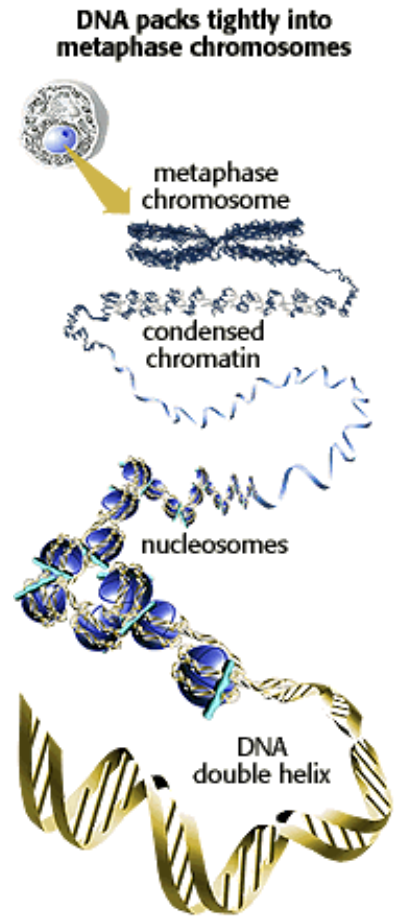


# Introduction to Physical Biology:

## Models and Numbers

Boulder Lectures on Soft Matter Physics  
July 2012  
Yitzhak Rabin



R. Phillips, J. Kondev, J. Theriot, "*Physical Biology of the Cell*" (Garland, 2009)  
A.D. Bates and A. Maxwell, "*DNA Topology*" (Oxford, 2005)

# Building blocks of cells – 4 types of macromolecules made of simple repeat units:

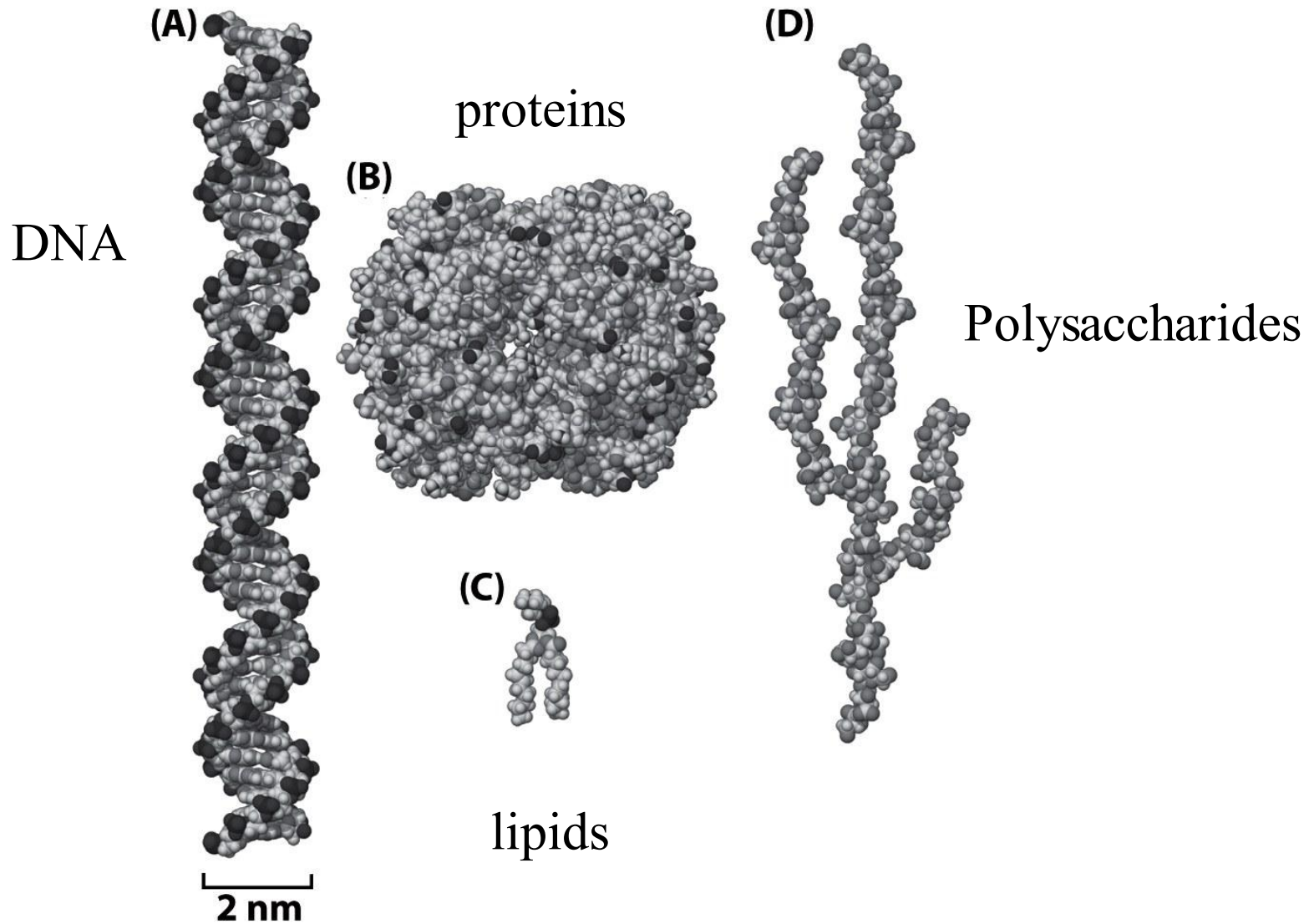


Figure 1.1 Physical Biology of the Cell (© Garland Science 2009)

# Distribution of macromolecules in E. coli bacterium

Substance	% of total dry weight	Number of molecules
Macromolecule		
Protein	55.0	$2.4 \times 10^6$
RNA	20.4	
23S RNA	10.6	19,000
16S RNA	5.5	19,000
5S RNA	0.4	19,000
Transfer RNA (4S)	2.9	200,000
Messenger RNA	0.8	1,400
Phospholipid	9.1	$22 \times 10^6$
Lipopolysaccharide	3.4	$1.2 \times 10^6$
DNA	3.1	2
Murein	2.5	1
Glycogen	2.5	4,360
<b>Total macromolecules</b>	<b>96.1</b>	
Small molecules		
Metabolites, building blocks, etc.	2.9	
Inorganic ions	1.0	
<b>Total small molecules</b>	<b>3.9</b>	

Table 2.1 Physical Biology of the Cell (© Garland Science 2009)

# Language of DNA and proteins:

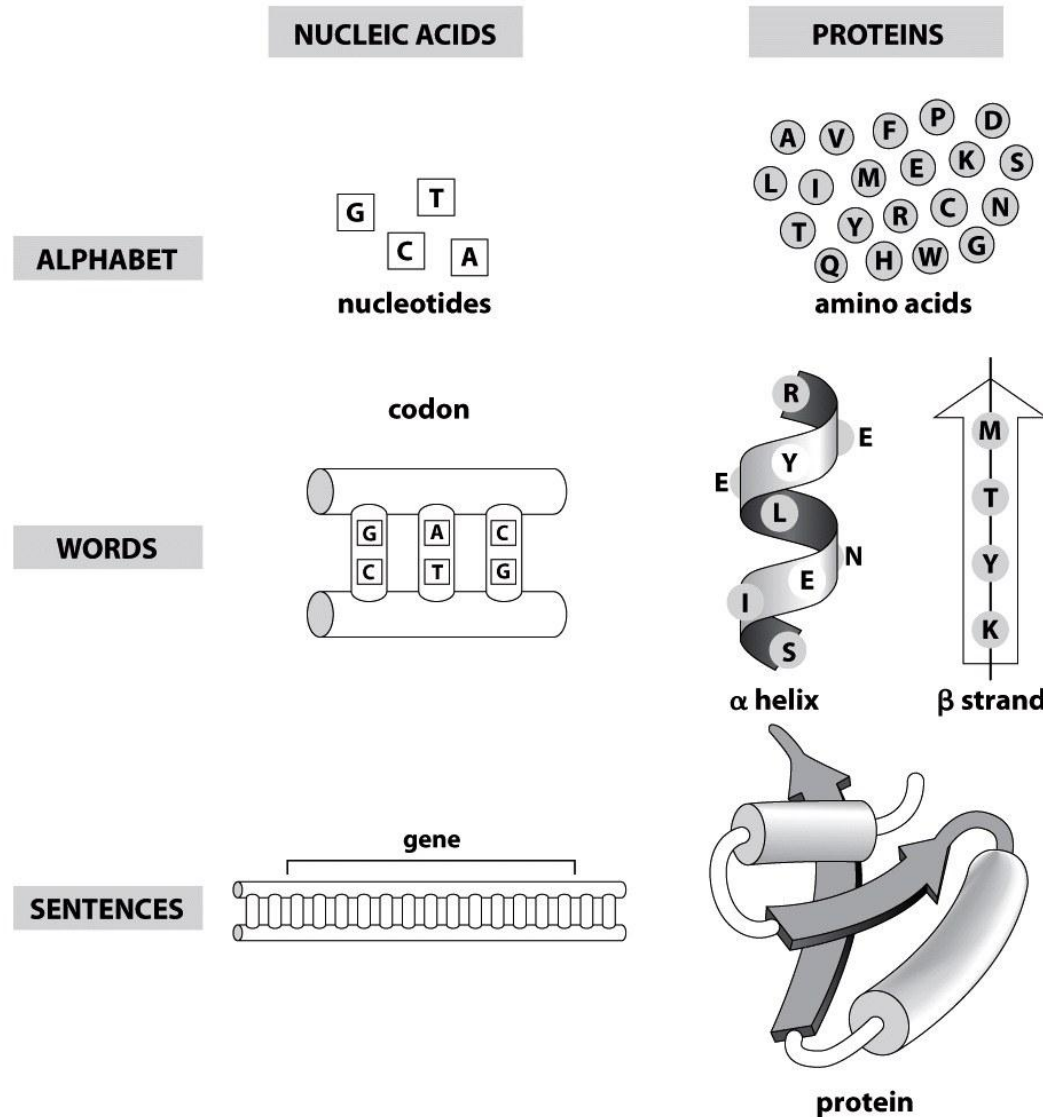


Figure 1.2 Physical Biology of the Cell (© Garland Science 2009)



# Nucleotides:

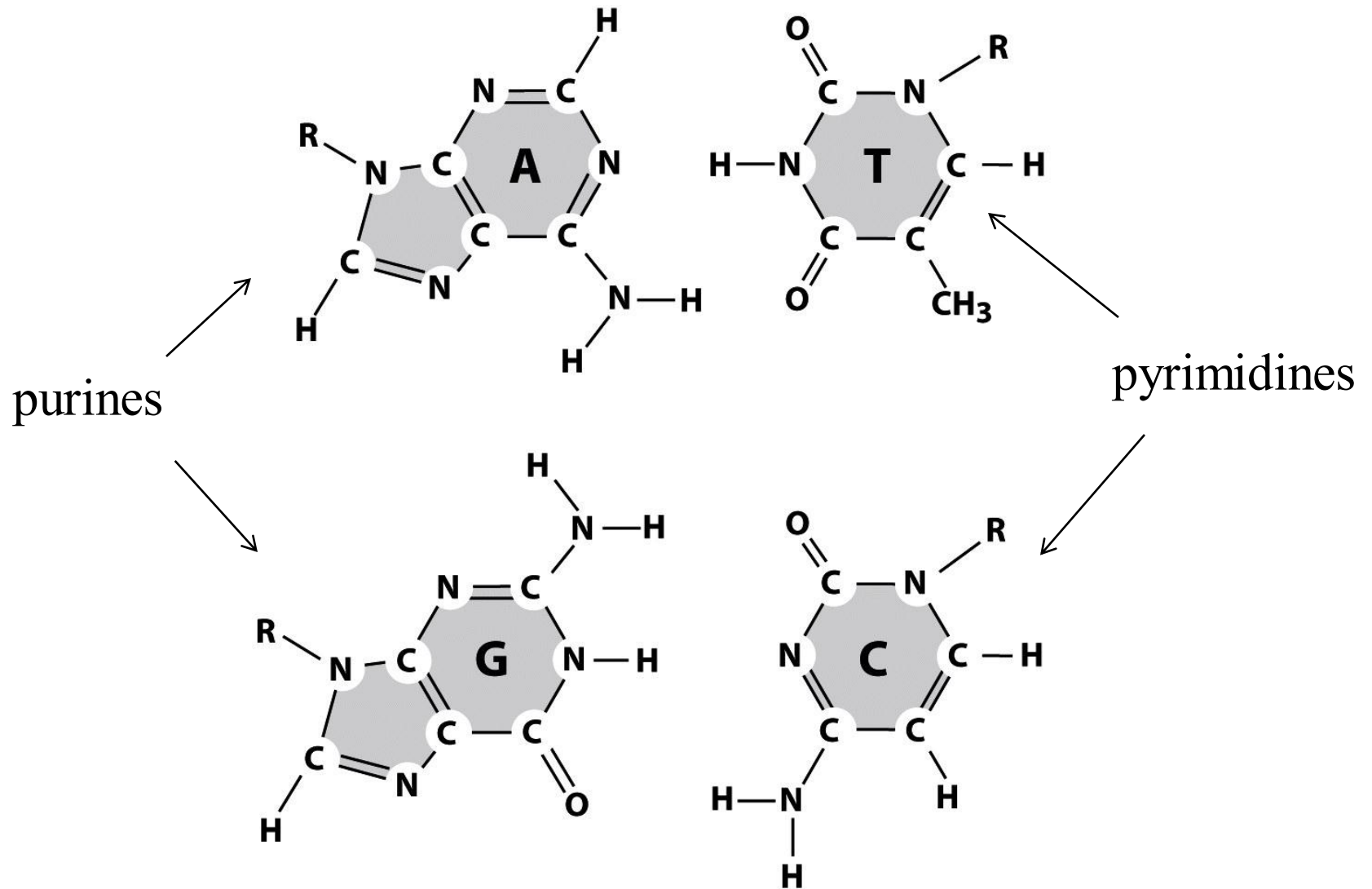


Figure 1.3a Physical Biology of the Cell (© Garland Science 2009)

# Chemical structure of DNA

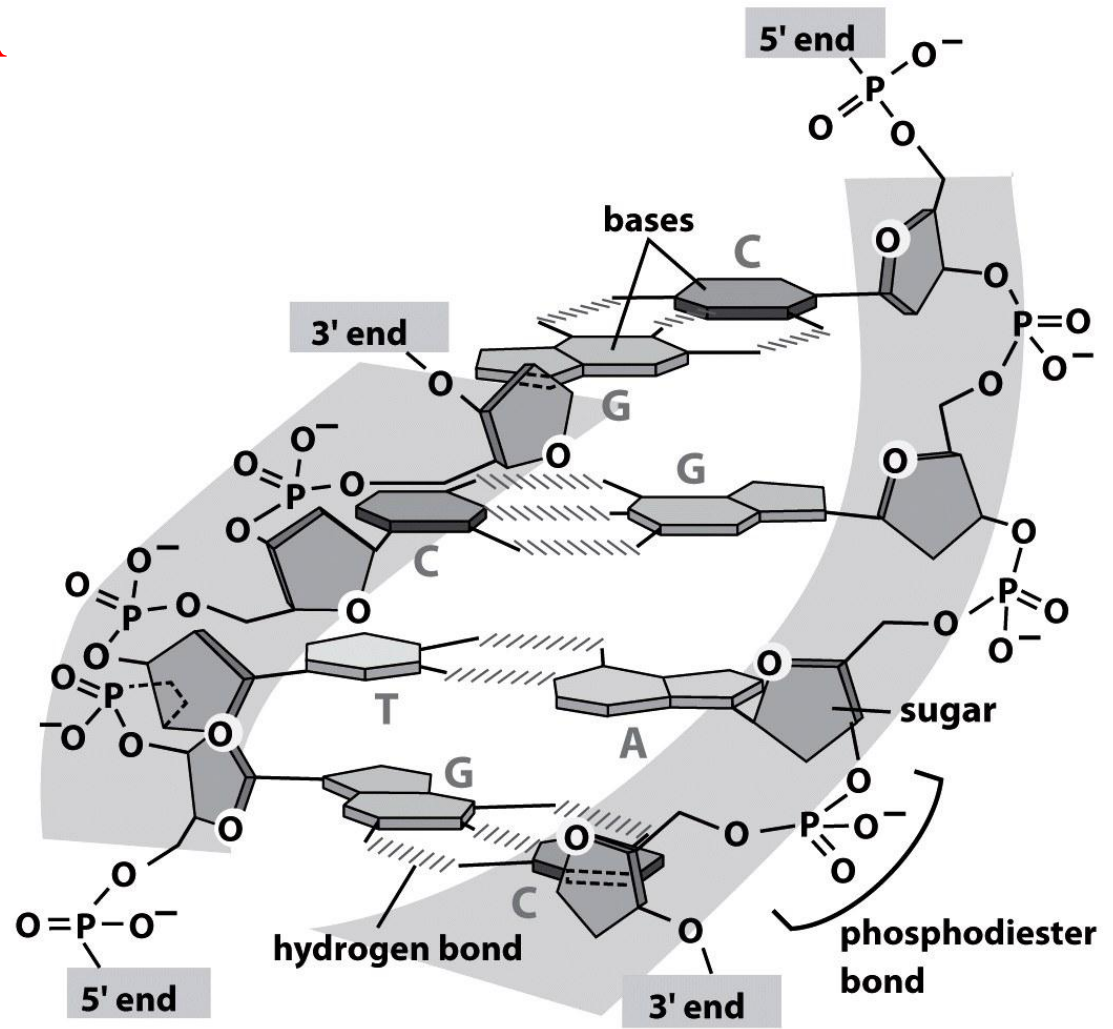


Figure 1.3b Physical Biology of the Cell (© Garland Science 2009)

## Conclusions at a glance:

1. Each strand has a direction (3' to 5')
2. GC bp more stable than AT bp – 3 h-bonds instead of 2!
3. Bases turn inwards – how do proteins recognize sequence?

## Standard B form of DNA

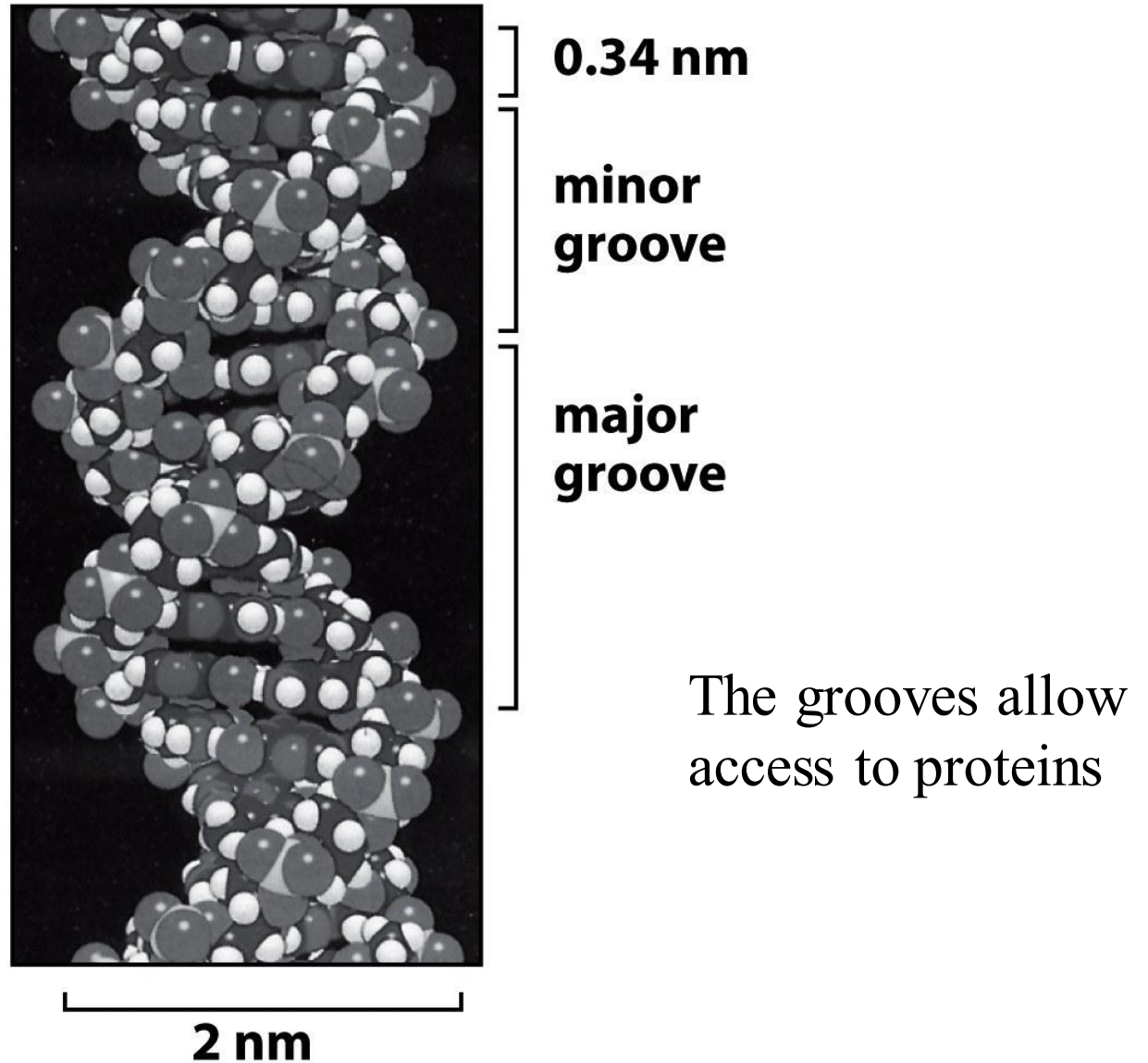


Figure 1.3c Physical Biology of the Cell (© Garland Science 2009)

# Rosetta stone - translating bp code to aa sequence

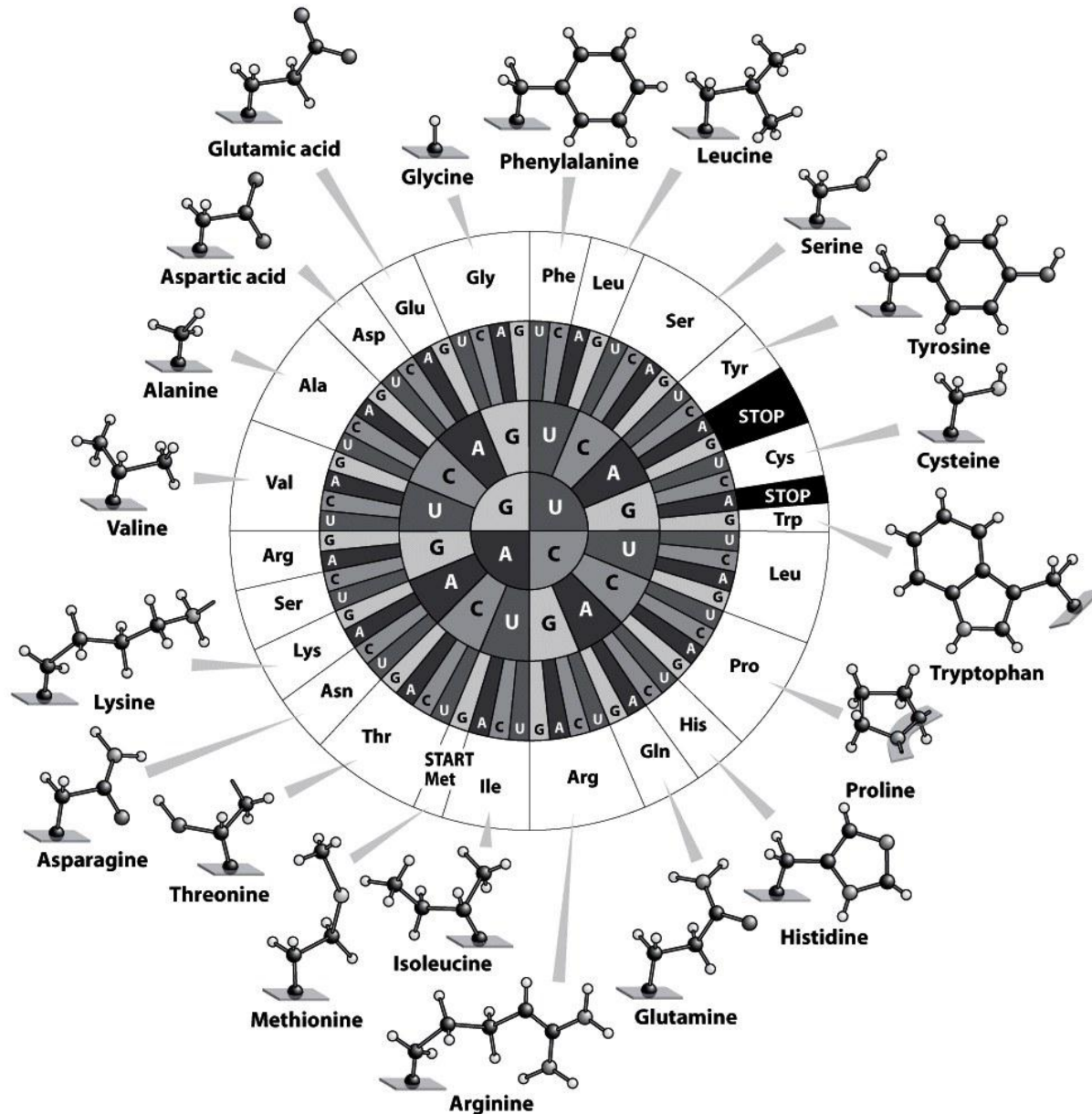


Figure 1.4 Physical Biology of the Cell (© Garland Science 2009)

# Models of DNA – what is the question?

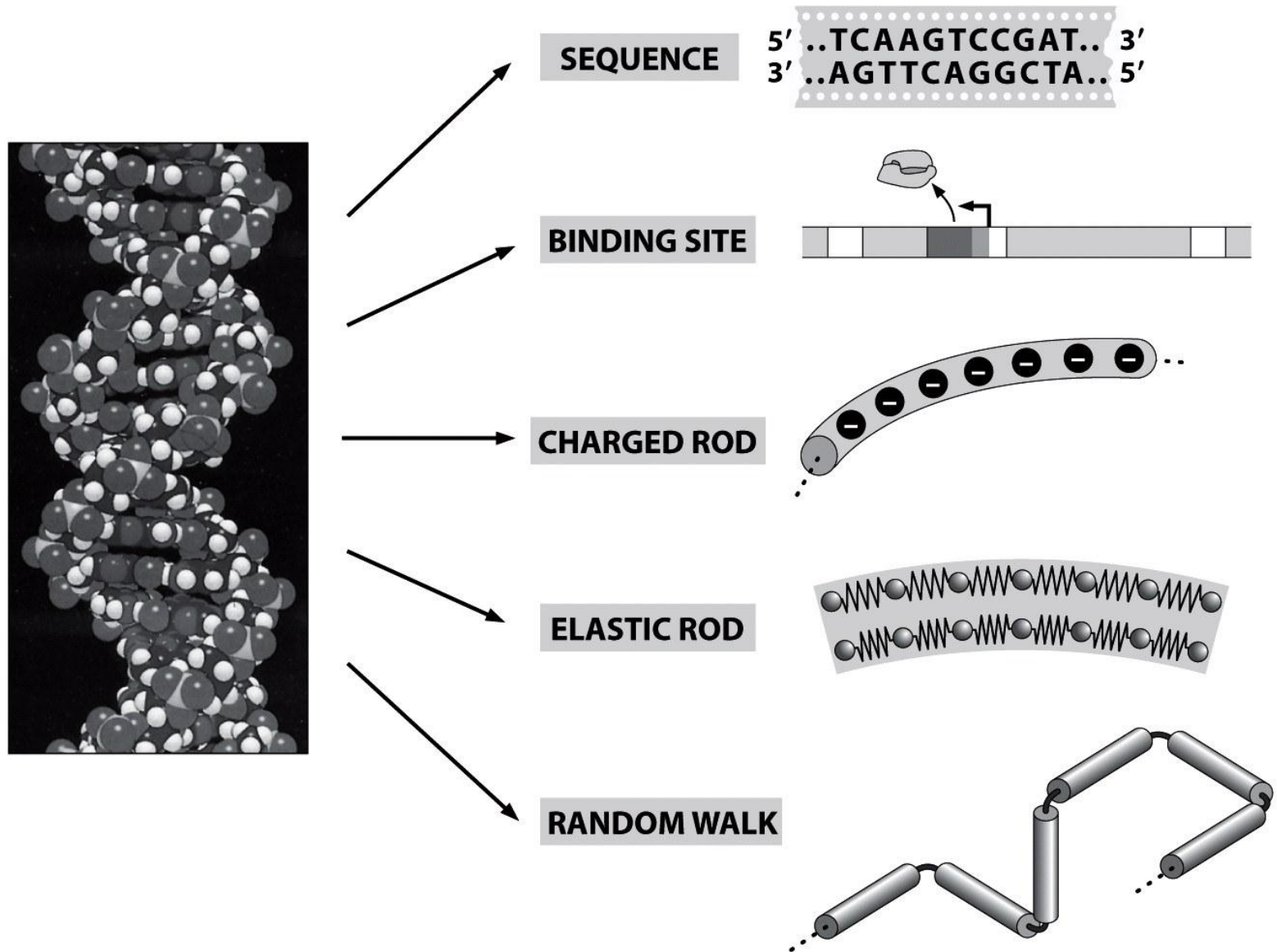


Figure 1.5 Physical Biology of the Cell (© Garland Science 2009)



# Models of proteins:

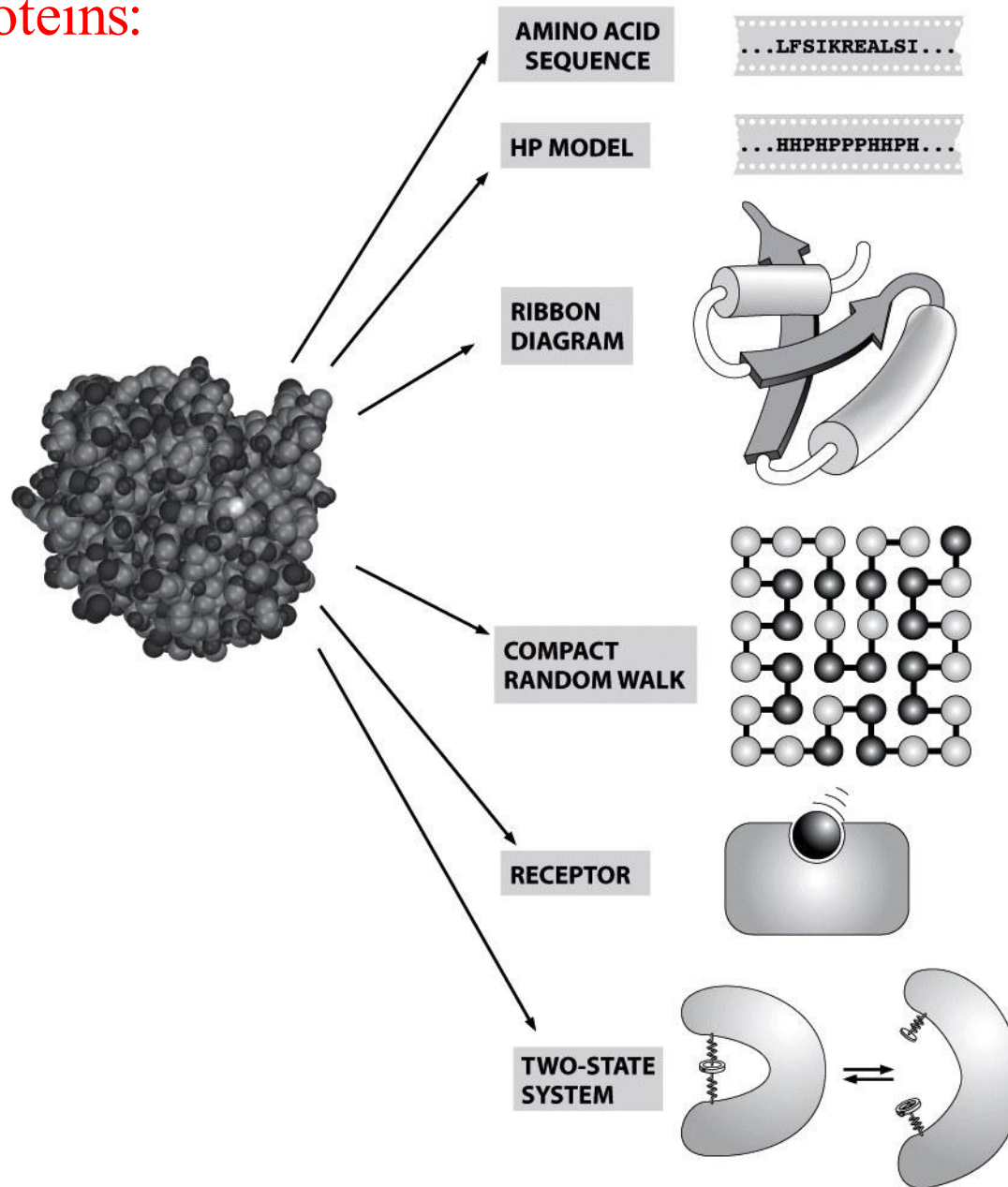


Figure 1.6 Physical Biology of the Cell (© Garland Science 2009)

# Models of membranes:

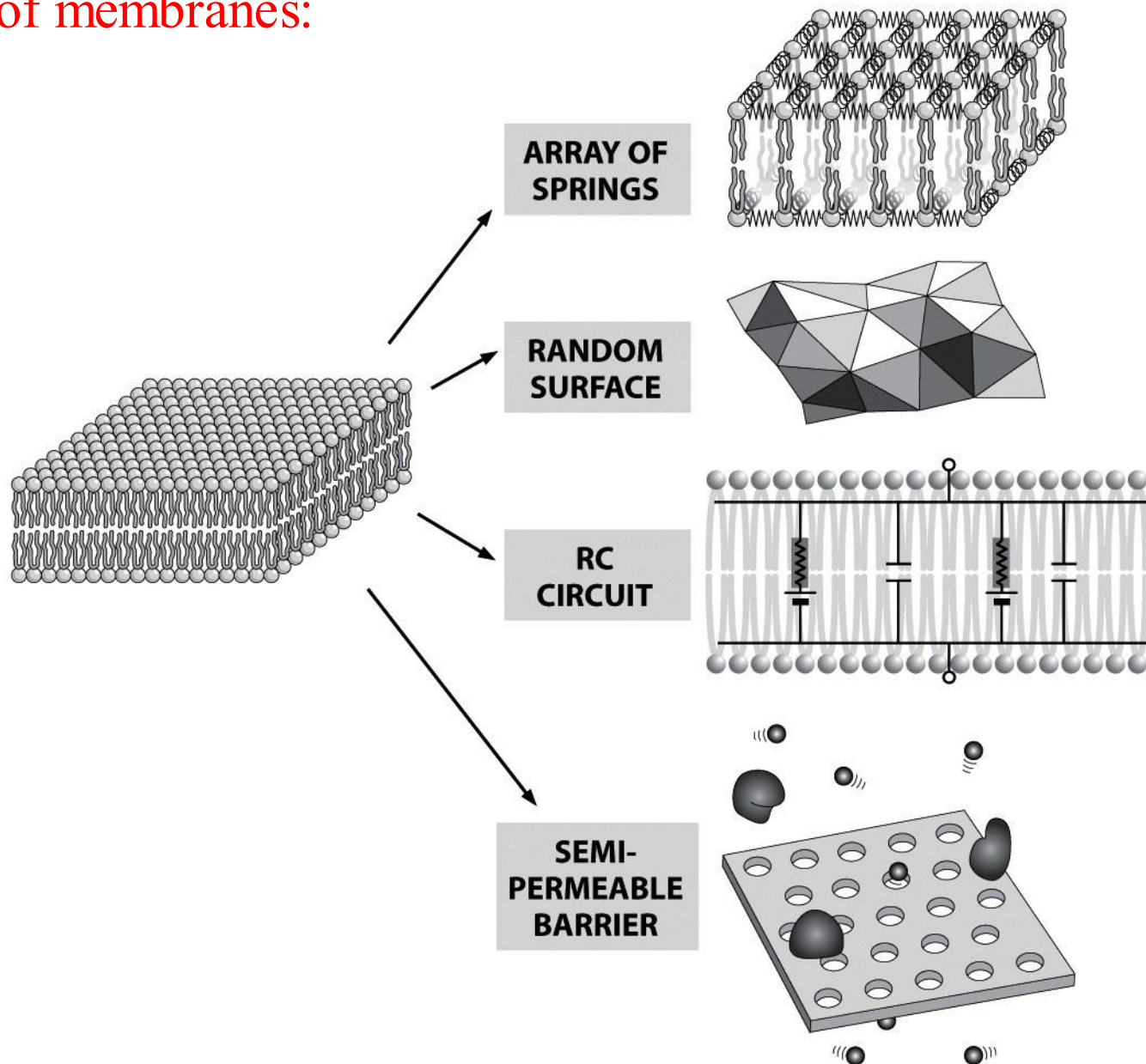


Figure 1.7 Physical Biology of the Cell (© Garland Science 2009)



# Models of Bacteria

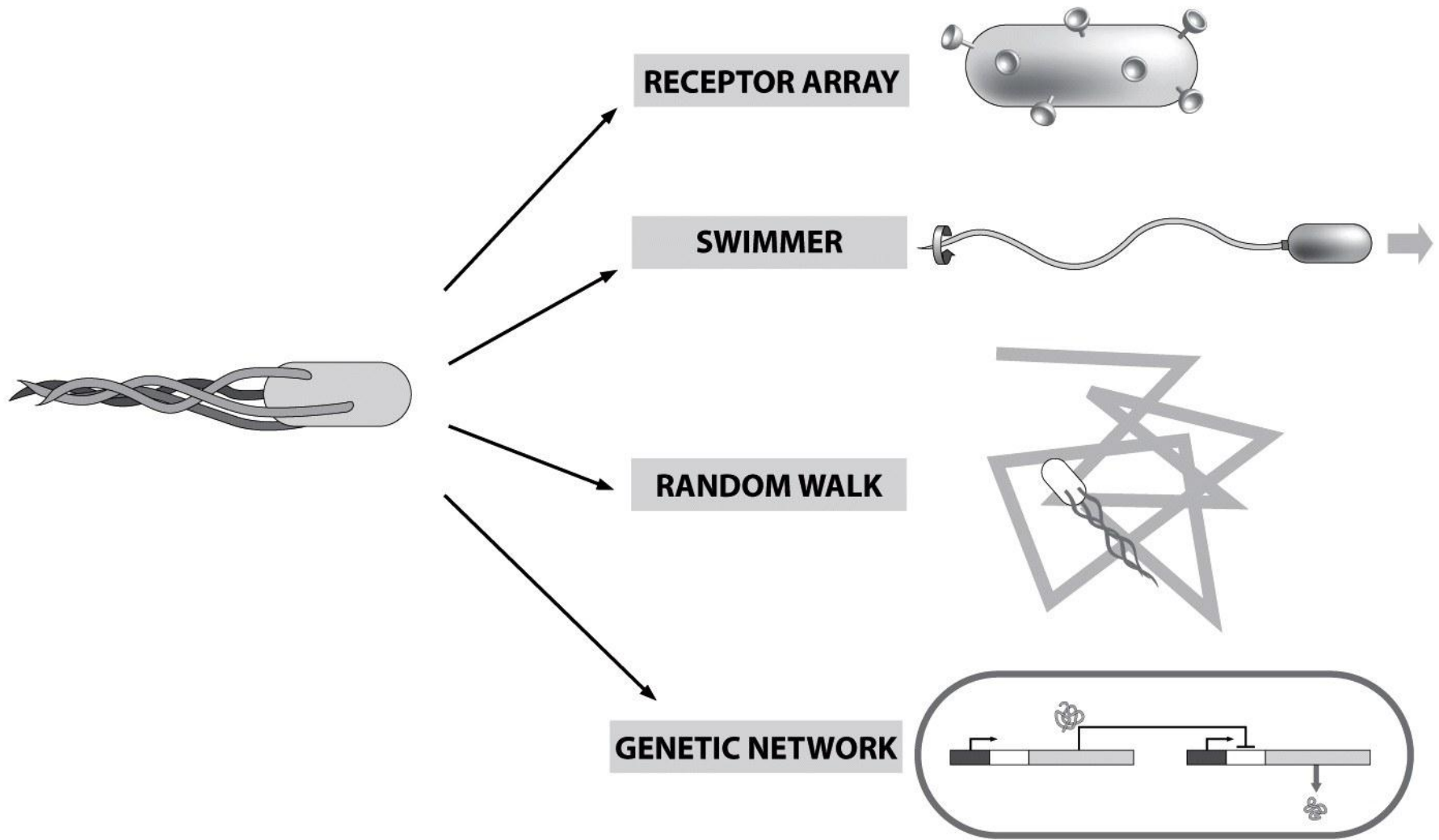


Figure 1.8 Physical Biology of the Cell (© Garland Science 2009)

Pictures are nice - but quantitative data requires quantitative models!

