For Boulder Summer School July, 2010

Strongly-correlated quantum many-body physics with ultracold <u>neutral atoms</u>.

(people also cool <u>molecules</u>, <u>ions</u>)

Realized so far:

- Paired-fermion superfluids (I'll focus here, mostly)
- Mott insulators
- Luttinger liquids, (1D)

with <u>some</u> aspects of <u>control</u> not available in condensed matter systems.

Atoms: Bosons or Fermions (fermionic atoms can "imitate" electrons)

mass range $^6Li \leftrightarrow {}^{133}Cs$ (or more, but alkalis are easiest to work with)

internal "spin" states are hyperfine states of nuclear spins + electrons in incomplete shells.

densities $10^{11}-10^{14}\ \frac{\text{atoms}}{\text{cm}^3}$ <u>dilute</u> gas by usual standards

interatom spacing $\sim \mu m$

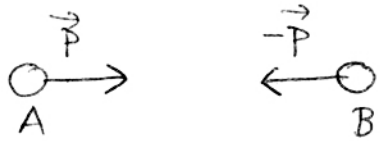
but cooled to quantum degeneracy $10^{-6}-10^{-9}\ {}^\circ K$,

thus not dilute on their own terms.

Thermal de Broglie wavelength $\gtrsim$ interatomic spacing $\cong 1\ \mu m$

Gas is <u>metastable</u> against forming ~~diatomic~~ molecular bound states:

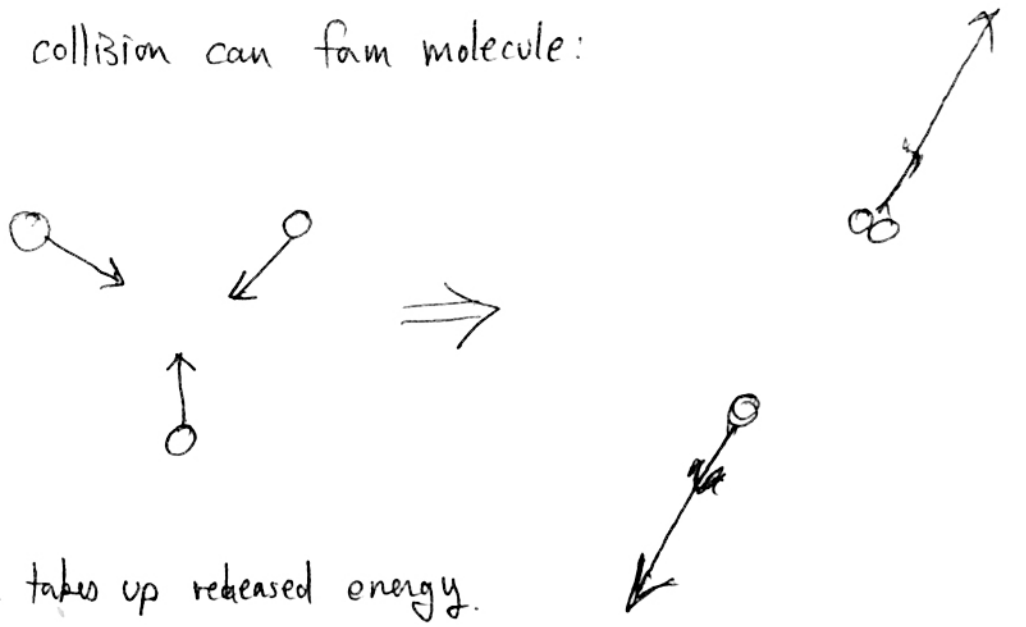
2 atom collision:



$E > 0$

Molecule formation is forbidden by conservation of $E + \vec{p}$

But 3-atom collision can form molecule:



and 3rd atom takes up released energy.

This limits the density of the gas.

Experiments last up to 10's of seconds, but often less than 1 second.

2-atom scattering time $\tau \gtrsim 10^{-6}$ seconds is ~~fastest~~ shortest "microscopic" time.

Makes far-from-equilibrium behavior quite accessible, and sometimes makes equilibrium difficult to access.

Good control of optical potential: (single-atom "external" potential)

$$V_\alpha(\vec{r}) = -\sum_{\omega} P_\alpha(\omega)\, I(\vec{r},\omega)$$

atomic species α

lasers

real part of α's polarizability

intensity of "light" at $\vec{r}$

Use:
- focussed beams to make smooth potentials
- ~~interference~~ standing waves, speckles, holograms to make optical lattices or other potentials that vary rapidly with $\vec{r}$ (on scale $\gtrsim \lambda$ of photons)

Keep ω away from absorption lines to avoid absorption, scattering, + heating

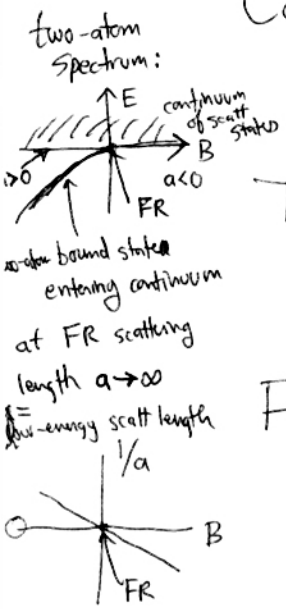
Can change $I(\vec{r},\omega)$'s quickly.

