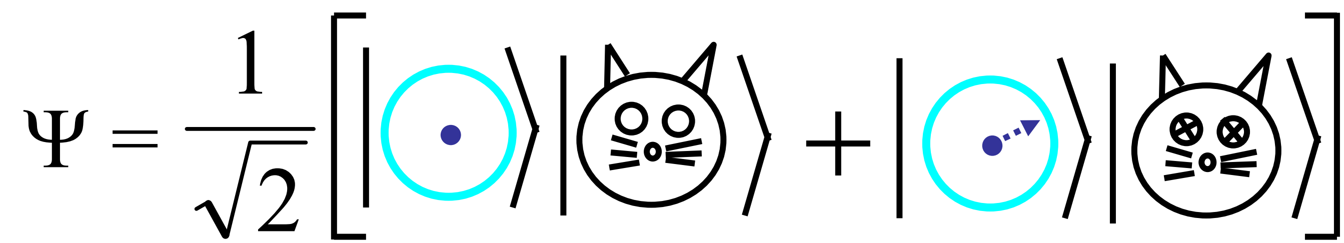
Erwin Schrödinger's Cat (1935)



sealed box

after one
half life:



- cat is simultaneously dead and alive!
- state of cat is "entangled" with radioactive particle $\Psi \neq \psi_{particle} \otimes \psi_{cat}$

Deterministic entanglement of trapped atomic ions I

NIST, Boulder, Ion Storage group:

M. Barrett (postdoc, Georgia Tech.)†
J. C. Bergquist (**NIST**)
B. Blakestad (student, CU)
J. J. Bollinger (**NIST**)
J. Britton (student, U. Colorado)
J. Chiaverini (postdoc, Stanford)
B. DeMarco (postdoc, U. Colorado)‡
W. Itano (**NIST**)
B. Jelenković (guest, Blegrade)¶
M. Jensen (U. Colorado)
J. Jost (student, U. Colorado)
E. Knill (**NIST**, computation Div.)
C. Langer (student, U. Colorado)
D. Leibfried (**NIST**)
W. Oskay (postdoc, U. Texas)
R. Ozeri (postdoc, Weizmann)
T. Rosenband (U. Colorado)
T. Schätz (postdoc, MPQ)
P. Schmidt (postdoc, Stuttgart)
D. J. Wineland (**NIST**)

† Present address: Otago University, NZ
‡ Present address: U. Illinois
¶ Present address: J.P.L.

Other ion groups pursuing entanglement:

Aarhus
Garching (MPQ)
Hamburg
Innsbruck
LANL
London (Imperial)
Michigan
Ontario (McMaster)
Oxford
Teddington (NPL)

Deterministic entanglement of trapped atomic ions I

NIST, Bo

M. Barrett (
J. C. Bergq
B. Blakesta
J. J. Bolling
J. Britton (st
J. Chiaverin
B. DeMarco
W. Itano (NI
B. Jelenkov
M. Jensen (U. Colorado)

D. J. Wineland (NIST)

ups
anglement:
Q)
ial)
ster)

† Present addre
‡ Present addre
¶ Present addre

Summary:

- basic ion trapology and entangling
- is it useful? maybe
  - quantum computers
    baby steps with ions
  - cats and metrology (next lecture)
- future

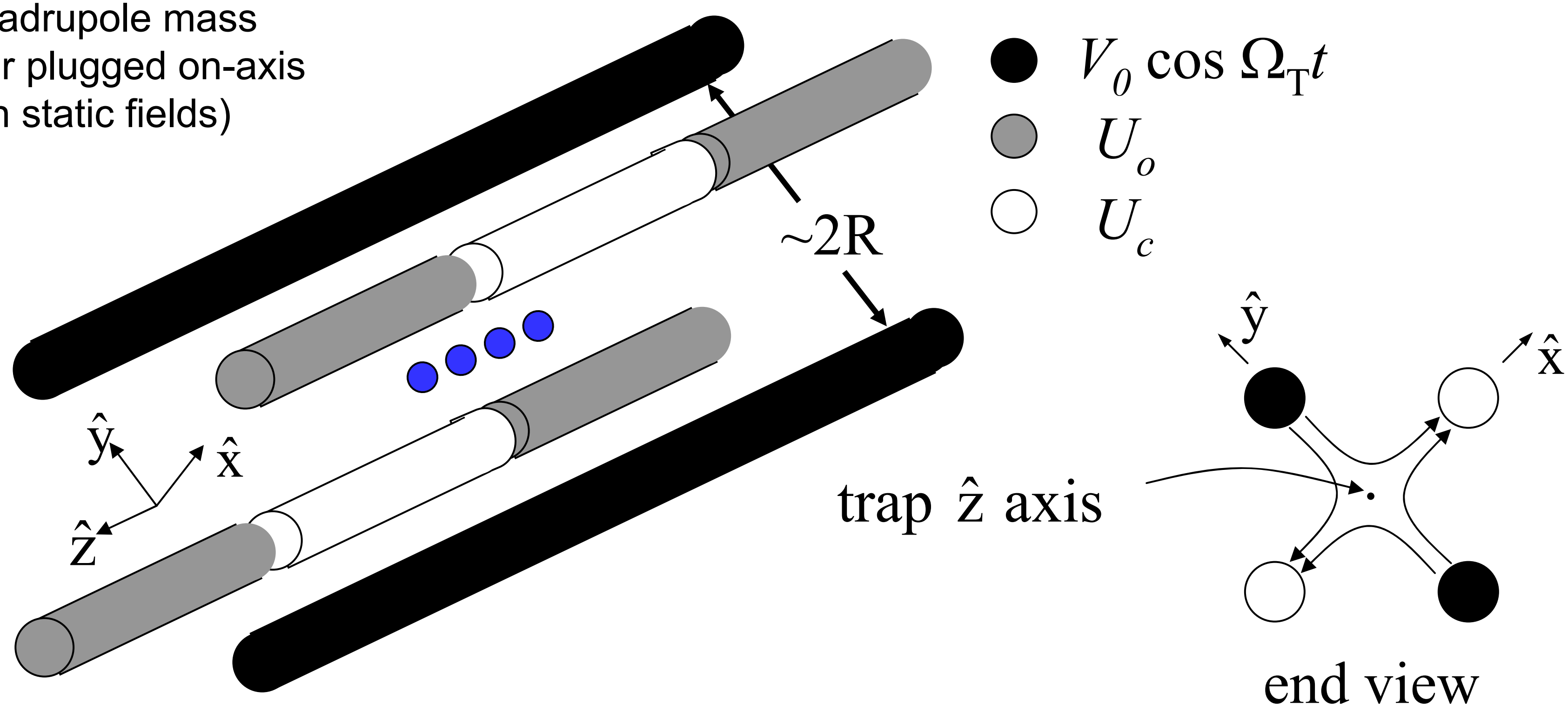
Caveat:

- many body quantum statistics not inherent,
  must be "engineered"

Ion trapping, particular case:

"linear" RF trap

(quadrupole mass filter plugged on-axis with static fields)



$$\Phi = \frac{(x^2 - y^2)}{2R^2} V_0 \cos \Omega_T t + \frac{(U_x x^2 + U_y y^2 + U_z z^2)}{2R^2}, \quad \boxed{\sum_{i=x,y,z} U_i = 0}$$

$$U_x = \alpha_{xo} U_o + \alpha_{xc} U_c, \quad U_y = \alpha_{yo} U_o + \alpha_{yc} U_c, \quad U_z = \kappa(U_o - U_c)$$

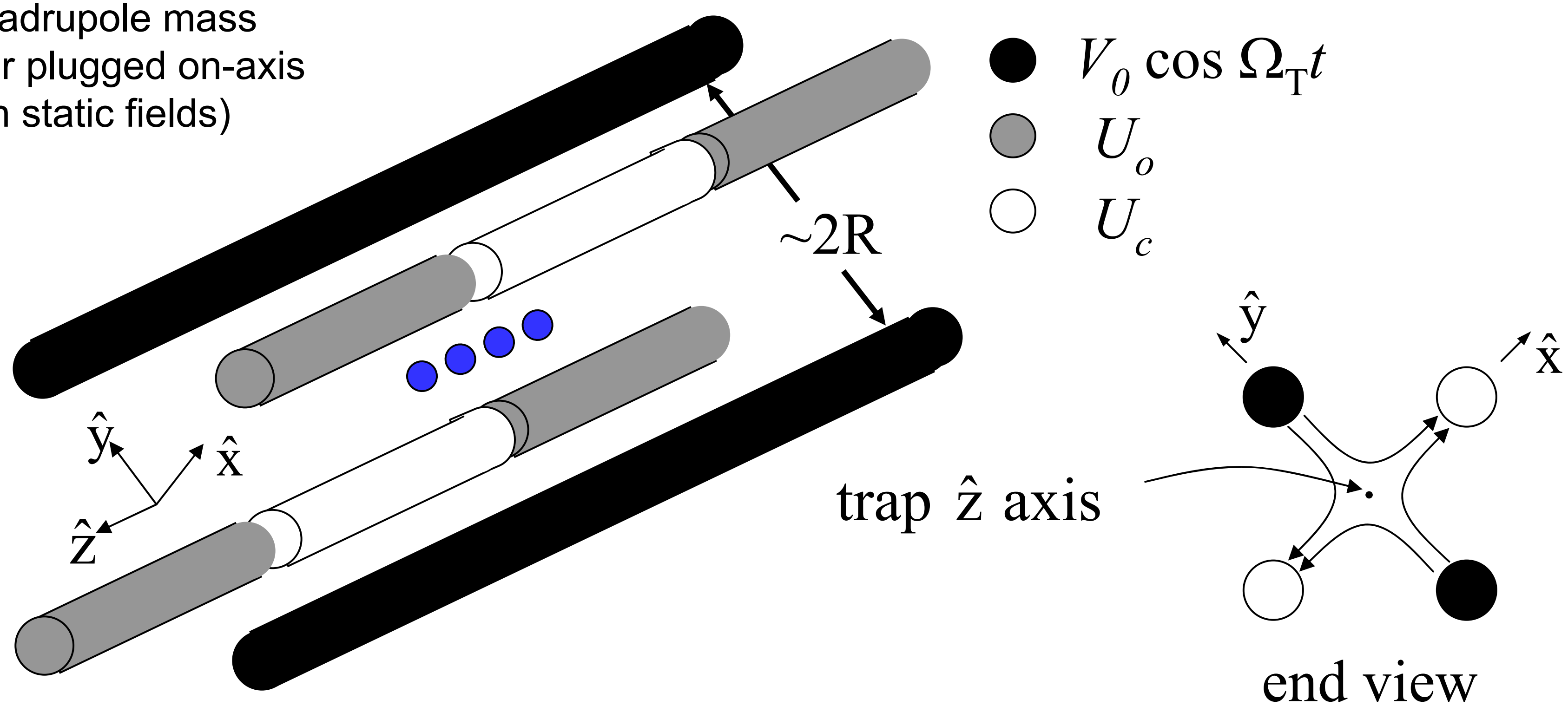
(Get α's and κ numerically)

more trap details: http://www.lkb.ens.fr/recherche/qedcav/houches/houches79.html

Ion trapping, particular case:

"linear" RF trap

(quadrupole mass filter plugged on-axis with static fields)



Neglecting "RF micromotion," (at Ω_T)

trap looks like 3-D harmonic well.

For linear trap:

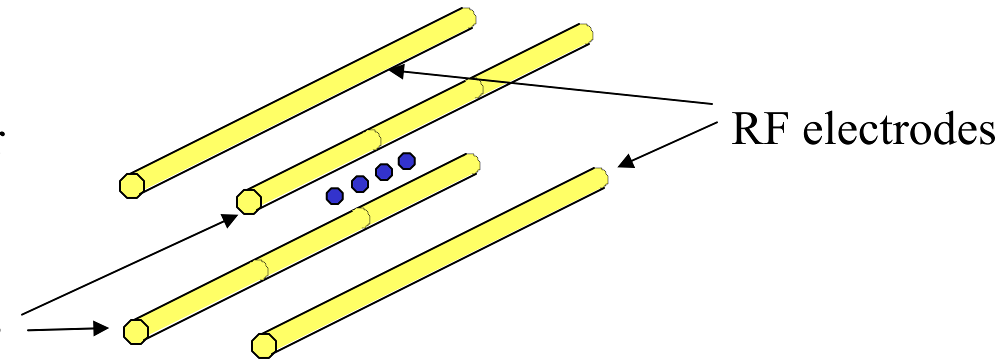
$$\omega_x \simeq \omega_y \simeq \frac{qV_0}{\sqrt{2}\Omega_T m R^2} \qquad (\omega_z < \omega_{x,y} < \Omega_T/2)$$

Fast (entangling) gates: Speed $\propto \omega_{x,y} \propto$ (dimensions)$^{-2}$