## The toolbox of physics and chemistry:

**Simple tools** (we understand them well):

- Harmonic oscillator
- Ideal gas
- Two level systems (Ising model)
- Diffusion and random walks
- Polymer physics
- DH and PB models of charges in solution
- Elasticity of rods and plates
- Low Reynolds number hydrodynamics
- Rate equations
- Newton's equations (MD simulations)

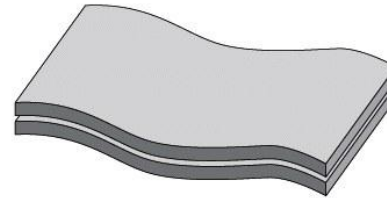
## **Complex tools** (we do not understand them well enough):

- Non-linear dynamics (attractors)
- Reaction-diffusion equations
- Many body systems of complex elements
- Quantum chemistry of large molecules
- Topology
- .....

## Example – simple applications of harmonic oscillator:



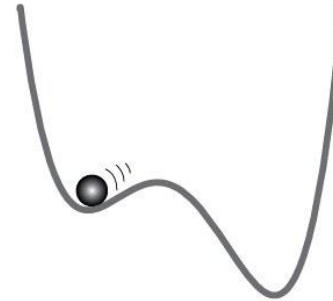
**beam bending**  
(e.g., AFM cantilever)



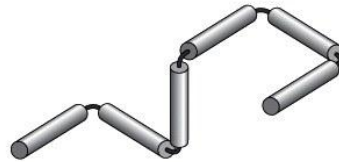
**cell membrane fluctuating**



**bead pulled to center**  
**of an optical trap**



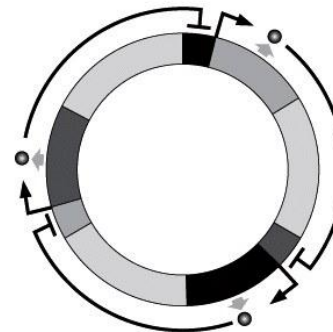
**molecules in an**  
**energy landscape**



**DNA polymer wiggling**  
**in solution**



**flagellum beating on**  
**a swimming sperm**



**genetic network changing**  
**expression levels over time**

Choice of model requires knowing the order of magnitude of some characteristic numbers!

## Examples:

### 1. Is inertia important for bacterial propulsion?

Estimate the Reynolds number of E. coli's motion in water

Size:  $L=1\mu\text{m}$ , velocity:  $V=10\mu\text{m/s}$ , kinematic viscosity:  $\nu=0.01\text{cm}^2/\text{s}$

$$\text{Re} = \frac{V \cdot L}{\nu} = 10^{-5}$$

Inertia is irrelevant – velocity is proportional to the force and responds instantaneously to it!

(Aristotle beats Newton for bacteria)

## 2. Confinement of DNA:

Estimate the radius of gyration of the lambda phage DNA:

Kuhn segment length of DNA:  $a=100\text{nm}$ ;

Length of DNA:  $L=5 \cdot 10^4 \text{ bp} \cdot 0.3\text{nm} = 1.5 \cdot 10^4 \text{nm}$

DNA radius of gyration in solution

$$R=(a \cdot L/6)^{1/2}=125 \text{ nm}$$

Radius of capsid:  $27\text{nm}$

$$\text{Confinement ratio: } \frac{125^3}{27^3}$$

100 fold confinement (by volume) –

DNA will rapidly spread when capsid is removed

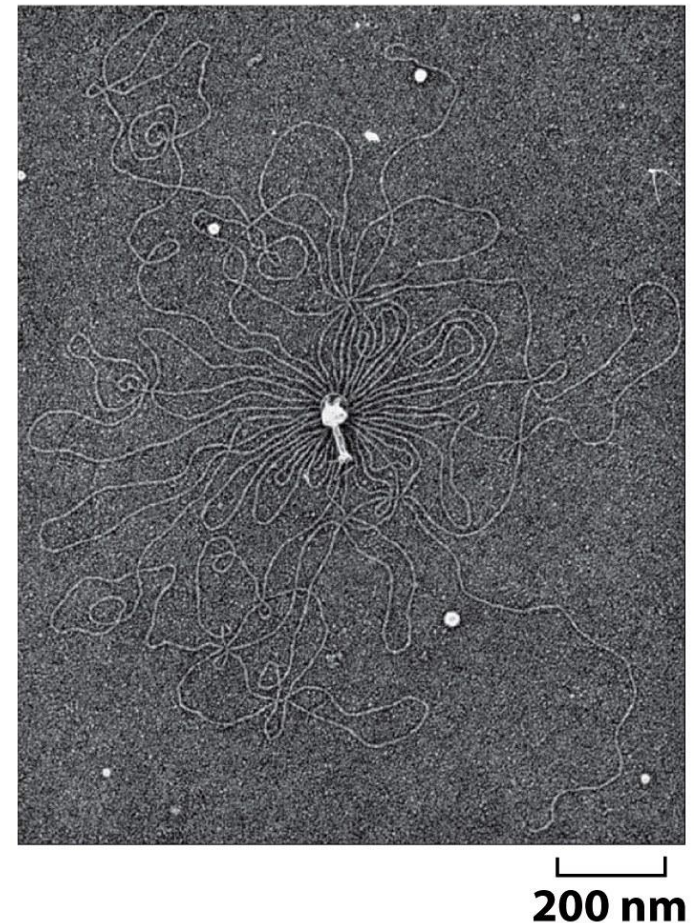


Figure 1.13 Physical Biology of the Cell (© Garland Science 2009)

# What is the volume fraction occupied by DNA?

Volume per bp of DNA =  $1 \text{ nm}^3$

Packing ratio: 
$$v_{\text{lambda}} = \frac{1 \cdot N_{\text{bp}}}{\frac{4\pi}{3} \cdot r^3}$$

Confining radii:  $r_{\text{lambda}} = 27 \text{ nm}$ ;  $r_{\text{nucleoid}} = 0.25 \mu\text{m}$ ;  
 $r_{\text{sperm head}} = 2.5 \mu\text{m}$ ;  $r_{\text{nucleus}} = 5 \mu\text{m}$

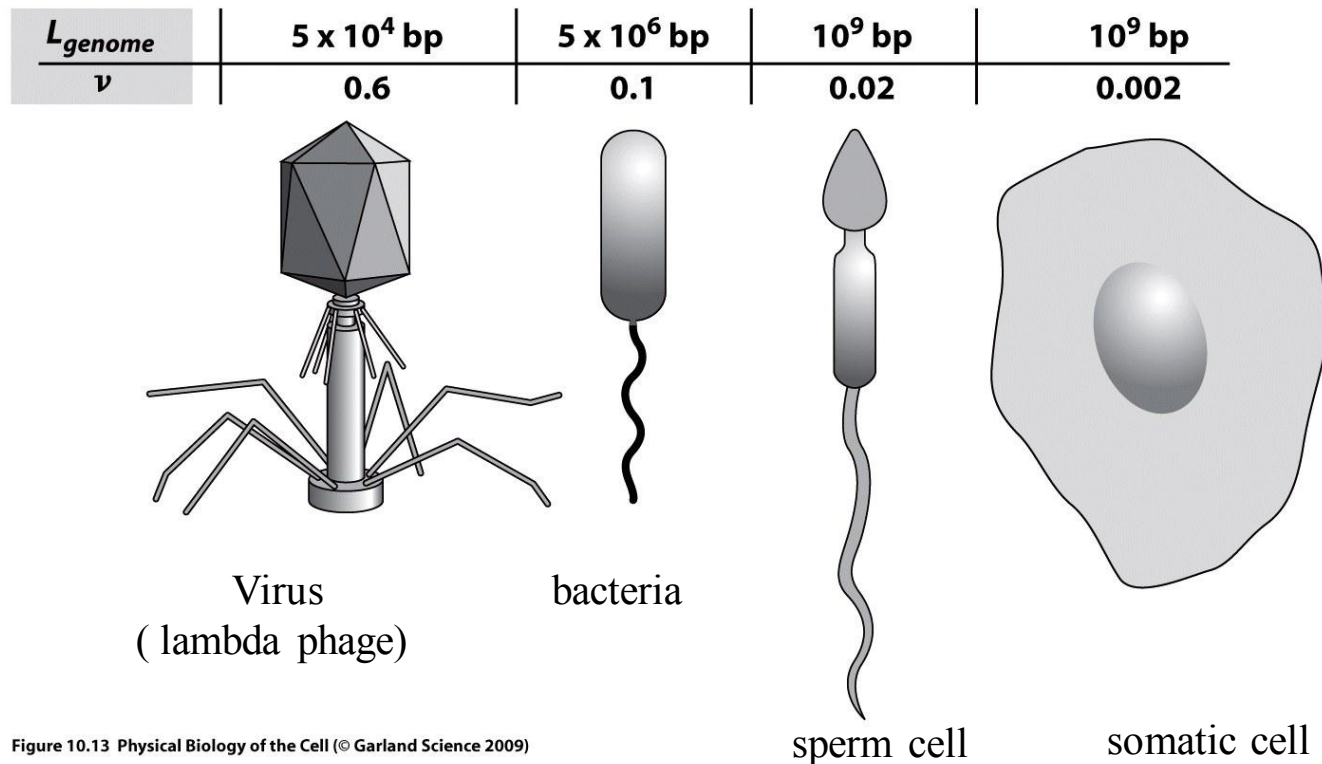


Figure 10.13 Physical Biology of the Cell (© Garland Science 2009)



# How is the highly confined state of DNA maintained?

**Viruses:** crystal-like density and arrangement

- need motors for packing
- No proteins inside to balance electrostatic repulsion – Wigner crystal –type stabilization by walls

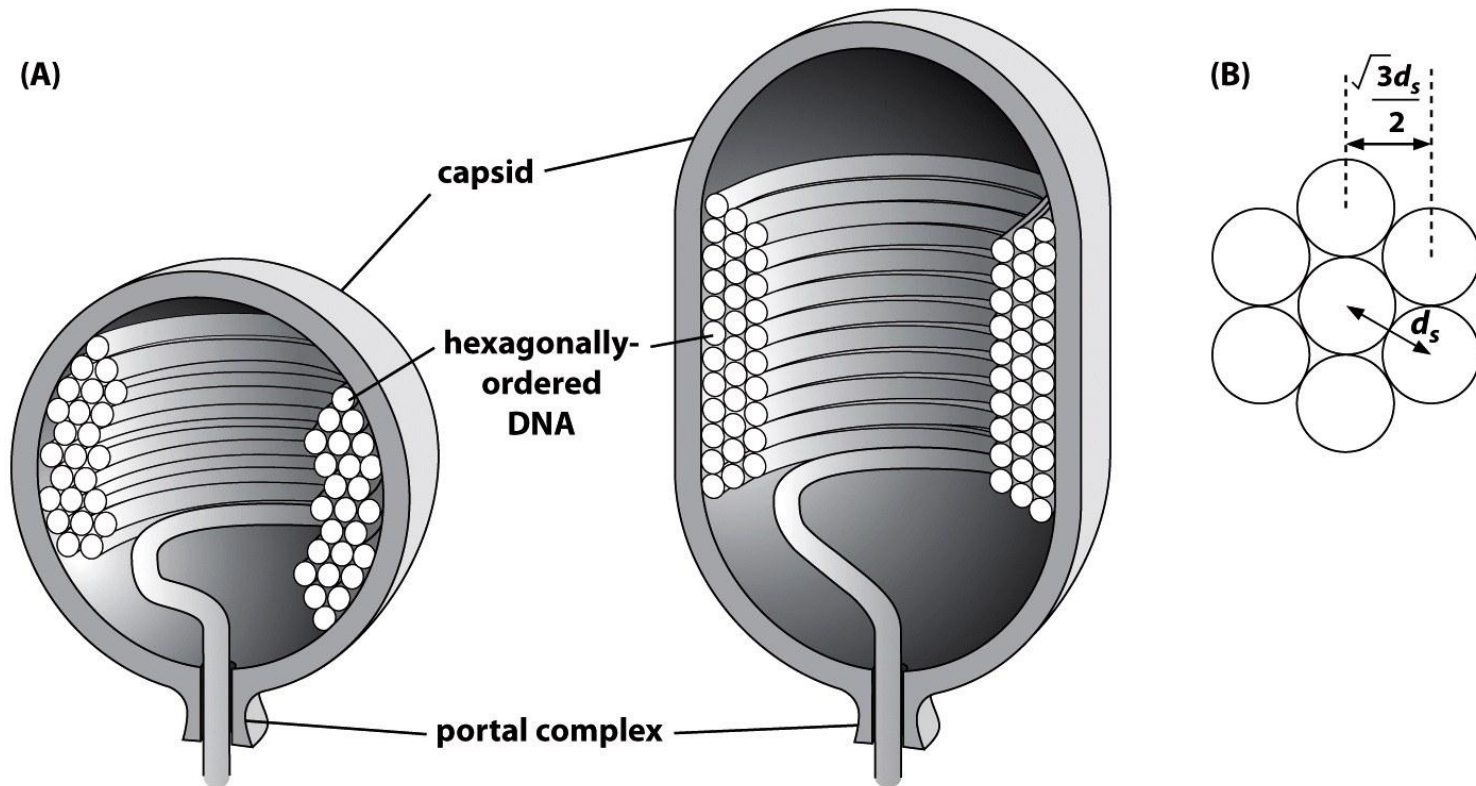
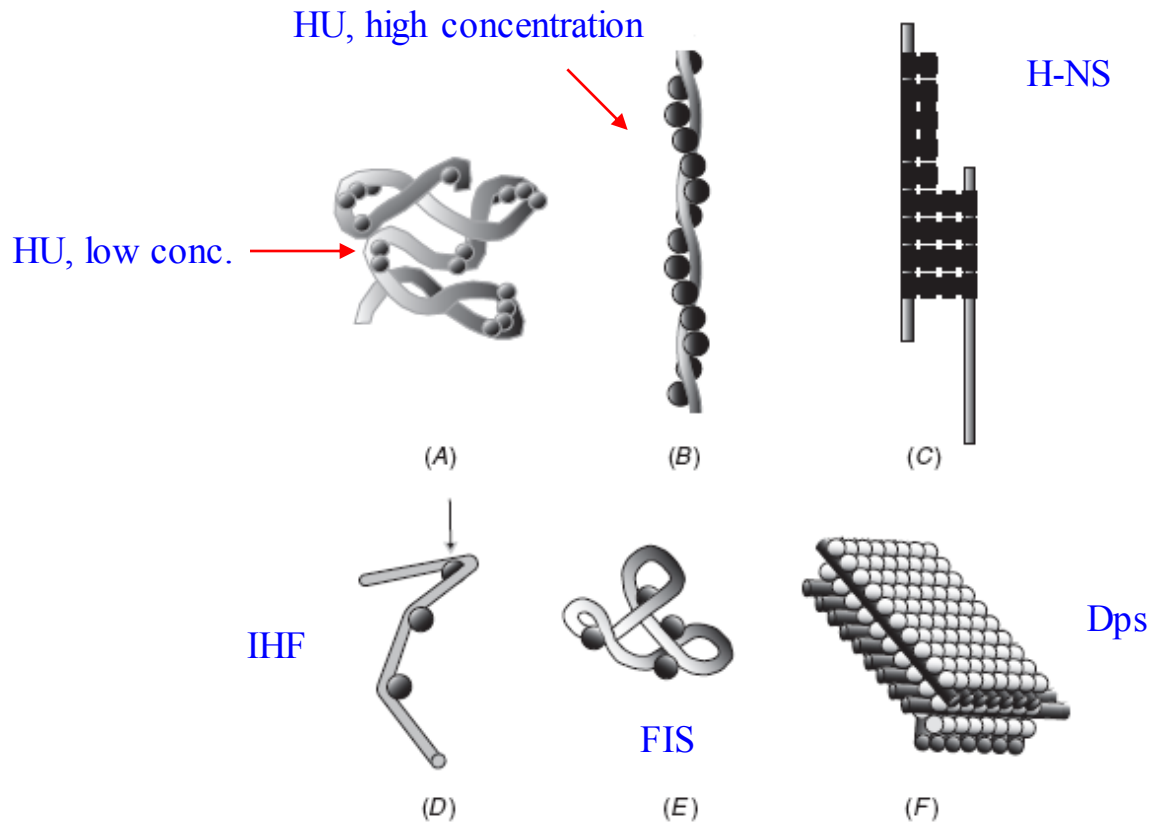


Figure 10.15 Physical Biology of the Cell (© Garland Science 2009)

**Bacteria:** single DNA is packed –without confining walls–  
in  $< 10\%$  of bacterial volume – need attractions!

1. DNA is packaged by about a dozen of nucleoid-associated proteins that bend and twist it



2. Topology: bacterial DNA is circular –  
conserved linking number  $Lk = \#$  of turns of double helix

Native linking number of DNA:

$$Lk_0 = \frac{\text{length of DNA}}{\text{helix repeat length (3.4nm)}} = \frac{N_{bp}}{10.4}$$

Topology-changing enzymes (gyrases, topoisomerases) can cut one DNA strand, overwind ( $Lk > Lk_0$ ) or unwind ( $Lk < Lk_0$ ) it, and glue it together again.

Bacterial DNA is typically underwound

$$\frac{Lk - Lk_0}{Lk_0} = -0.06$$

(this destabilizes the double helix – promotes recognition and reactivity of exposed bases?)

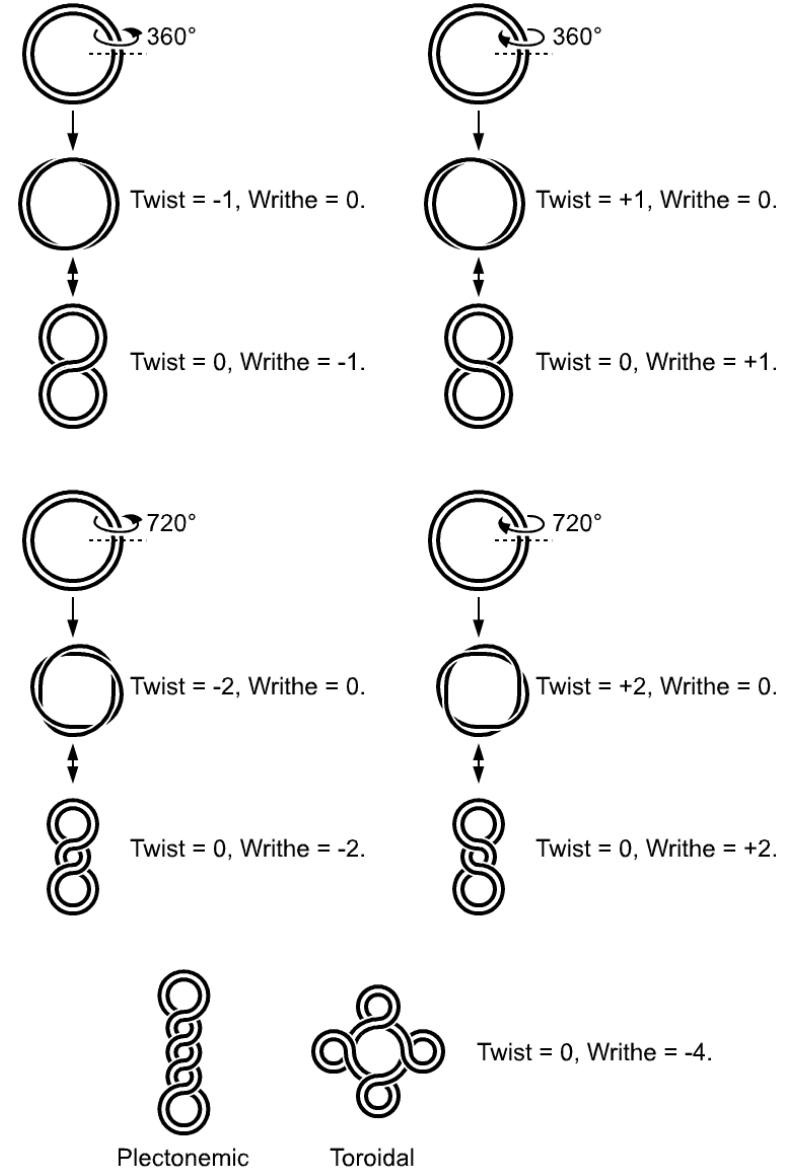
Linking number can be decomposed into twist and writhe:

$$L_k = Tw + Wr$$

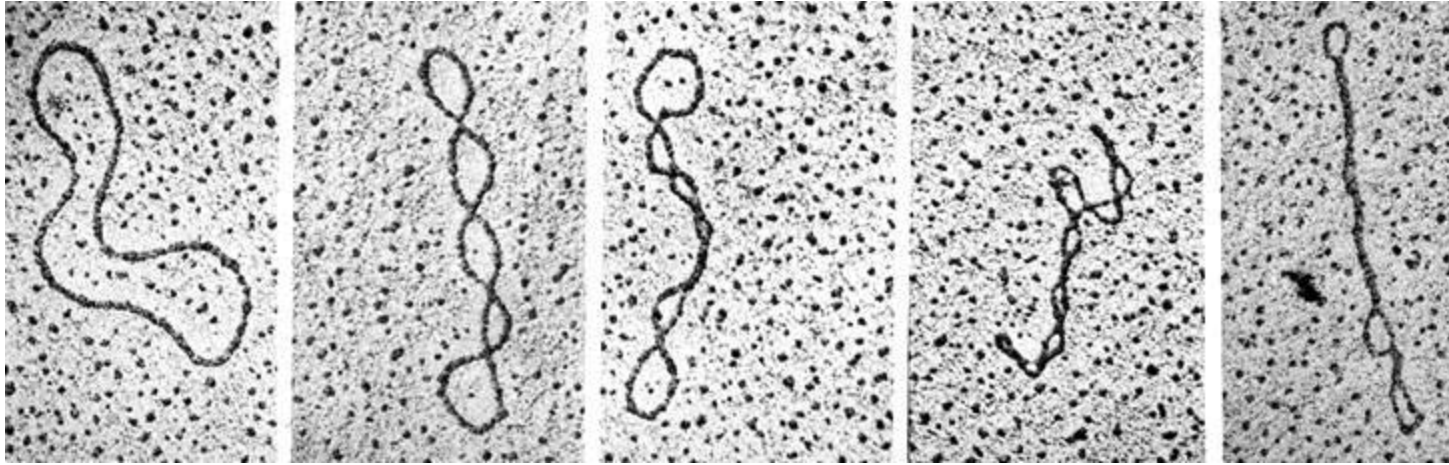
As there is energy penalty for twist

$$E_{tw}/kT = \frac{l_x}{2L} Tw^2$$

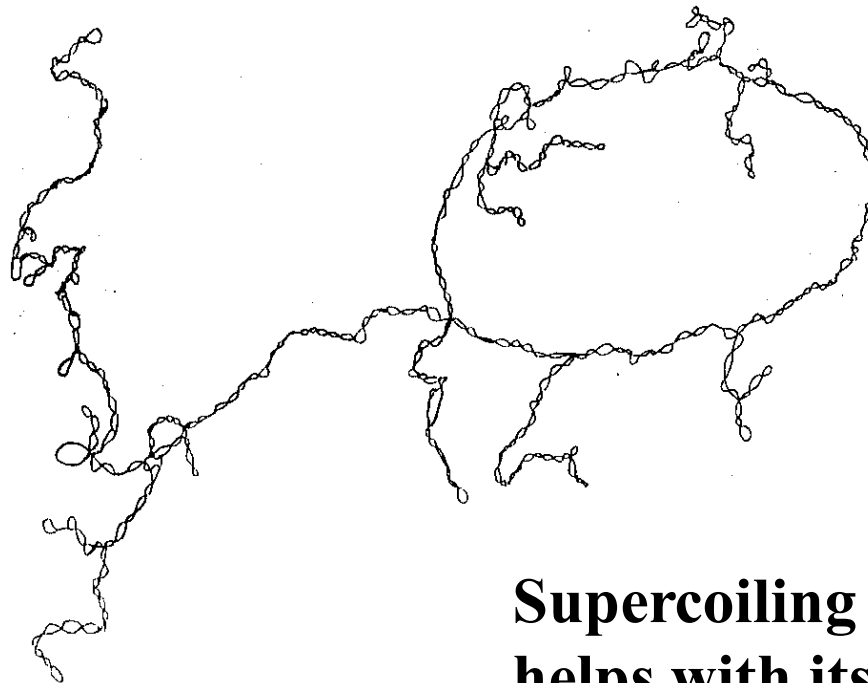
but no direct penalty for writhe,  
over/underwinding will generate  
high writhe configurations  
(supercoils, plectonemes)



Plasmids form right-handed (-) supercoils

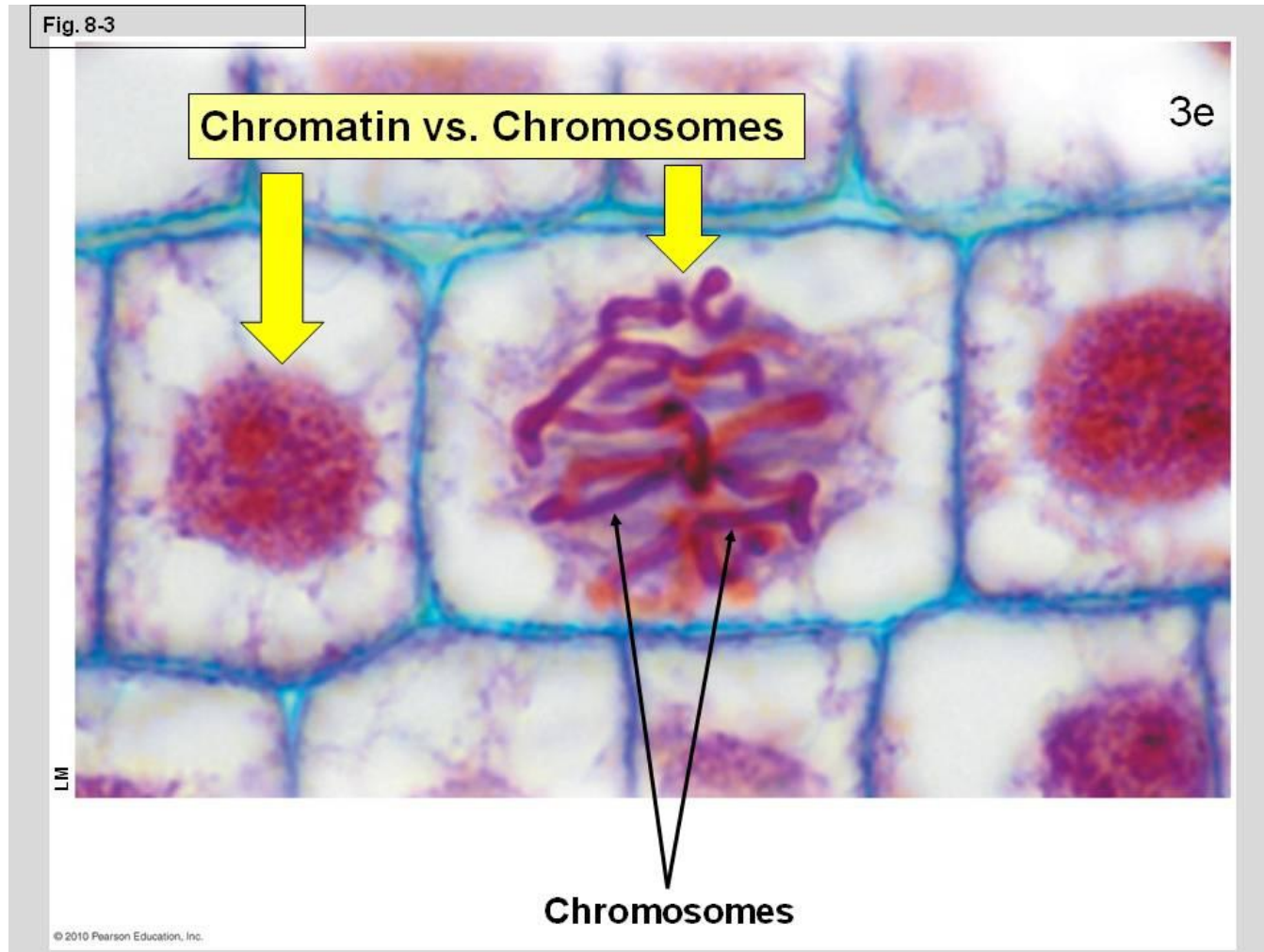


*E.coli* genome



**Supercoiling of bacterial DNA helps with its packing?**

# Eukaryotic (human) cells: 46 DNA molecules inside nucleus



Chromosomes are segregated during mitosis— what about interphase chromatin?



# Spaghetti soup “model” of chromatin?



Problem – numerous inter-chain entanglements for  $N > N_e$  ( $N_e=200$ )  
-like a polymer melt!

Even a single DNA molecule confined in a nucleus ( $R_G/R_{\text{nucleus}} > 10$ )  
will be strongly self-entangled!

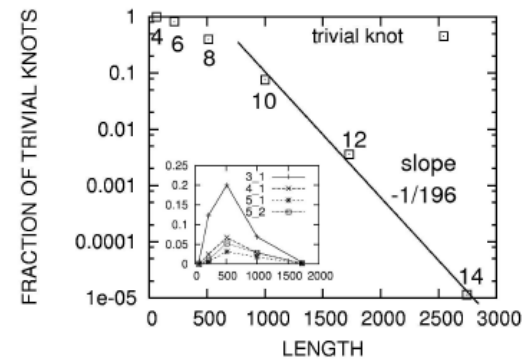


Fig. 2. Trivial knot probabilities for compact conformations of size  $4 \times 4 \times 4$  to  $14 \times 14 \times 14$ . Inset shows the probabilities of the non-trivial knots  $3_1$  (trefoil),  $4_1$  (figure-eight),  $5_1$  (star),  $5_2$ .



## Solution I: entanglements and knots can be removed by topo II

Sikorav and Janninck, C. R. Acad. Sci. Paris t. 316, serie II, p. 751, 1993

## Solution II: DNA molecules are organized in crumpled globules and segregated in space

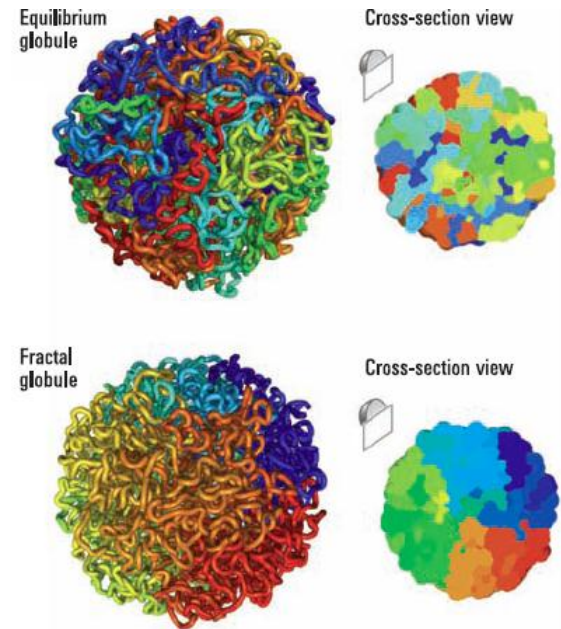
Grosberg et al, Europhys. Lett. **23**, 373, 1993

Crumpled (unknotted) globule:  $d_f=3$

Equilibrium (knotted) globule:  $d_f=2$

HiC experiments agree with crumpled/fractal globule:

$$R(s)=\text{const} \cdot s^{1/3}$$



# Chromatin territories:

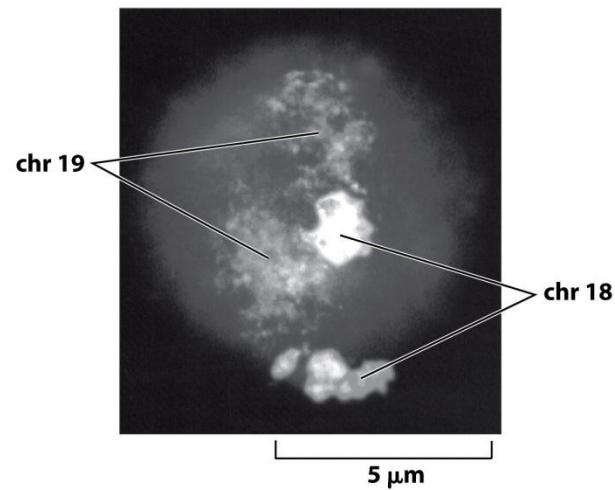
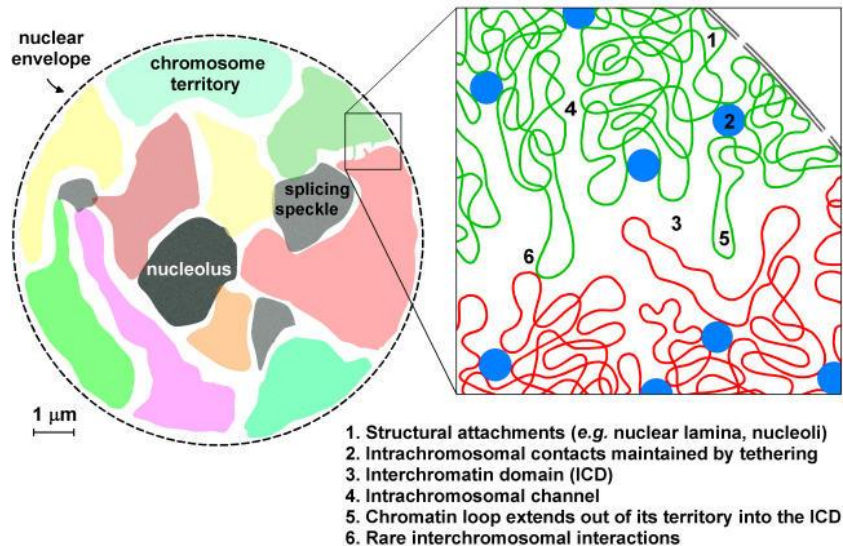


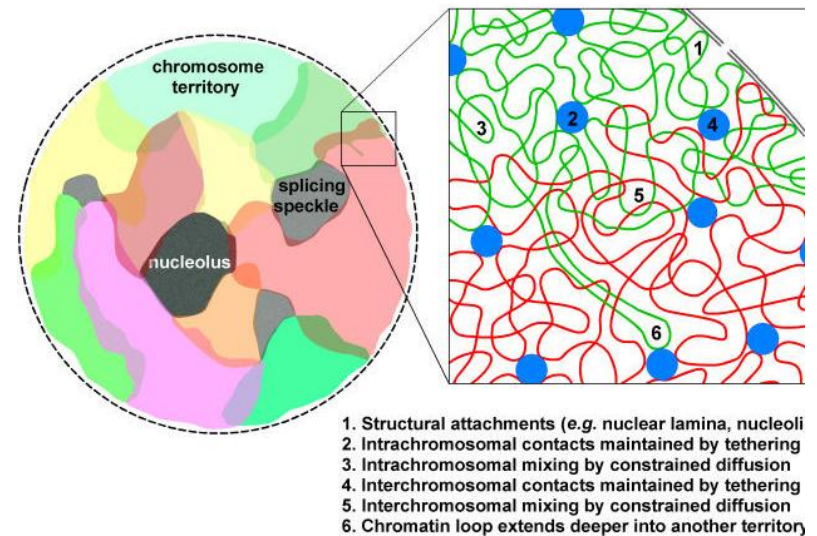
Figure 8.8 Physical Biology of the Cell (© Garland Science 2009)

## Segregation vs interpenetration?

### A. Interchromatin domain model



### B. Interchromosomal network model



# Small scale organization of chromatin

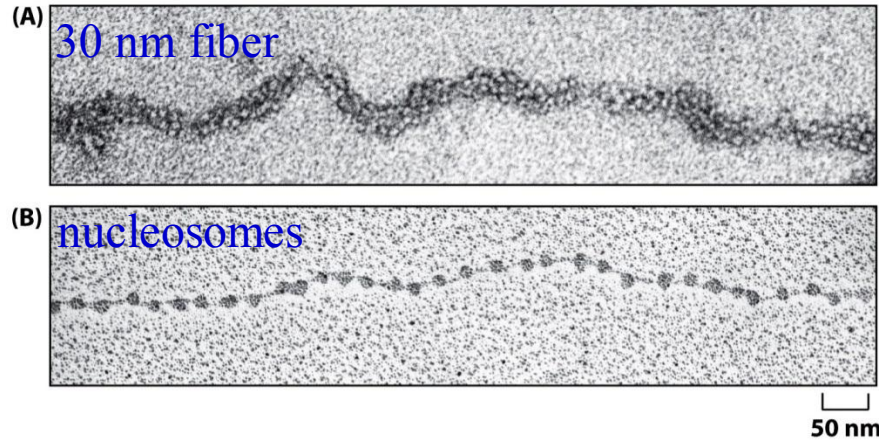


Figure 8.7 Physical Biology of the Cell (© Garland Science 2009)

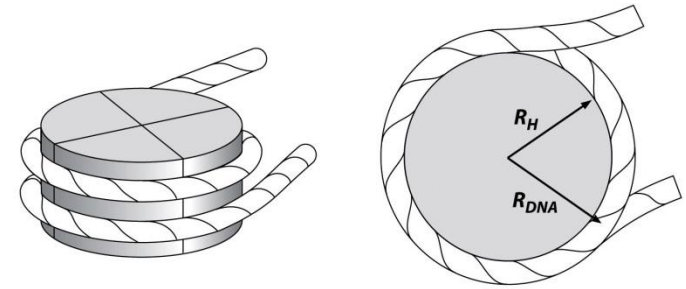


Figure 10.20c Physical Biology of the Cell (© Garland Science 2009)

Nucleosome chain— 147bp DNA wrapped around histone octamer with 50 bp linker

$$\text{Number of nucleosomes in genome} = \frac{3 \cdot 10^9 \text{ bp}}{200 \text{ bp}} = 10^7$$

Histone octamer: cylinder of  $r=3.5\text{nm}$  and  $h=6\text{nm}$

$$\text{Volume fraction of nucleosomes in nucleus: } \frac{\pi \cdot 3.5^2 \cdot 6 \cdot 10^7}{\frac{4\pi}{3} \cdot (5 \cdot 10^3)^3} = 0.5 \%$$

Very dilute!