Optical potential can "imitate" crystalline lattice of a material.

Interactions between neutral atoms are short-ranged (van der Waals or bonding). $r_{vdW} \sim nm \ll \lambda_{dB} \cong \mu m$

Usually negligible between identical fermions, due to anti-symmetry of wavefunction.

Can be greatly enhanced near a Feshbach resonance (FR) where they are on verge of forming a weakly-bound state.

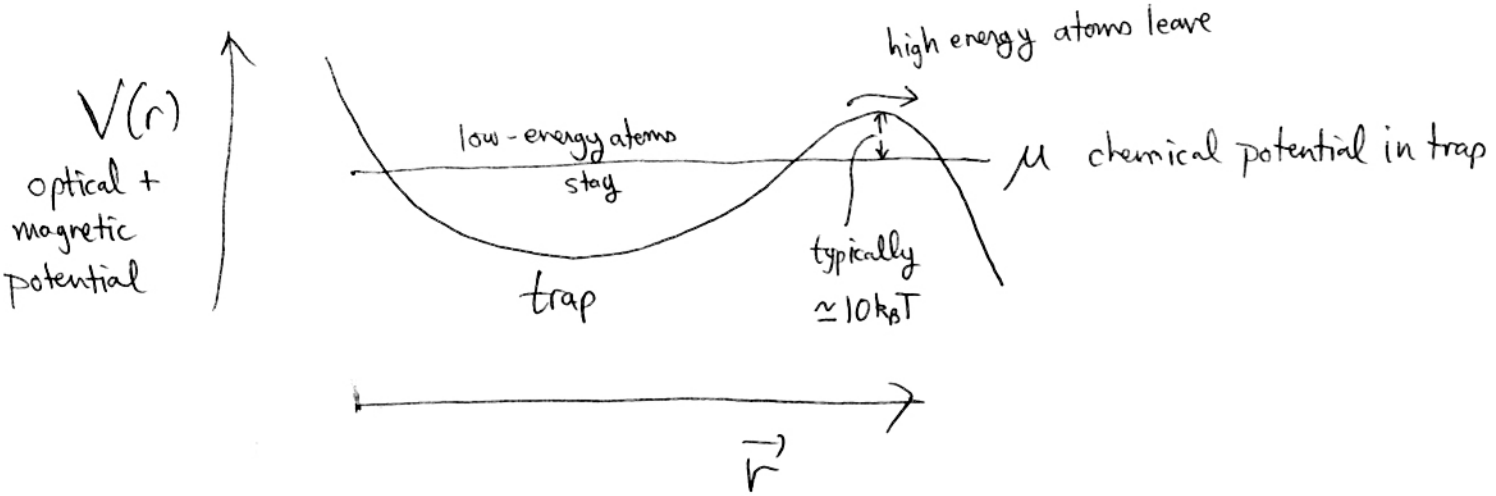


This (FR) makes the <u>interactions controllable</u> (via B-field) <u>in certain cases</u>

For bosons, this enhancement of interactions also enhances 3-atom loss processes, but for case of two species of fermions "Pauli blocking" (at least 2 of the 3 must be identical) prevents this, allowing tuning to strong interaction (divergent scattering length = "unitarity")

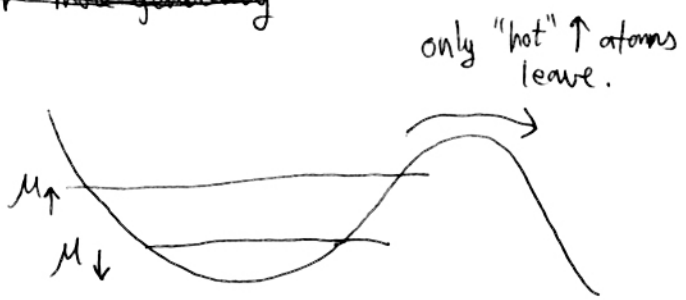
Evaporative cooling + depolarization

Final stage of cooling to lowest temperature/entropy is "simple" evaporative cooling:



need atom-atom interaction to continuously produce high-energy atoms via scattering. Doesn't work for <u>one</u>-component Fermi gas. ~~Or more generally~~

Two component gas (e.g. Fermions):



evaporation here cools + depolarizes, towards $\mu_\uparrow \to \mu_\downarrow$

Cooling + Thermometry here are <u>not</u> accurately controlled/measured compared to doing low-T physics with materials.
This is an area where good new ideas/techniques are needed.

One-body + 3-body losses make <u>holes</u>.

μ

0 — states below μ

For fermions this produces a lot of heating + entropy.

