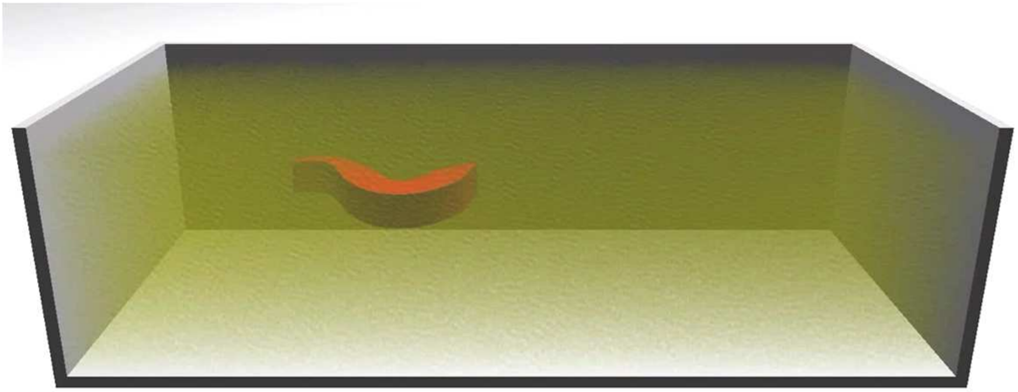
Swimming in Sand, part 2



Daniel I. Goldman
School of Physics
Georgia Institute of Technology

Boulder Summer School on Hydrodynamics

July 25-27

Lectures on the mechanics of interaction with granular media including biological & physics experiments, numerical, theoretical and physical robot models

Topics in the lectures

(revised)

- General principles in terrestrial locomotion
- Intro to granular media
- Drag, lift and flow fields during localized intrusion in granular media

L1

- Modeling approaches: DEM & RFT
- Sandfish biological experiments
- Sandfish modeling: robot

L2

- Sandfish modeling: DEM
- Biological tests of model predictions
- RFT modeling of sand-swimming

L3

Drag Induced Lift

Yang Ding, Nick Gravish, DG, *PRL*, 2010

PRL **106,** 028001 (2011) PHYSICAL REVIEW LETTERS week ending 14 JANUARY 2011

Drag Induced Lift in Granular Media

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(Received 31 August 2010; published 13 January 2011)

Laboratory experiments and numerical simulation reveal that a submerged intruder dragged horizontally at a constant velocity within a granular medium experiences a lift force whose sign and magnitude depend on the intruder shape. Comparing the stress on a flat plate at varied inclination angle with the local surface stress on the intruders at regions with the same orientation demonstrates that intruder lift forces are well approximated as the sum of contributions from flat-plate elements. The plate stress is deduced from the force balance on the flowing media near the plate.

DOI: 10.1103/PhysRevLett.106.028001 PACS numbers: 45.70.Mg, 47.50.−d, 83.10.Rs

Objects moved through media experience drag forces opposite to the direction of motion and lift forces perpendicular to the direction of motion. The principles that govern how object shape and orientation affect these forces are well understood in fluids like air and water. These principles explain how wings enable flight through air and fins generate thrust in water [1].

Lift and drag forces are also generated by movement within dry granular media—collections of discrete particles that interact through dissipative contact forces. Generation and control of these forces while moving

Following the method of [6], forces on the connecting rod were determined in separate measurements and subtracted from F_x and F_z. The grain bed was 75 PD wide by 53 PD deep by 75 PD long. The initial packing state of the grains was prepared by shaking the container moderately in the horizontal direction before each run. The volume fraction was determined through measurements of ρ, total grain mass (M), and occupied volume (V) to be $\frac{M}{\rho V} = 0.62 \pm 0.01$.