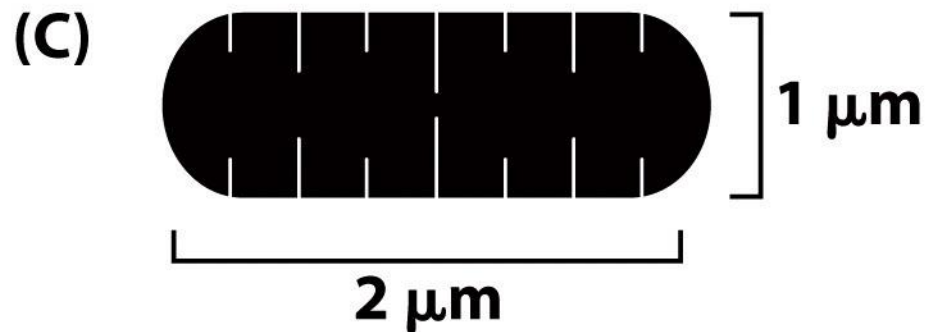
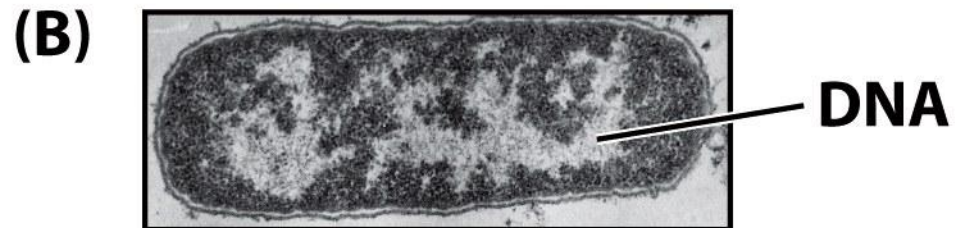
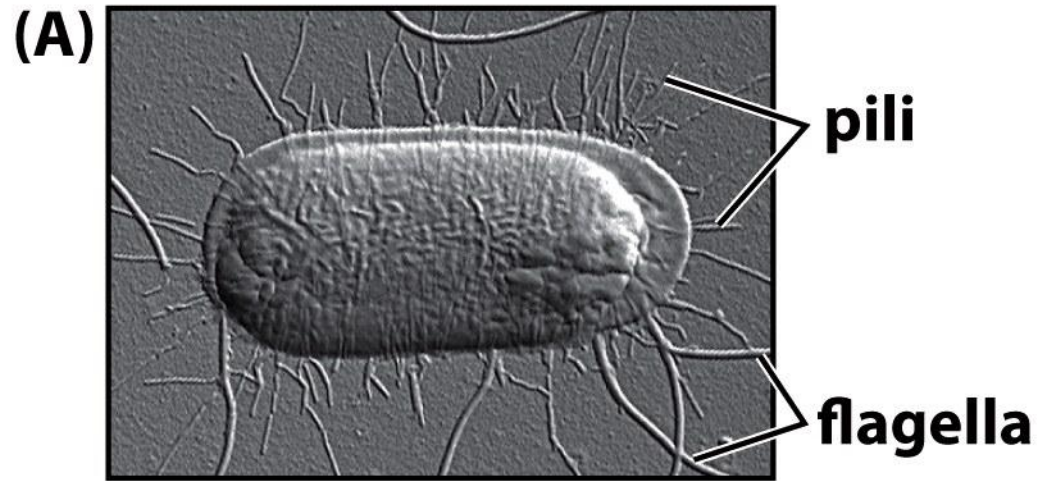


Figure 2.1 Physical Biology of the Cell (© Garland Science 2009)



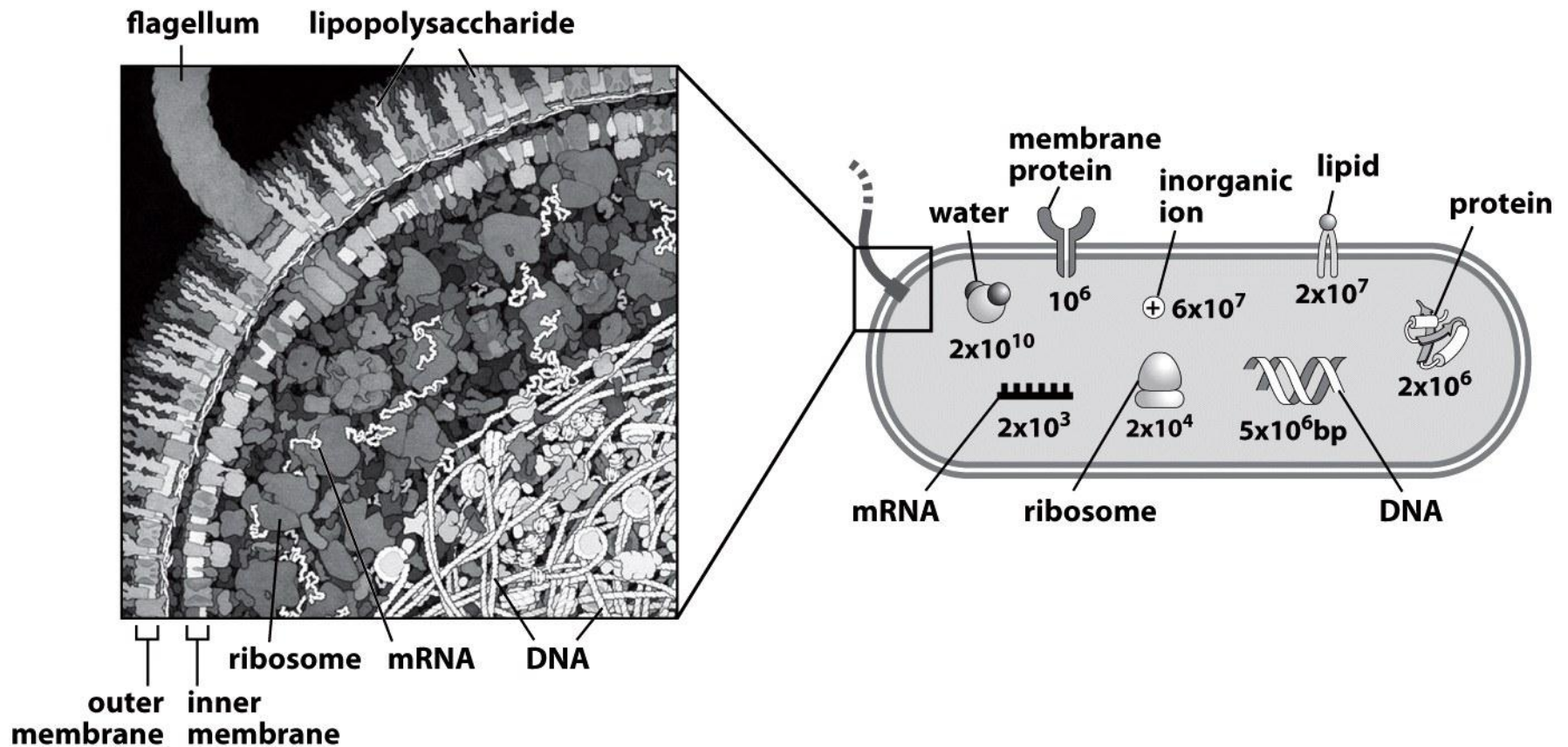


Figure 2.2 Physical Biology of the Cell (© Garland Science 2009)



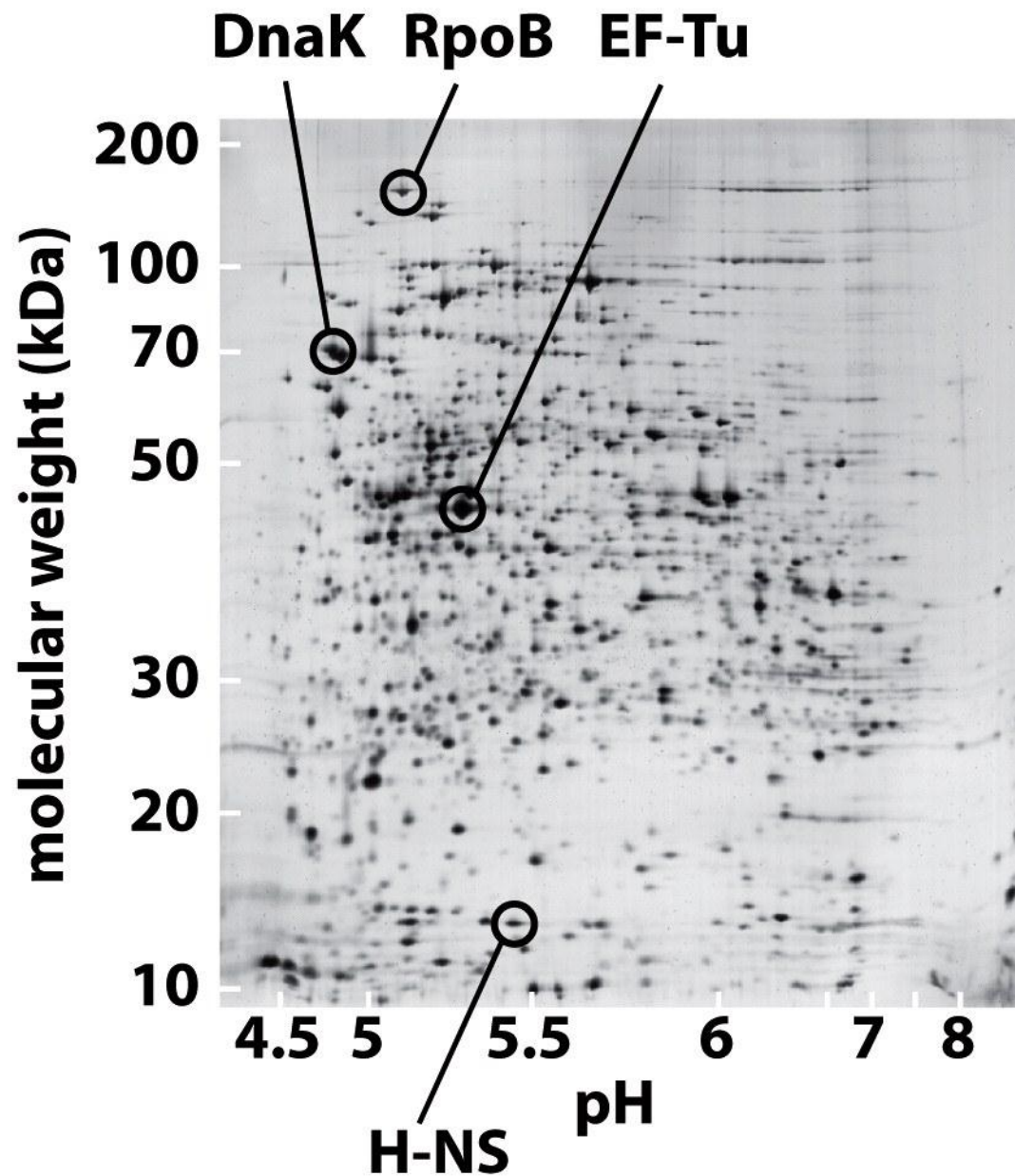


Figure 2.3 Physical Biology of the Cell (© Garland Science 2009)

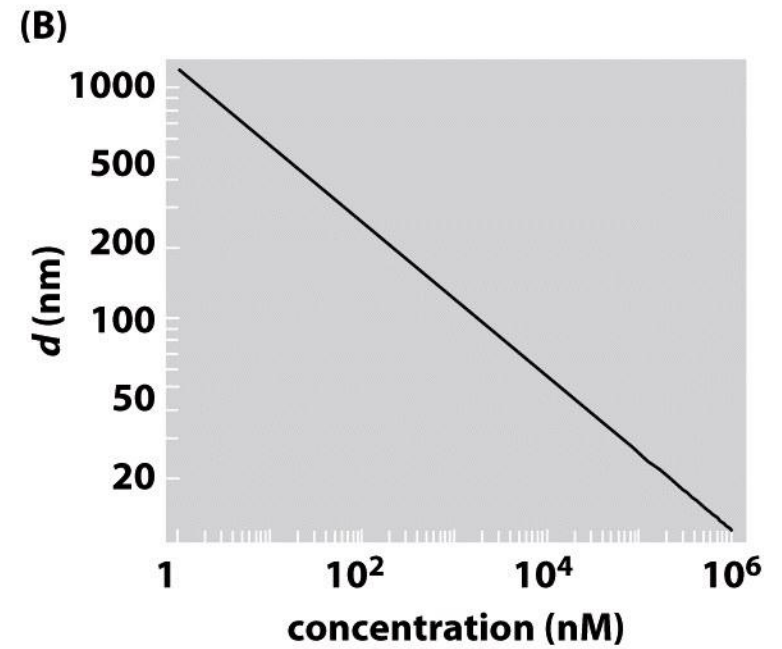
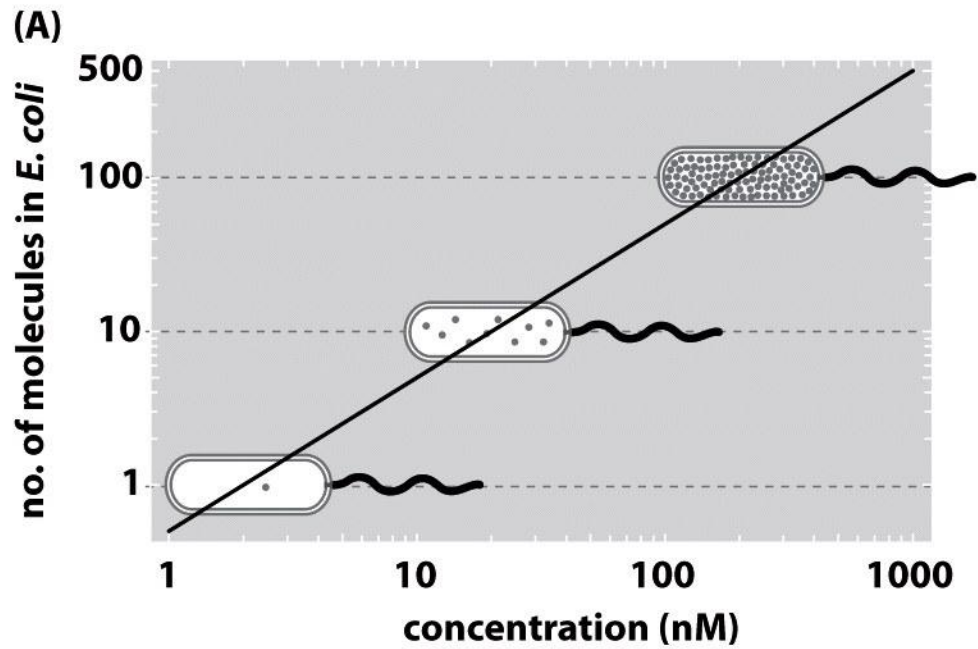


Figure 2.4 Physical Biology of the Cell (© Garland Science 2009)

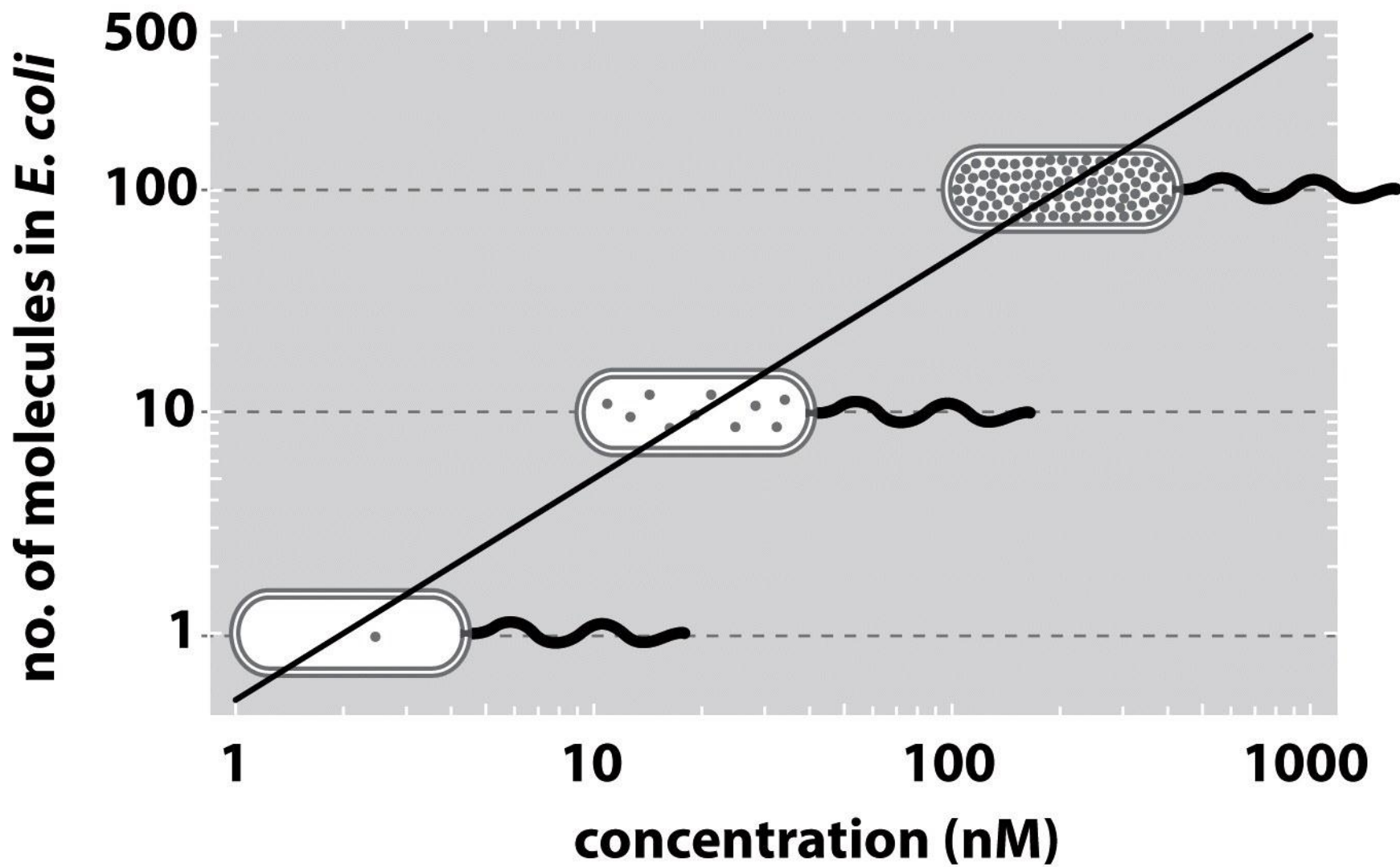


Figure 2.4a Physical Biology of the Cell (© Garland Science 2009)

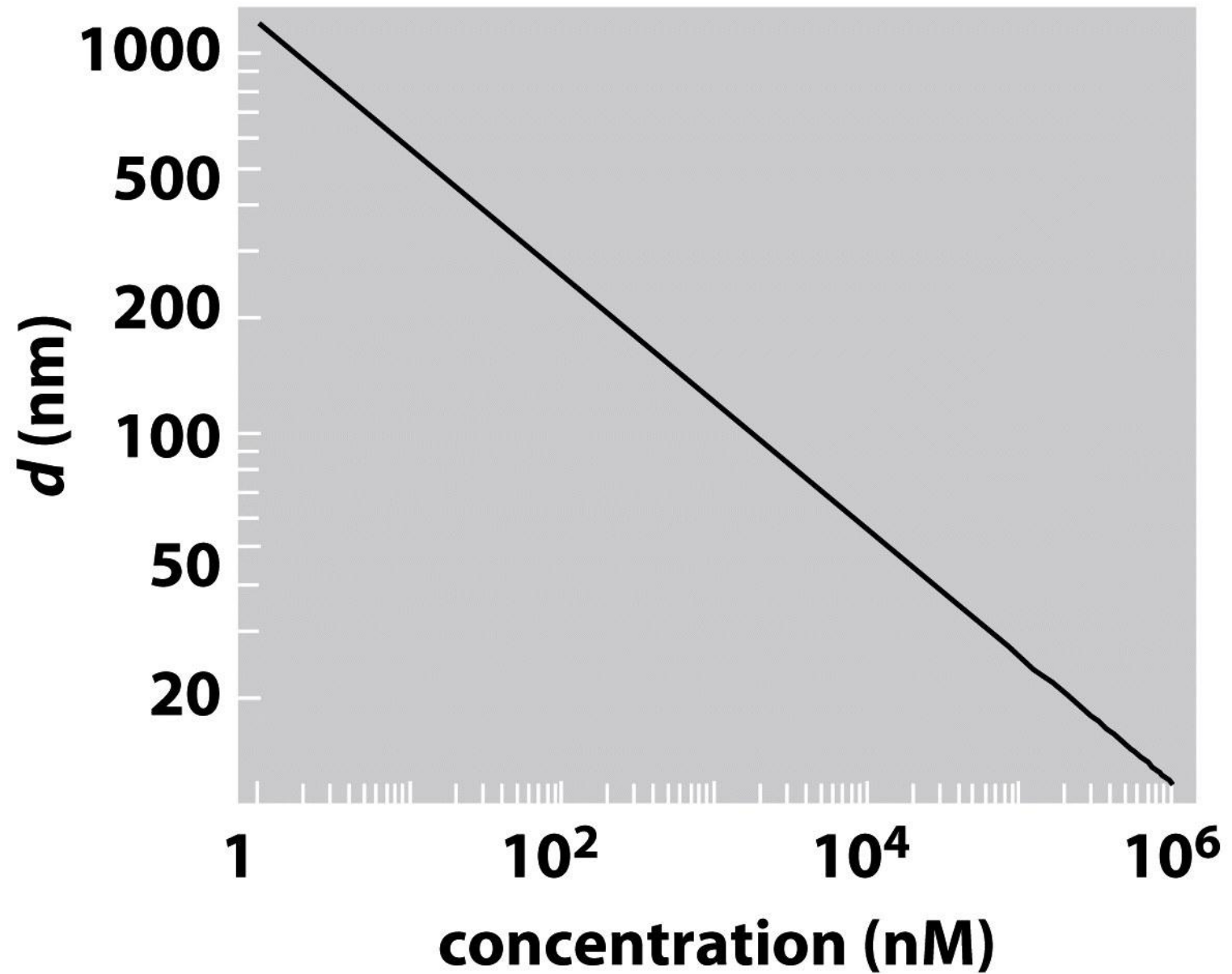
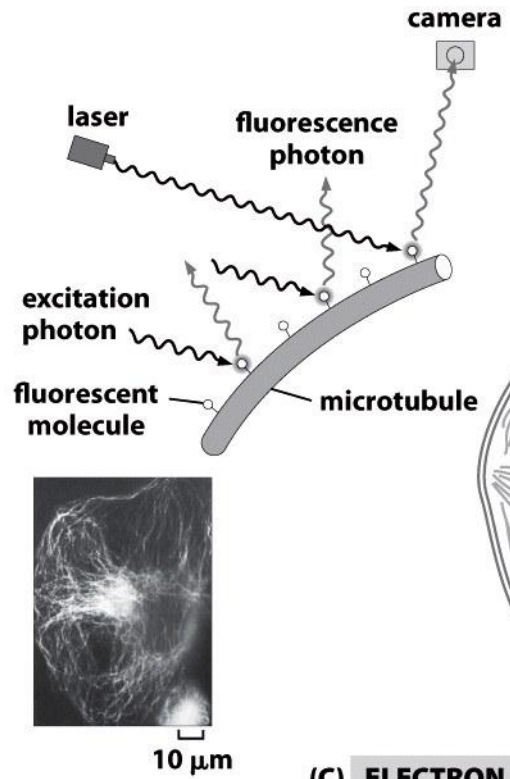
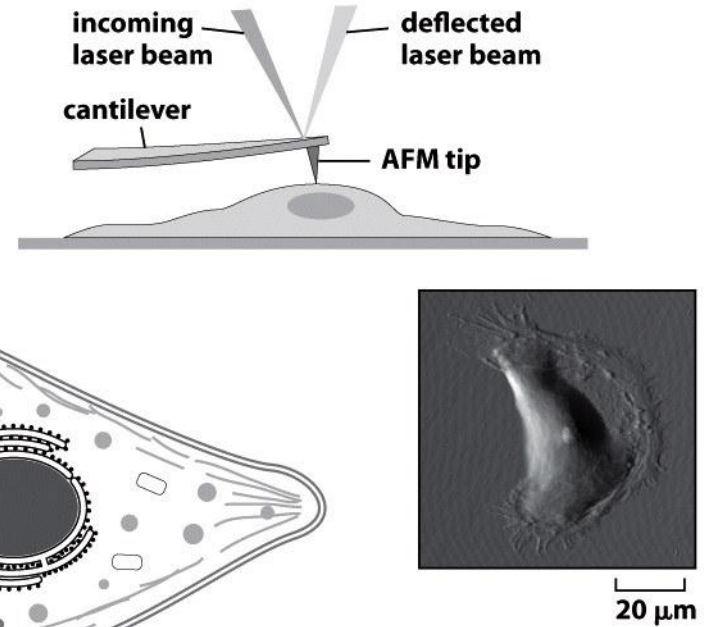


Figure 2.4b Physical Biology of the Cell (© Garland Science 2009)

**(A) FLUORESCENCE MICROSCOPY**



**(B) ATOMIC-FORCE MICROSCOPY**



**(C) ELECTRON MICROSCOPY**

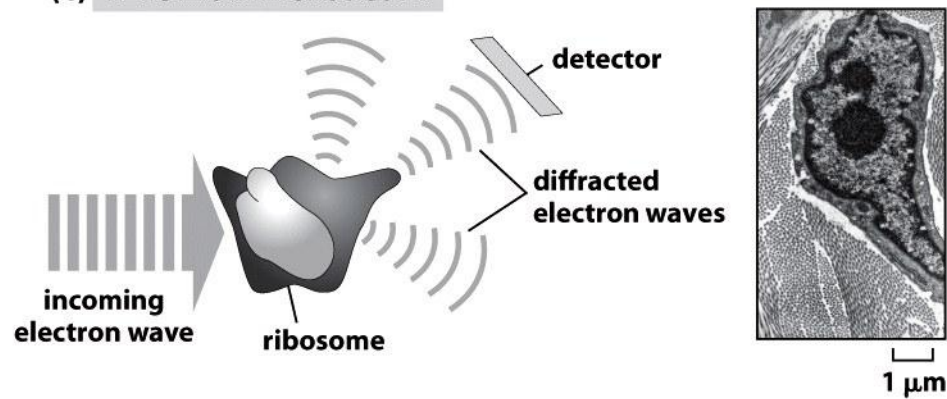


Figure 2.5 Physical Biology of the Cell (© Garland Science 2009)



## FLUORESCENCE MICROSCOPY

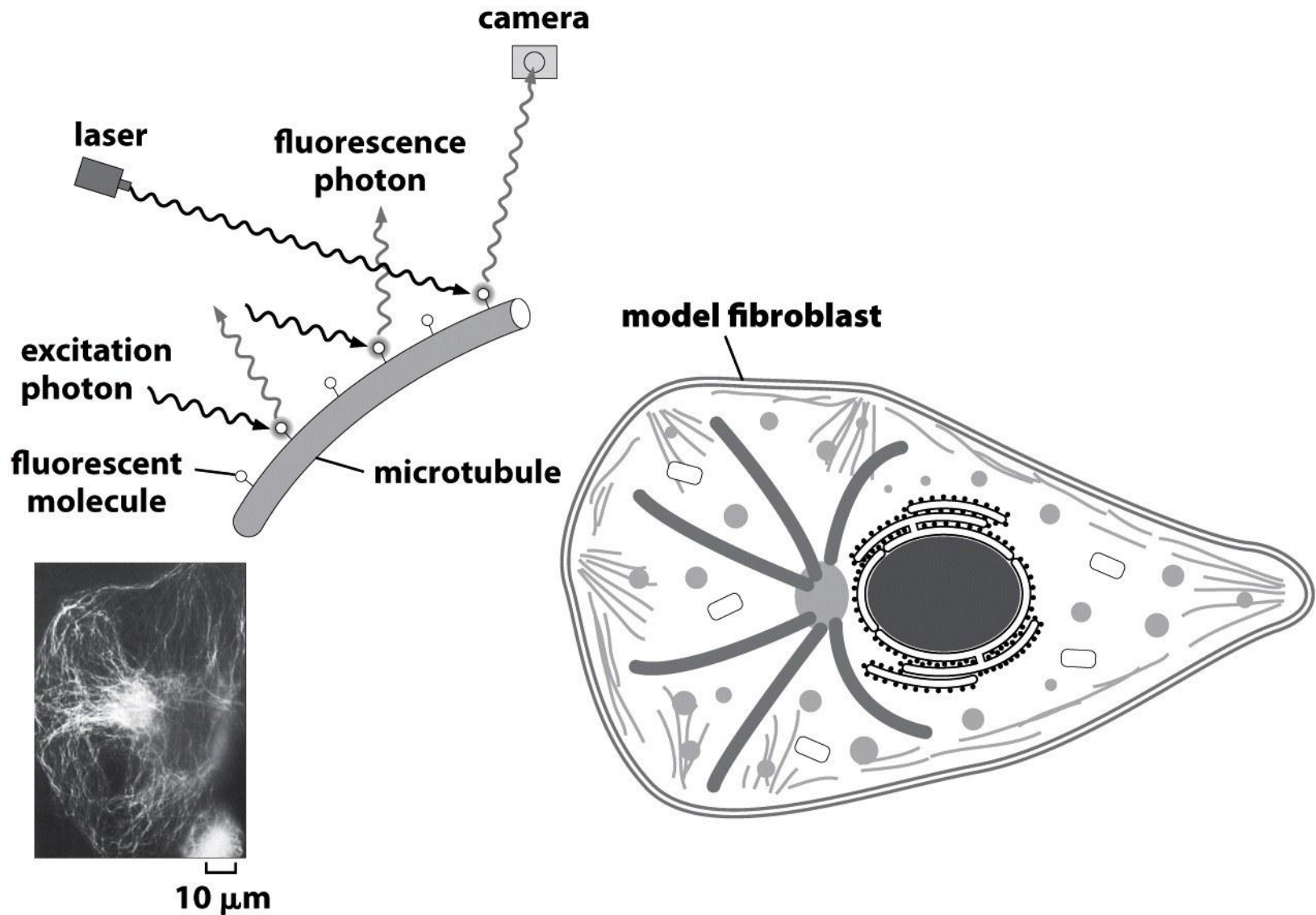


Figure 2.5a Physical Biology of the Cell (© Garland Science 2009)



## ATOMIC-FORCE MICROSCOPY

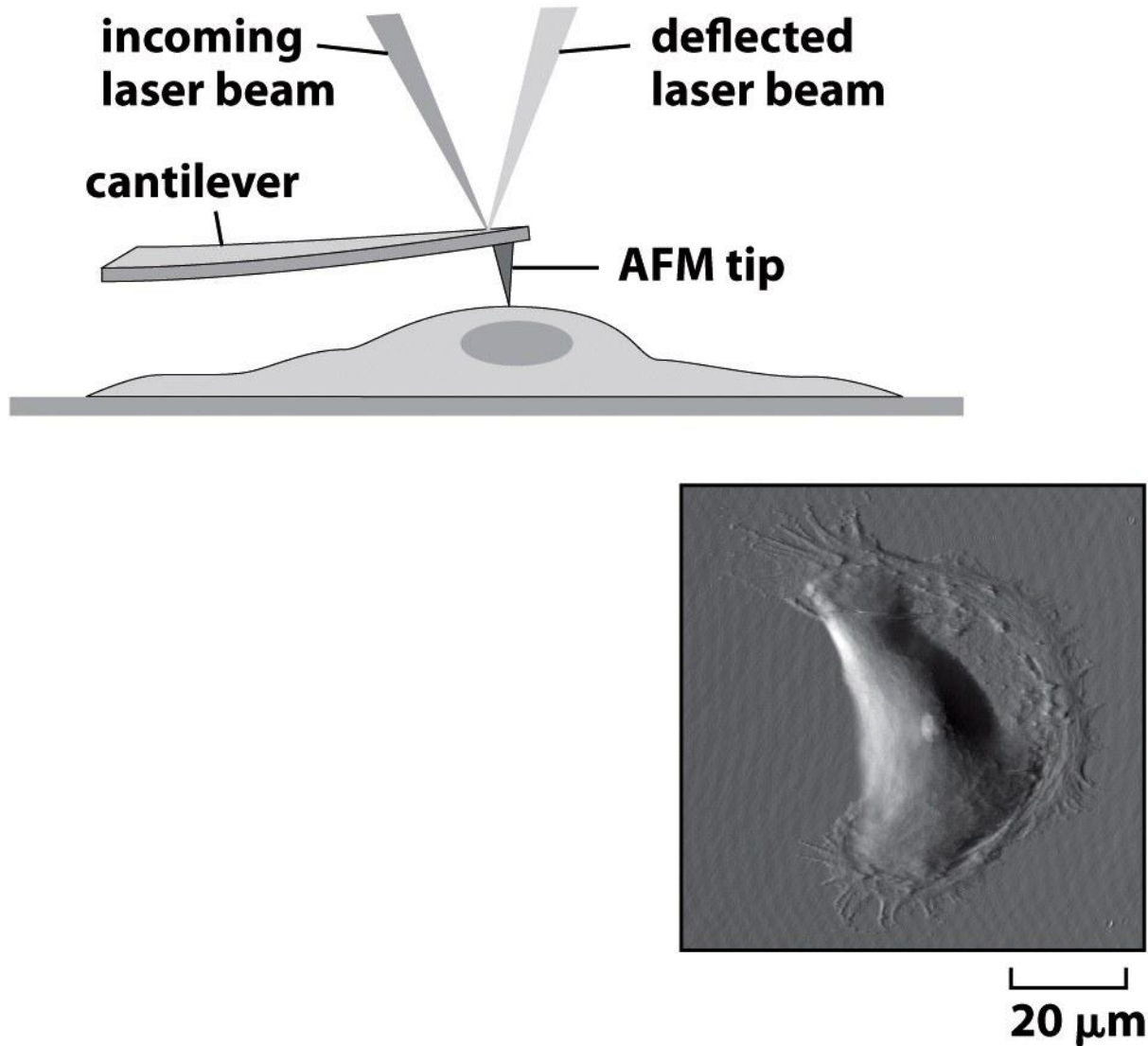


Figure 2.5b Physical Biology of the Cell (© Garland Science 2009)

