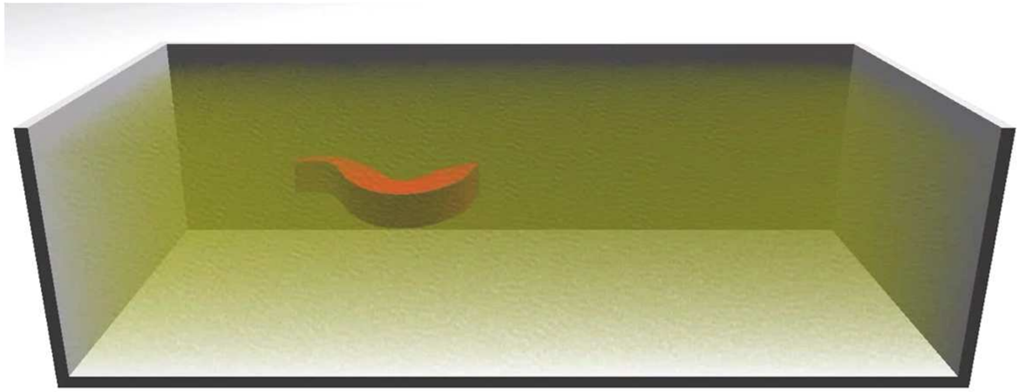
Swimming in Sand, part 3



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School of Physics
Georgia Institute of Technology

Boulder Summer School on Hydrodynamics

July 25-27

Lectures on the mechanics of interaction with granular media including biological & physics experiments, numerical, theoretical and physical robot models

Topics in the lectures

(revised)

- General principles in terrestrial locomotion
- Intro to granular media
- Drag, lift and flow fields during localized intrusion in granular media
- Modeling approaches: DEM & RFT
- Sandfish biological experiments
- Sandfish modeling: robot
- Sandfish modeling: DEM
- Biological tests of model predictions
- RFT modeling of sand-swimming

L1: General principles in terrestrial locomotion; Intro to granular media; Drag, lift and flow fields during localized intrusion in granular media

L2: Modeling approaches: DEM & RFT; Sandfish biological experiments; Sandfish modeling: robot

L3: Sandfish modeling: DEM; Biological tests of model predictions; RFT modeling of sand-swimming

Swimming in Sand

Papers:

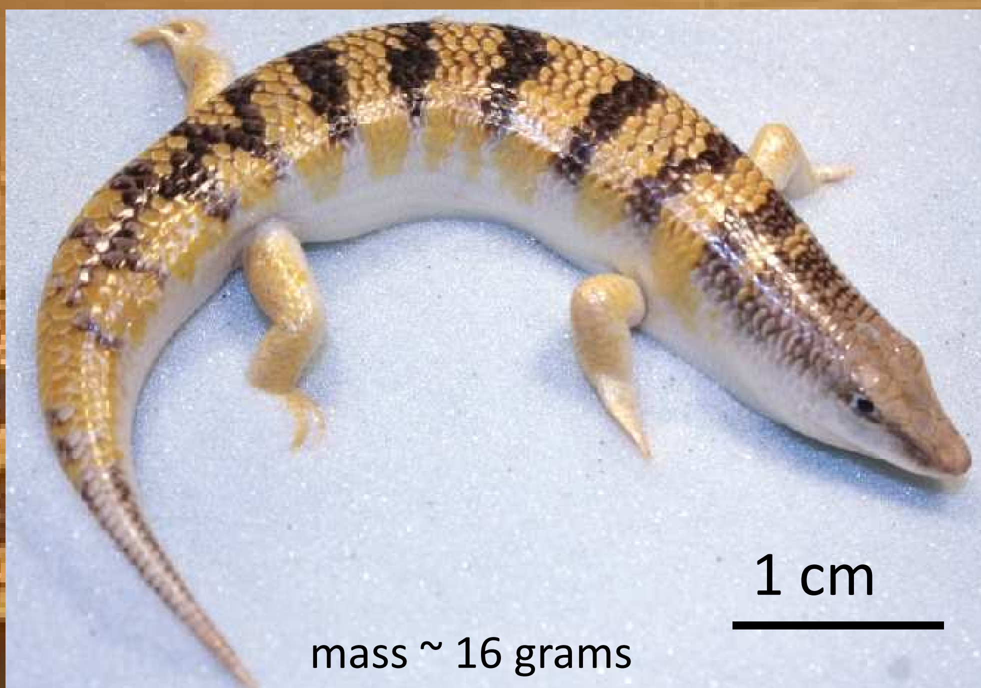
Maladen et al, Science, 2009
Maladen et al, Robotics: Science & Systems conference 2010 (Best paper award)
Maladen et al, J. Royal Society Interface, 2011
Maladen et al, International Journal of Robotic Research, 2011
Maladen et al, ICRA, 2011

Pdfs and links to movies here:

http://crablab.gatech.edu/pages/publications/index.htm

The sandfish lizard

Sandfish (*Scincus scincus)*

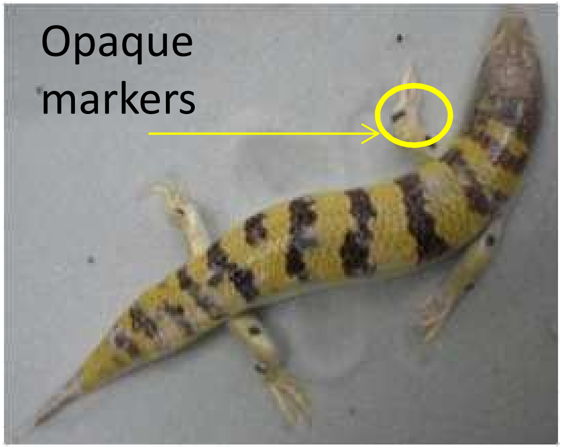


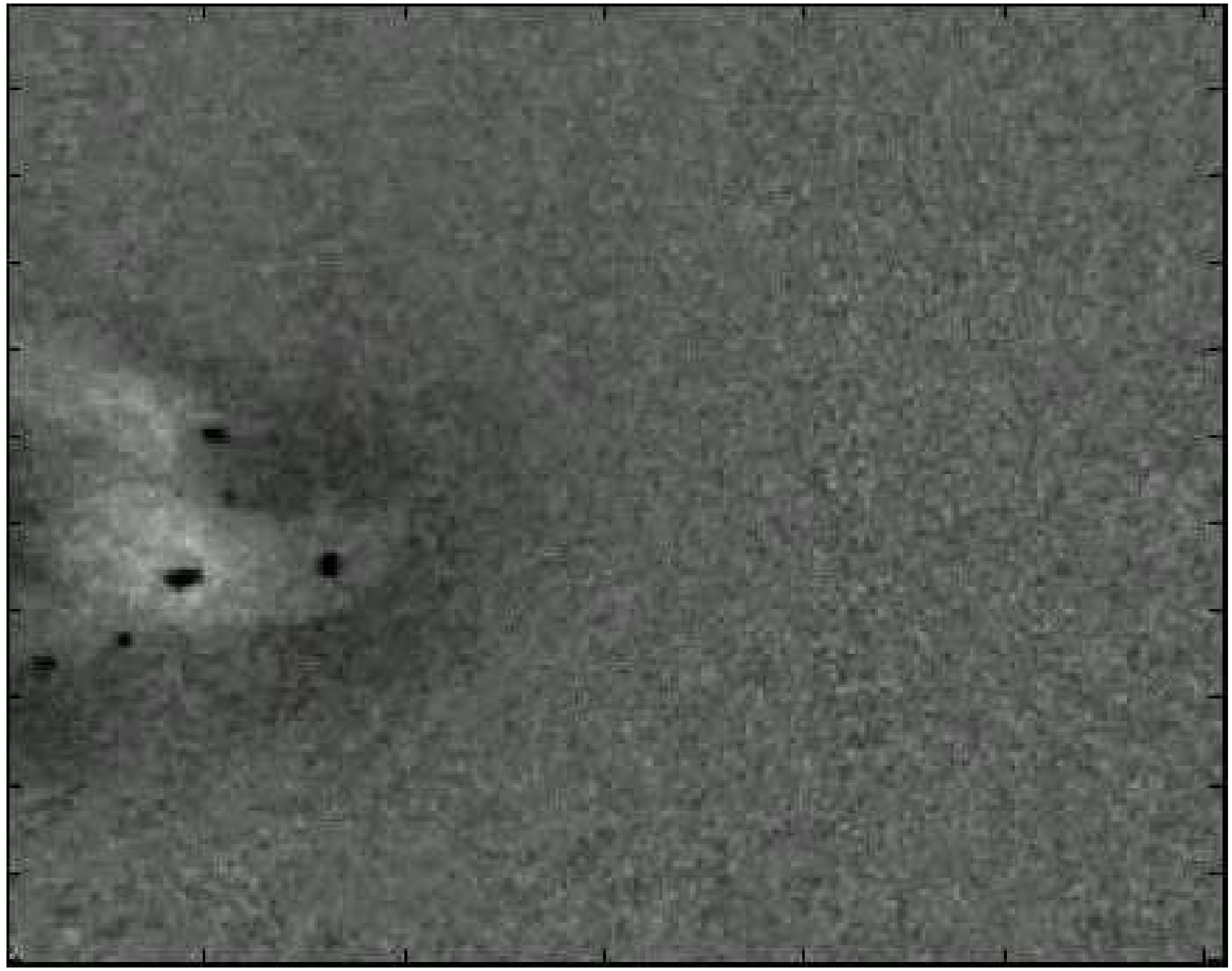
- Native to Sahara desert

- Adaptations for living in sand: countersunk jaw, fringe toes, smooth scales, flattened sidewalls

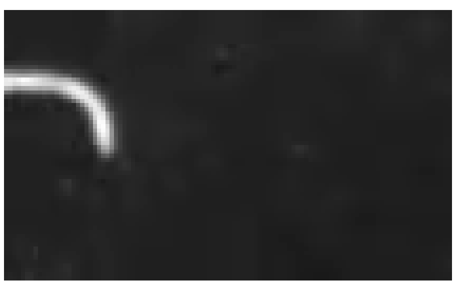
- One of ~10 species classified *subarenaceous*: “swims” within sand

Swimming without use of limbs



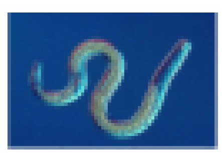


1 mm

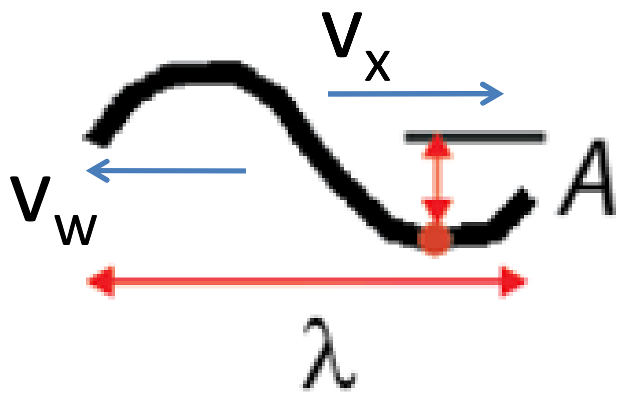


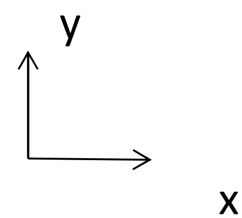
1 cm

Nematode (*C. elegans*) in fluid
Hang Lu, Georgia Tech



Kinematics during steady swimming





fit

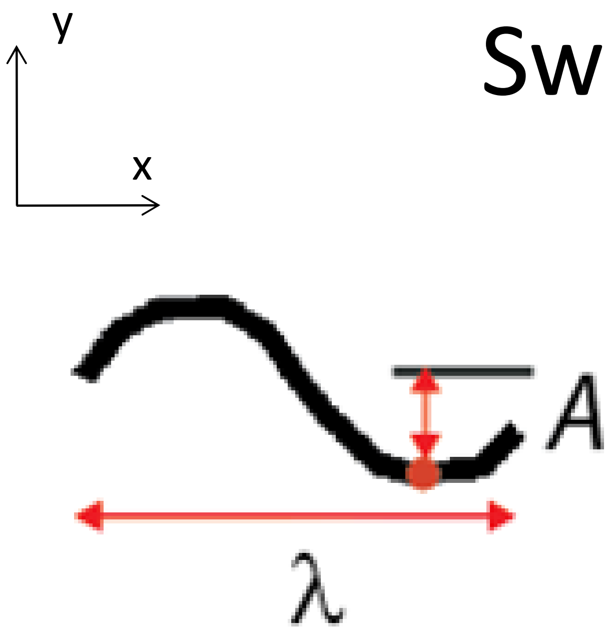
$$y = A\sin\frac{2\pi}{\lambda}(x + v_w t) \qquad v_w = \lambda f$$

R^2>0.95 at all phases in cycle

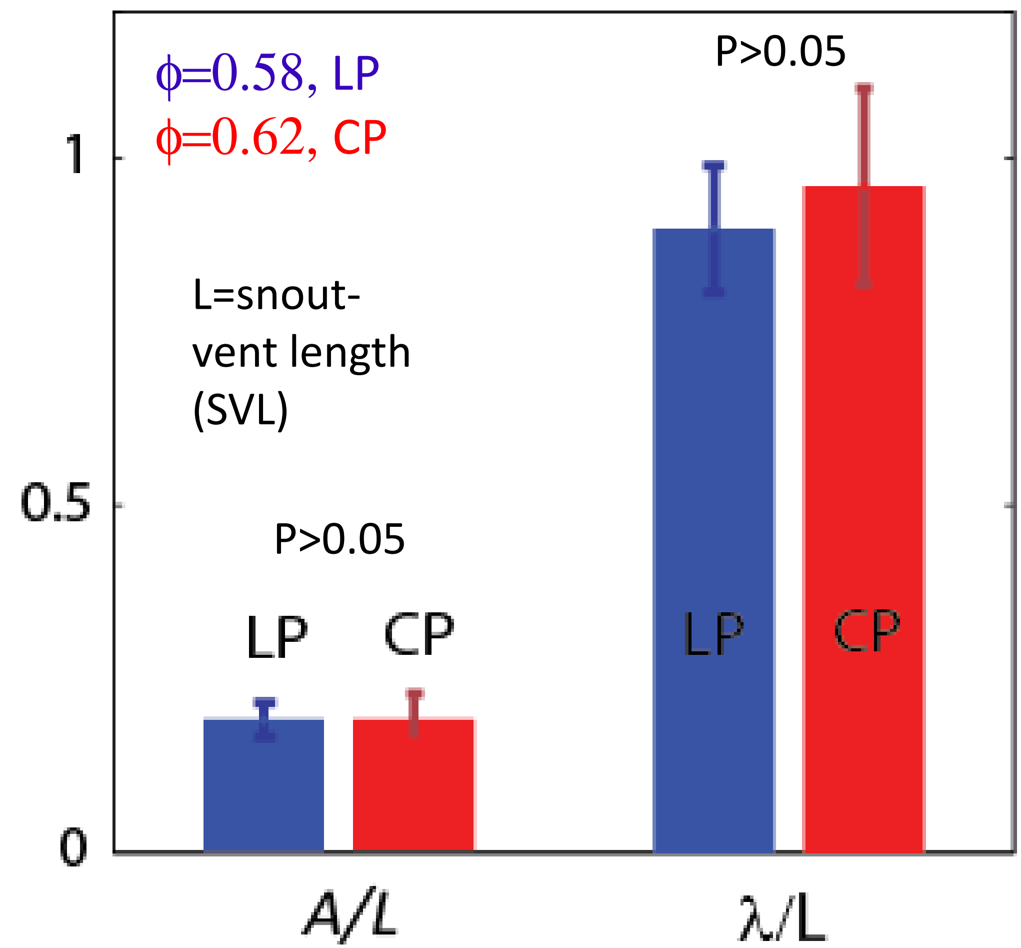
Single period sinusoidal wave, traveling head to tail

n=11 animals
mass=16.2 ± 4 g

Swimming kinematics



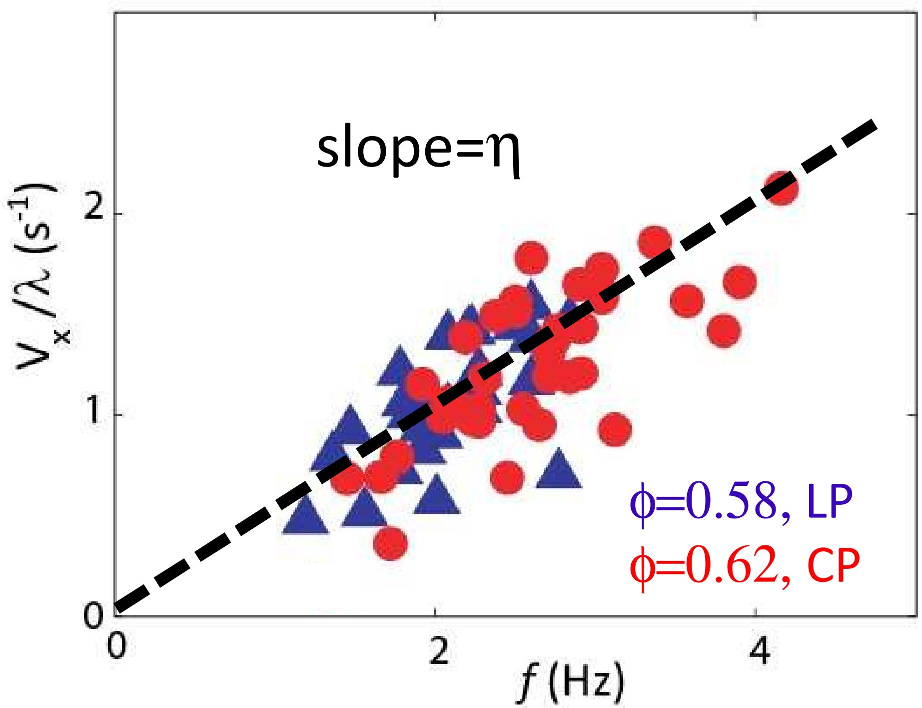
Travelling sinusoidal wave, kinematics independent of ϕ



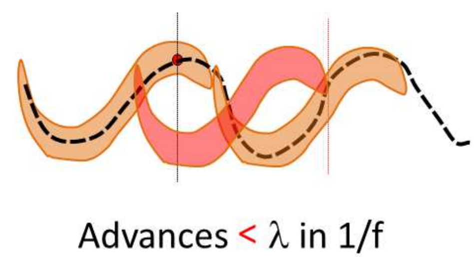
A/λ = 0.20 ± 0.04 LP
0.22 ± 0.06 CP

n=11 animals
mass=16.2 ± 4 g

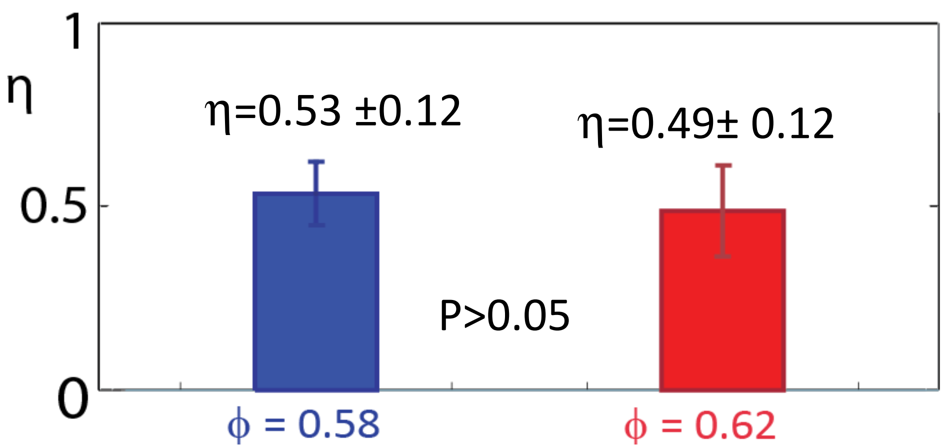
Swimming speed vs frequency & wave efficiency



$$\eta = \frac{v_x}{v_w} = \frac{v_x}{f\lambda} = \frac{v_x/\lambda}{f}$$



Measures amount of "slip" relative to movement in a tube



- Wave efficiency (η~0.5) is independent of ϕ

n=11 animals
mass=16.2 ± 4 g

Swimming by the sandfish inspired robot

10 cm

Robot on the surface

ξ=1,
A/λ=0.2
f=1 Hz

Robot sub-surface

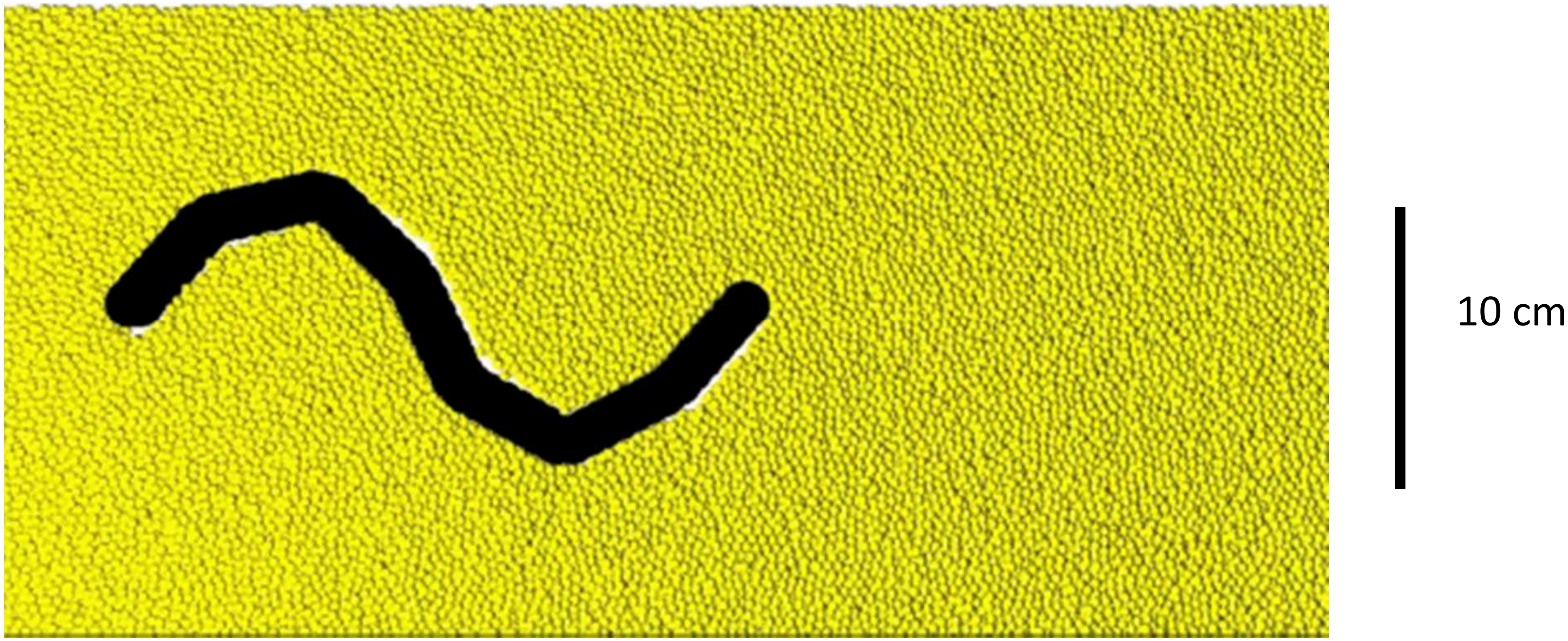
Real time

ξ=1,
A/λ=0.2
f=0.25 Hz

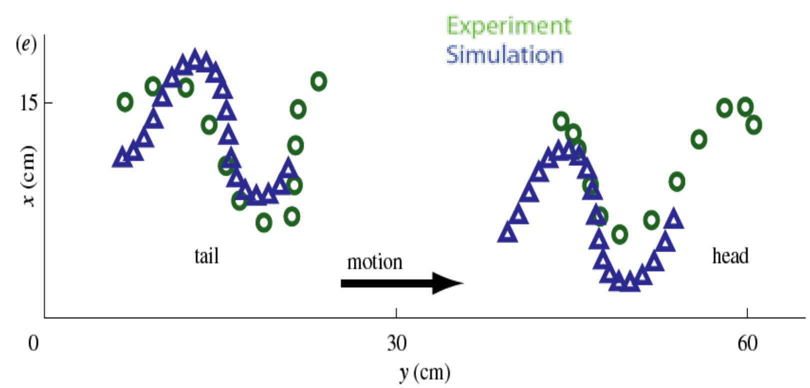
Submerge robot to a depth of 4 cm in closely packed bed

Integrating WM with DEM simulation

Particles above the robot rendered transparent



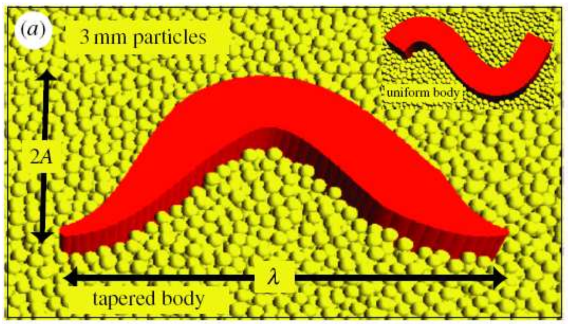
Box dimensions: 108cm x 40cm x 15cm
Number of particles: 3e5
Particle size : 0.6cm

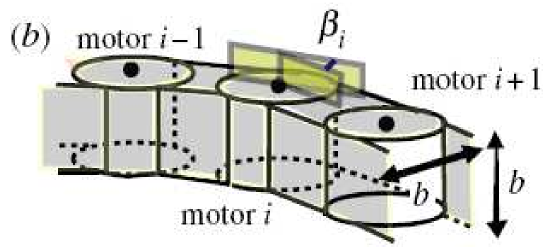


Sandfish scale simulation

Maladen, Ding, Umbanhowar, Goldman, J. Royal Soc. Interface, 2011

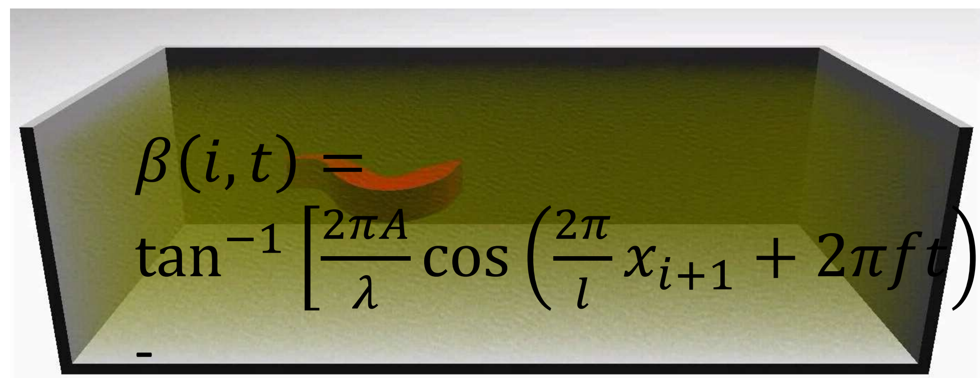
50 segment “sandfish” model





← l=10 cm →

Motors controlled to generate sandfish’s traveling sinusoidal wave *kinematics.*



$$\beta(i,t) = \tan^{-1}\left[\frac{2\pi A}{\lambda}\cos\left(\frac{2\pi}{l}x_{i+1} + 2\pi f t\right)\right] - \tan^{-1}\left[\frac{2\pi A}{\lambda}\cos\left(\frac{2\pi}{l}x_{i} + 2\pi f t\right)\right]$$

