BSS2023 D. Huse, rough notes. (this version is scanned after all lectures were given).

more questions?: huse@princeton.edu

topics: • Unitary Many-body quantum dynamics (closed systems)

• Thermalization, emergence of dissipation in unitary dynamics.

• Many-body localization (MBL).

Focus on unitary dynamics.

Systems of N quantum degrees of freedom

$\boxed{1 \ll N < \infty}$ spins, qubits, trapped atoms, ions, molecules, etc.

In a model, or in the lab. [Many such systems are built and studied. These are interesting new physical systems. They are not simulations.] [Perhaps they can be used as quantum simulators. they are realizations.]

Many-body state of system is a density matrix $\rho(t)$.

[$\rho(t)$ is a conditional probability distribution.

Conditional on the procedures used to prepare and select this state, which is done many times repeatedly in any useful experiment. [Thus: "frequentist" probabilities.]]

[$\rho(t)$ gives the probabilities of the outcomes of any observations (measurements) that could be made.]

A useful idealization is a pure state:

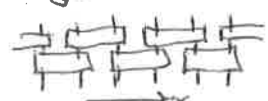
$$\rho(t) = |\Psi(t)\rangle\langle\Psi(t)|$$

Pure states are very useful theoretically and conceptually. Experimentally, one can try to make $\rho(t)$ as close to pure as one can, but it will always have some small (or large) impurity (infidelity).

We consider unitary dynamics.

Due to a Hamiltonian $H(t)$.

e.g.

$t\uparrow$ 

Or due to a unitary $U(t)$ produced by a quantum circuit. (gates)

$$i\frac{d\rho(t)}{dt} = [H(t), \rho(t)] \quad \text{or} \quad \rho(t+1) = U(t)\rho(t)U^{\dagger}(t)$$

$H(t)$: continuous time, for pure state: circuit: discrete time

$$i\frac{d|\Psi(t)\rangle}{dt} = H(t)|\Psi(t)\rangle \quad \text{or} \quad |\Psi(t+1)\rangle = U(t)|\Psi(t)\rangle$$

If there are important effects due to a quantum "environment", include part of that "environment" as part of your system.

Special case: $H(t) = H$. System is fully isolated from any external dynamics. Has eigenstates that are stationary. Has an extensive conserved energy than can be transported.
Could have other extensive conserved quantities setting its thermodynamics, and being transported.

More generally system is <u>driven</u>, drive is autonomous and classical producing $H(t)$, or $U(t)$
(Really, drive is a <u>large</u> quantum system in a coherent state in the <u>classical</u> + autonomous <u>limit</u>.)
general $H(t)$, or $U(t)$ does not have stationary pure eigenstates. ($\rho = \mathbb{1}/Z$ is always stationary)

Another special case:

Floquet system or Floquet unitary

$H(t) = H(t+\tau)$ is periodic

$$U_F = T\, e^{-i\int_0^\tau dt\, H(t)}$$

"advances time" by τ, one period.

U_F has <u>stroboscopically</u> stationary eigenstates. These eigenstates have within-period "micromotion" dynamics.

for circuits:

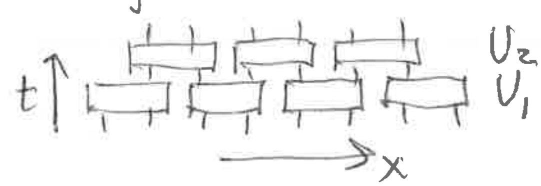
$U(t+1) = U(t) = U_F$, $\tau = 1$

same unitary every step.

U_F might have substructure within one period

$U_F = U_n \cdots U_2 U_1$

e.g.:



Unitary dynamics

General $H(t), U(t)$
random unitary circuits
gives "simplest" dynamics.

(with additional extensive conserved quantities).
add a conserved charge, "simplest" models of transport due to unitary dynamics (random circuits)

restrict: $H(t) = H(t+\tau)$
$U(t+1) = U(t)$

Floquet. $H(t)$ or $U_{circuit}$
Has eigenstates
Can have MBL

Floquet with transport

restrict: $H(t) = H$
(no circuits)

Has energy transport,
a temperature if it thermalizes.

Has "thermopower", "cross-transports".
(time-independent H, with other extensive conserved quantity(s) to transport)

Other "axes": locality in $\underline{\underline{d}}$ dimensions.

or nonlocal $H(t), U(t)$, involving
few-body interactions or fully many-body nonlocal interactions,

Fully nonlocal limit: $H(t), U(t)$ are random matrices, Thermalization:

Does system with unitary $H(t)$ or $U(t)$ act as a "bath" for itself and bring (at long time) all of its small subsystems to thermal equilibrium with each other?

If "yes", this is thermalization.

For this question to have sharp yes/no answer, need to take limits $N \to \infty$, $t \to \infty$ (like for phase transitions).

But some small systems can thermalize very well (e.g., Jensen + Shankar PRL (1985) 7-spin chain), and strong changes in thermalization can be seen in small systems and/or at moderate time scales. So we can ask: How well does our finite system thermalize? (or finite time)

Thermal equilibrium: maximizing thermodynamic entropy, given a few (or zero) extensive conserved quantities.

Most interacting many-body systems do thermalize.

Exceptions: Integrability, Many-body localization } instead an extensive number of conserved quantities (later this week)

Many-body "scars"

Certain Other constrained systems ...

Consider N spin-$\frac{1}{2}$'s (qubits), Hilbert space dimension 2^N.

Number of linearly independent operators $= 4^N$

(Pauli I, X, Y, Z for each spin.)

