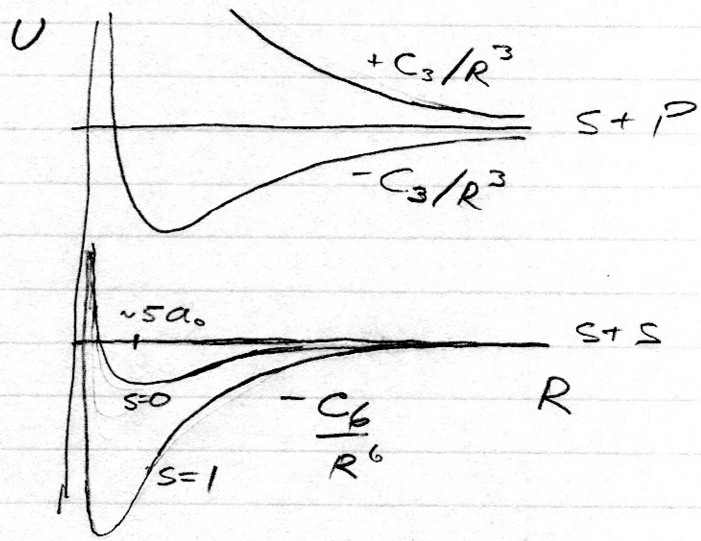
B

# ULTRACOLD COLLISIONS

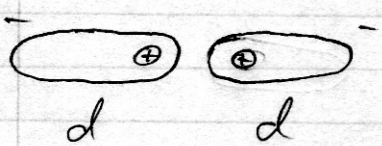
- J. WEINER ET AL, REV. MOD PHYS. 71, 1 (1999)
- PETHICK AND SMITH
- METCALF AND VAN DER STRATEN

ELASTIC - NEEDED FOR EVAPORATIVE COOLING
INELASTIC - LIMITS DENSITY

POTENTIALS




GROUND STATE:
INDUCED DIPOLE
(VAN DER WAALS)




$$U = -\int_0^E \vec{d}\cdot d\vec{E}$$

$$= -\alpha\int_0^E \vec{E}\cdot d\vec{E} = -\tfrac{1}{2}\alpha E^2, \quad \text{BUT } \vec{E} \simeq \vec{d}/R^3$$

$$\Rightarrow U = -\frac{C_6}{R^6}$$

EXCITED STATES: RESONANT DIPOLE INTERACTION

DIPOLE MOMENT BETWEEN THE S AND P STATES

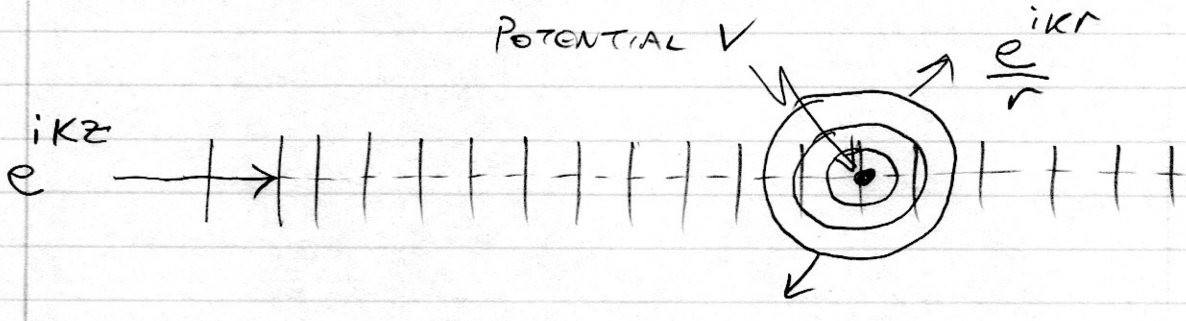
$$\Rightarrow U = \pm C_3/R^3 \quad \text{LONG-RANGE}$$

FOR $S_1 = \frac{1}{2}$, $S_2 = \frac{1}{2}$ $\Rightarrow$ $S = 0, 1$ $\Rightarrow$ SINGLET OR TRIPLET

- THESE POTENTIALS GENERALLY SUPPORT BOUND VIBRATIONAL STATES
- $H_{HF} \propto \vec{I}\cdot\vec{S}$ $\Rightarrow$ $[H_{HF}, S] \neq 0$

$\therefore$ INDIVIDUAL COLLISION CHANNELS ARE GENERALLY NOT PURE TRIPLET OR SINGLET.

