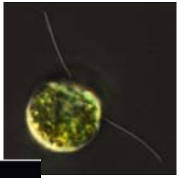
Fluid Dynamics of Swimming Cells – Jerry Gollub

- Began as a collaboration with Ray Goldstein's group: M. Polin, I. Tuval, K. Drescher, K Leptos, Adriana Pesci.
- Haverford participants: Jeff Guasto (Haverford); Huseyin Kurtuldu, Karl Johnson, Ivy Tao.
- Supported by NSF-DMR (Haverford) and several UK grants, and a Leverhulme trust Visiting Professorship.
- Fluid dynamics at small scales involving microorganisms.

Chlamydomonas reinhardtii



- •Green alga isolated from soil
- •Unicellular; Biflagellated
- •8-10 µm body size, 10-12 µm flagella length
- •Swims using breast strokes with frequency 50-60 Hz
- •Phototactic for λ < 620 nm

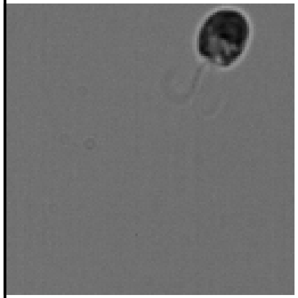
Some Questions

- How do the contraints of fluid dynamics at low Re affect the behavior of swimming cells?
- How do eukaryotic swimming microorganisms control their swimming to explore space (diffuse)?
- What flows are induced in the surrounding fluid, and what mixing is produced by this?
- What are the forces between the flagella and the fluid?

Background-Flagellar Coupling and Dynamics

- Cilia and flagella are highly conserved structures with many functions in biology.
  - » *Fluid transport in respiratory system*
  - » *Embryonic left-right asymmetry*
- Coordination or synchronization of the flagella can strongly influence their function.
- For *Chlamydomonas* algae, one of those functions is to explore space, either in the dark or in response to light.
- Exploration is understood for prokaryotes (bacteria), but not for eukaryotic cells.

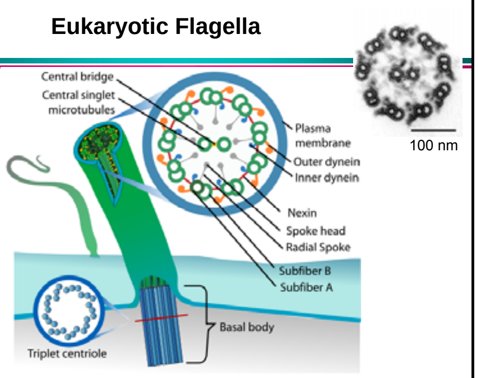
Flows Induced by Swimming Algae
J. Guasto, K. Johnson, and JPG



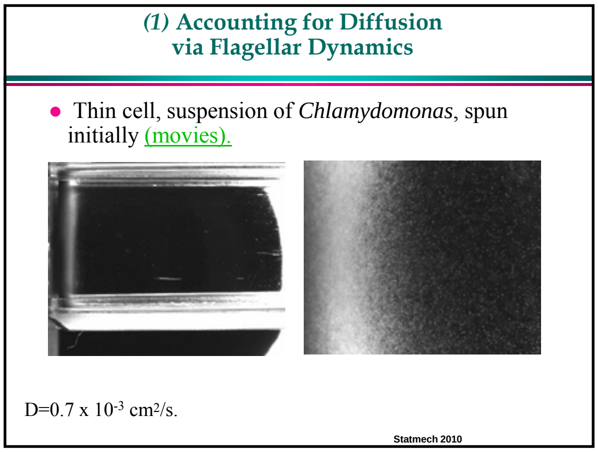
Swimming on a film, 500 fps

- •*Chlamydomonas reinhardtii* Green alga isolated from soil
- •Unicellular; Biflagellated
- •8-10 µm body size, 10-12 µm flagella length
- •Swims using breast strokes with frequency 50-60 Hz
- •Phototactic for λ < 620 nm

Eukaryotic Flagella



Mitchell, David R. : 'Chlamydomonas Flagella'. *Journal of Phycology* (2000)



(1) Accounting for Diffusion via Flagellar Dynamics

- Thin cell, suspension of *Chlamydomonas*, spun initially (movies).