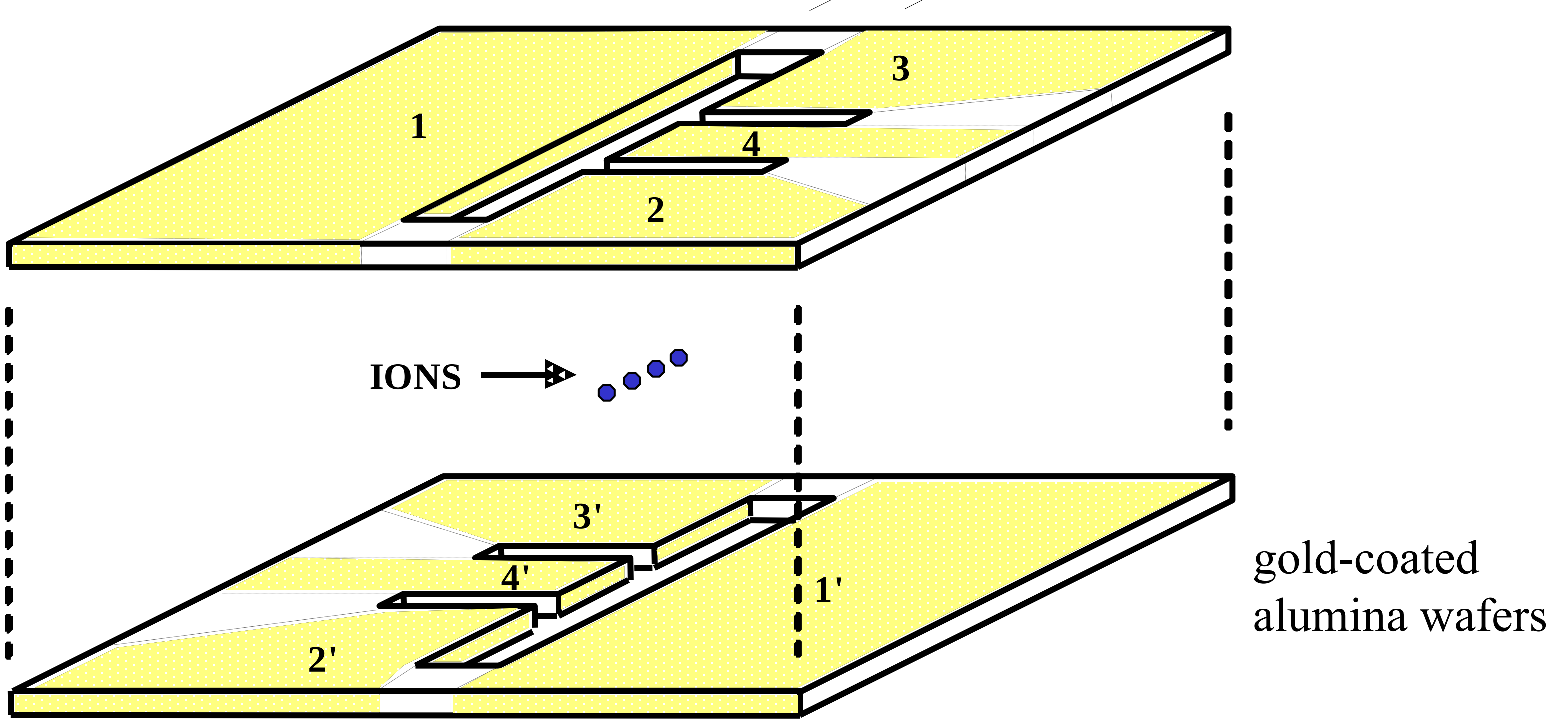
$$\omega_{x,y} \simeq \frac{qV_0}{\sqrt{2}\Omega_T m R^2}$$

Idealized trap:

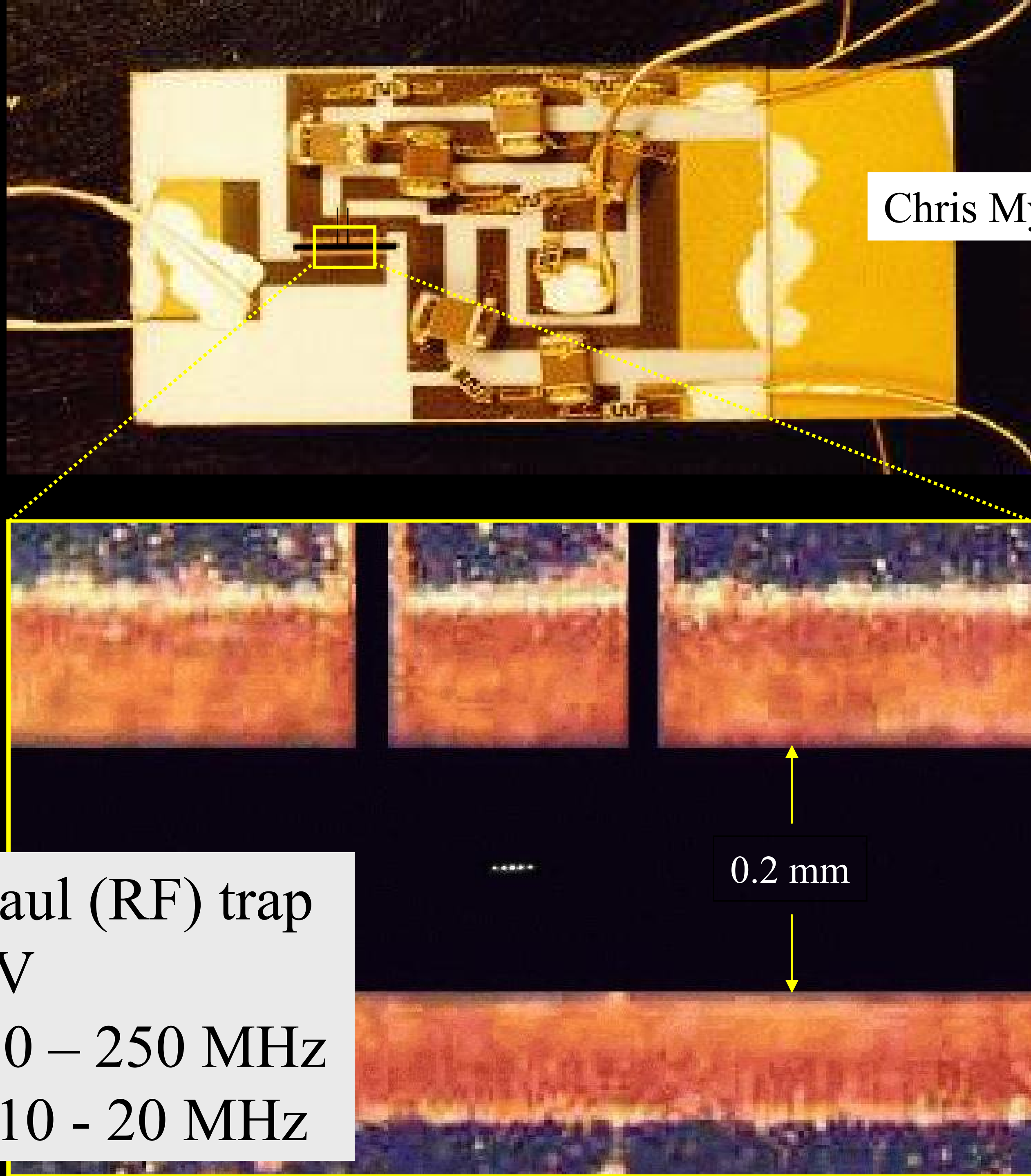
(RF quadrupole mass analyzer with trapping zones)



Approximation:



Chris Myatt *et al.*



“linear” Paul (RF) trap

$V_0 \sim 500$ V

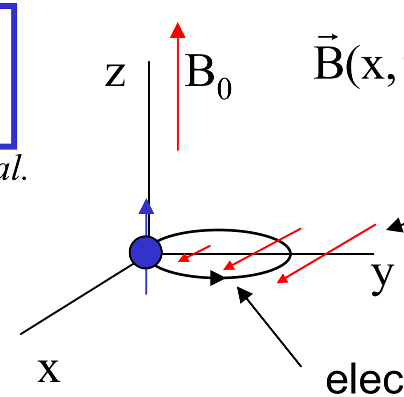
$\Omega_T/2\pi \sim 50 - 250$ MHz

$\omega_{x,y}/2\pi \cong 10 - 20$ MHz

Motion/spin entanglement:

e.g., electron g – 2 experiment

Dehmelt, Van Dyck, *et al.*



$$y(t) = Y_0 \cos(\omega_{cyclotron} t)$$

$$\partial B_x/\partial y(t) = B' \cos([\omega_{spin} - \omega_{cyclotron}]t)$$

$$\Rightarrow \quad \vec{B}(t) = \hat{z}B_0 + \hat{x}(\tfrac{1}{2}B'Y_0)(\underbrace{\cos(\omega_{spin}t)}_{\text{resonant term flips spin}} + \cos([\omega_{spin} - 2\omega_{cyclotron}]t)$$

Quantum mechanically:

$$H = -\vec{\mu}\cdot\vec{B} \qquad y = y_0(a + a^\dagger), \quad (y_0 = \text{zero-point amplitude})$$

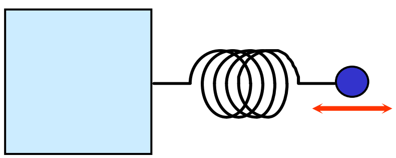
$$\vec{\mu} = g_J \mu_B \vec{S} \quad \Rightarrow$$

$$H = -\tfrac{1}{2} g_J \mu_B \{B_0 \sigma_z + (\partial B_x/\partial y) y_0 \underbrace{(a + a^+)(\sigma^+ + \sigma^-)}\}$$

resonant interaction = $a\sigma^+ + a^\dagger\sigma^-$ entangles spin and motion

details about laser couplings: http://www.lkb.ens.fr/recherche/qedcav/houches/houches79.html

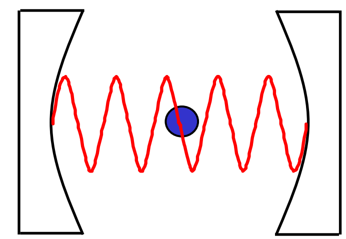
Trapped ions
(or neutral atoms)



quantized oscillator =
mode of motion

⇔

Cavity-QED



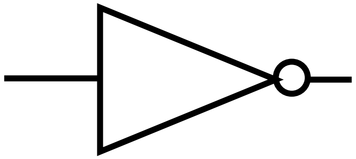
quantized oscillator =
mode of electromagnetic field

QUANTUM COMPUTERS: UNIVERSAL LOGIC GATE SETS

DiVincenzo, PRA **51**, 1015 ('95)
Barenco *et al*. PRA **52**, 3457 ('95)

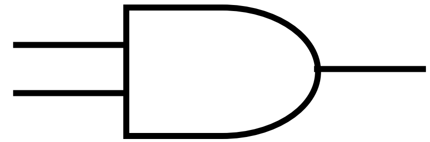
- Classical:

1-bit NOT



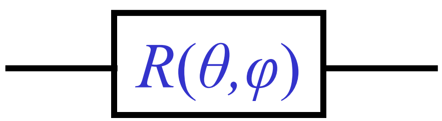
$0 \rightarrow 1$
$1 \rightarrow 0$

2-bit AND



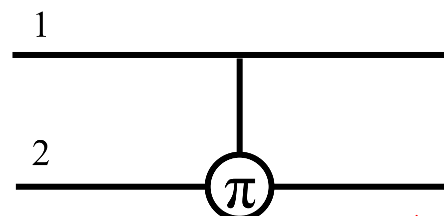
$00 \rightarrow 0$
$01 \rightarrow 0$
$10 \rightarrow 0$
$11 \rightarrow 1$

- Quantum: rotation



$$|0\rangle \rightarrow \cos(\theta/2)|0\rangle + e^{i\varphi}\sin(\theta/2)|1\rangle$$
$$|1\rangle \rightarrow \cos(\theta/2)|1\rangle - e^{-i\varphi}\sin(\theta/2)|0\rangle$$

π-phase gate $U_{1,2}(\pi)$



$|0\rangle_1|0\rangle_2 \rightarrow |0\rangle_1|0\rangle_2$
$|0\rangle_1|1\rangle_2 \rightarrow i|0\rangle_1|1\rangle_2$
$|1\rangle_1|0\rangle_2 \rightarrow i|1\rangle_1|0\rangle_2$
$|1\rangle_1|1\rangle_2 \rightarrow |1\rangle_1|1\rangle_2$

$$\Psi = \psi_1 \otimes \psi_2 = \tfrac{1}{2}\big(|0\rangle + |1\rangle\big)\big(|0\rangle + |1\rangle\big) \rightarrow U_{1,2}(\pi) \rightarrow$$

$$\tfrac{1}{2}\big(|0\rangle|0\rangle + i|0\rangle|1\rangle + i|1\rangle|0\rangle + |1\rangle|1\rangle\big) \neq \psi_1 \otimes \psi_2$$

entanglement!

general ref: *Quantum Computation and Quantum Information*
Michael Nielsen and Isaac Chuang (Cambridge University Press, Cambridge, 2000) ("Mike and Ike")

Peter Shor (AT&T, ~1995): Quantum Computer algorithm to efficiently factorize large numbers