Consider a local operator $A = A(0)$.

e.g. $Z_n \otimes$ (identity on all other spins)

$$\langle A \rangle_{t=0} = \text{Trace}\{\rho(0) A(0)\}$$

time-evolve in Schrodinger picture:

$$= \text{Trace}\{U(t)\rho(0)U^\dagger(t)U(t)A(0)U^\dagger(t)\}$$

$U(t)$ evolves from $t=0$ to t

$$= \text{Trace}\{\rho(t) A(t)\} = \langle A(t) \rangle_t$$

time-evolved operator $A(t) = U(t)A(0)U^\dagger(t)$

expectation value $\langle A(t) \rangle_t$ is t-independent (by definition of $A(t)$).

If $\rho(0)$ is thermal equilibrium, so is $\rho(t)$, so

$\langle A(t) \rangle_t = \langle A(t) \rangle_{eq'm}$ is time-independent.

If $\rho(0)$ and $\langle A \rangle_{t=0}$ are not at thermal equilibrium,

then $\langle A(t) \rangle_t$ remains nonthermal at all times.

dynamics is unitary, so no features of $\rho(0)$ are lost, only "re-arranged".

So, if we consider all operators to be observables, then there is no thermalization. (no "forgetting" of any properties of initial state)

Lychkovskiy, PRA (2013). To define thermalization we need to consider only some operators to be our observables, and not consider $\langle A(t) \rangle_t$ to remain observable for all time, as $A(t)$ becomes a very complex nonlocal operator, at late times for large N.

unitary dynamics "hides" nonthermal properties of ρ, In the limit of large N, it is true that only a zero fraction of all 4^N operators are observable. For small N, to examine thermalization, we need to specify which operators we will consider as observables (e.g. one-spin + two-spin operators).

In the limit of large N, almost all operators are not observables.

for any state $\rho(t)$, almost all operators ~~have~~ A have $\langle A \rangle_\rho = \langle A \rangle_{eq'm}$ so are thermal.

But there are observable operators, and ~~they~~ states $\rho_{n'eq}$ exist (and are readily prepared) where most observables have nonthermal $\langle A_{obs} \rangle_{\rho_{n'eq}} \neq \langle A_{obs} \rangle_{eq'm}$.

How does thermalization (dissipation) happen? : (in one perspective)

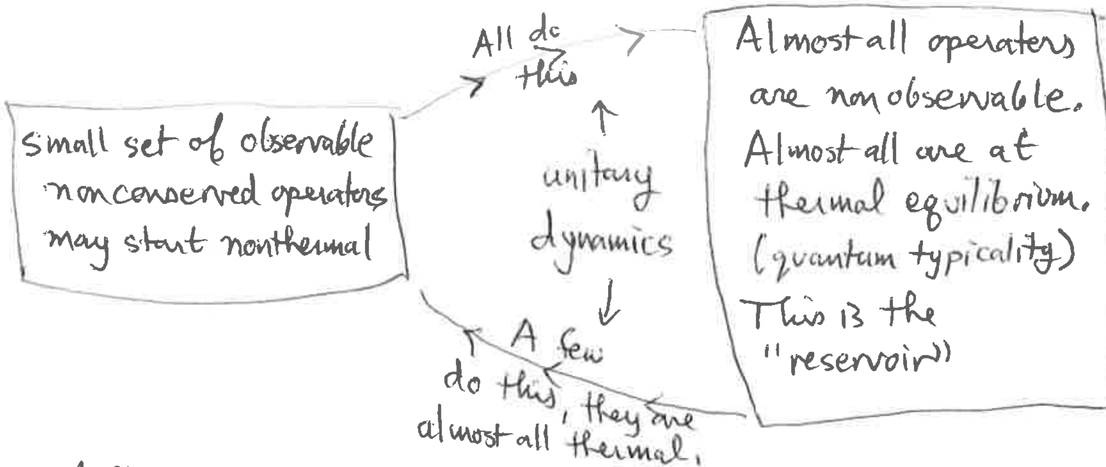
Initially observable, nonconserved, and non-thermal $\langle A(0) \rangle_{t=0}$ are all "moved" by $U(t)$ to $\langle A(t) \rangle_t$ with $A(t)$ no longer observable at long time (thus "hidden").

~~[illegible]~~ E.g. n-spin operators are observable for $n \leq k$ for some finite k.

are "non-observable" for $n > k$.

For a thermalizing system with large $N \gg k$, any nonconserved $A(0)$ with spread to complex nonlocal $A(t)$, which at long times is almost all $n > k$, so non-observable. Choice of k sets time scale when this happens.

Thermalization + Dissipation in a thermalizing system:
for large N:



Is a unitary "flow" of operators between "simple" observables and complex, nonlocal, non-observables. The initial nonthermal observables get moved to non-observables, so the "northermality" is "hidden".

start with bottom of pill
precise S(t) does depend on choice of "observables"

Boulder Mandy

Tuesday

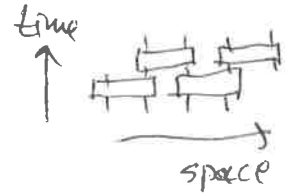
Some aspects of The spreading of operators can be exactly calculated for random unitary circuits in 1+1 dimensions, → next page

Refs: von Keyserlingk, et al PRX $\underline{8}$, 021013 (2018); Nahum et al 021014

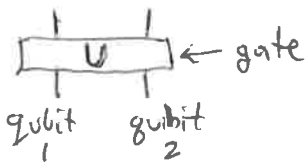
Khemani et al ~~PRX 8~~, 031057

Random unitary circuits ~~are a~~ allow analytic study of operator dynamics (operator spreading) ~~scrambling~~).

Qubits in 1D.



"brickwork" circuit



Each qubit has 4 operators (Pauli)

$I, \underbrace{X, Y, Z}_{N = \text{non identity}}$ $A = \text{all}$

For 2 qubits there are 16 operators: $A_{12} = A_1 A_2$

One is special: $I_{12} = I_1 I_2 = U I_1 I_2 U^{+}$ remains invariant.

15 non-identity (traceless) operators A_{12}, trace $A_{12} = 0$, so A_{12} does not contain identity

A ~~rand~~ "Haar-random" unitary gate U (has probability distribution that is invariant under any other unitary operation V $P(U) = P(VU)$ for any unitary V.)