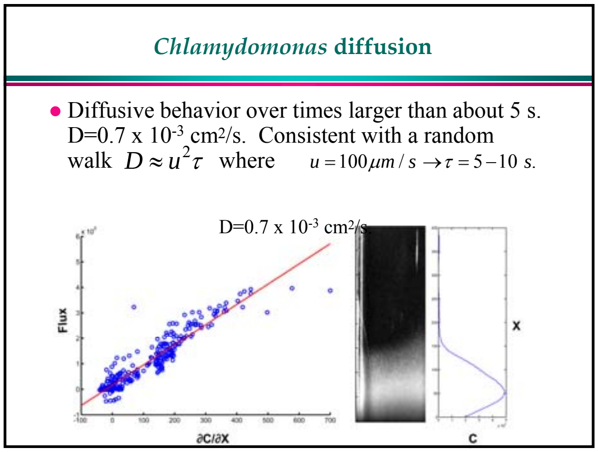
D=0.7 x 10^{-3} cm^2/s.

Statmech 2010

Demonstrating Diffusive Behavior

- How can we determine whether this spreading is a diffusive process?

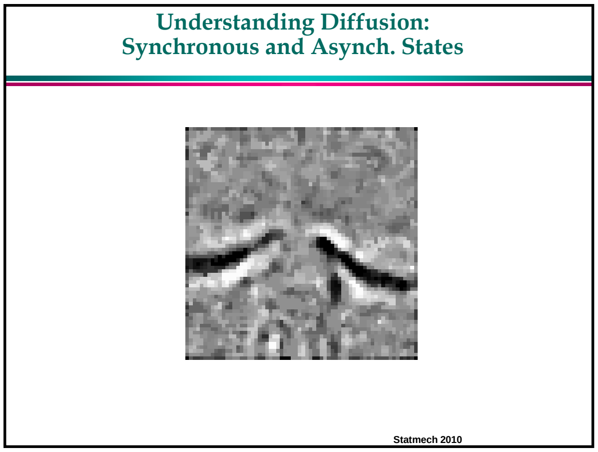
Statmech 2010



Chlamydomonas diffusion

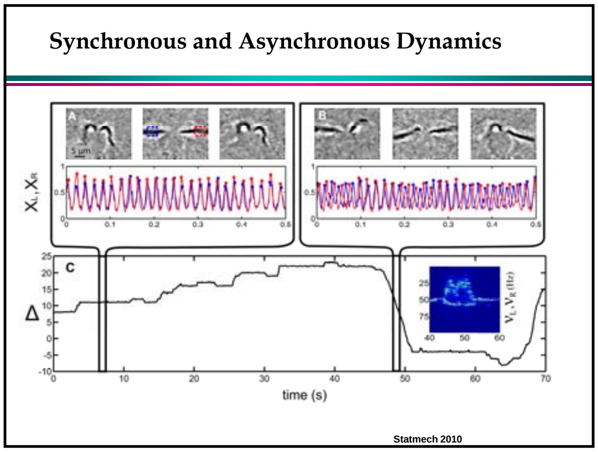
- Diffusive behavior over times larger than about 5 s. D=0.7 x 10^{-3} cm^2/s. Consistent with a random walk $D \approx u^2\tau$ where $u = 100\mu m / s \rightarrow \tau = 5-10\ s.$

D=0.7 x 10^{-3} cm^2/s



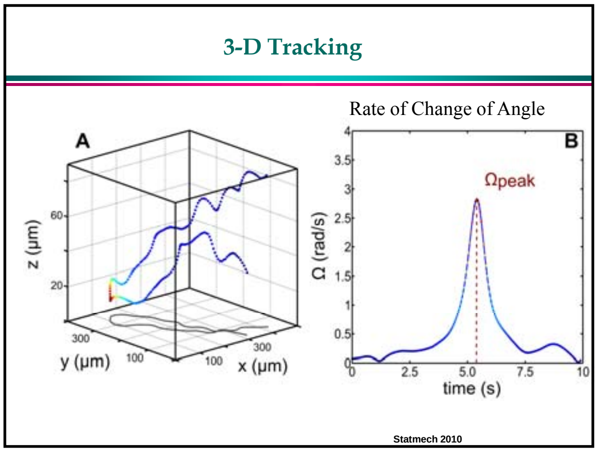
Understanding Diffusion: Synchronous and Asynch. States

