A

FERMI DEGENERACY

ANTI-SYMMETRIC WRT PARTICLE EXCHANGE:

$$\Psi(1,2) = -\Psi(2,1) \Rightarrow \tfrac{1}{2}\text{-INTEGER SPIN PARTICLES.}$$

eg
ELECTRONS, PROTONS, NEUTRONS } BUILDING BLOCKS OF MATTER
^{2}D, ^{6}Li, ^{40}K } COMPOSITE FERMIONS

^{6}Li AND ^{40}K ARE THE ONLY STABLE ALKALI METAL FERMIONS

PAULI EXCLUSION PRINCIPLE:
- NO TWO IDENTICAL FERMIONS CAN OCCUPY THE SAME QUANTUM STATE
- CONSEQUENCE OF EXCHANGE SYMMETRY

PARTICLE COORDINATES

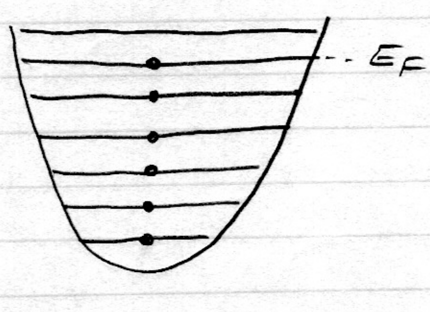
$$\Psi(1,2) = \frac{1}{\sqrt{2}}\left[\psi_a(1)\,\psi_b(2) - \psi_a(2)\,\psi_b(1)\right]$$

QUANTUM #'S

(ANTI-SYMMETRIC)

$\therefore$ IF $a = b$, $\Psi(1,2) = 0$

$\therefore$ AT $T=0$



$$E_F = \frac{\hbar^2 K_F^2}{2m} \qquad K_F = \text{FERMI WAVEVECTOR}$$

HOW MANY STATES ARE CONTAINED WITHIN THE FERMI SPHERE OF RADIUS K_F?

$$N = \int \rho_k \, d^3k = \frac{V}{(2\pi)^3}\left(\frac{4}{3}\pi K_F^3\right)$$

$$\Rightarrow K_F = \left(6\pi^2 \frac{N}{V}\right)^{1/3} = \left(6\pi^2 n\right)^{1/3}$$

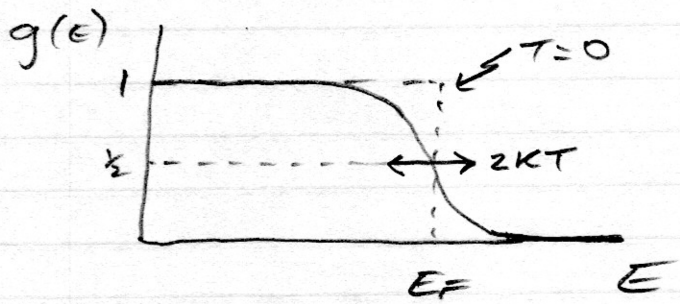
$$\therefore \boxed{E_F = \frac{\hbar^2}{2m}\left(6\pi^2 n\right)^{2/3}} \Rightarrow K_F = \left(6\pi^2 n\right)^{1/3}$$

FOR ^6Li @ $n = 10^{13}\ cm^{-3}$, $T_F = \frac{E_F}{K_B} \simeq 3\mu K$

DEGENERACY $\Rightarrow$ $T \ll T_F$

FD DISTRIBUTION FUNCTION:

$$g(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$



FOR FERMIONS $T_F \propto n^{2/3}$

FOR BOSONS $n\Lambda^3 = \text{CONST} \Rightarrow T_C \propto n^{2/3}$

$$\frac{T_F}{T_C} = 2.29 \quad \text{FOR } m_B = m_F$$

$\therefore$ ALTHOUGH THERE IS NO PHASE TRANSITION, DEGENERACY OCCURS AT APPROXIMATELY THE SAME T.