Any Non-identity traceless A_{12} is a linear combination of the 15 non-identity Pauli operators $I_1 X_2, \cdots I_1 Z_2, X_1 I_2, \cdots Z_1 Z_2$. P_{12}

Of these, 3 are one-site operators on site 1, 3 on site 2, and 9 are two-site operators.

Haar-Random unitary U acting on one of these 2-qubit non-identity Paulis: $U P_{12} U^{+}$ produces a linear combination of all 15 (with equal weights on average). This is, by weight, on average

$\frac{1}{5}$ a one-site operator on site 1 $N_1 I_2$

$\frac{1}{5}$ " " " " " site 2 $I_1 N_2$ in the Pauli basis.

$\frac{3}{5}$ a two-site operator $N_1 N_2$

Say $A(0)$ is a single-qubit operator;
after time t,
$A(t)$ between gates is a sum of many "Pauli strings"
that have ends at x_L, x_R: (last non-identity in string)

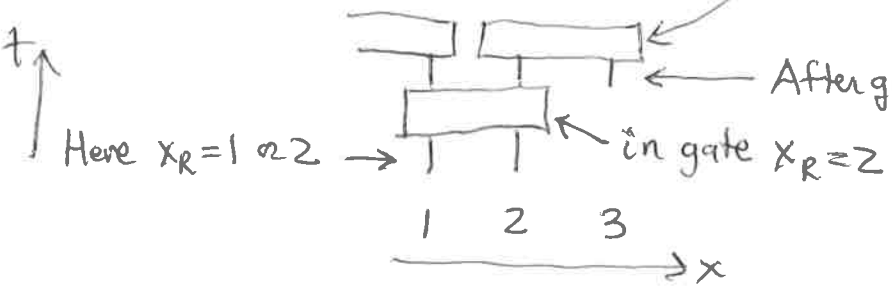
$\xrightarrow{\quad x \quad}$

$\cdots I\,I\,I\,N\,A\,A \cdots\cdots \quad \cdots A\,A\,N\,I\,I\,I\,I \cdots$

$\qquad\qquad x_L \qquad\qquad\qquad\qquad x_R$

The ends x_L, x_R do biassed random walks:
Let's follow x_R for one string:

In the next gate: $\frac{1}{5}$ has $x_R = 1$
$\frac{4}{5}$ has $x_R = 3$



After gate becomes 15 Pauli strings.
$\frac{1}{5}$ of weight has $x_R = 1$
$\frac{4}{5}$ of weight has $x_R = 2$

So from one layer of gates to the next:
String shortens by 1 with weight $1/5$
" lengthens " " " " $4/5$.

Random walk has mean drift speed ("butterfly speed") $3/5$. $\frac{\text{sites}}{\text{layer of gates}}$

Typical operator lengthens at both ends, spreads. → "nonobservable"
Also, gets entangled: becomes a sum of many different Pauli strings.
(Clifford gates produce spreading, but no operator entanglement.)
Very special operators do get shorter: these are the rare operators
that time evolve non-observable → observable.

An example of:

Emergence of dissipation (e.g. in energy transport due to a weak temperature gradient):

Example: Time independent (generic H) $H = \sum_{\vec{r}} h(\vec{r})$ (so no longer a unitary circuit)

$\uparrow$ local terms in energy

Consider initial state

$$\rho = Z^{-1} e^{-\beta H} \left(1 - \sum_{\vec{r}} \delta\beta(\vec{r})\, h(\vec{r})\right)$$

β: average inverse temperature.

$\delta\beta(\vec{r})$: very small smooth gradual temperature gradient. $\vec{\nabla}\delta\beta(\vec{r})$ very small. $\int \delta\beta(\vec{r})\, d\vec{r} = 0$.

$$i\frac{d\rho(t)}{dt} = [H, \rho(t)]$$ is nonzero only due to $\vec{\nabla}\delta\beta(\vec{r})$ so very small

$\rightarrow$ produces a small energy current, and other operators in ρ

$\vec{\nabla}\beta \rightarrow \vec{J}_E \rightarrow \cdots \rightarrow$ nonobservables.

That energy current operator also does not commute with H

$\rightarrow$ produces small time derivatives of $\delta\beta(\vec{r})$ and more operators in ρ ~~of the energy current, etc.~~ (a flow of operators from simple to complex)

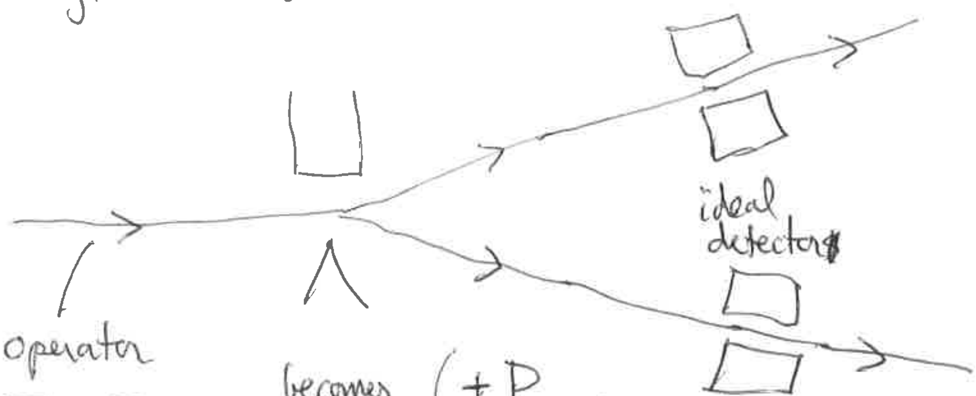
Initial "simple" operator $\sum_{\vec{r}} \delta\beta(\vec{r})\, h(\vec{r})$ (also energy current) spreads and "scrambles" and eventually "hides" as a non-observable operator. ($\delta\beta(\vec{r})$ relaxes by diffusion equation)

$\rightarrow$ dissipation has happened, the entropy has been increased, all with unitary reversible dynamics.