Statmech 2010



Synchronous and Asynchronous Dynamics

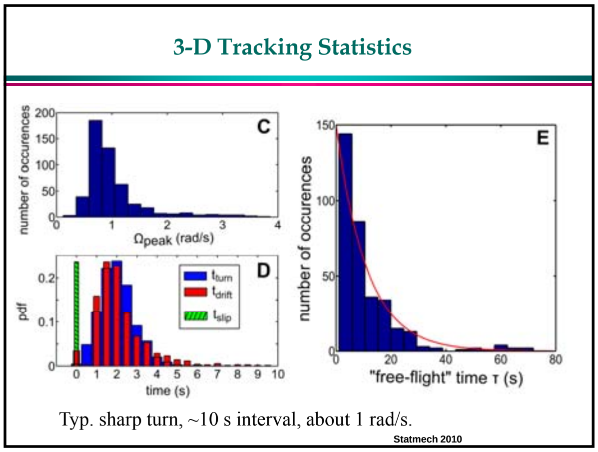
Statmech 2010



3-D Tracking

Rate of Change of Angle

Statmech 2010



3-D Tracking Statistics

Typ. sharp turn, ~10 s interval, about 1 rad/s.

Experiments explain the effective diffusivity

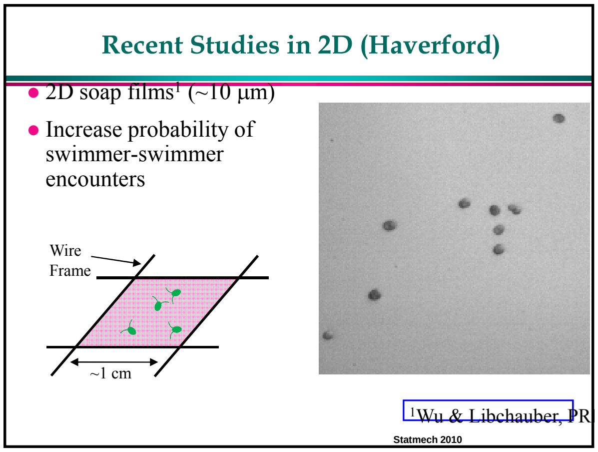
Random walk models:

$$\boxed{D = \frac{u^2\tau}{3(1-\cos\theta)}}$$

$$\left\{\begin{array}{l} u \sim 100\ \mu m/s \\ \tau \sim 10\ s \\ \theta \sim 100\ ^\circ \end{array}\right. \Rightarrow D_{est} \sim 10^{-3}\ cm^2/s$$

Summary So Far – *Science* 2009

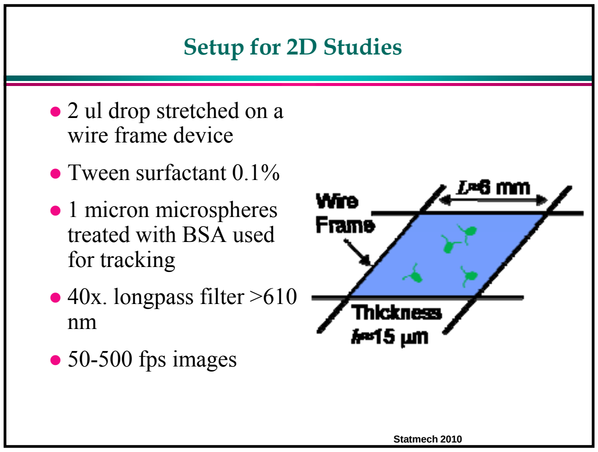
- We can understand how these organisms explore space in terms of the dynamics of the coupled flagella.
- Two states, with sharp turns in the asynchronous state. Similar to “run & tumble” of bacteria, but these are eukaryotes.
- Questions: How does the cell control or regulate the synchronization? Is it connected to photo-taxis?