N-qubits: $|i\rangle \equiv |001\ldots.101\rangle \equiv |0\rangle|0\rangle|1\rangle\ldots..|1\rangle|0\rangle|1\rangle$

$$\psi_{in} = \sum_{i=0}^{2^N-1} C_i |i\rangle$$

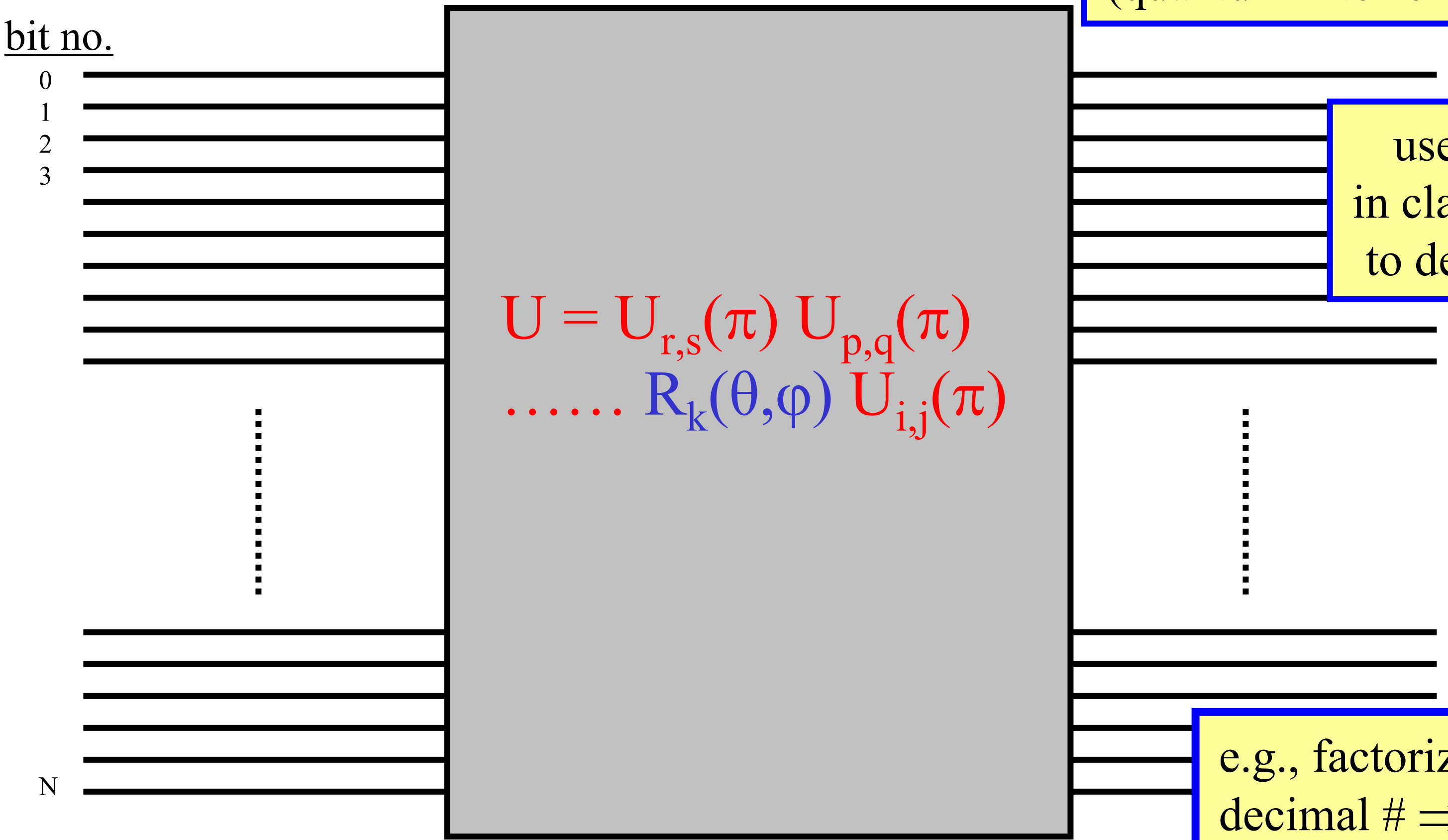
$C_i = 2^{-N/2}\ \forall\ i$

Process all possible inputs simultaneously

$$\psi_{out} = \sum_{i=0}^{2^N-1} C_i |i\rangle$$

$C_i = 0$ for most i (quantum interference)

measure qubits



use measured "i" in classical algorithm to determine factors

e.g., factorize 150 digit decimal # $\Rightarrow$ ~ 10^9 ops

Peter Shor (AT&T, ~1995): Quantum Computer algorithm to efficiently factorize large numbers

N-qubits: $|i\rangle \equiv |001\ldots.101\rangle \equiv |0\rangle|0\rangle|1\rangle\ldots..|1\rangle|0\rangle|1\rangle$

$$\psi_{in} = \sum_{i=0}^{2^N-1} C_i|i\rangle$$

$C_i = 2^{-N/2}\ \forall\ i$

$$\psi_{out} = \sum_{i=0}^{2^N-1} C_i|i\rangle$$

measure qubits

Process all possible

inp

bit

0
1
2
3

”

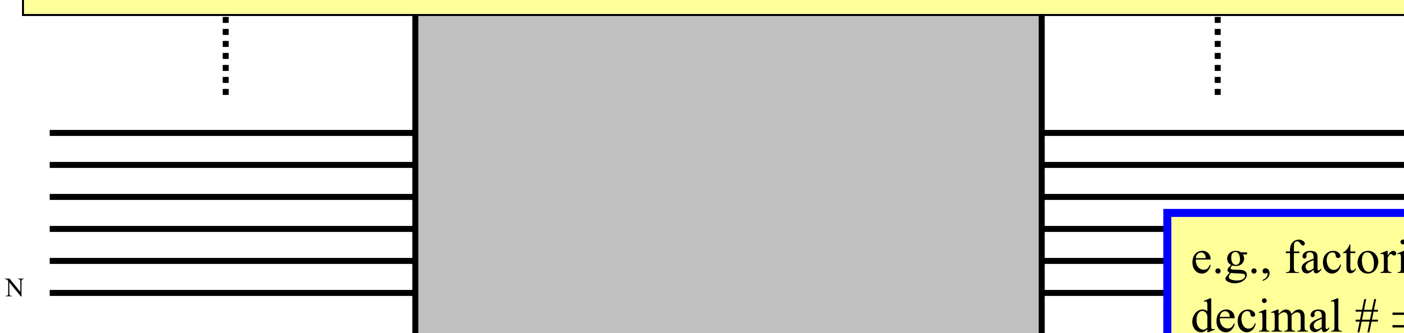
hm

ors

At some intermediate time:

$$\psi_{in}(t > t_0) = |0\rangle_0 \sum_{i=1}^{2^N-1} C_i|i\rangle + |1\rangle_0 \sum_{i=1}^{2^N-1} D_i|i\rangle$$

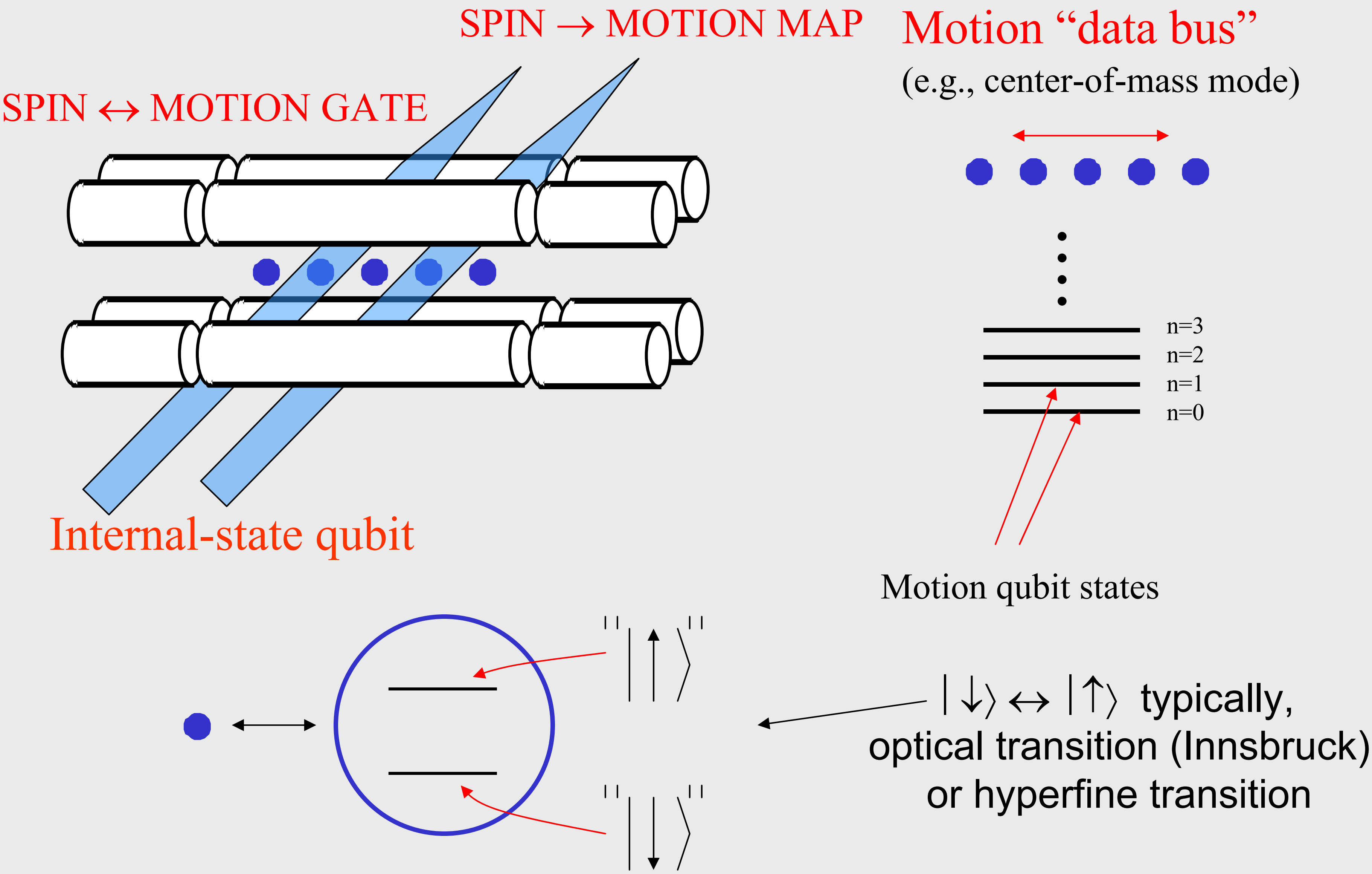
"live cat" "dead cat"



e.g., factorize 150 digit decimal # $\Rightarrow \sim 10^9$ ops

Atomic ion entanglement factory:

Basic Idea for ion quantum computer: Cirac and Zoller, PRL**74**, 4091 (1995)



Example (NIST):

$^2P_{3/2}$

$^2P_{1/2}$

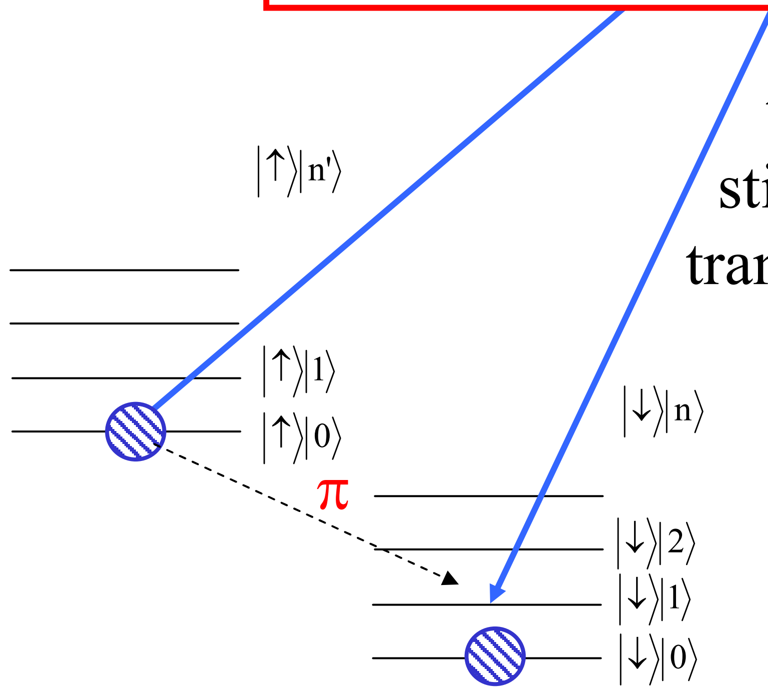
$^9Be^+$ ($^2S_{1/2}$ electronic ground state)

$|\downarrow\rangle \equiv |F = 2, m_F = -2\rangle$

$|\uparrow\rangle \equiv |F = 1, m_F = -1\rangle$

Mapping:

$$[\alpha|\downarrow\rangle + \beta|\uparrow\rangle]\otimes|0\rangle \rightarrow |\downarrow\rangle\otimes[\alpha|0\rangle + \beta|1\rangle]$$



two-photon stimulated-Raman transitions

- laser beams ⇒ addressability
- laser beams ⇒ strong gradient

$$E \propto exp(i(\vec{k}_2 - \vec{k}_1) \cdot \vec{x})$$

- transition frequency ⇒ RF modulator

details about Raman transitions: http://www.lkb.ens.fr/recherche/qedcav/houches/houches79.html

Example (NIST):

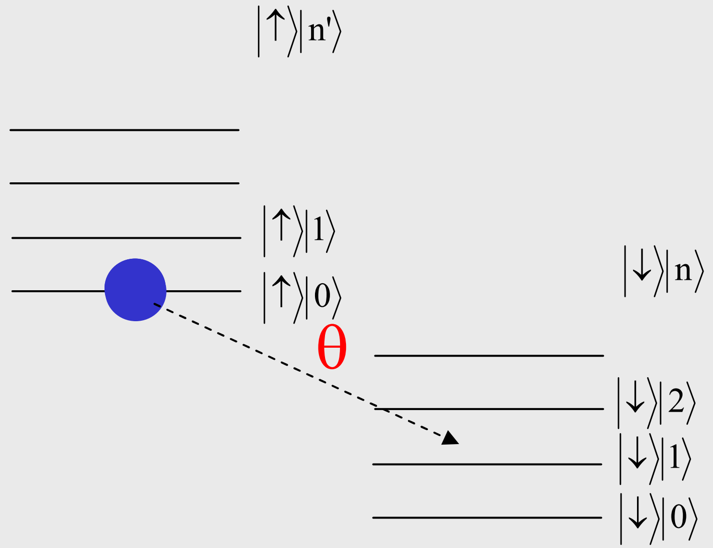
$^9Be^+$ ($^2S_{1/2}$ electronic ground state)

$|\downarrow\rangle \equiv |F = 2, m_F = -2\rangle$

$|\uparrow\rangle \equiv |F = 1, m_F = -1\rangle$

Entanglement:

$$|\uparrow\rangle \otimes |0\rangle \rightarrow \left[\cos(\theta/2)|\uparrow\rangle|0\rangle + \sin(\theta/2)|\downarrow\rangle|1\rangle\right]$$