What is the entropy of $\rho(t)$?: Find the ρ (among all possible ρ's) that has the same $\langle A \rangle$ for all "observables" but has the maximum von Neumann entropy $S = \mathrm{Trace}(-\rho \log \rho)$. This S is the entropy of $\rho(t)$. $\mathrm{Trace}(-\rho(t) \log \rho(t))$ might be much smaller, or even zero if ρ is pure.

In some special situations, some simple local operators do not fully "scramble" and hide, but they remain observable to long times and even become more easily observable. This is what happens in a measurement:

E.g. Stern-Gerlach:



"separate before scrambling".

operator $Z_{spin} P_{beam} P_{\text{detector ready}}$ $\xrightarrow{\text{becomes}}$ $\left(+P_{\text{spin}\uparrow,\,\text{beam}\uparrow} - P_{\text{spin}\downarrow\,\text{beam}\downarrow}\right) \cdot P_{\text{detector ready}}$

projector onto correct incoming beam

this operator only starts scrambling and becoming complex when a detector is triggered.

after detector becomes

$+P_{\text{spin}\uparrow,\,\text{beam}\uparrow,\,\uparrow\text{ detector triggered}}$

$-P_{\text{spin}\downarrow,\,\text{beam}\downarrow,\,\downarrow\text{ detector triggered}}$.

so it becomes readily observable whether spin ~~was~~ is ↑ or ↓. (Not other components of spin)

State ρ (which is a conditional probability distribution) becomes a linear combination of the two results. (say, spin was initially along X)

Then we can choose to post-select on one of the two outcomes and thus "collapse" the state.

(add another condition on ~~the~~ the conditional probability distribution that defines ρ)

Thermalization:

Is a dynamical question: Does initially non-thermal state become thermal at almost all long times?

For Floquet system, or time-independent H can also ask: Are eigenstates of U_F, H thermal?

These questions are sharply defined only in $N \to \infty$ limit (and $t \to \infty$ for dynamics).

To thermalize to T, need initial distribution of E to be narrow, so distribution of $\frac{E}{N} \to$ a Dirac delta function for $N \to \infty$.

A thermal state for $N \to \infty$ is the limit of a sequence of states so that for each observable $\langle A \rangle_\rho \to \langle A \rangle_T$ in the limit.

(e.g. consider all k-spin operators to be observables, take limit $N \to \infty$ with $k/N \to 0$ in limit)

Almost all states with $\langle H \rangle_\rho = \langle H \rangle_T$ are thermal (quantum typicality)

Eigenstate Thermalization Hypothesis (ETH):

Eigenstates are thermal. Appears to be true for most systems that thermalize, with all states thermal
scars: almost all states thermal.

When ETH is true: Microcanonical ensemble middle of a single eigenstate is thermal.

and almost all states are thermal, then

If ETH is true, how do we make non-thermal states?
at ~~temperature~~ energy E? T!?

In basis of eigenstates of H $\qquad H|n\rangle = E_n|n\rangle$

$$\rho(t) = \underbrace{\sum_n \rho_{nn}|n\rangle\langle n|}_{\text{this is thermal}} + \underbrace{\sum_{m\neq n} \rho_{mn}(0)\, e^{i(E_n-E_m)t}|m\rangle\langle n|}$$

this is thermal
weight in ρ_{nn} concentrated near E

to be non-thermal these need to add up coherently to make $\langle A\rangle_{\rho(t)} \neq \langle A\rangle_T$ for some A (very special states)

But at other times (much later) these will dephase and $\rho(t)$ becomes thermal:

Thermal equilibration is dephasing (in the basis of the eigenstates of H or U_F), ~~when~~ for systems for which ETH is true.

Diagonal part of ETH:

$$\langle n|A|n\rangle \to \langle A\rangle_{T(E_n)}$$ in limit, for all observables A
(generically for all n, but can have exceptions like "scars")

Off-diagonal part of ETH:

informally: $\langle n|A|m\rangle$ are small enough and large enough so we can make non-thermal states and pseudorandom enough so system does thermalize, from all nonthermal initial states. (with narrow dist of E) ~~(not fully understood)~~
some sufficient conditions are known, but what is necessary is still unclear.

to next page

Srednicki PRE 50, 888 (1994) ignored locality. Foini + Kurchan PRE 99, 042139 (2019)
Chan, DeLuca, Chalker PRL 112, 220601 (2019) showed how locality constrains off-diagonal ETH. But minimal criteria needed for $\{\langle n|A|m\rangle\}$ is not known.

PRE
Srednicki 1994 Off-diagonal part of ETH:

eigenstates

$$\langle m | A | n \rangle = e^{-S((E_m+E_n)/2)} f(|E_m - E_n|) R_{mn}$$

observable — entropy — nonuniversal function — order-one pseudorandom numbers,

Foini + Kurchan PRE
Chan, De Luca, Chalker PRL
} 2019 locality requires certain correlations among the $\{R_{mn}\}$.

More work in progress about this by Chalker + collaborators, to explore what is universal in off-diagonal ETH

Tues

Thurs?

"Thermalization" of Floquet systems.

Bukov et al PRB 2016,
Morningstar, H, Khemani 2210.13444

System driven with period τ, frequency $\omega = \frac{2\pi}{\tau}$

System ~~may~~ exchanges energy quanta $\simeq \omega$ with drive at rate Γ.

If Γ is small enough we have

$$U_F \cong e^{-iH_{eff}\tau}$$

any difference here is the processes that cause

$$E_{eff} \longleftrightarrow E_{eff} \pm \omega$$

H_{eff} is the quasi-local effective H that gives best (quasi-local) such approximation to U_F

"Prethermal" early or intermediate time regime:

Γ is small enough so that in this time regime system thermalizes to H_{eff} (with a slowly increasing temperature if $\Gamma > 0$).