Recent Studies in 2D (Haverford)

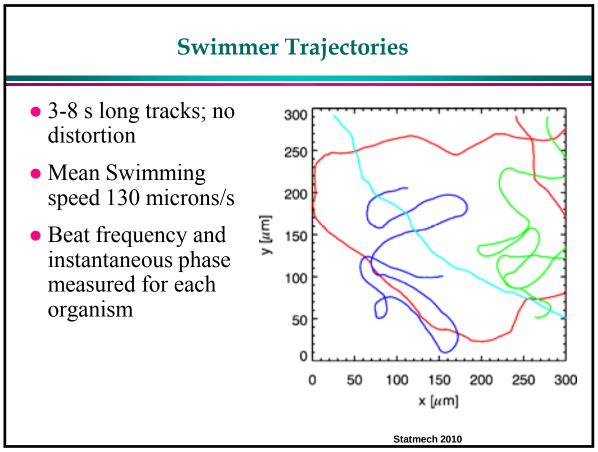
- 2D soap films[1] (~10 µm)
- Increase probability of swimmer-swimmer encounters

[1] Wu & Libchauber, PR



Setup for 2D Studies

- 2 ul drop stretched on a wire frame device
- Tween surfactant 0.1%
- 1 micron microspheres treated with BSA used for tracking
- 40x. longpass filter >610 nm
- 50-500 fps images

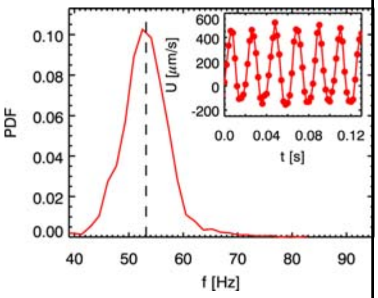


Swimmer Trajectories

- 3-8 s long tracks; no distortion
- Mean Swimming speed 130 microns/s
- Beat frequency and instantaneous phase measured for each organism

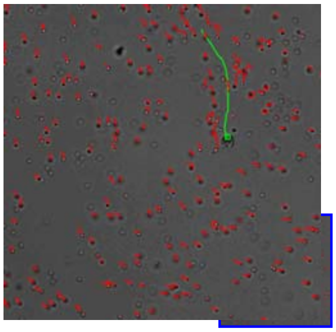
Beat Frequency

- 500 fps imaging
- Mean beat frequency 53 Hz +/-5 Hz
- Inset: oscillations of a single swimmer (~10 frames/cycle)
- Velocity negative during recovery

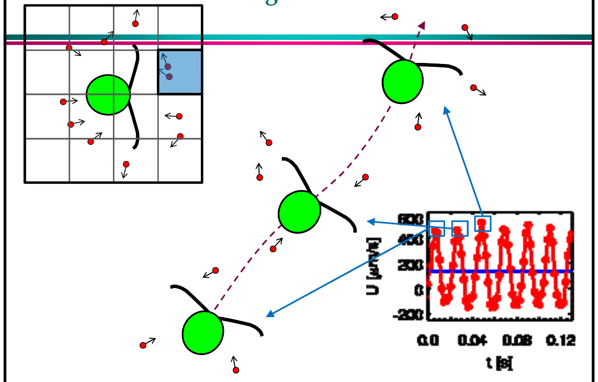


Tracking Swimmers and Tracers

- 40X, 50 fps
- Particle tracking of both swimmers and tracers
- Predictive algorithm (N.T. Ouellette, Yale)

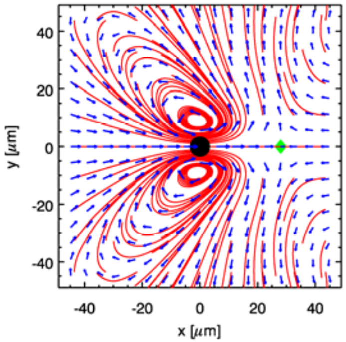


Measuring the Flow Field



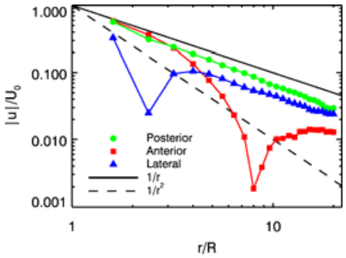
Velocity Field (Mean)