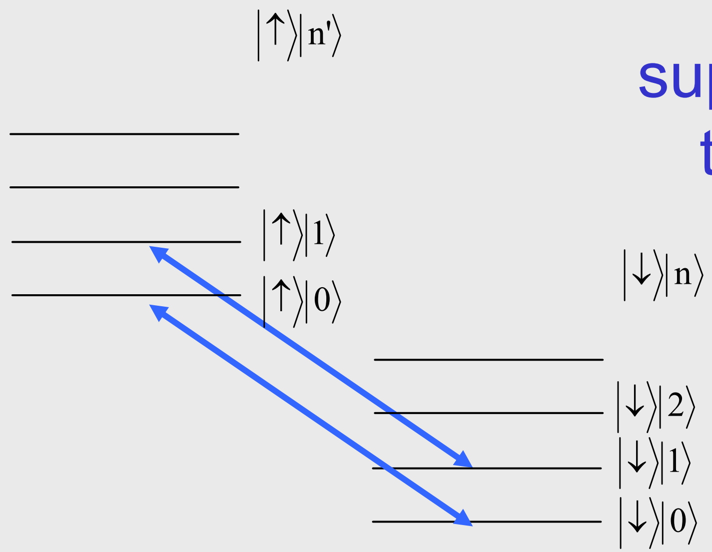
$^9Be^+$ ($^2S_{1/2}$ electronic ground state)

$|\downarrow\rangle \equiv |F = 2, m_F = -2\rangle$

$|\uparrow\rangle \equiv |F = 1, m_F = -1\rangle$

R(θ,φ):

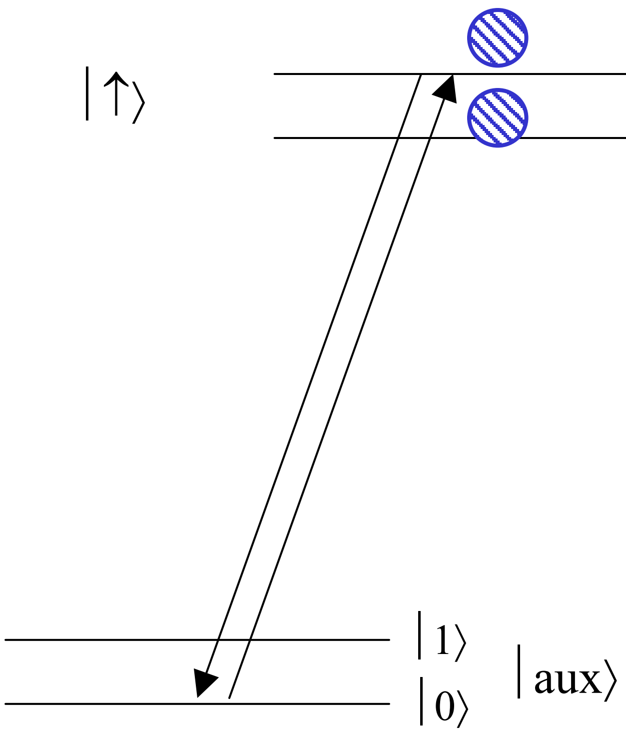
$$|\downarrow\rangle|n\rangle \rightarrow \cos(\theta/2)|\downarrow\rangle|n\rangle + e^{i\phi}\sin(\theta/2)|\uparrow\rangle|n\rangle$$
$$|\uparrow\rangle|n\rangle \rightarrow -e^{-i\phi}\sin(\theta/2)|\downarrow\rangle|n\rangle + \cos(\theta/2)|\uparrow\rangle|n\rangle$$



superposition coherence times τ_1, τ_2 > 10 min observed

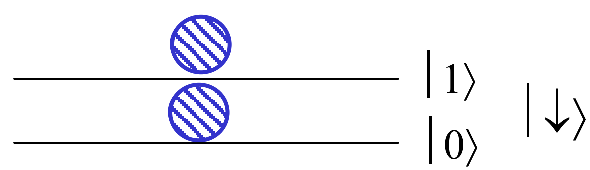
Gates, example 1:

conditional dynamics:
$\Rightarrow$ gates!



π phase shift, $|\uparrow\rangle|1\rangle \rightarrow - |\uparrow\rangle|1\rangle$

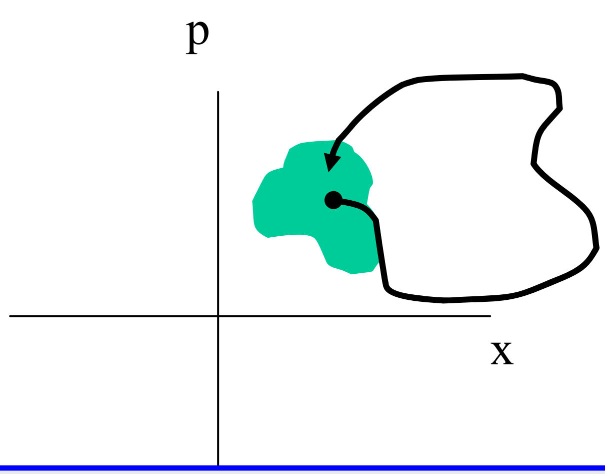
Chris Monroe *et al.*, PRL **75**, 4714 (1995)
(complete Cirac Zoller gate: Schmidt-Kaler *et al.*, *Nature* **422**, 408 (2003))



Gates, example 2:

Geometrical phase gate: (Didi Leibfried *et al.)*

phase-space diagram for (mode of axial) motion



$\Psi \rightarrow e^{i\phi}\Psi$

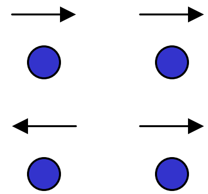
$\phi \propto$ enclosed area

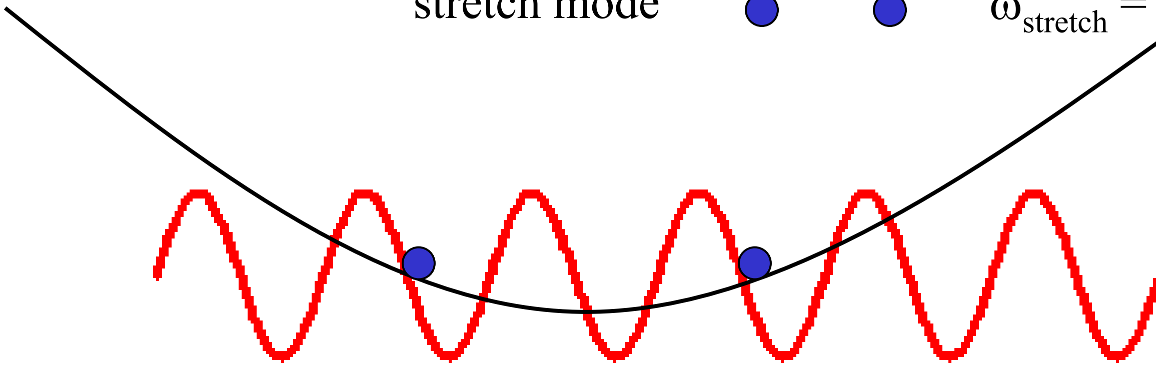
- use optical dipole forces to implement displacement
- make displacement state-dependent

special case of more general formalism by:
Milburn, Schneider, James, Forschr. Physik **48**, 801 (2000)
Sørensen & Mølmer, PRA**62**, 02231 (2000)

center-of-mass mode $\omega_{C.O.M}$

stretch mode $\omega_{stretch} = 3^{1/2}\,\omega_{C.O.M.}$



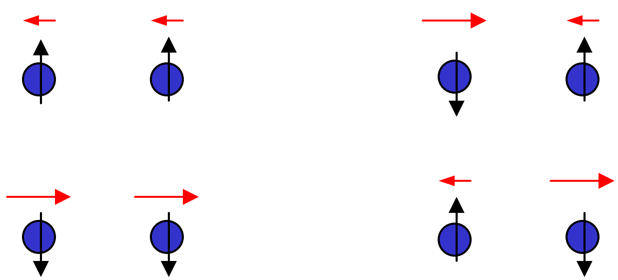


walking-wave polarization gradient

$$\vec{E} = \vec{E}_1 \sin(kx - \omega t) + \vec{E}_2 \sin(-kx - (\omega - \omega_{diff})t)$$

$$\omega_{diff} \cong \omega_{stretch}$$

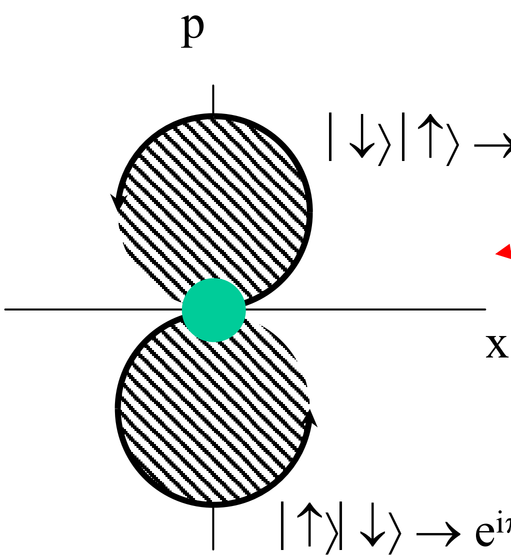
Optical-dipole (Stark shift) force, $F_{\downarrow} = -2F_{\uparrow}$



AC version of neutral-atom displacement gates (e.g., exps of Bloch, Greiner *et al.*)

$|\downarrow\rangle|\downarrow\rangle,\ |\uparrow\rangle|\uparrow\rangle$ no displacement

$|\downarrow\rangle|\downarrow\rangle \rightarrow |\downarrow\rangle|\downarrow\rangle,\quad |\uparrow\rangle|\uparrow\rangle \rightarrow |\uparrow\rangle|\uparrow\rangle$



$|\downarrow\rangle|\uparrow\rangle \rightarrow e^{i\pi/2}|\downarrow\rangle|\uparrow\rangle = i|\downarrow\rangle|\uparrow\rangle$

$|\uparrow\rangle|\downarrow\rangle \rightarrow e^{i\pi/2}|\uparrow\rangle|\downarrow\rangle = i|\uparrow\rangle|\downarrow\rangle$

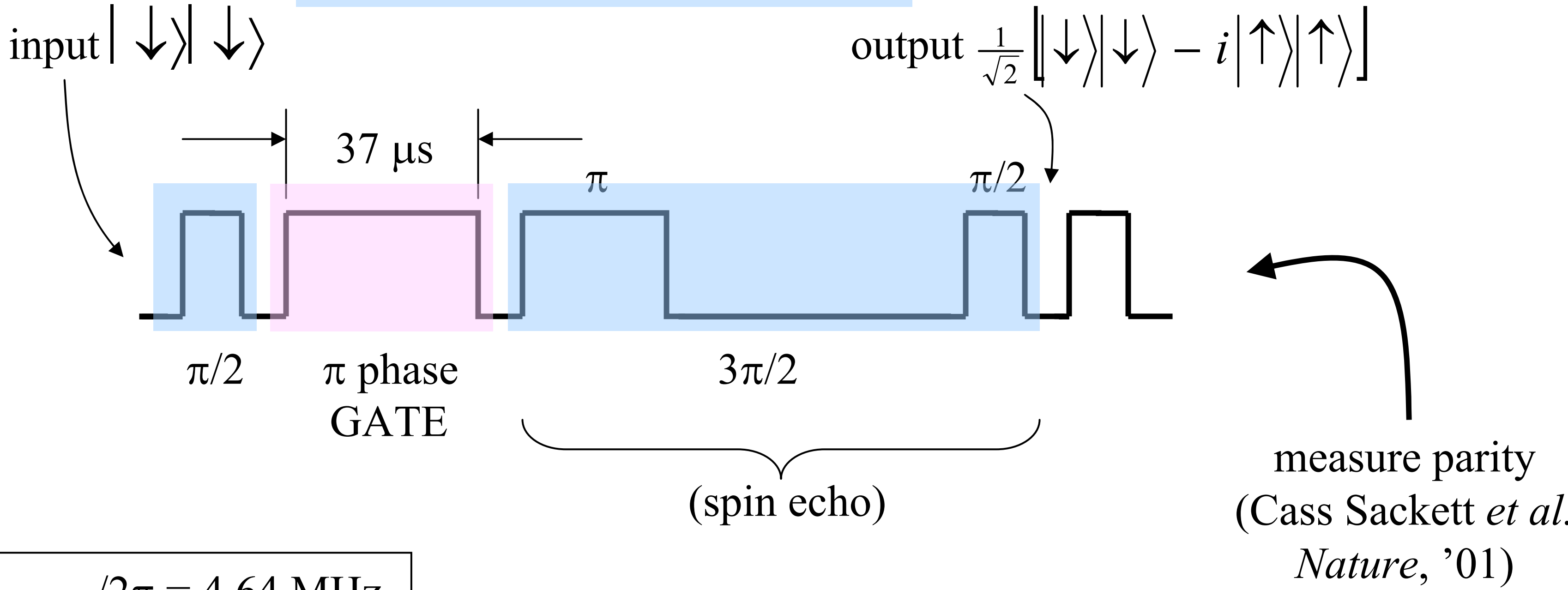
state vector

$$\begin{pmatrix} C_{\downarrow\downarrow} \\ C_{\downarrow\uparrow} \\ C_{\uparrow\downarrow} \\ C_{\uparrow\uparrow} \end{pmatrix}$$