PARTIAL WAVE ANALYSIS OF POTENTIAL SCATTERING
(JOACHAIN, "QUANTUM COLLISION THEORY")




ASSUME THAT $V$ IS CENTRAL (SPHERICALLY SYMMETRIC):

$$\psi(r,\theta,\phi) = R(r)\,Y_{\ell m}(\theta,\phi)$$

FOR $r \to \infty$, i.e. OUTSIDE THE RANGE OF $V$:

$$\psi \to Ae^{ikz} + A\,f(\theta,\phi)\frac{e^{ikr}}{r}$$

$f(\theta,\phi) = f(\theta)$ FOR SPH. SYMM IS <u>SCATTERING AMPLITUDE</u>

$f(\theta)$ ACCOUNTS FOR THE EFFECT OF THE POTENTIAL

ALSO, FOR $r \to \infty$,

$$R(r) \simeq A\frac{e^{i(kr+\frac{\ell\pi}{2})}}{r} - B\frac{e^{-i(kr-\frac{\ell\pi}{2})}}{r}$$

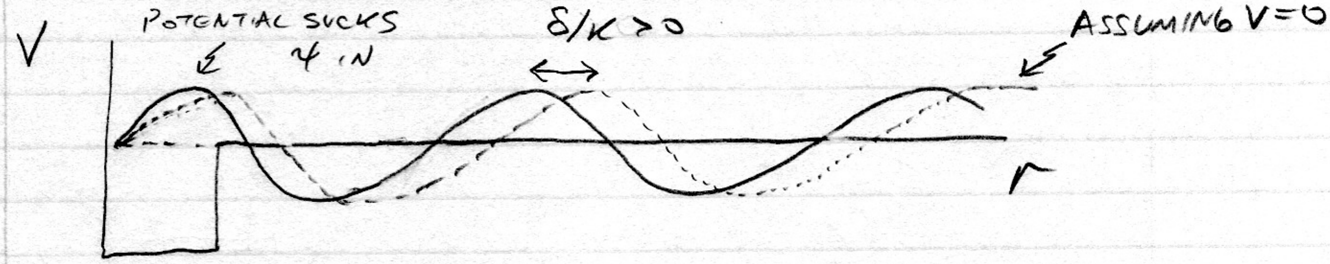
INCOMING SPHERICAL WAVE — OUTGOING

INCOMING AND OUTGOING FLUXES ARE EQUAL (ELASTIC)

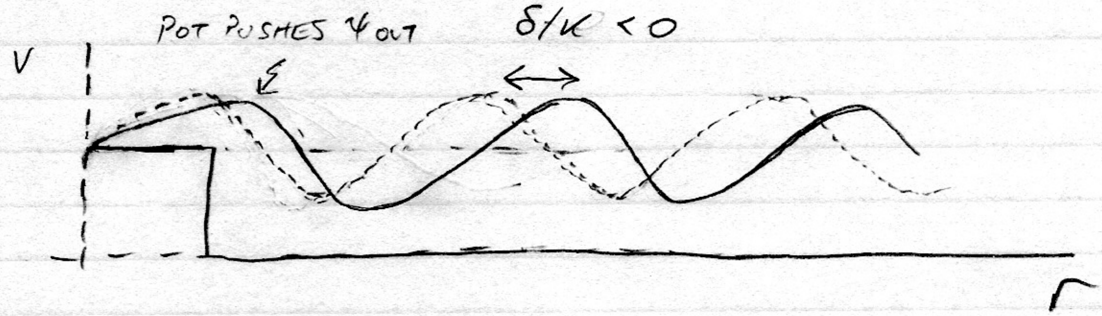
$$\Rightarrow R(r) \simeq e^{i\delta_\ell}\,\frac{\sin(kr - \frac{\ell\pi}{2} + \delta_\ell)}{kr}$$

$\delta_\ell$ = PHASE SHIFTS; THESE CONTAIN THE EFFECT OF $V$.