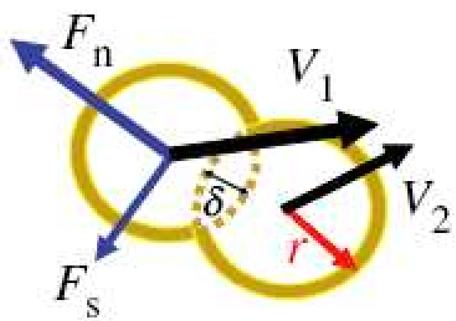
← 35 cm (~200 PD) →

~10^5 , 3 mm “glass” particles

Simulate granular medium: Discrete Element Method

(e.g, see book by Rappaport)

Specify particle-particle/particle-intruder interaction rule



elasticity dissipation

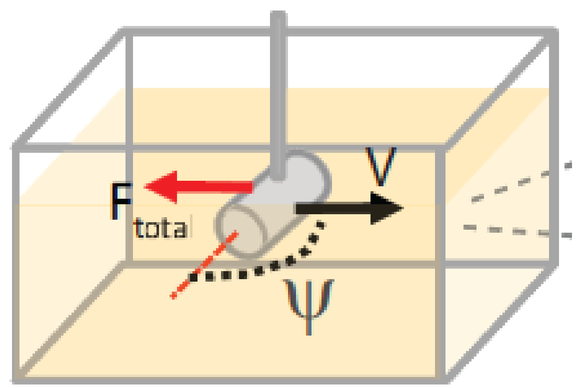
$$F_n = k\delta^{3/2} - G_n v_n \delta^{1/2}$$

$$F_s = \mu F_n$$

friction

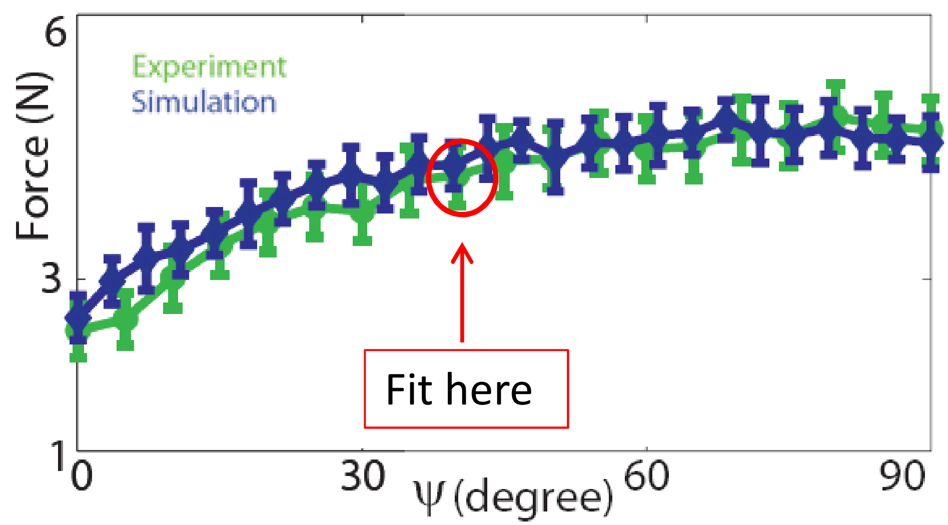
k=2 × 10^6 kg s^{-2} $m^{-1/2}$
G_n = 15 kg s^{-1} $m^{-1/2}$
μ_{pp} = 0.1

Model validation: rod drag



3 mm diameter glass beads

3 cm long SS cylinder

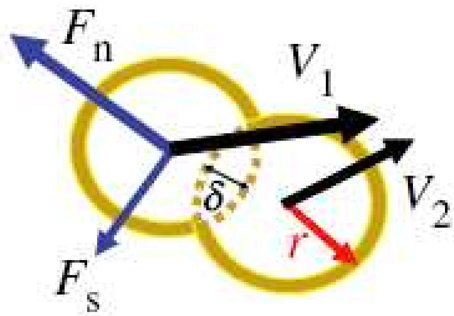


50:50 mix of 3.0,3.4 mm "glass spheres"

Simulate granular medium: Discrete Element Method

(e.g, see book by Rappaport)

Specify particle-particle/particle-intruder interaction rule



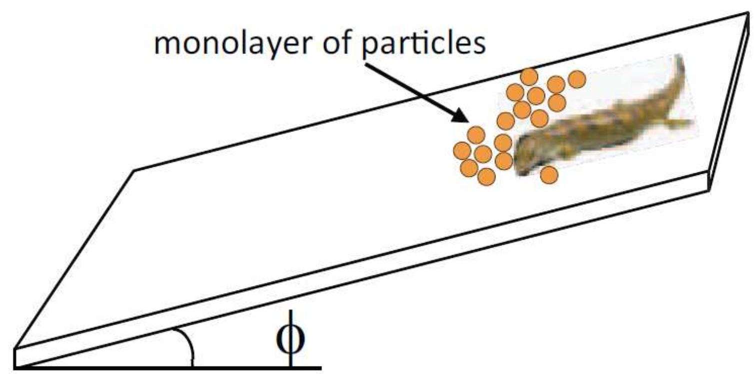
Anesthetize animal,
tilt platform until it slides
down, obtain μ_{pb}

elasticity dissipation

$$F_n = k\delta^{3/2} - G_n v_n \delta^{1/2}$$

$$F_s = \mu F_n$$

friction



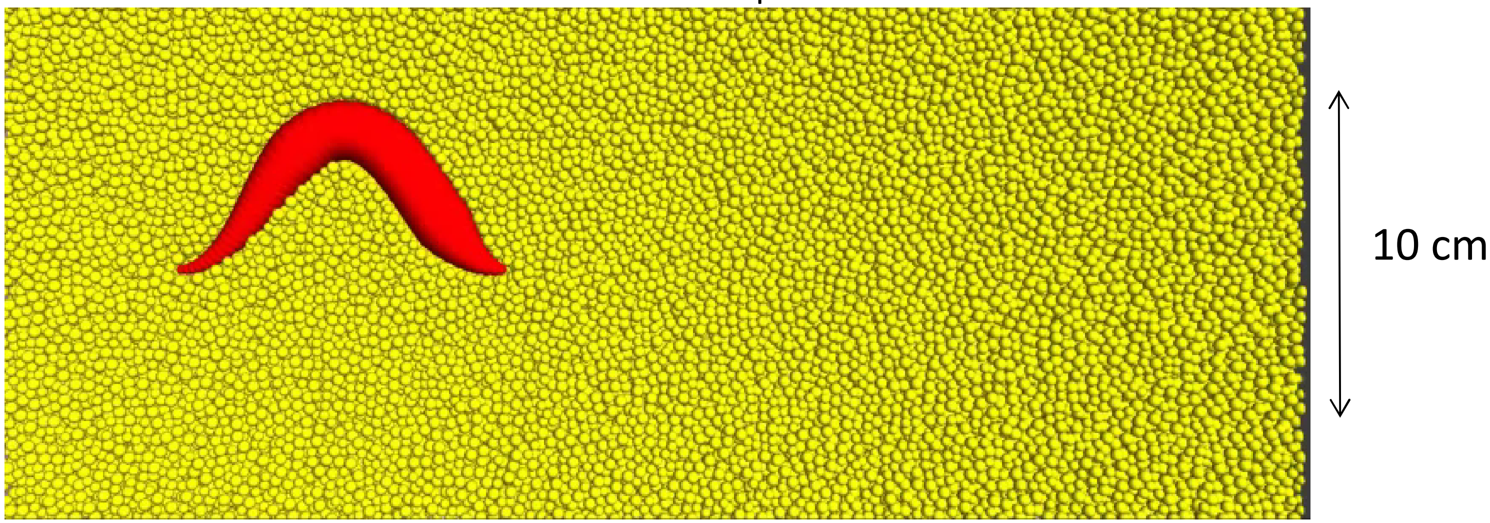
$k=2 \times 10^6$ kg s^{-2} $m^{-1/2}$
$G_n = 15$ kg s^{-1} $m^{-1/2}$
$\mu_{pp} = 0.1$
$\mu_{pb} = 0.27$

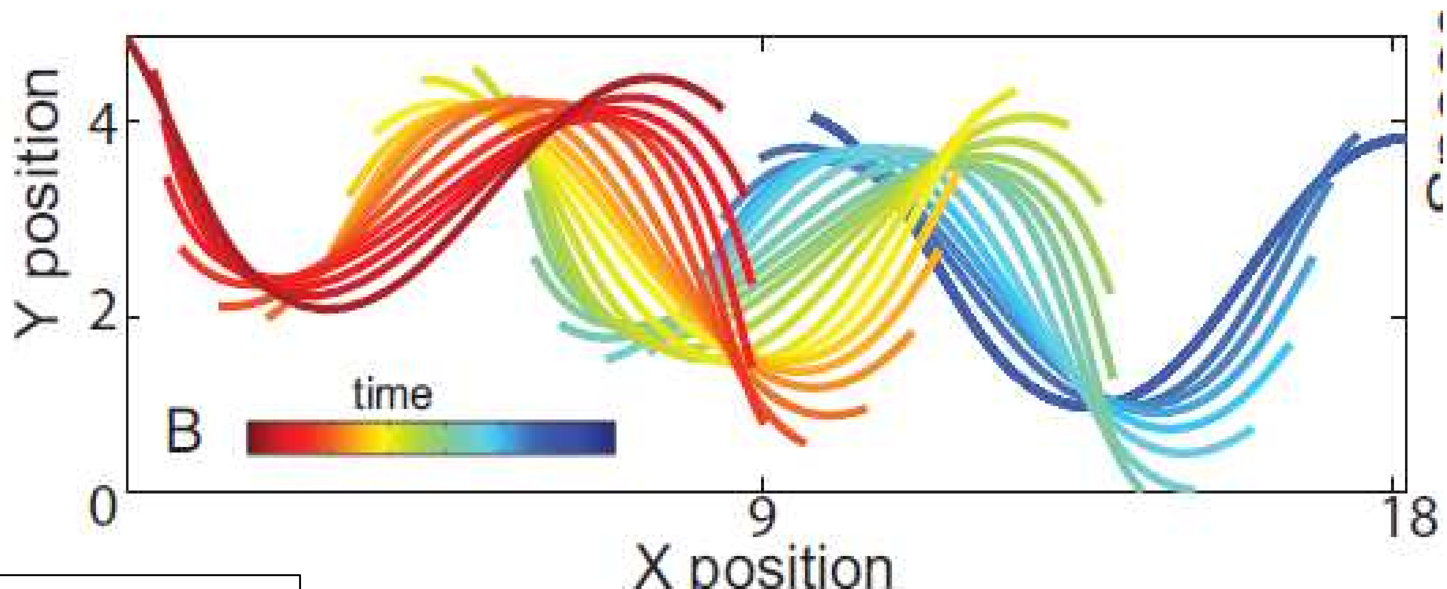
50:50 mix of
3.0,3.4 mm "glass
spheres)

Animal-particle friction = 0.27

Simulated sand-swimming

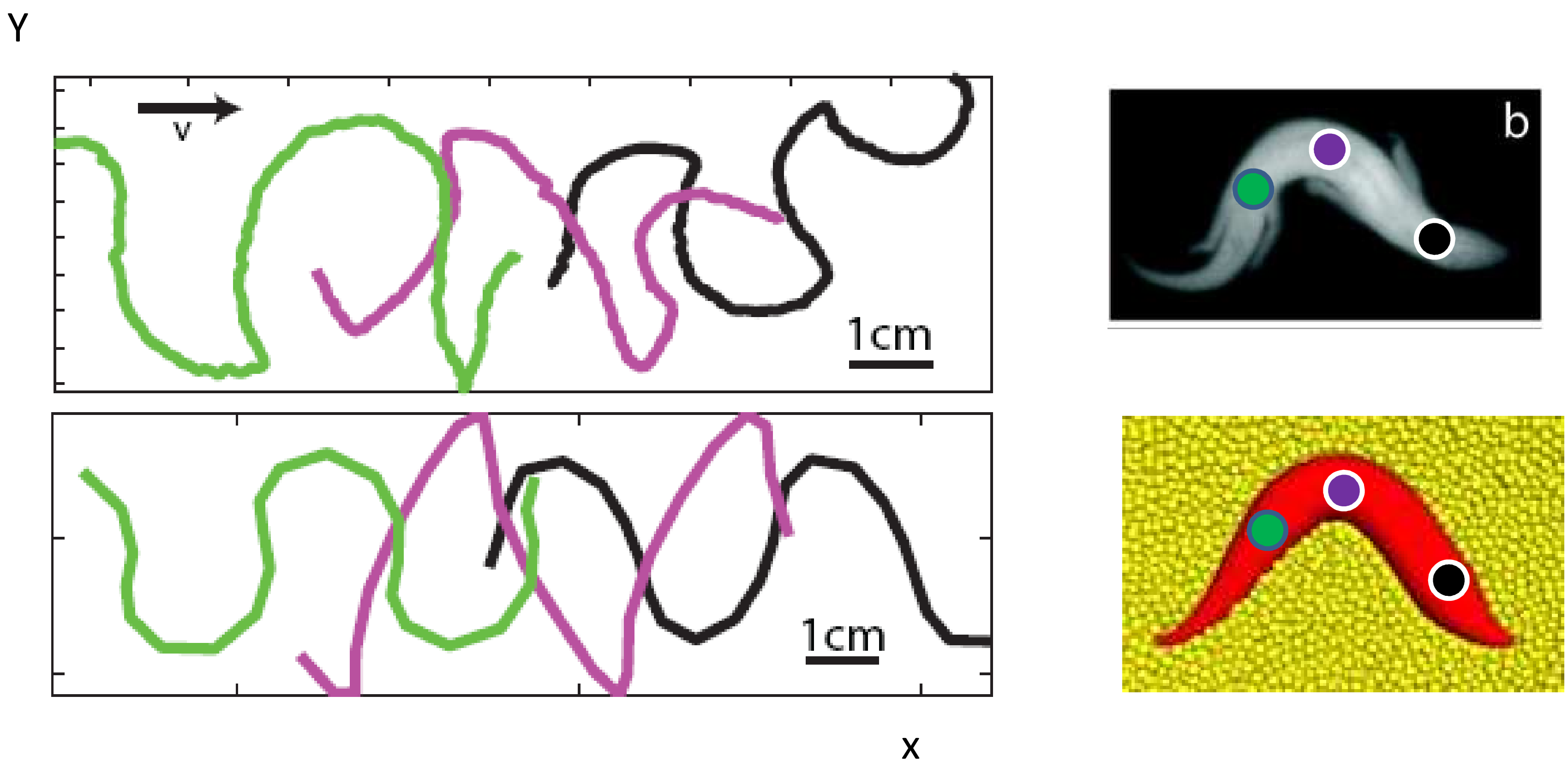
Particles above rendered transparent





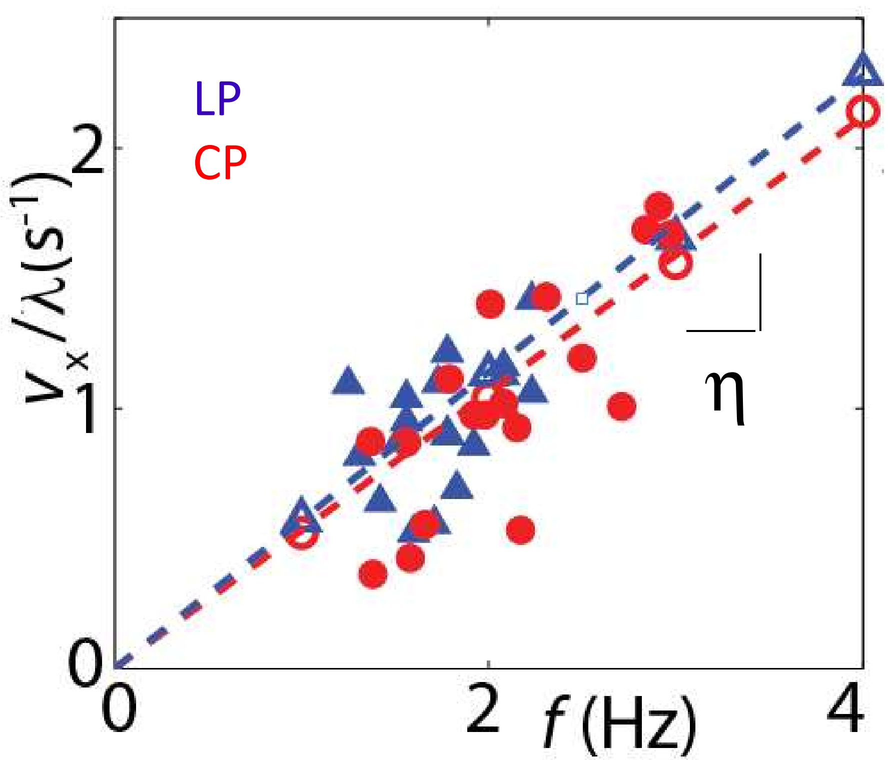
A/λ=0.25, ξ=1, f=1 Hz

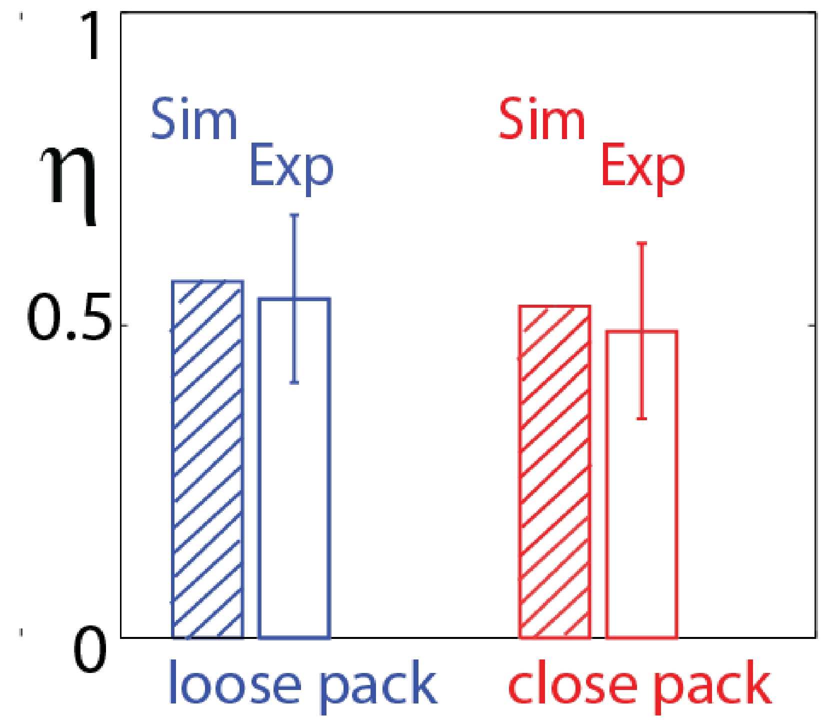
Trajectories of body markers



Speed vs frequency and η

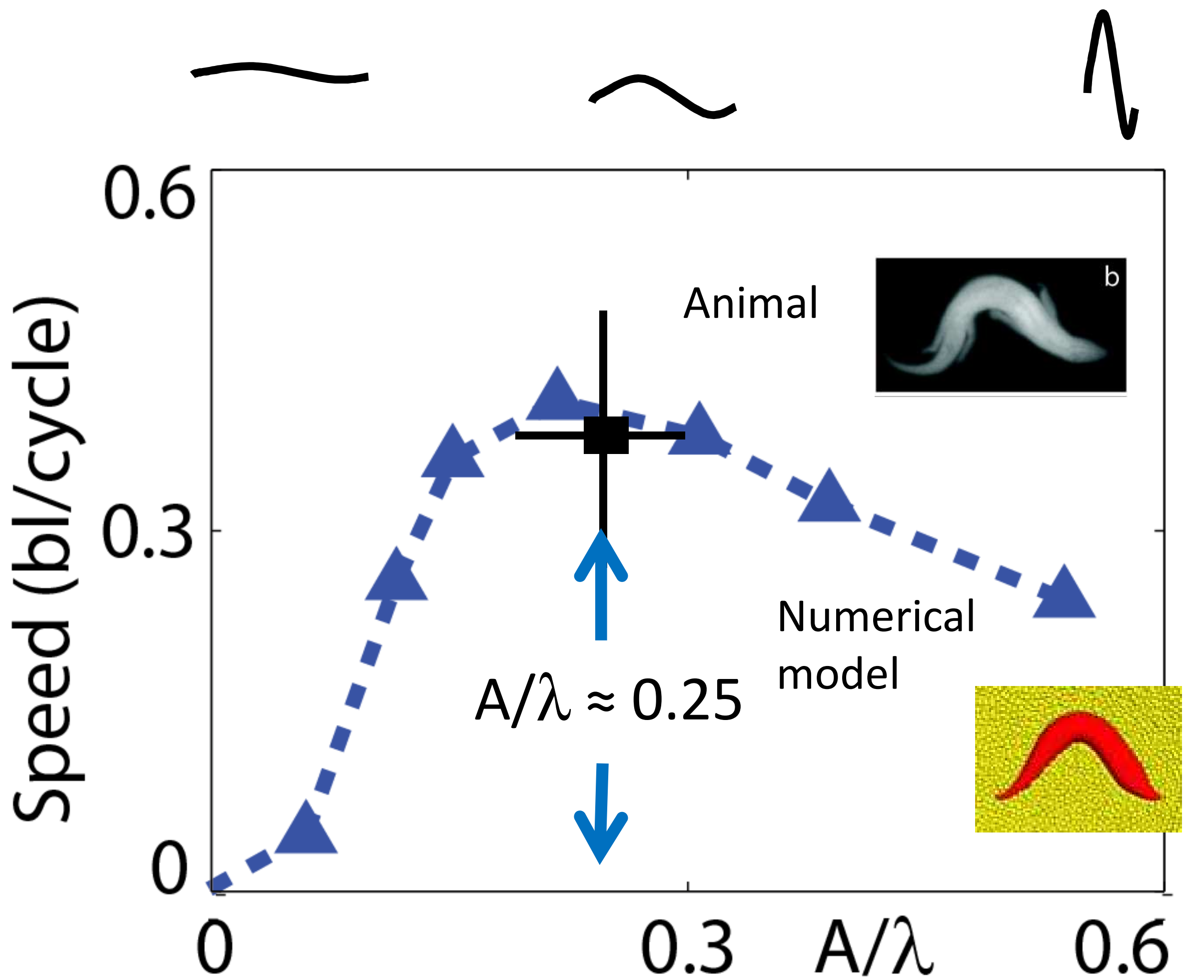
Swimming in 3 mm glass particles, in experiment and simulation





A/λ=0.25, ξ=1

Variation of amplitude-> optimal swimming in sand



Peak at
$A/\lambda = 0.23 \pm 0.01$

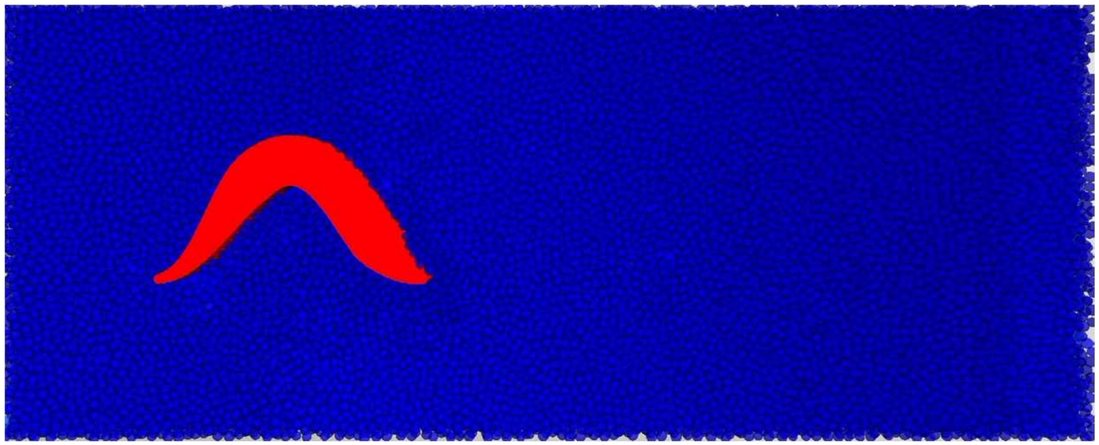
Fixed
f=2 Hz
$\xi=1$

Hypothesis: animal utilizes swimming kinematics which maximize escape into the sand→ a template!