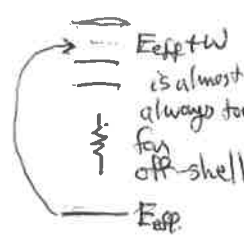
Regimes of long-time or eigenstate Floquet thermalization.

N and ω are large. To make these regimes sharp, can take limit $N \to \infty$, $\omega \to \infty$ together in various ways. But can also consider finite N, finite ω

regime 1 (highest ω) No heating: $U_F = e^{-iH_{eff}\tau}$ This is prethermal regime extending to $t \to \infty$ (no longer "pre")

Exchange of energy with drive is negligible.

U_F and H_{eff} have same eigenstates.



An extensive conserved energy, in spite of drive.

Processes that change $E_{eff} \to E_{eff} \pm \omega$ have matrix element $\ll$ many-body level spacing, so do not happen

regime 3: (lowest ω) Heats to ∞ temperature with $\Gamma \gg \omega$

(note: time to heat to $T=\infty \sim N/\Gamma$ so can still be slow in this regime when N is large)

No conserved energy, eigenstates of U_F and H_{eff} ~~have~~ are very different. Eigenstates of U_F are linear combos of all eigenstates of H_{eff}.

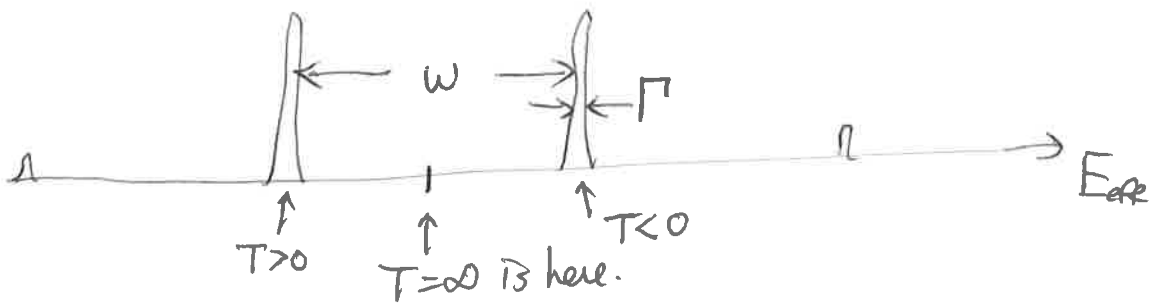
regime 2 intermediate ω has

$$\delta = \begin{matrix}\text{many-body} \\ \text{level spacing}\end{matrix} \ll \Gamma \ll \omega$$

E_{eff} is conserved modulo ω to precision Γ

Eigenstates of U_F have E_{eff} probabilities:

e.g.



If initial state has uncertainty in E_{eff} that is $< \omega$,

~~the~~ system will thermalize to this

"ladder ensemble" with E_{eff}

conserved modulo ω to a precision Γ

A "new" type of thermal equilibrium ensemble, with multiple temperatures present.

OBB 14

18 Weds Boulder

Many-body localization (MBL)
"avalanche" instability
many-body resonances, ~~"avalanches"~~.

MBL is Anderson localization with many interacting degrees of freedom, in highly-excited states that at thermal equilibrium would have non-zero entropy density.
Anderson 1958, Altshuler et al 1997, Basko, Aleiner, Altshuler 2005, then many.

As in Anderson 1958, let's consider a spin model.
~~As in 2020's, let it be a Floquet model with no extensive conservation laws, energy conserved only modulo $\omega = 2\pi$ $(\tau=1)$~~

Trivial limit of MBL: N non-interacting spins: $H_0 = \sum_n h_n S_{nz}$

$\cancel{U_{F0} = e^{iH_{F0}}}$ $\cancel{H_{F0} = \sum_n h_n Z_n \text{ (modulo } 2\pi)}$

(every Z_n is conserved, so $\langle Z_n \rangle$ will not relax to thermal equilibrium, which is $\langle Z_n \rangle = 0$)

$Z_n \to S_{zn}$

h_n uniform in ~~$[-\pi, \pi]$~~ $[-W, W]$

h_n may be random, or in a deterministic pattern such as quasiperiodic, but without degeneracies under low-order spin flip processes.

Eigenstates of U_F: any product state of $\uparrow$'s and $\downarrow$'s.

2^N eigenstates, ~~energies uniform at $[-\pi, \pi]$~~: typical many-body level spacing $\sim 2^{-N}$: gapless

These eigenstates are not thermal (thermal with ~~no conservation laws~~ maximal entropy $(E=0, \sum_n S_{nz}=0)$ is $\langle S_{nz} \rangle$ ~~$\langle Z_n \rangle$~~ $= 0$) ~~$\langle Z_n \rangle = \pm 1$ does not relax~~, Now add weak 2-spin interaction:

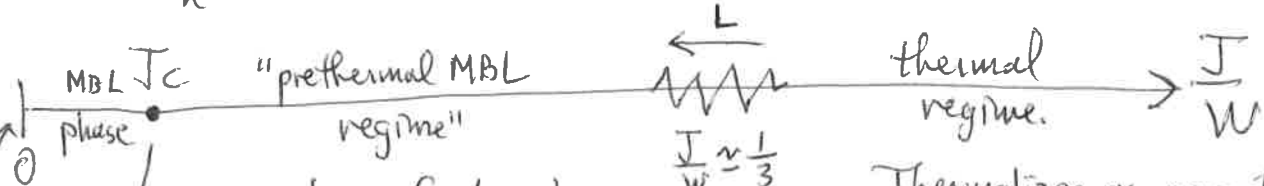
$$H = \sum_{n=1}^{L} h_n S_{nz} + J \sum_{n=1}^{L-1} \vec{S}_n \cdot \vec{S}_{n+1} \qquad \underline{h_n \text{ random}} \text{ in } [-W, W]$$

this is the "standard model" of MBL.