The simulation employed the soft-sphere discrete element method (DEM) [10] in which particle-particle and

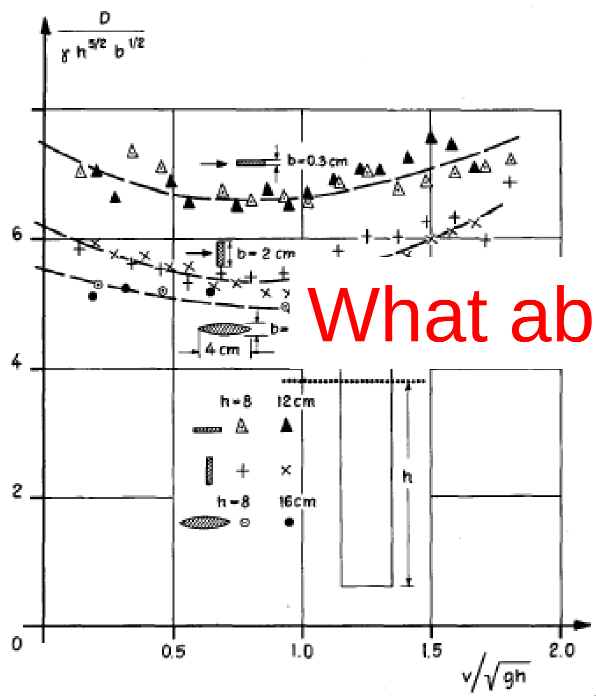
Features of granular drag

*Insensitive to speed

*Increases with depth

*Insensitive to shape

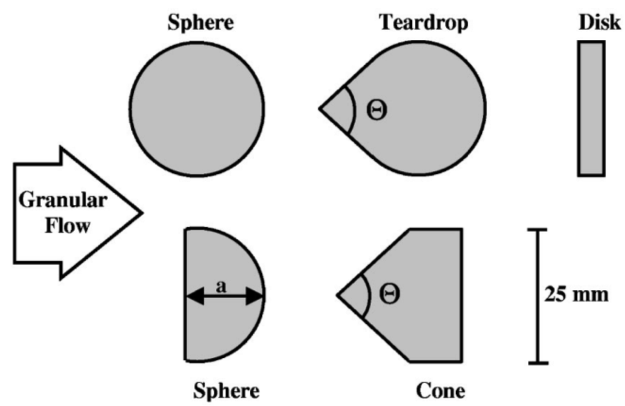
Drag force

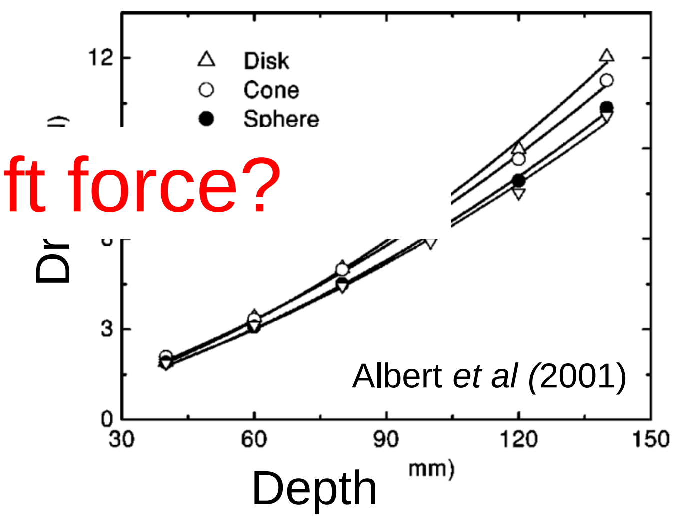


v

Wieghardt (1975)

What about the lift force?