For weakly-interacting Bose condensate losing atoms from the condensate does not produce much heating or entropy (no states below μ)

So cooling fermions to very low T / low S has been less successful than bosons.

Mott insulator was first produced with bosons in an optical lattice, only later ~~[illegible]~~ (~6 years) with fermions.

Magnetically-ordered phases of Mott insulator ~~multi-component gases~~ ~~in an optical lattice~~ have not yet been produced. Will it again be easier with bosons? People are pushing on both.

Paired-fermion superfluids.

The simplest BCS model for paired-electron superconductivity has a momentum-independent attraction between ↑ + ↓ electrons at the Fermi surface.

Momentum-independent = short-range in real space (δ-function)

This not accurate for electrons, but <u>is</u> realized accurately for two-species Fermi gases ^6Li, ^{40}K, e.g.

The two species are usually called ↑ and ↓, although they are really 2 different hyperfine states + usually differ more in the nuclear spin state than the electron's.

Control: interaction (via Feshbach resonance)

densities $n_\uparrow, n_\downarrow$

New:

Can go to much <u>stronger</u> attraction than electrons, or liquid ^3He (comparing interaction energy to E_F)

Can explore highly-polarized Fermi gas $n_\uparrow \gg n_\downarrow$ (spin flip rate is very slow)

attractive

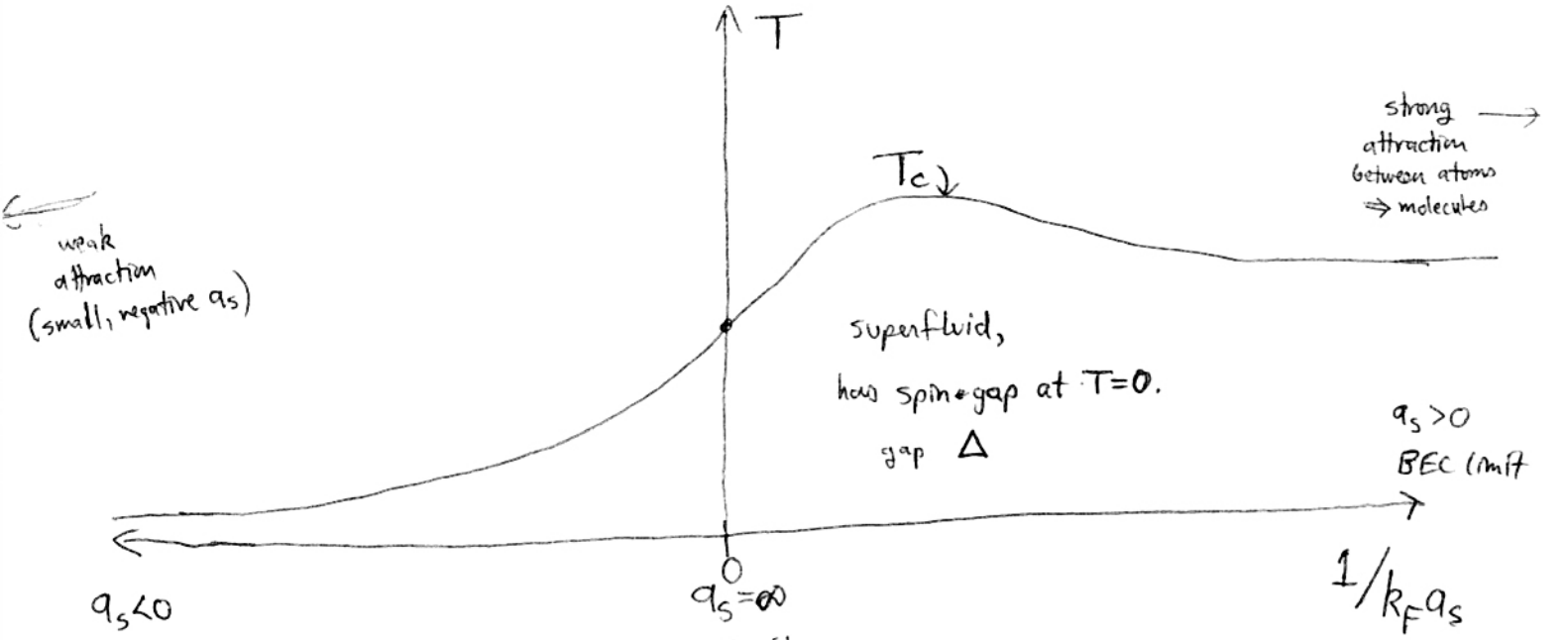
$$H = \sum_{\vec{k},\sigma} \frac{\hbar^2 k^2}{2m} c^\dagger_{\vec{k},\sigma} c_{\vec{k}\sigma} - g_\Lambda \int d\vec{r}\, c^\dagger_\uparrow(\vec{r}) c_\uparrow(\vec{r}) c^\dagger_\downarrow(\vec{r}) c_\downarrow(\vec{r})$$

U.V. cutoff

"universal" limit:

$g_\Lambda \to 0$ as $\Lambda \to \infty$ with s-wave scattering length ($\ell=0$) (2 atoms at $E=0$) a_s fixed.