## ELECTRON MICROSCOPY

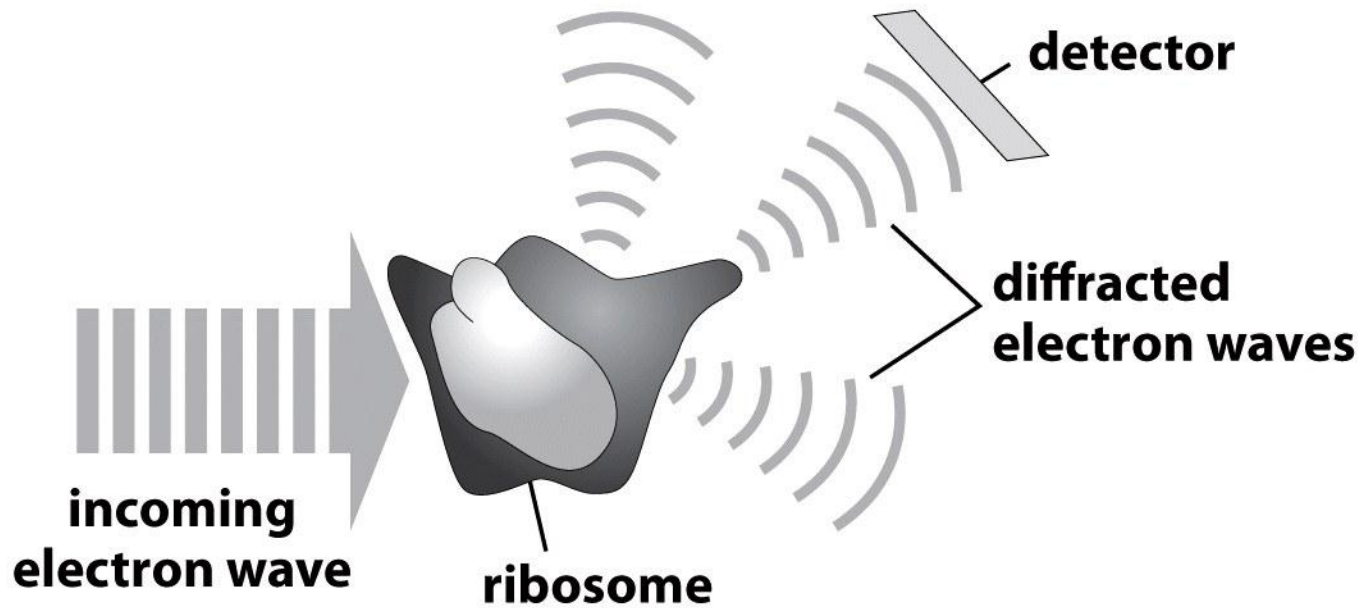
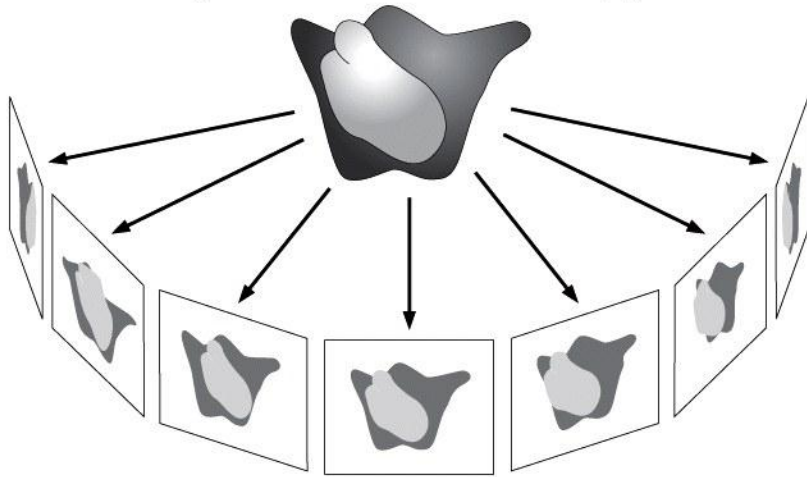


Figure 2.5c Physical Biology of the Cell (© Garland Science 2009)

**(A) cryo-electron microscopy**



**(B) image reconstruction**

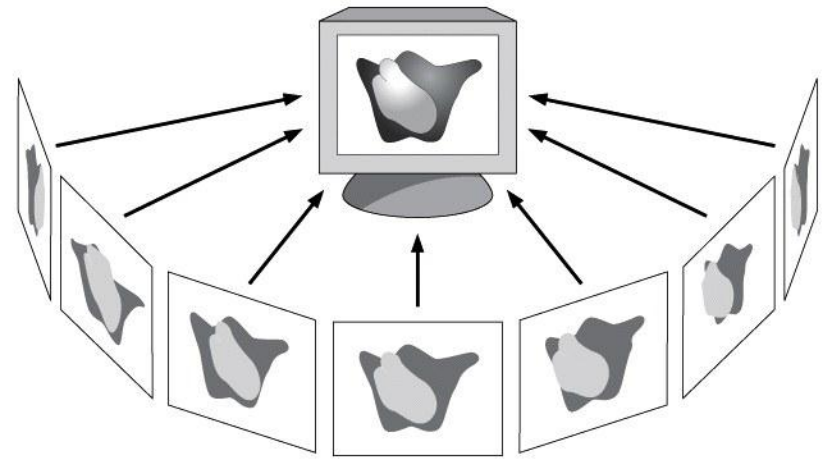


Figure 2.6 Physical Biology of the Cell (© Garland Science 2009)

# cryo-electron microscopy

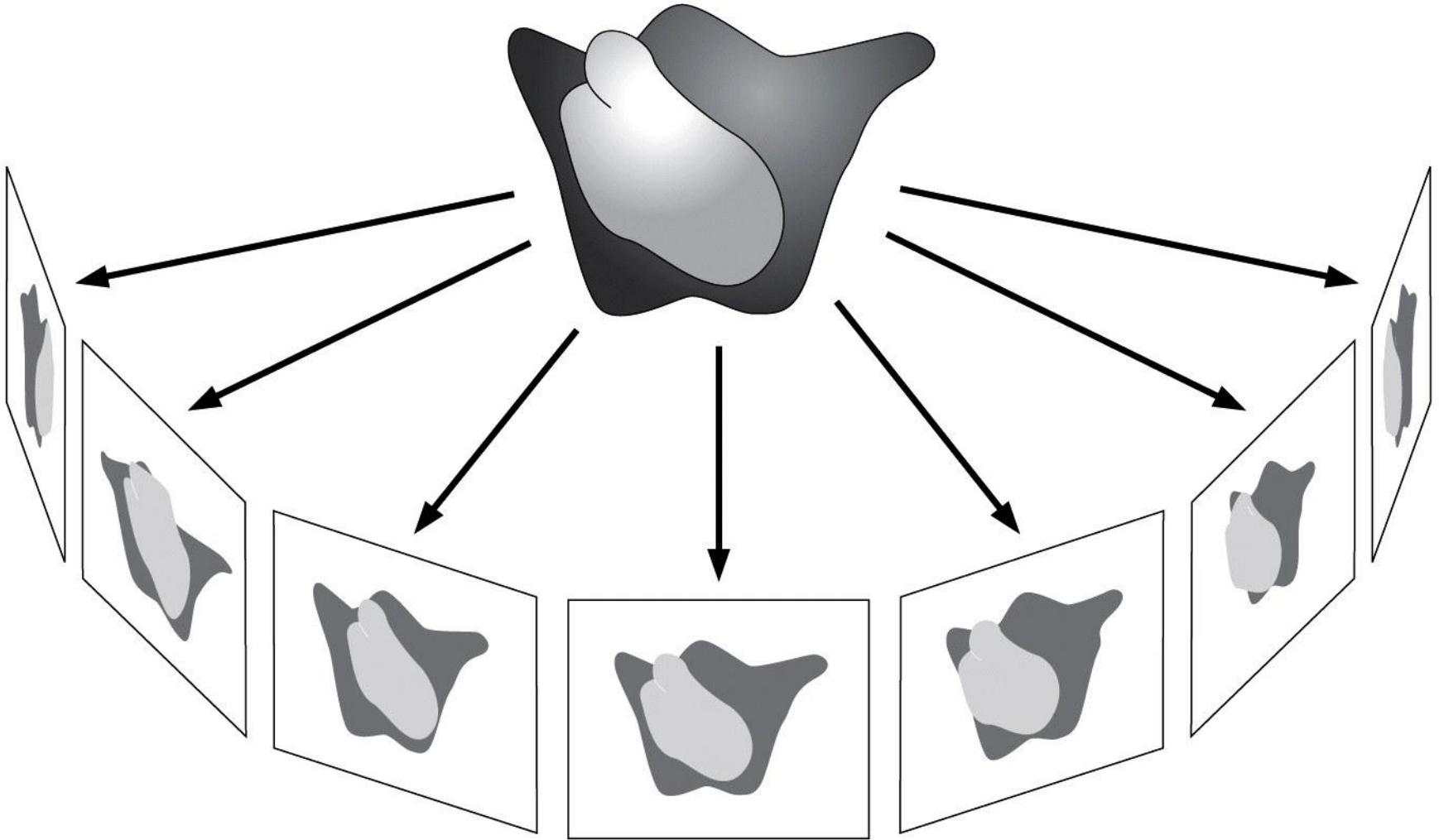


Figure 2.6a Physical Biology of the Cell (© Garland Science 2009)

# image reconstruction

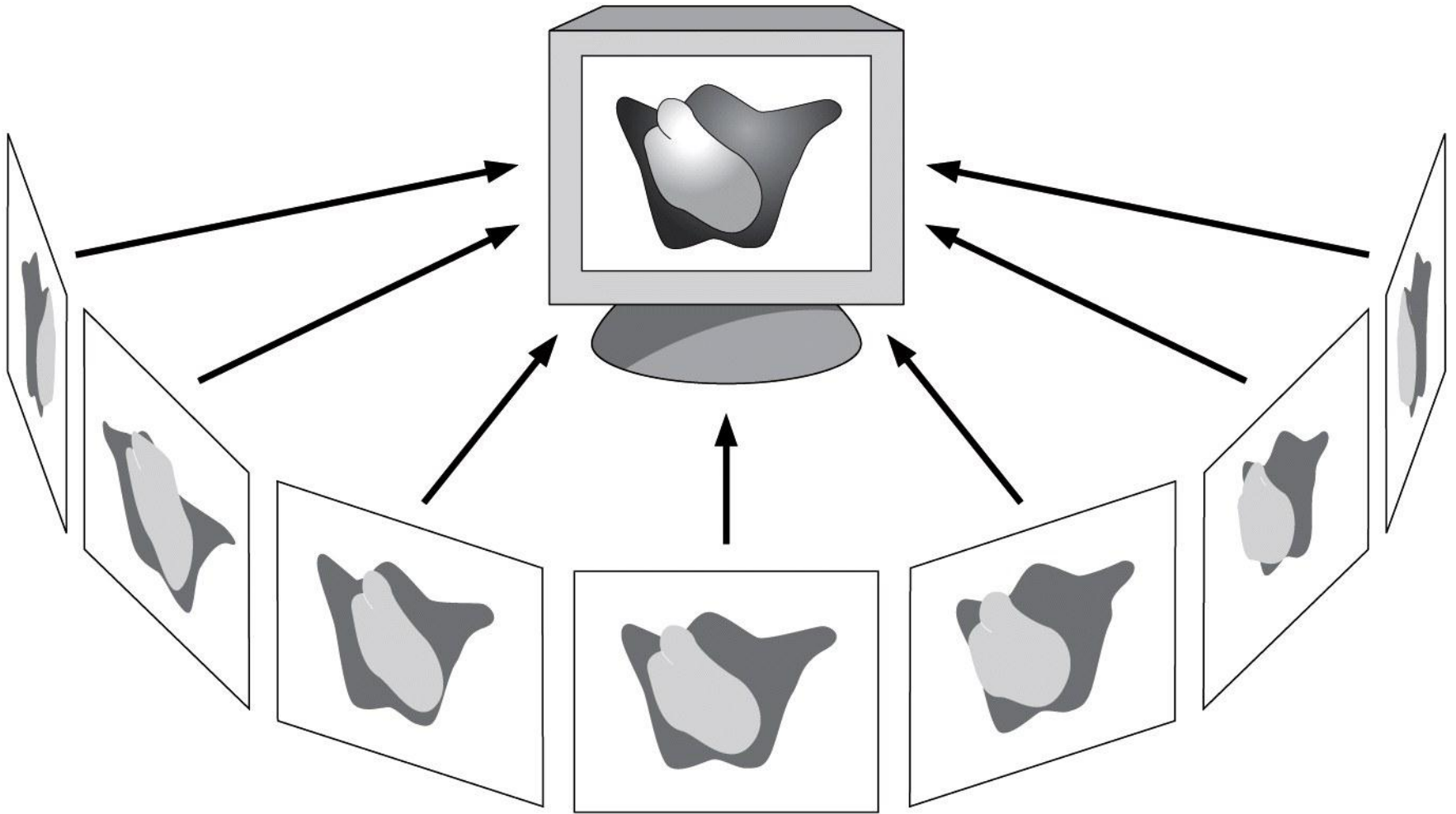


Figure 2.6b Physical Biology of the Cell (© Garland Science 2009)

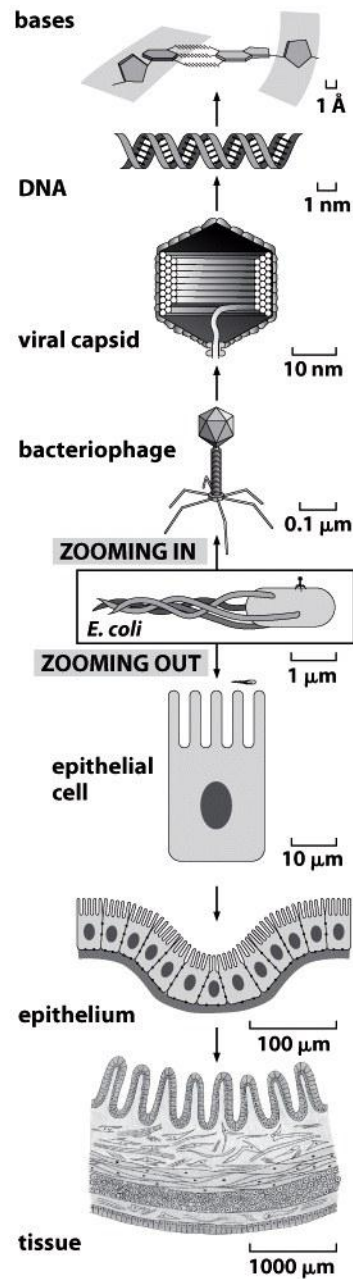


Figure 2.7 Physical Biology of the Cell (© Garland Science 2009)



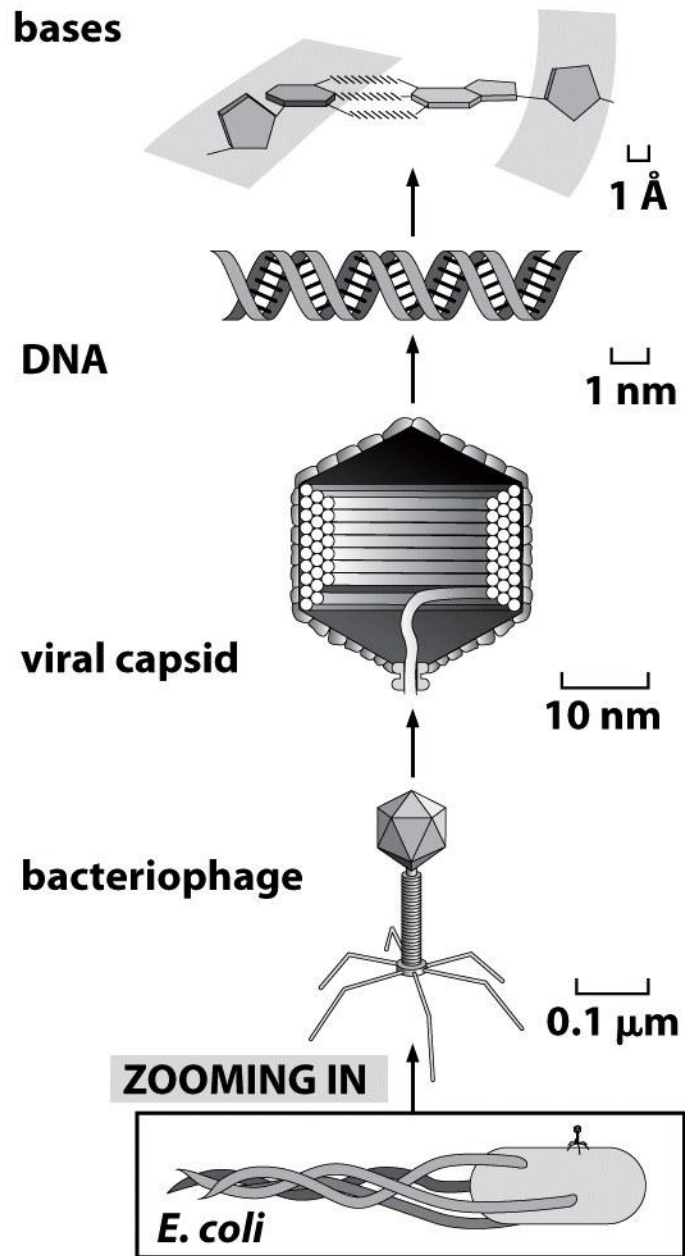


Figure 2.7 (part 1) Physical Biology of the Cell (© Garland Science 2009)

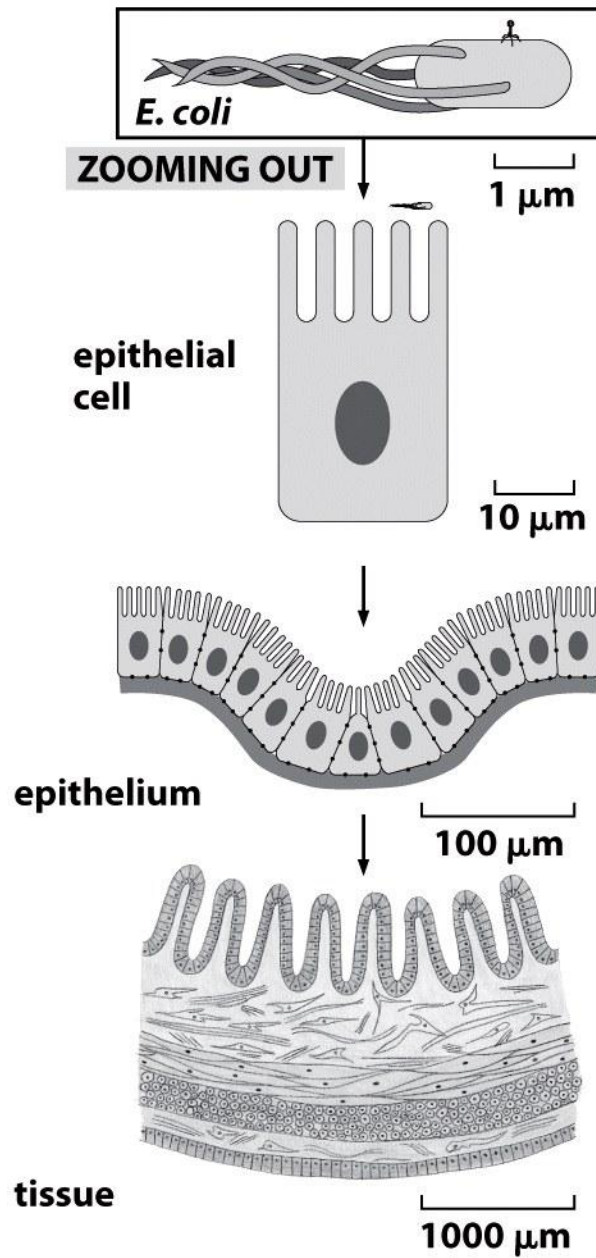


Figure 2.7 (part 2) Physical Biology of the Cell (© Garland Science 2009)

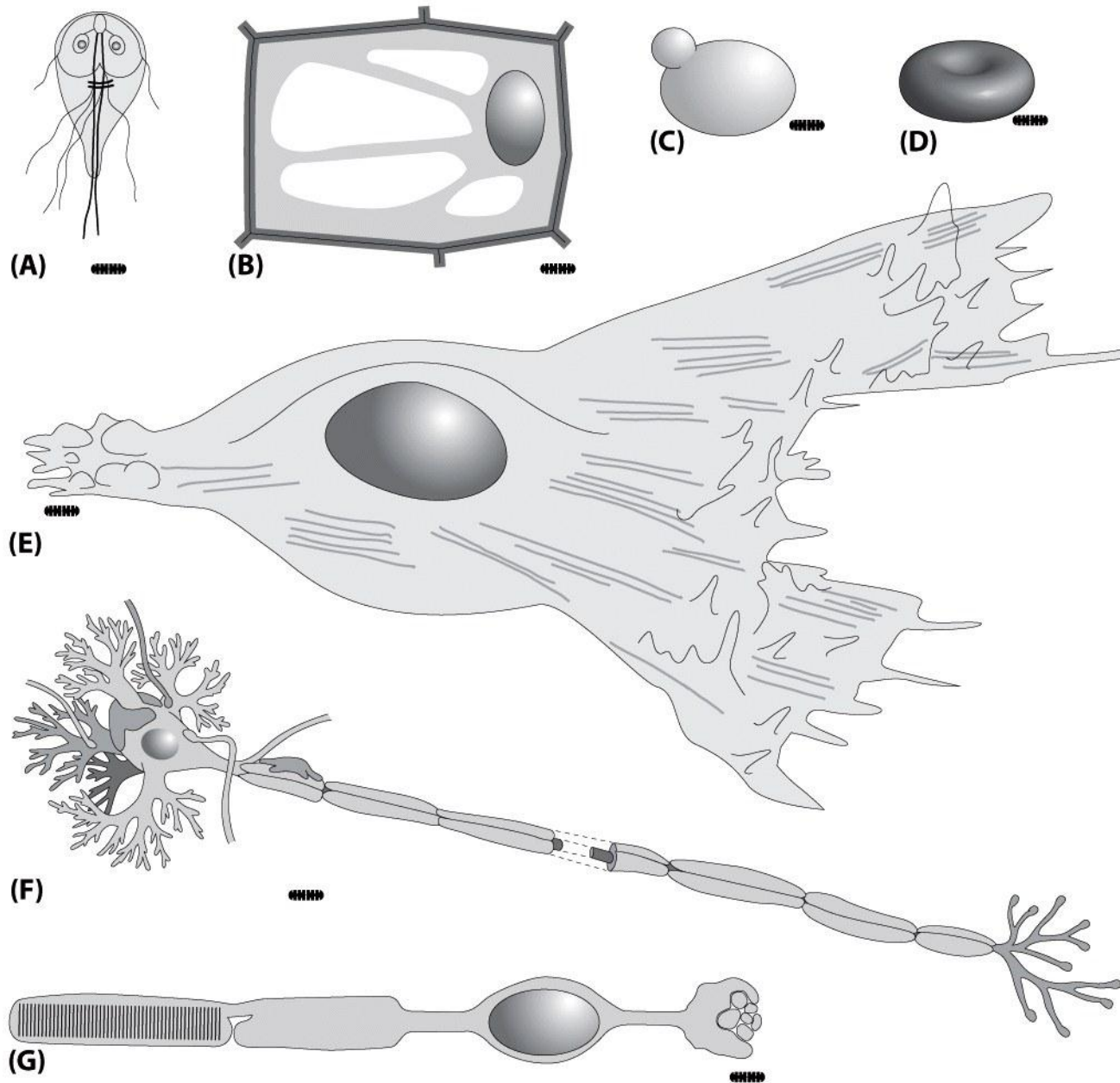
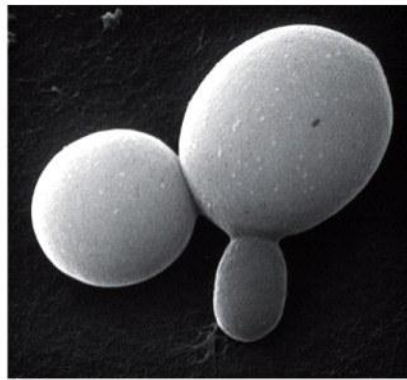
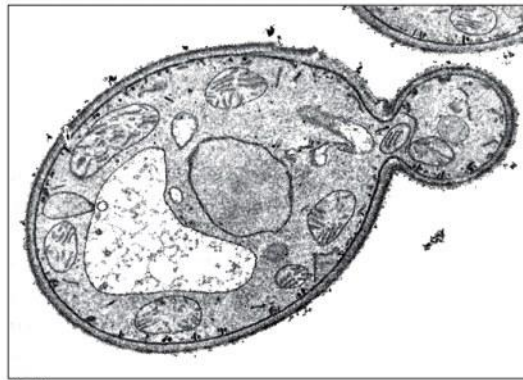


Figure 2.8 Physical Biology of the Cell (© Garland Science 2009)



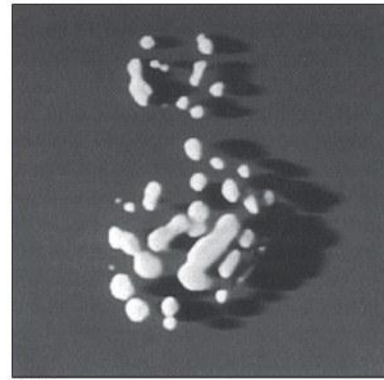
(A)

2  $\mu\text{m}$

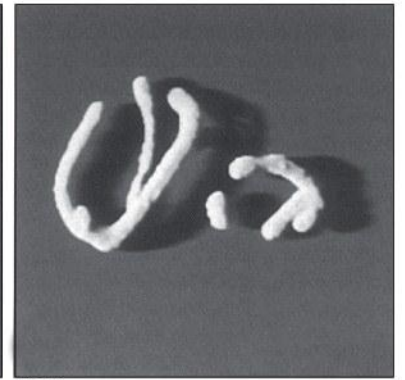


(B)

1  $\mu\text{m}$



(C)



(D)

5  $\mu\text{m}$

Figure 2.9 Physical Biology of the Cell (© Garland Science 2009)

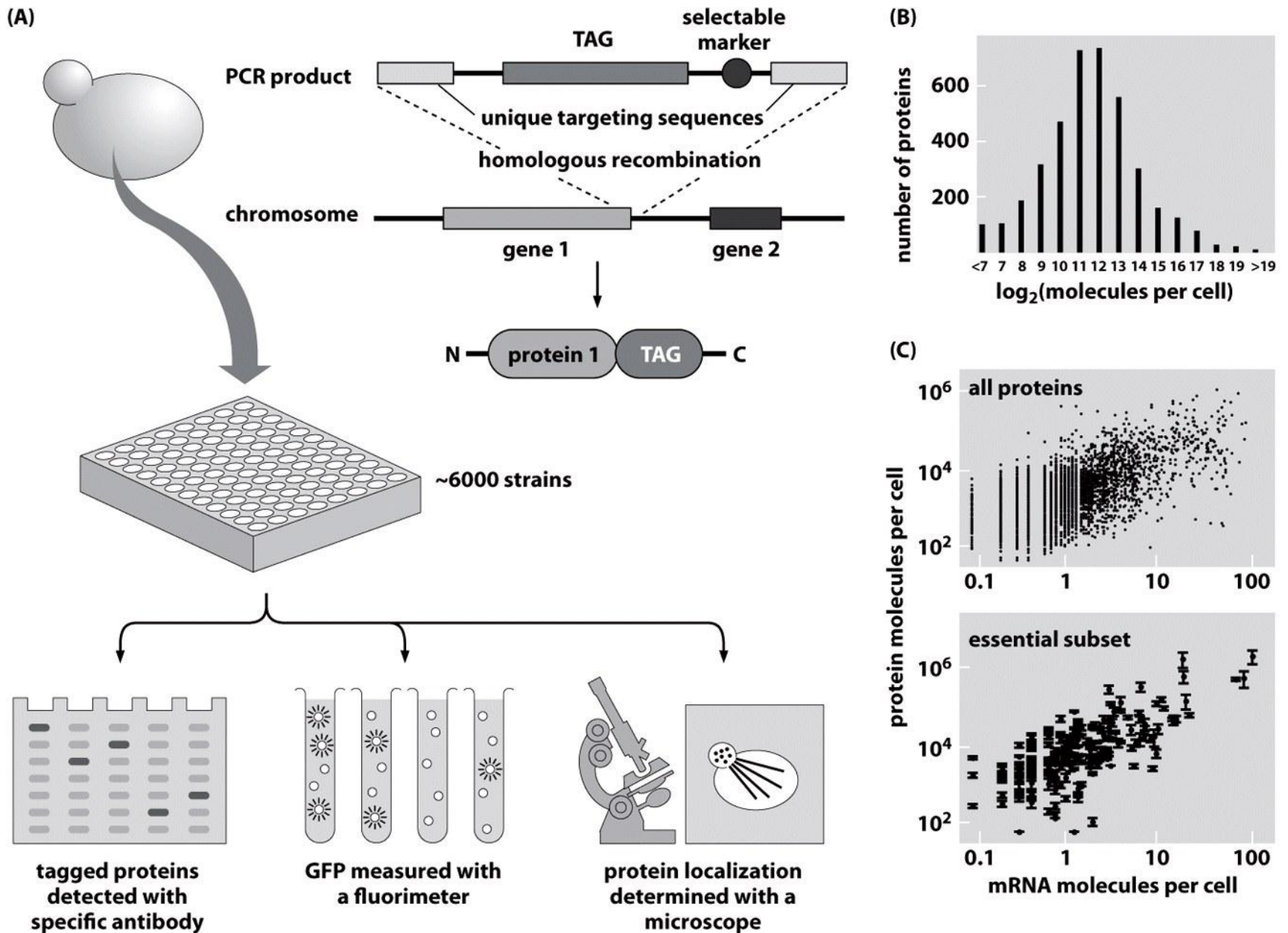


Figure 2.10 Physical Biology of the Cell (© Garland Science 2009)



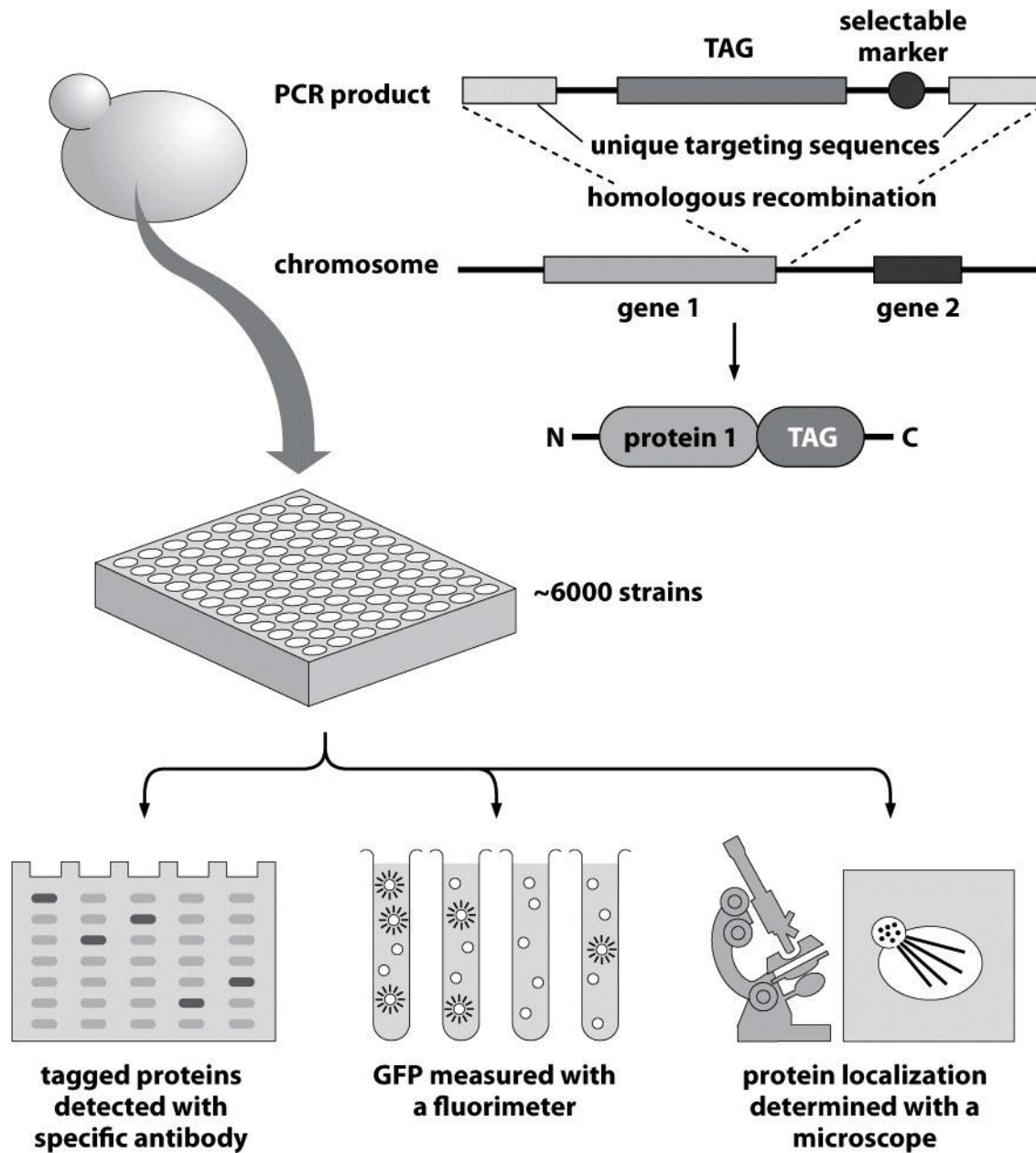


Figure 2.10a Physical Biology of the Cell (© Garland Science 2009)



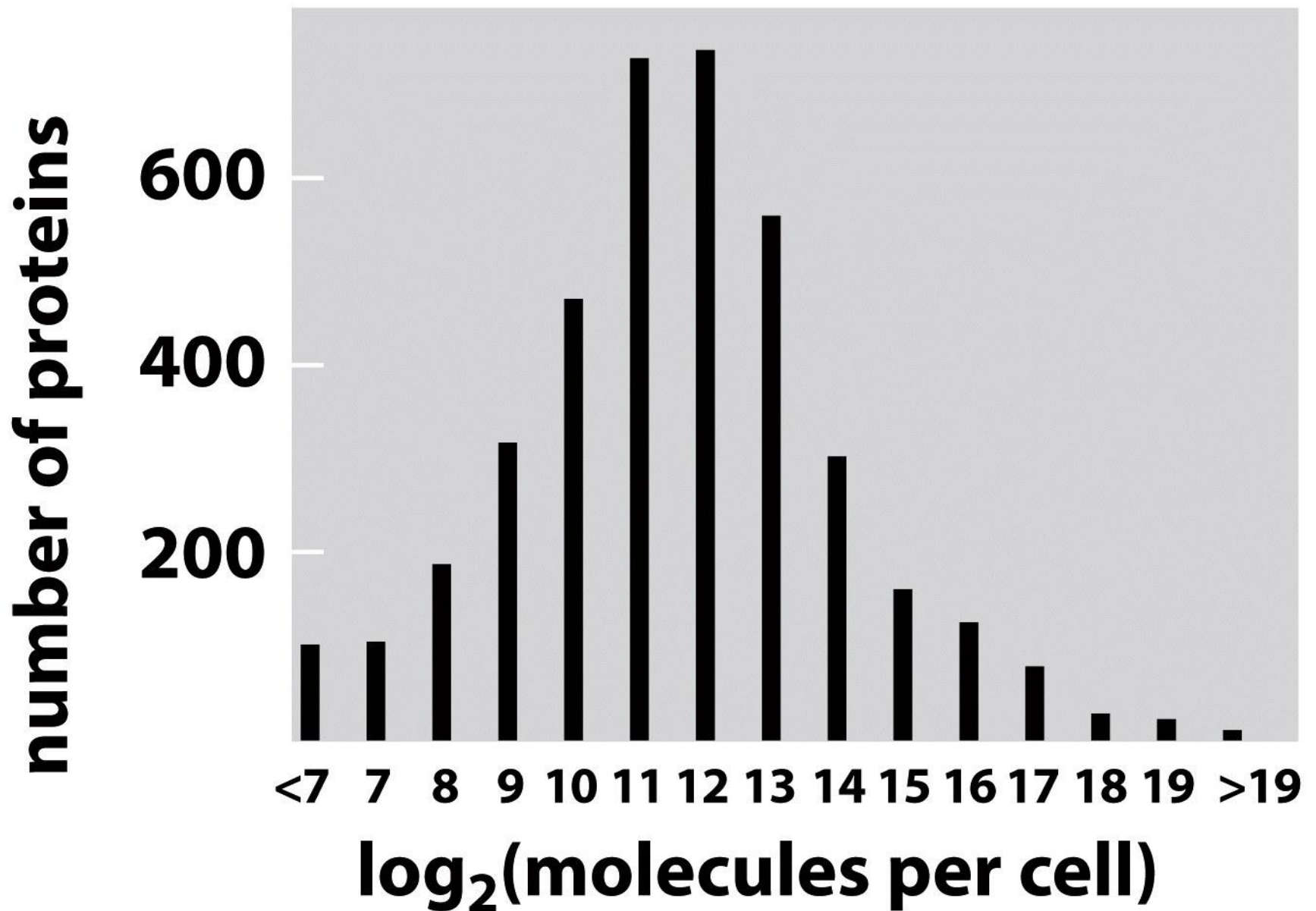


Figure 2.10b Physical Biology of the Cell (© Garland Science 2009)