I discuss it here because it is familiar to many.

I now prefer Floquet MBL model with no conservation laws.

Fix $\sum_n S_{nz} = 0$, $E \cong 0$. Phase diagram:



trivial limite

$\frac{J_c}{W} \lesssim 0.06$

here, for L where numerics can be done, eigenstates show area-law entanglement, very weak level repulsion: appears MBL

$\frac{J}{W} \simeq \frac{1}{3}$

depends on L

the transition was long thought to be near here.

Thermalizes on accessible time scales. Eigenstates show volume-law entanglement RMT energy-level statistics.

But we have good arguments that this regime thermalizes for ~~N~~ $L \to \infty$, $t \to \infty$

Kiefer-Emmanoulidis et al PRB 2021 motivated a more careful look in MBL regime.

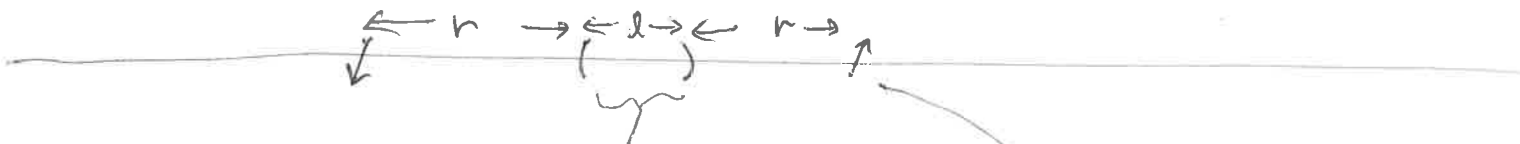
Morningstar, Colmenarez, Khemani, Luitz, H PRB 2022 (also D Sels, PRB 2022)

we moved this "bound" on J_c/W down by a lot.

Based on "avalanche" instability of MBL phase:

The "avalanche"; De Roeck + Huveneers PRB 2017.

Consider the case of h_n random, 1D, large systems, small J/W



thermal rare region, here, by chance, all spins have same h_n within J, so locally thermalizes ~~listen~~ a local "bath"

There are other ways to make a small locally thermalizing region?

spin at distance r spins from "bath" relaxes at rate $\underset{\Gamma(r)}{\Gamma} \sim k^{-r}$ (if bath is big enough to have many-body level spacing $\ll \Gamma$ so it can do this) namely, $k \sim \# \frac{W}{J}$

"Avalanche": "bath" has thermalized and thus "recruited" spins out to distance r spins.

Now many-body level spacing of "bath" is $\delta \sim 2^{-l-2r} \sim 2^{-l} 4^{-r}$

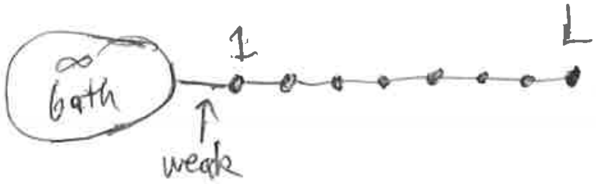
This "avalanche" will stop if it reaches $\delta(r) = \Gamma(r)$.

Starts with $\delta \ll \Gamma$ (due to 2^{-l} factor).

If $k<4$ it never stops. So transition is at $k_c = 4$.

→ more than 1D avalanche never stops once started. Transition is instead when avalanch starts (Gopalakrishnan + H PRB 2019)

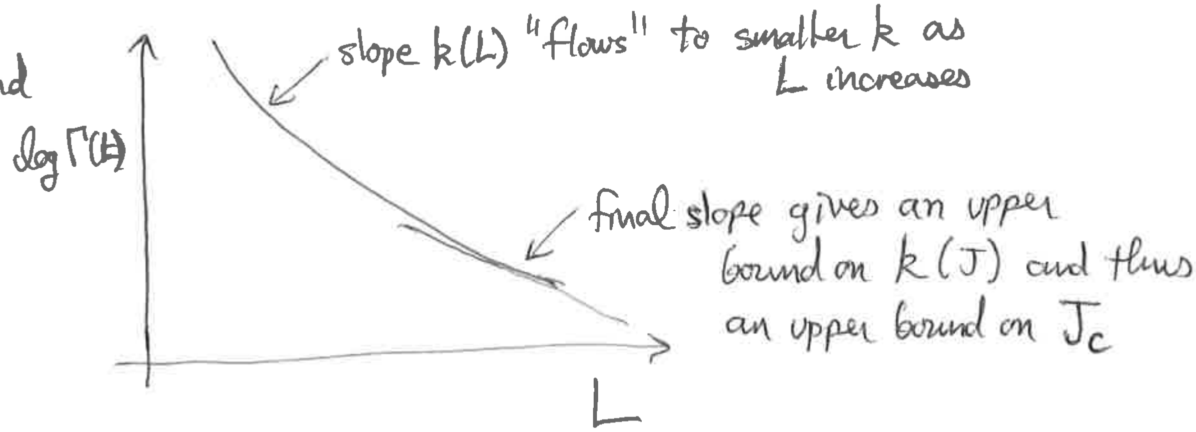
To explore this numerically, we do open system;

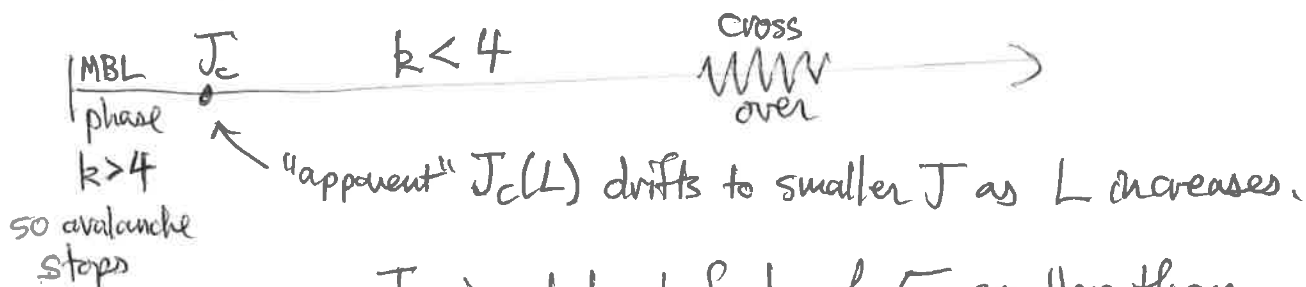


weak coupling

get relaxation rate $\Gamma(L)$ of farthest (slowest) spin (gap in spectrum of open-system super-operator)

We find





J_c is at least factor of 5 smaller than J at crossover.