BCS – BEC crossover: $n_\uparrow = n_\downarrow$ $k_{F\uparrow} = k_{F\downarrow} = k_F$



$a_s < 0$
BCS limit

$a_s = \infty$
unitarity
(where 2-atom bound state forms at $k_F = 0$)

Bose condensate of weakly-interacting, strongly bound (↑↓) molecules

$T_c \sim \Delta$

$\sim T_F e^{-\pi/|2k_F a_s|}$

size of Cooper pairs $\gg \frac{1}{k_F} \sim$ interatomic spacing

weak pairing

$T_c \sim \Delta \sim T_F$

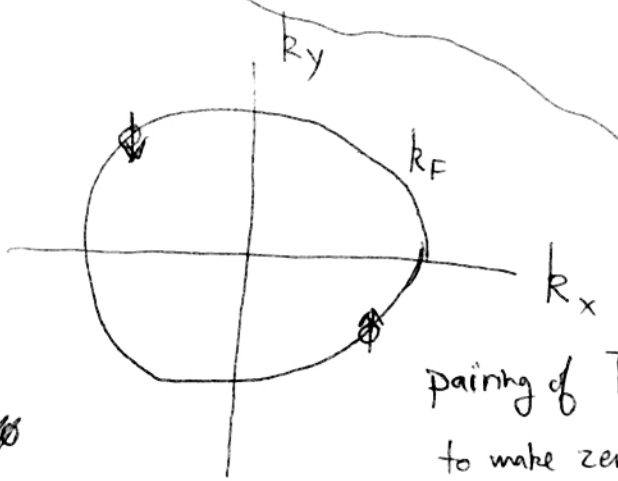
size of Cooper pairs $\sim$ interatomic spacing $\sim \frac{1}{k_F}$

$T_c \sim T_F$

spin gap $\Delta \sim \frac{\hbar^2}{m a_s^2} \gg T_F$

is binding energy of molecules

size of Cooper pairs (molecules) $\ll$ spacing between Cooper pairs



For $k_{F\uparrow} = k_{F\downarrow}$
Fermi surfaces are perfectly nested:
superfluid at T=0 for any $-g_\Lambda < 0$

pairing of $\vec{k}, \uparrow$, with $-\vec{k}, \downarrow$
to make zero-momentum, spin-singlet bosonic Cooper pairs that condense.

Polarized superfluid: $n_\uparrow > n_\downarrow$ $\quad k_{F\uparrow} > k_{F\downarrow}$

well over

<u>On "BEC side"</u> makes Bose-Fermi mixture of

$(\uparrow\downarrow)$ and excess $\uparrow$

bosons fermions.

<u>all</u> minority atoms are in molecules at $T=0$

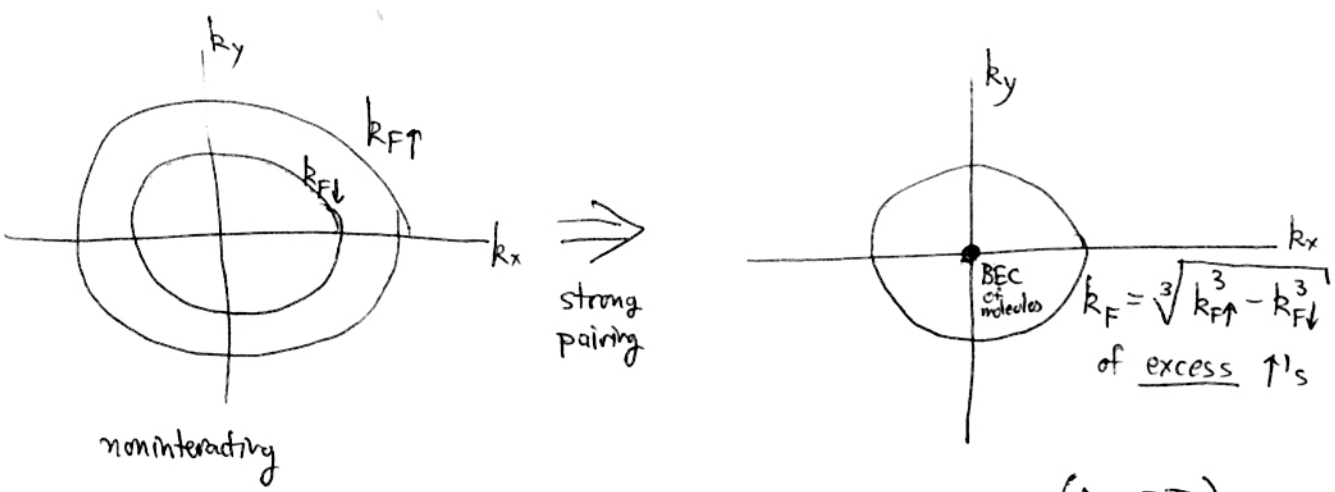
when bosons are strongly bound: $0 < k_F a_s \ll 1$

strongest "repulsion" is $\uparrow - \uparrow$ Pauli exclusion.