EX SQUARE WELL




EX HARD SPHERE




IT TURNS OUT THAT

$$f(\theta) = \frac{1}{K} \sum_{l=0}^{\infty} (2l+1) e^{i\delta_l} \sin\delta_l \, P_l(\cos\theta)$$

AND $\frac{d\sigma}{d\Omega} = |f(\theta)|^2$ = # PARTICLES SCATTERED INTO $d\Omega$ PER INCIDENT FLUX
= AREA

$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega = \frac{4\pi}{K^2} \sum_{l=0}^{\infty} (2l+1) \sin^2\delta_l = \sum_l \sigma_l$$

$$= \frac{4\pi}{K} \operatorname{Im} f(0) \quad \text{OPTICAL THEOREM}$$

THE USEFULNESS OF PARTIAL WAVE ANALYSIS COMES IF ONLY A FEW $\delta_l(K) \neq 0$

EACH $l$ CORRESPONDS TO A PARTICULAR ANGULAR SOLUTION TO THE S.E. WITH ANGULAR MOMENTUM $l$

$\therefore$ THERE IS A MAXIMUM $l$ DEPENDING ON THE RANGE $r_0$ OF $V$ AND INCIDENT ENERGY

$$l_{MAX}\hbar = \hbar K r_0 \Rightarrow l_{MAX} = K r_0$$

CONSIDER Na WITH $r_0 \simeq 100\, a_0$ :

$$K_B T = \frac{\hbar^2 K^2}{2m} \Rightarrow K r_0 \simeq 1.6 \text{ AT } 1\text{mK} \Rightarrow l_{MAX} \simeq 2$$

$$K r_0 \simeq .05 \text{ AT } 1\mu\text{K} \Rightarrow l_{MAX} \simeq 0$$
(S-WAVE)

$\therefore$ PARTIAL WAVE ANALYSIS IS VERY USEFUL FOR ULTRACOLD COLLISIONS

FOR S-WAVE SCATTERING :

$$\boxed{\begin{aligned} f(\theta) &= \frac{1}{K} e^{i\delta_0} \sin\delta_0 \\ \sigma &= \frac{4\pi}{K^2} \sin^2\delta_0 \end{aligned}} \quad \begin{aligned} &\Rightarrow \text{ ISOTROPIC} \\ &\leq \frac{4\pi}{K^2} \end{aligned}$$

THE S-WAVE SCATTERING LENGTH IS MORE USEFUL THAN $\delta_0$ :

$$a \underset{\lim K\to 0}{=} -\frac{\tan\delta_0}{K} \Rightarrow \boxed{\delta_0 \simeq -Ka}$$

AS $K \to 0$

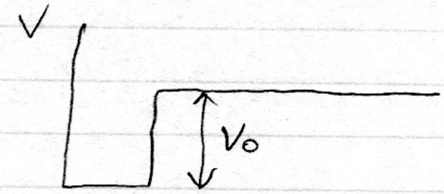
$$f(\theta) \simeq -a \quad \text{AND} \quad \boxed{\sigma \simeq 4\pi a^2}$$

$a > 0 \Rightarrow \delta < 0 \Rightarrow V$ REPULSIVE (eg HARD SPHERE)
$a < 0 \Rightarrow \delta > 0 \Rightarrow V$ ATTRACTIVE (eg SQUARE WELL)

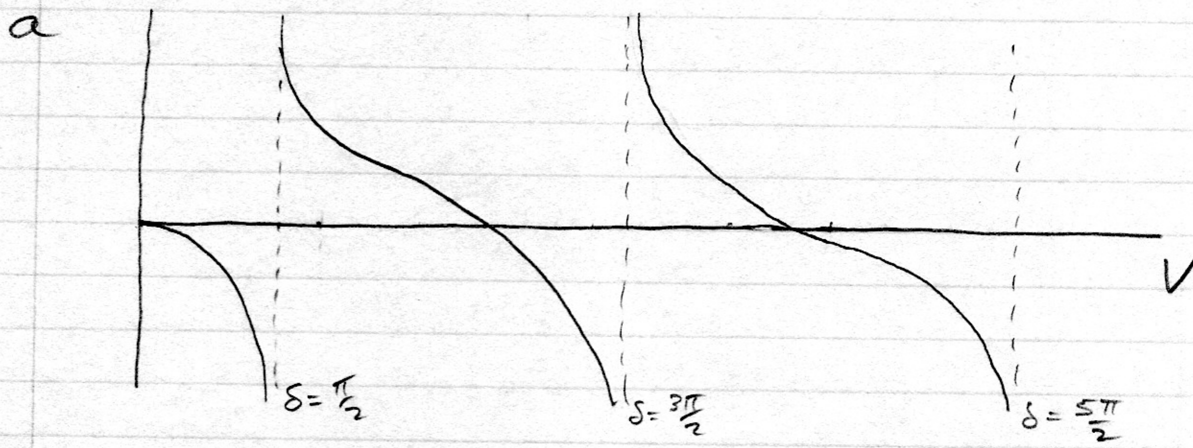
NOTE THAT $a \to \pm\infty$ FOR $\delta_0 = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