- 50 fps imaging (1 frame per beat cycle)
- Log scaling shown on grid
- Inst. streamlines
- 3 um resolution, up to 20 radii away
- Strong unexpected vortices
- Surprising location of hyperbolic stag. pt

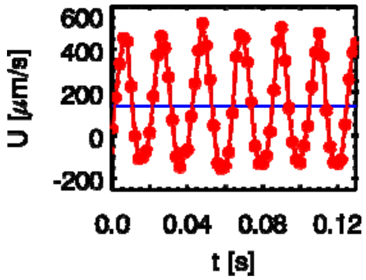


Velocity Decay

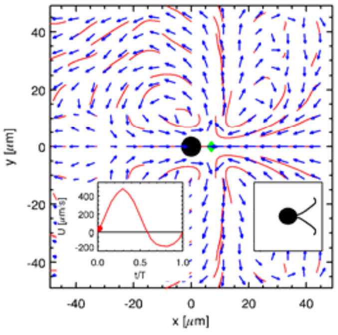
- Posterior shows expected 1/r decay for a force dipole in 2D
- Dips correspond to vortices and stagnation point
- 1/r scaling appears to be recovered in far field for all directions



Body Oscillation

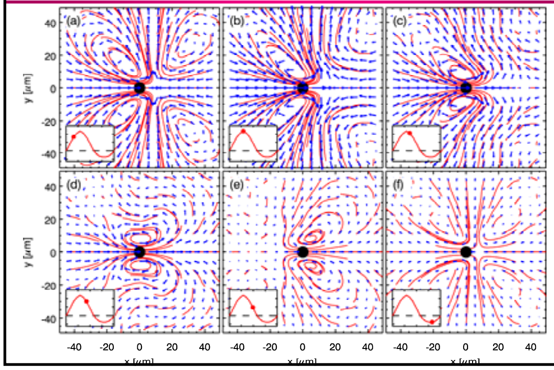


Time Resolved Velocity Field



Statmech 2010

Phase-resolved Velocity Field

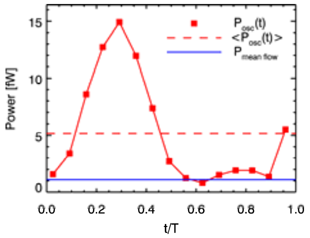


Viscous Dissipation

- Power dissipation per unit volume calculated from velocity fields
- Total power results from integration over field of view
- Dashed red line=mean of time resolved results
- Blue line=calculated from mean velocity field

$$P = \int 2\mu(\underset{\sim}{\Gamma}:\underset{\sim}{\Gamma})hdA$$

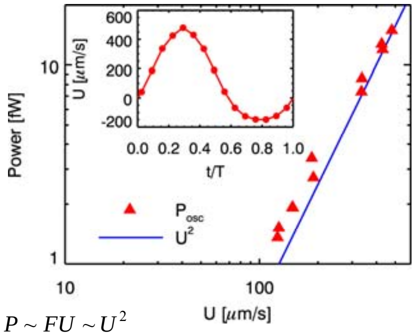
$$\mathbf{\Gamma} = 1/2\left[\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right]$$



Q: How should the power depend on the inst. body speed V\U?

Statmech 2010

Speed Dependence of Power



$F \sim U;\ P \sim FU \sim U^2$

Statmech 2010

Interpretation

- Observed power production requires about 1 ATP molecule per 15 nm of flagellar length per cycle, neglecting heat production. (Estimate from RG based on our data)

Statmech 2010

Summary - Swimmers

- First fully resolved space and time-resolved velocity fields for C. *reinhardtii*; complex structures varying over the beat cycle
- r^{-1} scaling of induced velocity in 2D.
- Power P~U^2 scaling with 15 fW maximum.
- Actual mean mechanical power expended is far larger than the time-averaged fields would suggest.
- J.S. Guasto, K.A. Johnson, and JPG, PRL 2010.