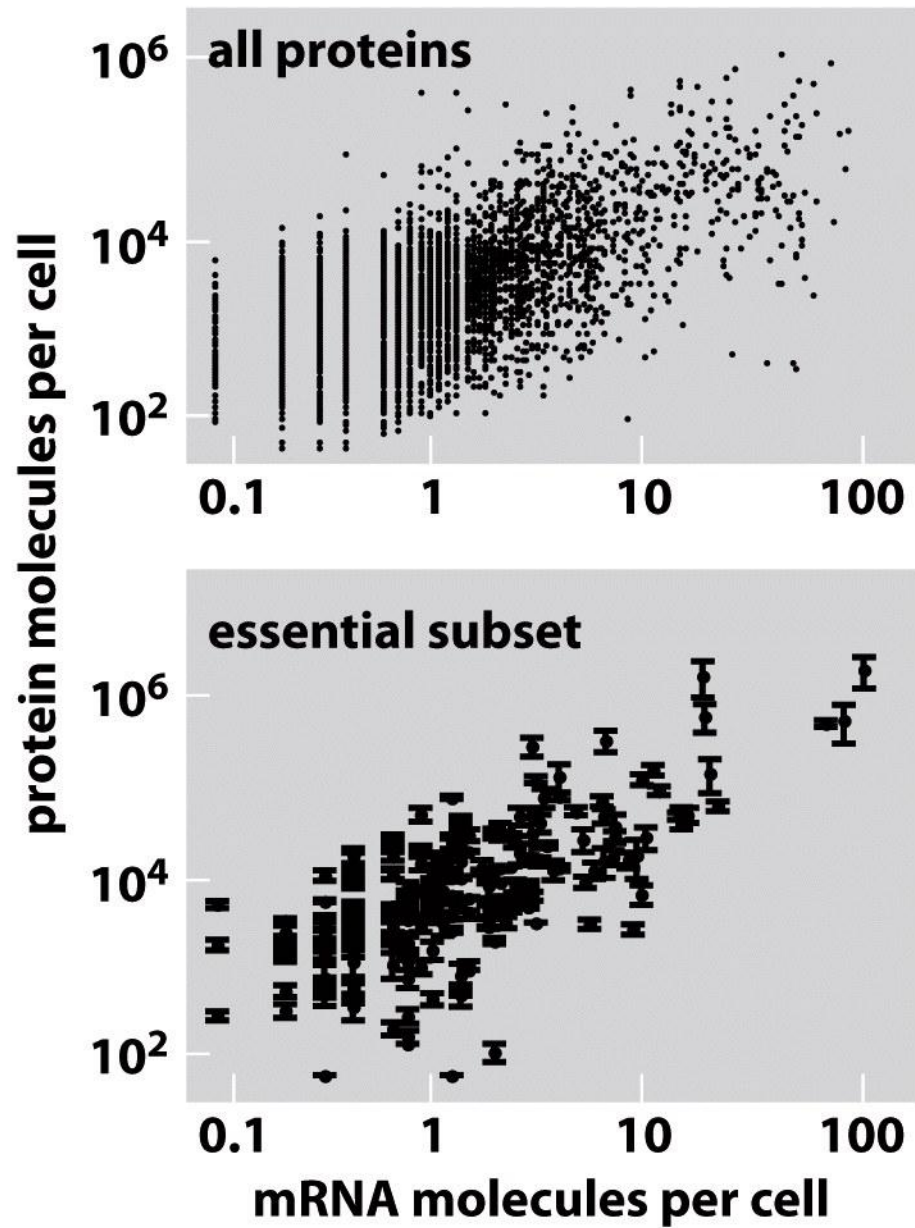


Figure 2.10c Physical Biology of the Cell (© Garland Science 2009)

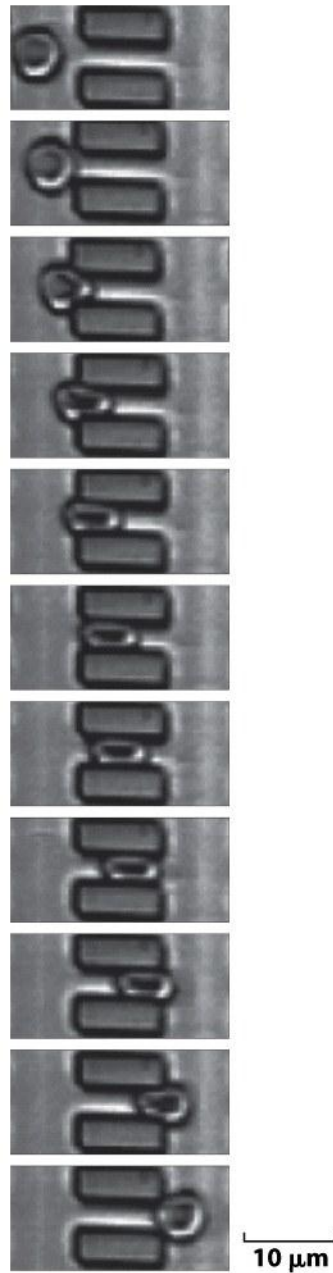


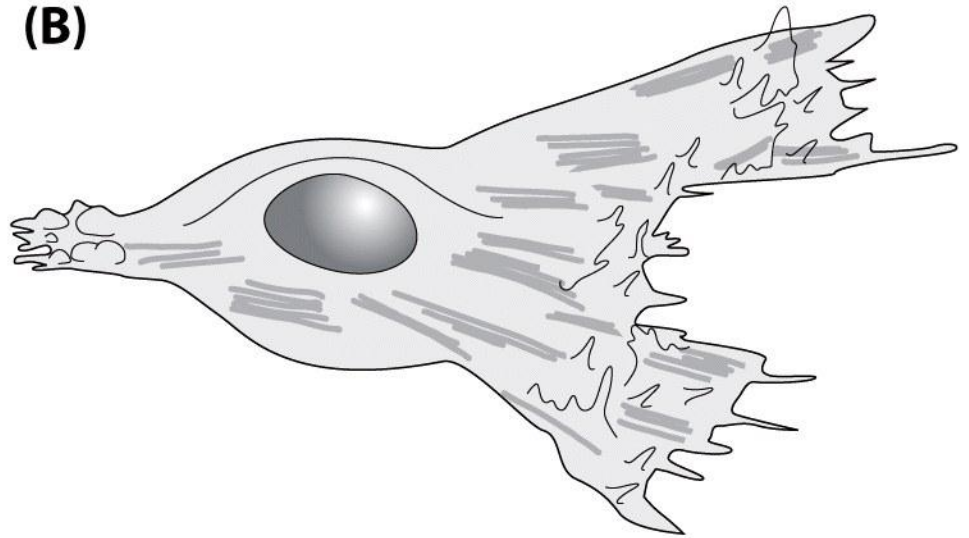
Figure 2.11 Physical Biology of the Cell (© Garland Science 2009)

**(A)**



20 μm

**(B)**



**(C)**

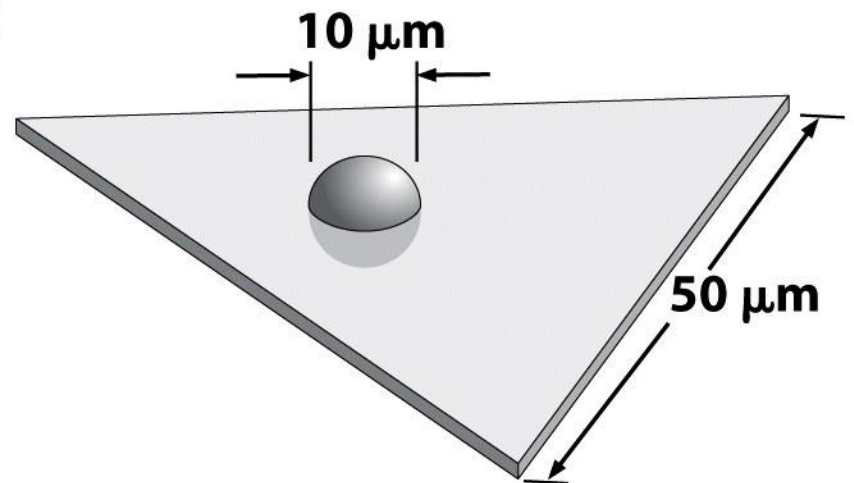


Figure 2.12 Physical Biology of the Cell (© Garland Science 2009)



20  $\mu\text{m}$

Figure 2.12a Physical Biology of the Cell (© Garland Science 2009)

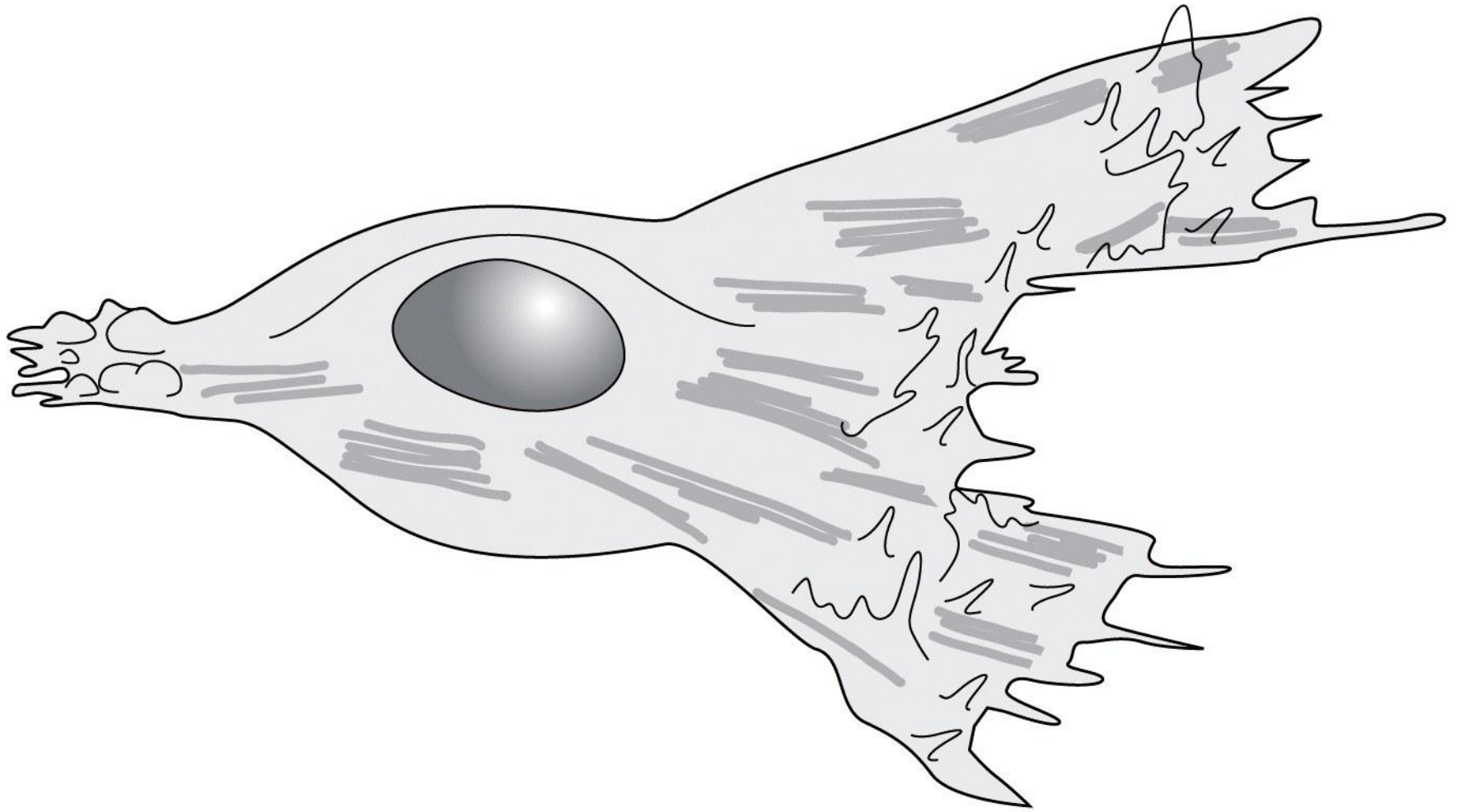


Figure 2.12b Physical Biology of the Cell (© Garland Science 2009)



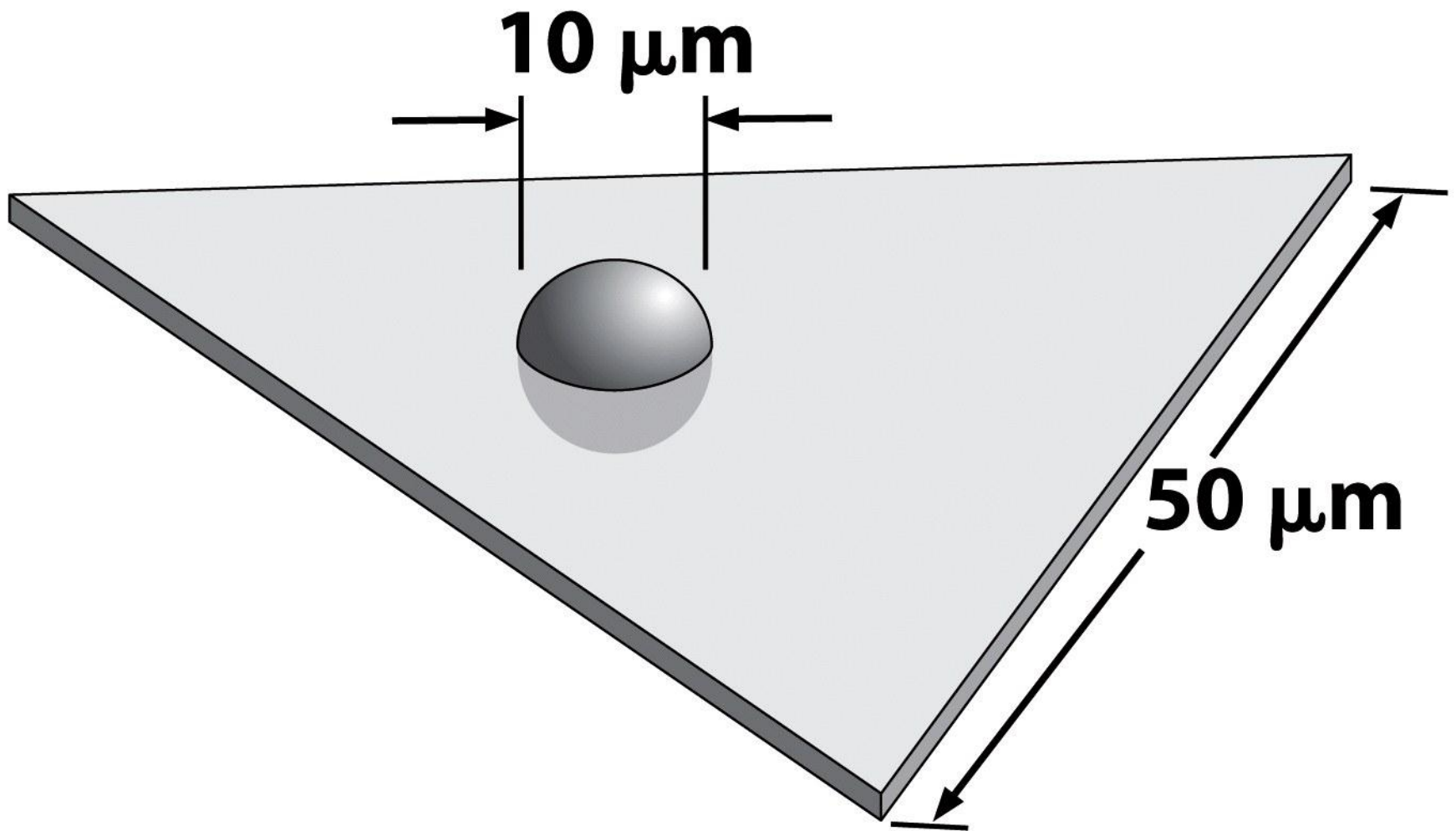


Figure 2.12c Physical Biology of the Cell (© Garland Science 2009)

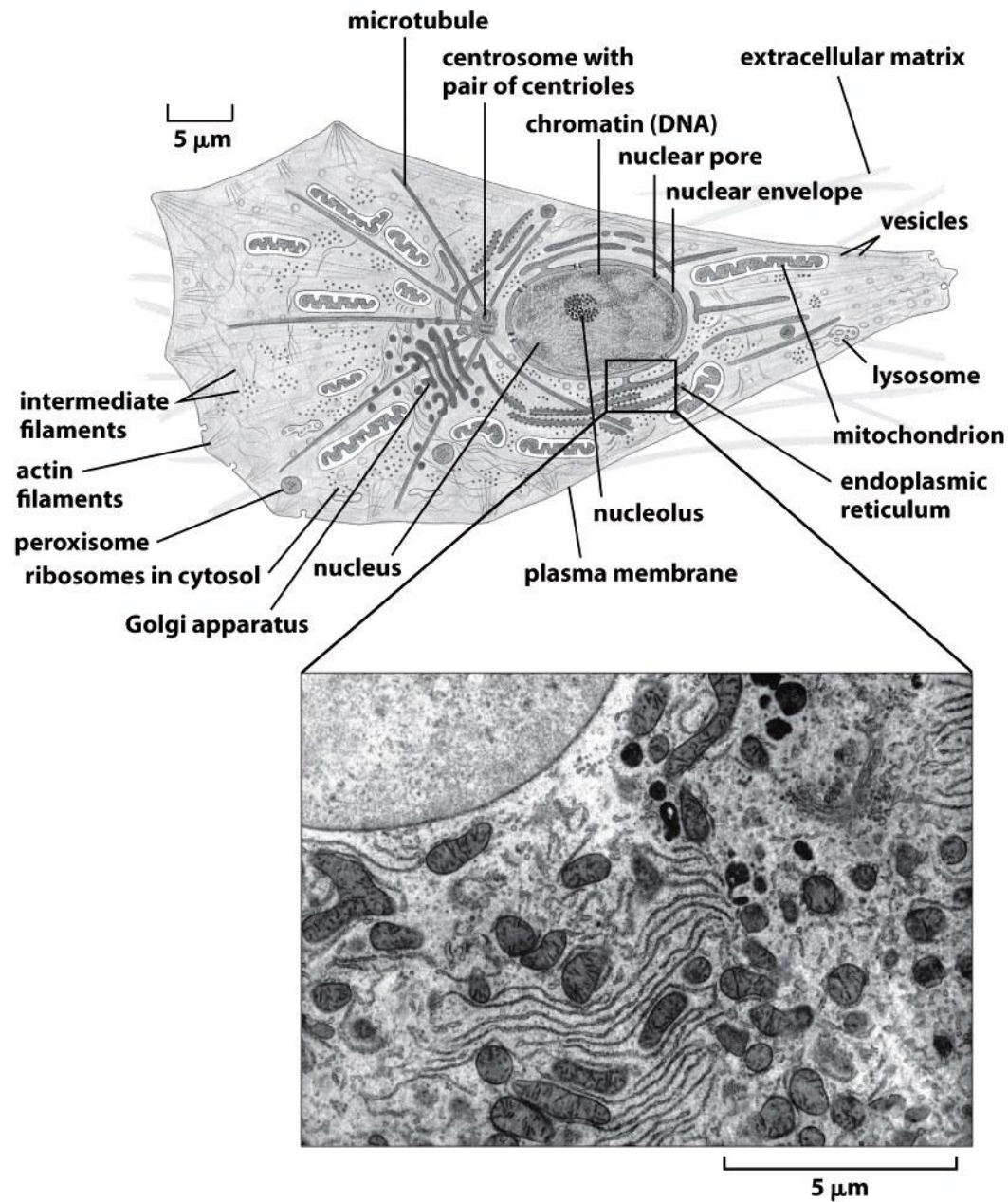


Figure 2.13 Physical Biology of the Cell (© Garland Science 2009)

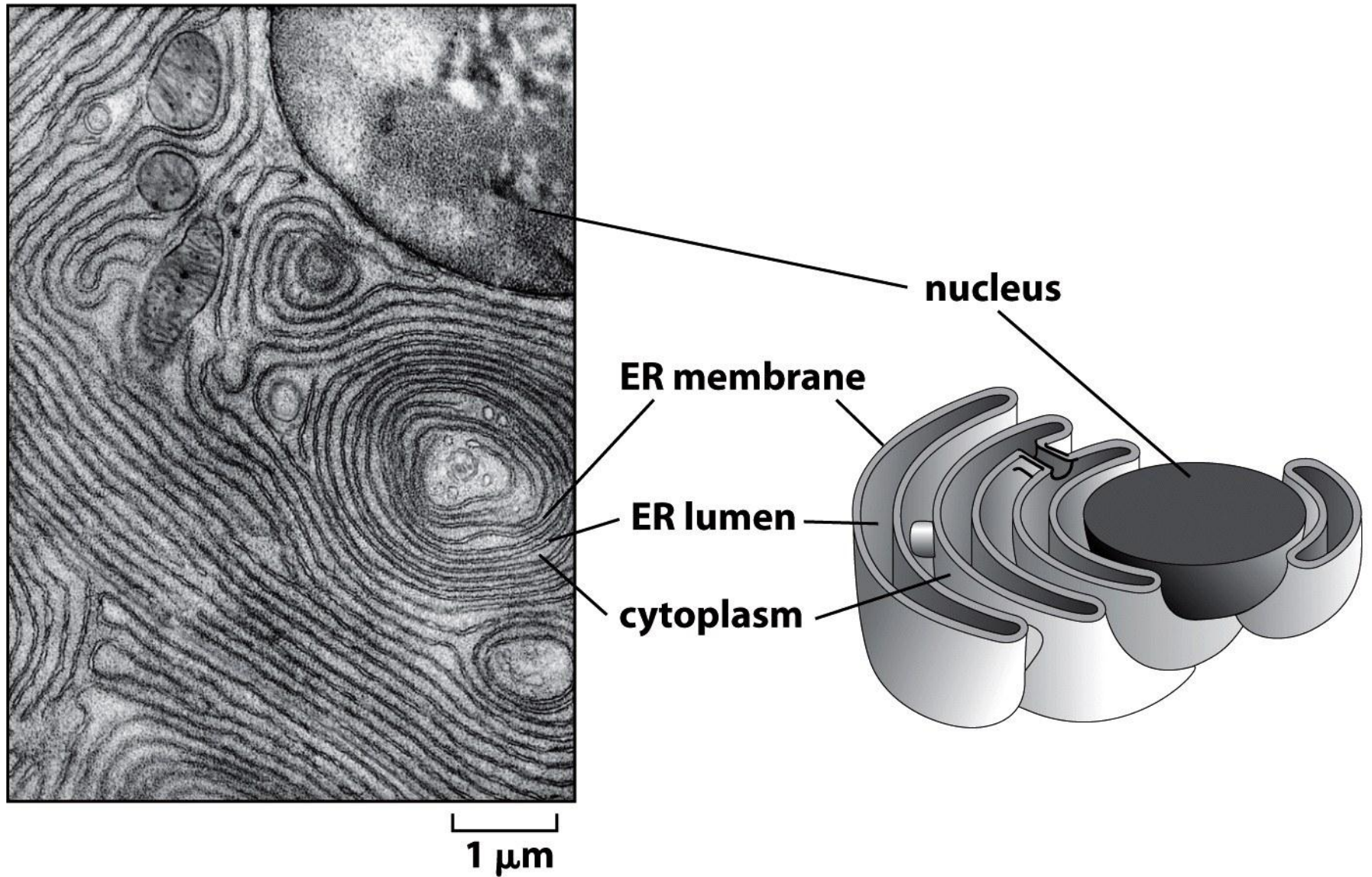
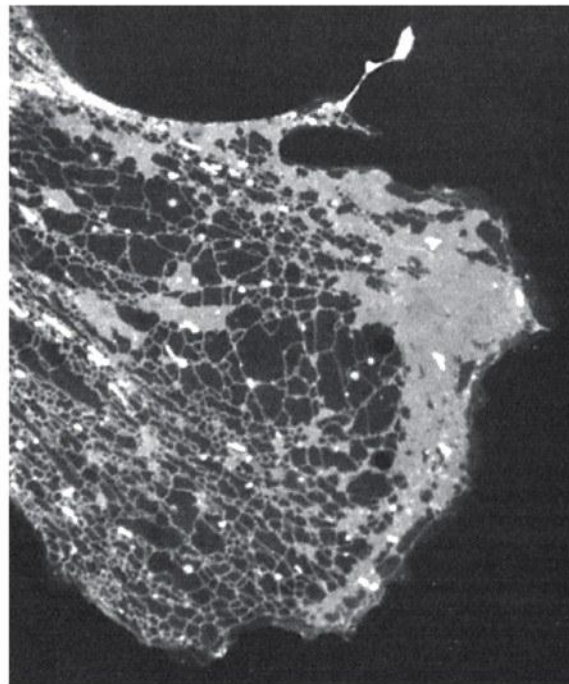


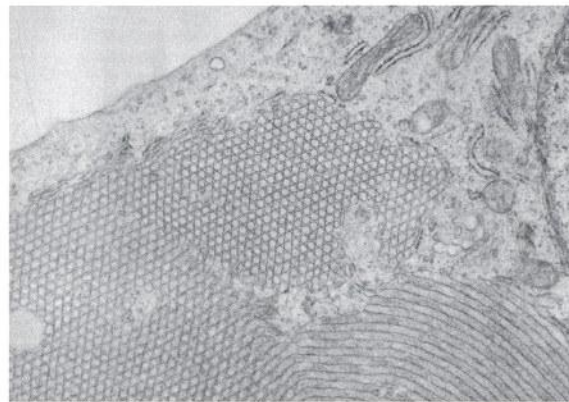
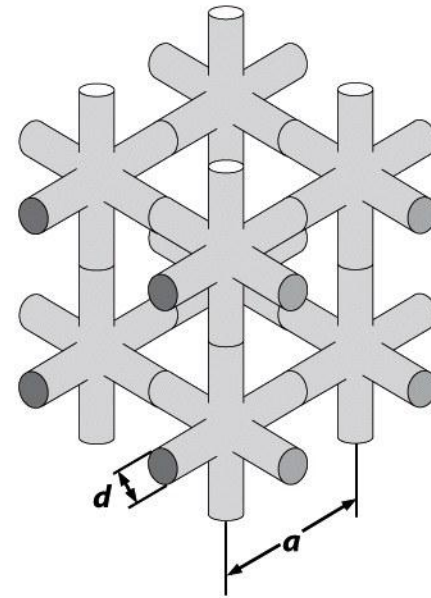
Figure 2.14 Physical Biology of the Cell (© Garland Science 2009)





(A)

10  $\mu\text{m}$



(B)

1  $\mu\text{m}$

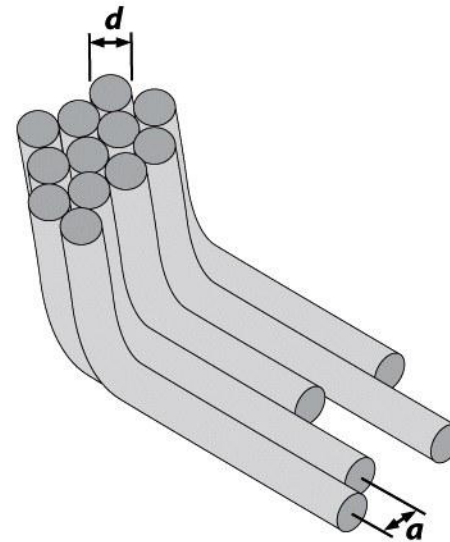
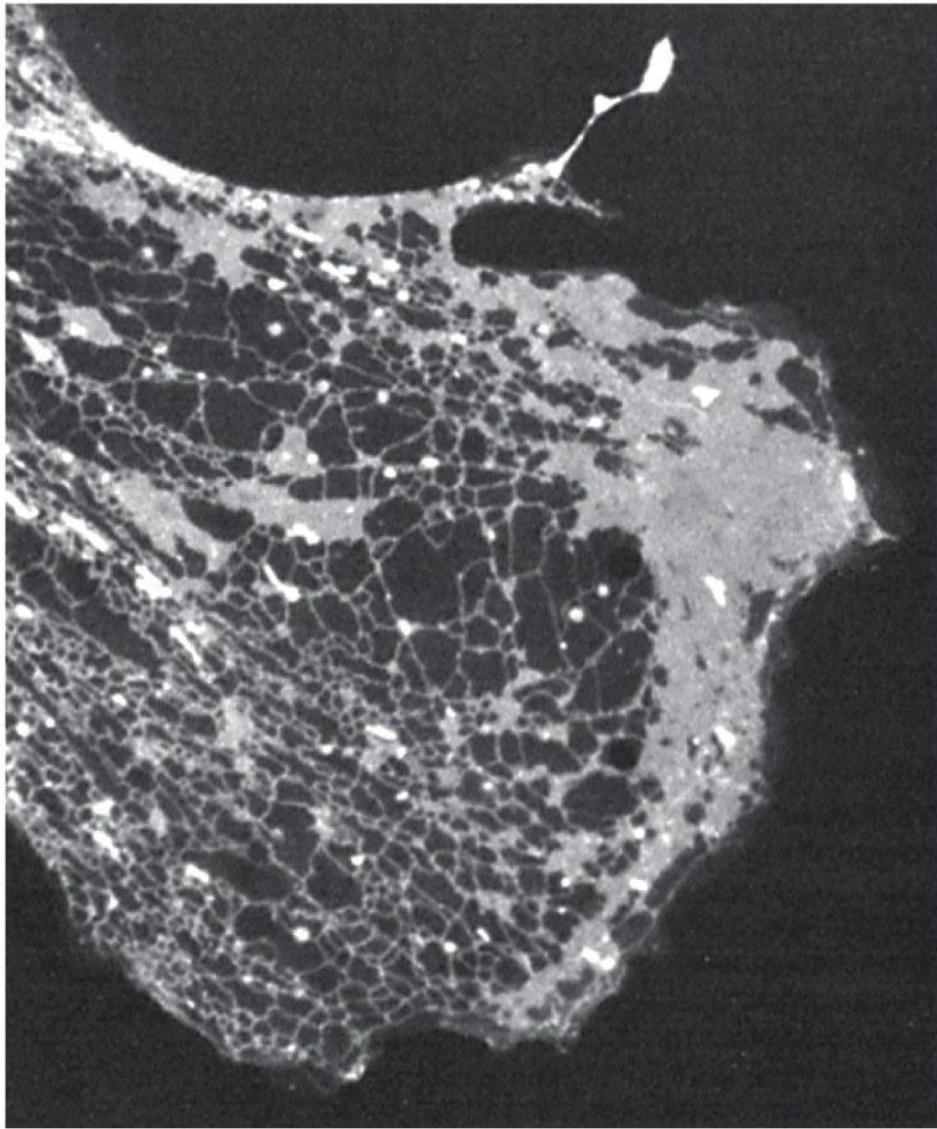


Figure 2.15 Physical Biology of the Cell (© Garland Science 2009)



10  $\mu\text{m}$

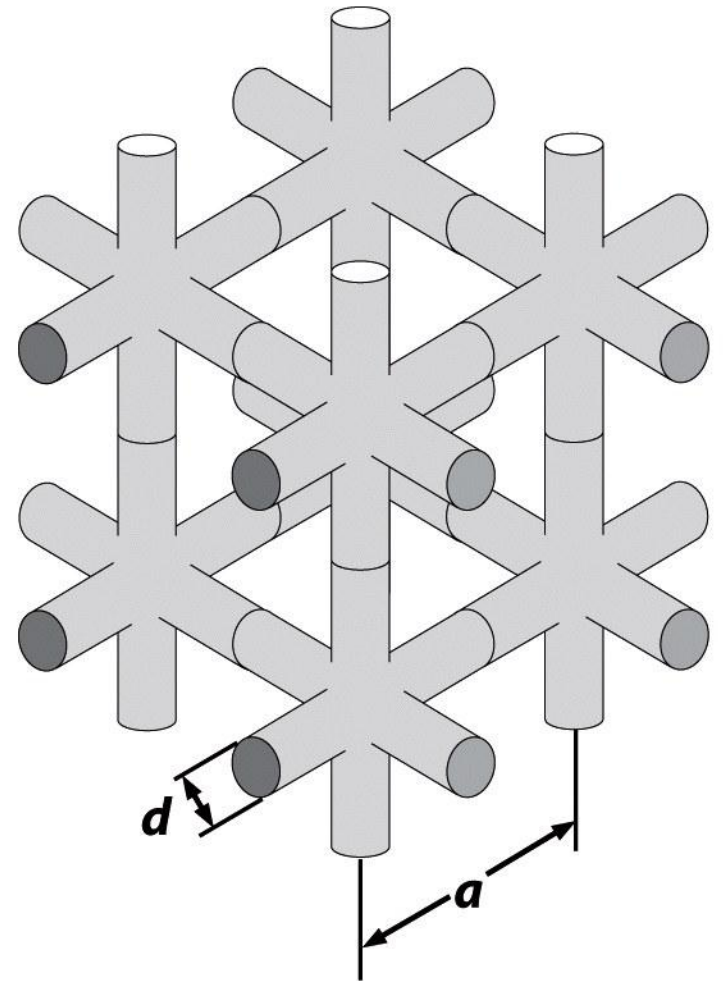


Figure 2.15a Physical Biology of the Cell (© Garland Science 2009)

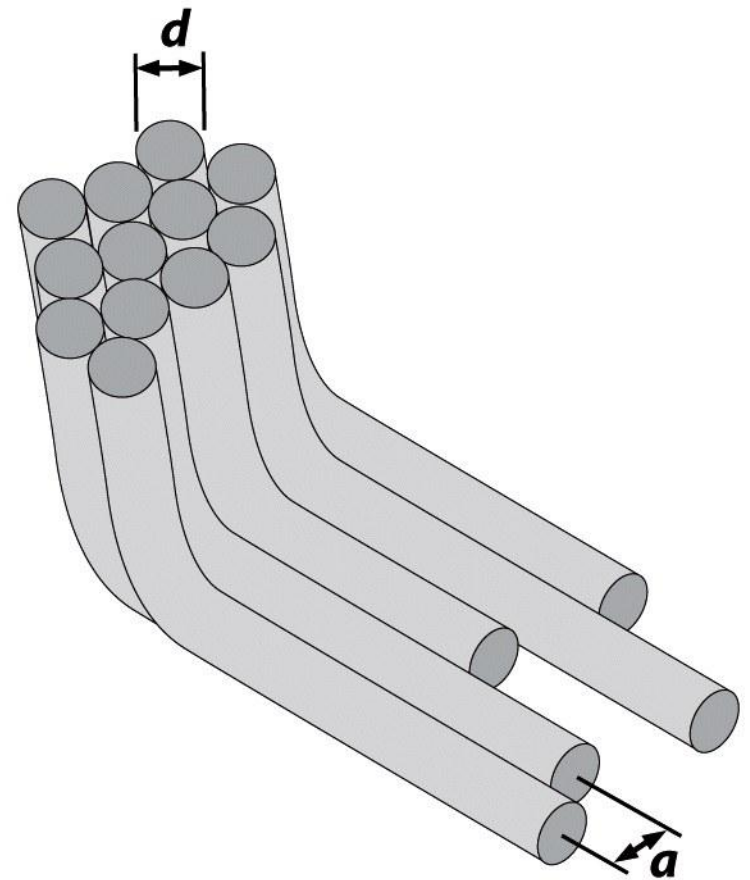
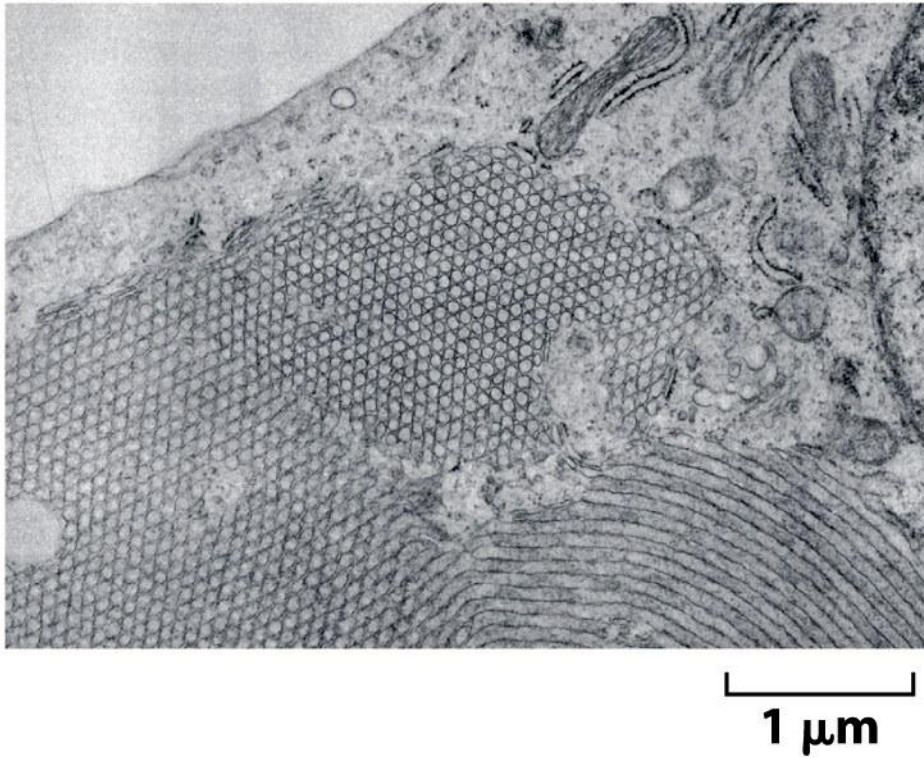
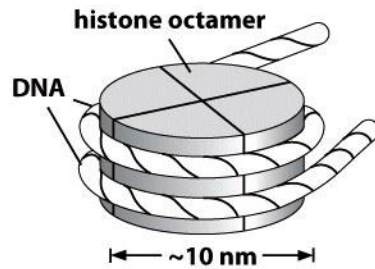
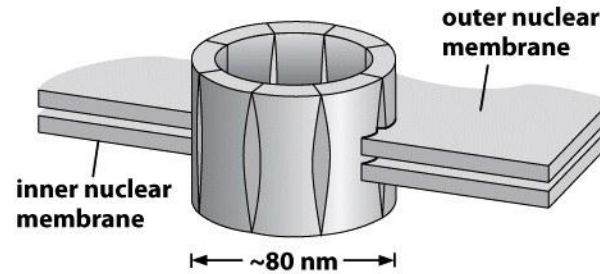