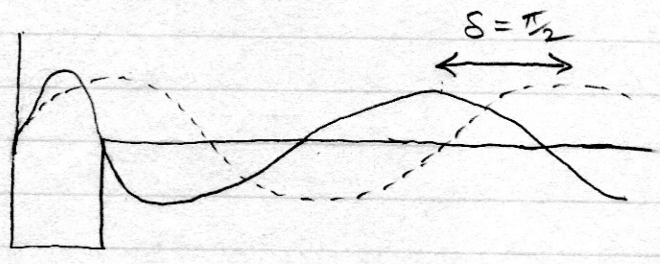
$\Rightarrow$ "ZERO-ENERGY RESONANCES."

CONSIDER SQUARE WELL




WHAT HAPPENS TO $a$ AS $V_0$ IS INCREASED?




$\Rightarrow$ 1 NODE APPEARS IN WELL

$\Rightarrow$ BOUND STATE

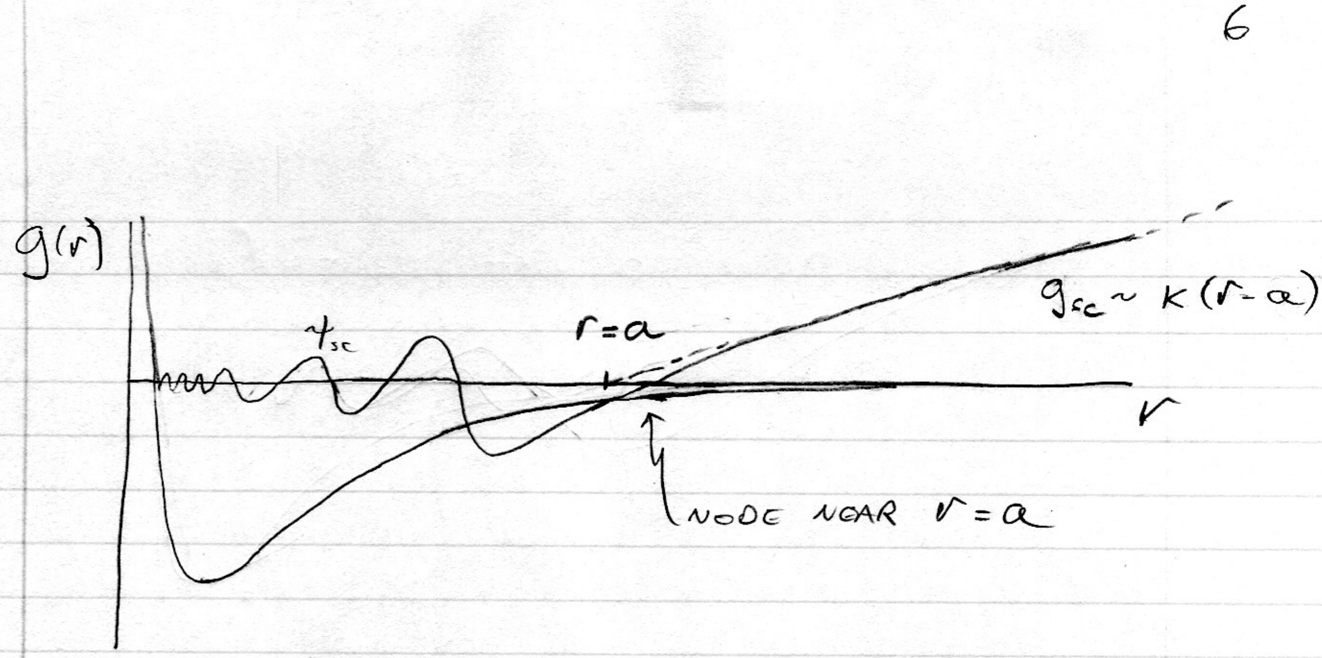
| | | | |
|---|---|---|---|
| $0 < \delta < \frac{\pi}{2}$ | 0 | BOUND STATES | $\Rightarrow$ ATTRACTIVE |
| $\frac{\pi}{2} < \delta < \pi$ | 1 | " " | REPULSIVE |
| $\pi < \delta < \frac{3\pi}{2}$ | 1 | " " | ATTRACTIVE |
| $\frac{3\pi}{2} < \delta < 2\pi$ | 2 | " " | REPULSIVE |

