QUANTUM HALL EFFECTS

Lecture 2

- Results for $\nu = 1/2$ & environs
- Fractional Charges in FQHE
- Relation between quasi-particles & C.S. Fermions
- Other fractions with even denominator
- EFFECTS of BOUNDARIES and IMPURITIES on the INTEGER QHE.
- QHE in a strip of finite width: Role of Edge States
- Role of localized states due to disorder
- Why the Hall Quantization is "exact"
- Other topics

RESULTS of FCS Theory.

At $\nu = 1/2$: (In absense of impurities)

- Electron system is "compressible".
- Fluctuations in the electron density relax very slowly at long wavelengths:

- $\omega \propto i q^2$, for unscreened Coulomb Interactions.

- $\omega \propto i q^3$ for screened interactions: systems with a nearby gate; or bilayer systems.

- Longitudinal electrical conductivity vanishes as $\sigma_{xx}(q) \sim \frac{e^2}{8\pi\hbar} \frac{q}{k_F}$.

Longitudinal Conductivity $\sigma_{xx}(q)$ is measured in Surface Acoustic Wave experiment. q = wavevector of SAW.

Anomaly in SAW propagation at $\nu = 1/2$ first seen by Willett et al, in 1990:

- High-frequency SAW, wavelength small compared to mean free path of quasiparticles due to impurity scattering, see enhanced $\sigma_{xx} \propto q$ as predicted by theory for pure system.

- Low frequencies, wavelength long compared to mean free path; see ordinary dc conductivity, shows no anomaly at $\nu = 1/2$.

Slightly away from $f = 1/2$:

Fermions see effective magnetic field

$$\Delta B \equiv B - 4\pi n_e \equiv B - B_{1/2}$$

with $|\Delta B| \ll B$.

Fermions move in circular orbits with radius:

$$R_c^* = \frac{\hbar c}{e} \frac{k_F}{|\Delta B|}$$

- Orbit Diameter $2R_c^*$ has been measured via geometric resonance experiments:

using:
- Surface Acoustic Waves
- Superimposed Periodic Gate Structures

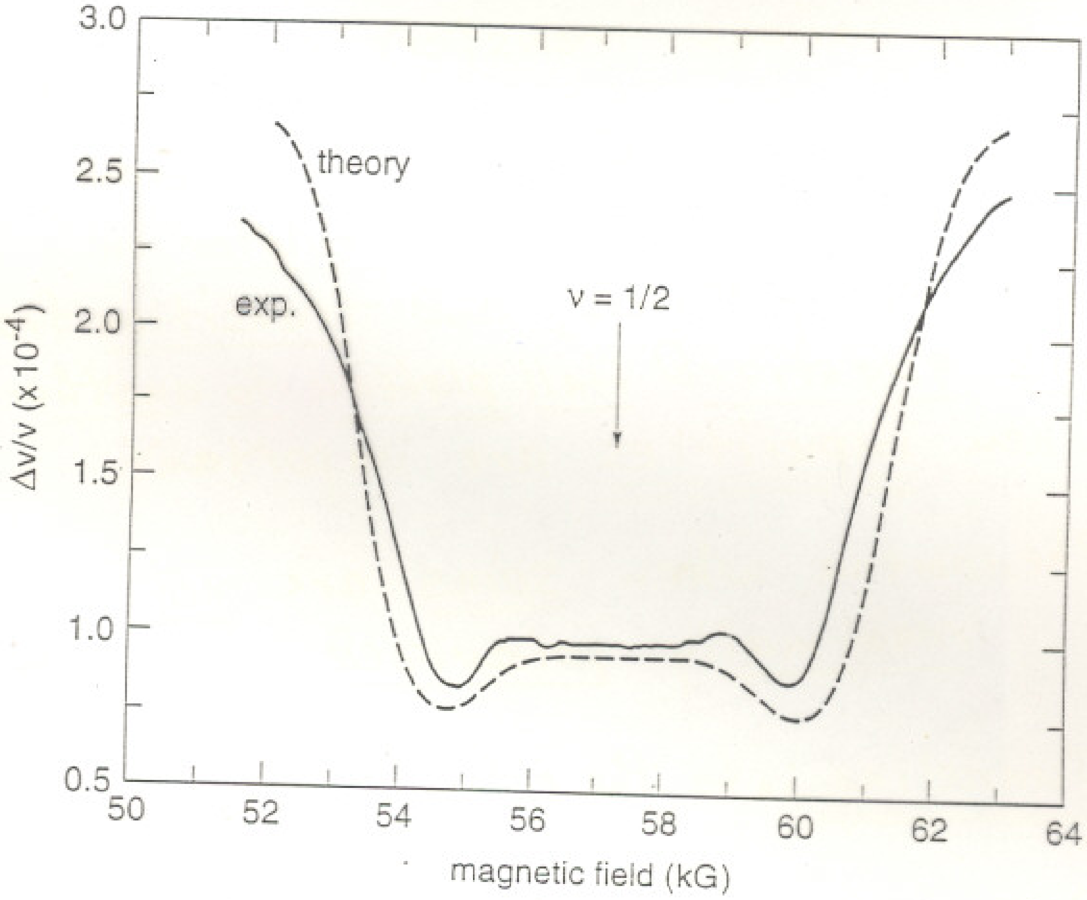
$\therefore$ Results agree with theoretical prediction.

- Measured values of $2R_c^*$, up to $\simeq 1\,\mu m$, order 100x actual cyclotron radius of electrons
- Dramatic confirmation of one aspect of composite Fermion theory.

Surface Acoustic Wave Data & "Theory"

(R. Willett, R. Ruel, K. West & L. Pfeiffer) (1993)

· 8,5 GHz



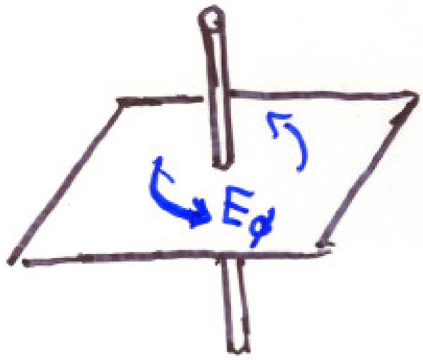
—— Data

-----Theory including sample inhomeneity (1.5% FWHM) + adjustment to overall conductivity scale of theory.*

* Theory = Halperin, Lee & Read (1993). Minima predicted at $\Delta B = |B - B_{1/2}| = 3.83 \frac{\hbar c}{e}$ q k_F

Existence of Fractional Charges

Laughlin Argument at $\nu = 1/3$



1. Insert SOLENOID of zero diameter at origin
2. Turn on current adiabatically until flux $\phi = \phi_0 \ (= \frac{1}{2\pi})$

- Azimuthal EMF $= 2\pi r E_\phi = \frac{d\phi}{dt}$

Far from origin:

- Radial current $j_r = \sigma_{xy} E_\phi = \frac{1}{3} \frac{E_\phi}{2\pi}$

- Cumulative charge near origin

$$q = -\int dt \ 2\pi r \, j_r = \frac{-e}{3}$$

But Hamiltonian with one extra flux at the origin is physically identical to original Hamiltonian H_0 (Differs only by a gauge transformation)

- So states with charge $\pm \frac{e}{3}$ are eigenstates of H_0.