Figure 2.15b Physical Biology of the Cell (© Garland Science 2009)

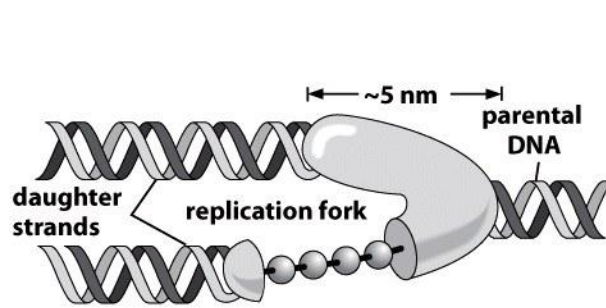




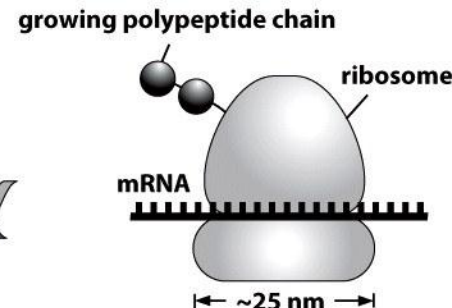
(A) nucleosome



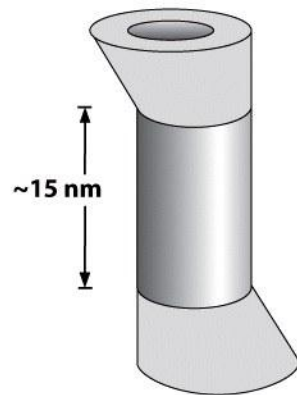
(B) nuclear pore complex



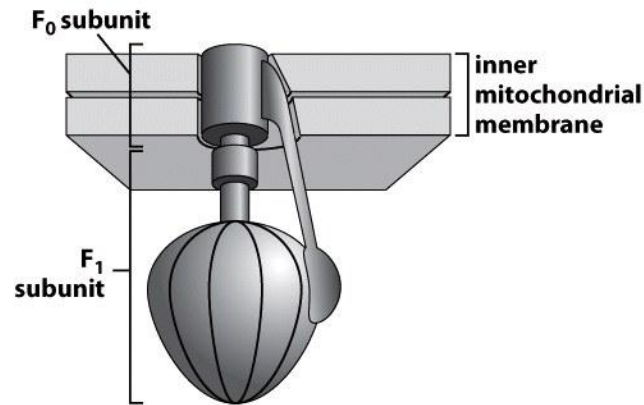
(C) replisome



(D) ribosome



(E) proteasome



(F) ATP synthase

Figure 2.16 Physical Biology of the Cell (© Garland Science 2009)

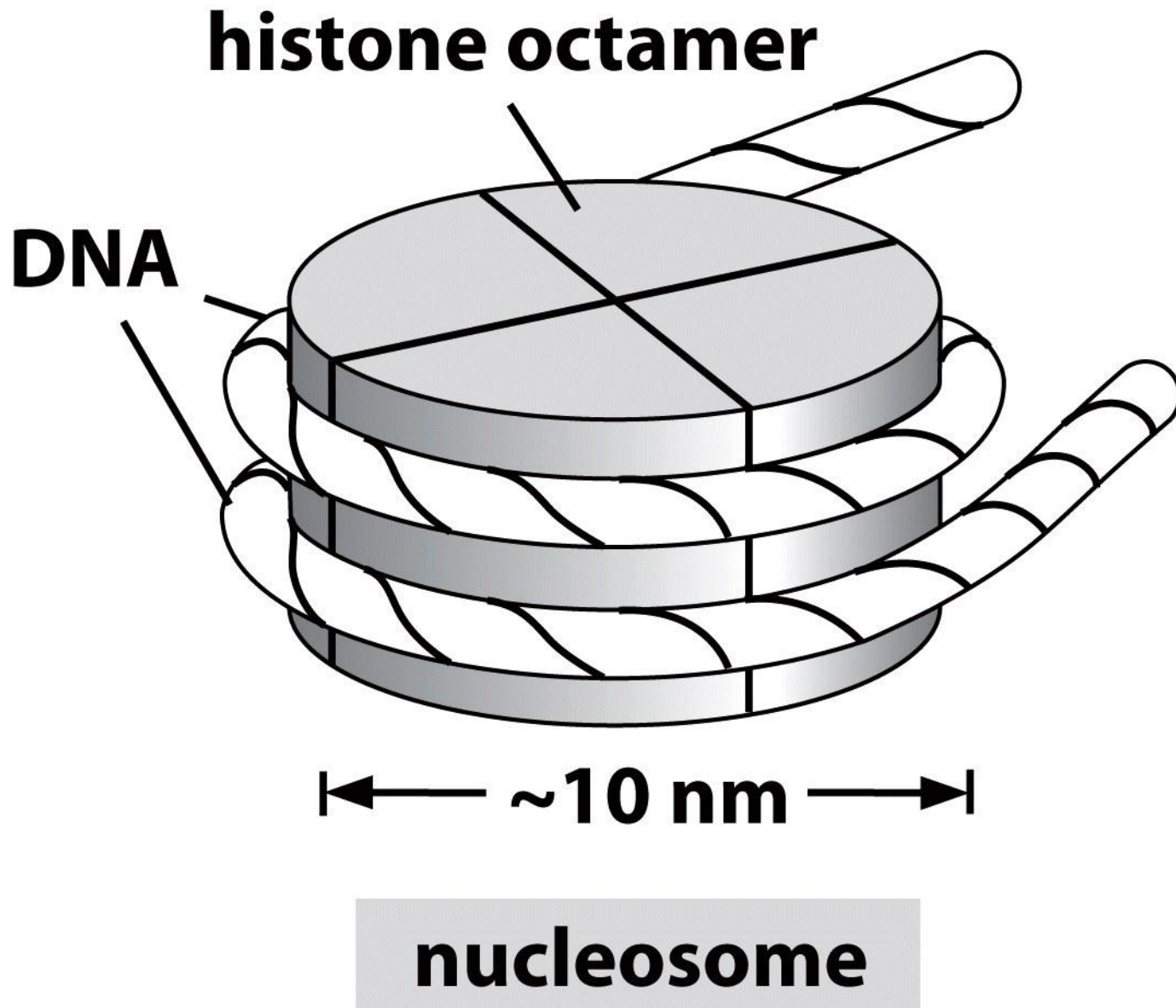


Figure 2.16a Physical Biology of the Cell (© Garland Science 2009)

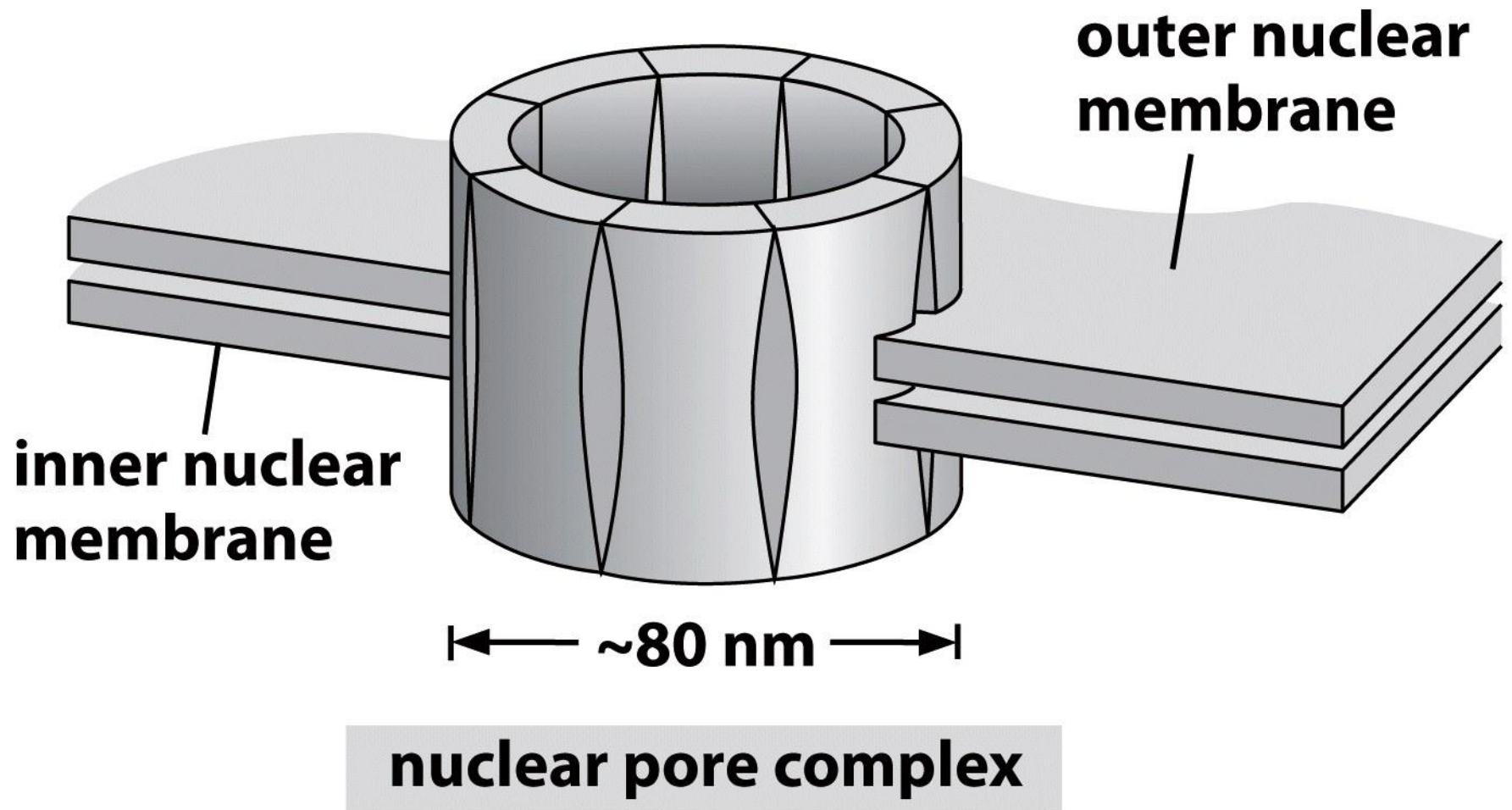


Figure 2.16b Physical Biology of the Cell (© Garland Science 2009)

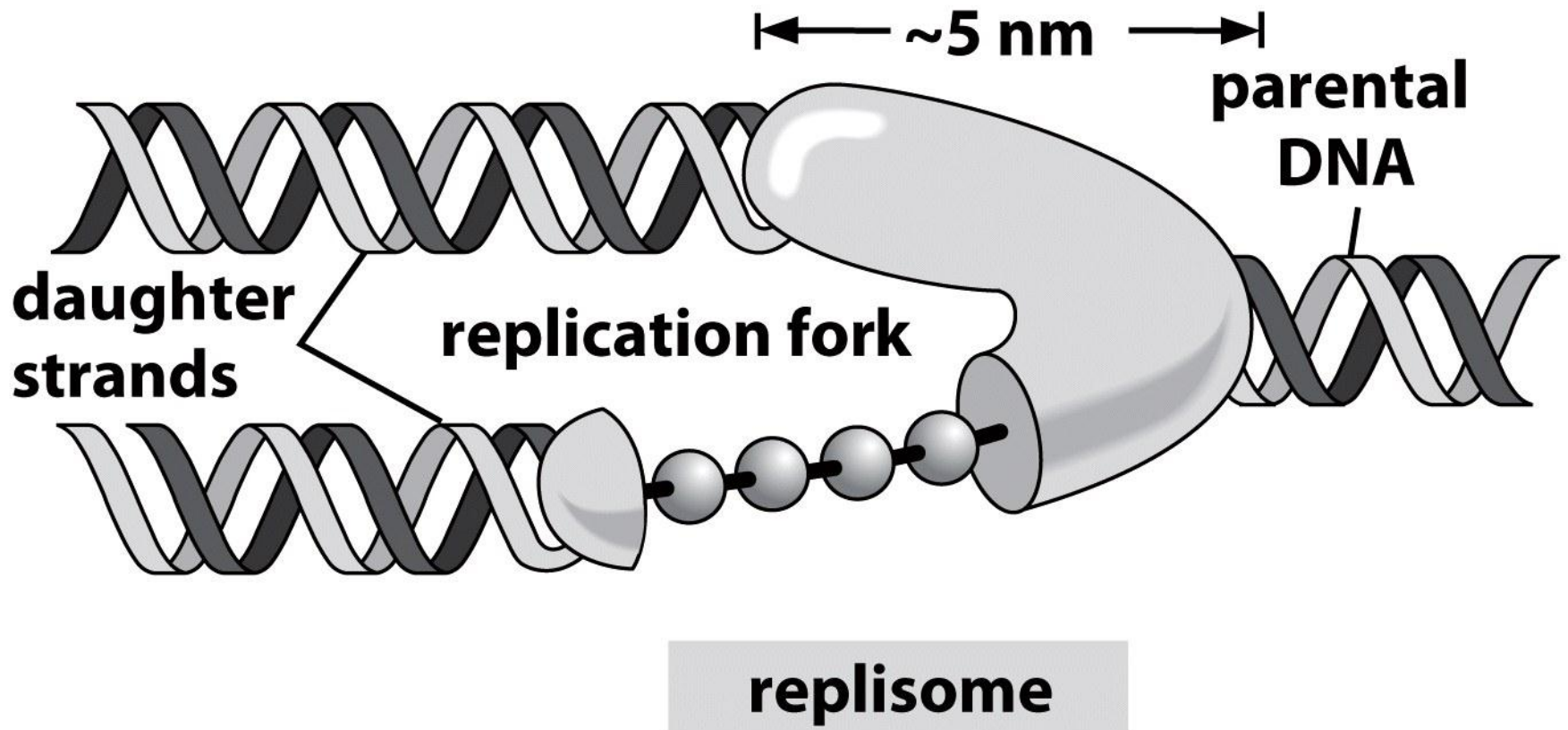


Figure 2.16c Physical Biology of the Cell (© Garland Science 2009)

**growing polypeptide chain**

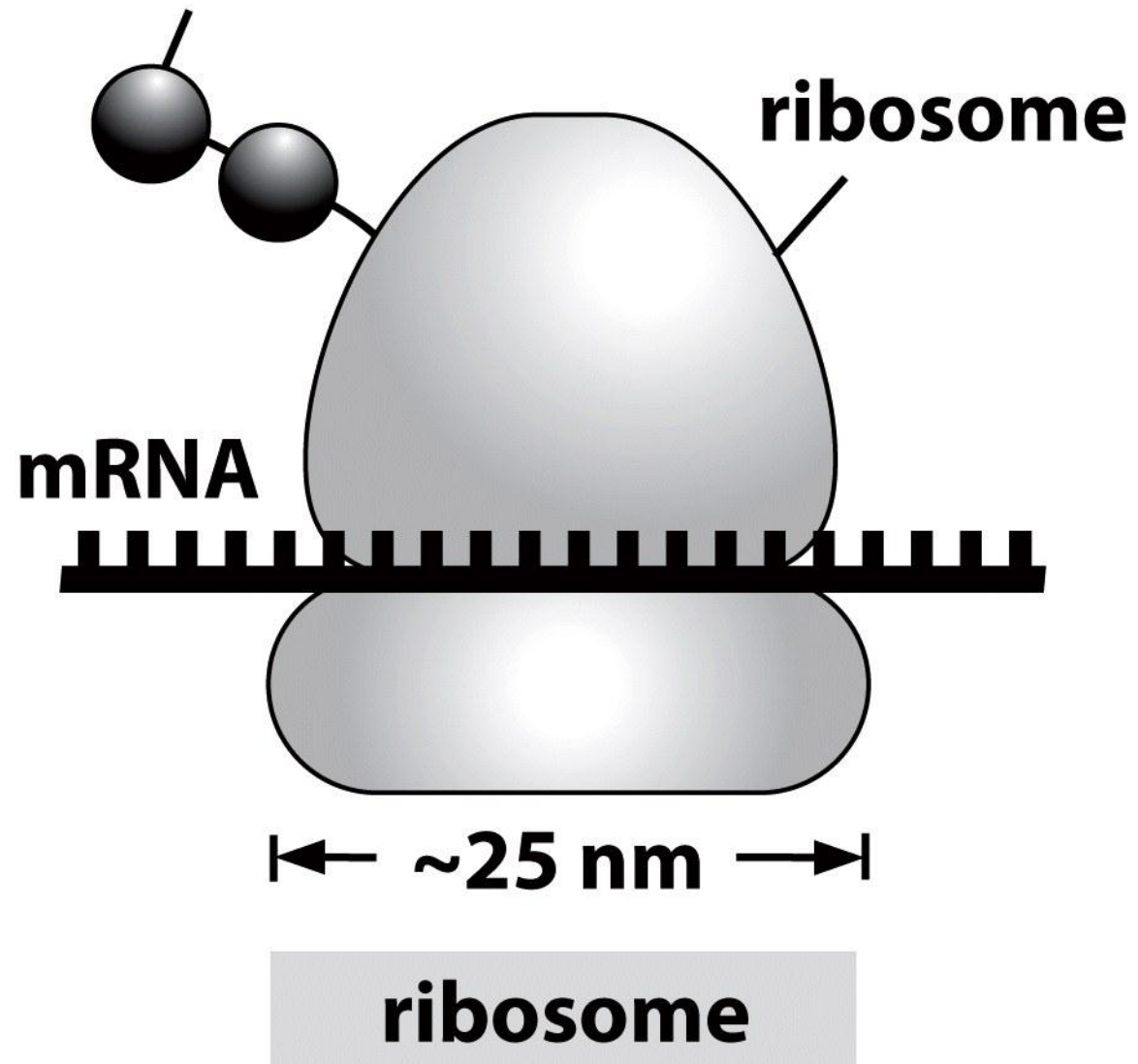
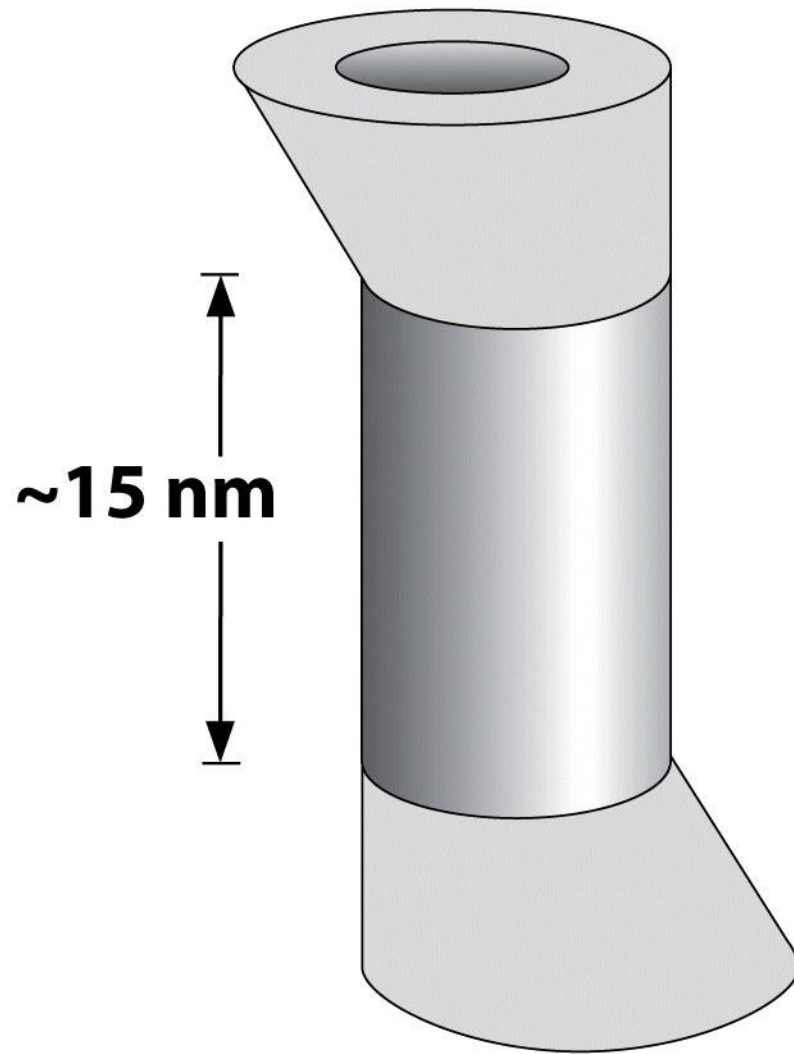


Figure 2.16d Physical Biology of the Cell (© Garland Science 2009)



**proteasome**



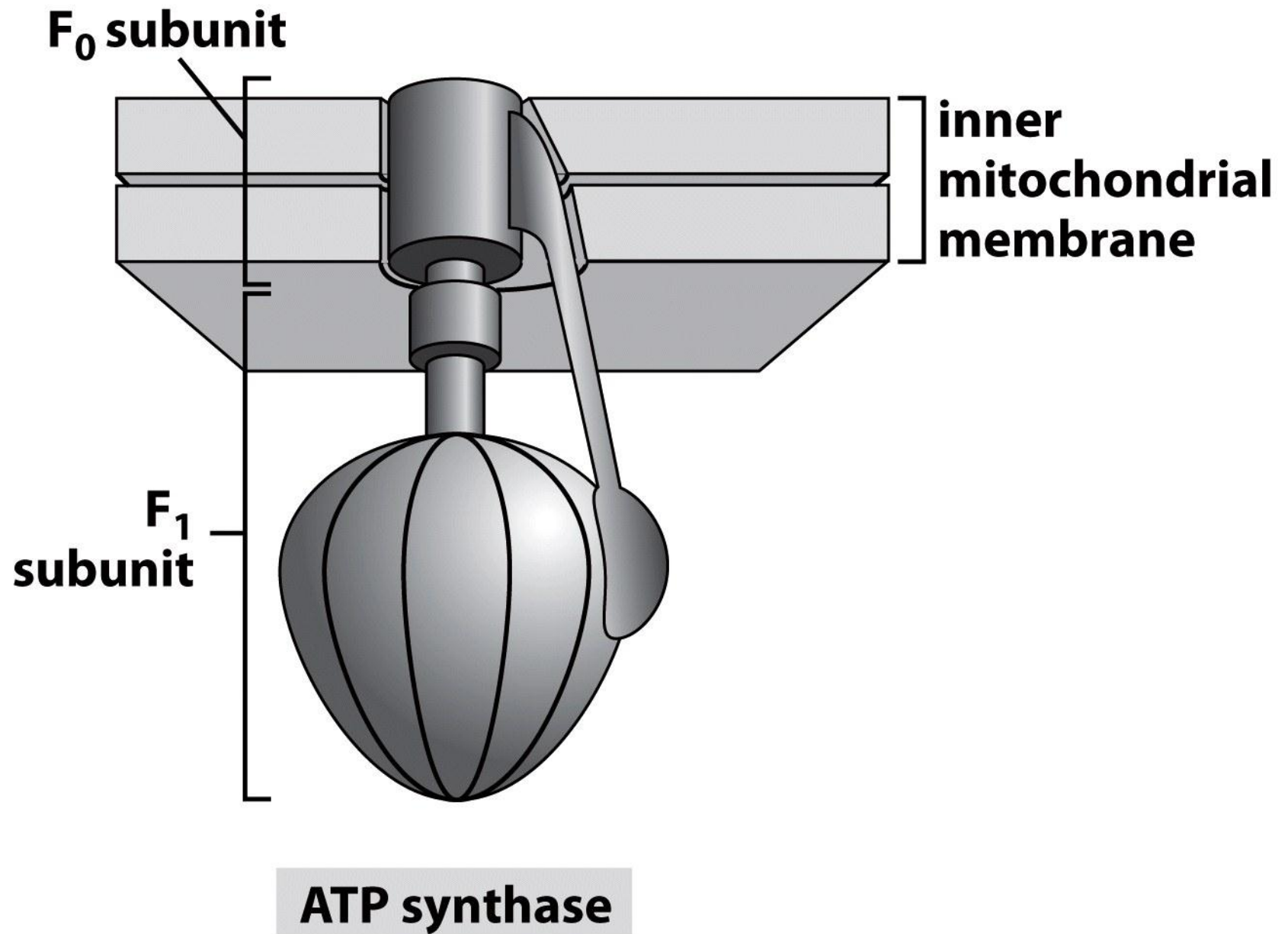
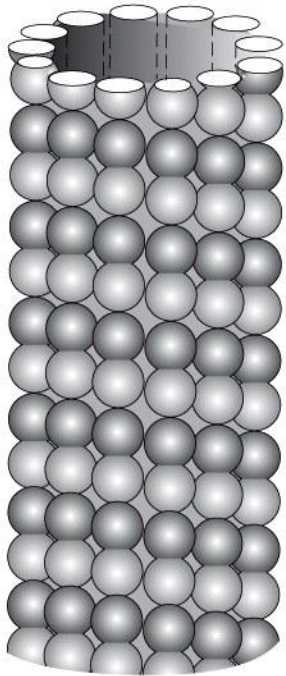


Figure 2.16f Physical Biology of the Cell (© Garland Science 2009)

(A)

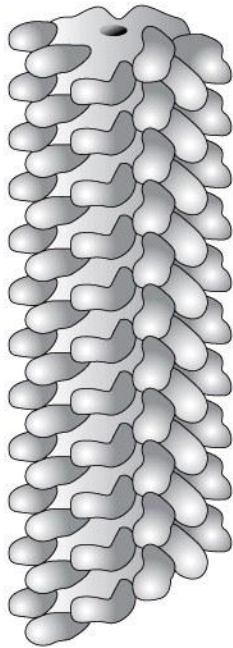
microtubule



25 nm

(B)

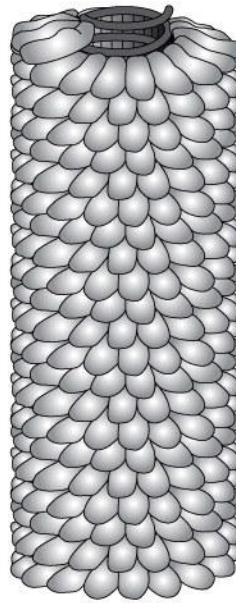
bacterial  
flagellum



20 nm

(C)

tobacco  
mosaic virus



20 nm

(D)

collagen  
fiber



1.5 nm

(E)

DNA



2 nm

Figure 2.17 Physical Biology of the Cell (© Garland Science 2009)

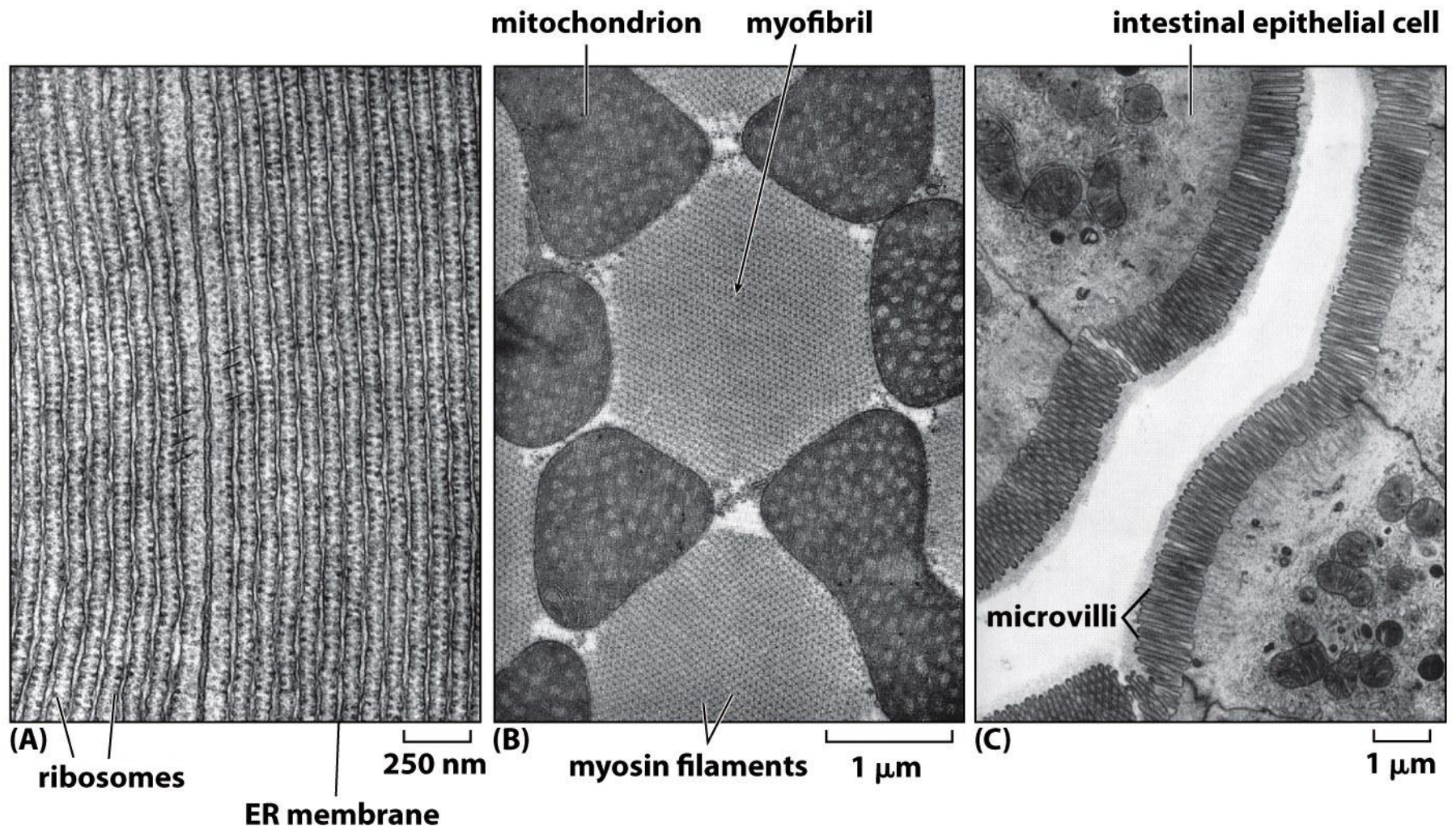


Figure 2.18 Physical Biology of the Cell (© Garland Science 2009)



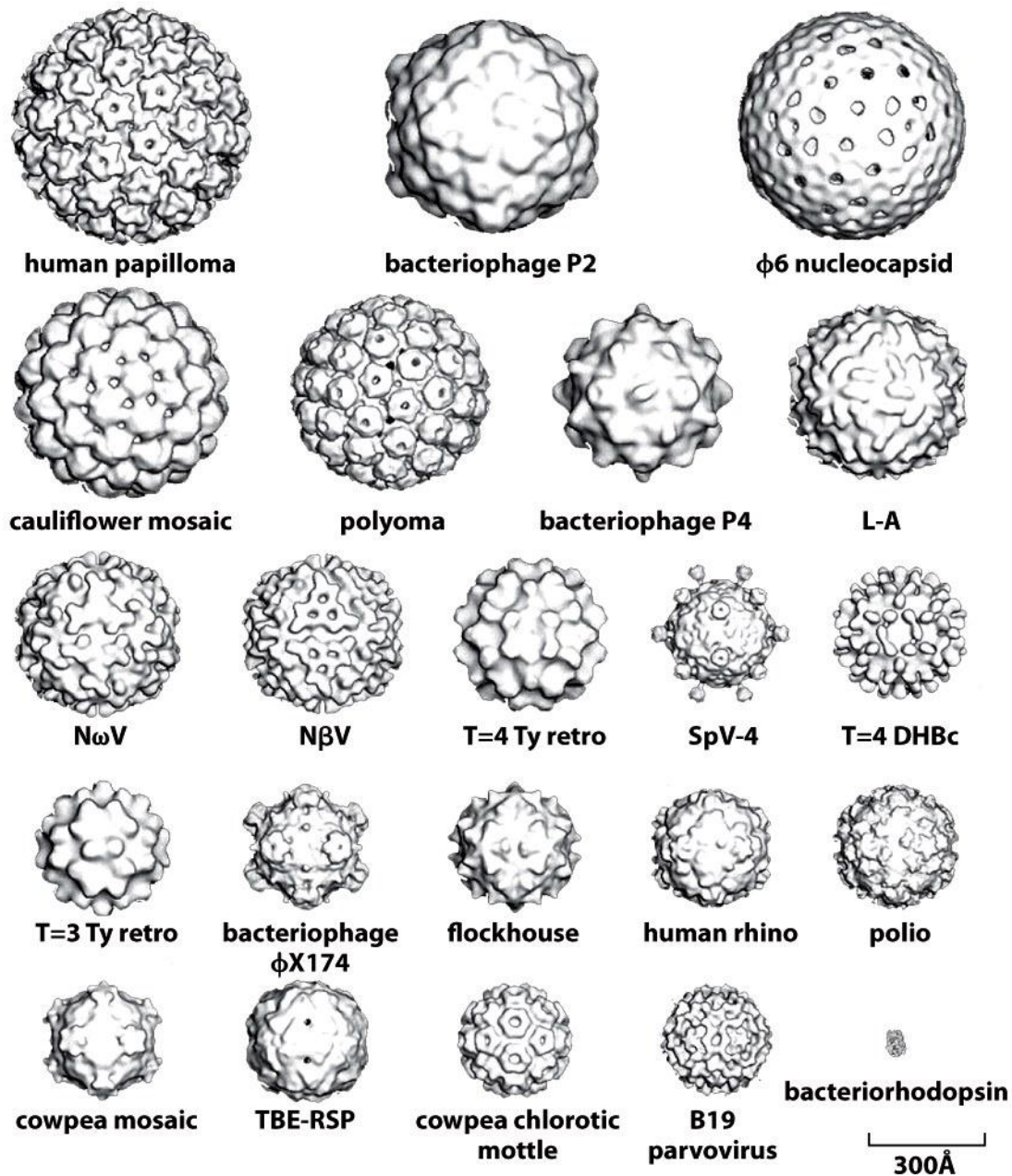
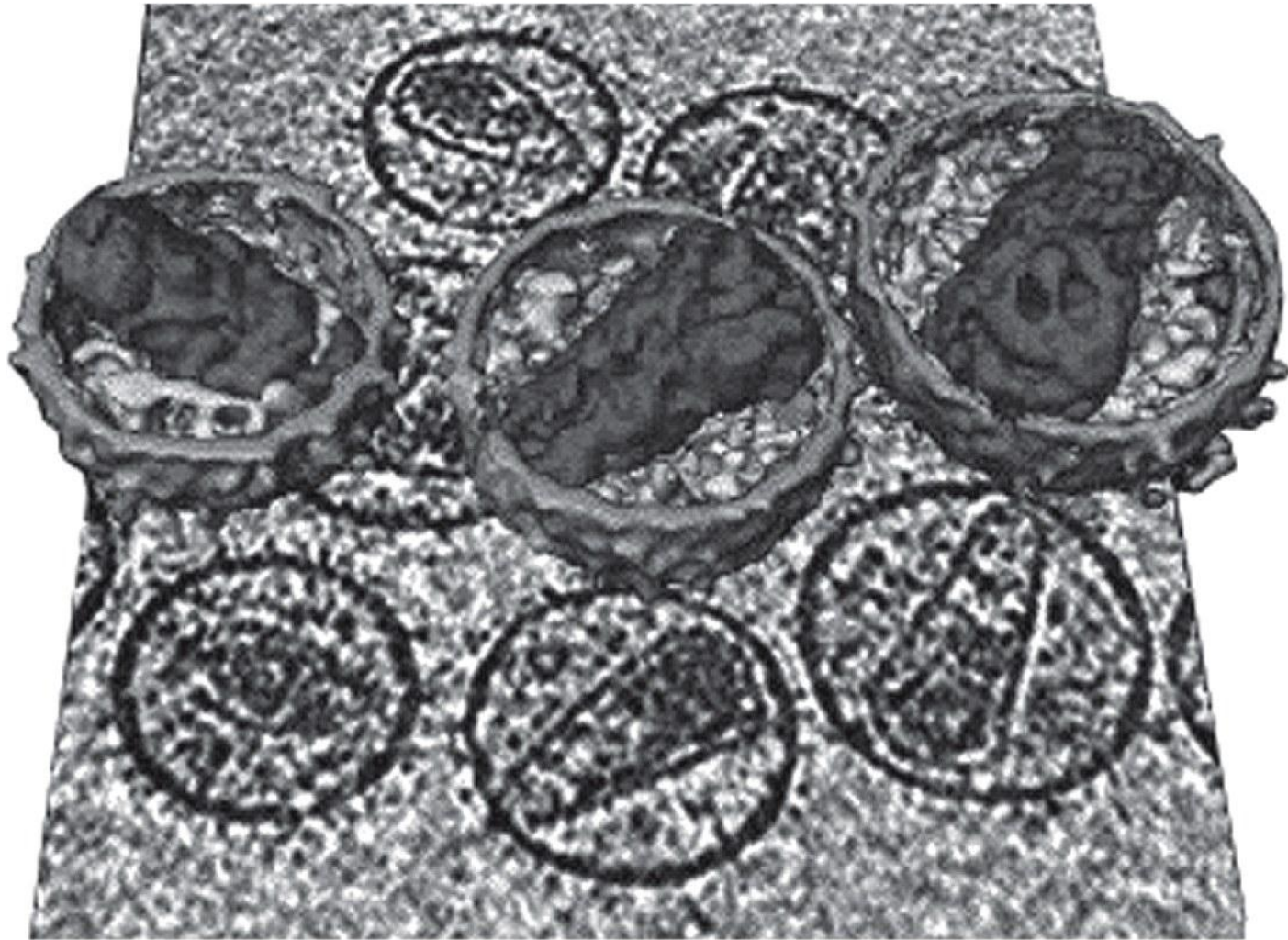