So we have this large intermediate prethermal MBL regime, that is MBL-like for accessible system sizes ~~and~~ time, but thermalizes for $L \to \infty$ and $t \to \infty$

Now add some weak 2-spin interactions, strength J

short-range in 1, 2, or 3-d.

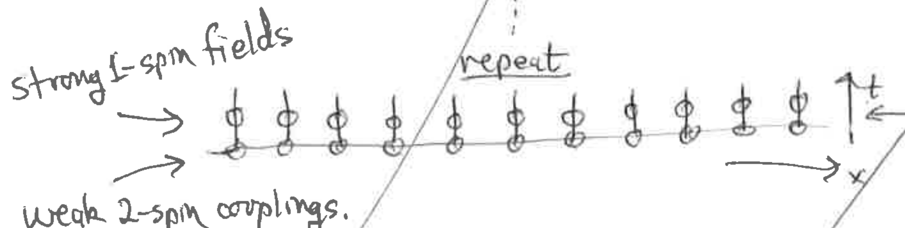
$$U_F = U_{F0}\, e^{-iJH_2}$$

$$H_2 = \sum_{\langle ij\rangle} h_{ij}$$

h_{ij}: order-one random matrix on that 2-spin state space, e.g. GUE

small J, small system, eigenstates remain localized, most $|\langle Z_n\rangle| \simeq 1$.

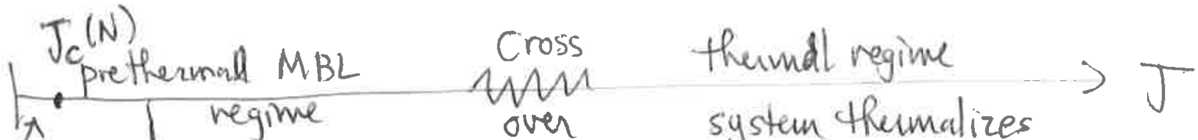
In 1D:



No conservation laws, so thermal-state = all states equally likely: $\langle X_n\rangle = \langle Y_n\rangle = \langle Z_n\rangle = 0$ + any few-spin traceless observables have $\langle A\rangle = 0$.

~~Naive description of many-bod~~

Phase diagram of this system:



$J_c(N)$: there might be a MBL phase that does not thermalize (discuss later)

prethermal MBL regime: on accessible length and time scales, system appears localized, but it will thermalize for $N \to \infty$, $t \to \infty$.

thermal regime: system thermalizes for small systems and/or short enough times to access in numerics and experiments. "decoherence"

MBL is a type of coherence, it is fragile.

→ What is physics of prethermal MBL regime, and the crossover to thermal regime?:

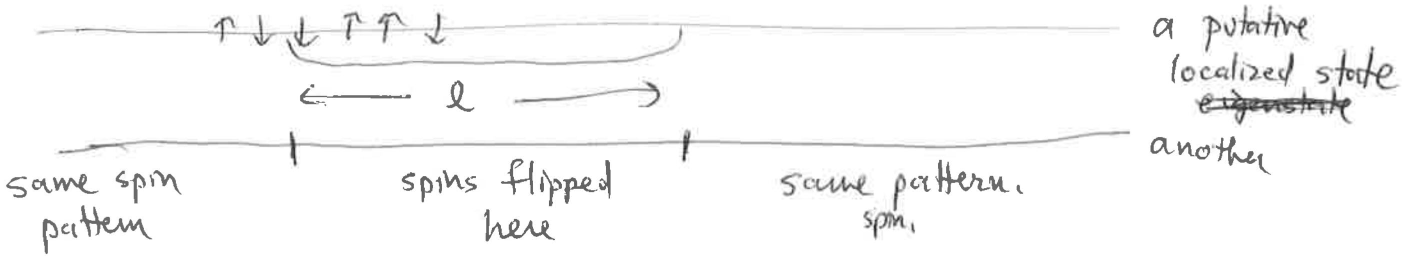
<u>many-body resonances.</u> Villalonga + Clark

Crowley + Chandran, SciPost 2022

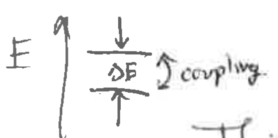
Garratt, Roy, Chalker, PRB 2021

Morningstar, et al PRB 2022

Naive estimate of many-body resonances (1D)



each state might be resonant at this patch of length l spins with $\sim 2^{l}$ other states. Closest in energy has