At $\nu = \frac{2}{5}$:

adding ϕ_0 through solenoid gives charge $-\frac{2}{5} e$.

But there also exist stable charges of size $q = \pm \frac{1}{5} e$

Eg. Add flux $-2\phi_0$ + add one electron: get

$$q = -e + \frac{4}{5} e = -\frac{1}{5} e$$

Distinction between bare

"Chern - Simons Fermions"

& Low Energy "Quasi Particles"

Bare CS Fermions have Charge = e

Quasiparticles have charge = e^*

At $\nu = \frac{p}{2p+1}$: $e^* = \frac{e}{2p+1}$

At $\nu = 1/2$ $(p \to \infty)$ $e^* = 0$

Also: At $\nu = \frac{p}{2p+1}$, quasiparticles have

Fractional Statistics: $\Theta = \pi \cdot \left(\frac{2p-1}{2p+1}\right)$

At $\nu = 1/2$ $(p \to \infty)$: $\Theta \to \pi$ (Fermions)

Note: At $\nu = 1/2$: Low Energy Quasiparticles have $e^* = 0$, but have electric dipole moment: $\vec{d}$

Momentum: $\vec{p} \equiv \hat{z} \times \vec{d} / \ell^2$

(N. Read)

For $\vec{p} \neq 0$: Electron is off-center in hole. Local electric fields cause electron & hole to move with drift-velocity:

$$\vec{v} = \frac{\vec{E} \times \vec{B}}{B^2}$$

For $|p| \approx k_F$, quasiparticle is stabilized by Pauli exclusion principal & existence of a Fermi Surface

Effective Cyclotron Radius R_c^*

(near $\nu = 1/2$)

Chern Simons Fermion Picture

$$R_c^* = \frac{k_F}{e\, B_{eff}} \qquad B_{eff} = B - 4\pi n_e \rightarrow \frac{B}{2p+1}$$
$$= B(1-2\nu)$$

Quasiparticle Picture:

Quasiparticles have charge e^* but see full B:

$$R_c^* = \frac{k_F}{e^* B} \qquad e^* = e(1-2\nu) \rightarrow \frac{e}{2p+1}$$

Get same answer for R_c^*

Relation between bare CS Fermions + low energy quasiparticles

$\psi_F^+(\vec{r}_0, t_0)$: Operator Creates a CS Fermion at point $\vec{r}_0$, time t_0: $\approx$ product of

$\psi_e^+(\vec{r}_0, t_0)$: Creates an Electron at $\vec{r}_0, t_0$.

$\times\ U(\vec{r}_0, t_0)$: Turns on a solenoid flux $2\phi_0$ at $\vec{r}_0, t_0$
(INSTANTLY!)

$U \rightarrow$ Impulse Electric Field: $|E| = \frac{2}{|\vec{r}-\vec{r}_0|}\delta(t-t_0)$

in azimuthal direction.

$\Rightarrow$ HALL CURRENT:
For $t - t_0 \gg 1/\omega_c$: find electric charge has moved outward by amount $\frac{2}{|\vec{r}-\vec{r}_0|} \times \sigma_{xy}$

Leaves hole at origin with charge: $-2\nu e$

$\therefore$ Net Charge of screened CS Fermion is

$$(1-2\nu)e = e^*$$

$(e^* = 0$ at $\nu = 1/2)$

Balance of charge: $(e - e^*)$ - goes to boundary
(if sample is homogeneous in interior)

- Would like to build Fermi-liquid theory directly in terms of low-energy (neutral, dipolar) quasiparticles
  (not coupled to Chern Simons Gauge Field)

- Can be done in several different ways
  Shankar & Murthy; Haldane & Pasquier; Nick Read; Dung-Hai Lee;
  - S. Simon, Ady Stern, F. von Oppen & BIH.

- Physical Results, Electron Response Functions are the same as predicted by Fermion Chern Simons Theory.

  Fermi Liquid has some special peculiar properties!

Strengths of Fermion Chern Simons approach

Can address analytically the low energy and long wavelength behavior at $\nu = 1/2$.

& asymptotic behavior of quantum Hall systems (e.g. gaps in FQHE states) in the limit $\nu \rightarrow \frac{1}{2}$.

Weakness Not good for calculating absolute value of energy gaps, other properties which depend on accurate description of short-distance behavior. Important physical fact: that energy gaps $\rightarrow 0$ if $\frac{e^2}{\epsilon} \rightarrow 0$ is hidden in FCS approach.

OTHER EVEN FRACTIONS

$\nu = 1/4$ $\quad [\tilde{\phi} = \underline{4}]$ ✓

$\nu = 1/6$ $\quad [\tilde{\phi} = \underline{6}]$ — but

experiments (+ calculations) suggest that for $\nu \lesssim 1/6$ ground state is a Wigner X'tal $\Rightarrow$ for $\tilde{\phi} \gtrsim 6$, Fermi surface is unstable to formation of Charge Density Waves

$\nu = \frac{3}{2} = [1] + 1/2$

$\nu = 3/4 = [1] - 1/4$

$\nu = \frac{5}{4} = [1] + 1/4$

$\nu = 3/8 = [1/3]$ + "Fermi liquid" of $\frac{1}{3}$-charge quasiparticles : Statistics $\theta = \pi/3$, with added $\tilde{\phi} = 8/3 \rightarrow$ converts quasiparticles to Fermions. $\langle B_{eff} \rangle = 0$ at $\nu = 3/8$.

Even-Denominator Fractions where Quantized Hall Effect Is Observed

BCS Pairing of Composite Fermions ??

1. Double Layer Systems: $\nu_{total} = 1/2$.

($\nu = 1/4$ in each layer)

Most likely explanation: "331 wavefunction"
$\approx$ pairing of electrons in different layers.

Fermion-Chern-Simons description (roughly):
BCS pairing - p-wave ("$p_x + ip_y$")
Pseudospin $S=1$, $S_z=0$ $\Rightarrow$
Energy gap at Fermi surface $\longrightarrow$ FQHE state.