Figure 2.19 Physical Biology of the Cell (© Garland Science 2009)



100 nm

Figure 2.20 Physical Biology of the Cell (© Garland Science 2009)

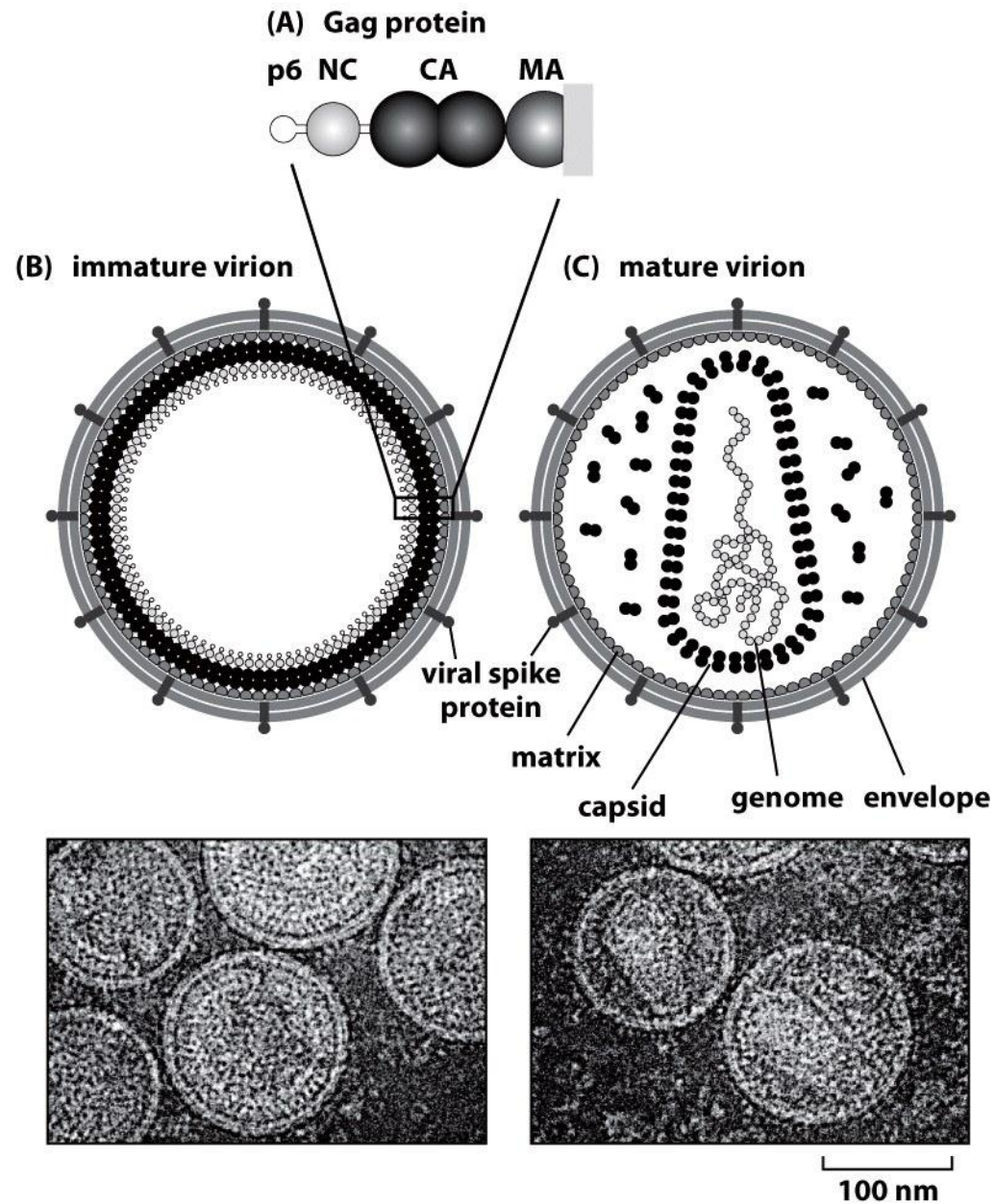
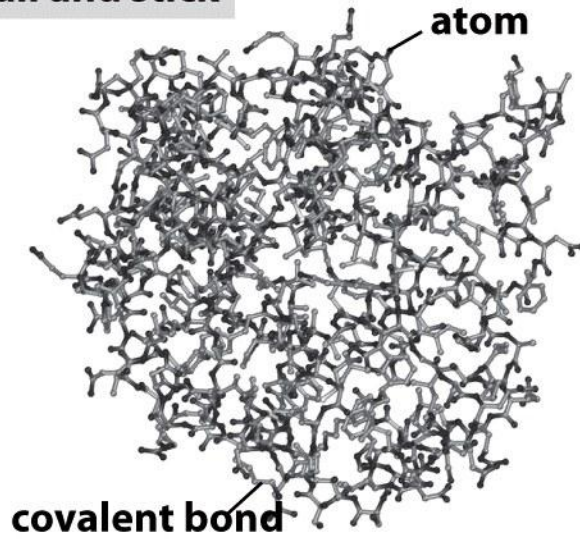


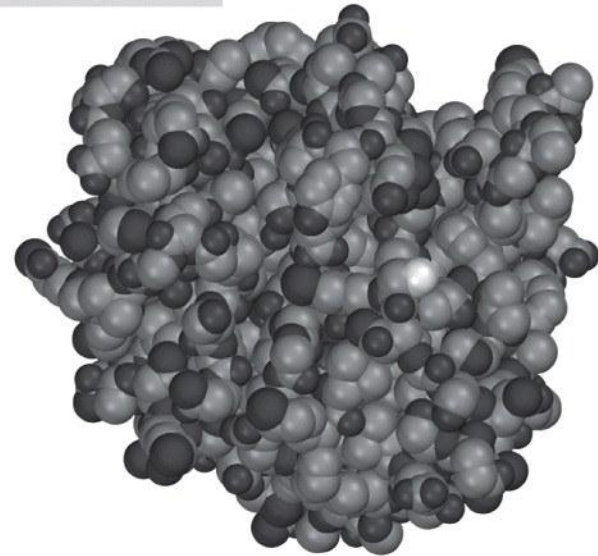
Figure 2.21 Physical Biology of the Cell (© Garland Science 2009)



**ball and stick**



**space-filling**



**ribbon**

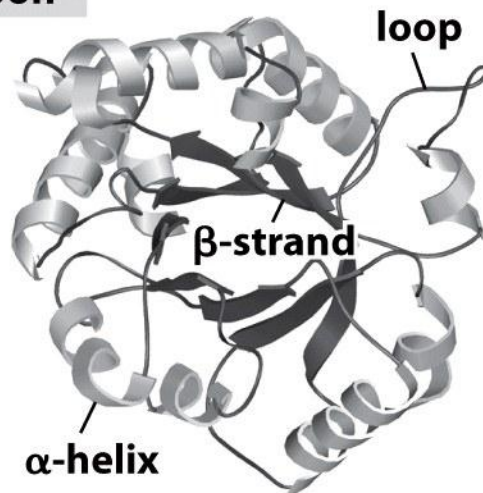
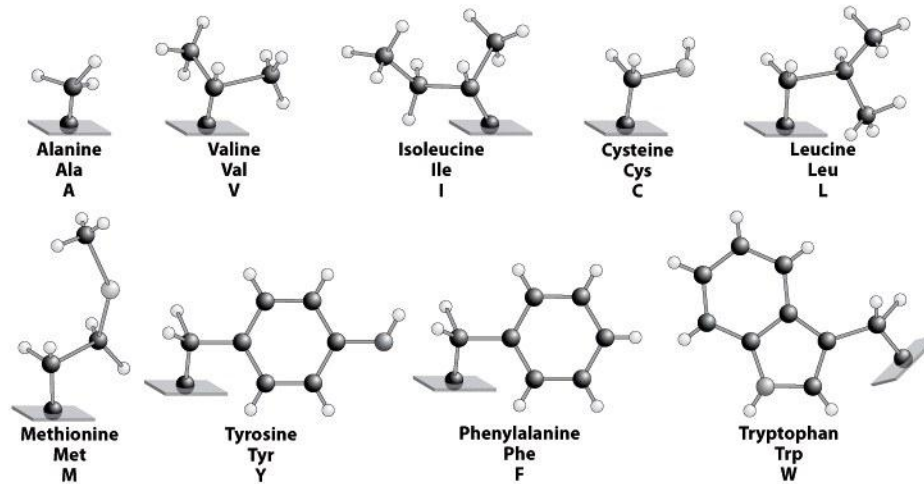
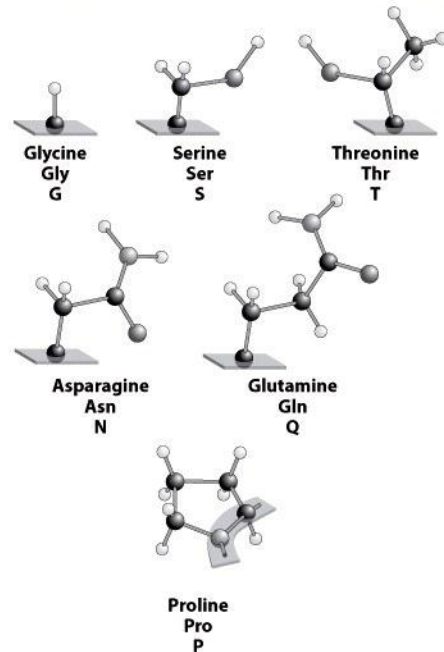


Figure 2.22 Physical Biology of the Cell (© Garland Science 2009)

## HYDROPHOBIC



## POLAR



## CHARGED

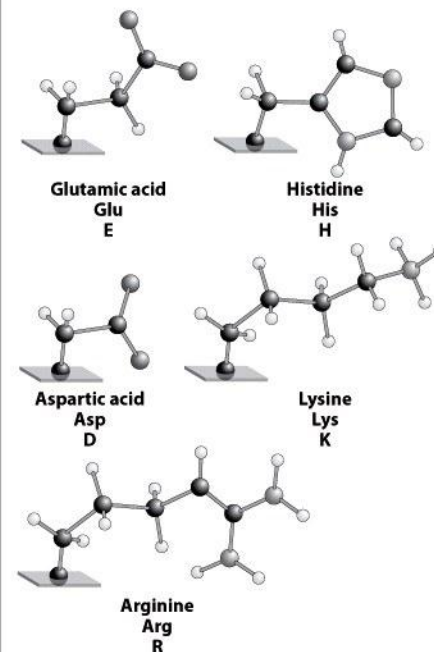


Figure 2.23 Physical Biology of the Cell (© Garland Science 2009)

## HYDROPHOBIC

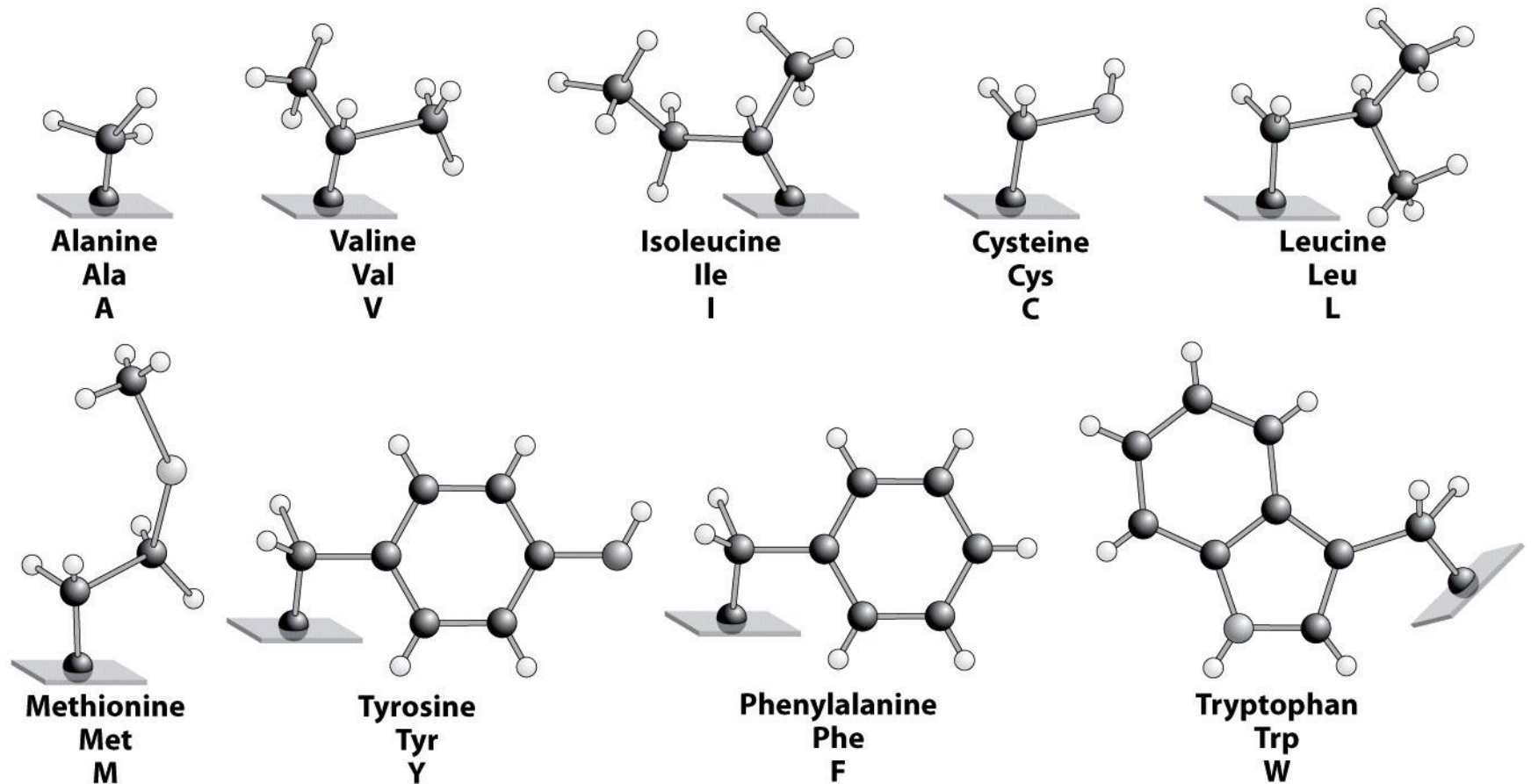


Figure 2.23 (part 1) Physical Biology of the Cell (© Garland Science 2009)

## POLAR

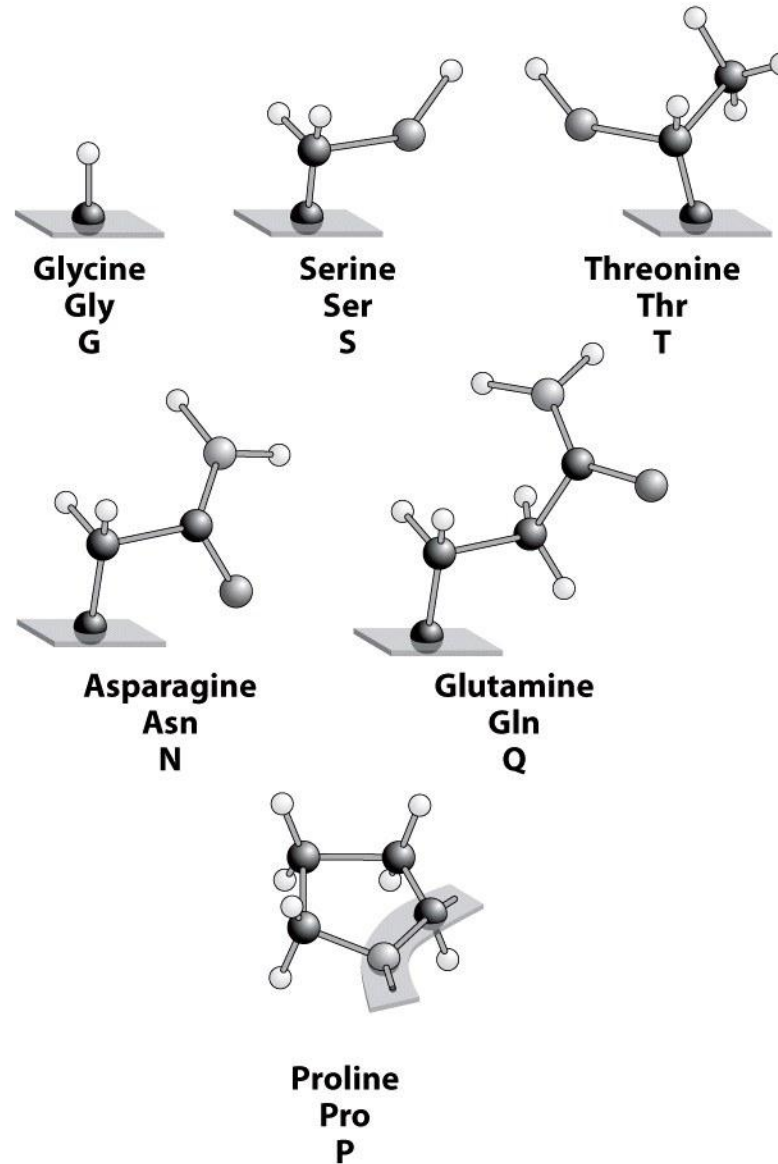
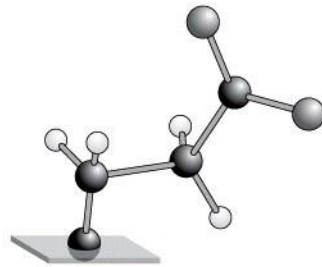
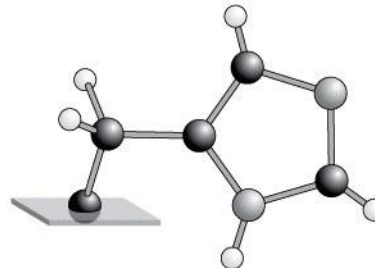


Figure 2.23 (part 2) Physical Biology of the Cell (© Garland Science 2009)

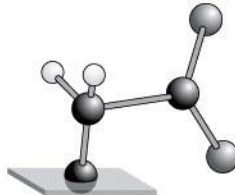
## CHARGED



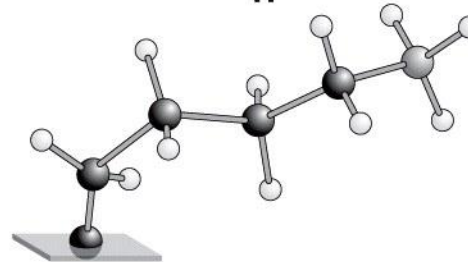
**Glutamic acid**  
**Glu**  
**E**



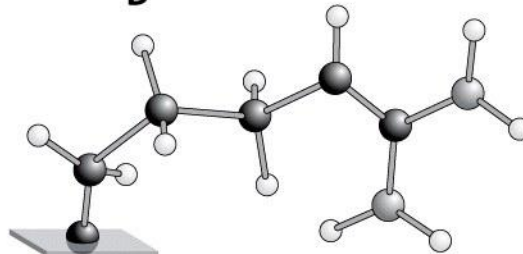
**Histidine**  
**His**  
**H**



**Aspartic acid**  
**Asp**  
**D**

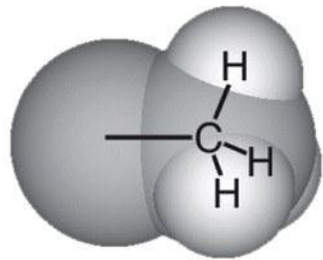


**Lysine**  
**Lys**  
**K**

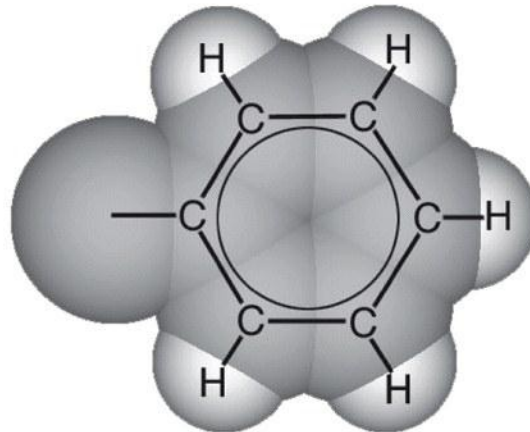


**Arginine**  
**Arg**  
**R**

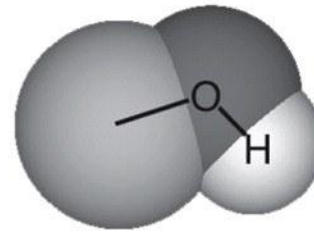
Figure 2.23 (part 3) Physical Biology of the Cell (© Garland Science 2009)



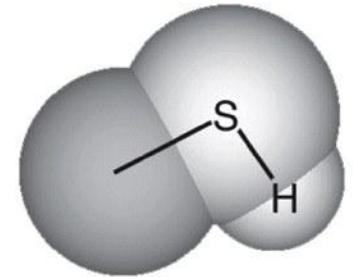
**methyl**



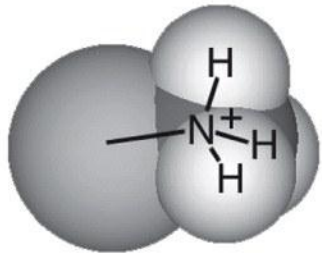
**phenyl**



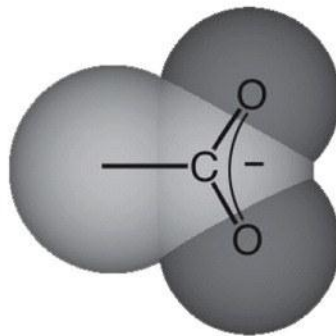
**alcohol (hydroxyl)**



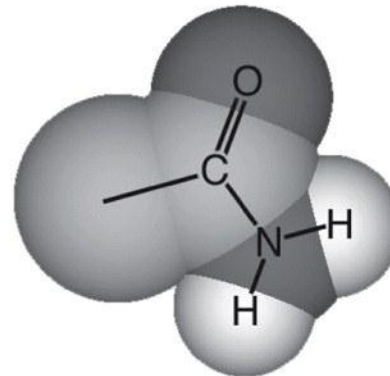
**thiol (sulfhydryl)**



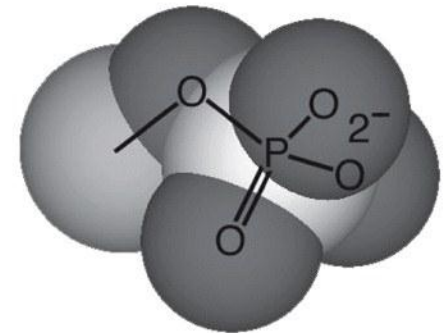
**amino**



**carboxyl**



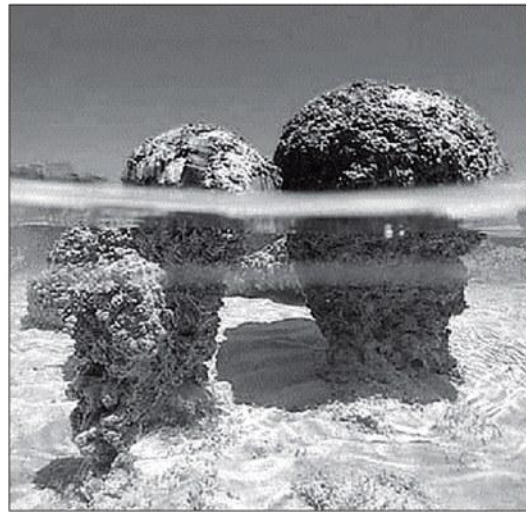
**amide**



**phosphoryl**

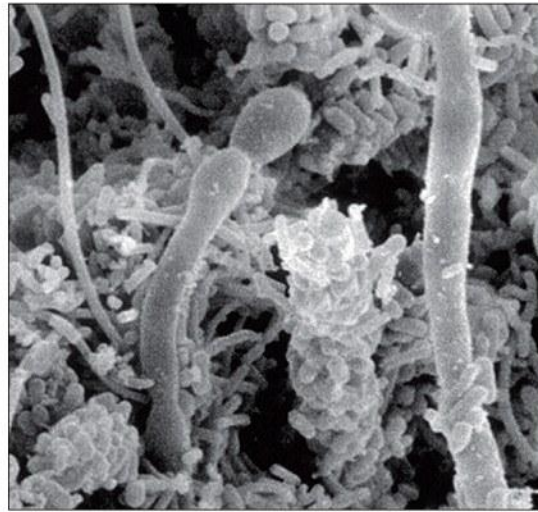
Figure 2.24 Physical Biology of the Cell (© Garland Science 2009)





(A)

0.5 m



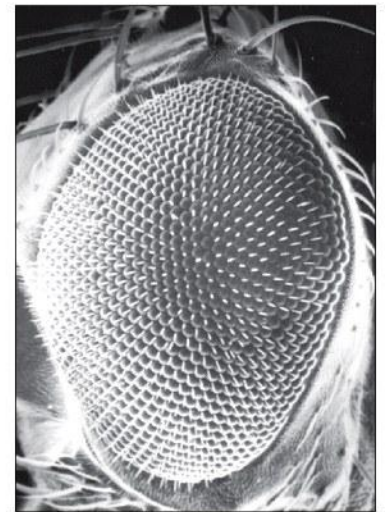
(B)

10  $\mu\text{m}$



(C)

100  $\mu\text{m}$

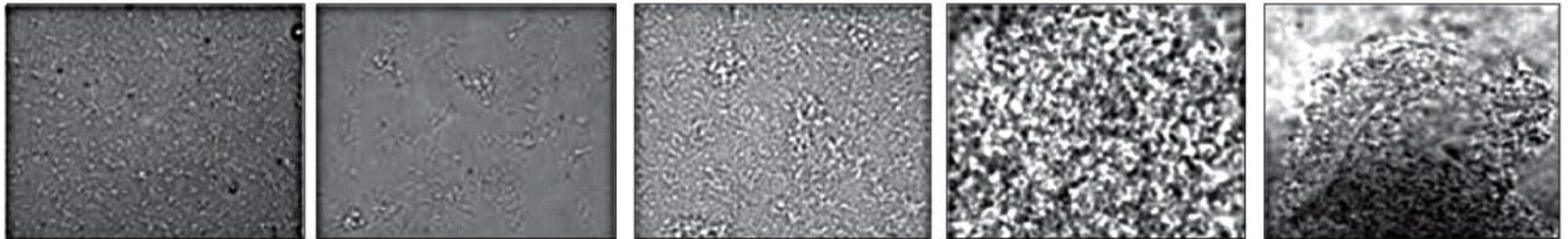
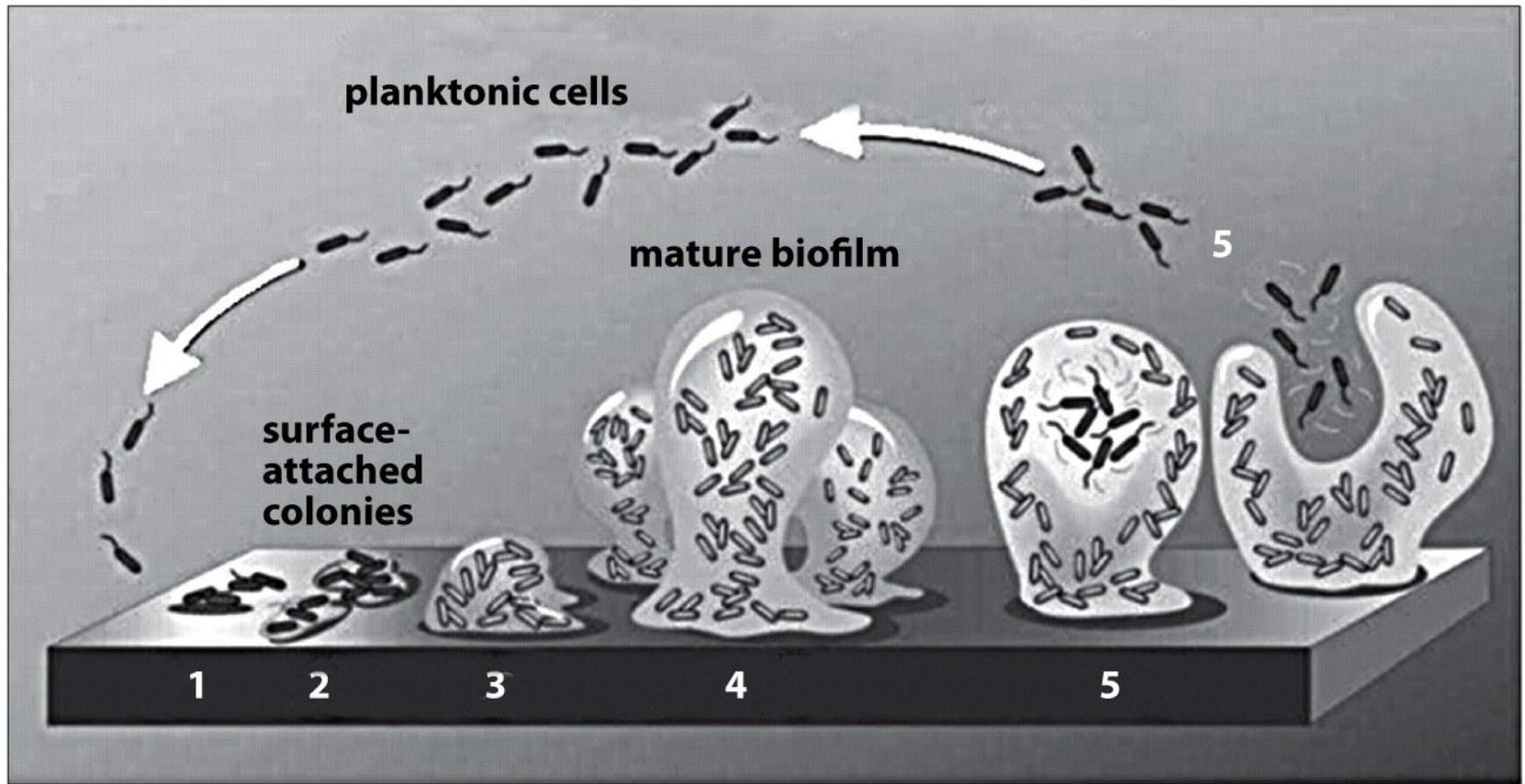


(D)

100  $\mu\text{m}$

Figure 2.25 Physical Biology of the Cell (© Garland Science 2009)

(A)



(B)

20  $\mu\text{m}$

Figure 2.26 Physical Biology of the Cell (© Garland Science 2009)

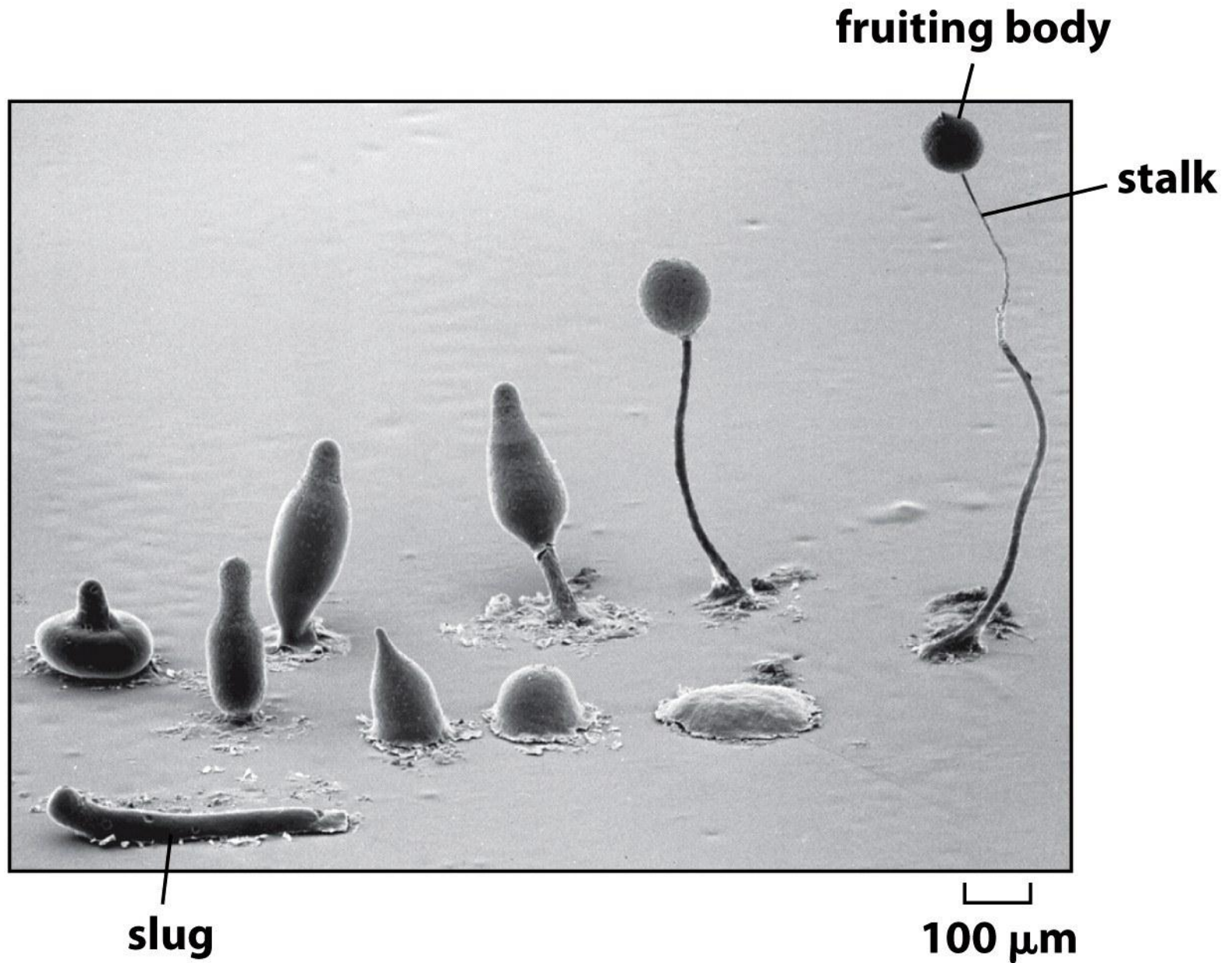


Figure 2.27 Physical Biology of the Cell (© Garland Science 2009)