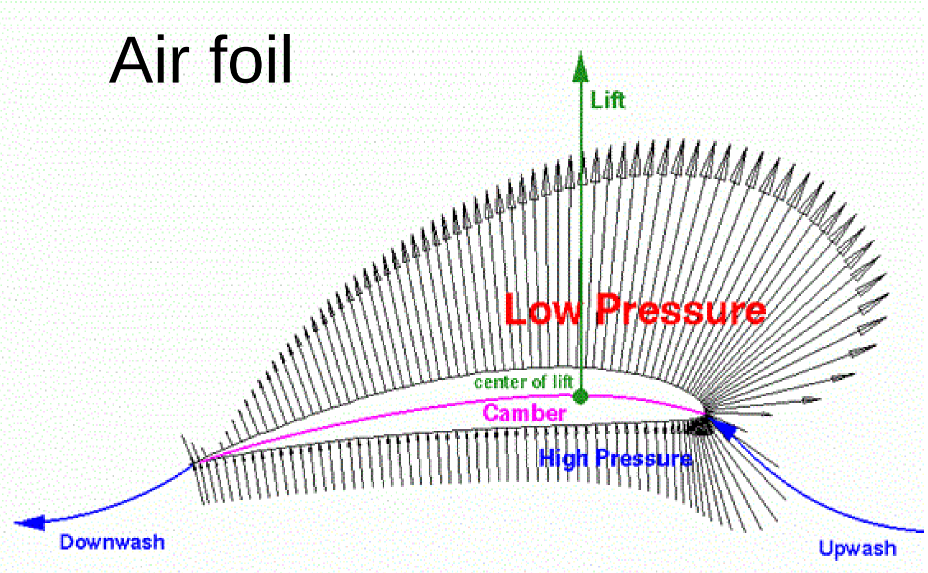


Depth

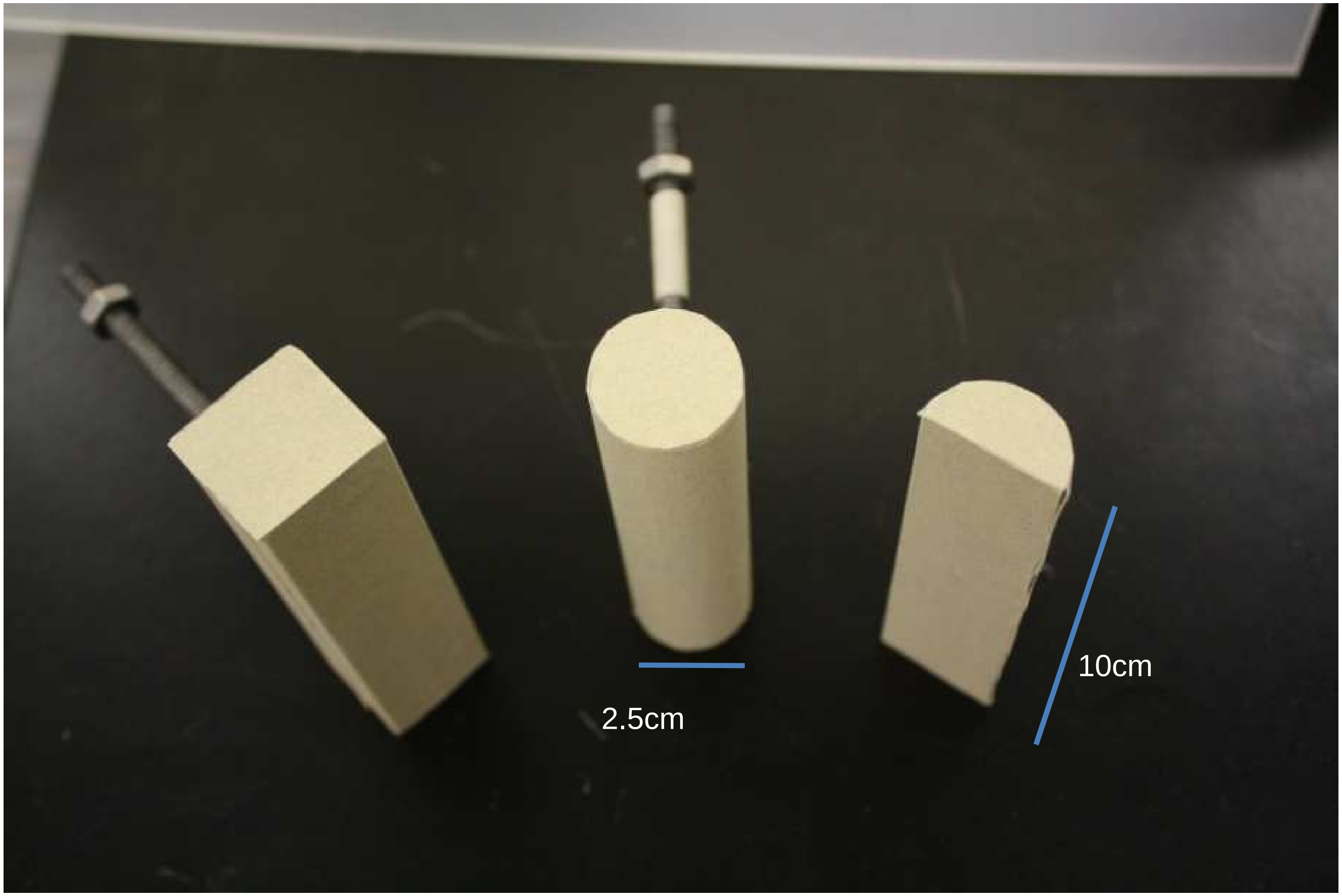
Albert *et al* (1999), Soller *et al* (2006), Chehata *et al* (2003)