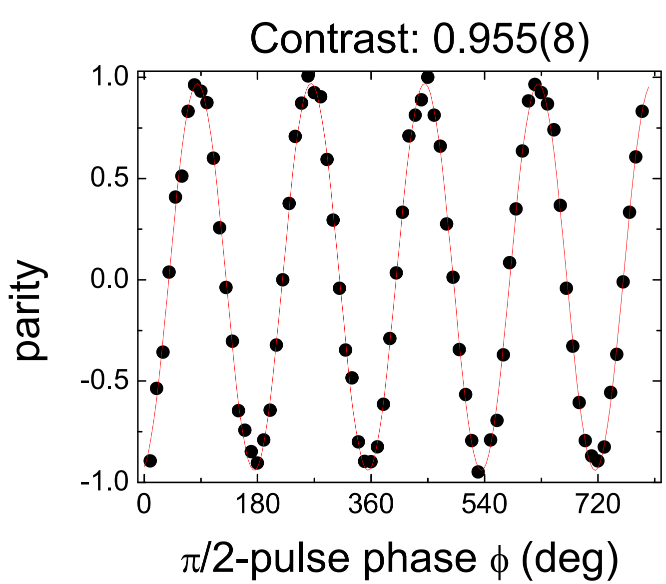
rotation — π phase gate

$$U = \begin{pmatrix} 1 & & & \\ & e^{i\pi/2} & & \\ & & e^{i\pi/2} & \\ & & & 1 \end{pmatrix} = \begin{pmatrix} 1 & & & \\ & e^{i\pi/2} & & \\ & & e^{i\pi/2} & \\ & & & e^{i\pi} \end{pmatrix} \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & e^{-i\pi} \end{pmatrix}$$

$\pi/2$, $3\pi/2$ Ramsey interferometer



$\omega_{C.O.M.}/2\pi = 4.64$ MHz

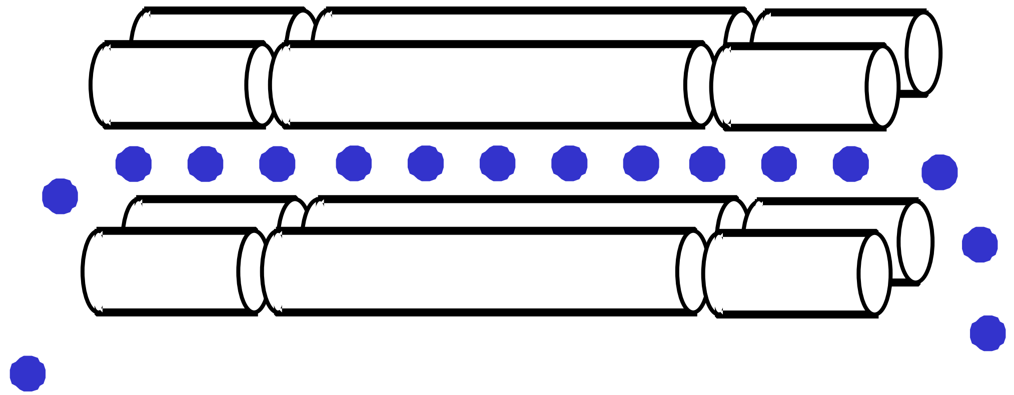


Fidelity of Bell states made with gate:

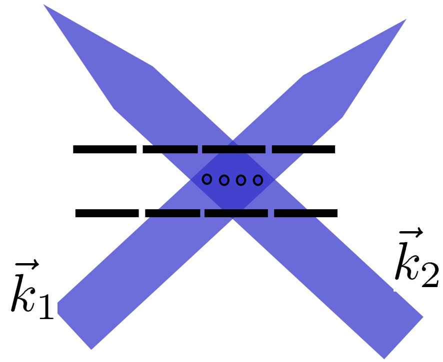
$$\mathcal{F} \equiv \tfrac{1}{2}\left\{\langle\downarrow|\langle\downarrow| + i\langle\uparrow|\langle\uparrow|\right\}\rho\left\{|\downarrow\rangle|\downarrow\rangle - i|\uparrow\rangle|\uparrow\rangle\right\}$$
$$\cong 0.97$$

Didi Leibfried *et al.*, *Nature* **422**, 412 (2003)

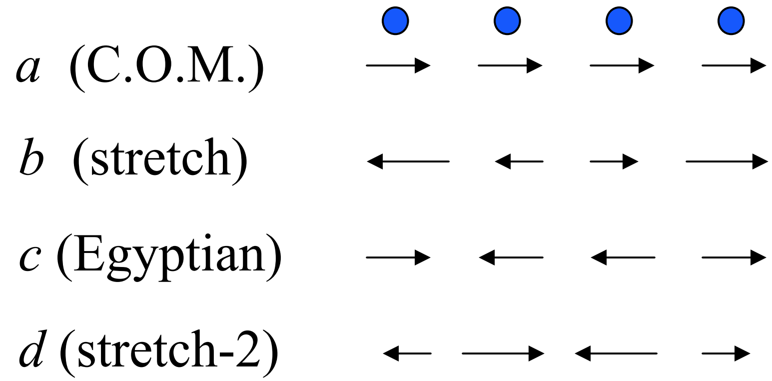
Scale up?



Many ions in one trap?

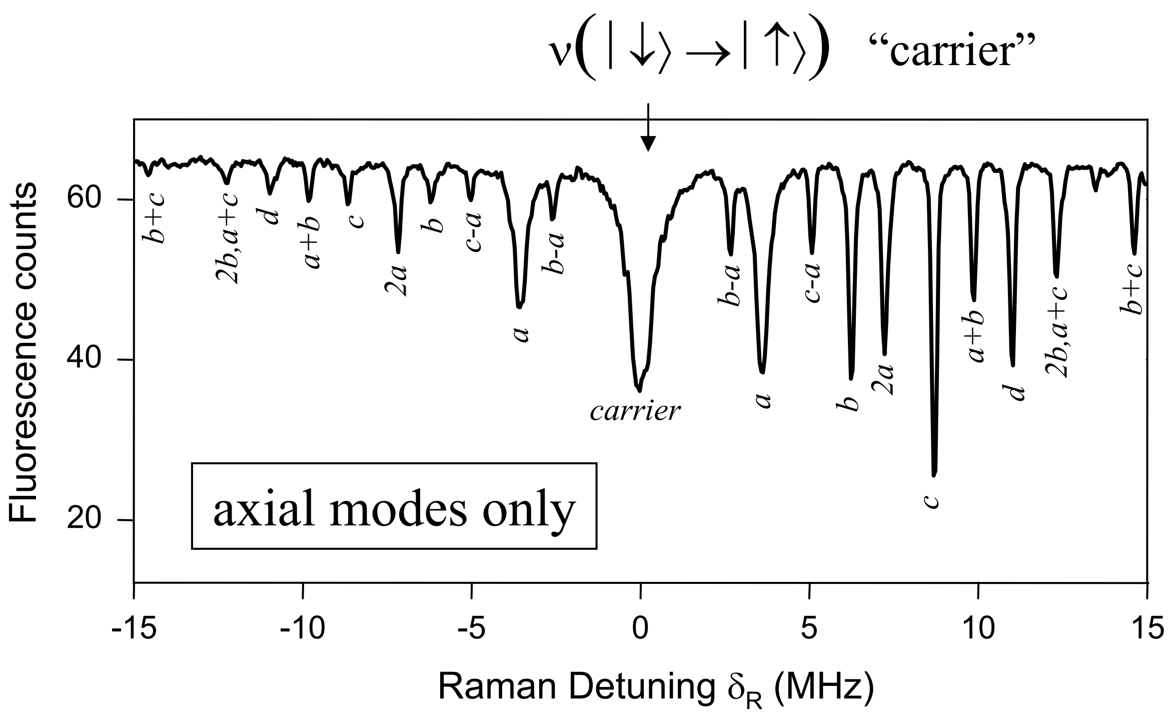


axial modes, N = 4 ions

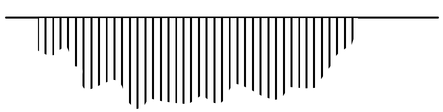


Four-ion excitation spectrum:

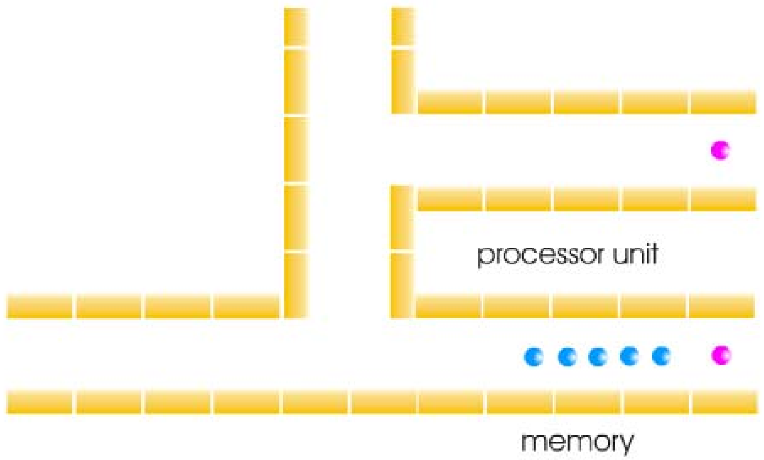
(Chris Monroe *et al.*, *Atomic Physics 17*, 2001)



N-ion spectrum:



multiplexed trap architecture



1. interconnected multi-zone structure
- subtraps decoupled

2. move ions with electrode potentials
3. logic ions sympathetically cooled
- few normal modes to cool
- weak cooling in memory zone

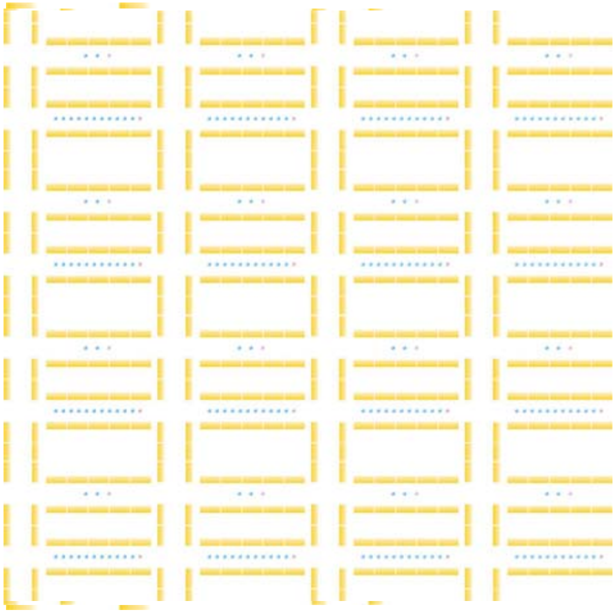
4. individual optical addressing during gates not required
- gates in tight trap ⇒ fast

5. readout, for error correction, in (shielded) subtrap
- no decoherence from fluorescence

- Wineland *et al.*, J. Res. Nat. Inst. Stand. Technol. **103**, 259 (1998);
- Kielpinski *et al.*, Nature **417**, 709 (2002).

Other proposals:
- Cirac *et al.*, Phys. Rev. Lett. **78**, 3221 (1997)
- DeVoe, Phys. Rev. A **58**, 910 (1998)
- Cirac & Zoller, Nature **404**, 579 (2000)
- L.-M. Duan, et. al., quant-ph/0401020 (2004)

Modularity

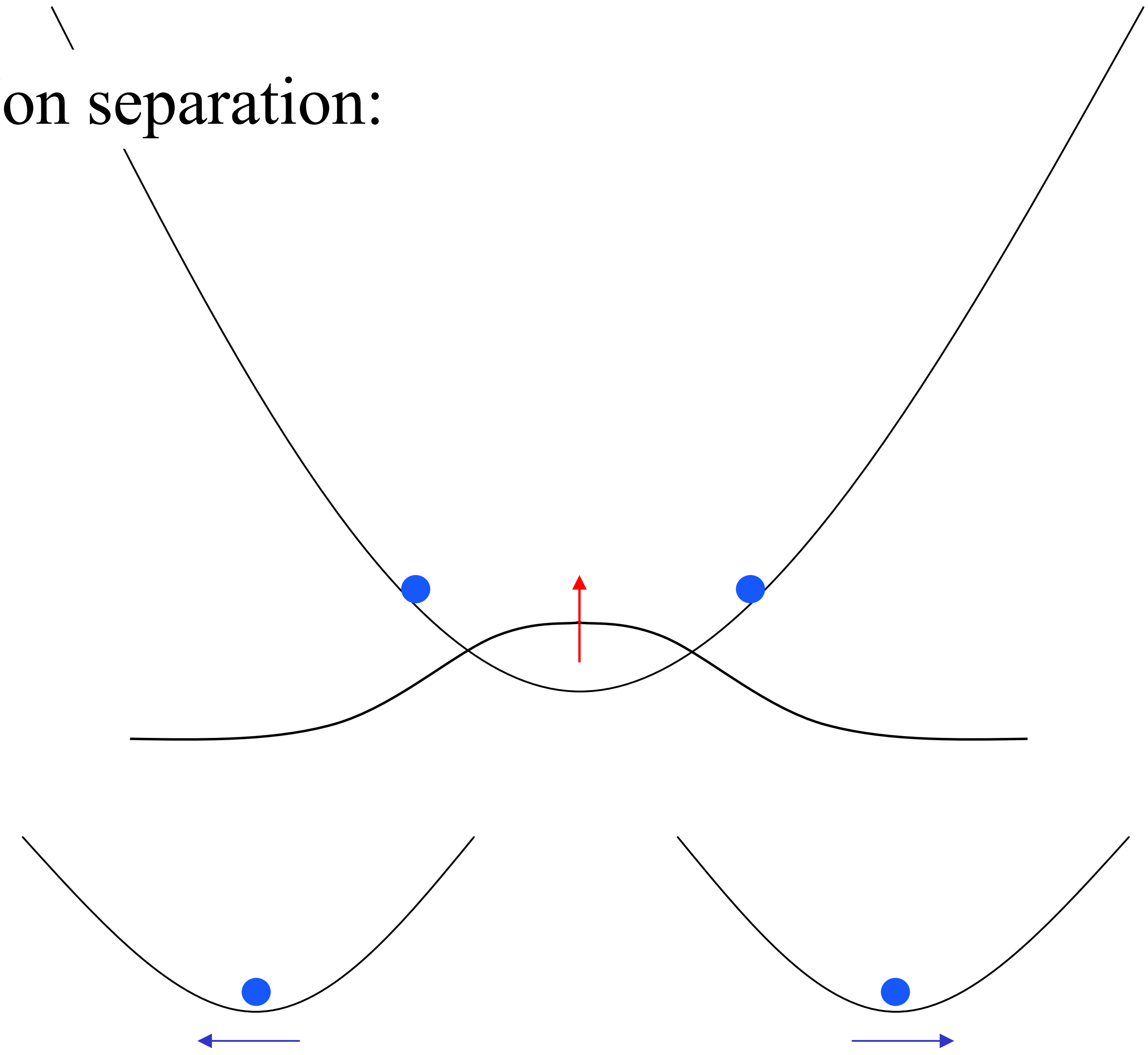


array N→4N:

- no additional motional modes
- mode frequencies same

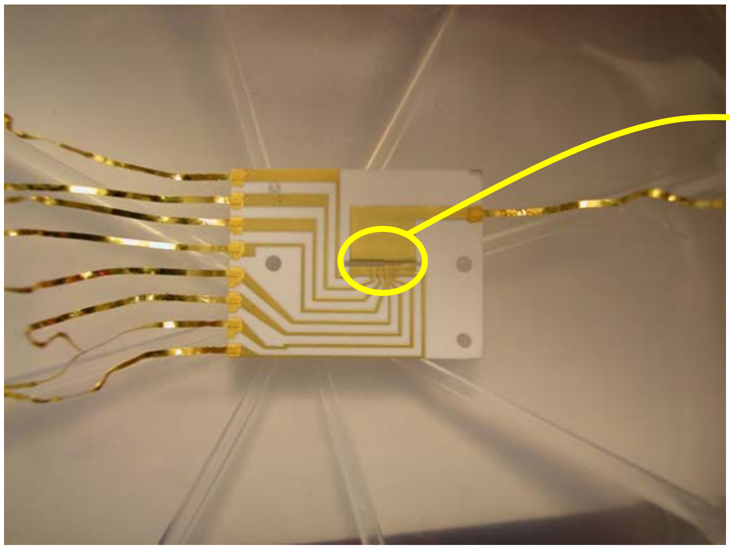
"only" have to demonstrate basic module

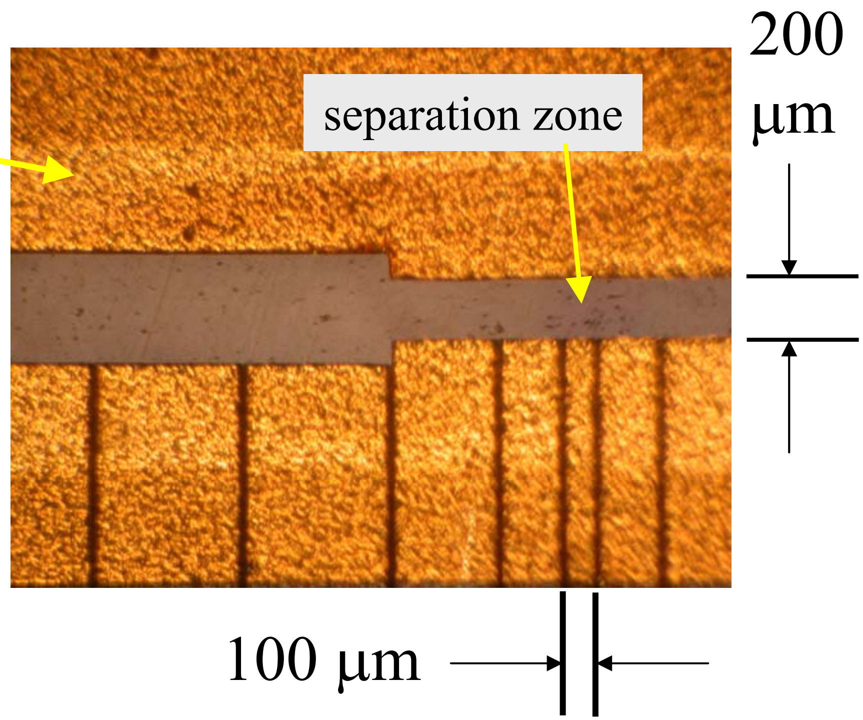
Ion separation:



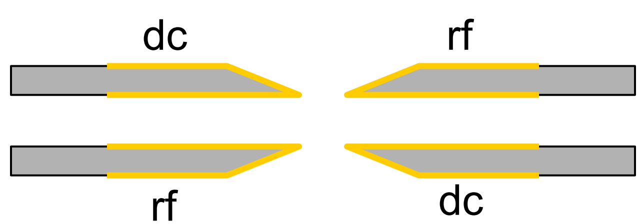
separation in six zone alumina/gold trap

(Murray Barrett, Tobias Schaetz *et al.*)

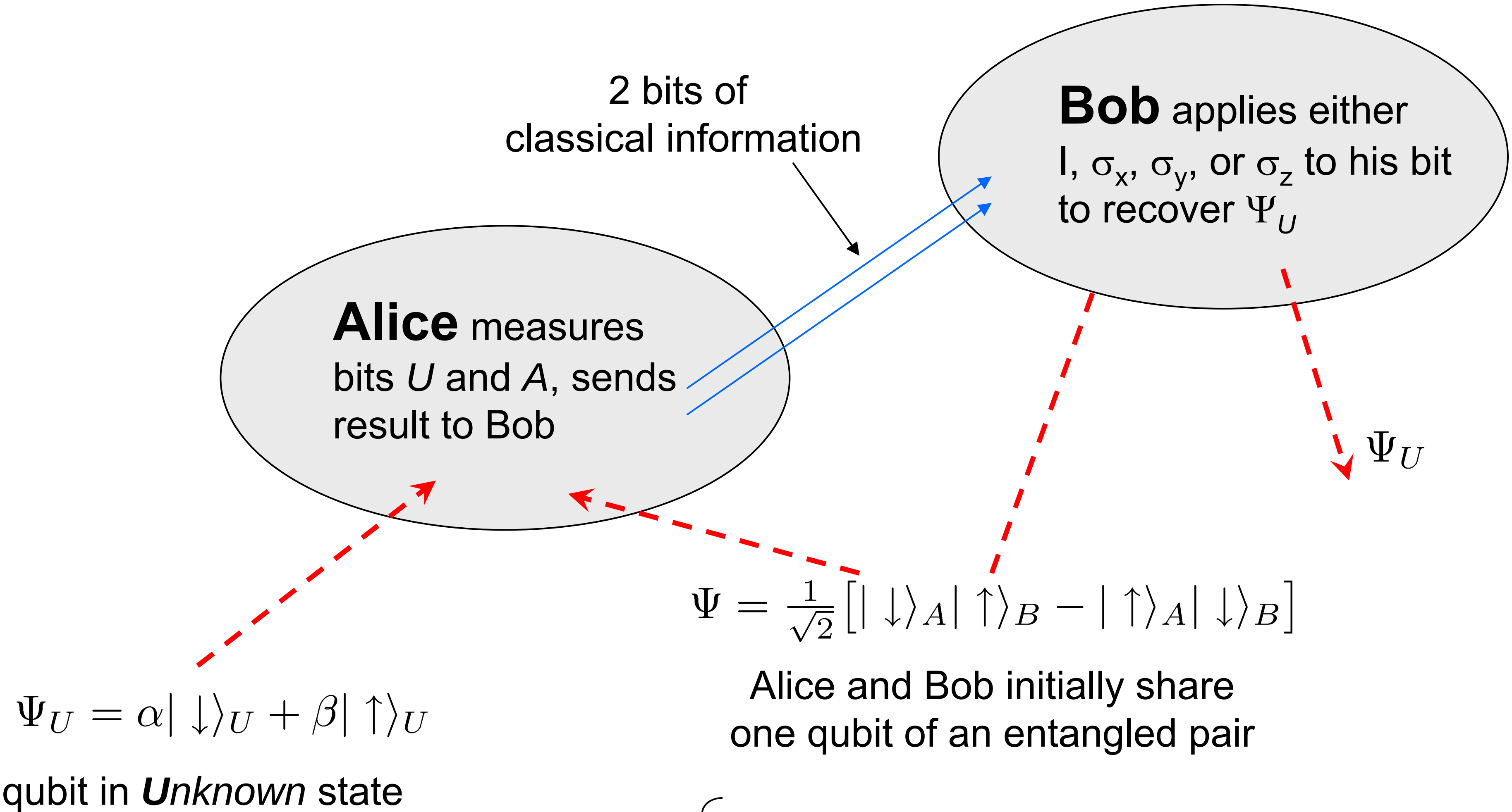




view along axis:



Quantum Teleportation (C. Bennett *et al.*, PRL 1993)



PHOTONS:

- Boumeester *et al*. *Nature*, '97
- Furusawa *et al. Science*, '98
- Boshi *et al.* PRL, '98
- Kim *et al.* PRL, '01
- Bowen *et al.* PRA '03
- Zhang *et al.* PRA '03

NMR: Nielsen *et al. Nature* '98

Teleportation protocol:

$\Psi_{A,B} = |\downarrow\rangle_A|\uparrow\rangle_B - |\uparrow\rangle_A|\downarrow\rangle_B$ (normalization omitted)

$\Psi_{unknown} \equiv \Psi_U = \alpha|\downarrow\rangle_U + \beta|\uparrow\rangle_U$

rewrite $\Psi = \Psi_{A,B} \otimes \Psi_{unknown}$

$\Psi = \Sigma^4_{k=1} \Psi_{A,U,k} \otimes (\tilde{O}_k \Psi_{unknown})_B$

$\Psi_{A,U,k}$ orthonormal & entangled ("Bell states")

$(\tilde{O}_k \Psi_{unknown})_B$ orthonormal

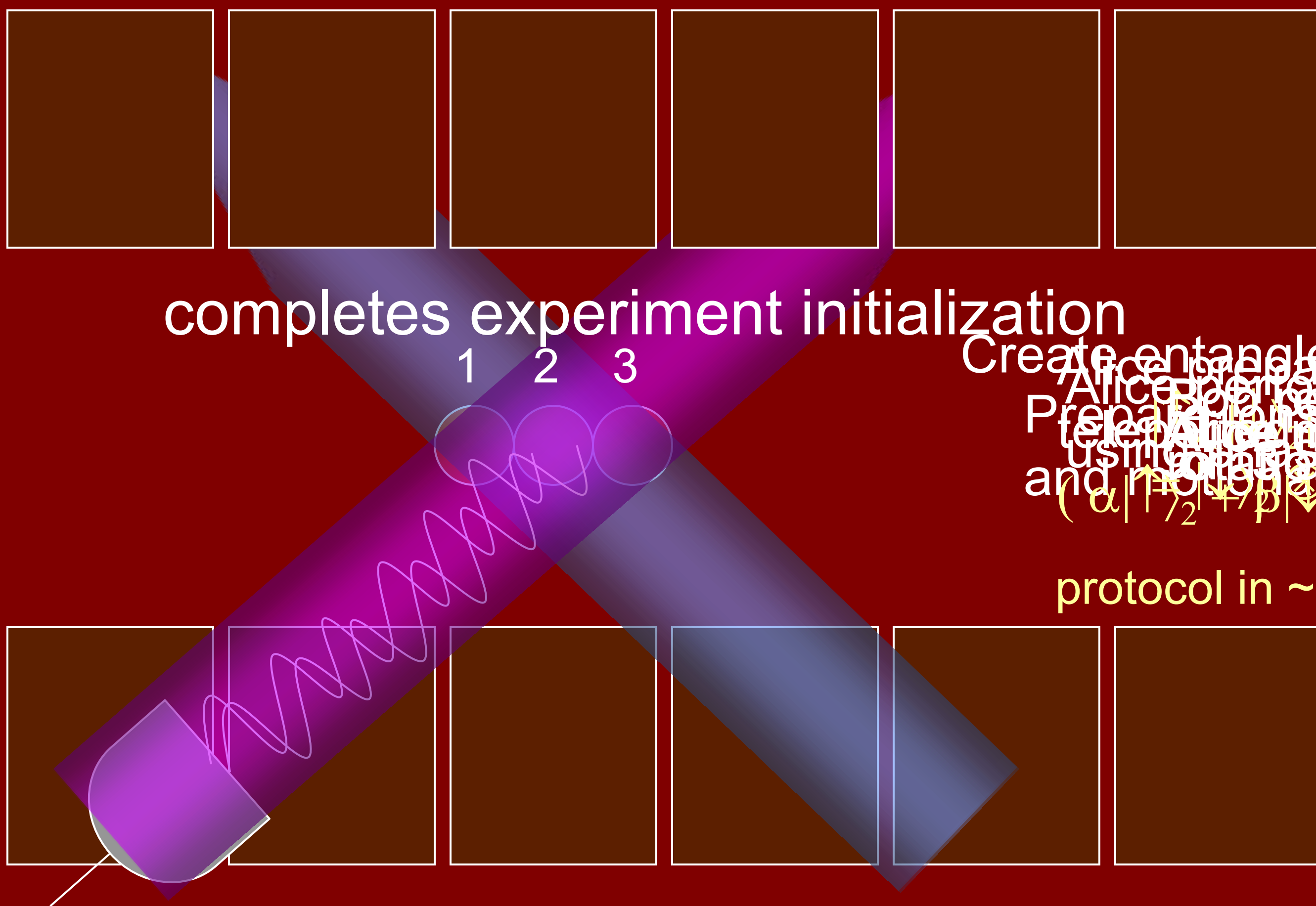
Bell states:

$\Psi_- = |\downarrow\rangle_A|\uparrow\rangle_B - |\uparrow\rangle_A|\downarrow\rangle_B$, $\Psi_+ = |\downarrow\rangle_A|\uparrow\rangle_B + |\uparrow\rangle_A|\downarrow\rangle_B$

$\Phi_- = |\downarrow\rangle_A|\downarrow\rangle_B - |\uparrow\rangle_A|\uparrow\rangle_B$, $\Phi_+ = |\downarrow\rangle_A|\downarrow\rangle_B + |\uparrow\rangle_A|\uparrow\rangle_B$

Bob applies $(\tilde{O}_k)^{-1}$

Quantum Teleportation with ions



completes experiment initialization

Create entangled state on outer ions

Alice performs Bell basis decoding

Alice measures ions 1 and 2

and motional ground state

$(\alpha|\uparrow\rangle_2+\beta|\downarrow\rangle_2)(|\uparrow\rangle_1|\downarrow\rangle_3+|\downarrow\rangle_1|\uparrow\rangle_3)$

protocol in ~2.5 msec

Barrett *et al.*, *Nature*, June, ’04)
(also demonstrated at Innsbruck with Ca^+ ions, *Nature*, June ‘04)

Teleportation (and other experiments) require lots of spin-echos!