WHAT DOES THE SCATTERED WAVE LOOK LIKE?

$$\psi_{sc}(r) \sim \frac{\sin(kr + \delta_0)}{r}$$ FOR S-WAVE SCATTERING $\Rightarrow k \to 0$

BUT $\delta_0 \simeq -ka$

$$\Rightarrow \psi_{sc}(r) \sim \frac{\sin k(r-a)}{r} \sim k(1 - a/r)$$




$a$ IS VERY IMPORTANT IN QUANTUM GASES BECAUSE IT DETERMINES WHETHER THE INTERACTIONS IN THE GAS ARE ATTRACTIVE OR REPULSIVE

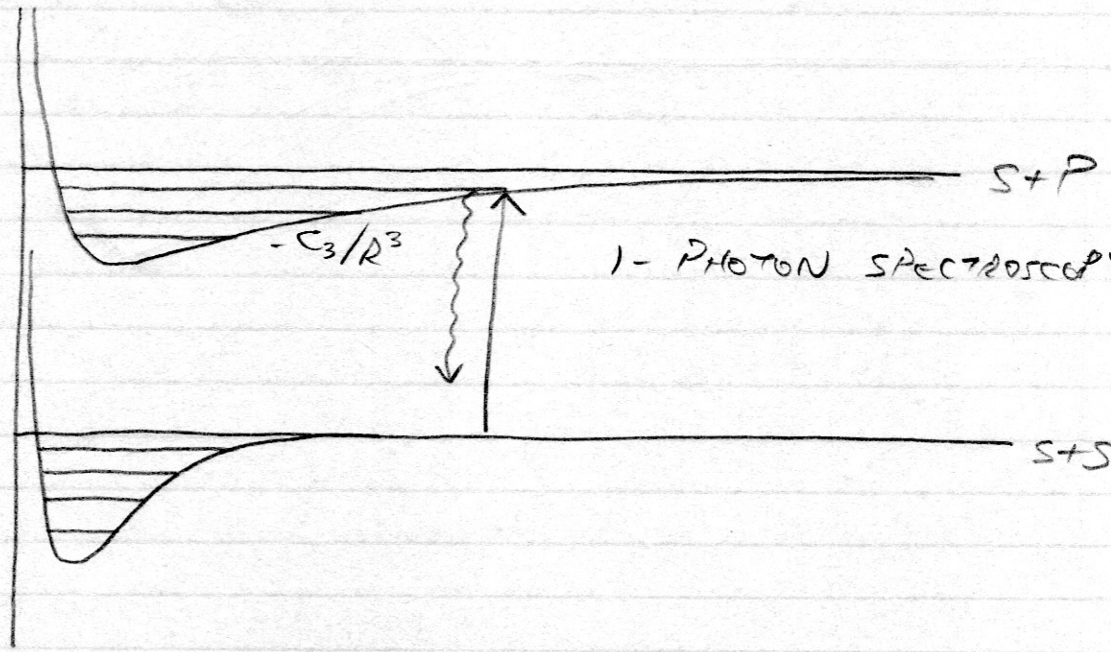
FOR BEC $a > 0$ : STABLE BEC
$a < 0$ : UNSTABLE TO COLLAPSE

FOR FERMIONS $a > 0$ : WEAKLY-BOUND BOSONIC MOLECULES
$a < 0$ : COOPER PAIRING

MEAN-FIELD ENERGY $U = \dfrac{4\pi\hbar^2 a n}{m}$

C

PHOTOASSOCIATION - BEST WAY TO MEASURE $a$




- ABSORPTION OF A LASER PHOTON TUNED TO A MOLECULAR RESONANCE CAUSES A SPONTANEOUS EMISSION

- S.E. LEADS TO LOSS OF ATOMS BY DECAYING TO BOUND MOLECULES OR BY HEATING ATOMS
  $\Rightarrow$ SIGNAL IS LOSS OF ATOMS.