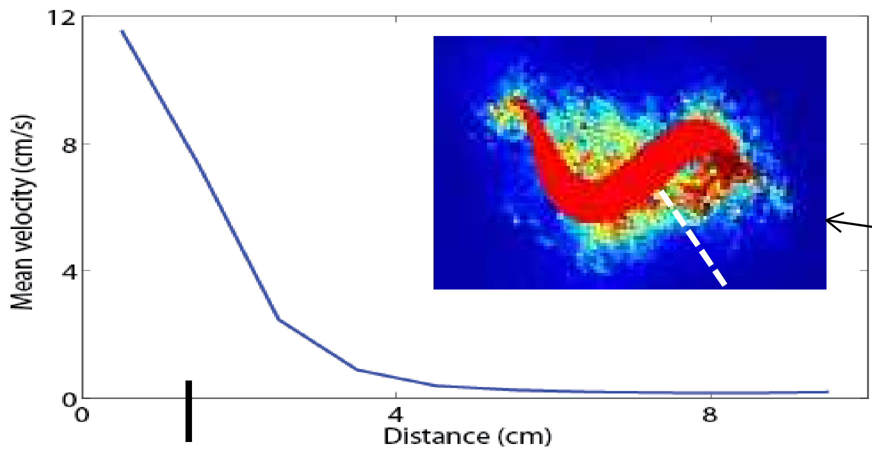
Localized fluid

Redder particles → higher speed

1 cm



Speed (cm/sec)

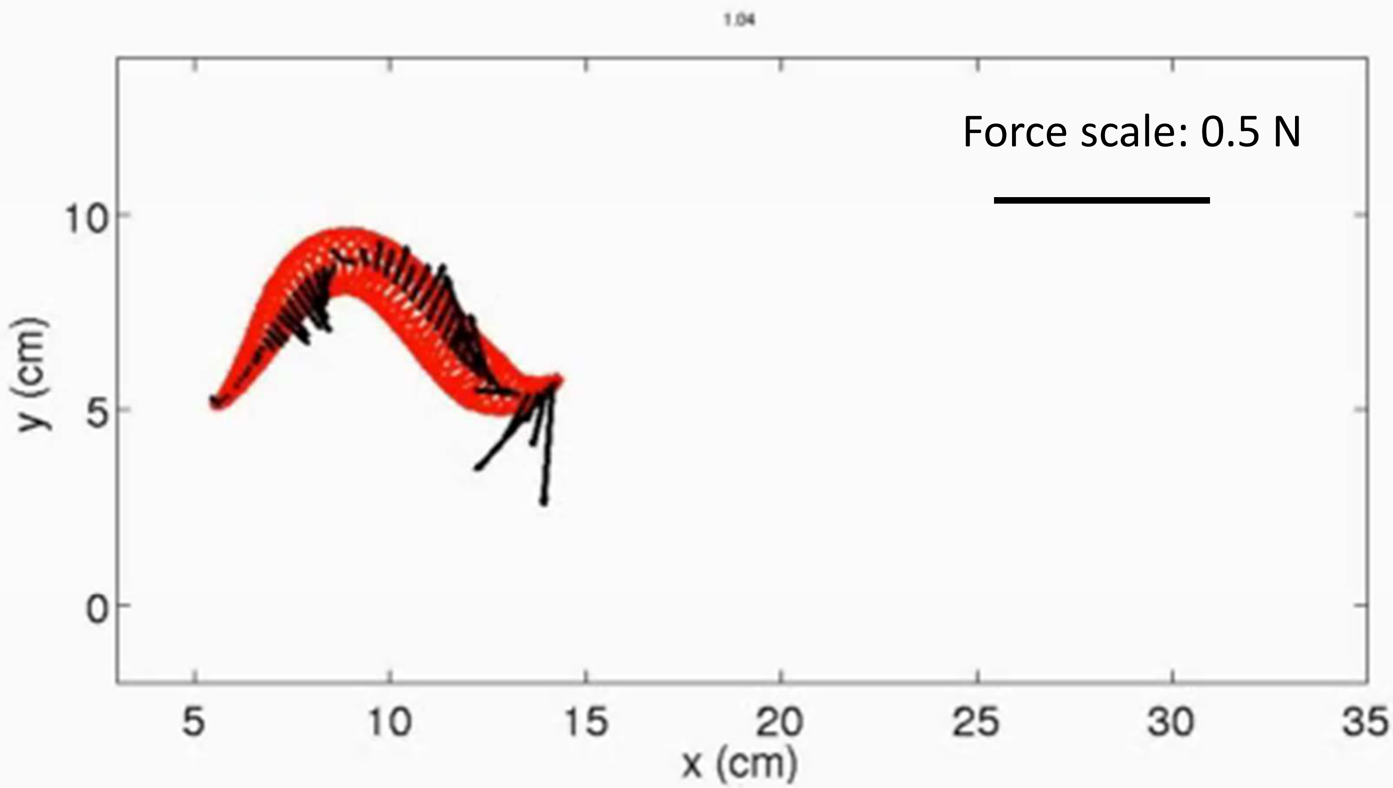


Calculate mean particle speed as a function of perpendicular distance from body, along body

max body width

Distance (cm)

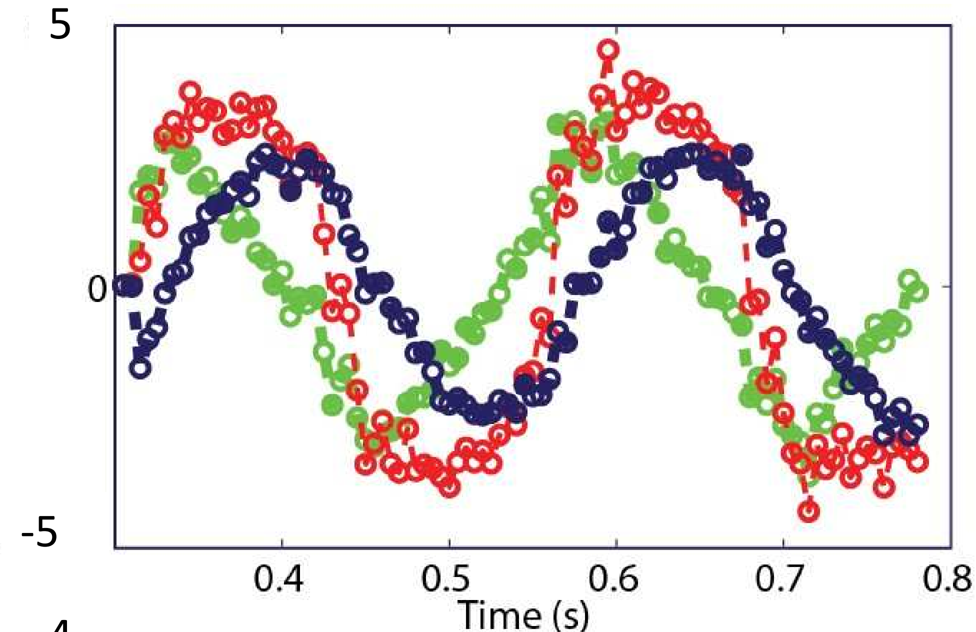
Resistive forces during swimming



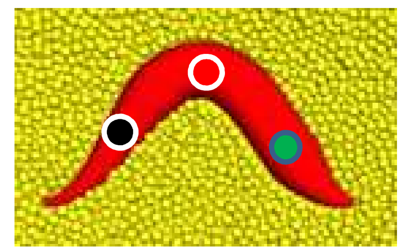
A/λ=0.25, ξ=1

Motor activation (torque) pattern

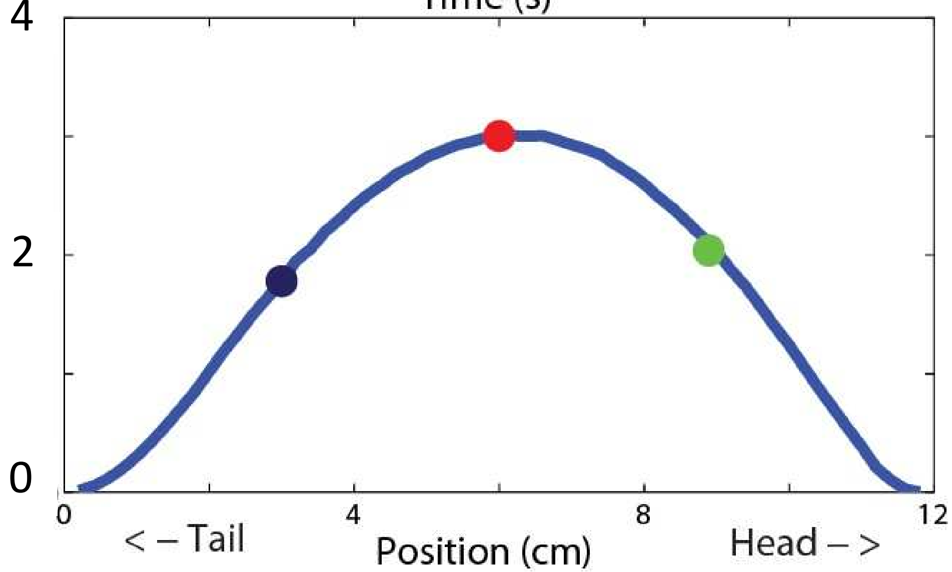
Torque (N-cm)



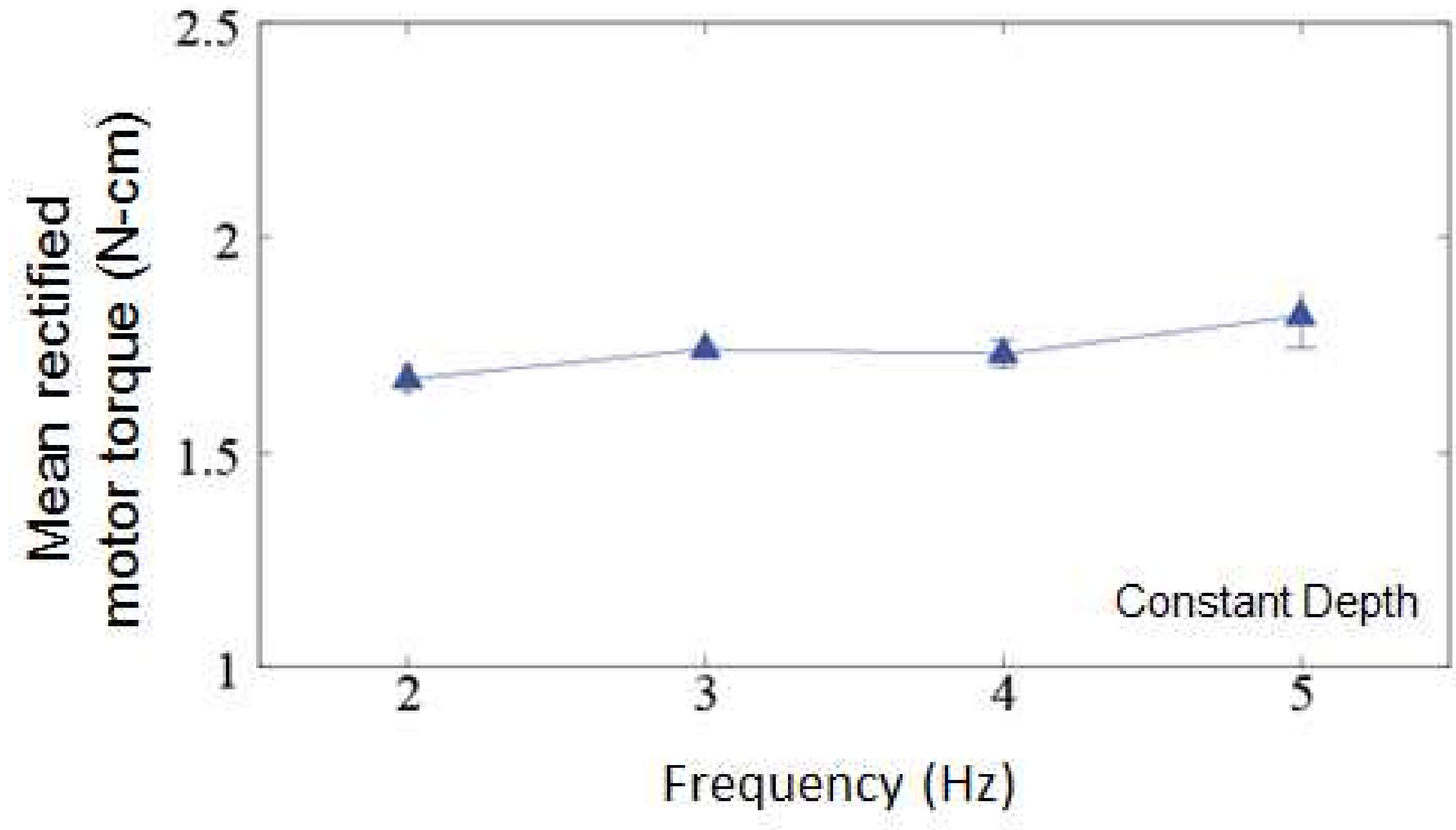
$A/\lambda=0.2$



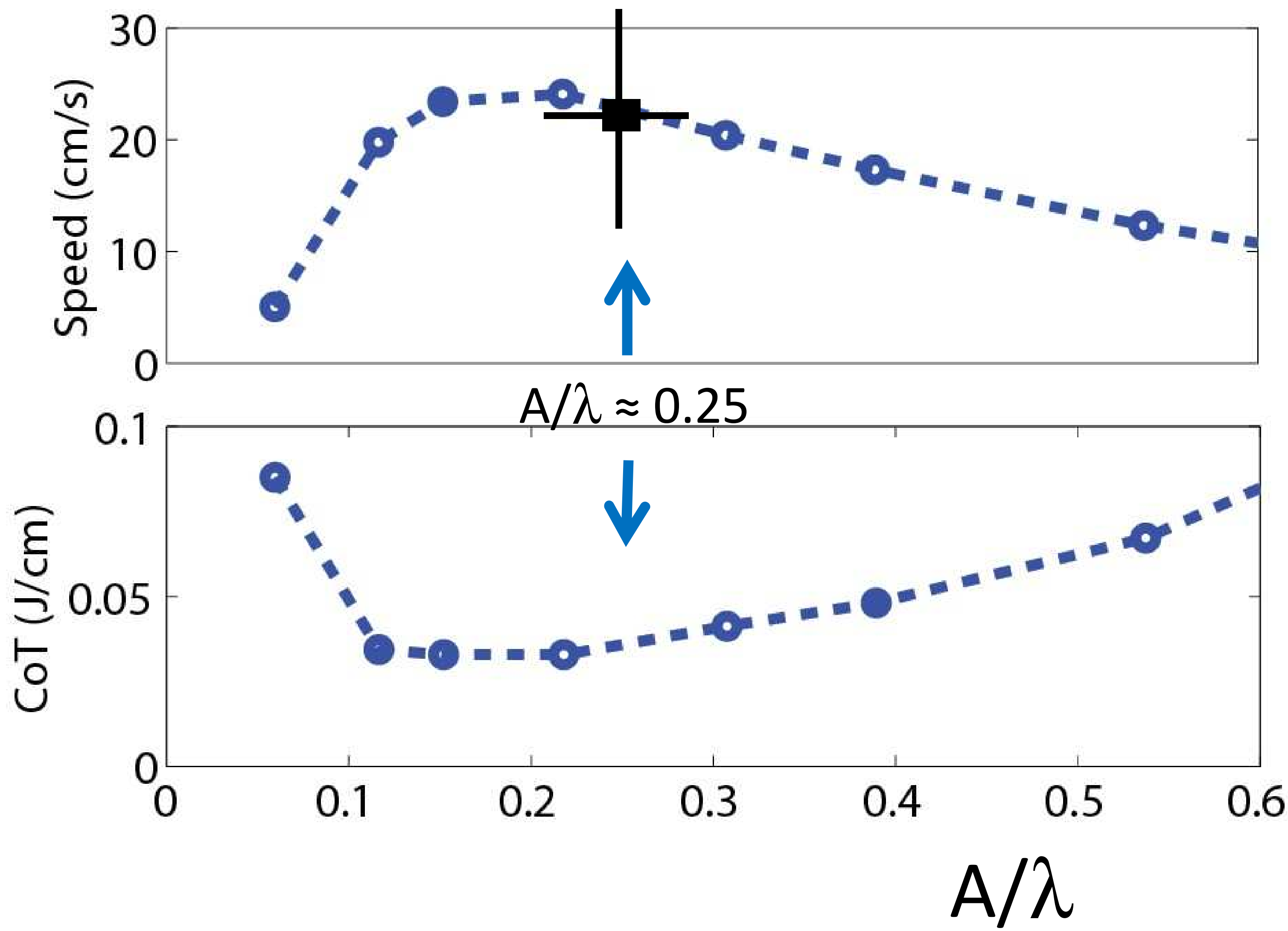
RMS of
torque (N-cm)



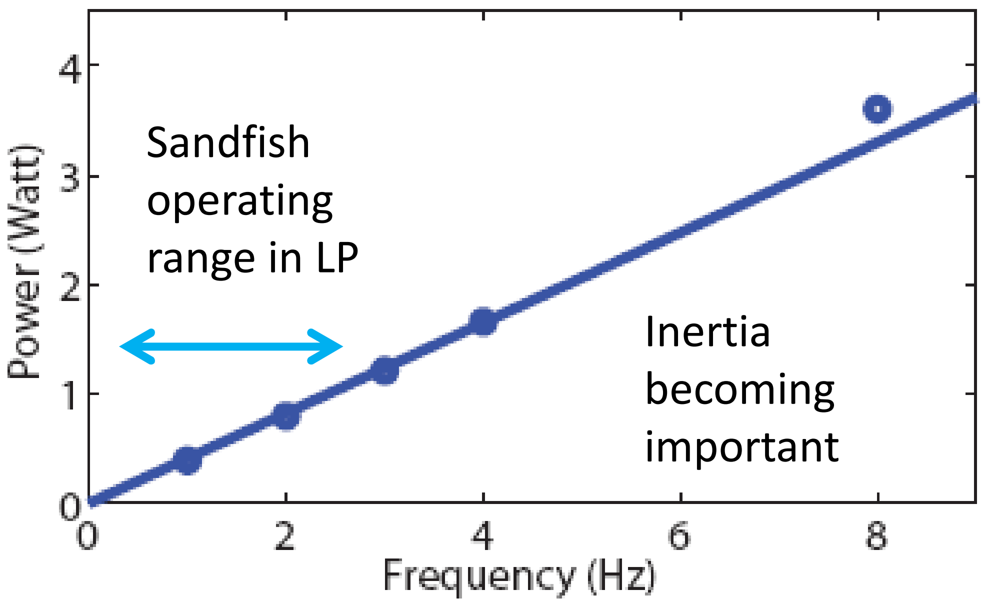
Torque is frequency independent-->
Frictional fluid



Minimum mechanical cost of transport



Power



1W/0.015 kg=

60 W/kg

Vertebrate muscle is capable of ~100 W/kg:

--Swoap et al, JEB, 1993 measured 154 W/kg at ~40 C in hind limb of desert iguana

--Carroll & Wainwright, Comp. Bio & Phys, 2006, max of 330 W/kg in epaxial musculature in a bass

so simulation is reasonable in this regard

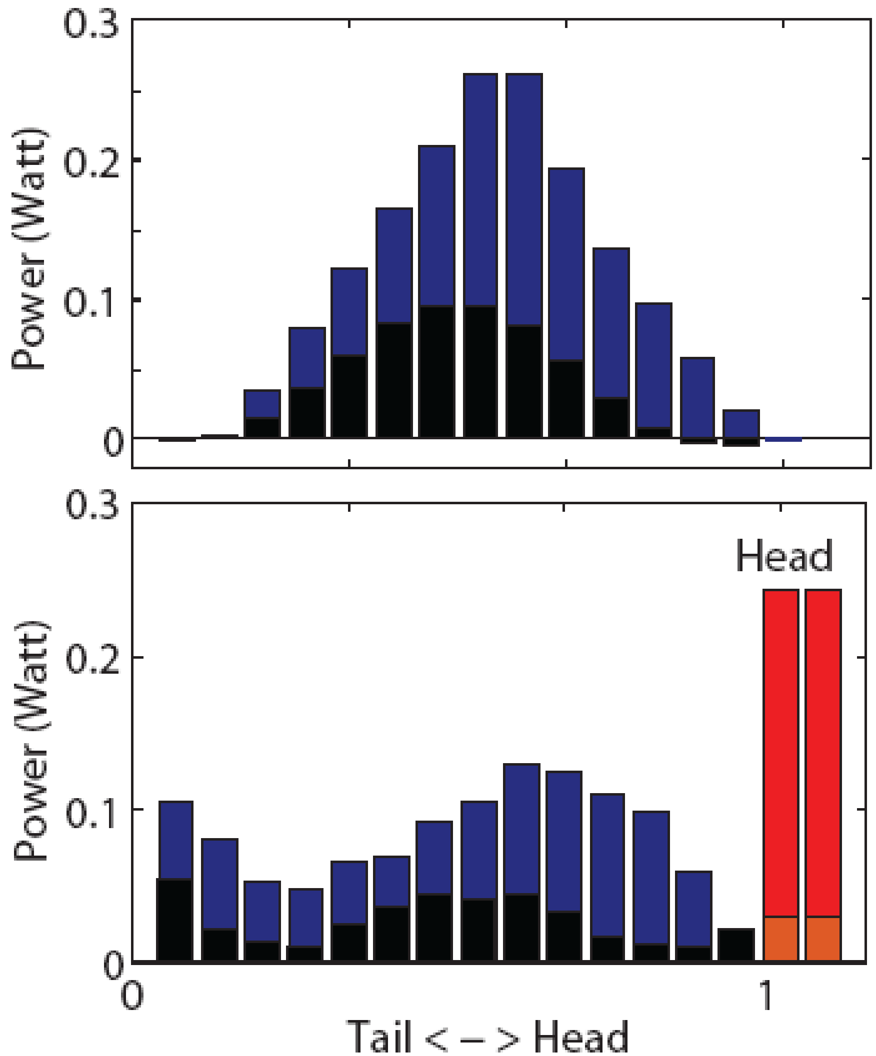
At f=2.5 Hz, total power developed in the 15 gram swimmer is ~1 W.

Top is 5 cm below surface

Power generation and dissipation on the body

Blue=A/λ=0.2, black=A/λ=0.06

f=4 Hz



Power calcuated

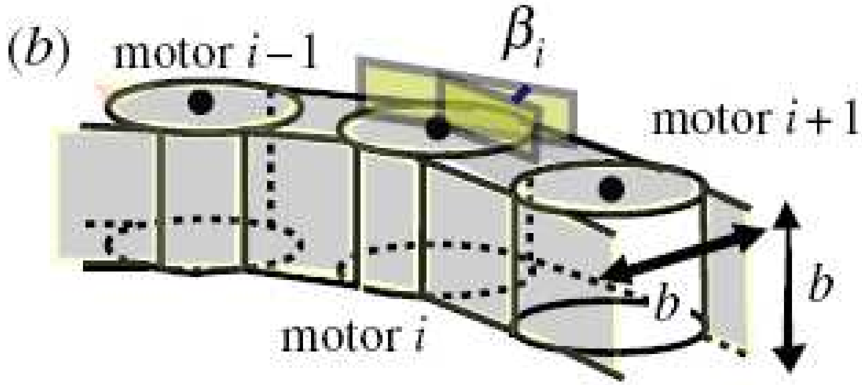
$$P = \boldsymbol{\tau} \cdot \boldsymbol{\omega}$$

$$P = \boldsymbol{F} \cdot \boldsymbol{v}$$

Each bar represents 0.8 cm × 1.6 cm cross-sectional area along the body or on the head.

Top is 5 cm below surface

Internal actuation generates kinematics

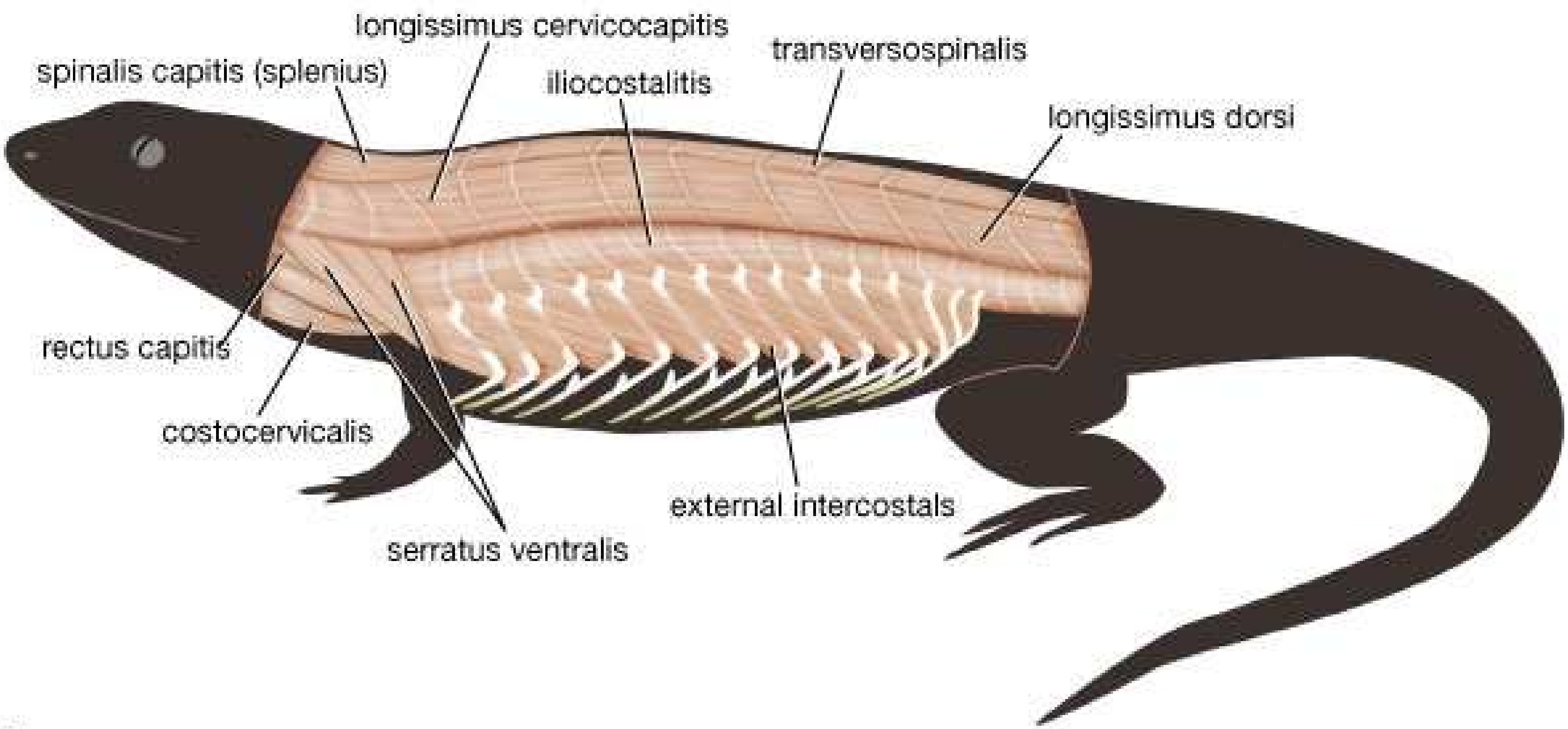


Motor driven

Muscle driven

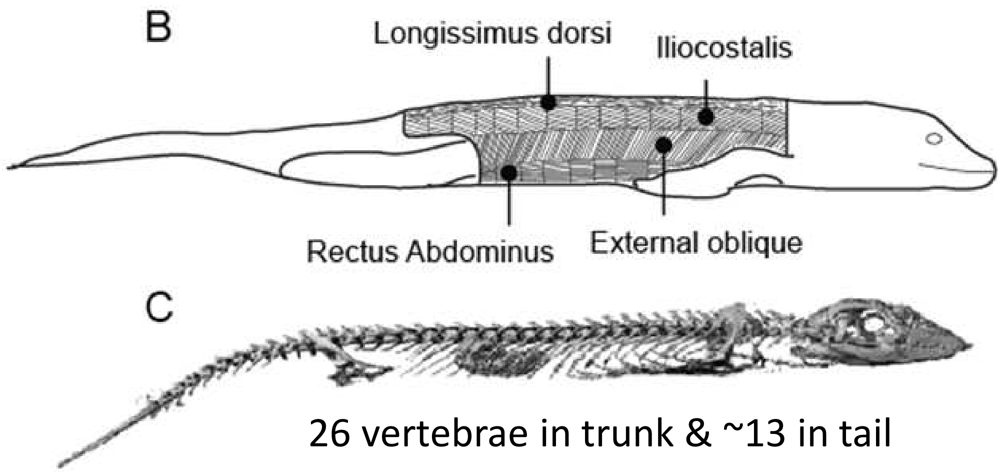
Can we use the model to predict how the sandfish "turns on" its muscles to move its body?