"Sarma phase":

Fermi sea of <u>excess</u> $\uparrow$'s fully mixed into superfluid (BEC) of $(\uparrow\downarrow)$'s (or vice versa)

(very low T: $(\uparrow\downarrow)$'s mediate P-wave ~~superfl~~ pairing of $\uparrow$'s, Bulgac et al.)



(in 3D)

this regime produced by Ketterle's group @ MIT

Strongly confine these molecules + atoms in 2 directions, with optical lattice
but let them move freely in the 3rd direction ⇒ 1D system

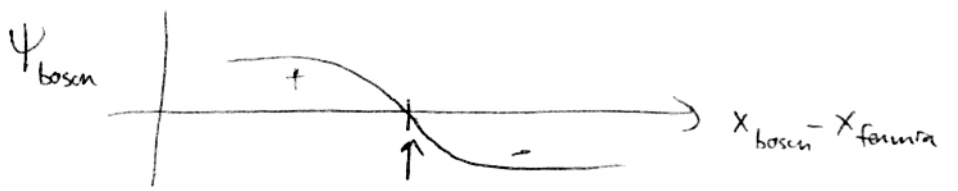
(Hulet's group, Rice U.) has produced this in the lab)

Still a Bose-Fermi mixture: but an important difference:

$(\uparrow\downarrow)$ $\uparrow$

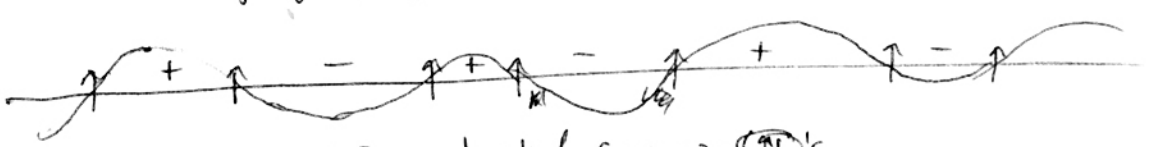
this boson contains one of these fermions

antisymmetry of fermions dictates wavefunction changes sign when boson passes fermion in 1D



Ground state of bosons in presence of the excess ↑ fermions in 1D

~~sign of~~ ψ_{boson} for a given set of fermion positions:



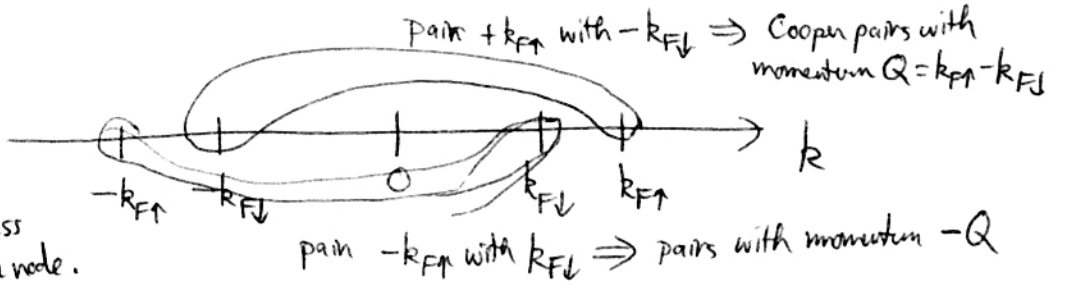
Bose-condensate of Cooper pairs $(\uparrow\downarrow)$'s

average wave ~~length~~ number of $Q = k_{F\uparrow} - k_{F\downarrow} = k_F$ of <u>excess</u> ↑'s

This is the ~~the~~ 1D version of Larkin-Ovchinikov-Fulde-Ferrell pairing (LOFF, or FFLO)

1D version is a type of Luttinger Liquid (divergent quantum fluctuations due to 1D)

Weak coupling view:



Two condensates $+Q, -Q$ make a standing wave: one excess fermion per node.

Back to 3D.

On "BEC side" at $T=0$, have molecules $\uparrow\downarrow$ bosons and Fermi sea of <u>excess</u> $\uparrow$'s fermions.