2. Single Layer $\nu = \frac{5}{2}$ ($\equiv \nu = \frac{1}{2}$ in second L.L.)

Possibilities A: Fully Spin Aligned in second L.L.: *

May be "Pfaffian" state of Read & Moore.
Something like p-wave BCS pairing with $S=1$, $S_z=1$.

B: Haldane-Rezayi Spin-Singlet State:

Equal numbers of spin up & spin down electrons,
$\approx$ s-wave pairing for electrons
$\approx$ d-wave BCS pairing for Chern-Simons Fermions

Even-Denominator Fractions where Quantized Hall Effect Is Observed

BCS Pairing of Composite Fermions ??

1. Double Layer Systems: $\nu_{total} = 1/2$.

($\nu = 1/4$ in each layer)

Most likely explanation: "331 wavefunction"

≈ pairing of electrons in different layers.

Fermion-Chern-Simons description (roughly):

BCS pairing - p-wave ("$p_x + ip_y$")

Pseudospin $S=1$, $S_z = 0$ ⟹

Energy gap at Fermi surface ⟶ FQHE state.

2 Single Layer $\nu = \frac{5}{2}$ ($\equiv \nu = \frac{1}{2}$ in second L.L.)

Possibilities A: Fully Spin Aligned in second L.L.: *

May be "Pfaffian" state of Read & Moore -

Something like p-wave BCS pairing with $S=1$, $S_z=1$.

B: Haldane-Rezayi Spin-Singlet State:

Equal numbers of spin up & spin down electrons,

≈ s-wave pairing for electrons

≈ d-wave BCS pairing for Chern-Simons Fermions

"Pfaffian" State at $\nu = \frac{5}{2}$

($\nu = \frac{1}{2}$ in second Landau Level)

Has elementary excitations with

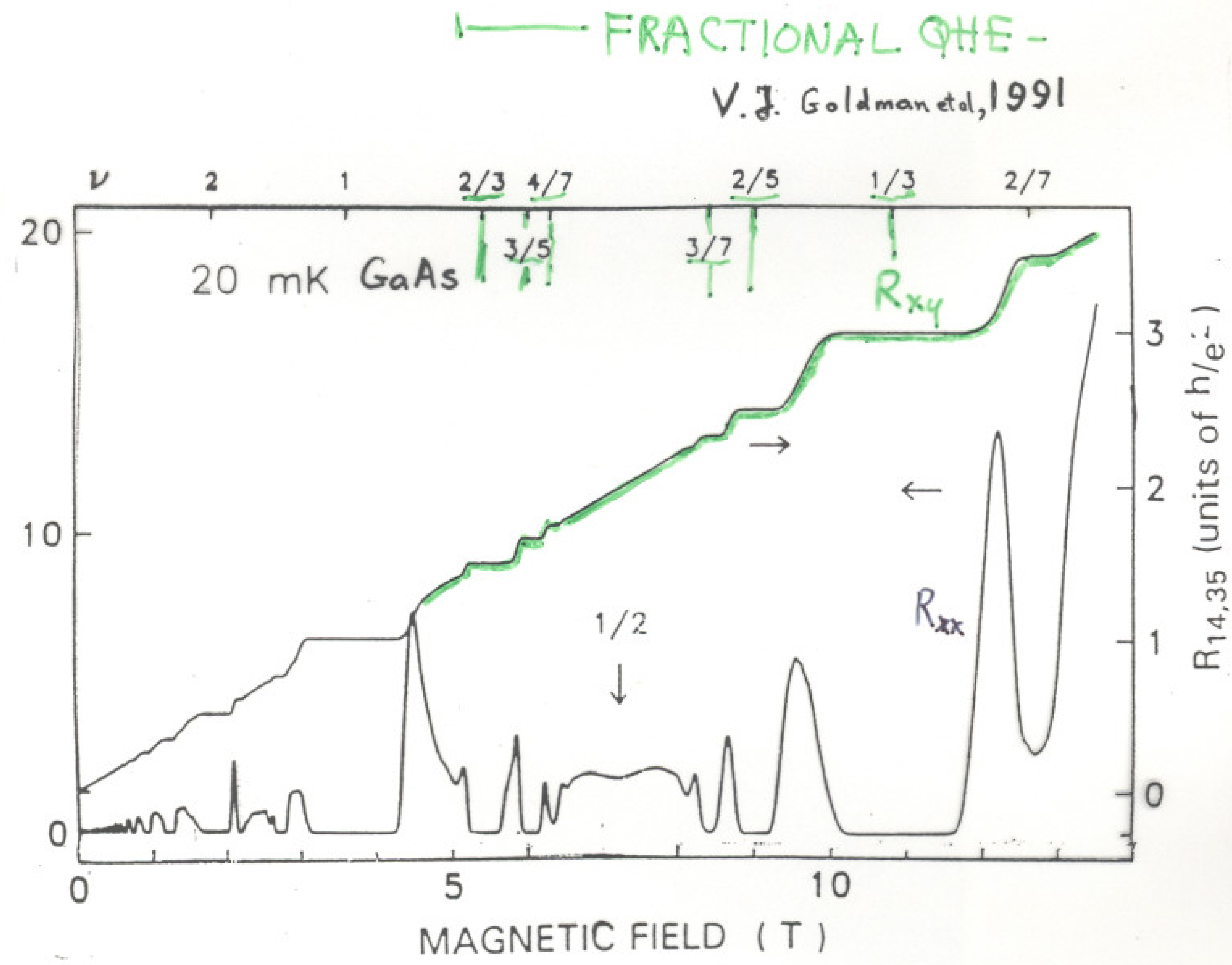
charge $= \pm \frac{e}{4}$

which obey "non-abelian" statistics

Interchanging quasiparticles in diffent orders can lead to physically different states (ie states which do not just differ by a phase factor.)

May be useful for quantum computing!