Trunk musculature in a lizard



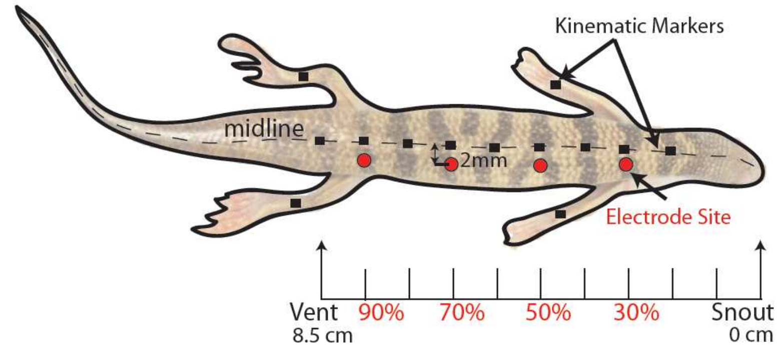
Muscle activity recordings during subsuface swimming

Musculature

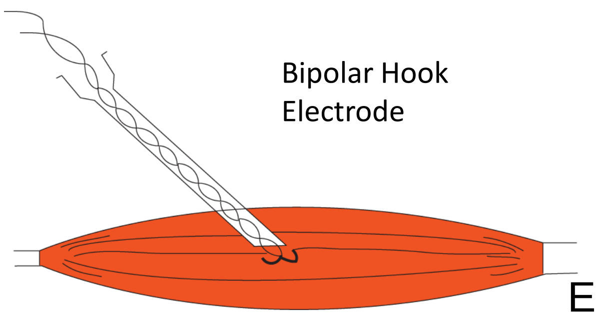


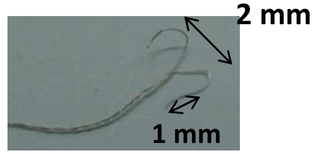
26 vertebrae in trunk & ~13 in tail

Implantation sites



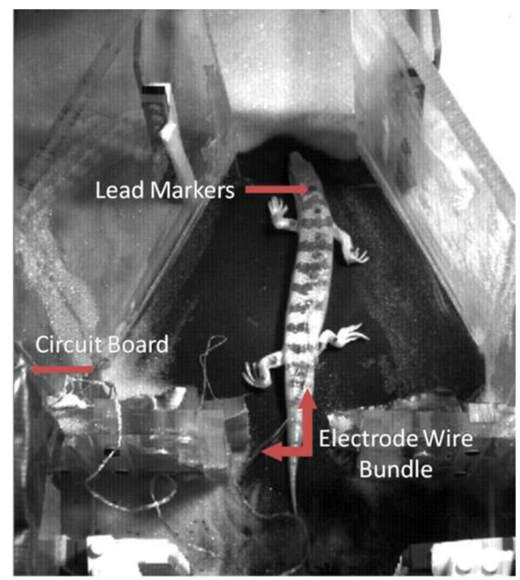
Recording Technique





50 µm diameter stainless steel wire

Apparatus

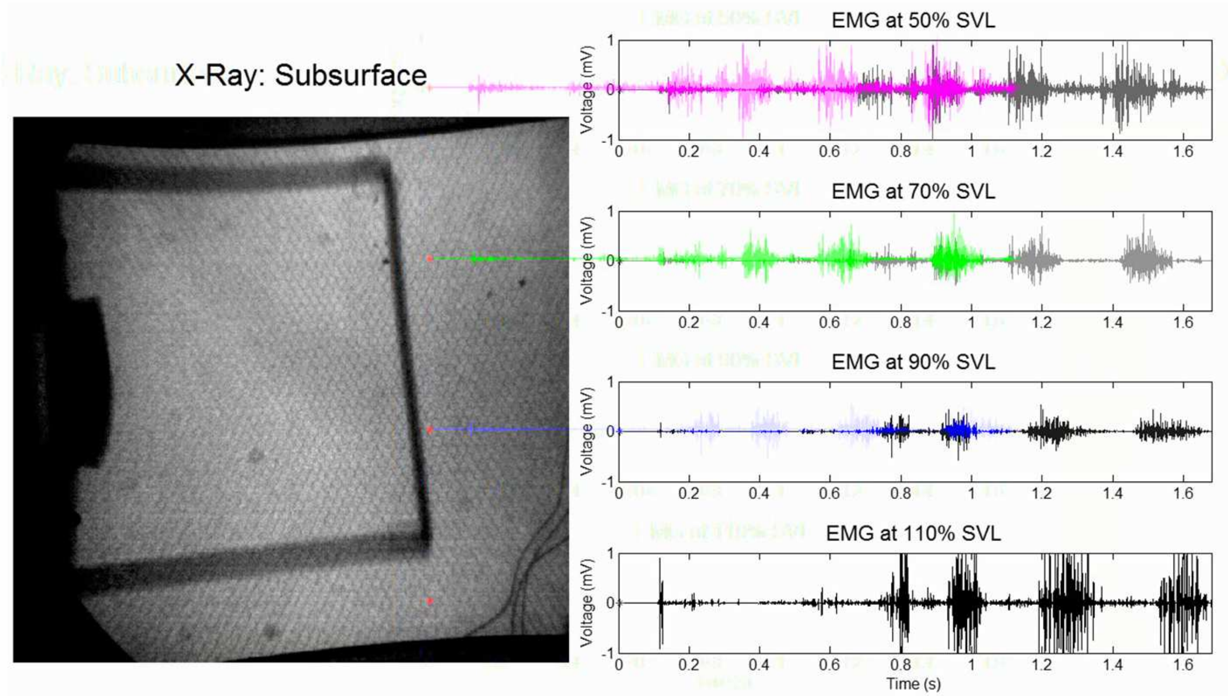


Steinmetz, Goldman, In prep, 2011

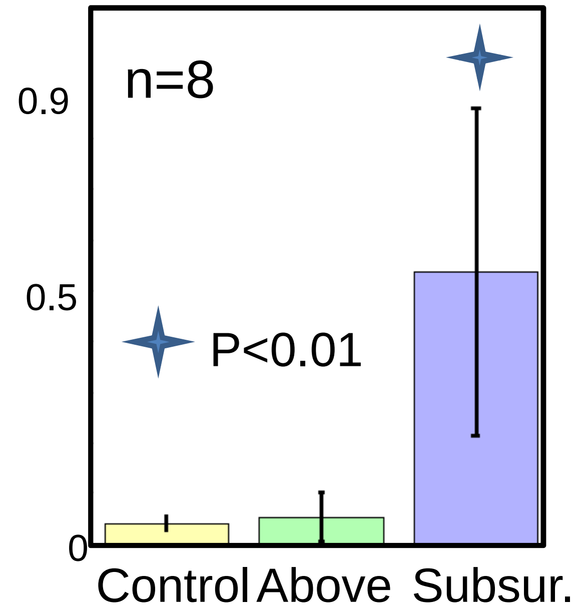
Swimming Muscle Activation (EMG)

Steinmetz, Goldman, In prep, 2011

Slowed x10



Mean Intensity (mV)

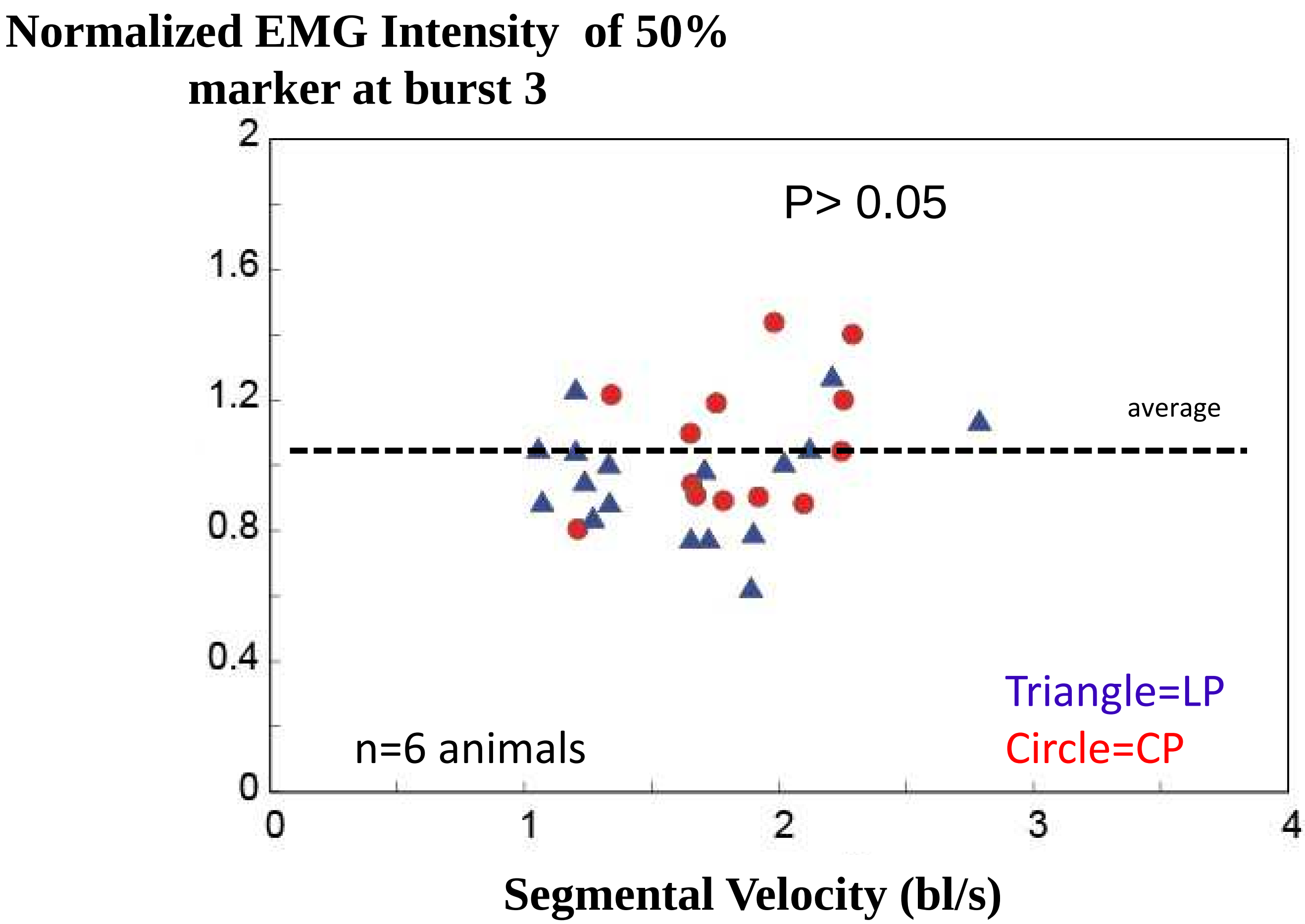


Intensity=EMG burst area/EMG duration

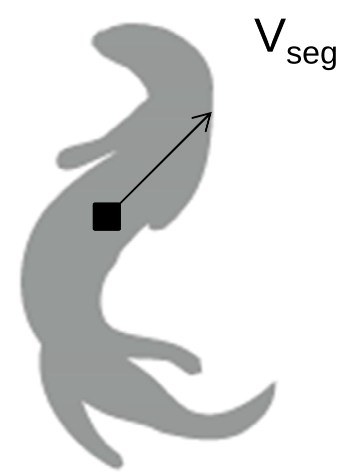
Control: Intensity is recorded when animal is not moving

Speed independence

Steinmetz, Goldman, In prep, 2011

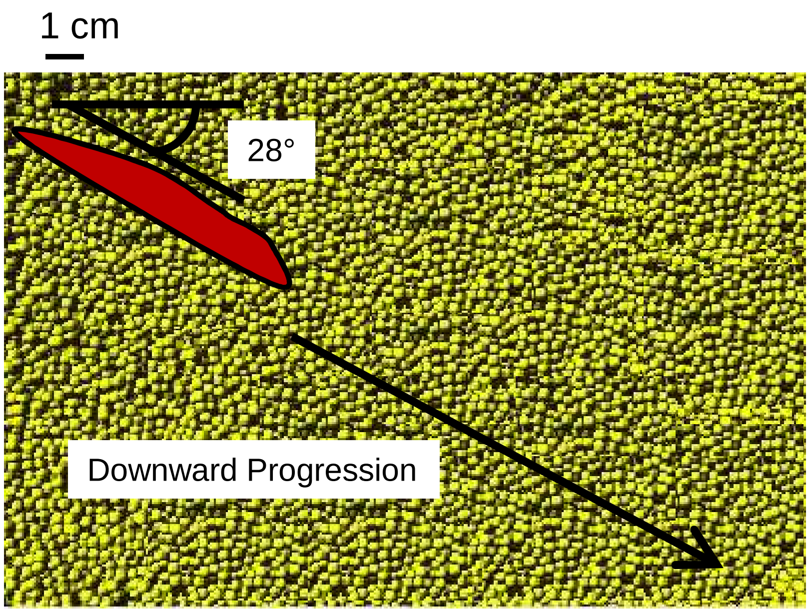


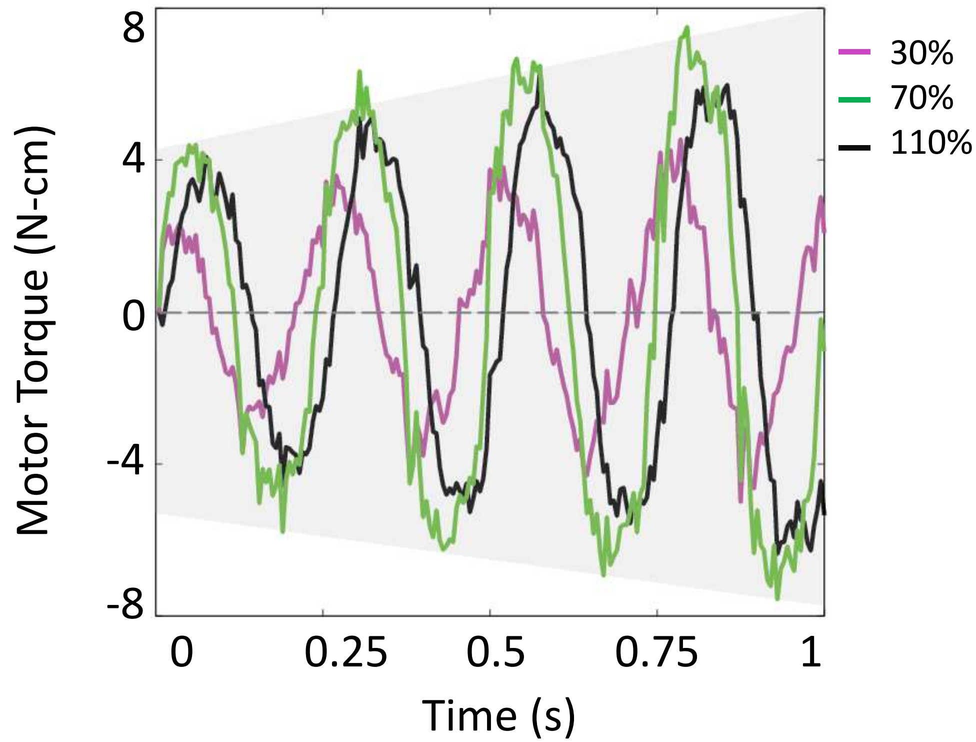
Mean segment velocity for half cycle of unduation



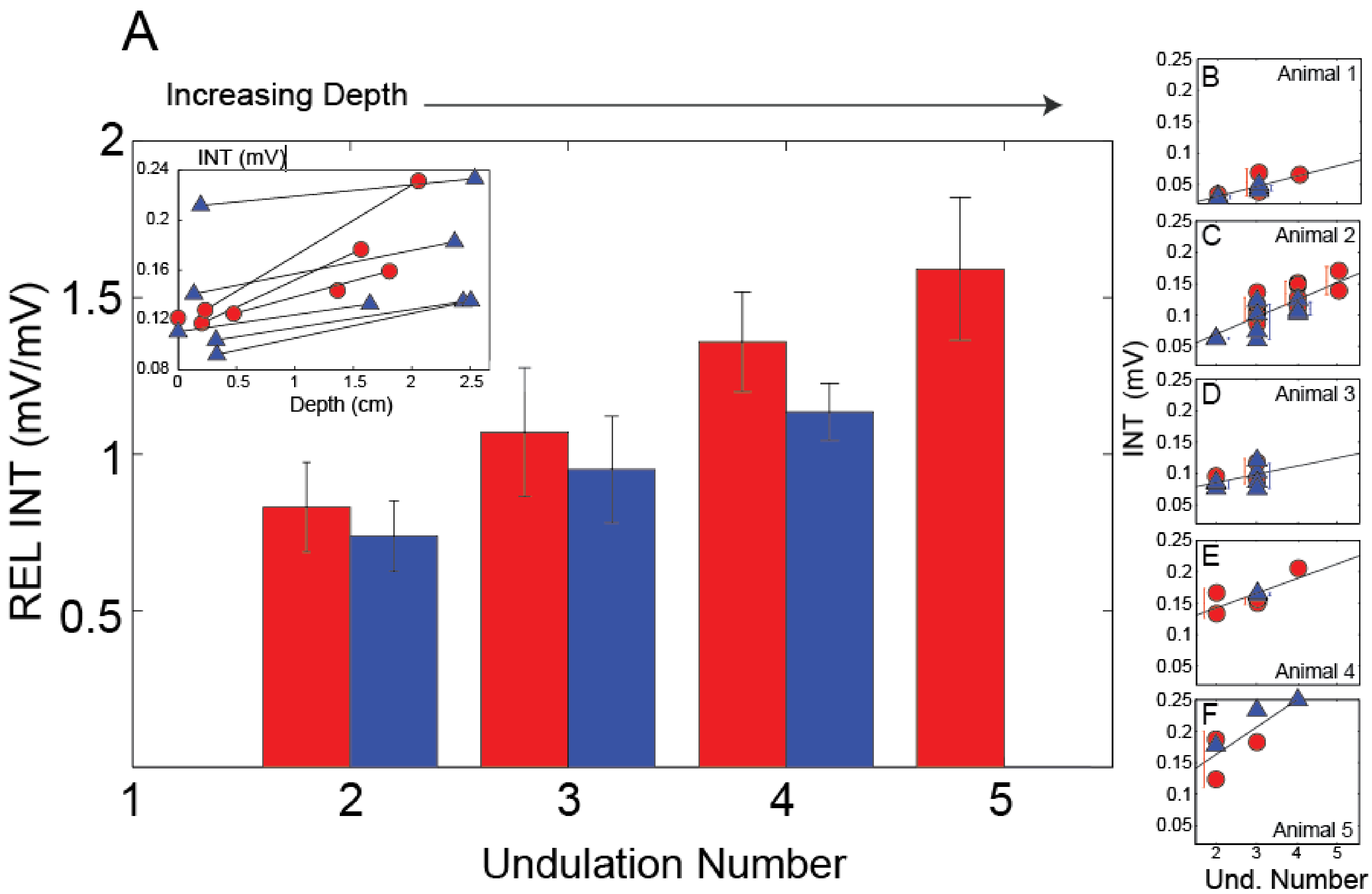
Biological support for frictional fluid picture

Numerical Simulation Predicts an Increase in Motor Torque with Depth



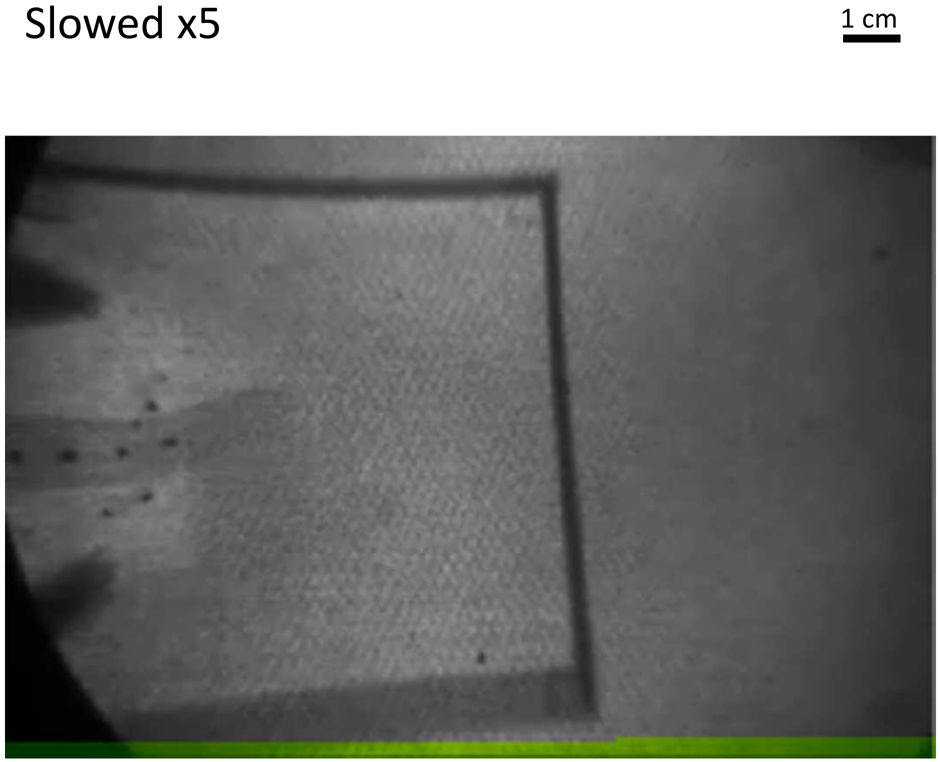


Intensity increases with depth

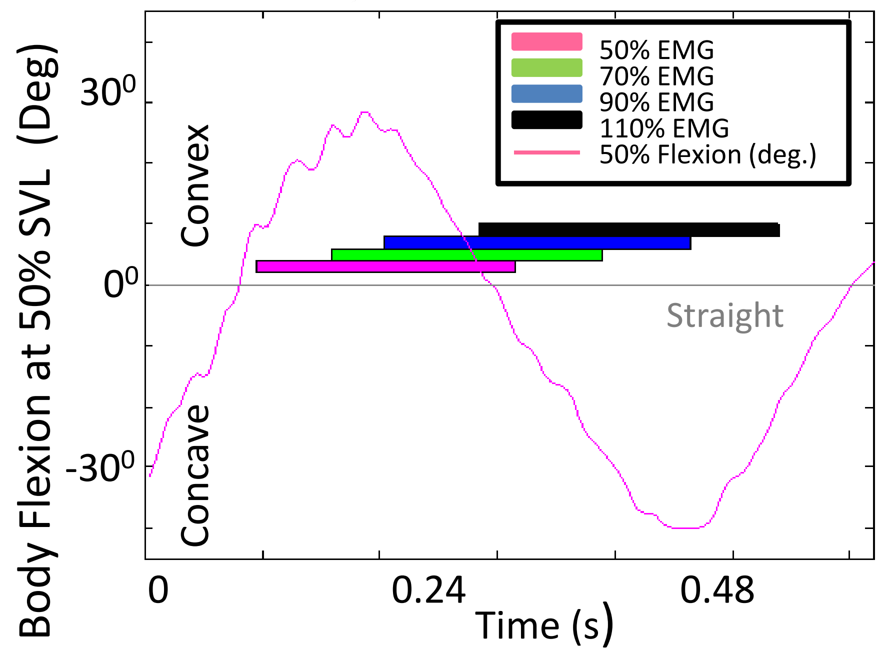


Activation timing of the wave

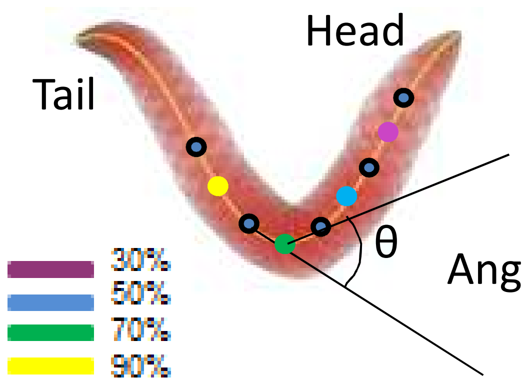
Slowed x5



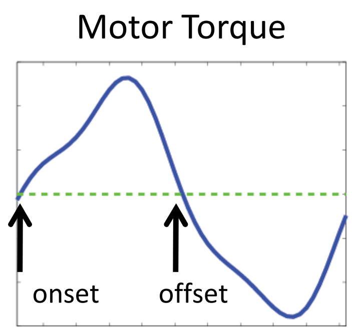
Flexion at
50%



Emergent Activation Pattern with Simple Model



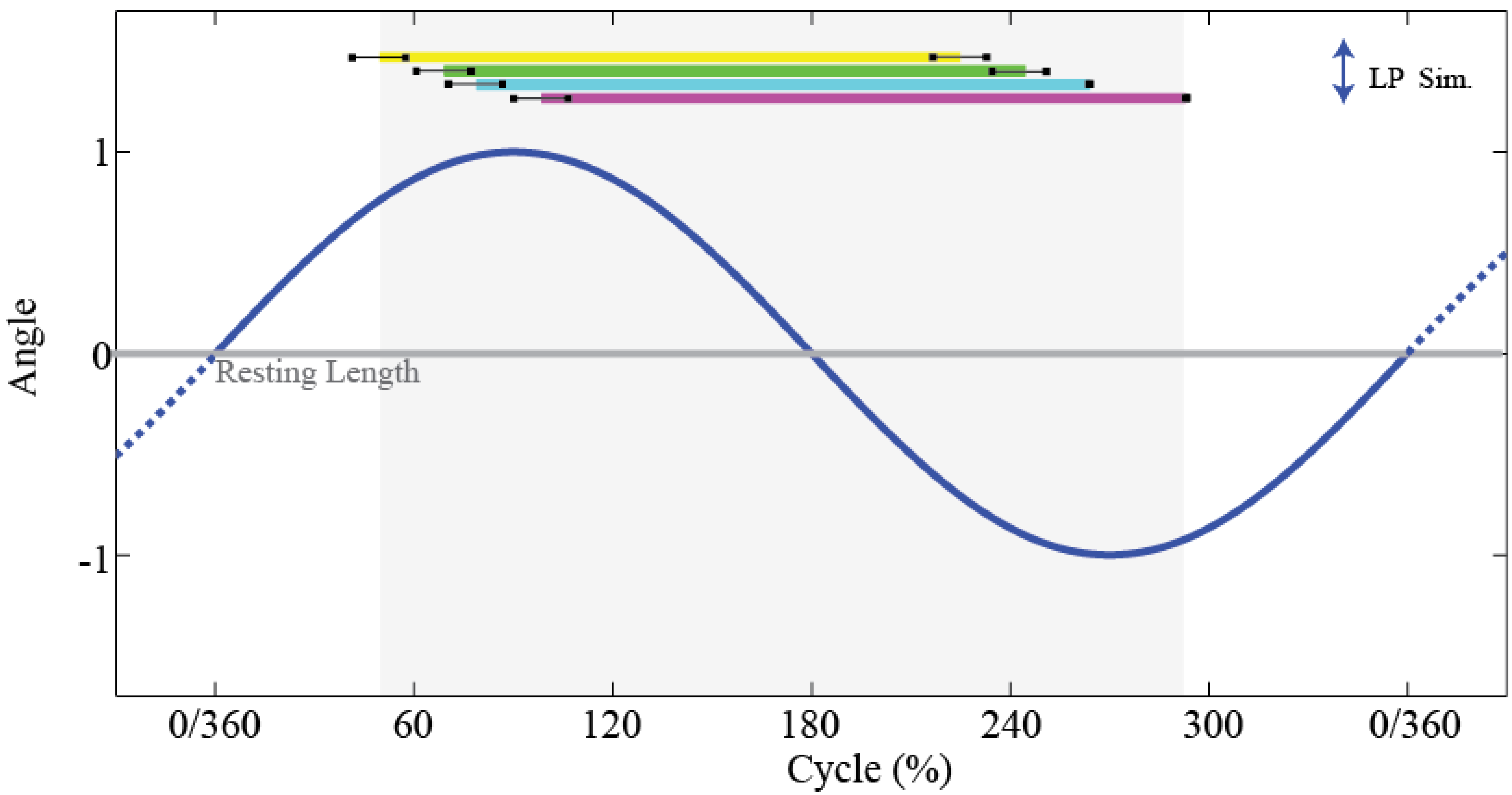
$$\text{Angle} = \frac{\theta_{seg}}{\theta_{max}}$$