~~Far from unitarity ($k_F a \ll 1$)~~ All interactions (at low energy) are repulsive due to "Pauli blocking"

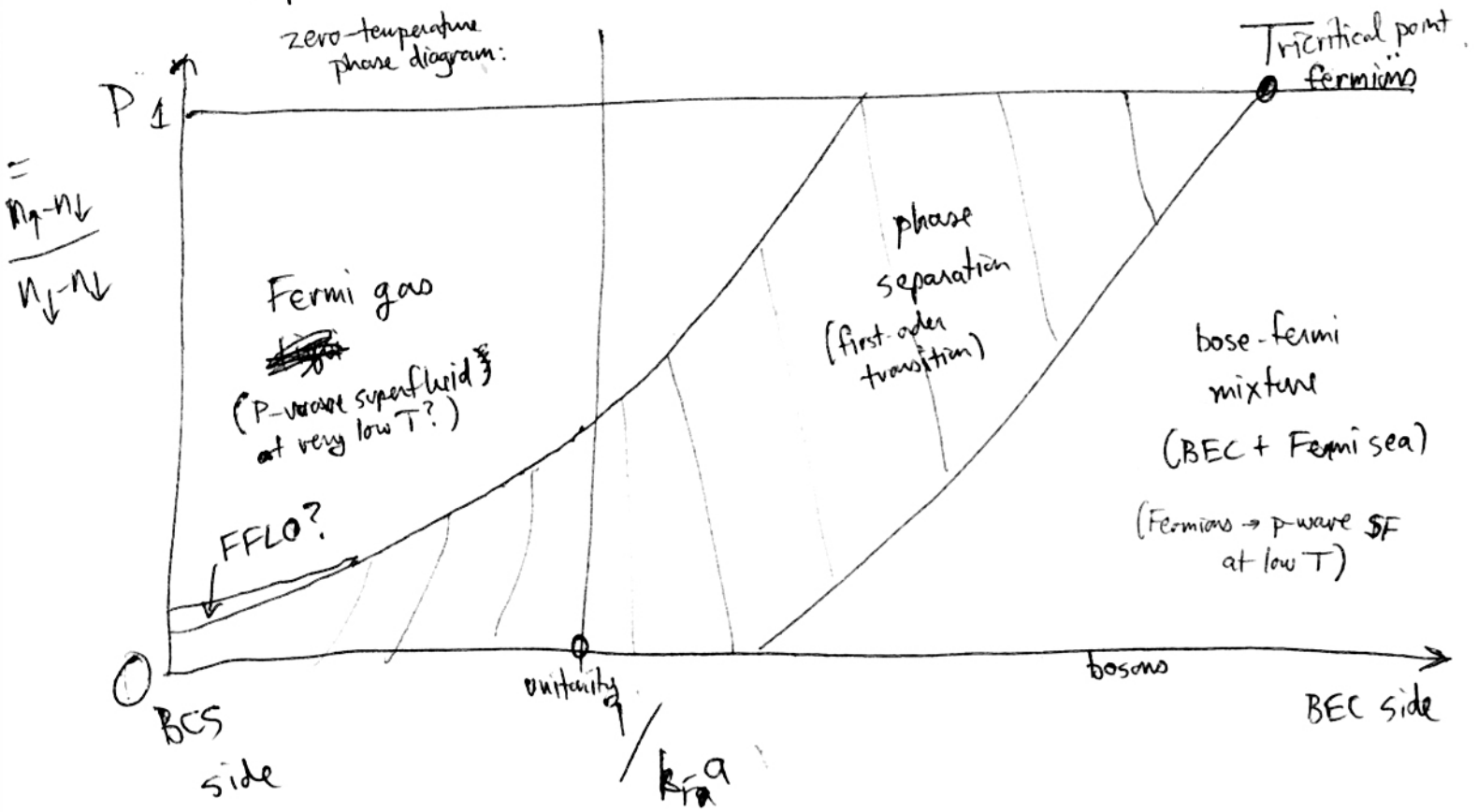
boson – boson
boson – fermion
fermion – fermion
} repulsive interactions ($\uparrow\downarrow$ attractions are all "used up" in making molecules.)

Far from unitarity molecule is tightly bound, so fermion in it is at high momentum + nearly orthogonal to ~~other~~ low-momentum fermions ⇒ weak repulsion.