WE CAN ALSO EXPRESS T_F IN TERMS OF N FOR A HARMONIC TRAP:

FOR A HARMONIC OSC $g(E) = \dfrac{E^2}{2\hbar^3 \bar{\omega}^3}$

WHERE $\bar{\omega} = (\omega_1 \omega_2 \omega_3)^{1/3}$

$$\therefore \quad N = \int_0^{E_F} g(E)\,dE = \frac{E_F^3}{6\hbar^3 \bar{\omega}^3}$$

$$\Rightarrow \boxed{E_F = \hbar\bar{\omega}(6N)^{1/3}}$$

(FOR BOSONS $K_B T_C = \hbar\bar{\omega}(N/1.2)^{1/3}$)

COOLING FERMIONS:

EVAPORATIVE COOLING OF FERMIONS IS NOT AS STRAIGHT FORWARD AS FOR BOSONS:

- NEED A WAY TO RETHERMALIZE ATOMS TO EVAPORATE THEM.

HOWEVER, SPIN-POLARIZED FERMIONS AT LOW TEMPERATURE ARE <u>NON</u>-INTERACTING.

WHY? LOOK AT ANY 2-PARTICLE QUANTUM STATE:

$$\Psi_{12} = \underbrace{\psi(\vec{r}_1, \vec{r}_2)}_{\substack{\text{S-WAVE} \\ \Rightarrow \text{EVEN SYMMETRY}}} \underbrace{\chi(s_1, s_2)}_{\substack{\text{SPIN POLARIZED} \\ \Rightarrow S = s_1 + s_2 \Rightarrow \text{EVEN SYMMETRY}}}$$

$\therefore \ \psi_{12} \rightarrow +\psi_{21}$ FORBIDDEN BY THE PAULI PRINCIPLE

$\therefore$ NEED SOMETHING ELSE TO ENABLE RETHERMALIZATION:

- 2-SPIN STATE MIXTURE

  ^6Li: $I = \frac{1}{2} \Rightarrow$ ONLY 3 MAGNETICALLY TRAPPABLE STATES

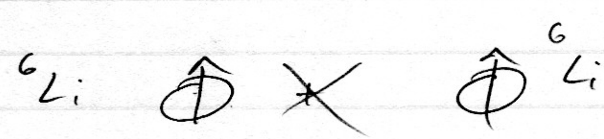
  SPIN MIXTURES OF THESE ARE NOT STABLE AGAINST SPIN-EXCHANGE.

  HOWEVER, COULD USE AN OPTICAL TRAP

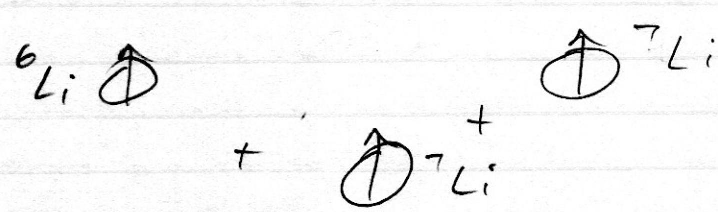
  ^{40}K: $I = 4 \Rightarrow$ 9 MAGNETICALLY TRAPPABLE STATES AND SOME OF THESE ARE STABLE

- SYMPATHETIC COOLING

  A SECOND SPECIES (USUALLY A BOSON eg ^7Li, Na, Rb) IS USED AS A "REFRIGERANT":



INSTEAD:

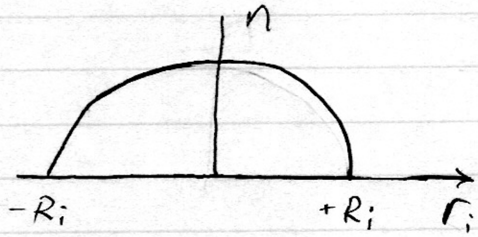


DENSITY DISTRIBUTION

FOR $T \ll T_F$, $$\underbrace{\frac{\hbar^2 k_F^2(\vec{r})}{2m}}_{\text{LOCAL FERMI ENERGY}} + \underbrace{V(\vec{r})}_{\text{TRAP POTENTIAL}} = \underbrace{\mu}_{\text{CHEMICAL POTENTIAL} = E_F}$$

THIS IS THE ENERGY COST OF ADDING ONE ATOM.