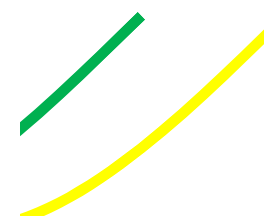


Speed of traveling wave of motor torque is faster than speed of mechanical wave

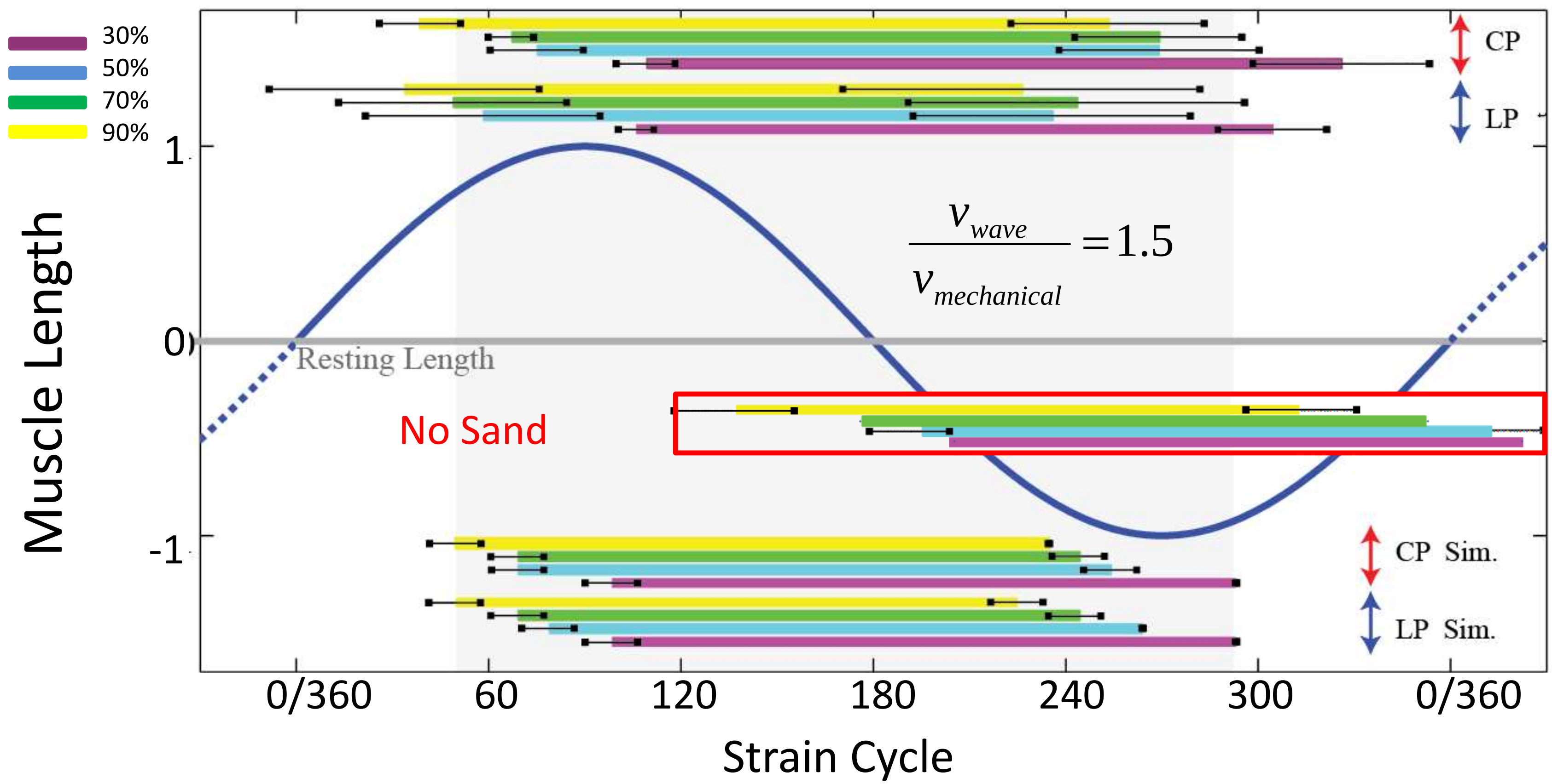
$$\frac{v_{wave}}{v_{mechanical}} = 2$$

Activation Timing

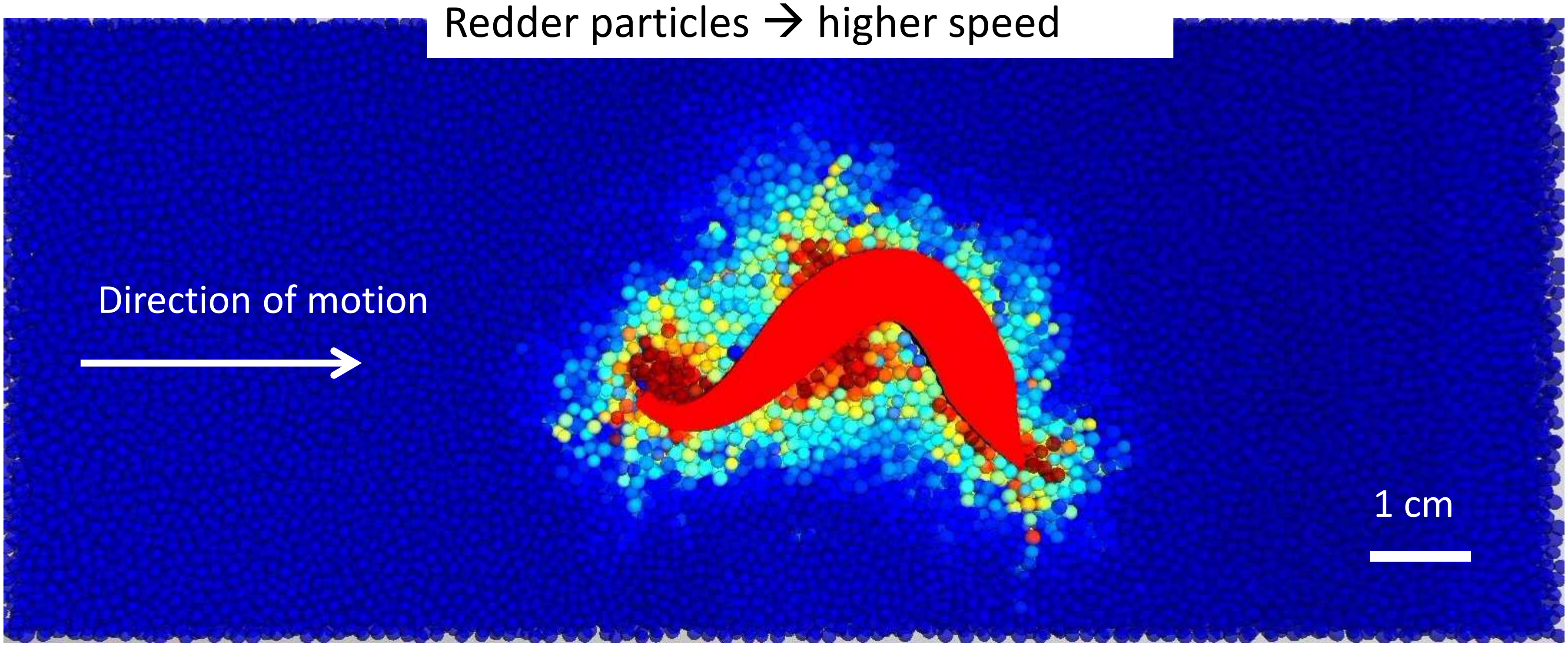




Timing is similar between experiment and simulation



Theory of sand-swimming

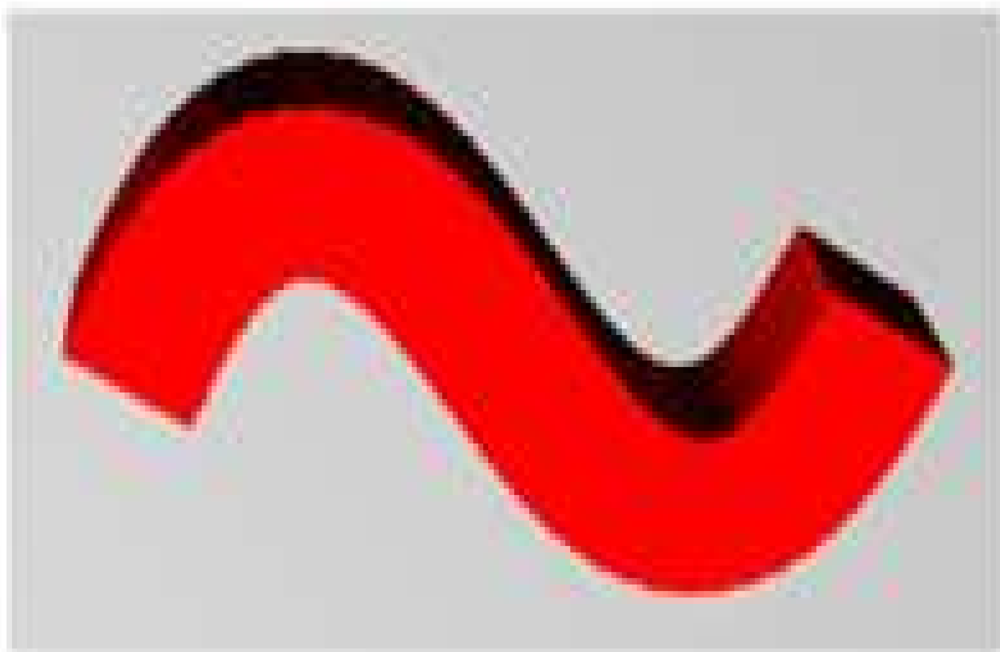


- **Goal**: gain analytic understanding using tools developed for small organisms swimming in fluids — *Resistive Force Theory*

- **Simplify**: no taper, flat head (in simulation η=0.45 for flat head, η=0.57 for tapered head, difference of ~20%)

Resistive force modeling

(after Gray and Hancock, 1954, Taylor 1952



$$\delta F_x = F_{\perp}(\psi)\sin\theta - F_{\parallel}(\psi)\cos\theta$$

$$\int_0^{\lambda}\left(\frac{F_{\perp}(\psi)}{area}\sin\theta - \frac{F_{\parallel}(\psi)}{area}\cos\theta\right)\sqrt{1+\tan^2\theta}\,b\,dx + \overline{F}_{head} = 0$$

- **Assume** square cross-section swimming at constant speed at fixed depth with waveform:

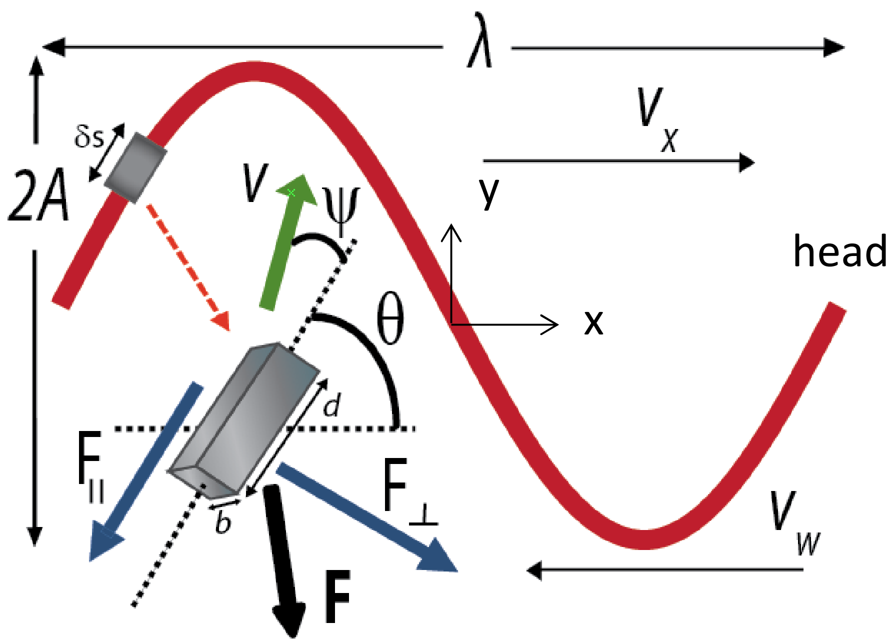
$$y = A\sin\frac{2\pi}{\lambda}(x+v_w t) \qquad \tan\theta = \frac{dy}{dx} = \frac{2A\pi}{\lambda}\cos\frac{2\pi}{\lambda}(x+v_w t)$$

$$v_y = \frac{dy}{dt} = \frac{2A\pi v_w}{\lambda}\cos\frac{2\pi}{\lambda}(x+v_w t) \qquad \psi = \tan^{-1}\left(\frac{v_y}{v_x}\right) - \theta.$$

- Non-inertial movement (net thrust=net drag)
- Head drag = flat plate (or for taper use 30% flat plate, Schiffer, 2001)
- Insert force laws to solve for $\eta = v_x / v_w$ for given A, λ and obtain $v_x = \eta v_w = \eta\lambda f$

Resistive force modeling

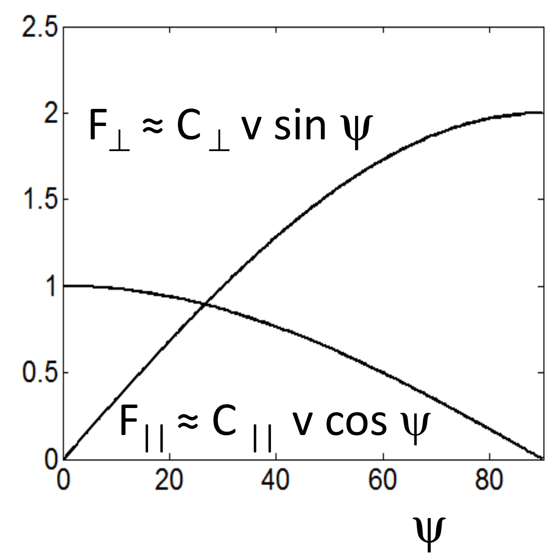
(after Gray and Hancock, 1954)



$$\delta F_x = F_\perp(\psi)\sin\theta - F_\parallel(\psi)\cos\theta$$

$$\int_0^\lambda \left(\frac{F_\perp(\psi)}{area}\sin\theta - \frac{F_\parallel(\psi)}{area}\cos\theta\right)\sqrt{1+\tan^2\theta}\,b\,dx + \overline{F}_{head} = 0$$

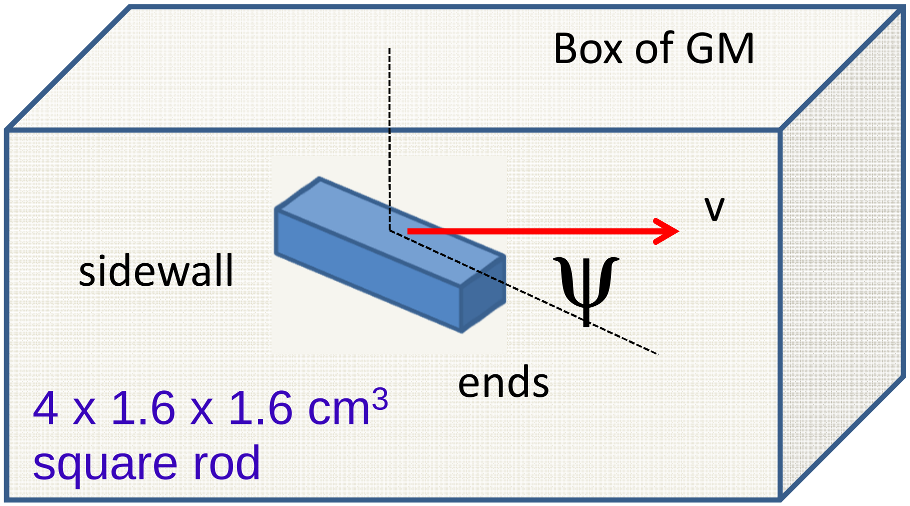
In low Re fluids, for long narrow element



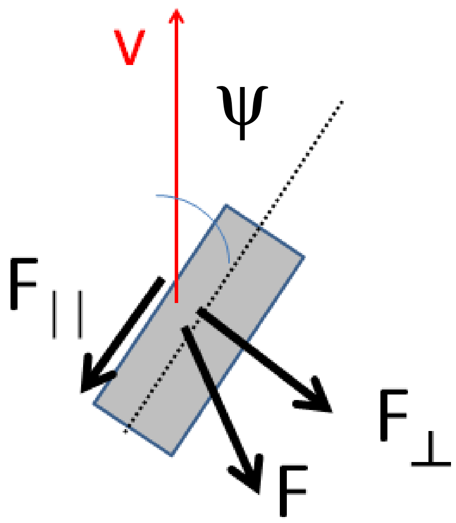
$C_\perp : C_{||} \approx 2:1$

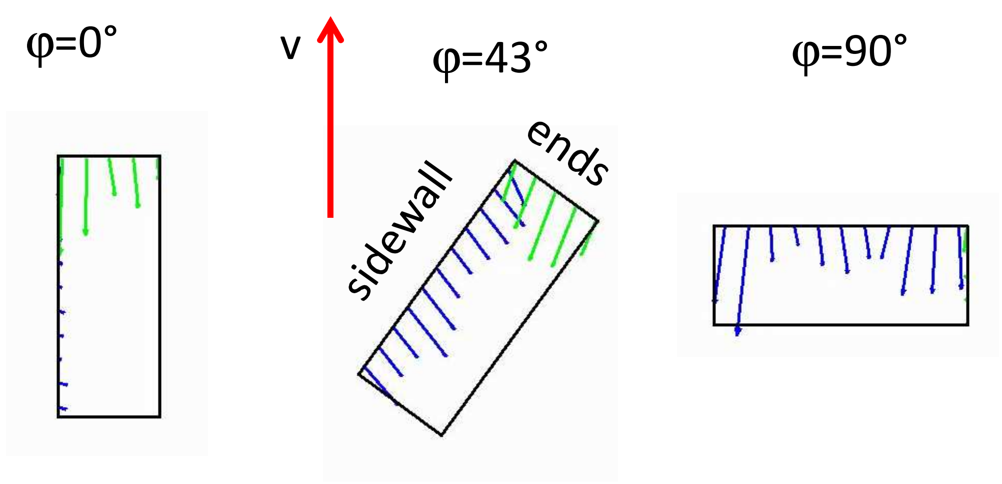
Granular resistive forces

Obtain empirical drag laws for $F_{\perp}$ and $F_{||}$

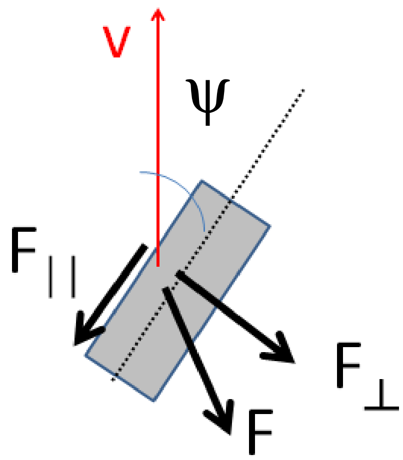


- Drag rod in **simulation** of 3 mm "glass" particles while varying φ
- Use simulation to resolve forces on all surfaces
- Average in space and time during **steady state**, divide by area to find surface stresses





Granular resistive forces



Empirical granular resistive force laws

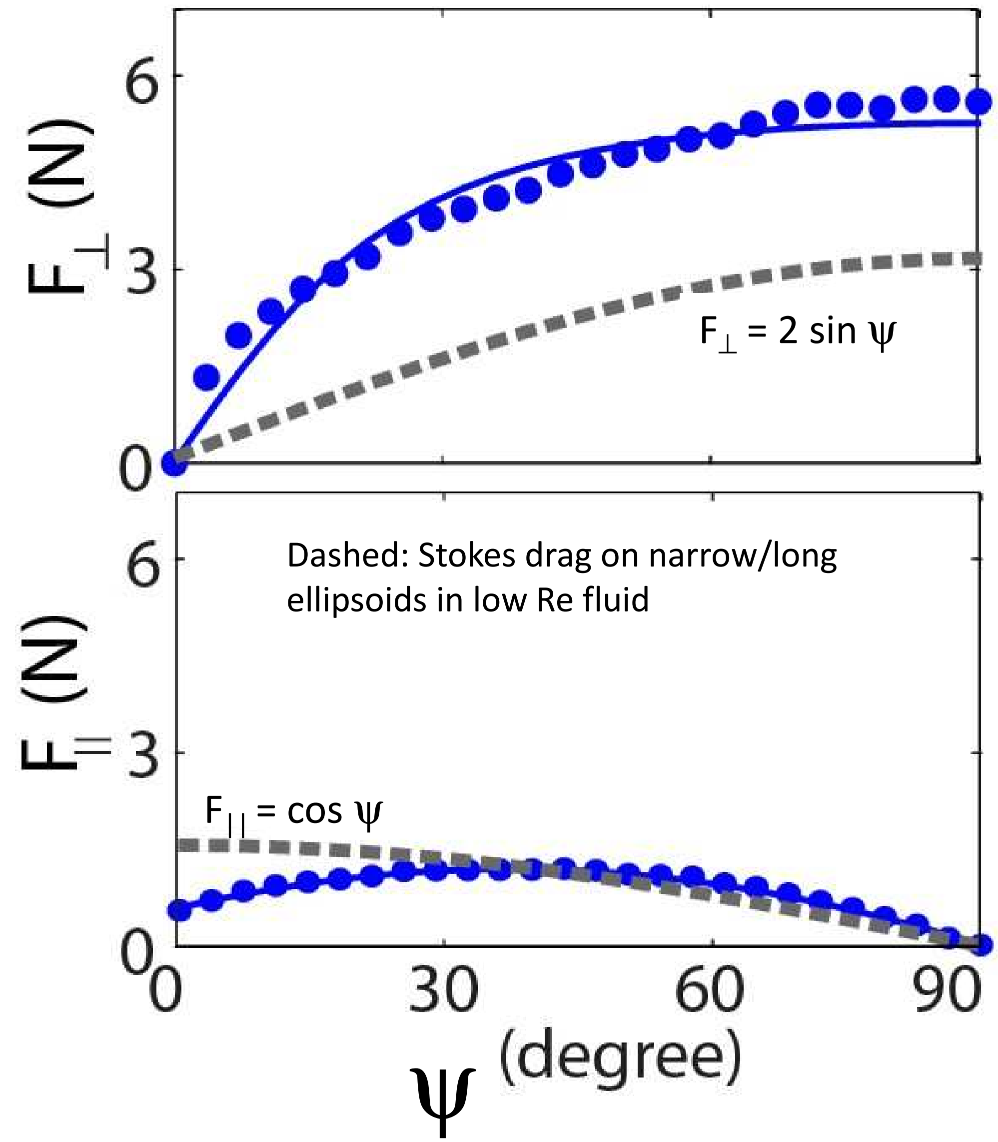
$$F_{\perp} = C_S \sin \beta_0$$

$$F_{\parallel} = [C_F \cos \psi + C_L (1 - \sin \psi)]$$

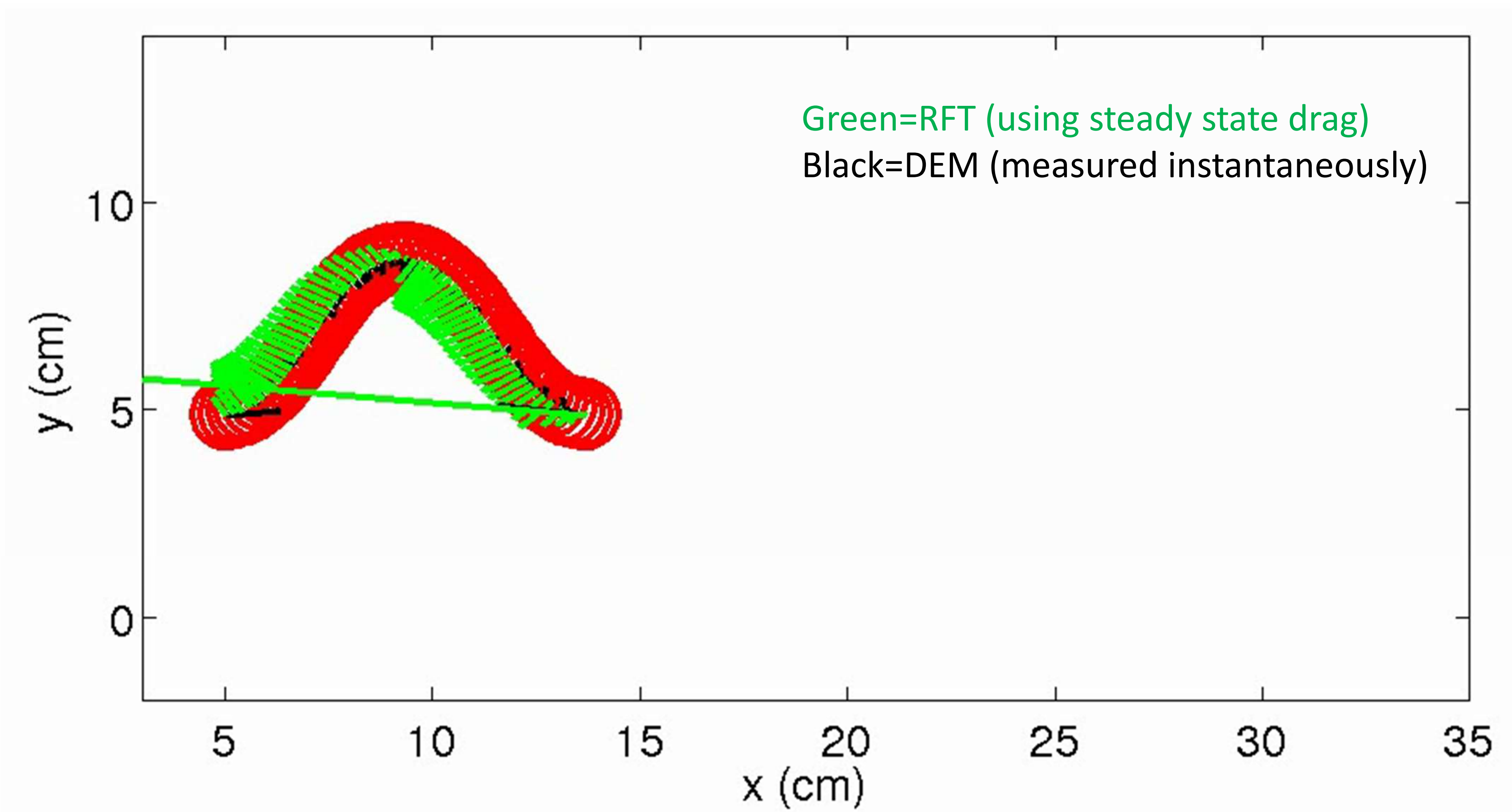
$$\tan \beta_0 = \gamma \sin \psi$$

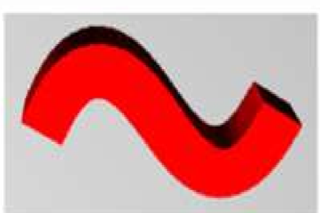
Independent of speed

(Forces shown for LP)