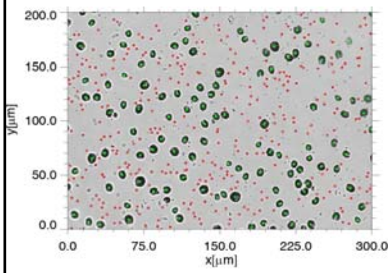
Statmech 2010

Enhancement of Biomixing by Swimming Cells

- H. Kurtuldu, J.P. Gollub,

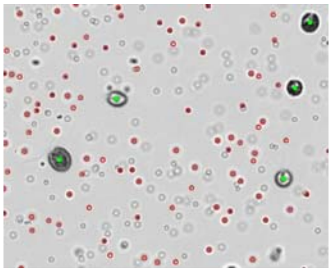
  and K.A. Johnson, Haverford
- J.S. Guasto, MIT (formerly Haverford)
- Supported by NSF DMR-0803153, PNAS, June 2011
- Motivation: Understanding how the induced velocity field affects the surrounding fluid. Possible effects on predation, and on incorporation of particles by swimmers.

Cells and Tracers in 2D



Algal cells (green) and 1 um tracers (red) in a thin film

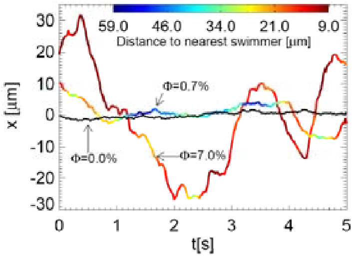
Movie – Swimmers and Tracers



10 μm diam. swimmers

1 μm diam. microspheres

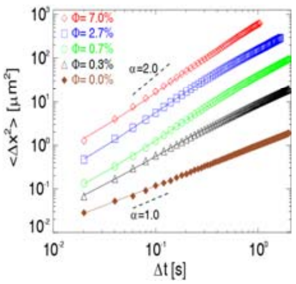
Induced Displacements



Induced tracer displacements for several cell concentrations

Fig. 2 : Tracer displacements for different swimmer concentrations; color shows the particle's distance from the nearest swimmer. Black = Brownian case

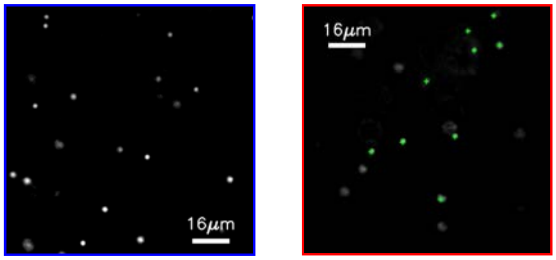
Diffusivity Enhancement for tracers



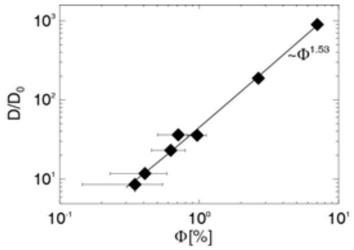
- Mean square tracer displacement vs. elapsed time, for various concentrations, fitted to a stochastic Langevin model.
- Ballistic for short times, diffusive for long times, when swimmers are present.

$$\langle \Delta x^2 \rangle = 2D\left[\Delta t + \tau\left(e^{-\Delta t/\tau} - 1\right)\right]$$

Movies – Contrasting Brownian and Swimmer-Induced Motion



Diffusivity Enhancement

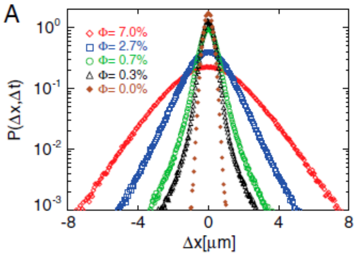


Strong concentration dependence of diffusivity, which we show arises from tracers being influenced by multiple swimmers. Enhancement by at least X10 compared to the 3D case.

Q: Keeping in mind the dimensionality of D, what experiment could we do to gain insight into this dependence?

Statmech 2010

Probability Distributions



PDFs of tracer displacements during a fixed short time interval ($\Delta t=0.04$ s) and at various Φ.