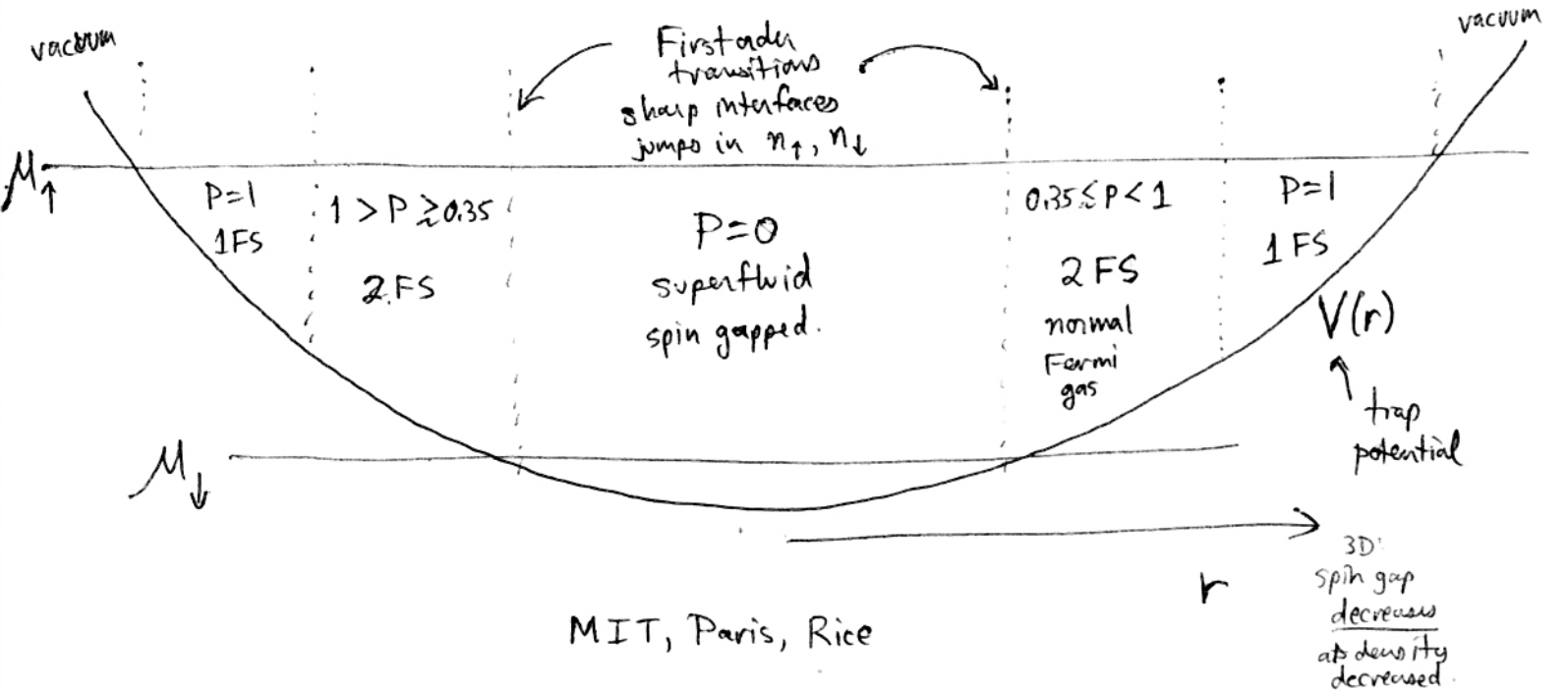
fermion-fermion repulsion dominates, forcing mixing of bosons + fermions.

Closer to unitarity boson-fermion repulsion dominates, causing phase separation

zero-temperature phase diagram:



With a large gas cloud in a large harmonic trap: see the phase diagram / equation of state: e.g. at unitarity $a_s \to \infty$ at global equilibrium



MIT, Paris, Rice

On BEC side, jump in density at interface SF/N can be more than factor of 2 in a dilute gas!

Like:

In 3D P=0 superfluid is denser (heavier) sits at bottom of trap — water

P>0 normal is lighter, sits on top — oil

In 1D it is the other way around:

1D:
spin gap decreases as density increases.

P>0 FFLO is denser than P=0 superfluid.

↓

in 1D fermions "fit in" to bose fluid + make it denser.

This "inversion" of phase separation seen in recent Rice experiments on 1D.

In 3D they don't: fermi-bose repulsion is stronger than bose-bose repulsion in 3D

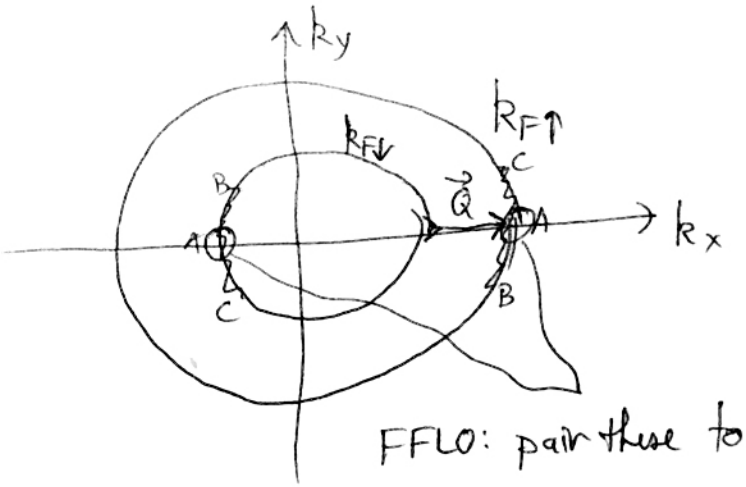
Cooper pairing is a Fermi surface nesting instability

For $k_{F\downarrow} = k_{F\downarrow}$ $(P=0)$, ~~the~~ $\uparrow + \downarrow$ Fermi surfaces match:

pair $(\vec{k}_\uparrow, \uparrow)$ with $(-\vec{k}, \downarrow)$ to make spin-singlet Pairs with total momentum $\vec{Q}=0$

~~at all~~ coherently at all points on Fermi Surface, gapping entire Fermi Surface.