Resistive forces in DEM and RFT

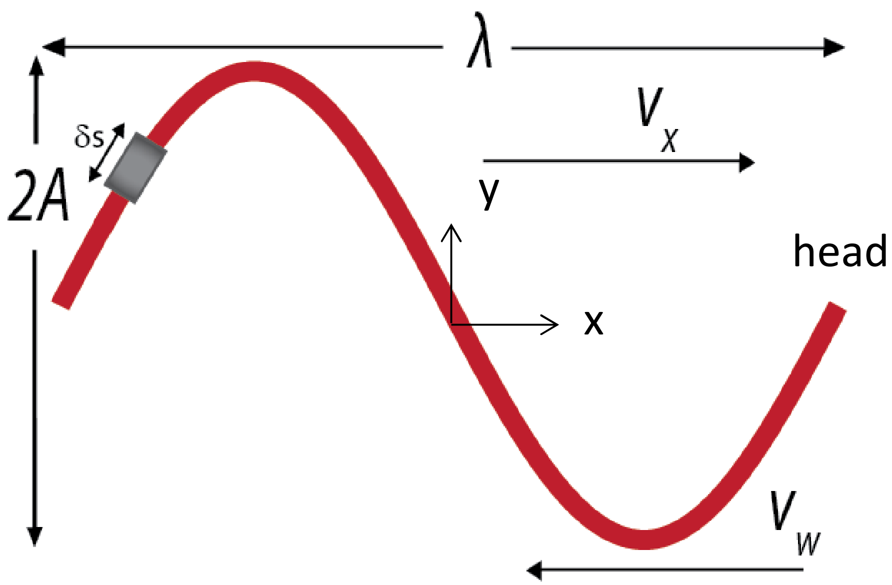




Square body, no taper, 3 mm particles

Resistive force modeling

(after Gray and Hancock, 1954)



- **Assume** square cross-section swimming at constant speed at fixed depth with waveform:

$$y = A\sin\frac{2\pi}{\lambda}(x+v_w t) \qquad \tan\theta = \frac{dy}{dx} = \frac{2A\pi}{\lambda}\cos\frac{2\pi}{\lambda}(x+v_w t)$$

$$v_y = \frac{dy}{dt} = \frac{2A\pi v_w}{\lambda}\cos\frac{2\pi}{\lambda}(x+v_w t) \qquad \psi = \tan^{-1}\left(\frac{v_y}{v_x}\right) - \theta.$$

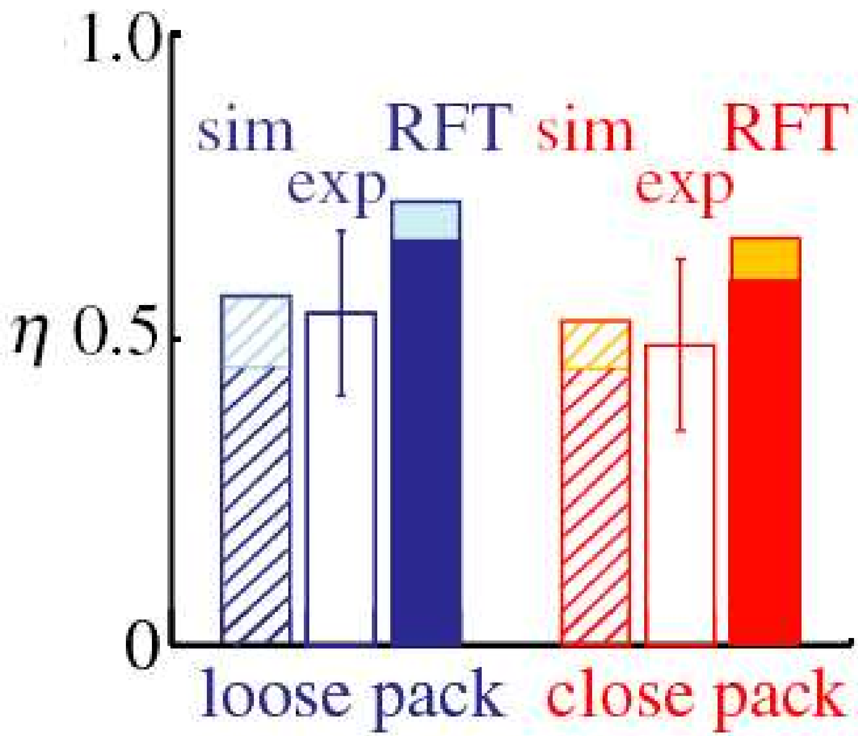
- Non-inertial movement (net thrust=net drag)
- Head drag = flat plate (or for taper use 30% flat plate, Schiffer, 2001)

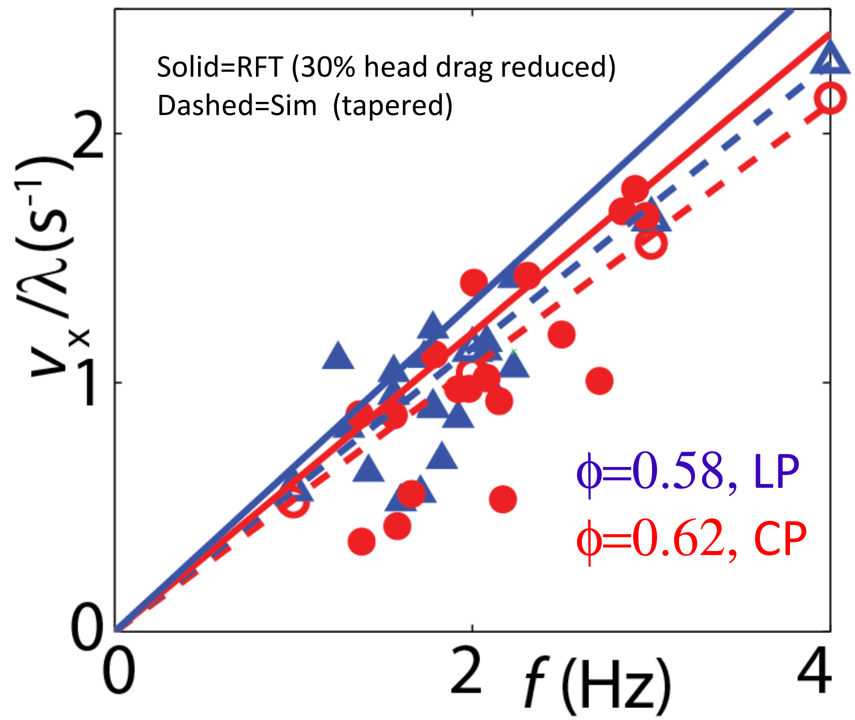
$$\delta F_x = F_\perp(\psi)\sin\theta - F_\parallel(\psi)\cos\theta$$

$$\int_0^\lambda \left(\frac{F_\perp(\psi)}{area}\sin\theta - \frac{F_\parallel(\psi)}{area}\cos\theta\right)\sqrt{1+\tan^2\theta}\,b\,dx + \overline{F}_{head} = 0$$

- Insert force laws to solve for $\eta = v_x / v_w$ for given A, λ and obtain $v_x = \eta v_w = \eta\lambda f$

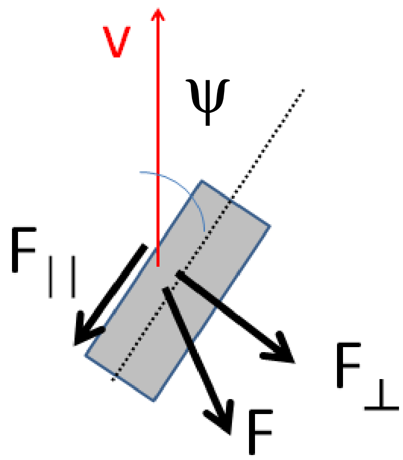
RFT solution





Range=from 30% flat plate drag on head to flat plate head

Granular resistive forces



Empirical granular resistive force laws

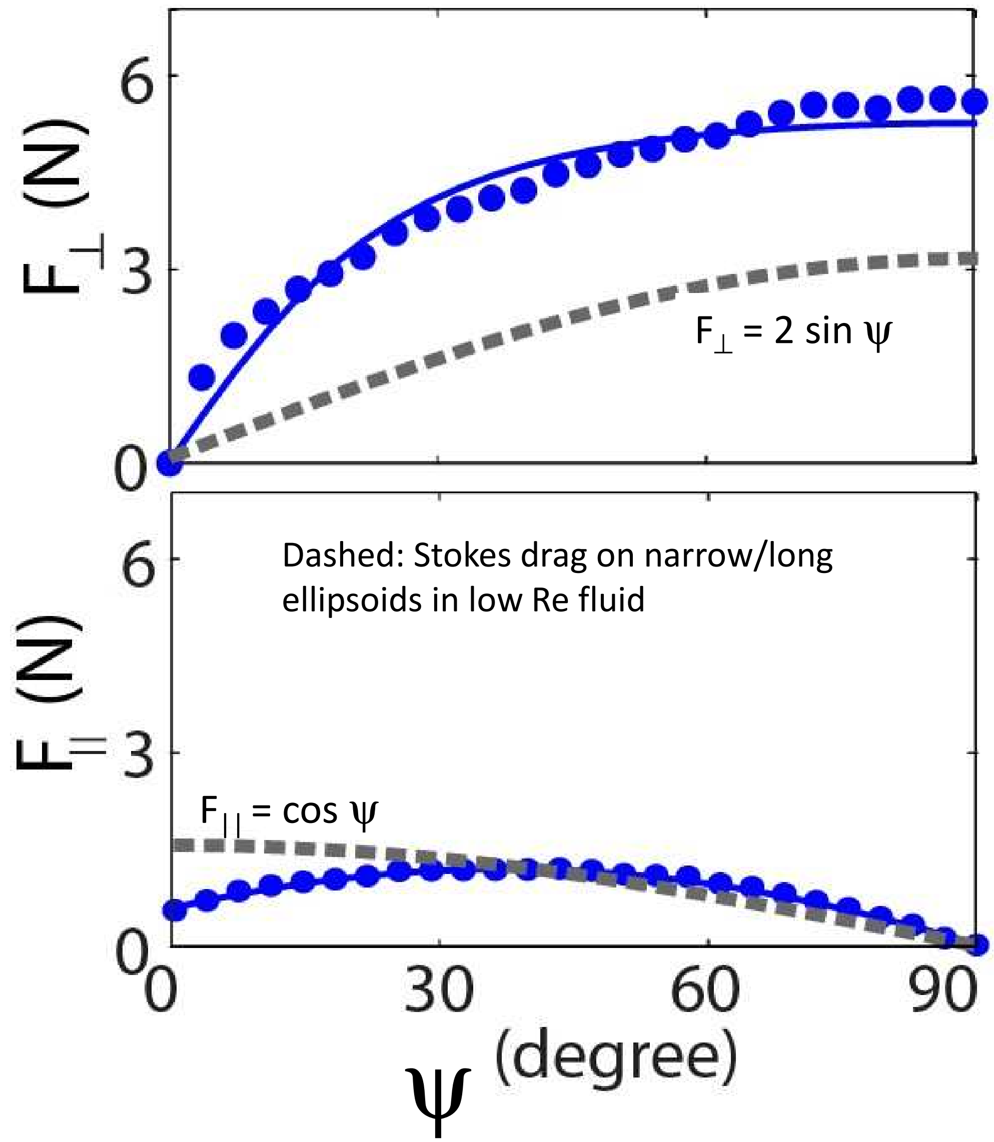
$$F_\perp = C_S \sin \beta_0$$

$$F_\| = [C_F \cos \psi + C_L (1 - \sin \psi)]$$

$$\tan \beta_0 = \gamma \sin \psi$$

Independent of speed

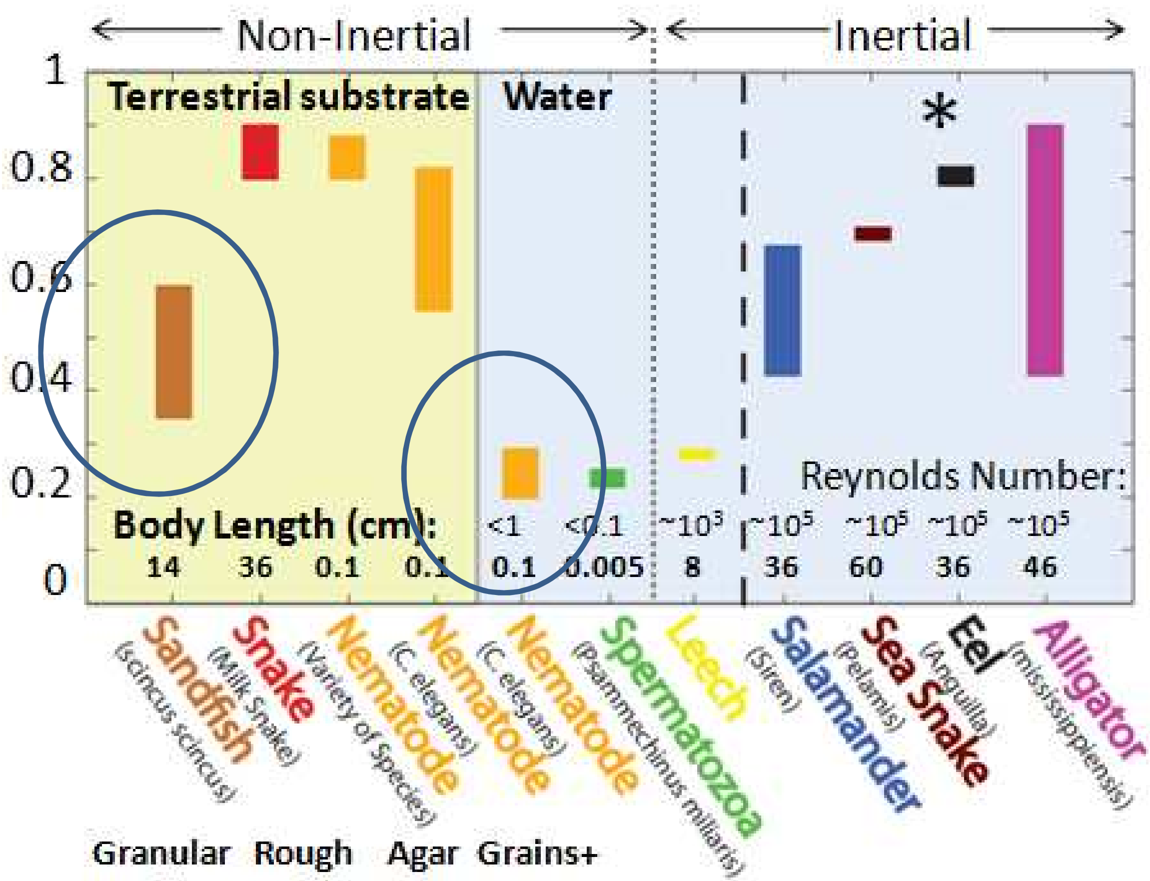
(Forces shown for LP)

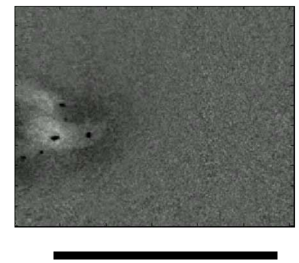


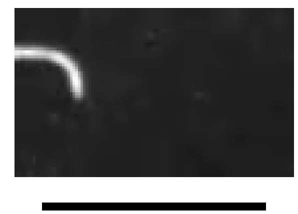
Wave efficiencies of undulatory swimmers

(see Alexander, Vogel, Gray & Hancock, Lighthill, etc..)

η





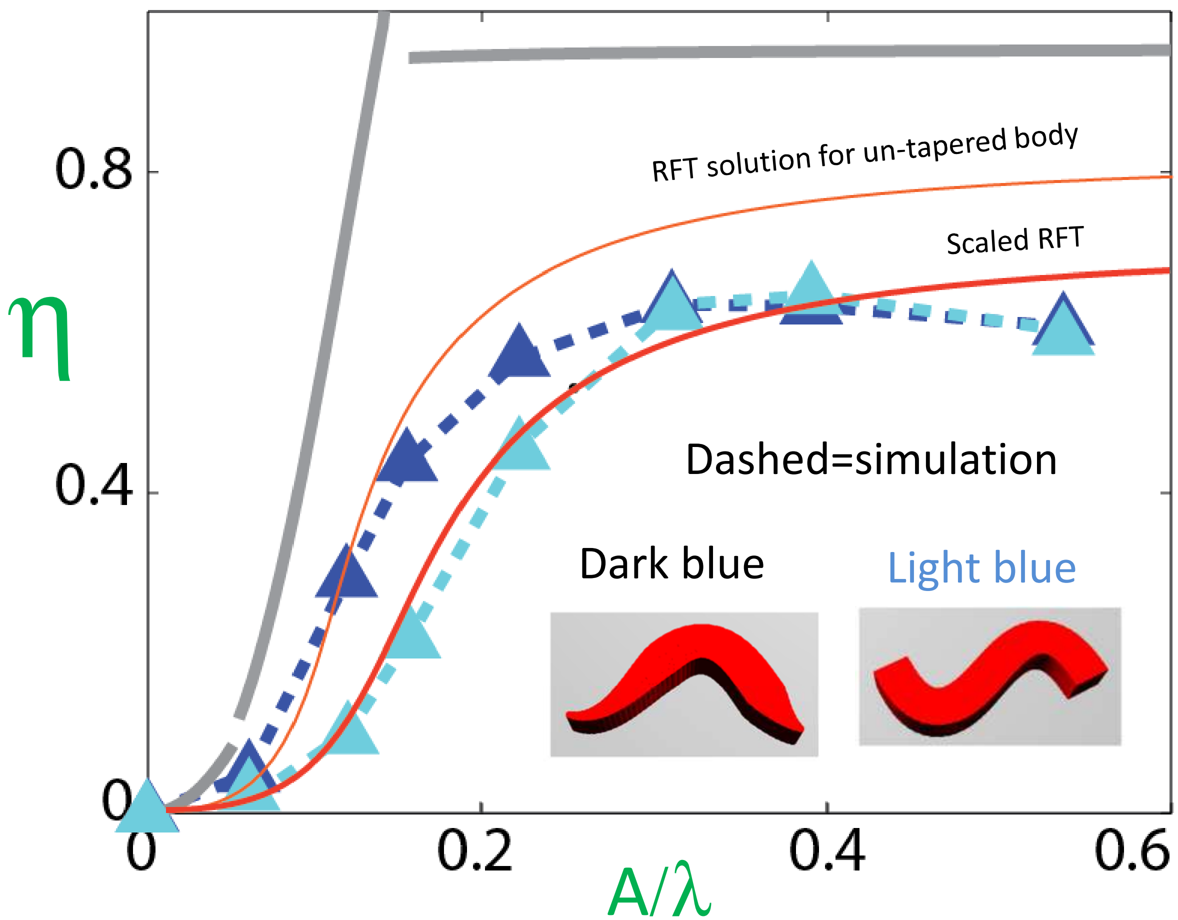


Sarah Steinmetz

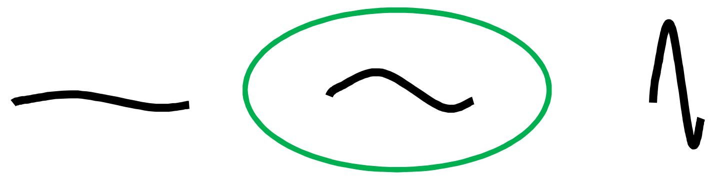
Maladen, et. al (2009), Hu (2010), Jung(2010),Gray and Lissman (1964),Gray and Hancock (1955),Gillis(1996),Fish (1984)

RFT captures form of η vs A/λ

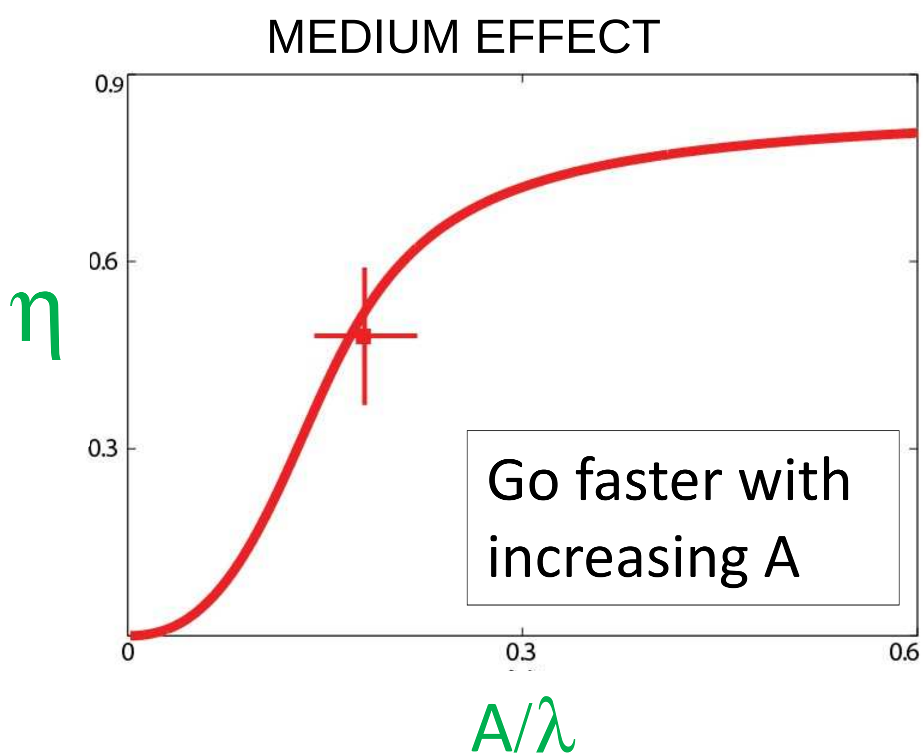
Gray=Analytic solutions (head drag neglected)