Statmech 2010

Discussion - Biomixing

- The net effect of stirring by a collection of swimmers is to cause tracer particles to move diffusively, simulating a random walk at long times.
- At short times, the tracer motion is ballistic rather than diffusive, as often happens with pseudo-random systems.
- Particle displacement distributions are generally not Gaussian.

Statmech 2010

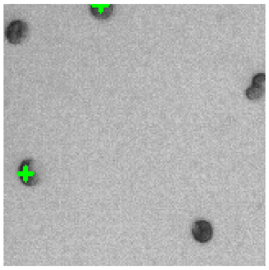
Swimmer Interactions – Current Work

- Swimmers interact with each other.
- Are these interactions mainly hydrodynamic?
- Or do the swimmers sense each other and respond?
- What are the effects of the time-dependence and complex near field structure on these interactions?
- What experiments would allow these questions to be answered?

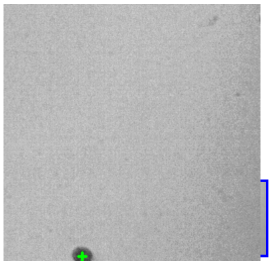
Statmech 2010

Scattering & Enhanced Hydrodynamic Interactions

Collisions

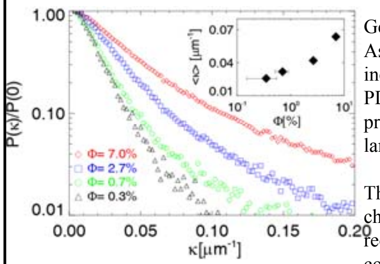


Drafting



[1]Ishikawa & Hota, JE
[2]Pooley et al., PRL (2
[3]Alexander et al., PRE

Statmech 2010



Geometry of trajectories: As concentration is increased, the curvature PDF of the trajectories is progressively extended to large mean curvature.

This shortens the characteristic time for reorientation, thus contributing to the diffusivity enhancement.

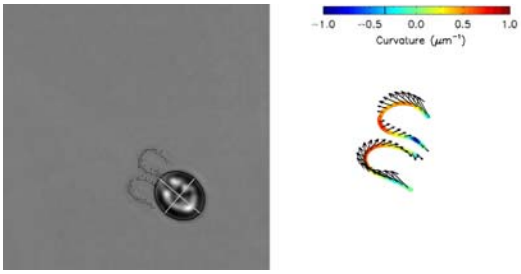
Statmech 2010

Studies of flagellar conformations:

- What determines the conformations? Are they optimizing something? (Collaboration with Tam and Hosoi.)
- Can we understand how the molecular motors generate the force distributions that are observed (a dream)?

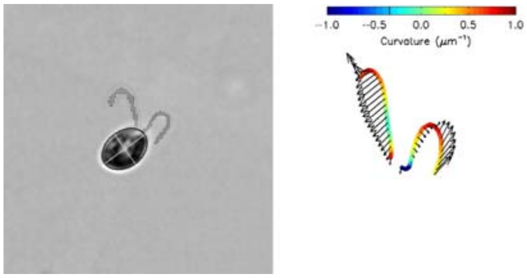
Statmech 2010

Studying Flagellar Conformations and Velocities



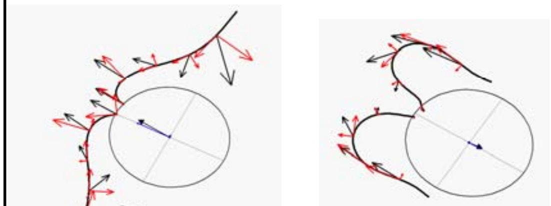
Statmech 2010

Asynchronous Case



Statmech 2010

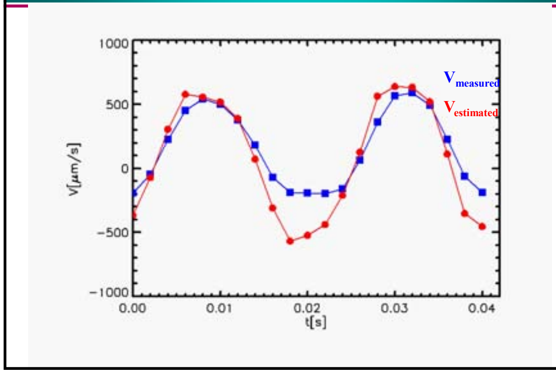
Using Velocity Components to Estimate Forces



Expect that total force on the cell is zero at low Re
→ Predict cell velocity.