- LINEWIDTHS GIVEN BY ENERGY OF COLLIDING ATOMS
  1 mK $\Rightarrow$ 21 MHz
  $\therefore$ HIGH PRECISION

- RESONANCE FREQUENCIES OF MOLECULAR TRANSITIONS GIVES NEW INFORMATION ON LONG-RANGE PART OF THE EXCITED STATE POTENTIAL

eg $C_3 \propto |\langle s|d|p\rangle|^2 \propto \gamma$

$\Rightarrow$ SPECTRAL MEASUREMENT OF ATOMIC LIFETIMES!
Li PRECISION OF $10^{-4}$ $\Rightarrow$ MOST PRECISE MEASUREMENT EVER.

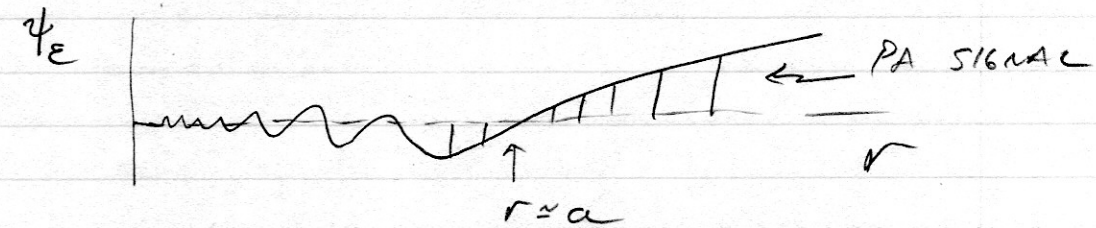
- 1-PHOTON PA ALSO GIVES INFORMATION ABOUT G.S. POTENTIAL

SIGNAL $\propto |\langle E | v' \rangle|^2$

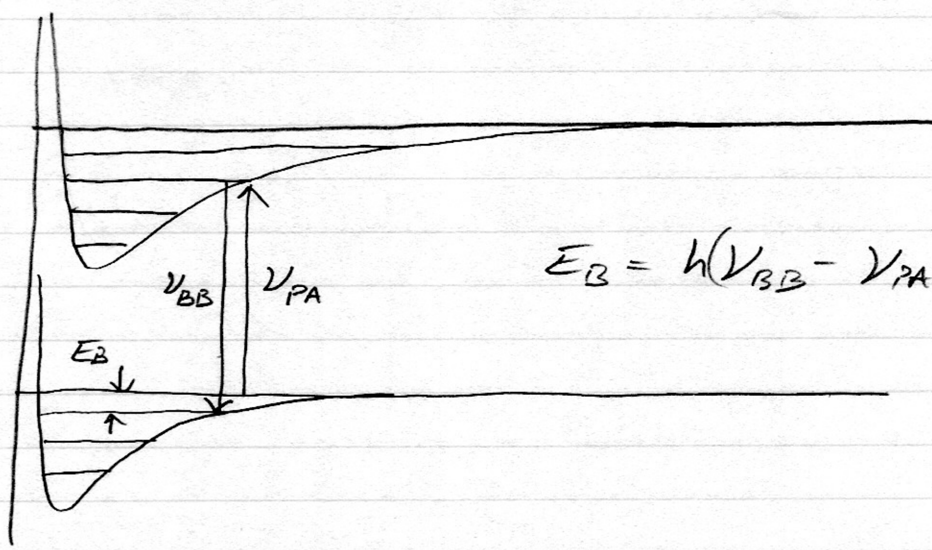
FREE ATOMS — VIBRATIONAL LEVEL OF EXCITED STATE

$\propto \int \psi_E \, \phi_{v'} \, d^3r$ OVERLAP INTEGRAL

$\therefore$ SIG GOES AWAY NEAR NODE'S IN $\psi_E$




- 2-PHOTON P.A. MEASURES G.S. LEVELS DIRECTLY




$$E_B = h(\nu_{BB} - \nu_{PA})$$

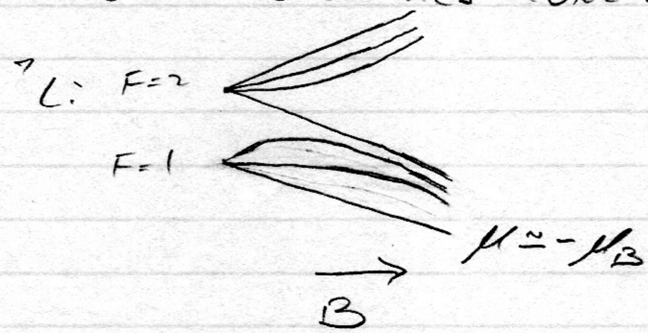
$a$ DEPENDS <u>SENSITIVELY</u> ON $E_B$ OF LEAST BOUND STATE