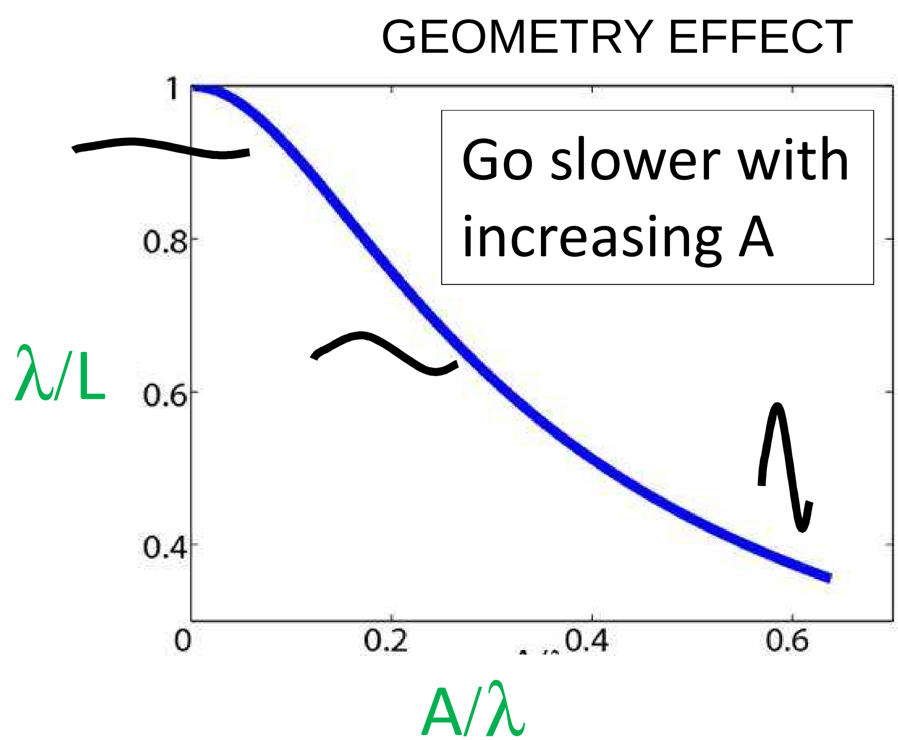


Competition of effects leads to maximum



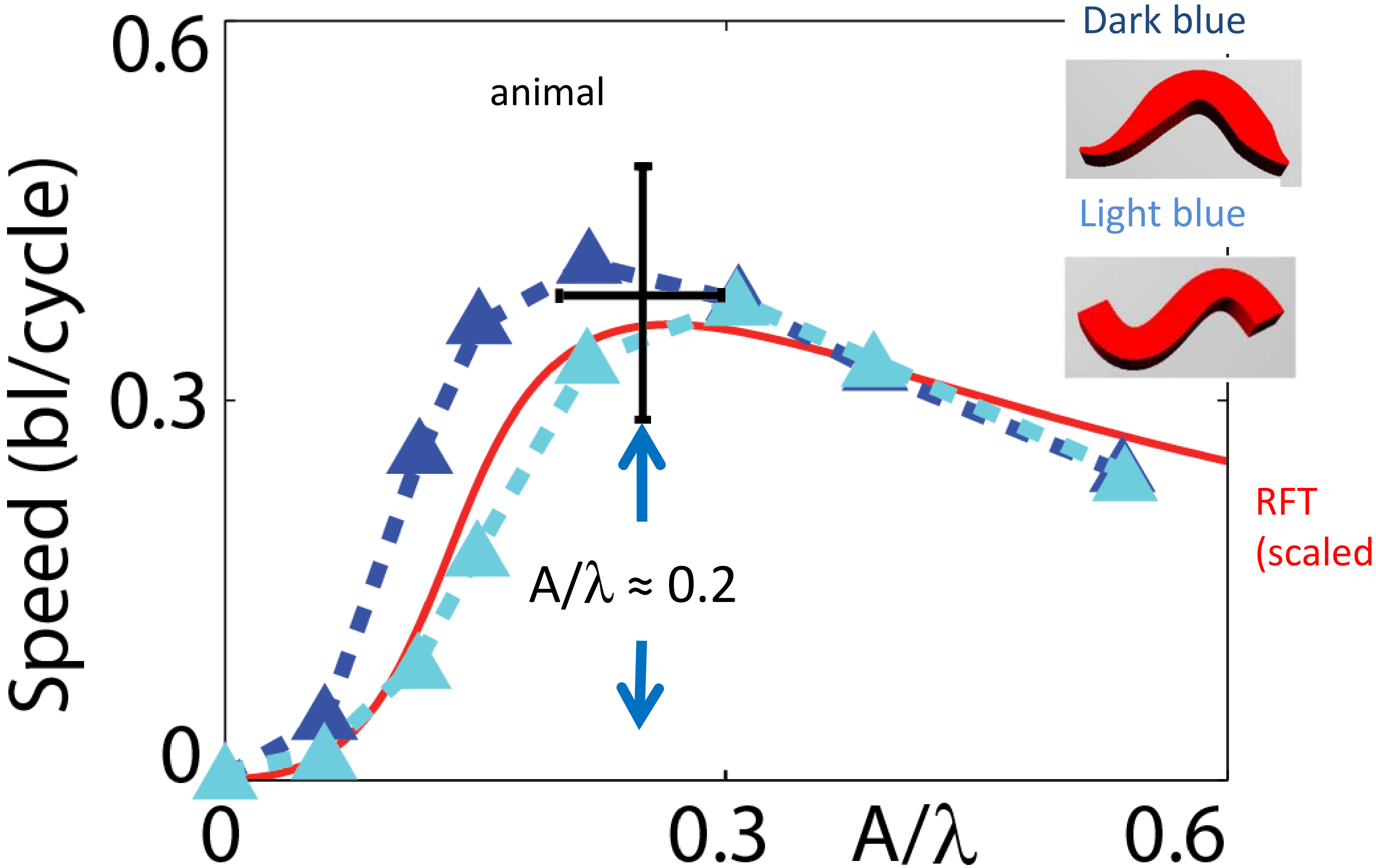
Body lengths/cycle= $\frac{v_x}{fL} = \frac{\eta\lambda f}{fL} = \eta \times \frac{\lambda}{L}$





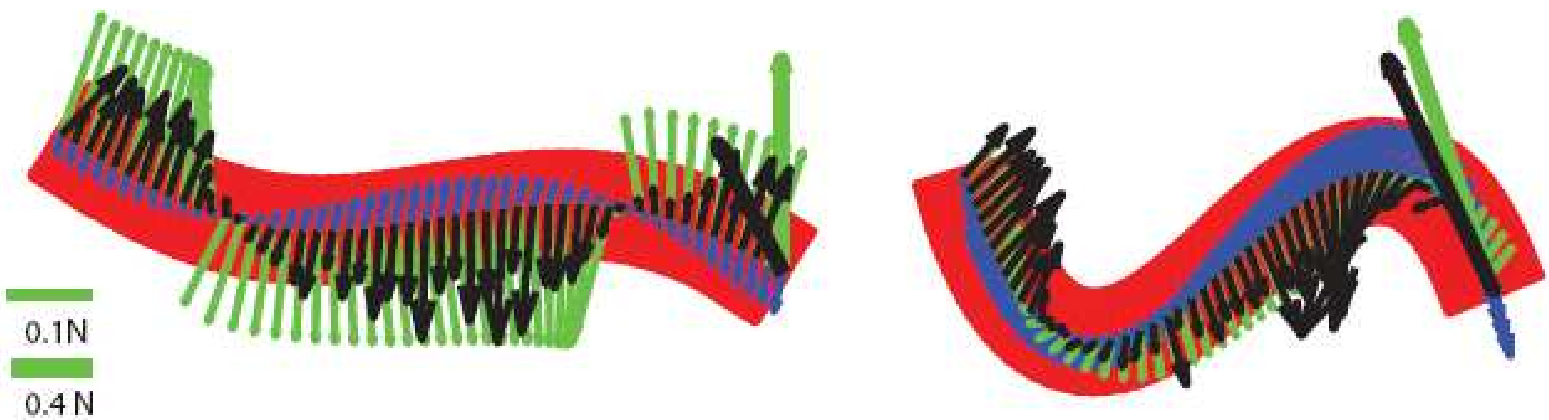
RFT captures functional form & location of optimum

Sandfish simulation in loose packed 3 mm glass beads



RFT force approximation is good at intermediate A/λ but not good instantaneously at small A/λ

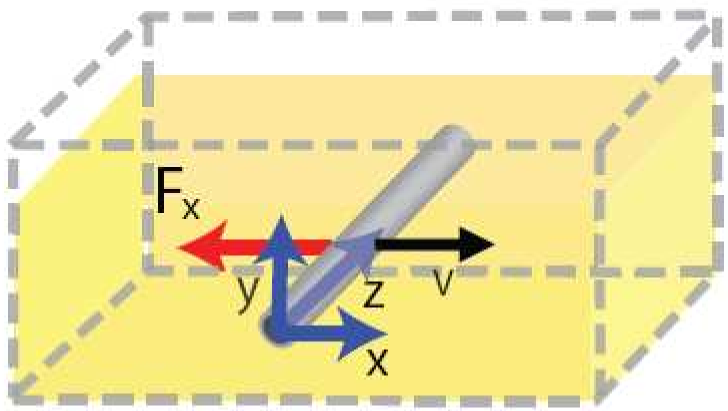
$A/\lambda = 0.06$ $A/\lambda = 0.22$

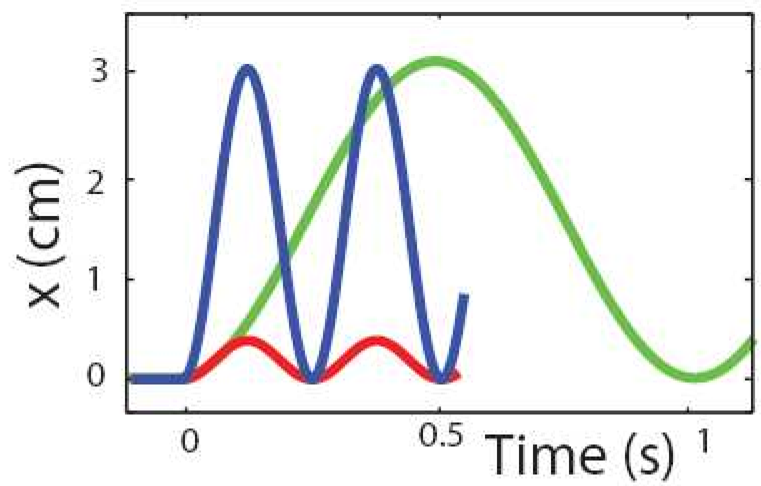


Green=RFT (using steady state drag)
Black=DEM (measured instantaneously)

Why thrust is over-estimated in RFT

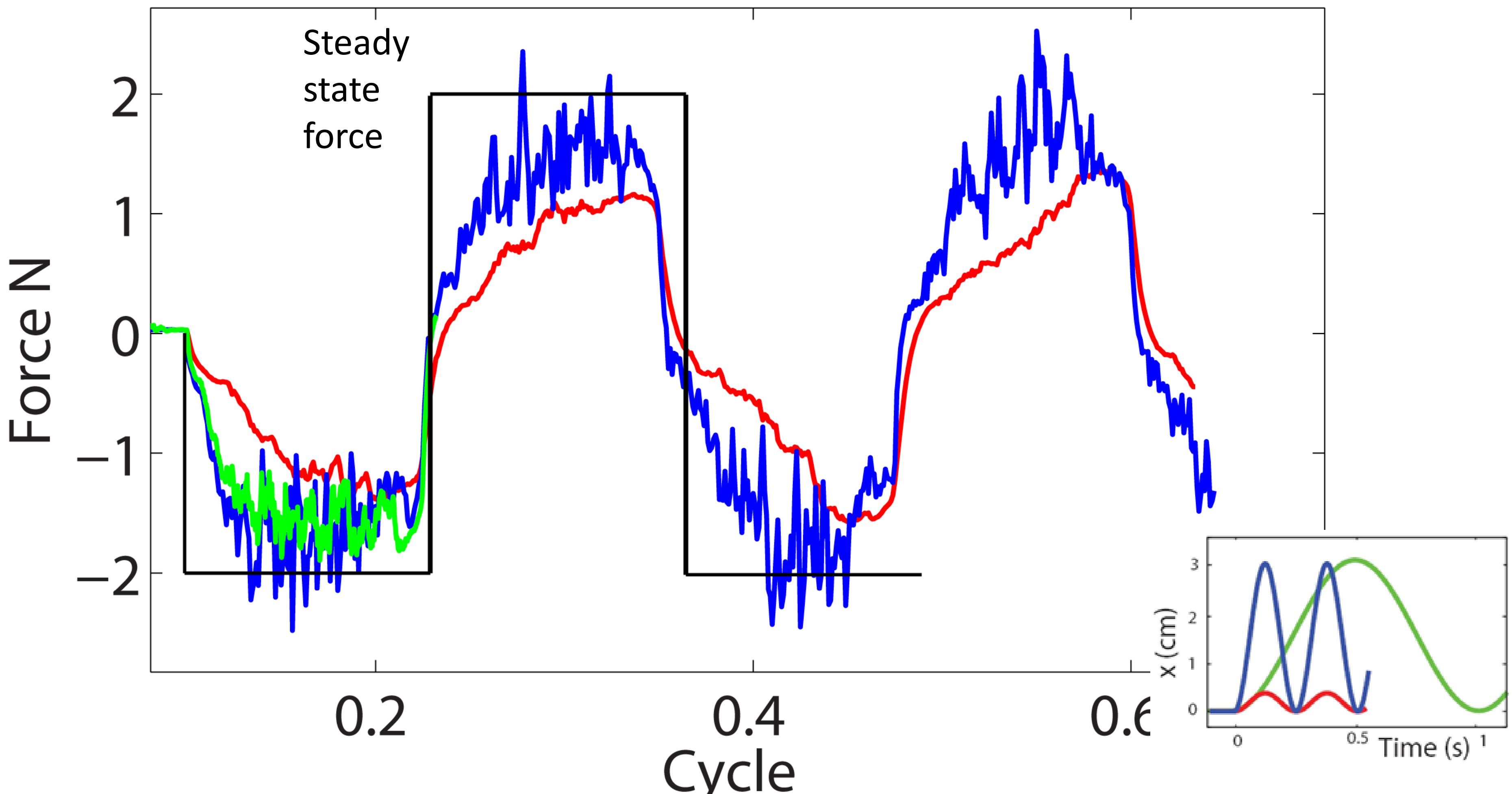
Examine *transient* response in rod drag





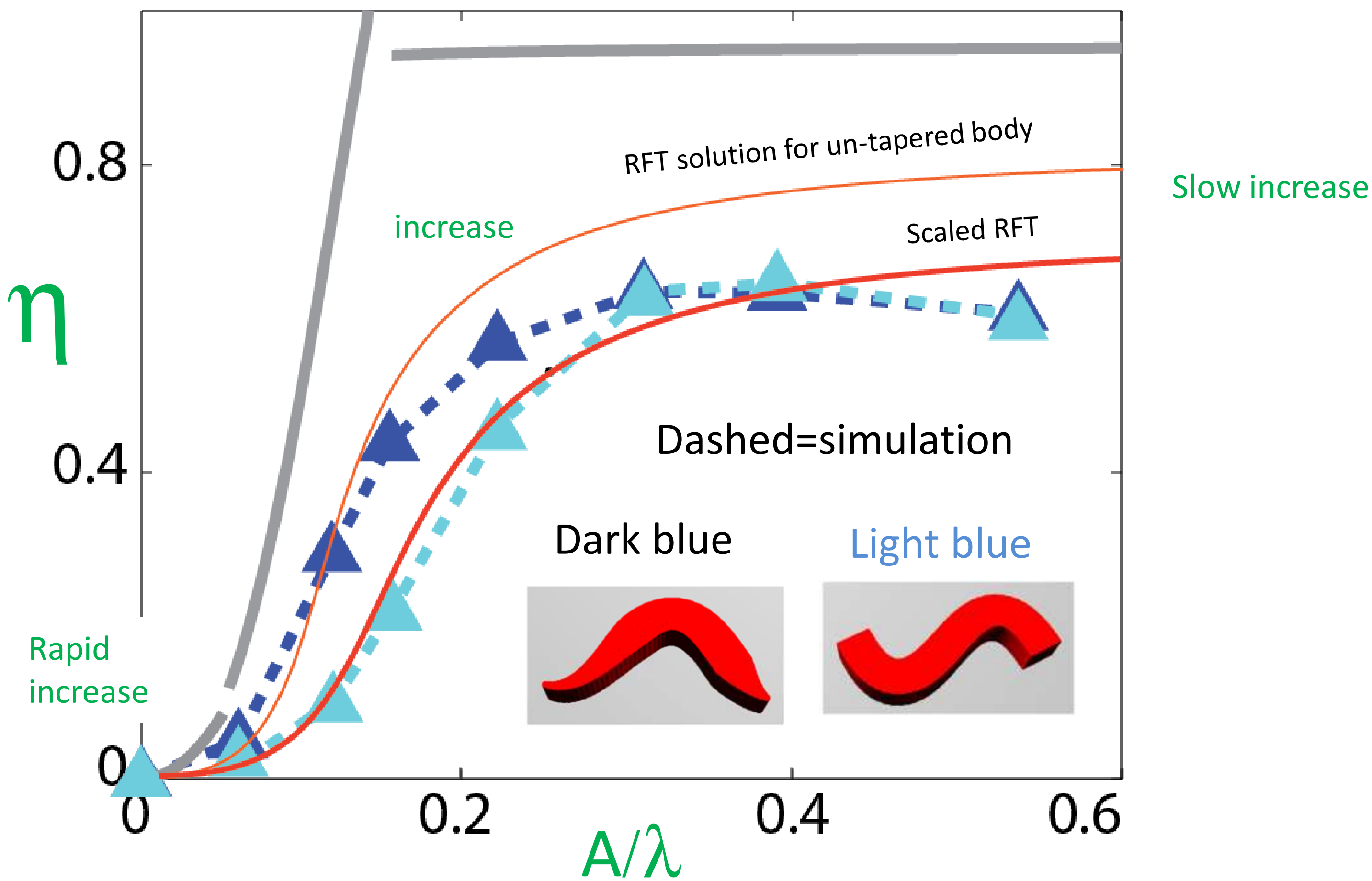
10 cm long rod, 4 cm deep

Force buildup occurs over a characteristic length



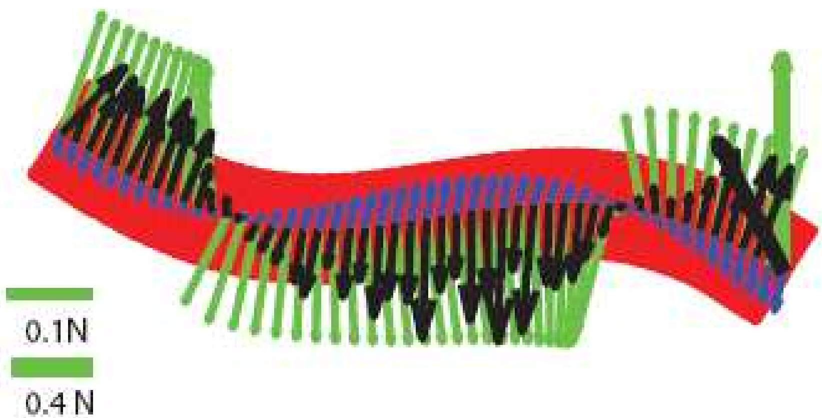
Analytic approximations

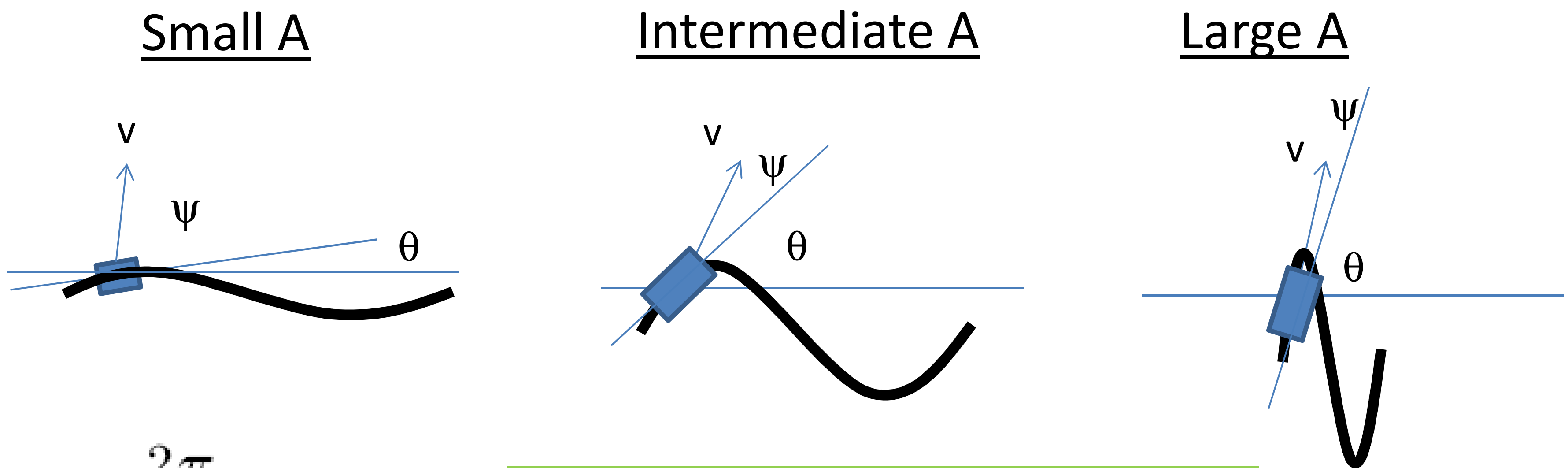
Gray=Analytic solutions (head drag neglected)



Direction of motion of segments

Blue arrows are velocity of each element



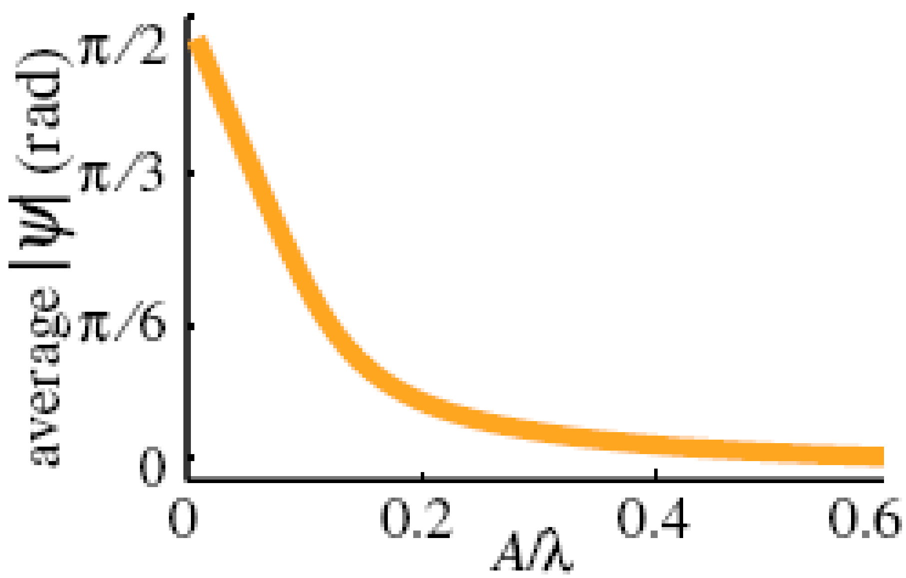


$$y = A \sin \frac{2\pi}{\lambda}(x + v_w t)$$

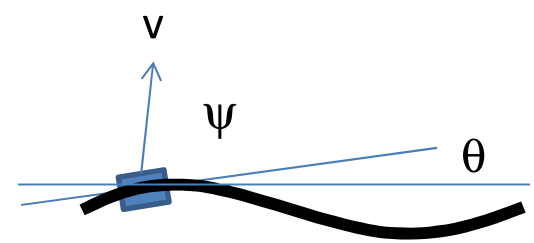
$$\delta F_x = F_\perp(\psi)\sin\theta - F_\parallel(\psi)\cos\theta$$

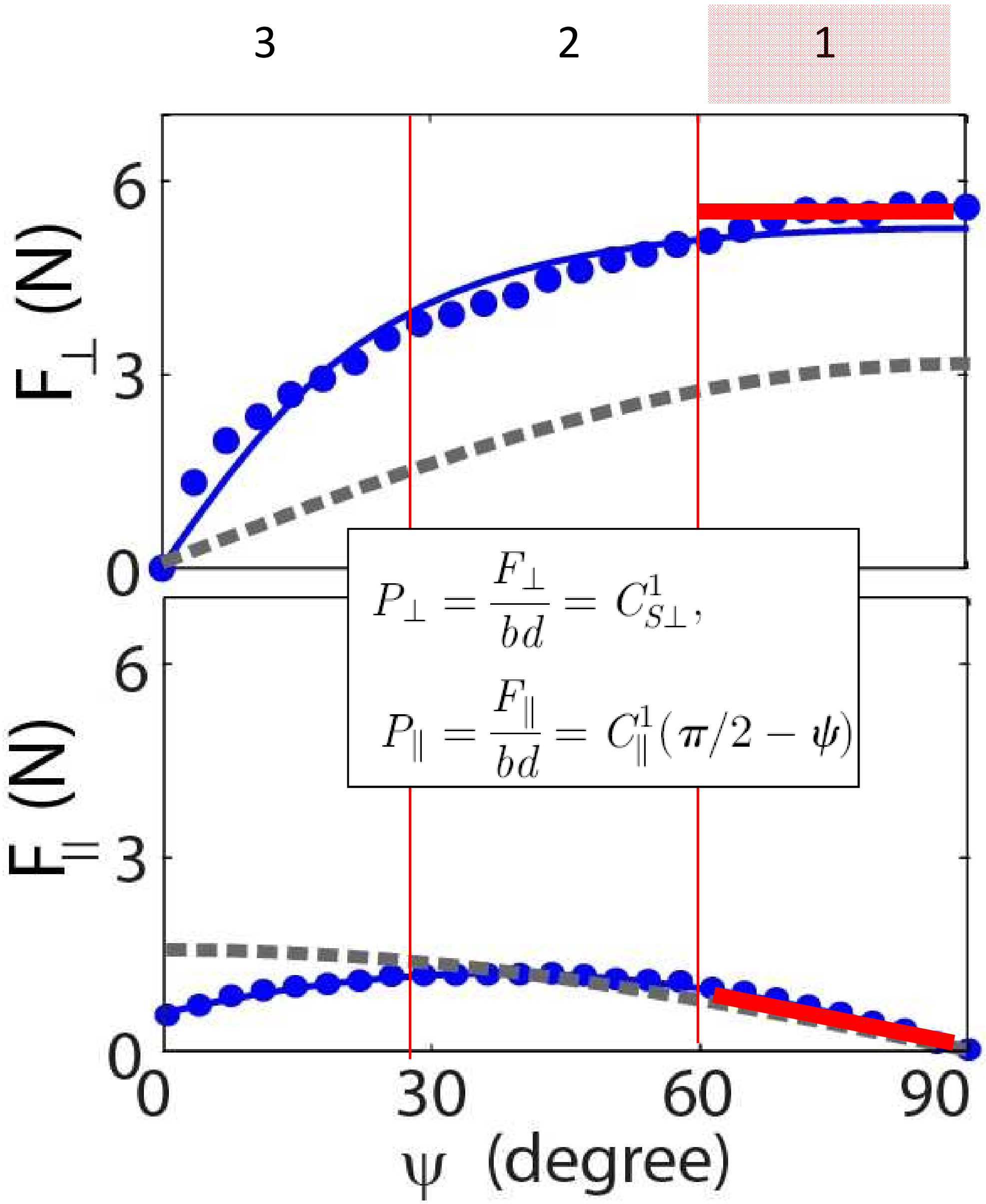
$$\tan\theta = \frac{dy}{dx} = \frac{2A\pi}{\lambda}\cos\frac{2\pi}{\lambda}(x + v_w t)$$

$$\psi = \tan^{-1}\left(\frac{v_y}{v_x}\right) - \theta.$$



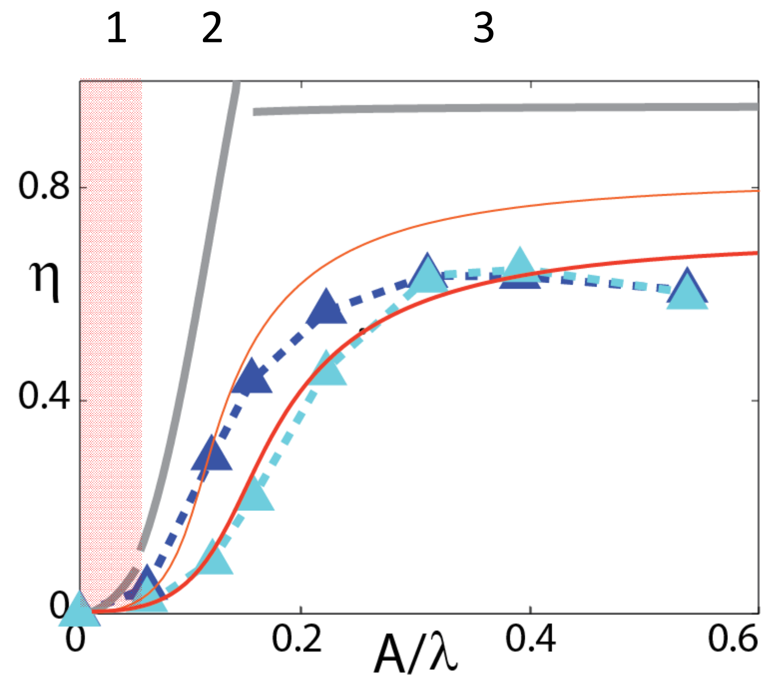
$$\delta F_x = F_\perp(\psi)\sin\theta - F_\parallel(\psi)\cos\theta$$