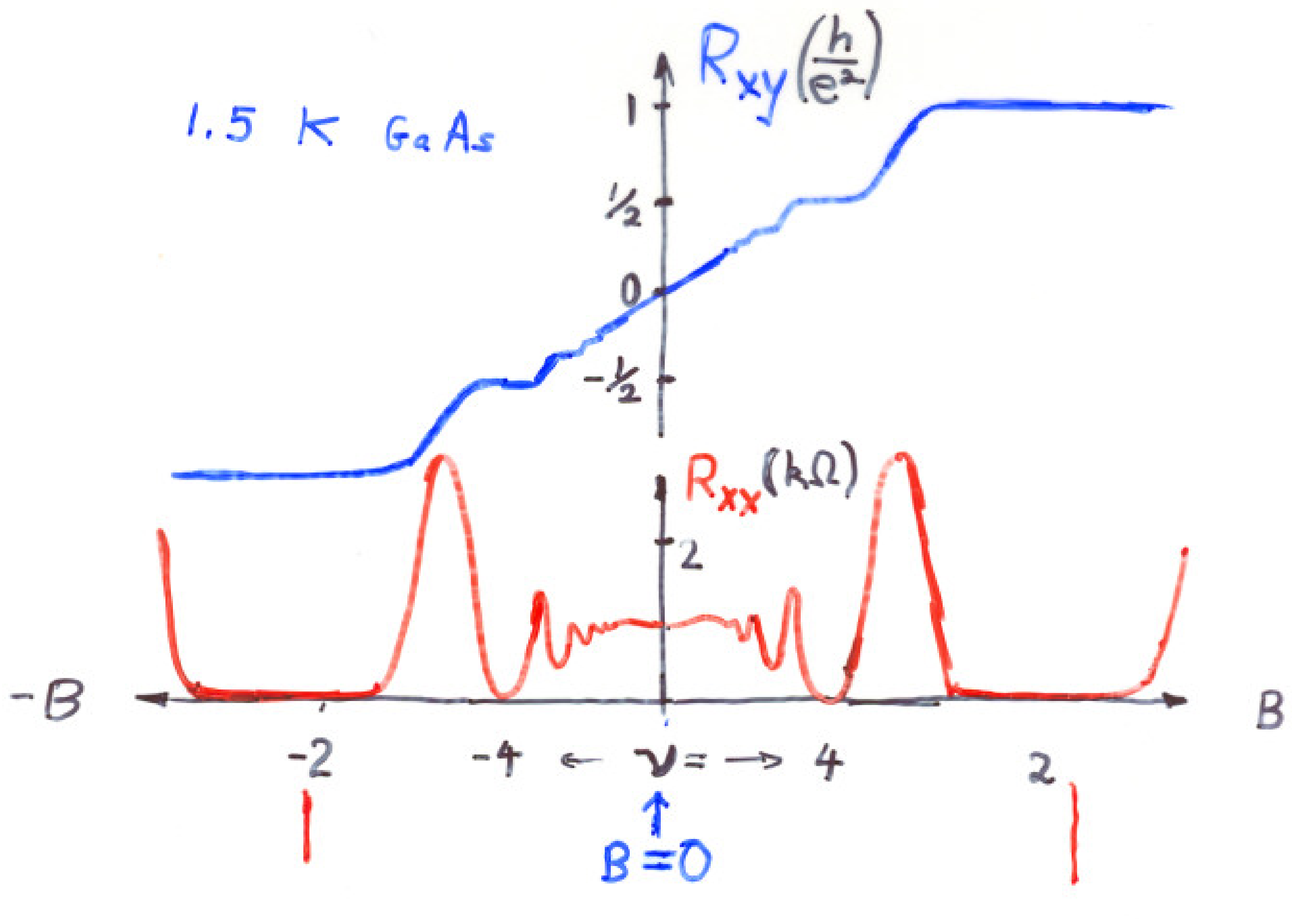
(Kitaev...)



FRACTIONAL QHE -
V. J. Goldman et al, 1991
ν
2
1
2/3
4/7
3/5
3/7
2/5
1/3
2/7
20 mK GaAs
R_{xy}
R_{xx}
1/2
20
10
0
3
2
1
0
0
5
10
MAGNETIC FIELD (T)
$R_{14,35}$ (units of h/e^2)

INTEGER QHE

K. von Klitzing, 1981



Replotted by D.B. Chklovskii

QUANTUM MECHANICS: UNIFORM BACKGROUND : LANDAU GAUGE

$$\vec{E}_0 \parallel \hat{y}, \quad \vec{B}_0 \parallel \hat{z} \qquad I \parallel \hat{x}$$

Let $\vec{A} \equiv A_x = B_0 y$

$$\Psi(\vec{r}) = e^{ikx} \phi_{k,\nu}(y)$$

$$\left[\frac{P_y^2}{2m^*} + \frac{1}{2m^*}\left(\frac{eB_0}{c}\right)^2 (y-y_k)^2 + V(y) \right] \phi_{k,\nu}(y)$$

$$= \epsilon_{k,\nu} \phi_{k,\nu}(y)$$

$\frac{e}{c} A_x(y_k) \equiv \hbar k$; $\Rightarrow y_k = \frac{k}{B_0} \frac{\hbar c}{e}$ $\qquad k = \frac{2\pi N_x}{L_x}$

Far from edges: Set $V_0(y) = 0$:

$\vec{E}_0 = 0 \Rightarrow \epsilon_{k,\nu} = (\nu + \frac{1}{2}) \hbar \omega_c$ $\qquad \nu = 0, 1, 2 \ldots$

$$\langle I \rangle_\psi = \frac{1}{m^*} \int \psi^* \left(\hbar k - \frac{eA_x}{c}\right) \psi \, dy$$

$$= \omega_c \int |\psi|^2 (y - y_k) \, dy$$

$\langle I \rangle_\psi = 0$ for $\vec{E}_0 = 0$ by symmetry about $y = y_k$

FINITE SYSTEM:

Near edges : $|\psi| = 0$ at edge

Pushes energy up: $\epsilon_{k,\nu} > \hbar\omega_c(\nu+\frac{1}{2})$

$$\langle I \rangle_\psi = \frac{e}{L_x} \frac{\partial \varepsilon_{k,\nu}}{\hbar\, \partial k} \neq 0$$

Sign different at two edges

Net current at edges $= 0$ if Fermi Level is the same at two edges.