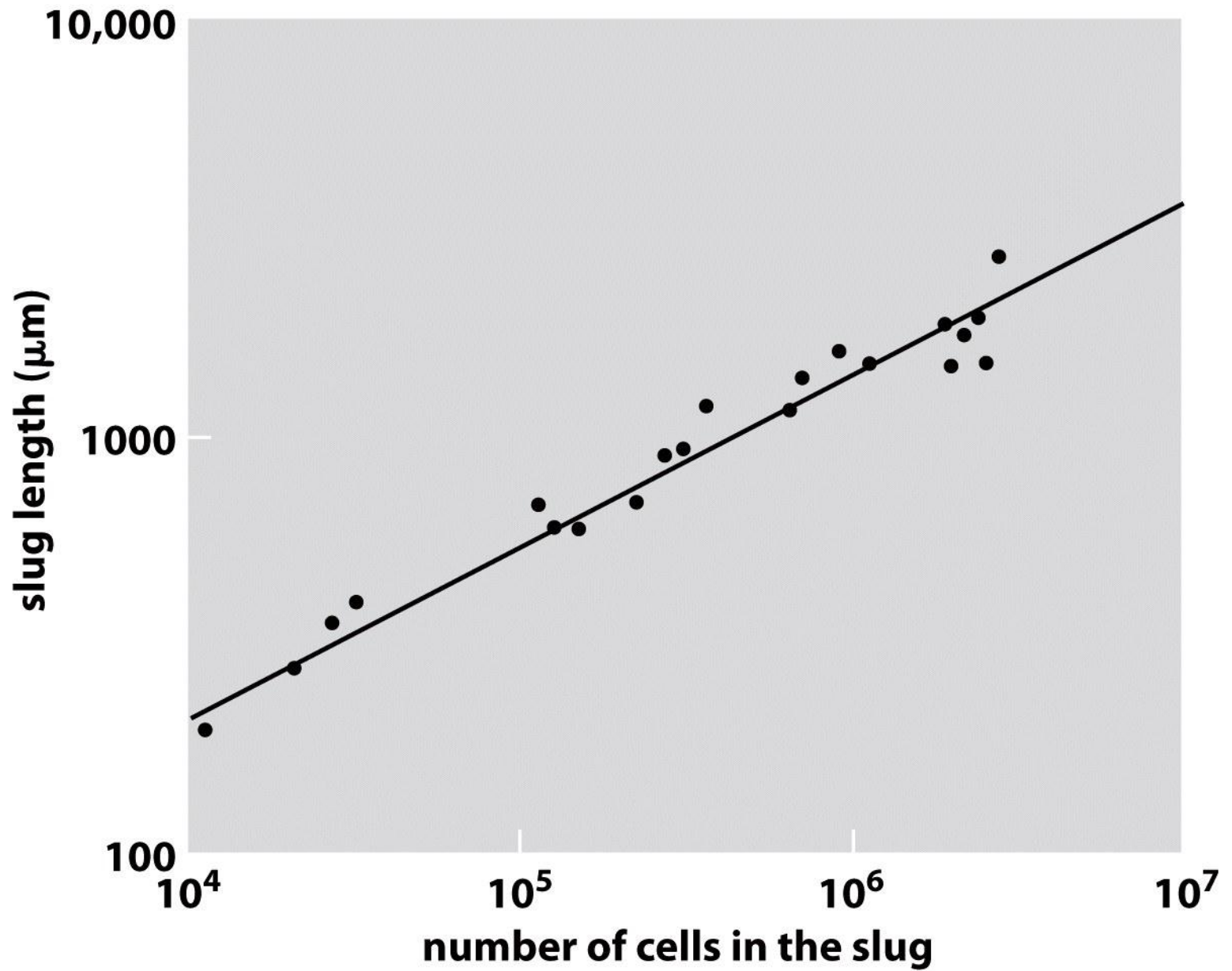
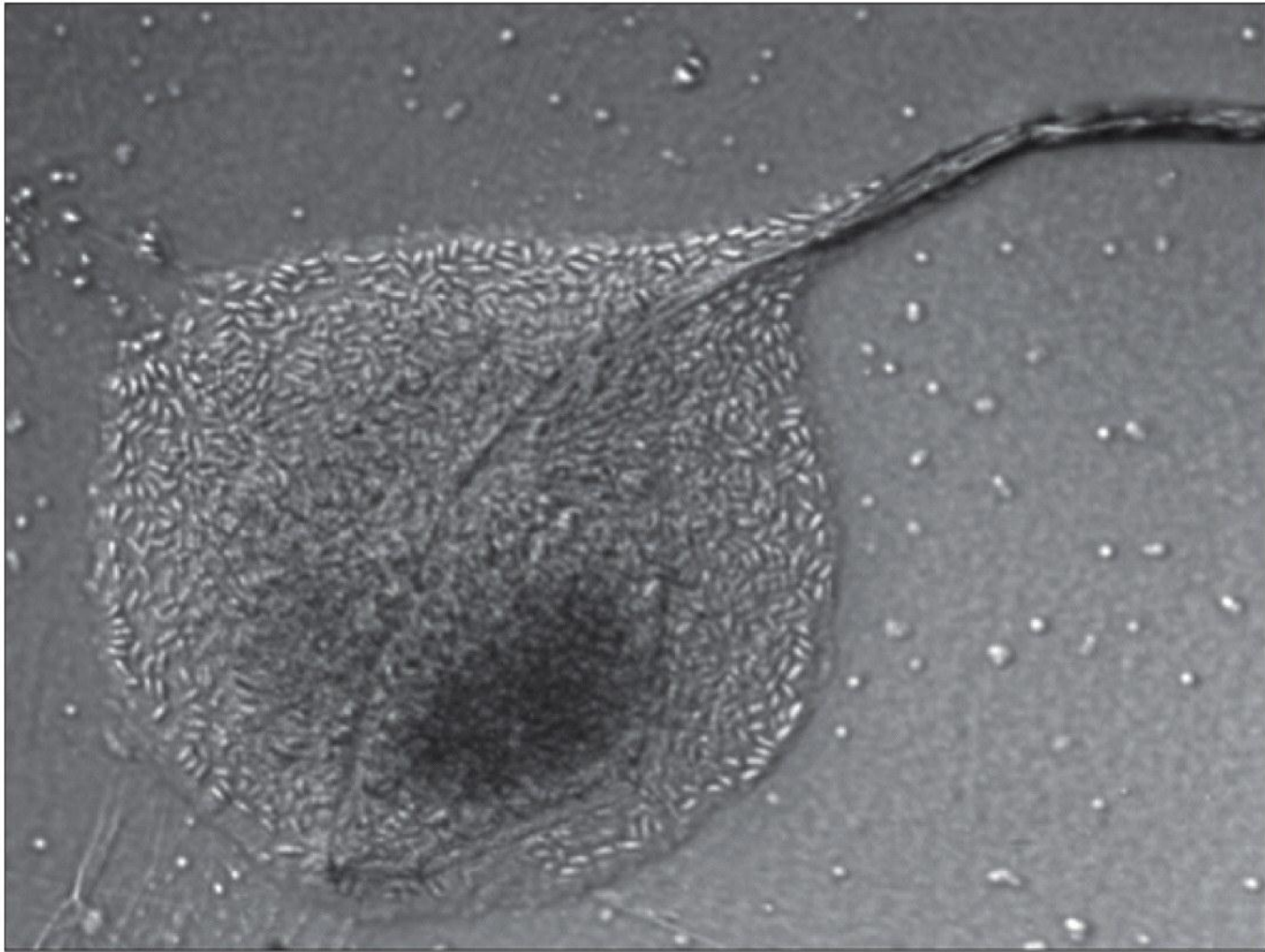


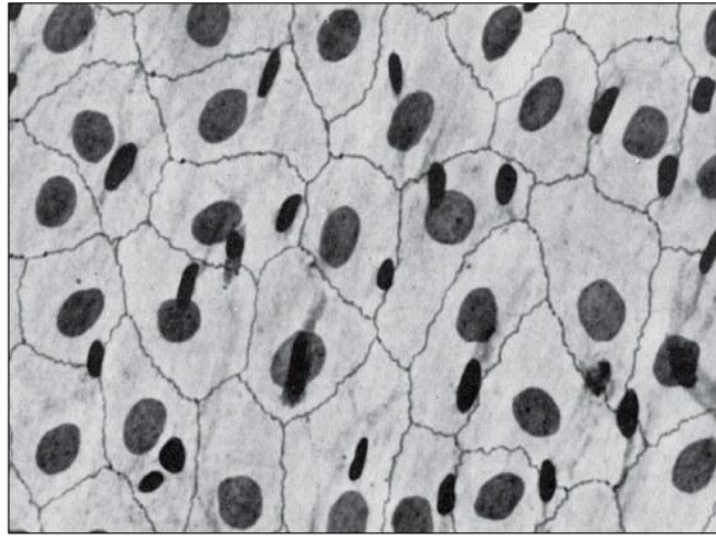
Figure 2.28 Physical Biology of the Cell (© Garland Science 2009)



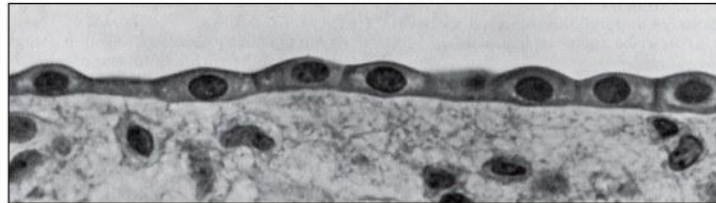
**50 μm**

Figure 2.29 Physical Biology of the Cell (© Garland Science 2009)

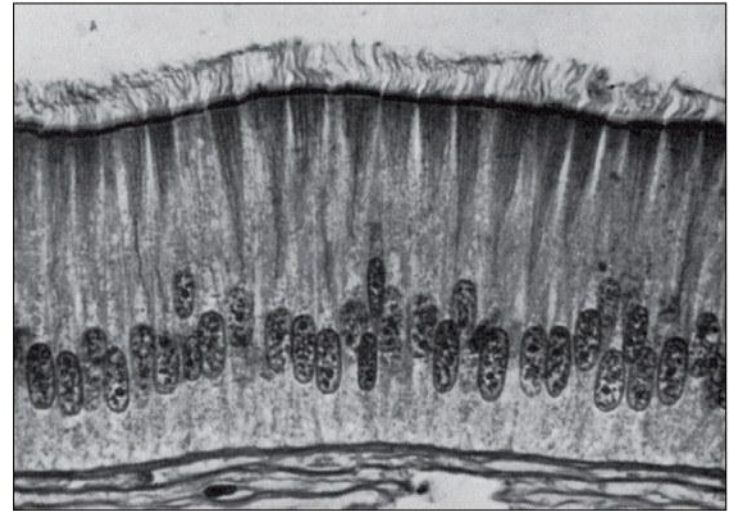




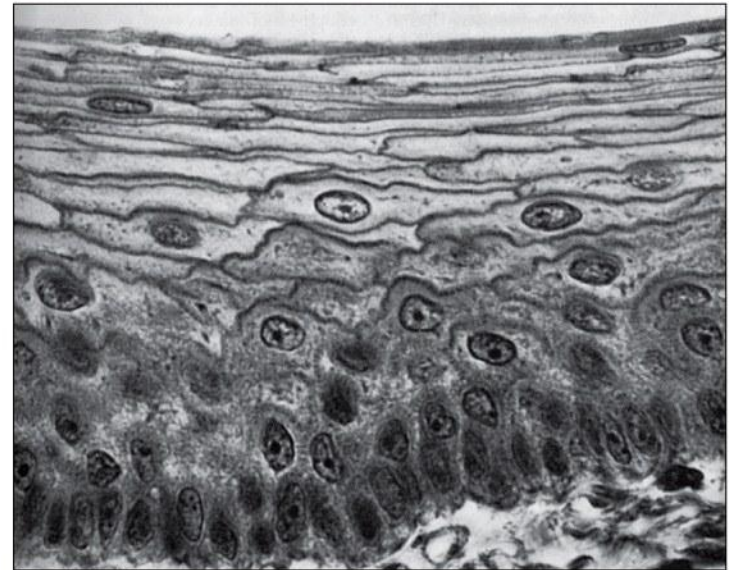
**(A)**



**(B)**



**(C)**



**(D)**

~10  $\mu\text{m}$

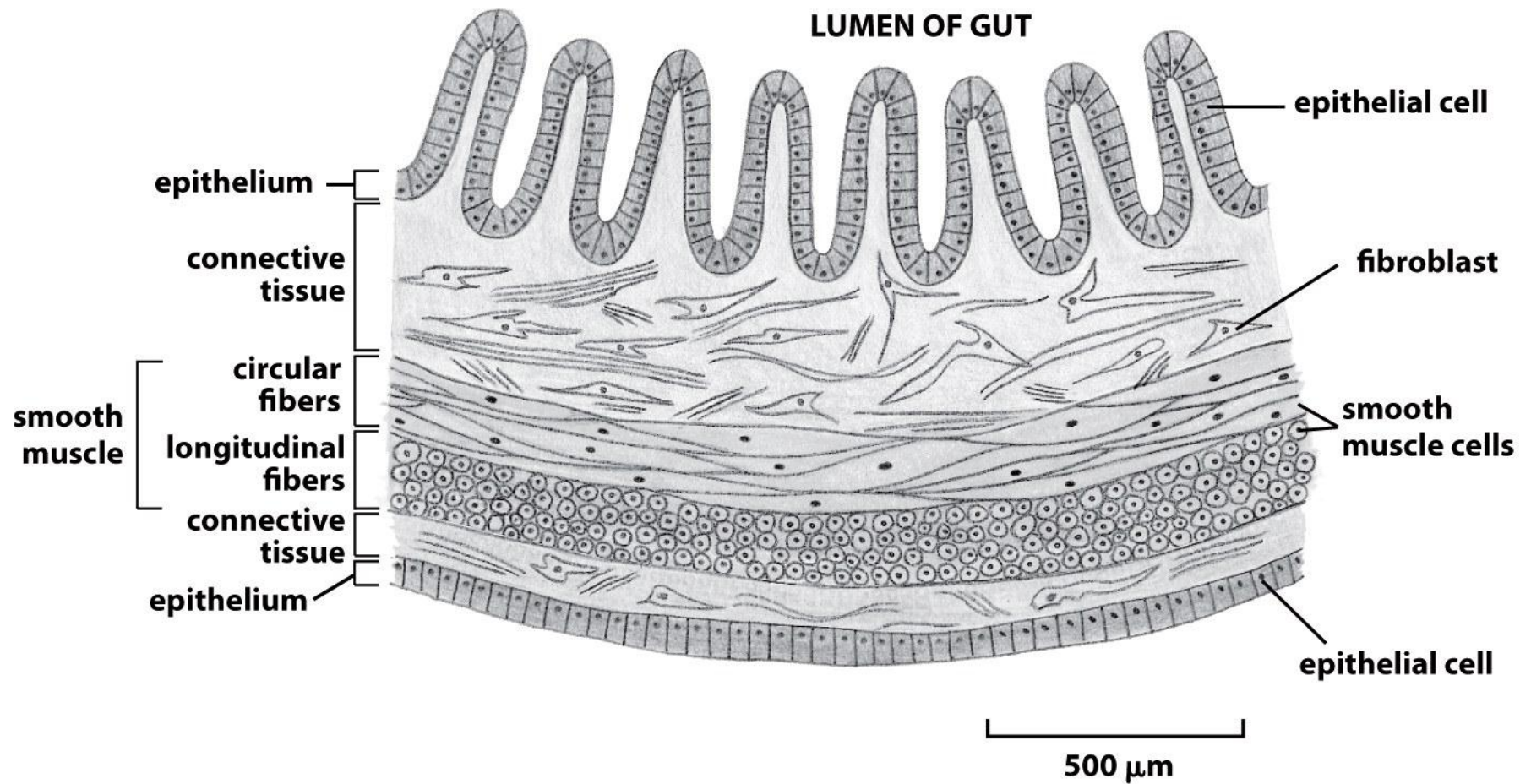
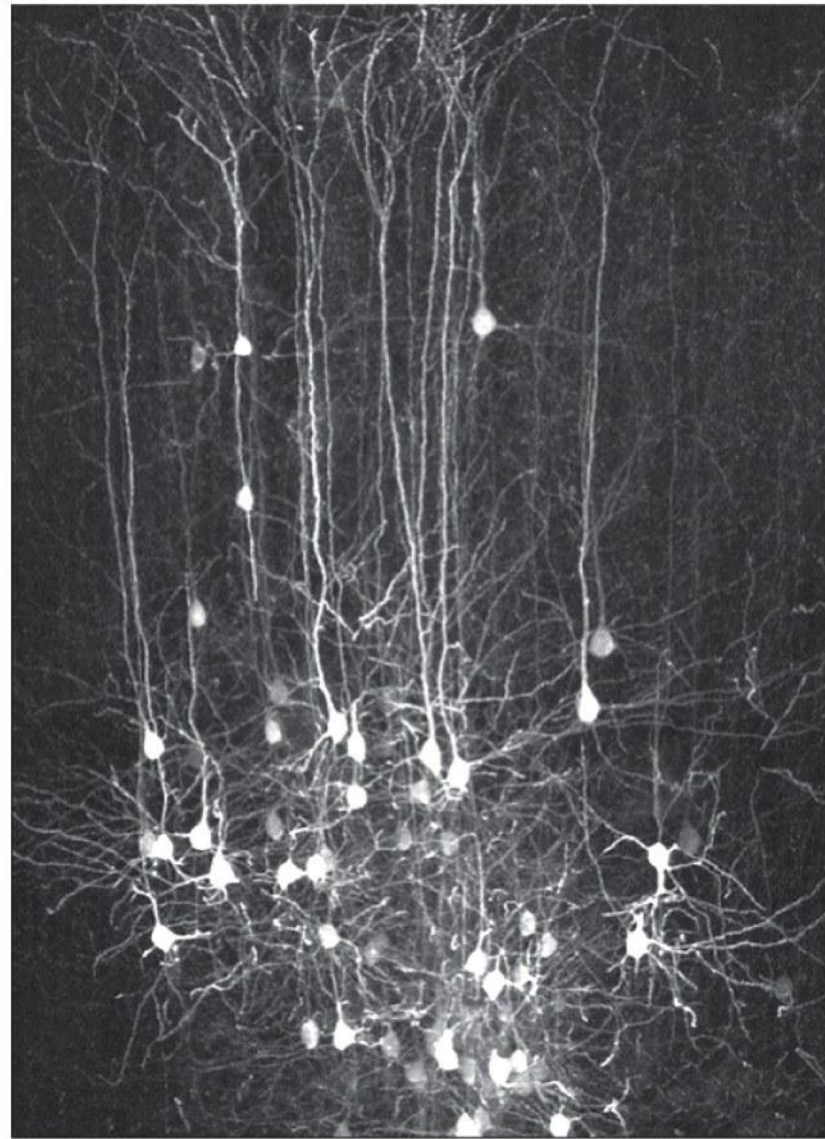


Figure 2.31 Physical Biology of the Cell (© Garland Science 2009)



50  $\mu\text{m}$

Figure 2.32 Physical Biology of the Cell (© Garland Science 2009)



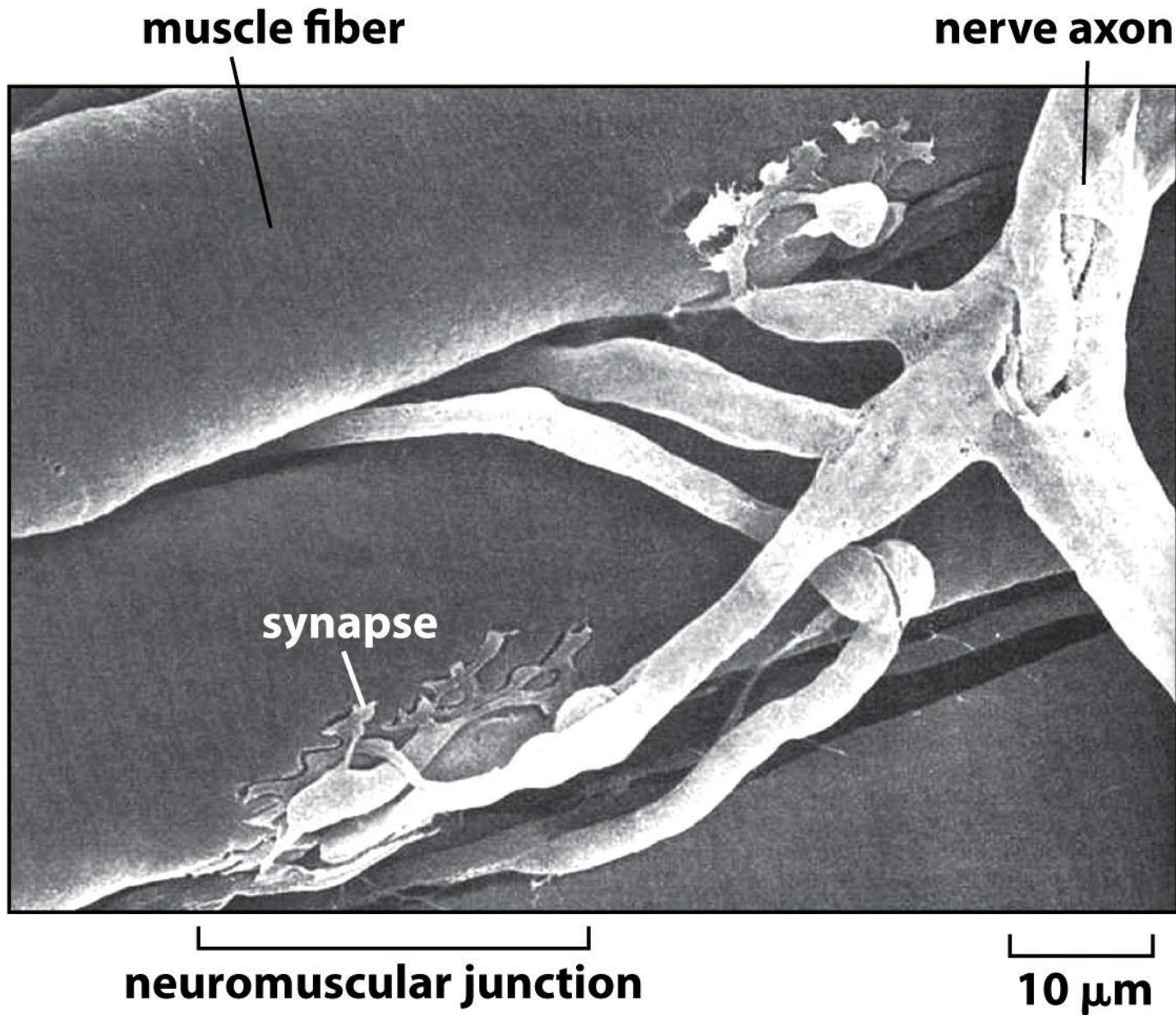
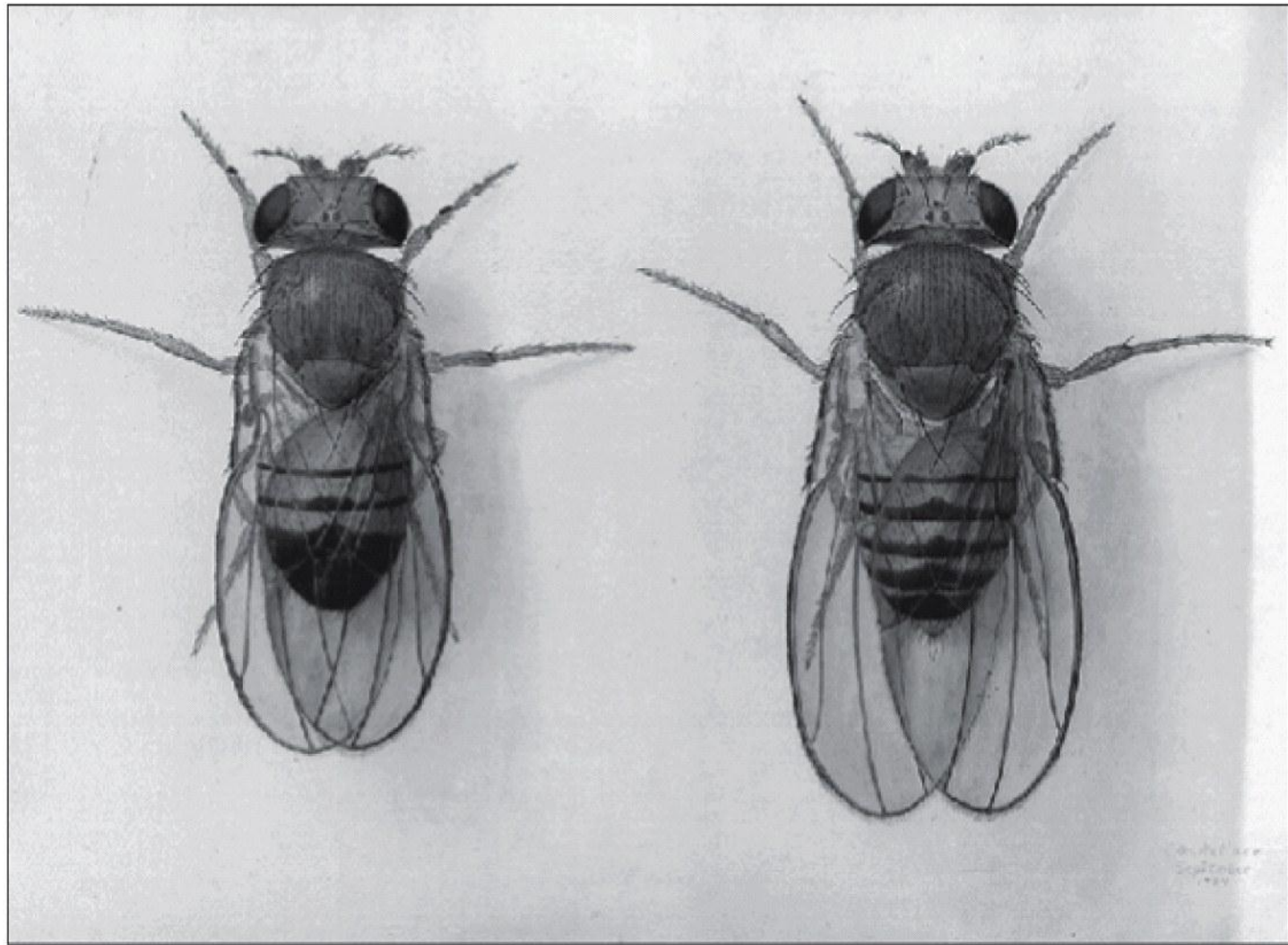


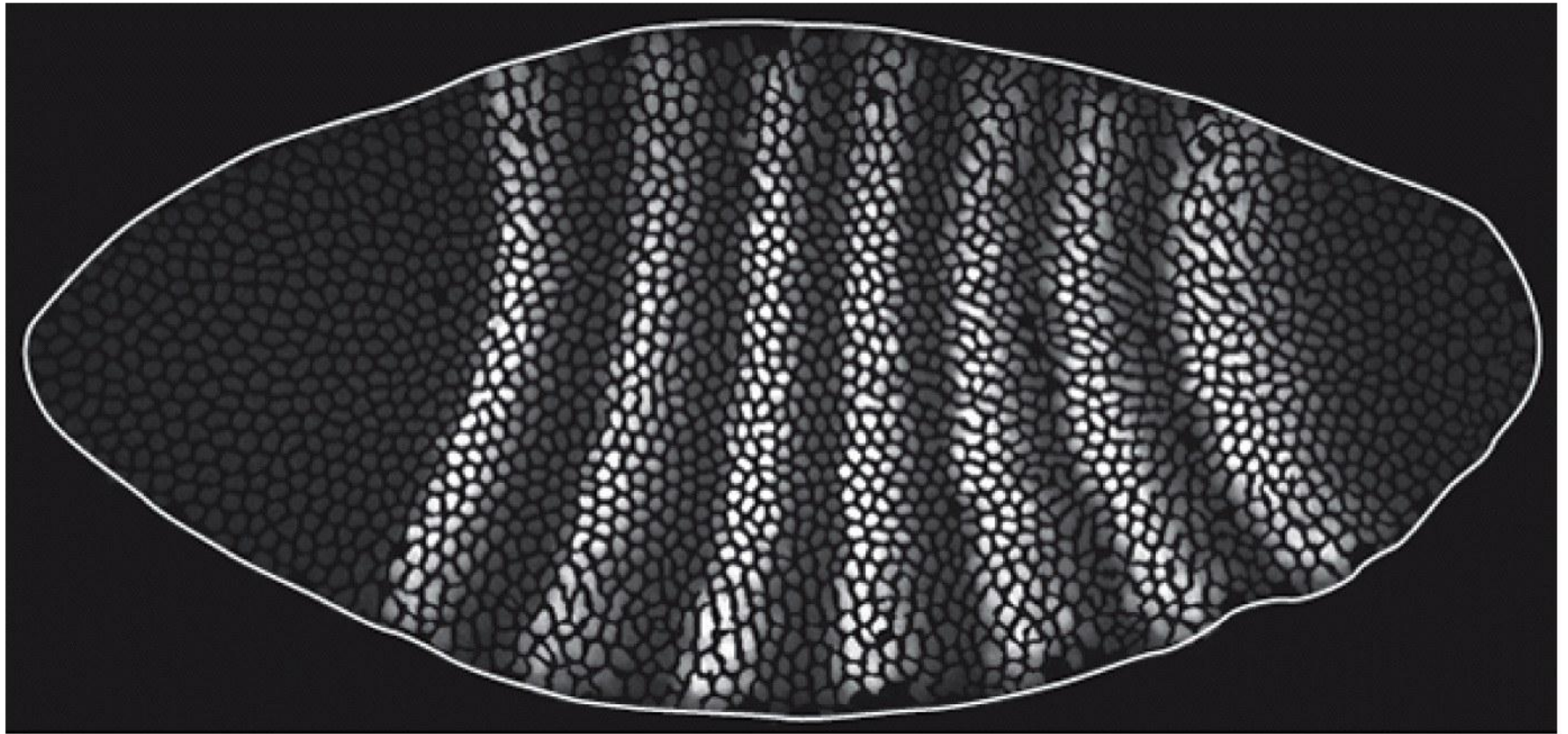
Figure 2.33 Physical Biology of the Cell (© Garland Science 2009)



**500  $\mu\text{m}$**

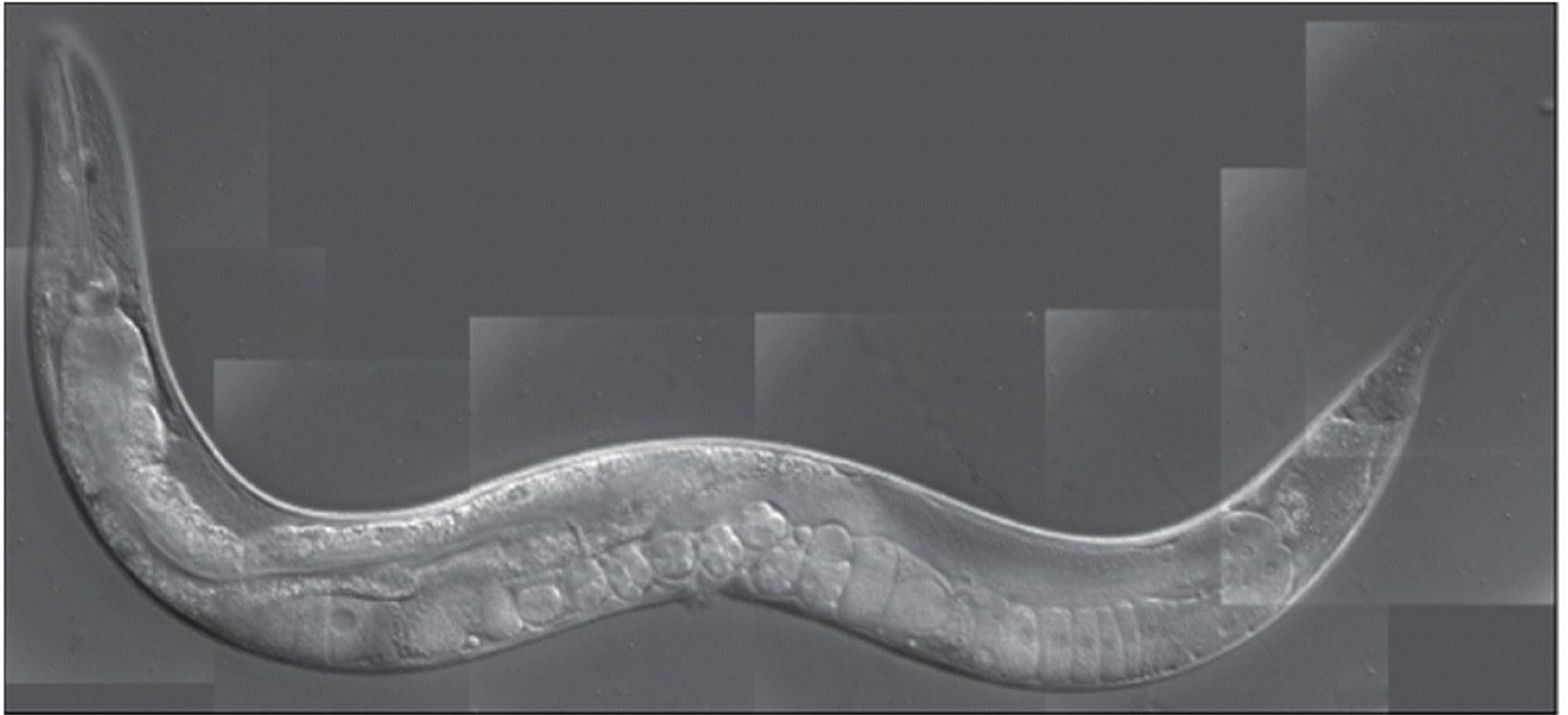
Figure 2.34 Physical Biology of the Cell (© Garland Science 2009)





**100  $\mu\text{m}$**

Figure 2.35 Physical Biology of the Cell (© Garland Science 2009)



100  $\mu\text{m}$

Figure 2.36 Physical Biology of the Cell (© Garland Science 2009)

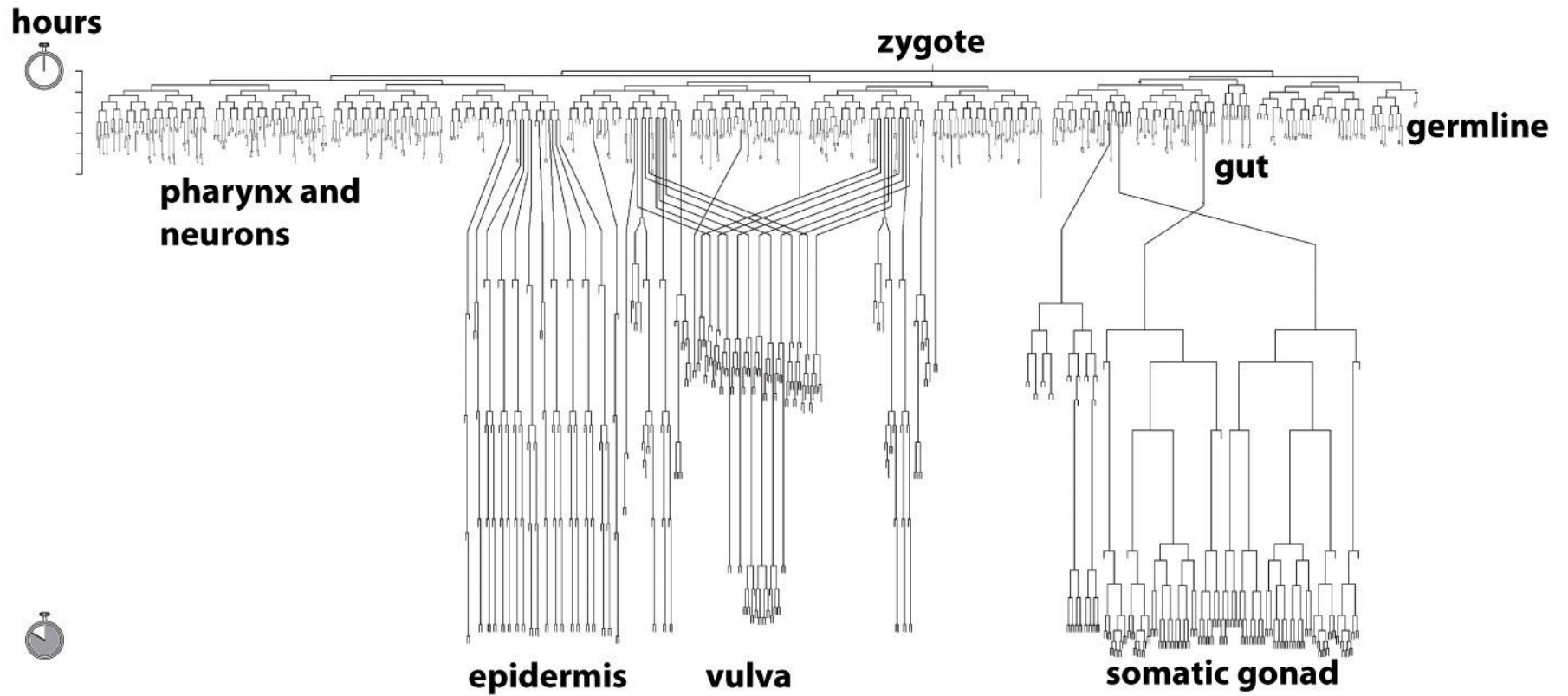


Figure 2.37 Physical Biology of the Cell (© Garland Science 2009)

**Table 1.1** Rules of thumb for biological estimates

	Quantity of interest	Symbol	Rule of thumb
<i>E. coli</i>	Cell volume	$V_{E. coli}$	$\approx 1 \mu\text{m}^3$
	Cell mass	$m_{E. coli}$	$\approx 1 \text{ pg}$
	Cell cycle time	$t_{E. coli}$	$\approx 3000 \text{ s}$
	Cell surface area	$A_{E. coli}$	$\approx 6 \mu\text{m}^2$
	Genome length	$N_{bp}^{E. coli}$	$\approx 5 \times 10^6 \text{ bp}$
	Swimming speed	$v_{E. coli}$	$\approx 20 \mu\text{m/s}$
Yeast	Volume of cell	$V_{yeast}$	$\approx 60 \mu\text{m}^3$
	Mass of cell	$m_{yeast}$	$\approx 60 \text{ pg}$
	Diameter of cell	$d_{yeast}$	$\approx 5 \mu\text{m}$
	Cell cycle time	$t_{yeast}$	$\approx 200 \text{ min}$
	Genome length	$N_{bp}^{yeast}$	$\approx 10^7 \text{ bp}$
Organelles	Diameter of nucleus	$d_{nucleus}$	$\approx 5 \mu\text{m}$
	Length of mitochondrion	$l_{mito}$	$\approx 2 \mu\text{m}$
	Diameter of transport vesicles	$d_{vesicle}$	$\approx 50 \text{ nm}$
Water	Volume of molecule	$V_{H_2O}$	$\approx 10^{-2} \text{ nm}^3$
	Density of water	$\rho$	$1 \text{ g/cm}^3$
	Viscosity of water	$\eta$	$\approx 1 \text{ centipoise}$ $(10^{-2} \text{ g/(cm s)})$
DNA	Hydrophobic embedding energy	$\approx E_{hydr}$	$25 \text{ cal/(mol } \text{\AA}^2)$
	Length per base pair	$l_{bp}$	$\approx 1/3 \text{ nm}$
	Volume per base pair	$V_{bp}$	$\approx 1 \text{ nm}^3$
	Charge density	$\lambda_{DNA}$	$2 \text{ e}/0.34 \text{ nm}$
	Persistence length	$\xi_P$	$50 \text{ nm}$
Amino acids and proteins	Radius of “average” protein	$r_{protein}$	$\approx 2 \text{ nm}$
	Volume of “average” protein	$V_{protein}$	$\approx 25 \text{ nm}^3$
	Mass of “average” amino acid	$M_{aa}$	$\approx 100 \text{ Da}$
	Mass of “average” protein	$M_{protein}$	$\approx 30,000 \text{ Da}$
	Protein concentration in cytoplasm	$c_{protein}$	$\approx 300 \text{ mg/mL}$
	Characteristic force of protein motor	$F_{motor}$	$\approx 5 \text{ pN}$
	Characteristic speed of protein motor	$v_{motor}$	$\approx 200 \text{ nm/s}$
	Diffusion constant of “average” protein	$D_{protein}$	$\approx 100 \mu\text{m}^2/\text{s}$
Lipid bilayers	Thickness of lipid bilayer	$d$	$\approx 5 \text{ nm}$
	Area per molecule	$A_{lipid}$	$\approx \frac{1}{2} \text{ nm}^2$
	Mass of lipid molecule	$m_{lipid}$	$\approx 800 \text{ Da}$

**Table 1.1** Physical Biology of the Cell (© Garland Science 2009)