Lift in fluids

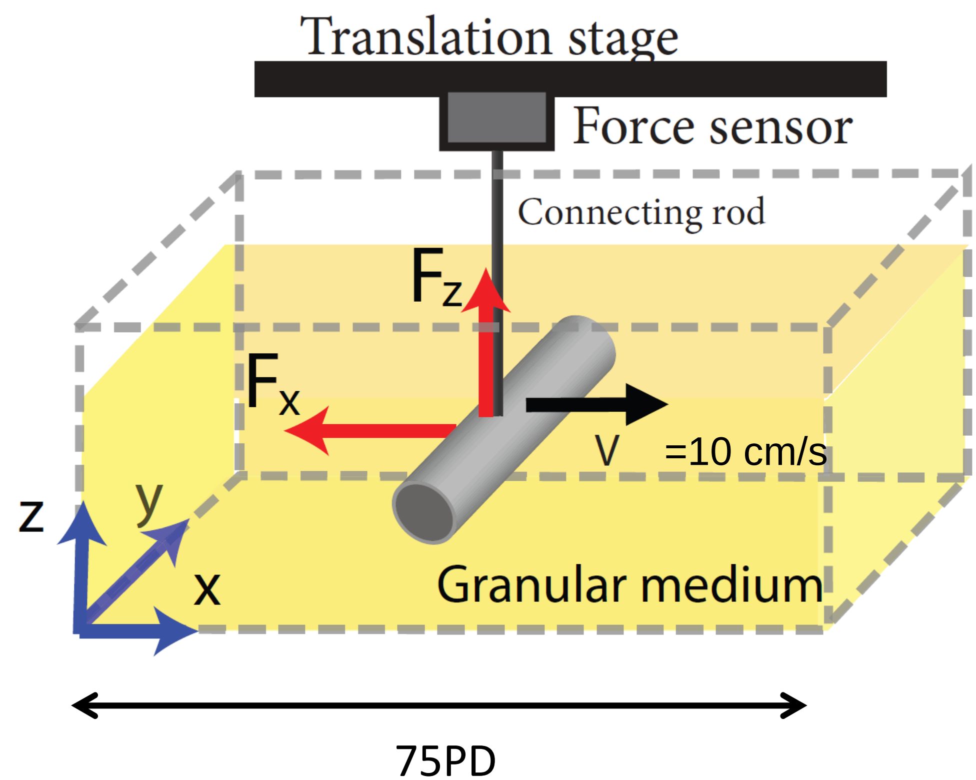
Air foil



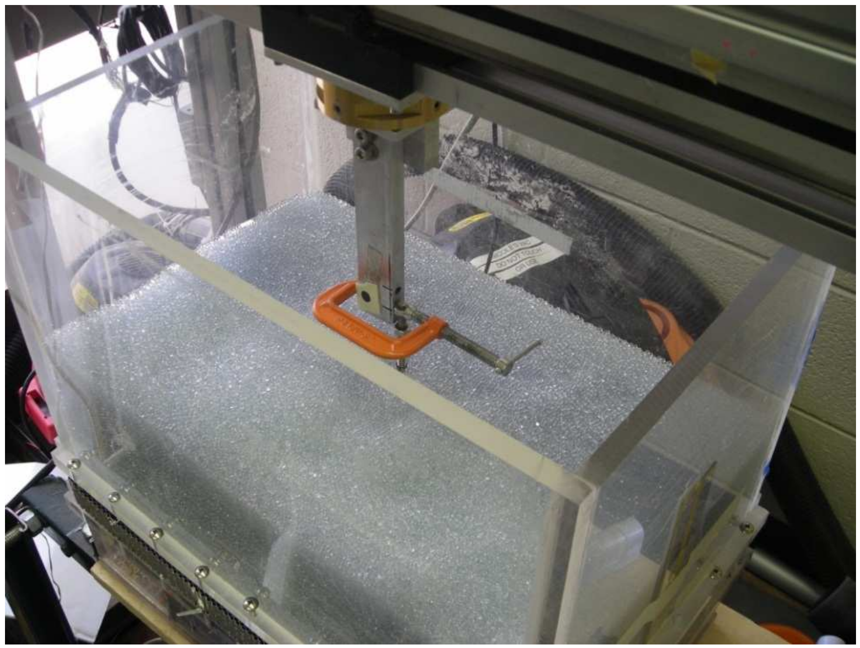
Measure lift force on simple shapes