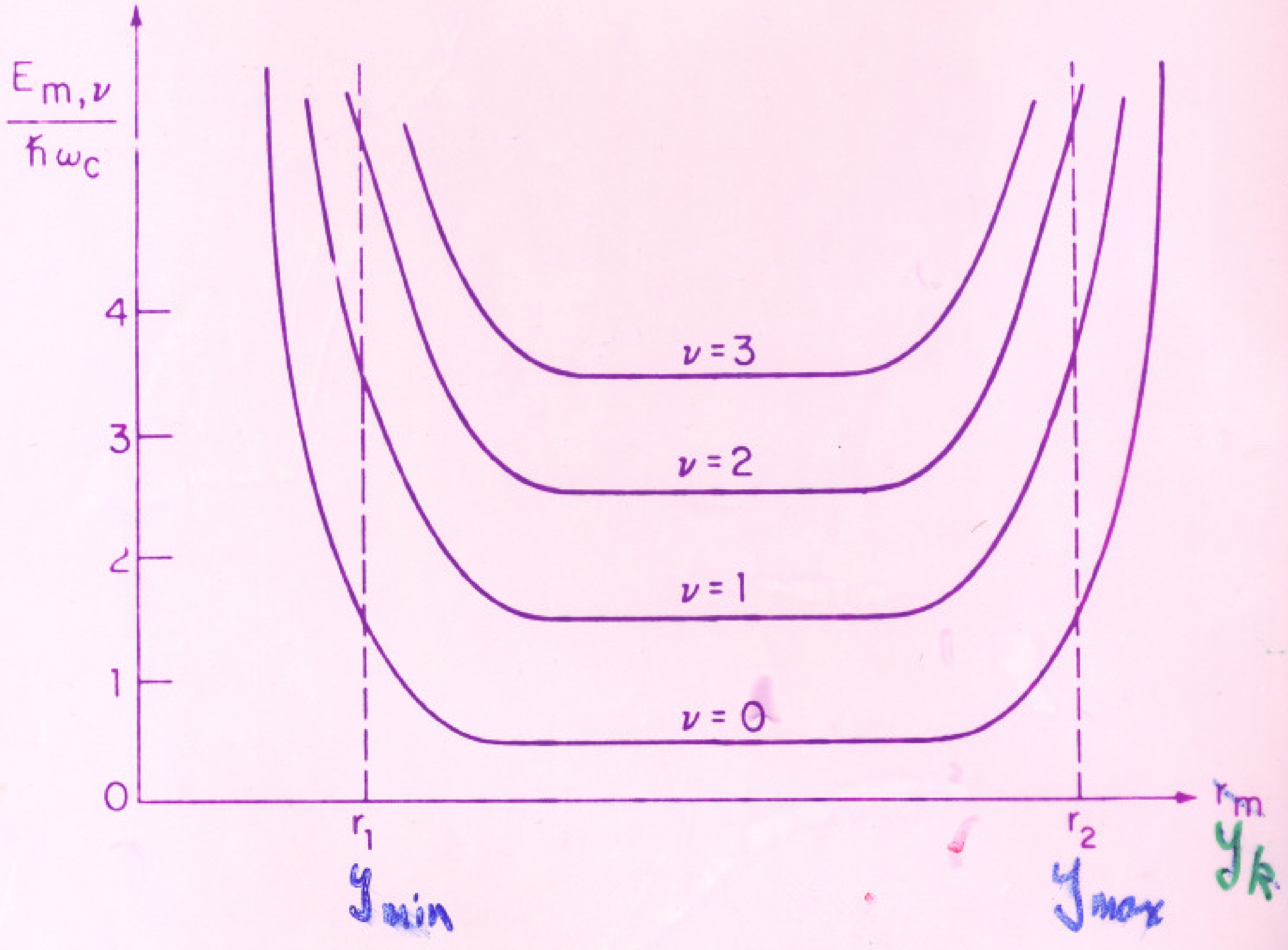
Separation between adjacent levels, $\delta k = \frac{2\pi}{L_x}$

$$\therefore \quad \delta\varepsilon = \frac{2\pi}{L_x} \left| \frac{\partial \varepsilon_{k,\nu}}{\partial k} \right|$$

$$\therefore |\langle I \rangle_\psi| = \frac{e}{h} |\delta\varepsilon|$$

If $\vec{E}_o = 0$, but Fermi Levels are unequal

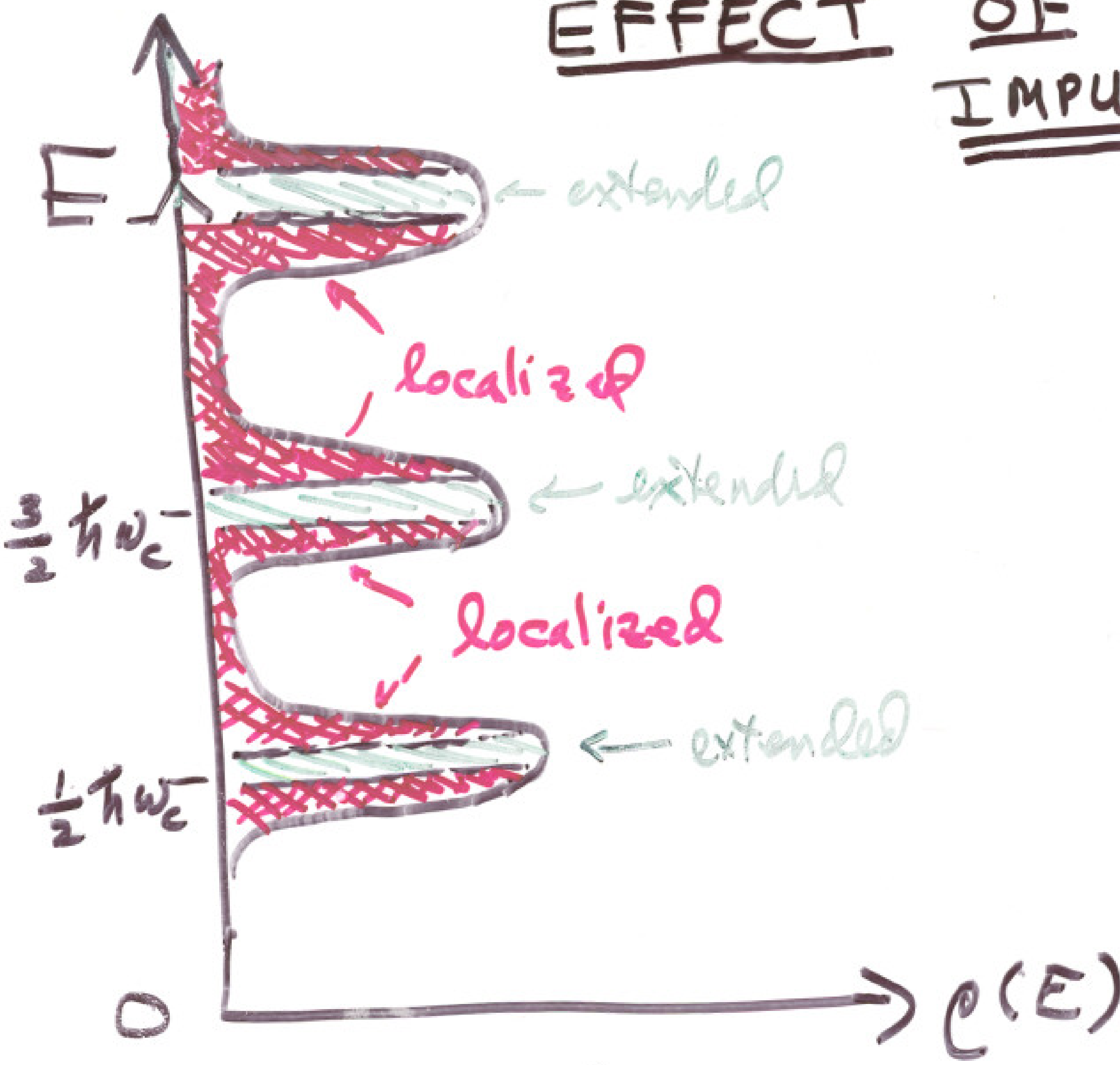
$$I = n \frac{e}{h} \left(E_F^{(1)} - E_F^{(2)} \right)$$