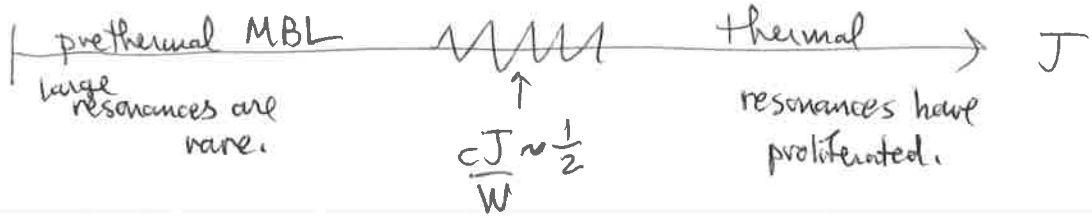


$\Delta E \sim 2^{-l}$, but distributed all the way to zero $= \Delta E$

Thinking about $J \vec{S}_i \cdot \vec{S}_{i+1}$ ~~H_2~~ perturbatively, the coupling between these two states $\sim \left(\frac{cJ}{W}\right)^{l}$ $(J<1)$ $H \sim \begin{pmatrix} 0 & \left(\frac{cJ}{W}\right)^{l} \\ \left(\frac{cJ}{W}\right)^{l} & 2^{-l} \end{pmatrix}$

c ← some constant

So for $c\frac{J}{W} < \frac{1}{2}$, these resonances become rare at large l, ~~so~~ most states are naively stable to such large many-body resonances at most locations. This is the naive estimate of the prethermal MBL – thermal crossover:



In more than 1D l is the number of spins in a patch where there might be a resonance. So this rough picture is not special to 1D.

Boulder Weds ↓

Our survey of many-body resonances

H closed system

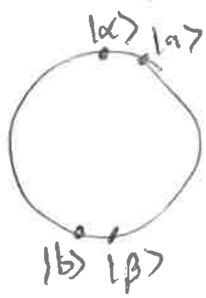
look at pairs of eigenstates adjacent in energy, $|\alpha\rangle, |\beta\rangle$
with end-to-end difference in spin orientations.

In prethermal MBL regime:

$$H = J \sum_n \vec{S}_n \cdot \vec{S}_{n+1} + \sum_n h_n S_{nz}$$

h_n in $[-W, W]$

find most localized $|a\rangle, |b\rangle$ on this Bloch sphere

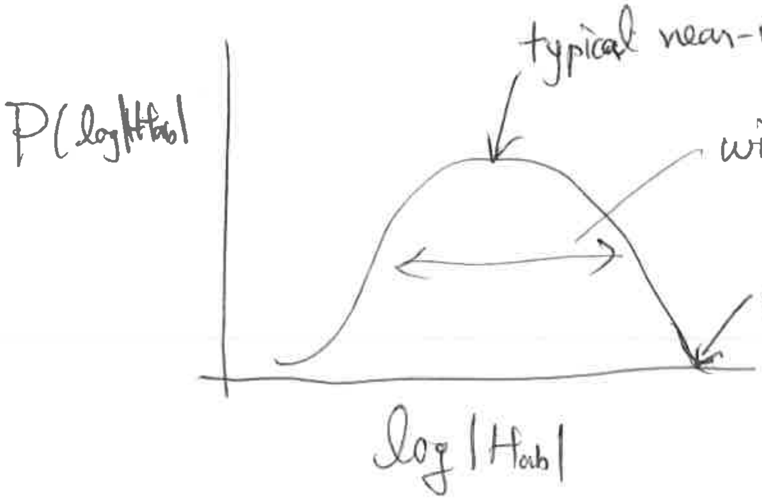


$$H = \begin{pmatrix} H_{aa} & H_{ab} \\ H_{ab}^* & H_{bb} \end{pmatrix}$$

Find ~~H_{ab} is~~ $\log|H_{ab}|$ is broadly distributed:

typical near-resonance $\sim \left(c_{typ} \frac{J}{W}\right)^L$

width grows strongly with L.



strongest resonances. (in each sample)

$\left(c_{max} \frac{J}{W}\right)^L$

appears that $c_{max} > c_{typ}$

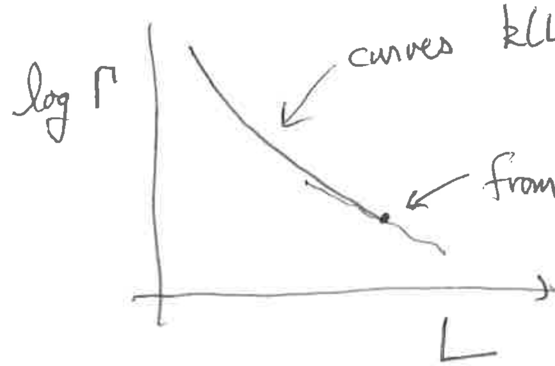
A next question (not answered yet):

do $C_{typ}(L)$, $C_{max}(L)$, "flow" with L?

If so, what is the physical mechanism?

(Possibly: large resonances are built out of smaller ones.)

We find

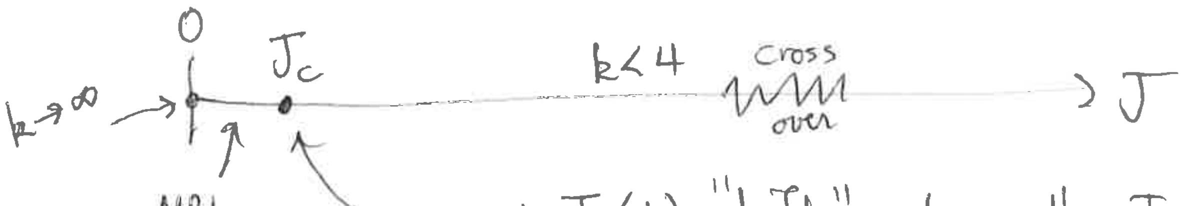


curves $k(L)$ "flows" larger L, smaller k

from slope at largest L we get an upper bound on $k(J)$

so upper bound on J_c

SKIP



MBL phase: avalanche stops. $k>4$

apparent $J_c(L)$ "drifts" to smaller J_c as L increases;

is factor of 5 or so smaller than location of crossover.

Ha, Morningstar, H 2301.04658

We looked in detail at the slowest mode of the open system (weakly coupled at one end to the bath) in terms of the eigenstates of the closed system:

spin L relaxes primarily via one (or a few) many-body near-resonances, so the avalanche spreads using some very special rare eigenstates.

This is a key reason J_c is so small: the avalanche instability uses resonances that are exponentially more coupled compared to the typical couplings that set the prethermal-thermal crossover.