- WE HAVE MEASURED $E_B$ IN Li (S=1) TO 1 kHz $\Rightarrow 10^{-7}$ PRECISION.
  $\Rightarrow a$ KNOWN TO $\sim 1\%$

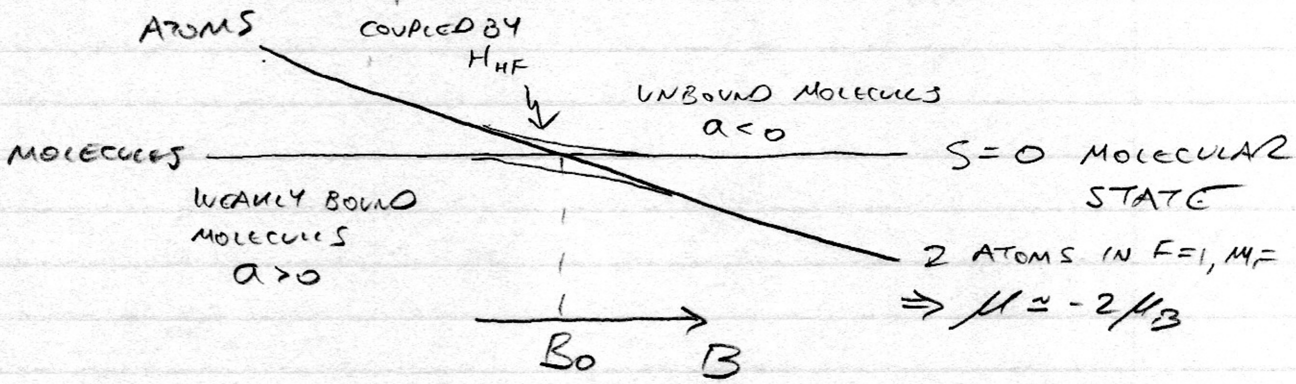
FESHBACH RESONANCE – "CONTROL KNOB" FOR $a$

$S=0$ & $S=1$ POTENTIALS TUNE DIFFERENTLY WITH B-FIELD

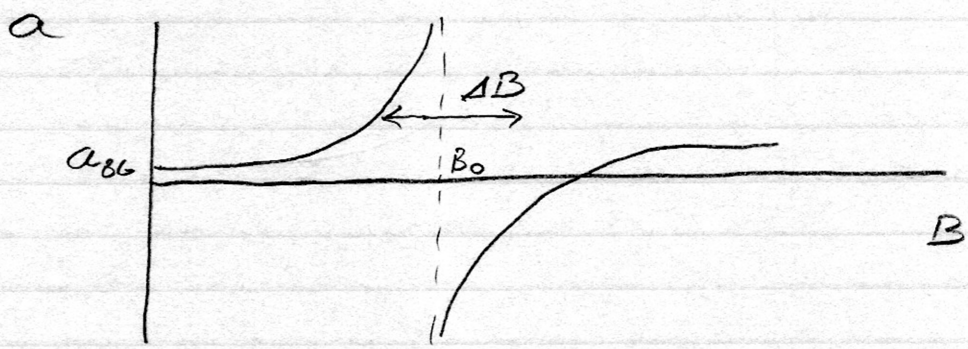
eg




ATOMS IN F=1, M=1 STATES ARE e SPIN-POLARIZED AT SUFFICIENTLY HIGH-FIELD $\Rightarrow S \simeq 1$




PAIR OF FREE ATOMS ARE RESONANT WITH A MOLECULAR LEVEL AT $B_0$




- $a = a_B\left(1 - \dfrac{\Delta B}{B - B_0}\right) \Rightarrow$ CONTROLLABLE INTERACTIONS
- ADIABATICALLY RAMP THROUGH RESONANCE $\Rightarrow$ CONVERSION OF ATOMIC GAS TO A MOLECULAR GAS – M. BEC.