$\frac{E_{m,\nu}}{\hbar\omega_c}$
4
3
2
1
0
$\nu = 3$
$\nu = 2$
$\nu = 1$
$\nu = 0$
r_m
r_1
r_2
y_{min}
y_{max}
y_k

Fig. 2

EFFECT OF IMPURITIES

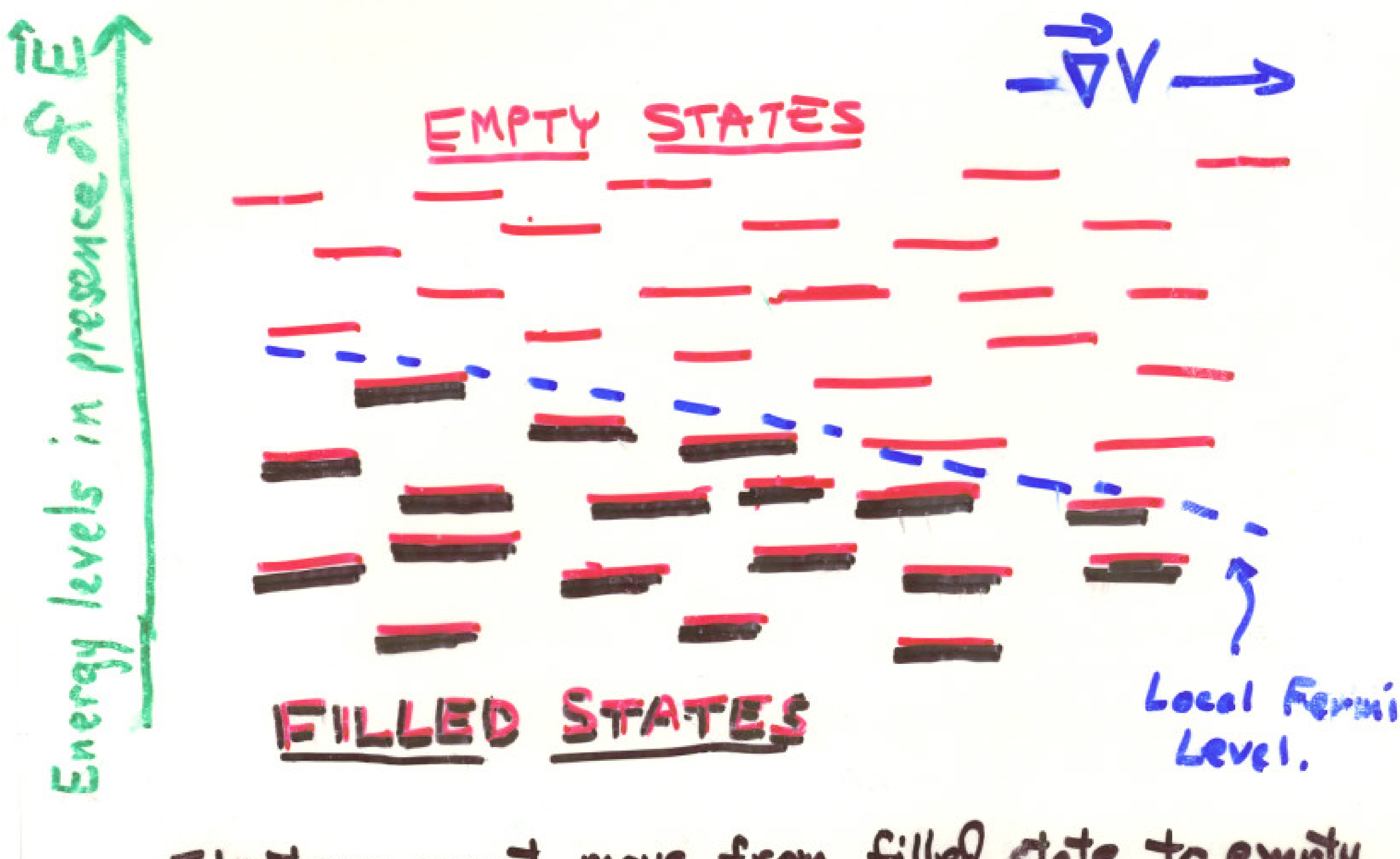


Density of States

- Integer Quantized Hall Effect occurs when the Fermi level is in a region of localized states.