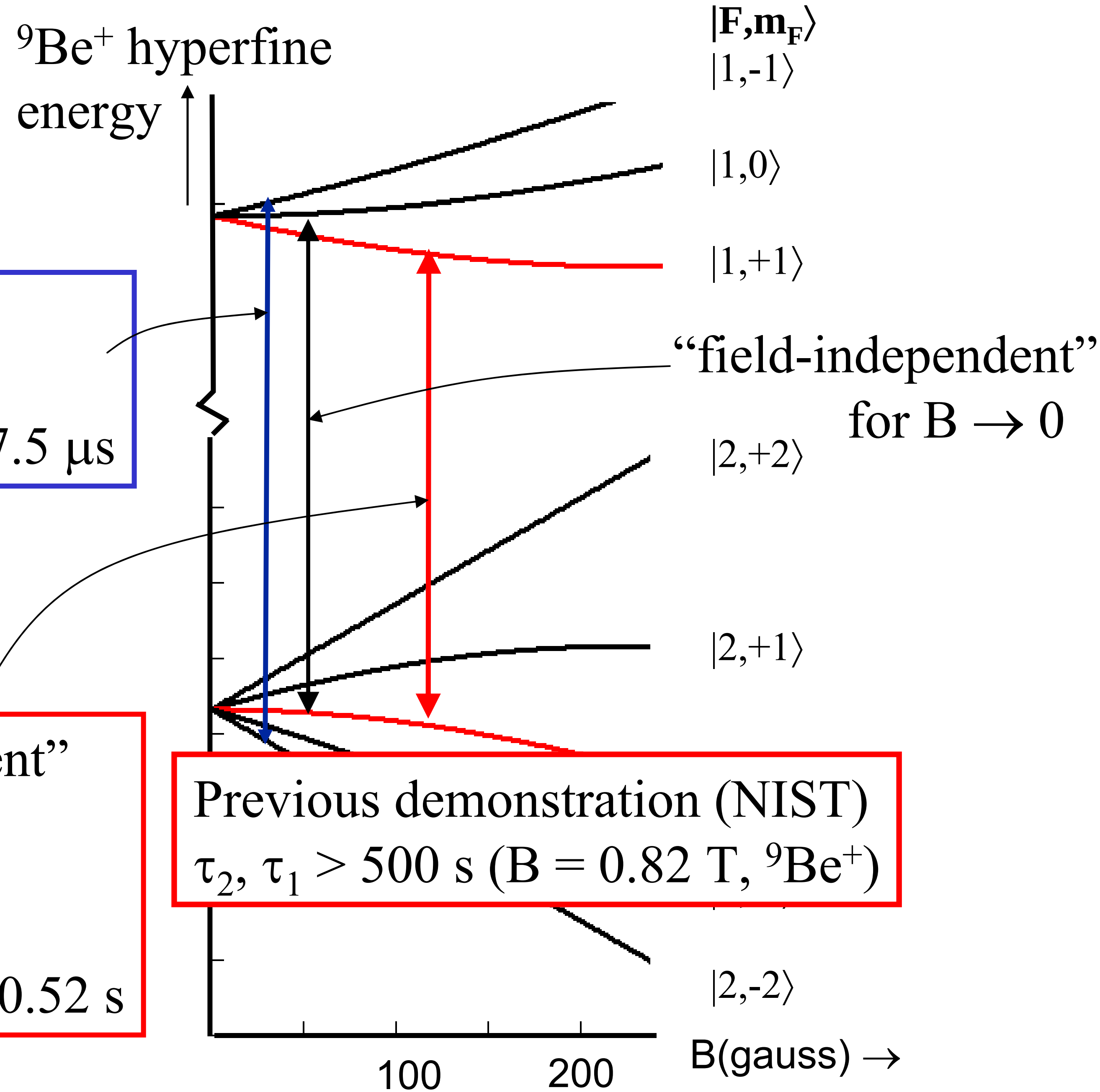
DFS qubits: (Dave Kielpinski *et al*., *Science*, **291**, (2001))

$$|0\rangle_{logical} = \alpha|\downarrow\rangle_1|\uparrow\rangle_2 + \beta|\uparrow\rangle_1|\downarrow\rangle_2$$

$$|1\rangle_{logical} = \beta^*|\downarrow\rangle_1|\uparrow\rangle_2 - \alpha^*|\uparrow\rangle_1|\downarrow\rangle_2$$

immune to magnetic field fluctuations
(but not gradients)

field-independent physical qubits?



for $\delta B = 0.01$ G
$|2,2\rangle \leftrightarrow |1,1\rangle$
$\tau_2(\delta\phi = 1 \text{ rad}) \cong 7.5\ \mu s$

"field-independent" at 119.5 gauss
(Chris Langer)
for $\delta B = 0.01$ G,
$\tau_2(\delta\phi = 1 \text{ rad}) \cong 0.52$ s

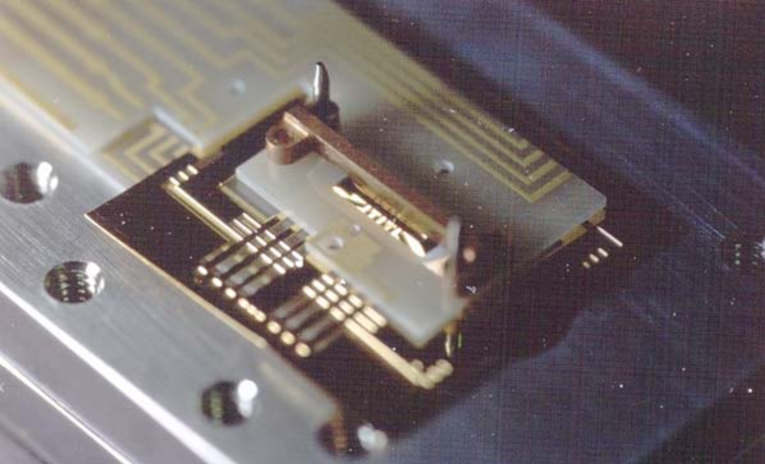
Previous demonstration (NIST)
$\tau_2, \tau_1 > 500$ s ($B = 0.82$ T, $^9Be^+$)

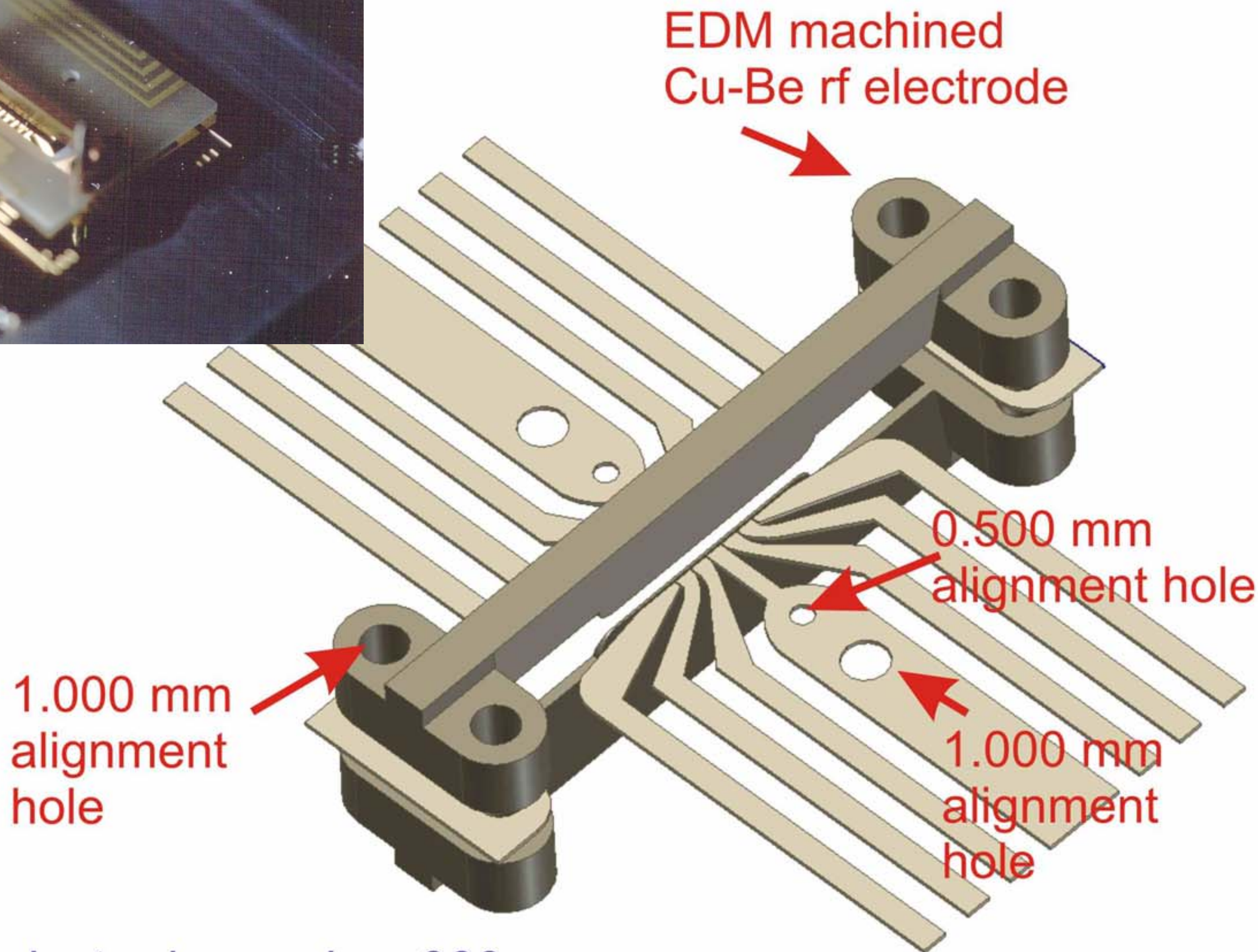
Bad news: Motional phase gate generally won't work on field independent hyperfine transitions

Trapology:

Requirements:

- small (~ 10 - 400 μm electrode separations)
- no RF breakdown (~ 500 V, ~ 100 MHz
  between RF and "control" electrodes)
- small RF loss tangent of insulators
- high vacuum compatibility
  (~ 10^{-11} Torr, room temp)
- bakeable (~ 300° C)
- CLEAN electrodes



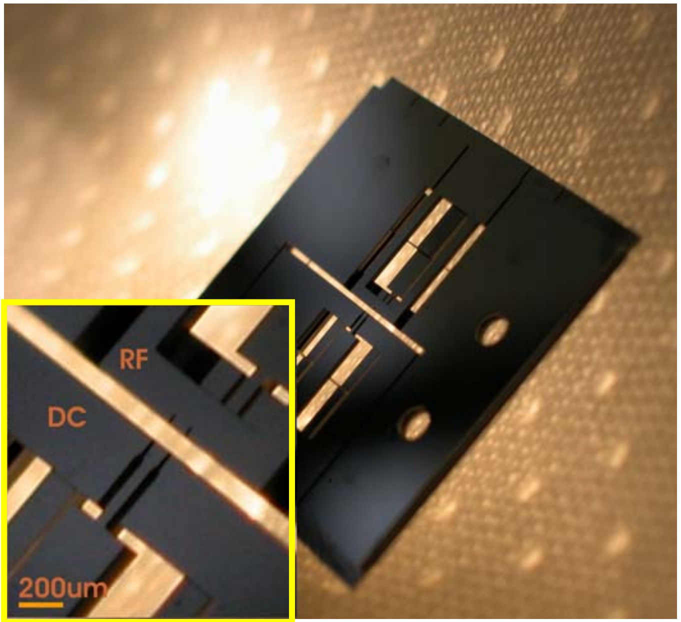


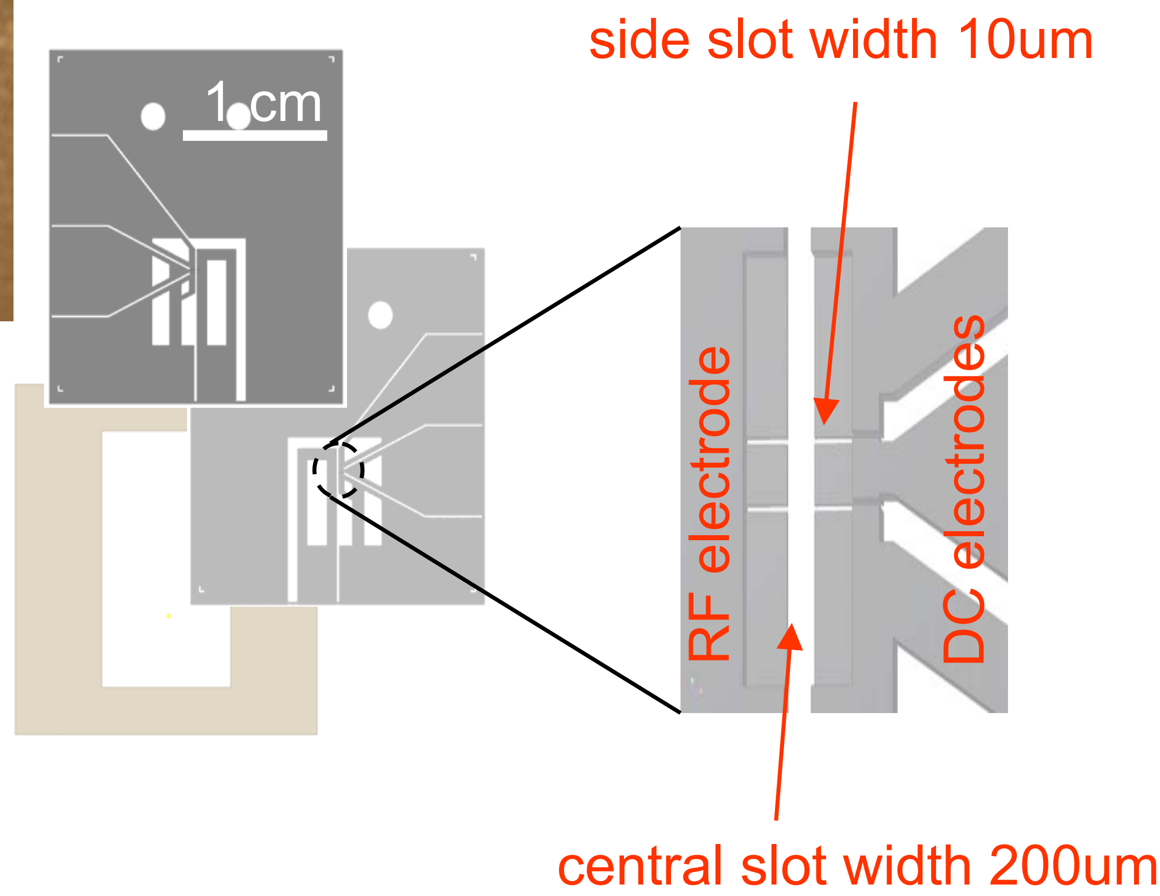
DC electrode spacing: 300 μm
rf electrode spacing: 630 μm