Statmech 2010

Compare Measured and Estimated Cell Velocity

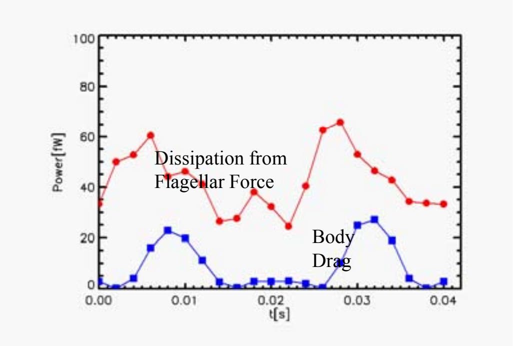


Reasons for Discrepancies?

- Drag coefficients perp = 5.7; par = 3.7 from "resistive force theory". (3, 1.7 also works pretty well. Lighthill.)
- Different segments of flagella may not be independent of each other.
- Interaction between flagellum and body.
- Hard to detect the tip of the flagellum.

Statmech 2010

Contributions to the Power Dissipation



Why Does it Swim This Way?

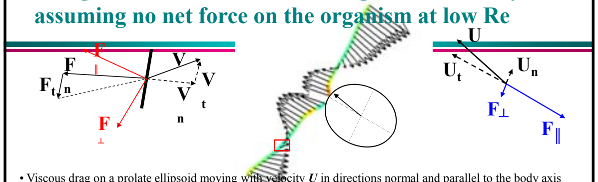
- Tam and Hosoi show that the stroke that we see experimentally is close to the one that minimizes power dissipation.

Summary – Flagellar / Fluid Interactions

- We are starting to have an understanding of the fluid dynamics of a typical swimmer
- Molecular motors generate curvature in the flagella, moving them relative to the fluid.
- Drag on both body and flagella transfers energy to the fluid, where it is dissipated by viscosity.
- Additional energy is converted directly to heat.
- The asymmetry of the forward and recovery stroke is essential in generating net motion at low Reynolds number.

END

Using the Data to Determine Drag Coefficients by assuming no net force on the organism at low Re



- Viscous drag on a prolate ellipsoid moving with velocity $\boldsymbol{U}$ in directions normal and parallel to the body axis $\boldsymbol{F}_\perp = -bD_n\boldsymbol{U}_n$, $\boldsymbol{F}_\parallel = -bD_t\boldsymbol{U}_t$, b: major axis length. ($D_n = D_t = 6\pi\mu$ for sphere (Chwang & Wu, 1975))
- Velocity of each segment of flagella ($\boldsymbol{V}$) is decomposed into velocities normal ($\boldsymbol{V}_n$) and tangent ($\boldsymbol{V}_t$) to the segment.
- Frictional drags $\boldsymbol{F}_n$ and $\boldsymbol{F}_t$ act in directions opposite to $\boldsymbol{V}_n$ and $\boldsymbol{V}_t$, respectively. $\partial\boldsymbol{F}_n/\partial s = -C_n\boldsymbol{V}_n$, $\partial\boldsymbol{F}_t/\partial s = -C_t\boldsymbol{V}_t$. ($C_n/C_t$ □ 2, Resistive Force Theory (Gray & Hancock, 1955))
- $\boldsymbol{F}_n$ and $\boldsymbol{F}_t$ integrated along flagella are decomposed into components normal ($\boldsymbol{F}_\perp$) and parallel ($\boldsymbol{F}_\parallel$) to the body axis.
- No net force $(Re\sim10^{-2})$: $\boldsymbol{F}_\perp + \boldsymbol{F}_\parallel + \boldsymbol{F}_\perp + \boldsymbol{F}_\parallel = 0$. Then C_n and C_t can be estimated.