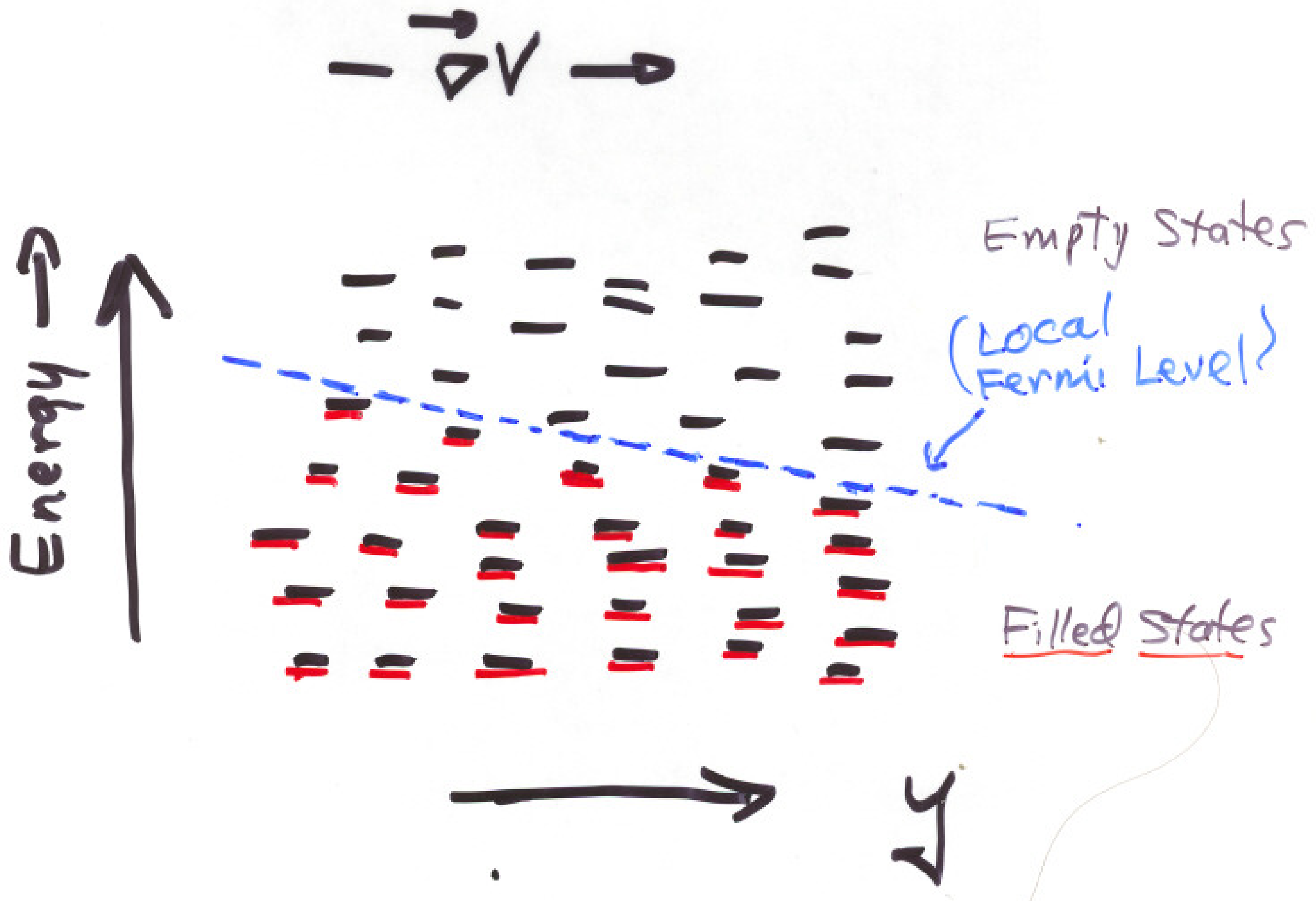
c8

If the Fermi Level is in an ENERGY REGION OF LOCALIZED STATES (or in an Energy Gap) there can be no current flow parallel to voltage drop and therefore no dissipation (for weak $\vec{E}$) at $T = 0$.



Electron must move from filled state to empty state of lower energy, a significant overlap. Zero probability for $T \to 0$, $\vec{\nabla}V \to 0$.

Localized states do not contribute to transport at T = 0.



An Extended State Below Fermi Energy can contribute to Hall Conductance $\vec{J} \perp \vec{E}$, but cannot contribute to longitudinal conductance ($\vec{J} \parallel \vec{E}$) because of energy conservation.