SINCE $$n(\vec{r}) = \frac{1}{6\pi^2} k_F^3(\vec{r})$$
$$= \frac{1}{6\pi^2}\left[\frac{2m}{\hbar^2}\left(\mu - V(\vec{r})\right)\right]^{3/2} = n_0\left[1 - \frac{V(\vec{r})}{\mu}\right]^{3/2}$$



WHERE $\frac{1}{2} m \omega_i^2 R_i^2 = \mu \Rightarrow R_i \simeq 1.9\, N^{1/6} \ell$

$$\ell_i = \sqrt{\frac{\hbar}{m\omega_i}}$$

$\therefore$ THE NON-INTERACTING $T=0$ FERMI DISTRIBUTION HAS A NON-ZERO SIZE $\Rightarrow$ FERMI PRESSURE

ALSO ENERGY/PARTICLE $= \epsilon \propto \beta E_F$ AT $T=0$, WHERE $\beta \sim 1$ AND DEPENDS ON POTENTIAL

INTERACTIONS

- SINGLE COMPONENT FERMI GAS : NON-INTERACTING AT LOW T

- TWO COMPONENT GAS : INTERACTION BETWEEN TWO UNLIKE FERMIONS $\Rightarrow a$

$$\therefore \quad \frac{n V_0}{E_F} = \frac{\frac{4\pi\hbar^2 a n}{m}}{\frac{\hbar^2}{2m} k_F^2} = \frac{8\pi a \, k_F^3/6\pi^2}{k_F^2}$$

$$= \frac{4}{3\pi} k_F a$$

SOME NUMBERS: ASSUME ^6Li, WITH $n \approx 10^{13}\ cm^{-3}$
$\Rightarrow T_F \simeq 3\ \mu K$ AND $a \simeq 100\ a_0$

$\therefore \ k_F \simeq 8.4 \times 10^4\ cm^{-1} \Rightarrow k_F a = 4.4 \times 10^{-2}$

AND $\frac{n V_0}{E_F} \simeq 2 \times 10^{-2} \Rightarrow$ INTERACTIONS ARE NORMALLY A SMALL CONTRIBUTION TO THE TOTAL ENERGY.

NOTE THAT $k_F |a| \ll 1 \Rightarrow n |a|^3 \ll 1$

i.e. THIS IS THE CRITERION FOR THE VALIDITY OF MEAN FIELD THEORY.

SUPERFLUIDITY:

FERMION PAIRING IS RESPONSIBLE FOR SUPERCONDUCTIVITY AND SUPERFLUIDITY IN ^3He.

IN SUPERCONDUCTORS, THE ATTRACTION BETWEEN e's IS INDUCED BY PHONONS IN THE CRYSTAL LATTICE.

SUPERFLUIDITY CAN ALSO OCCUR BY DIRECT S-WAVE ATTRACTION IN A TWO-COMPONENT FERMI GAS - BCS THEORY (LEGGETT) GIVES

$$T_C \simeq T_F \, e^{-\frac{1}{k_F |a|}}$$

(VALID FOR $k_F |a| \ll 1$ AND $a < 0$)

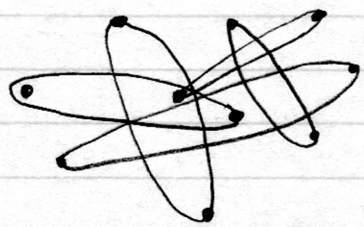
PUTTING IN NUMBERS:

FOR ^2H, $a = -8\,a_0 \Rightarrow T_c = 1\,fK$ @ $10^{19}\,cm^{-3}$

HOWEVER, FOR ^6Li, $a \approx -2200\,a_0 \Rightarrow T_c = 100\,nK$ @ $10^{13}\,cm^{-3}$

IN THE LATTER CASE, HOWEVER, $k_F|a| \sim 1$ AND BCS (MEAN-FIELD) THEORY ISN'T VALID.

FOR $k_F|a| \ll 1$, BCS THEORY GIVES COOPER PAIRS:



COOPER PAIRS LARGE
$\Rightarrow$ WEAK COUPLING LIMIT

FOR $k_F|a| \gtrsim 1$ STRONG COUPLING LIMIT
$\Rightarrow$ BEC OF MOLECULES

INTERMEDIATE REGIME $\Rightarrow$ BCS/BEC CROSSOVER