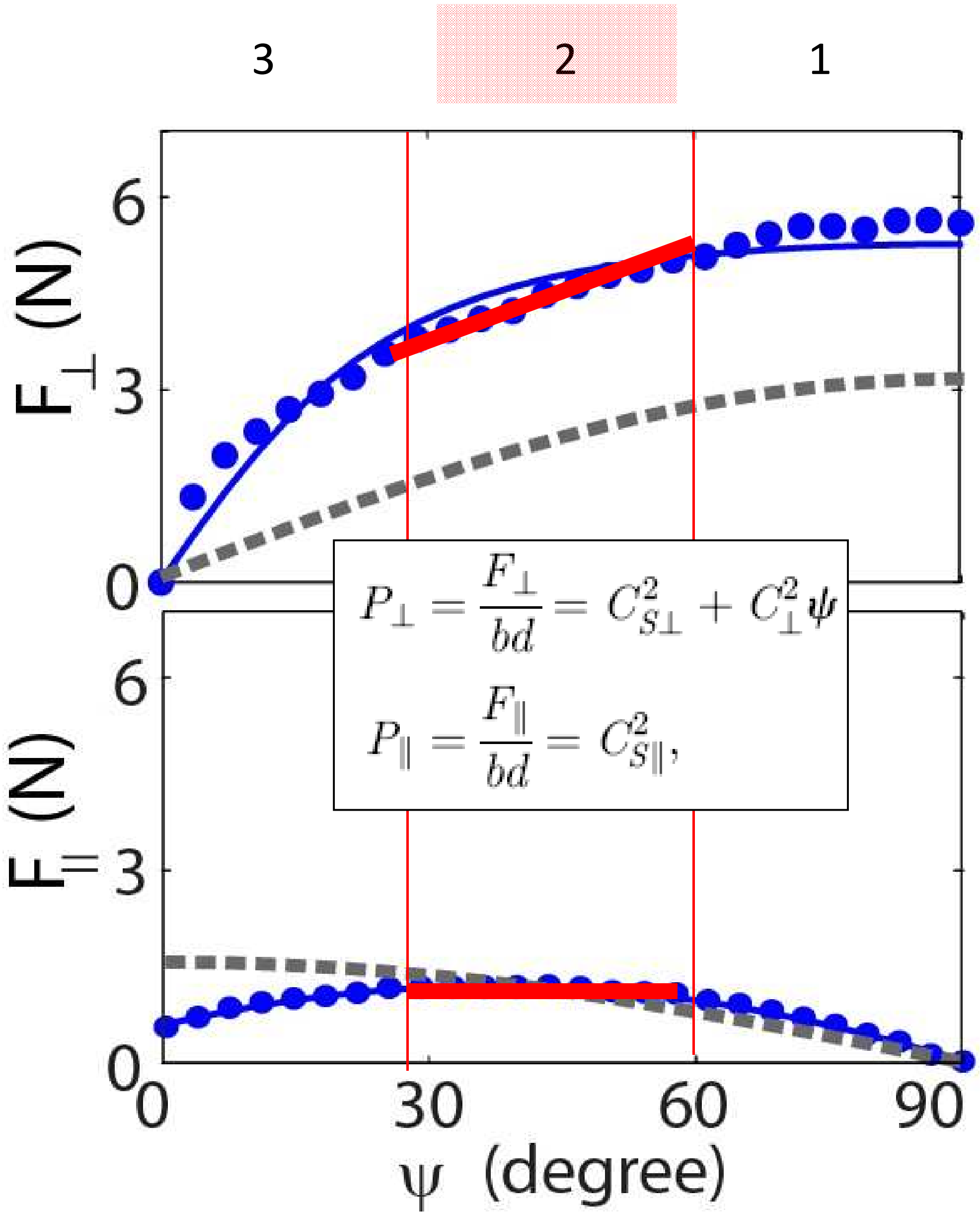
$$P_\perp = \frac{F_\perp}{bd} = C^1_{S\perp},$$

$$P_\parallel = \frac{F_\parallel}{bd} = C^1_\parallel(\pi/2 - \psi)$$



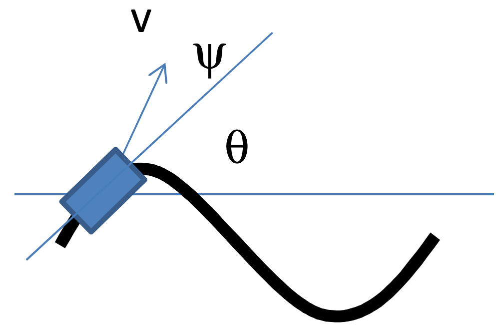
$$\eta = 2\pi^2\left(\frac{C^1_{S\perp}}{C^1_\parallel} - 1\right)\left(\frac{A}{\lambda}\right)^2$$

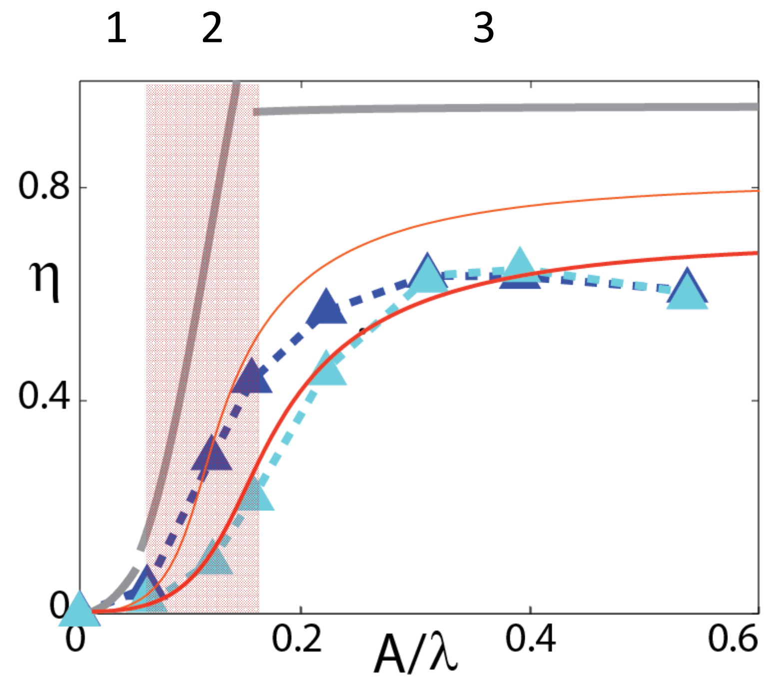
$$\delta F_x = F_\perp(\psi)\sin\theta - F_\parallel(\psi)\cos\theta$$



$$P_\perp = \frac{F_\perp}{bd} = C_{S\perp}^2 + C_\perp^2 \psi$$

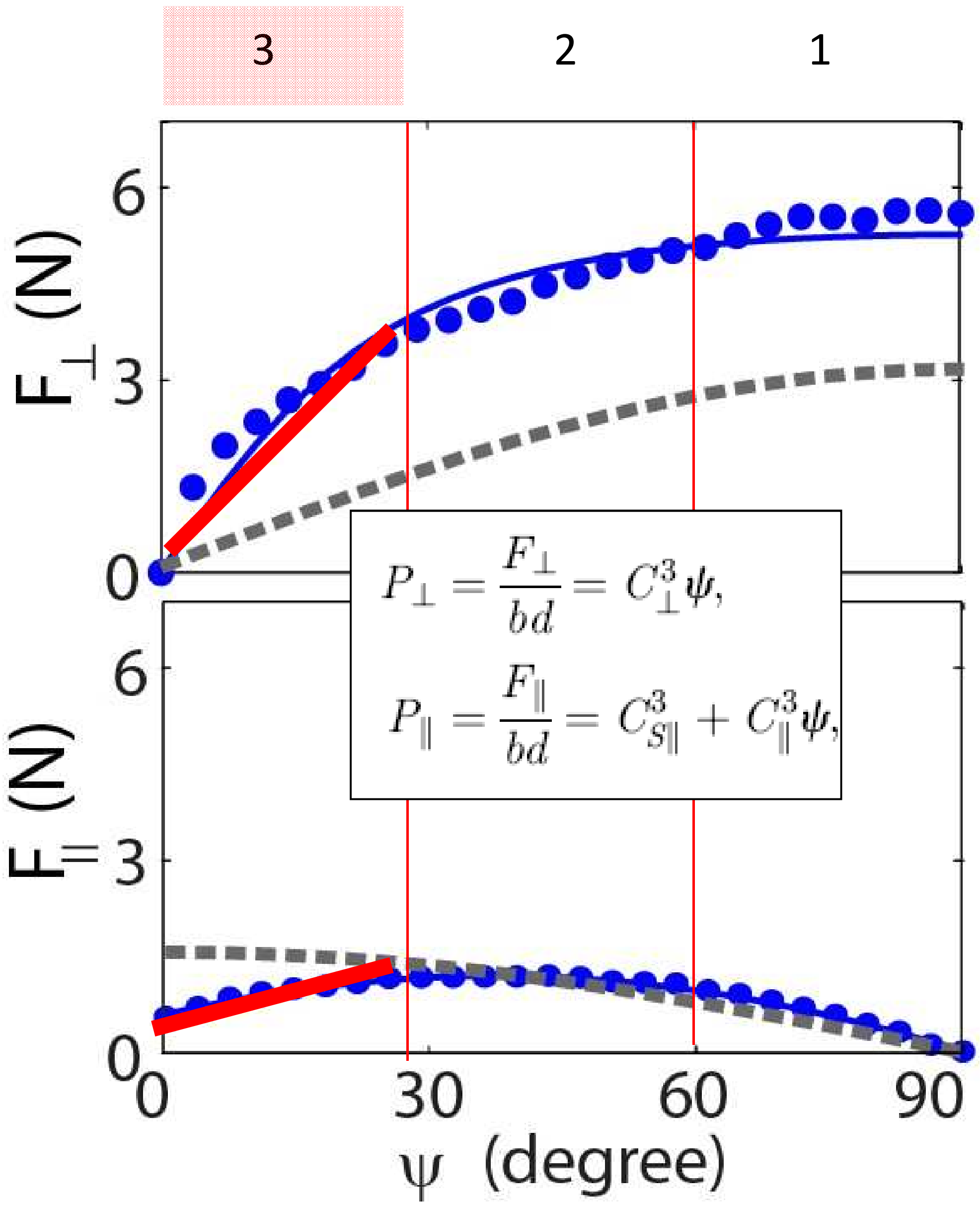
$$P_\parallel = \frac{F_\parallel}{bd} = C_{S\parallel}^2,$$





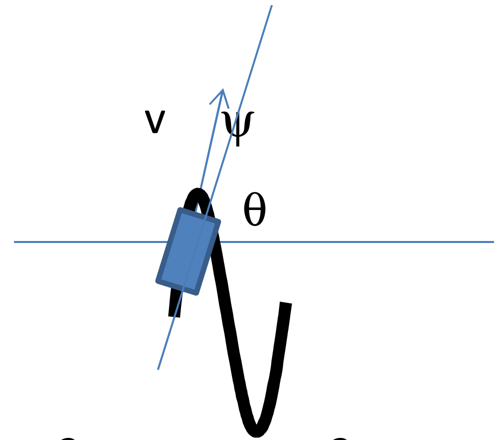
$$\eta = \left[\frac{C_{S\parallel}^2 - 4C_{S\perp}^2 \frac{A}{\lambda}}{16\, C_\perp^2 \left(\frac{A}{\lambda}\right)^2} + 1\right]^{-1}$$

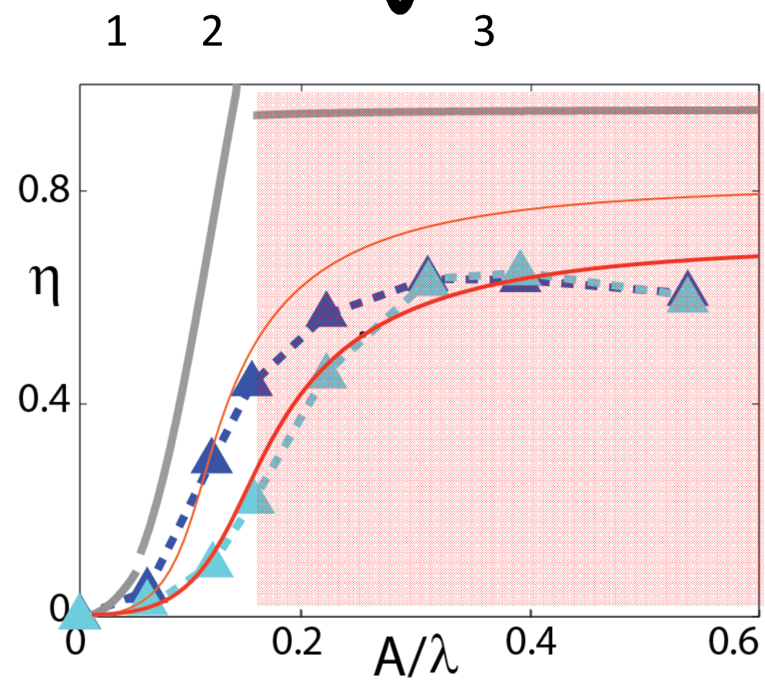
$$\delta F_x = F_\perp(\psi)\sin\theta - F_\parallel(\psi)\cos\theta$$



$$P_\perp = \frac{F_\perp}{bd} = C_\perp^3 \psi,$$

$$P_\parallel = \frac{F_\parallel}{bd} = C_{S\parallel}^3 + C_\parallel^3 \psi,$$





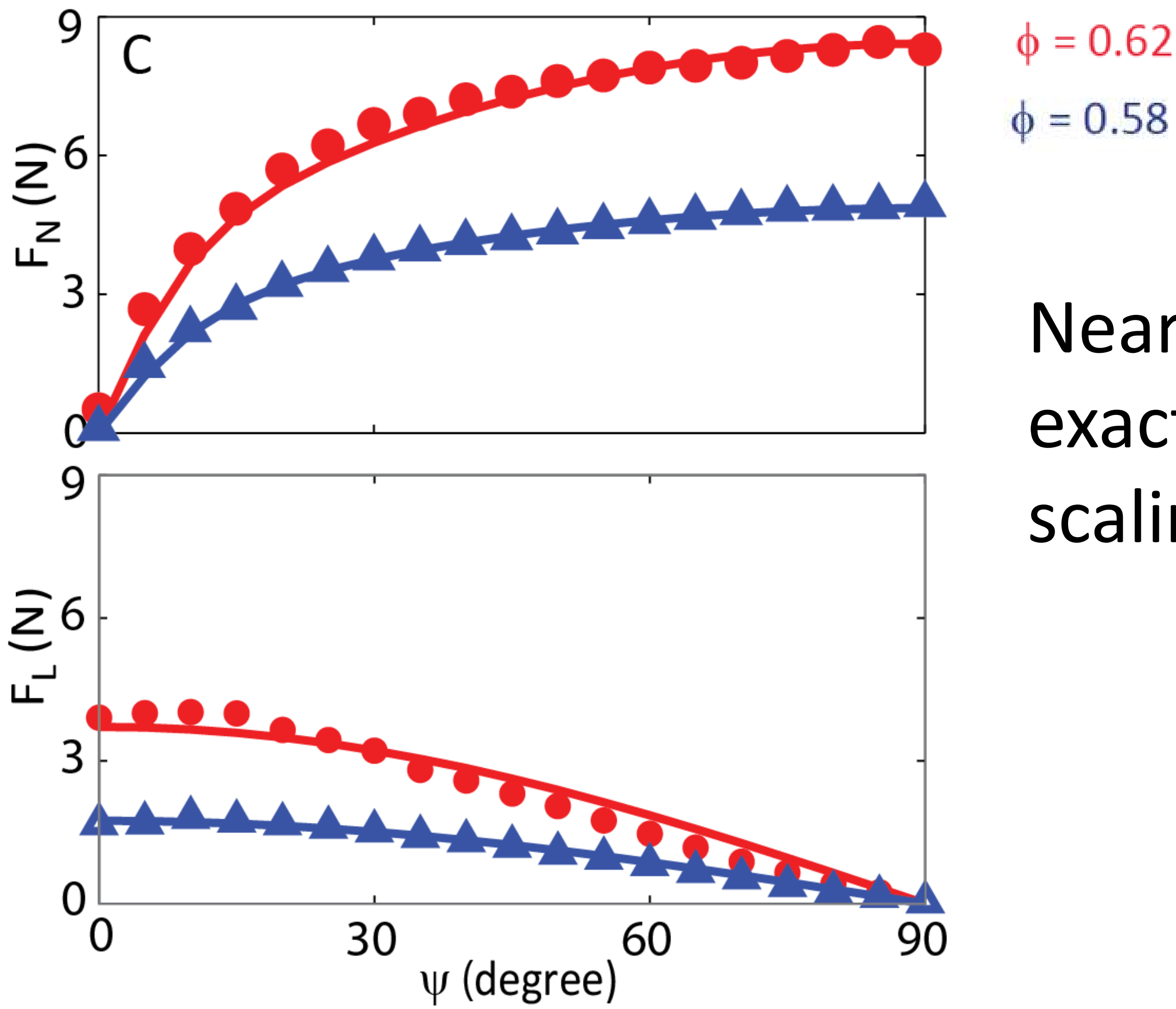
$$\eta = 1 - \frac{4C_{S\parallel}^3}{4C_\perp^3 - C_\parallel^3 (A/\lambda)^{-1}}$$

Why is η independent of ϕ?

Force laws for 0.3 mm particles

$F_{\perp}(N)$

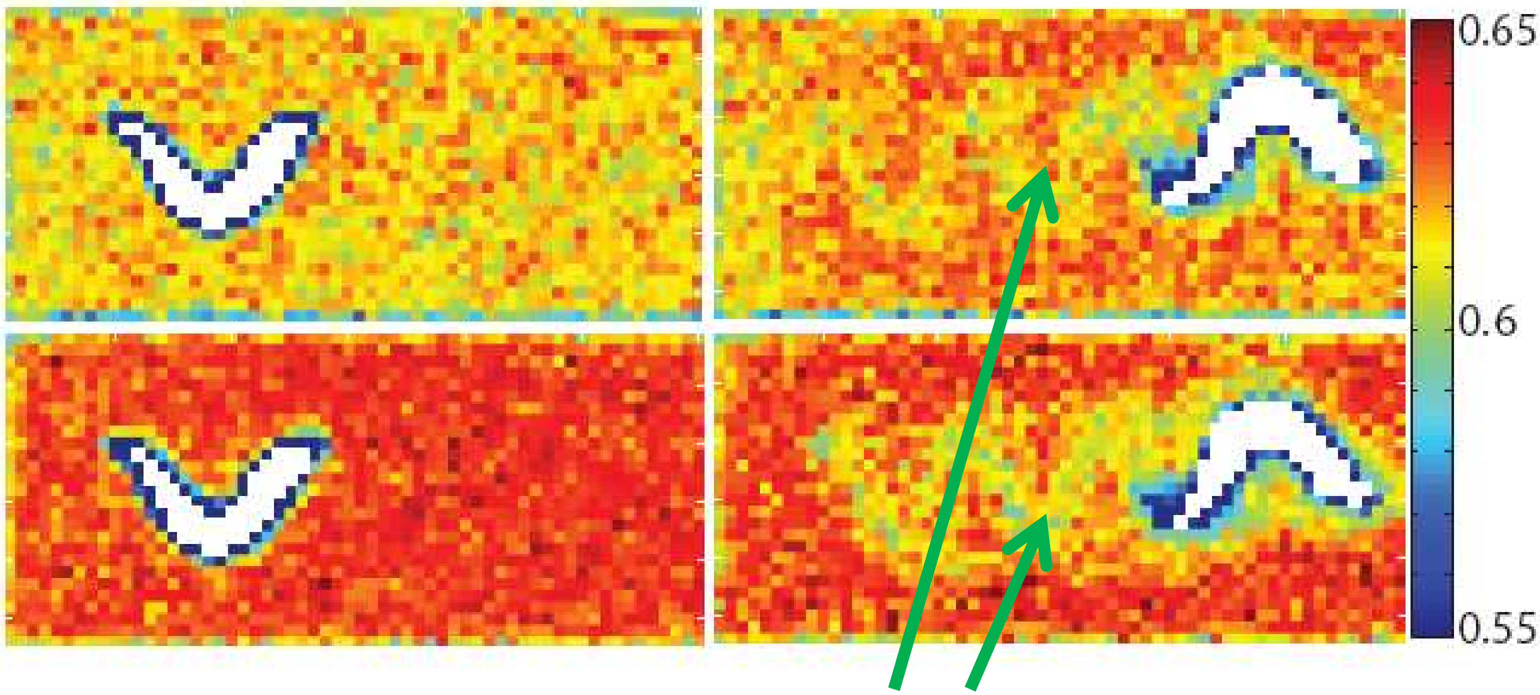
$F_{\parallel}(N)$



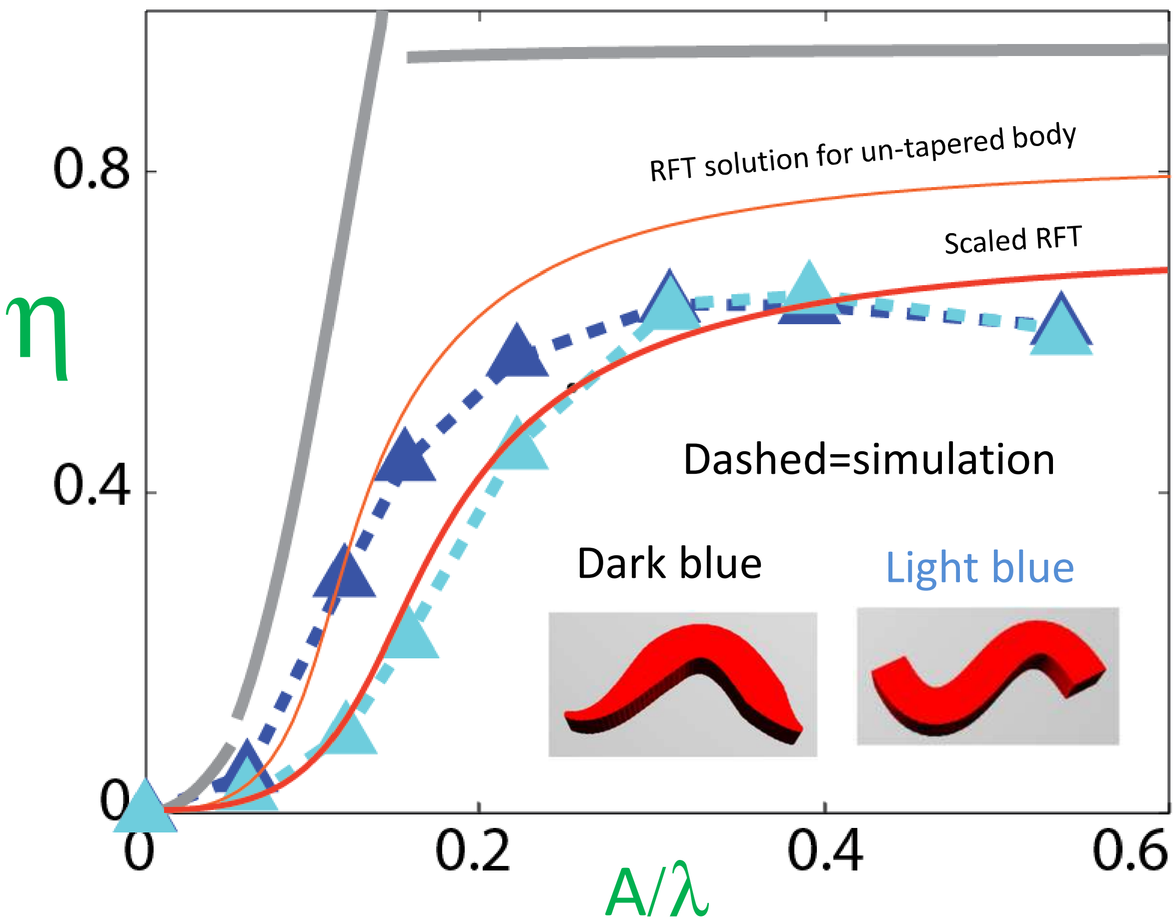
Near exact scaling!

OR…

Localized fluid achieves same state



RFT over-estimates η



Hypothesis
scale thrust
(but not
drag) by 50%

Summary

- Yielding terrestrial substrates---solid and fluid-like response to stress
  - many open locomotion questions
- Volume fraction qualitatively affects drag force: LP→fluid-like, CP→fracturing solid
- Granular lift forces are sensitive to shape dependent and can be approximated by summing plate elements
- Sandfish lizard swims within granular media ("frictional fluid") of different preparations using similar body undulation kinematics
  - Template for swimming in sand?
- DEM, robot and RFT models capture mechanics of sand-swimming:
  - v_x vs f, $\eta \approx 0.5$, optimality condition $A/\lambda = 0.2$
- RFT systematically deviates from DEM model
  - Ding et al, in prep, will show that instantaneous force=average drag force is not a good approximation