FINITE SYSTEM WITH IMPURITIES

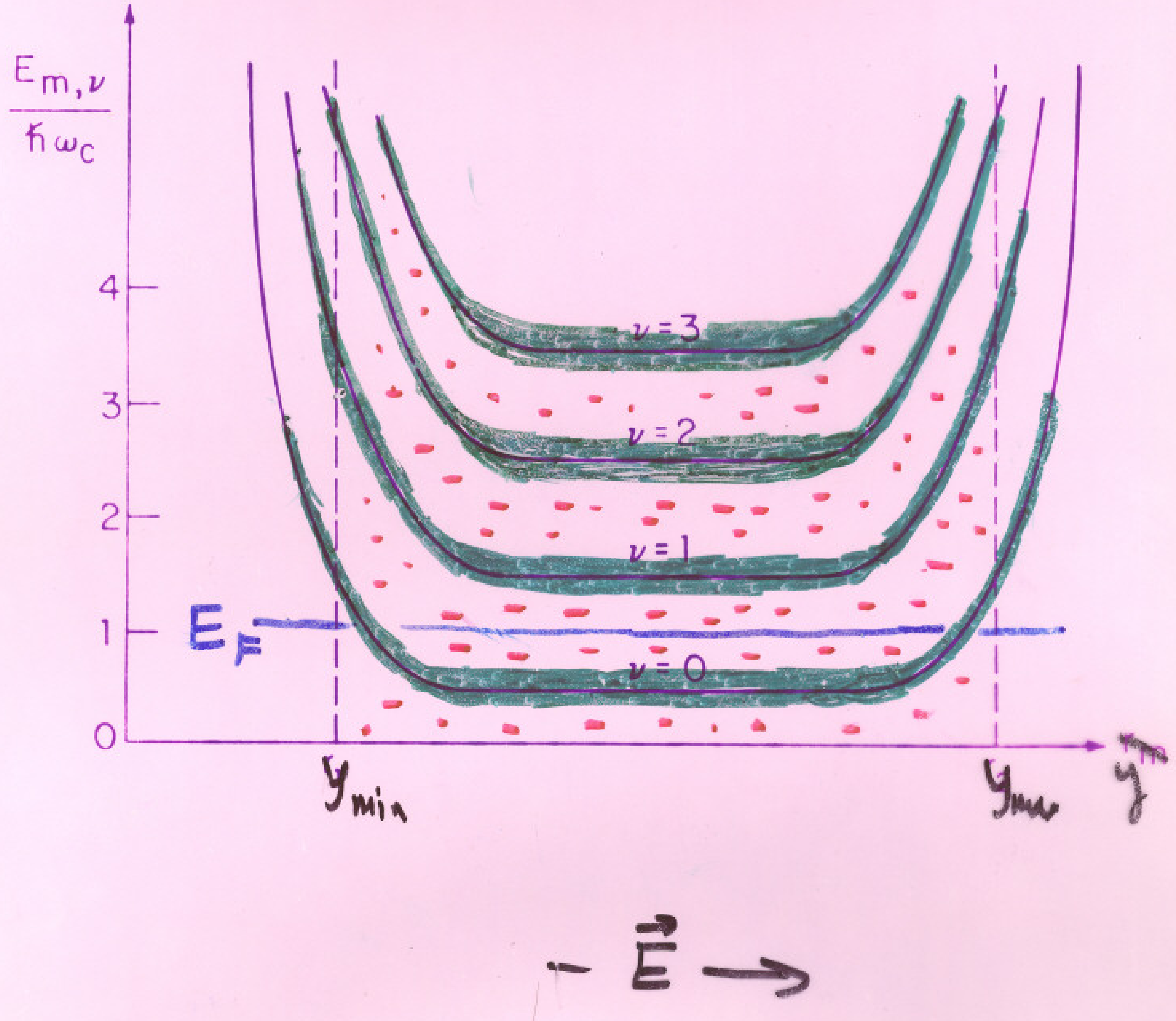
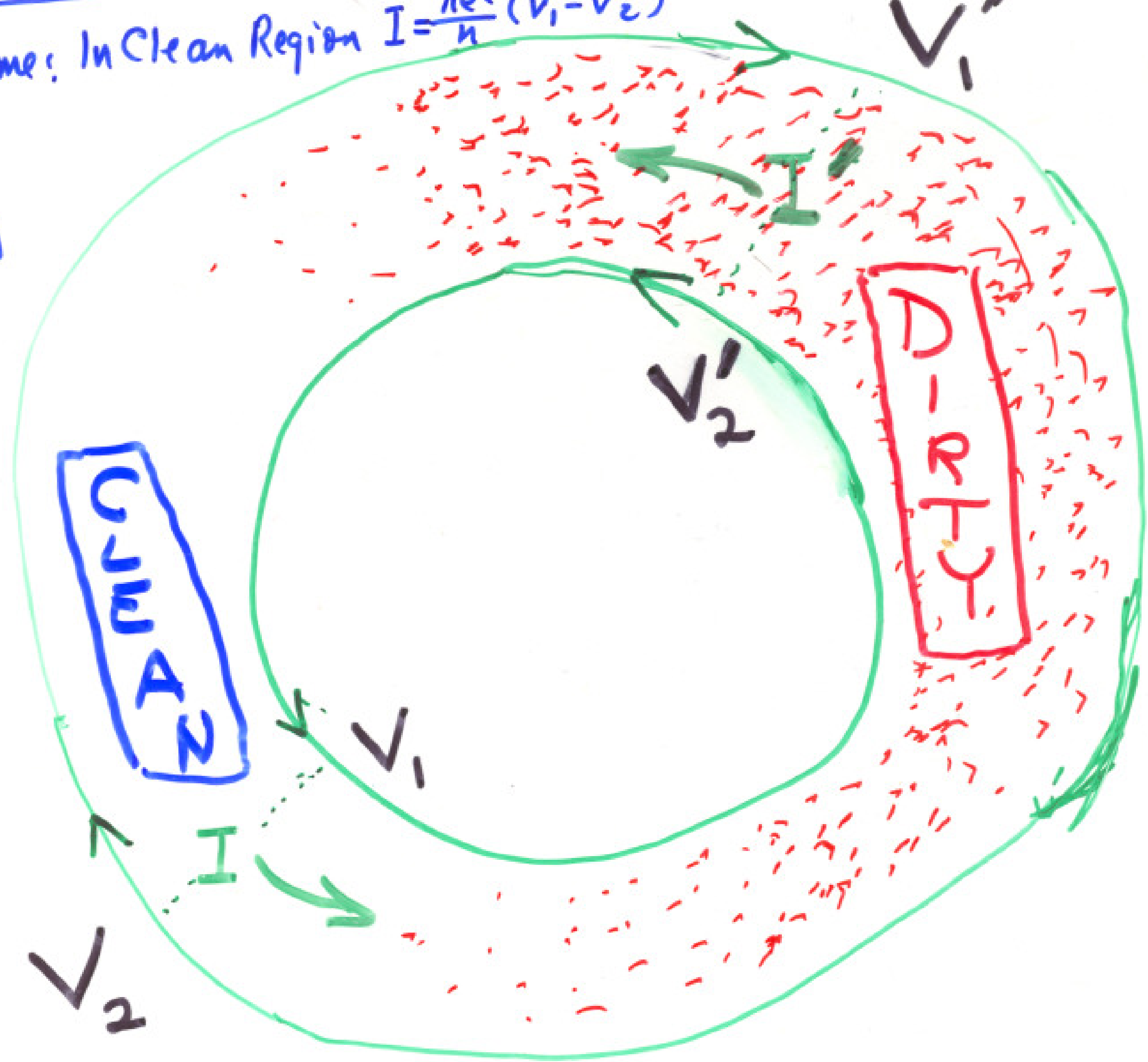


Fig. 2

QUANTIZATION of Hall Conductance in presence of Impurities

Gedanken Experiment

Assume: In Clean Region $I = \frac{ne^2}{h}(V_1 - V_2)$



Assume: In "bulk": at Fermi level:

Only localized states (or energy gap), including transition regions between clean and dirty:

$\therefore$ No dissipative flow (at $T=0$) from inside to outside edges $\Rightarrow$ Reach steady state with voltage a constant on inside edge; different constant on outside edge:

$V_1 = V_1'$, $V_2 = V_2'$; $I = I'$ by current conservation

$\therefore I' = \frac{ne^2}{h}(V_1' - V_2')$

Bi-Layer at $\nu_{total} = 1$

Small separation; Coulomb interactions between layers.

COHERENT PHASE: • "(111)" state

or: • "Quantum Hall (Pseudo)-Ferromagnet"

or "$\nu = 1$ with Bose Condensate of Excitons"

- Quantized Hall conductance for total current flow: $\left\{ \begin{array}{l} \sigma_{xx} = 0 \\ \sigma_{xy} = \frac{e^2}{h} \end{array} \right\}$
- Superfluid for antisymmetric currents (opposite in the 2 layers) $\{ \sigma_{xx} = \infty \}$
- Striking phenomena observed in drag experiments and in tunneling between layers.

Phenomena & Experimental Discoveries (Cont.)

- Phenomena Involving the Electron SPIN
  - Transitions between states of different spin polarization ($\approx$ 1987 +)
  - Spin depolarization near $\nu = 1$. (1995)
  - Coupling to nuclear spins.

- INSULATING STATE in very strong B (small ν):
  (1988+) • Wigner Crystal (?) in high mobility samples

- TRANSITIONS FROM QUANTIZED HALL STATE to INSULATOR, or Between Different Quantized HALL states when DISORDER is important. (1970's +)

- LARGE RESISTIVE ANISOTROPY observed in Higher Landau Levels (1998 ±)
  ($\nu \approx 9/2, 11/2, 13/2, 15/2, \ldots$)
  - Charge Density Wave Phase ? (Predicted 1996)

- MICROWAVE - INDUCED "Zero Resistance State"
  Moderate Magnetic Fields ($\nu \approx 50$) (2002)