Brian DeMarco, Amit Ben-Kish

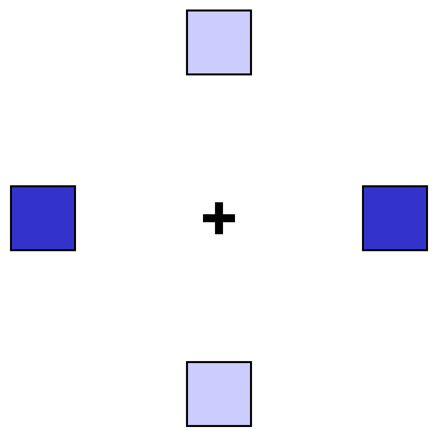
B-Silicon Trap

(Joe Britton, Dave Kielpinski)

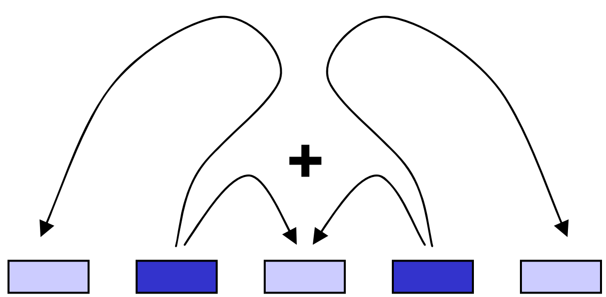




Read neutral atom papers: planar geometry: (John Chiaverini)

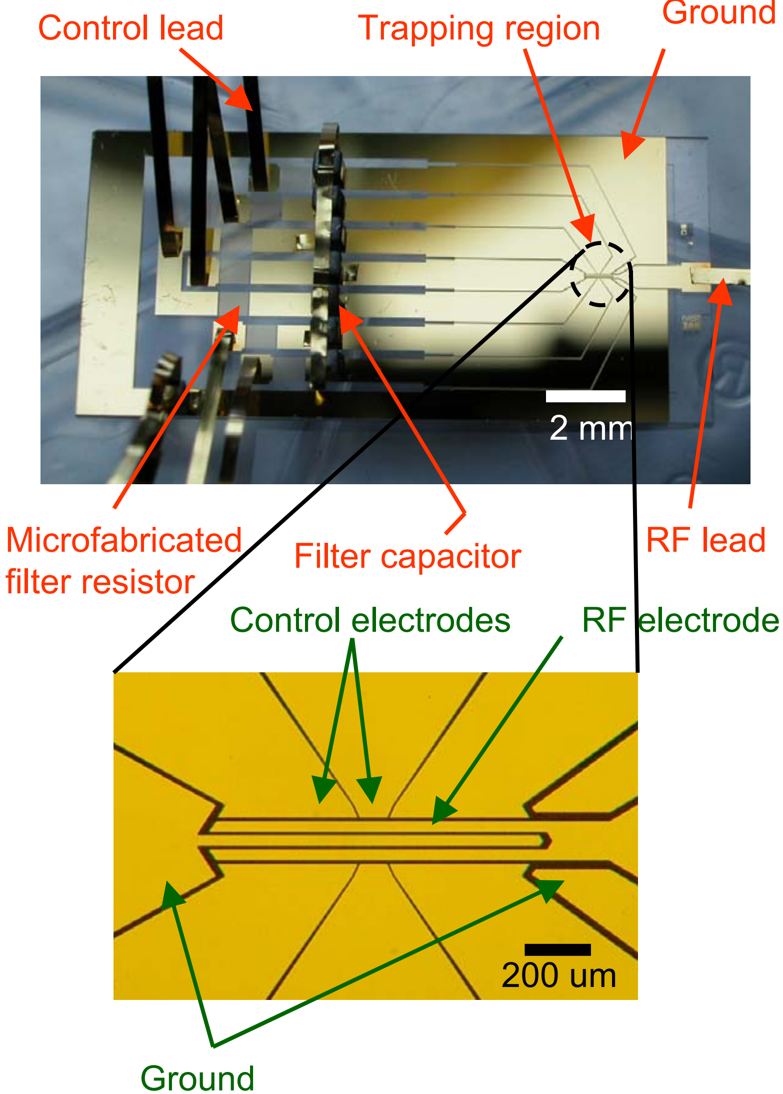


Field lines:

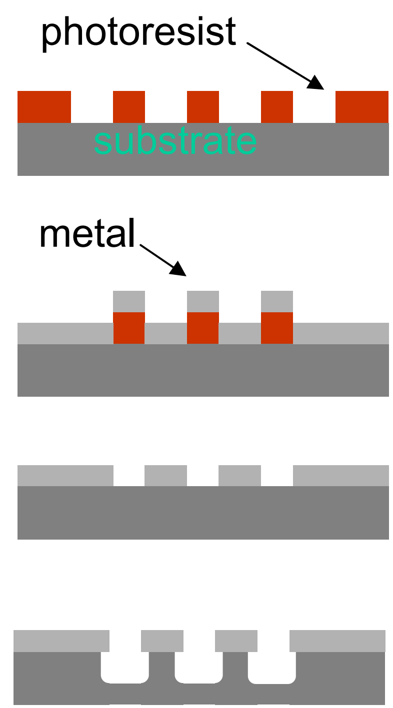


- fabrication steps
  - low loss substrate
  - deposit/pattern metal
- control electrodes on outside (connections straightforward)
- on-chip filtering

Microfabricated surface ion trap



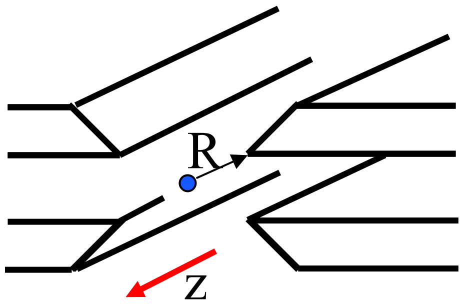
Current fabrication method: Liftoff with substrate etch



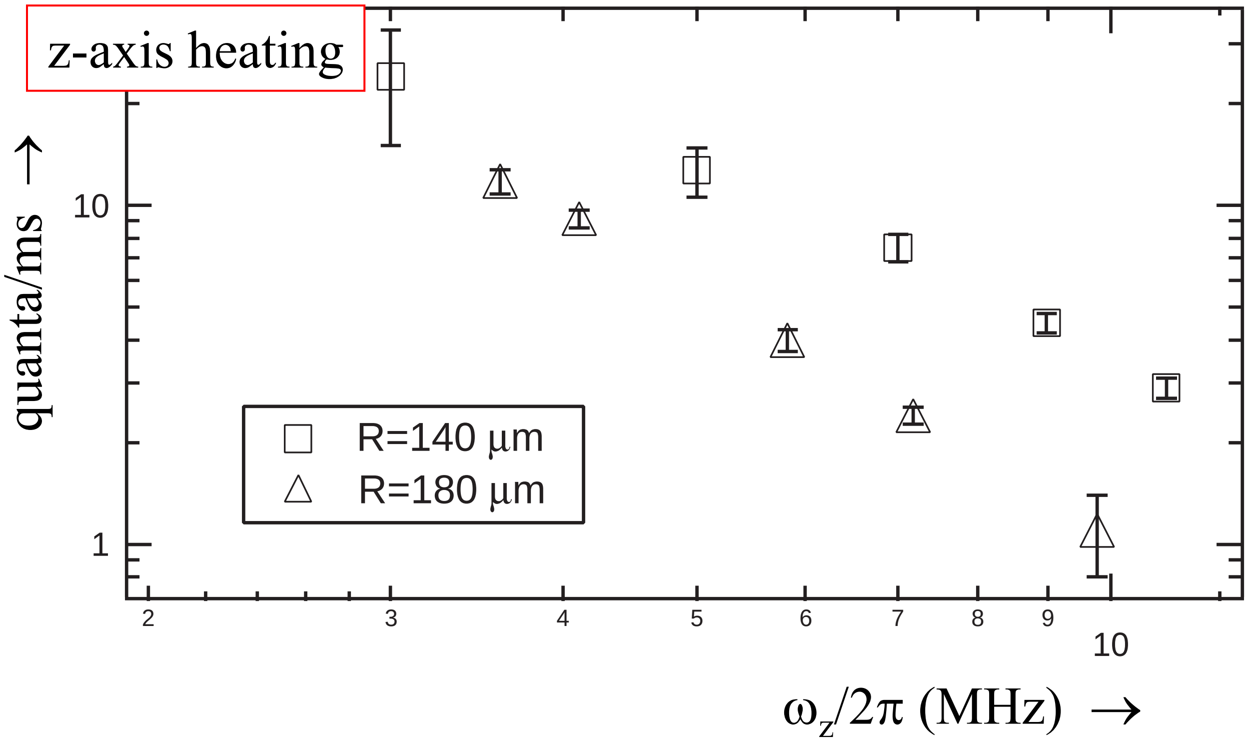
1. Coat substrate and define wire pattern in resist
2. Deposit metal
3. Remove resist
4. Etch trenches in substrate (RIE or HF) and remove resist

Heating in linear traps

(greater than Johnson noise heating !)



Turchette *et al.*, PRA **61**, 063418 (2000)

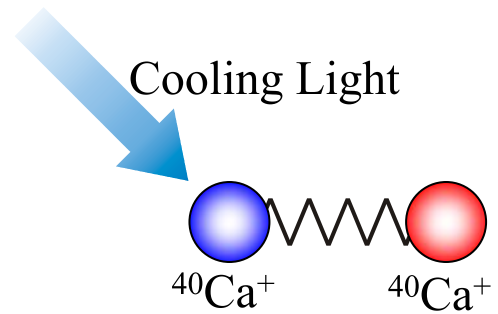


R = 270 μm (Mary Rowe *et al*. ‘02)

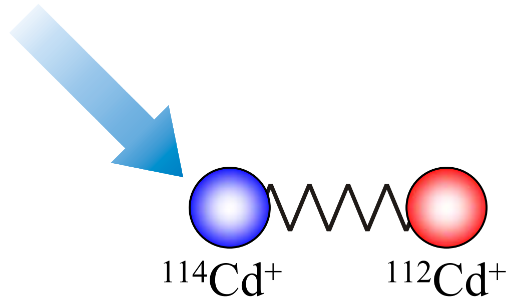
Sympathetic Cooling

Approaches:

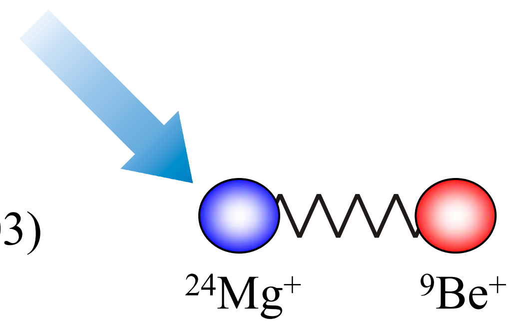
Cooling with same species
Innsbruck group: Rhode, *et al.*,
J. Opt. B **3**, S34 (2001)



Cooling with different isotopes
Michigan group: Blinov, *et al.*,
PRA **65**, 040304 (2002)



Cooling with different ion species
NIST (Barrett *et al.*, PRA**68**, 042302 (2003)



Next …

I. Quantum-information processing: put together all elements of multiplexed trap, improve fidelity, increase *N*

more complicated algorithms, quantum error correction, …

II. build larger (and more reliable) trap arrays

lithography, chemical machining, MEMS, ?

III. "scale" electronics and optics

integrated electronics and optics (multiplexers, DACs, MEMS mirrors, …)

V. Future?

- crack secret codes and make Schrödinger's cat?

or: discover fundamental source of decoherence!

- better clocks
- ???

NIST ions, March, '04

From left to right:

Joe Britton, Jim Bergquist, John Chiaverini, Windell Oskay, Marie Jensen, John Bollinger, Vladislav Gerginov, Taro Hasegawa, Carol Tanner, Wayne Itano, Jim Beall, David Wineland, Dietrich Leibfried, Chris Langer, Tobias Schaetz, John Jost, Roee Ozeri, Till Rosenband, Piet Schmidt, Brad Blakestad