For $P>0$ nesting is no longer perfect:



FFLO: pair these to make condensate of Cooper pairs with momentum $\vec{Q}$

But it only works over a "patch" of the Fermi surfaces.

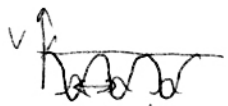
To enhance FFLO superfluid pairing need to enhance the Fermi surface nesting of $\uparrow$ and $\downarrow$ Fermi surfaces.

Put fermions in an optical lattice to make Fermi surfaces less "rounded", flatter.

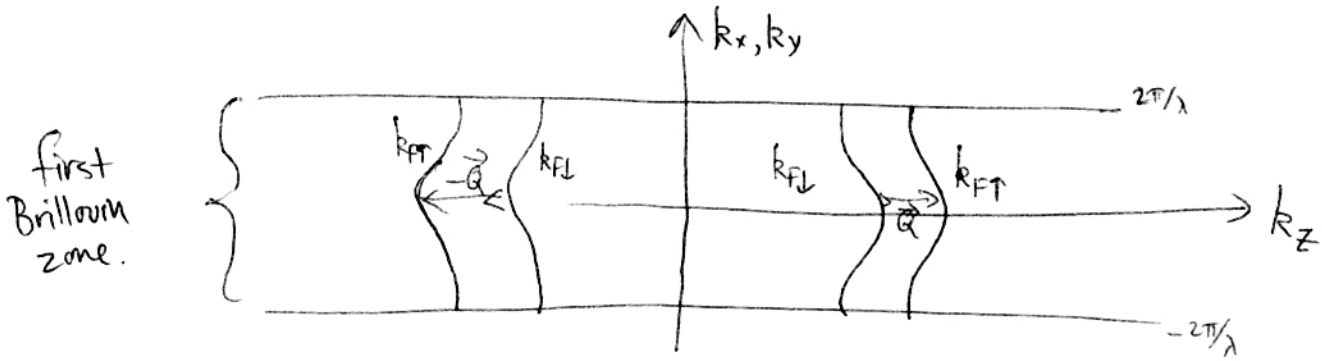
Back towards 1D: Put square optical lattice along x, y, not along z

$$V(x,y,z) = -V_0\left(\cos^2\left(2\pi\frac{x}{\lambda}\right) + \cos^2\left(2\pi\frac{y}{\lambda}\right)\right)$$

λ = wavelength of light making lattice



Now single-particle dispersion of fermions is free along z, but high effective mass for moving in bands along x, y, direction (tunneling in optical lattice



Makes Fermi surfaces more nested, (can also do this with other types of lattices)

enhances FFLO pairing,

pairs with total momentum $\vec{Q}, -\vec{Q}$ two condensates of Cooper pairs:

$$\langle C_\uparrow(\vec{r}) C_\downarrow(\vec{r}) \rangle \approx \Delta \cos(Qz) + \text{higher harmonics.}$$

But how to demonstrate that this type of pairing is present? Not clear.

Transport in unitary Fermi gas with $T \sim T_F$

Transport of: energy (thermal conductivity)
spin ← easiest to measure
momentum (viscosity)

Carriers are atoms moving with speed $\sim \frac{\hbar k_F}{m} = v_F$

strong scattering at unitarity: mean-free path $\ell \sim \frac{1}{k_F} \sim$ interatomic spacing

So diffusivity $D \sim v_F \ell \sim \frac{\hbar}{m} \sim 10^4 \frac{(\mu m)^2}{s.}$

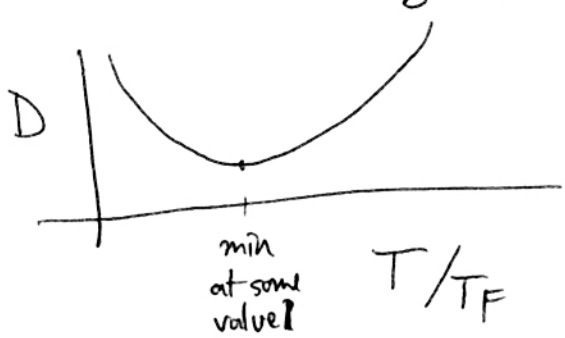
Go to higher T $\quad v \sim v_F \sqrt{\frac{T}{T_F}}$ $\quad$ mean-free path $\ell \sim \frac{1}{n\sigma} \sim \frac{1}{k_F}\left(\frac{T}{T_F}\right)$

↑ x-section

$\sigma \sim \frac{1}{E}$ at unitarity

so $D \sim \frac{\hbar}{m} \left(\frac{T}{T_F}\right)^{3/2}$ for $T \gg T_F$

at low T for $n_\uparrow = n_\downarrow \Rightarrow$ superfluid: spin is transported by dilute quasiparticles $\ell \to \infty$ as $T \to 0$
$D \to \infty$



Zwierlein (MIT) is measuring this by setting up spin-density gradient + watching it relax by diffusion.