Experiment



0.32 ±0.02 cm diameter glass particles



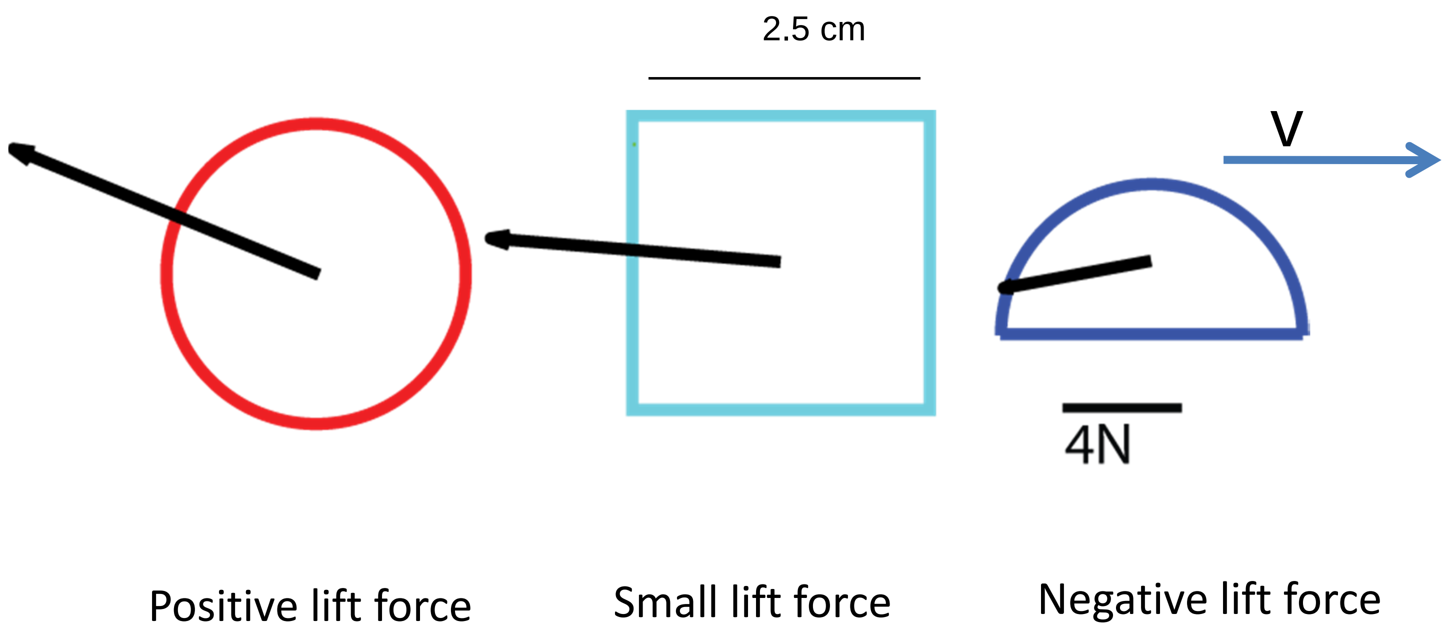
ϕ=0.62

rod=10 cm long

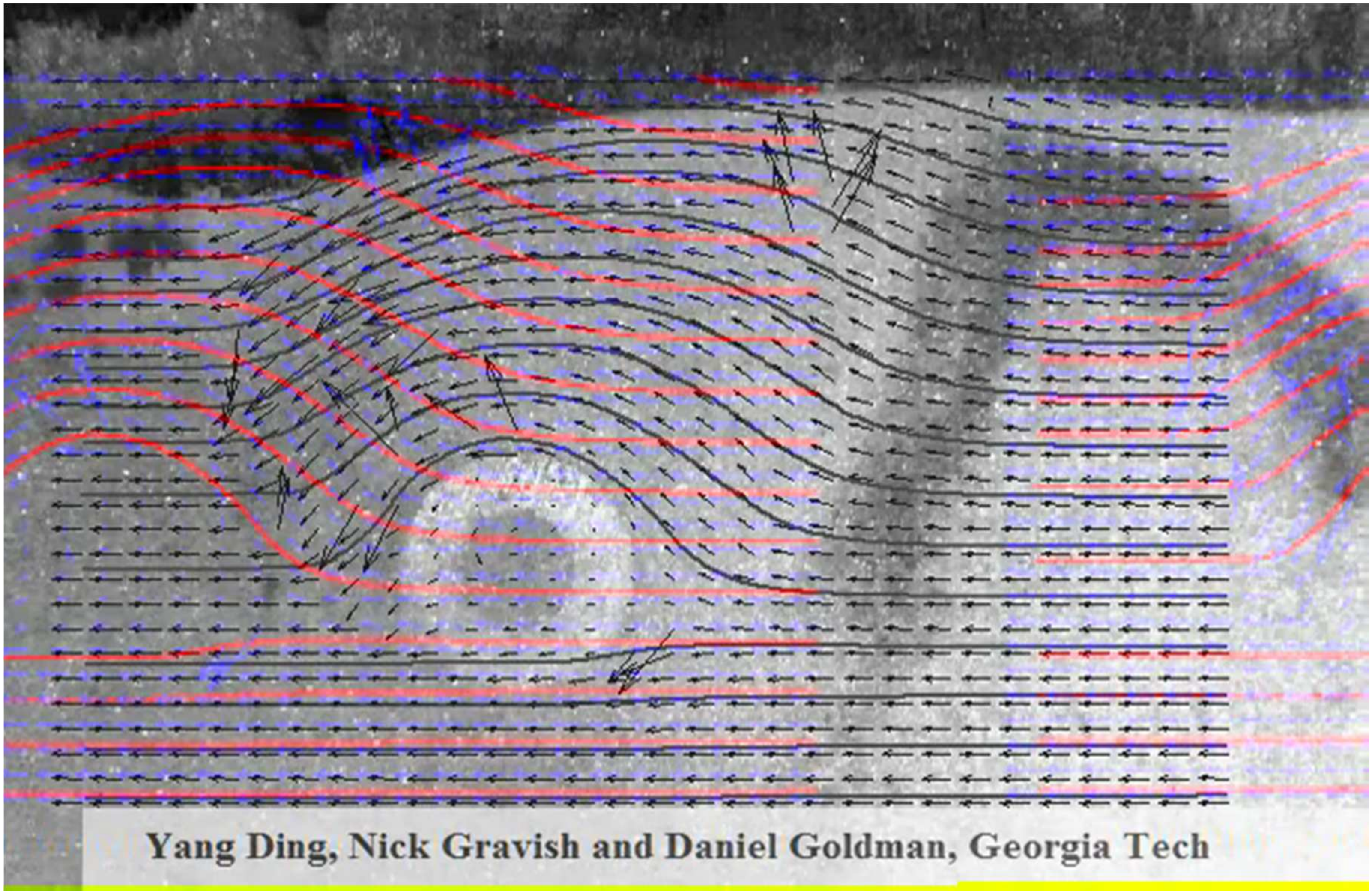
Note: larger particles (10x) than in previous drag experiments

Net forces on intruders

Net force in vertical plane



Velocity field (in co-moving frame)

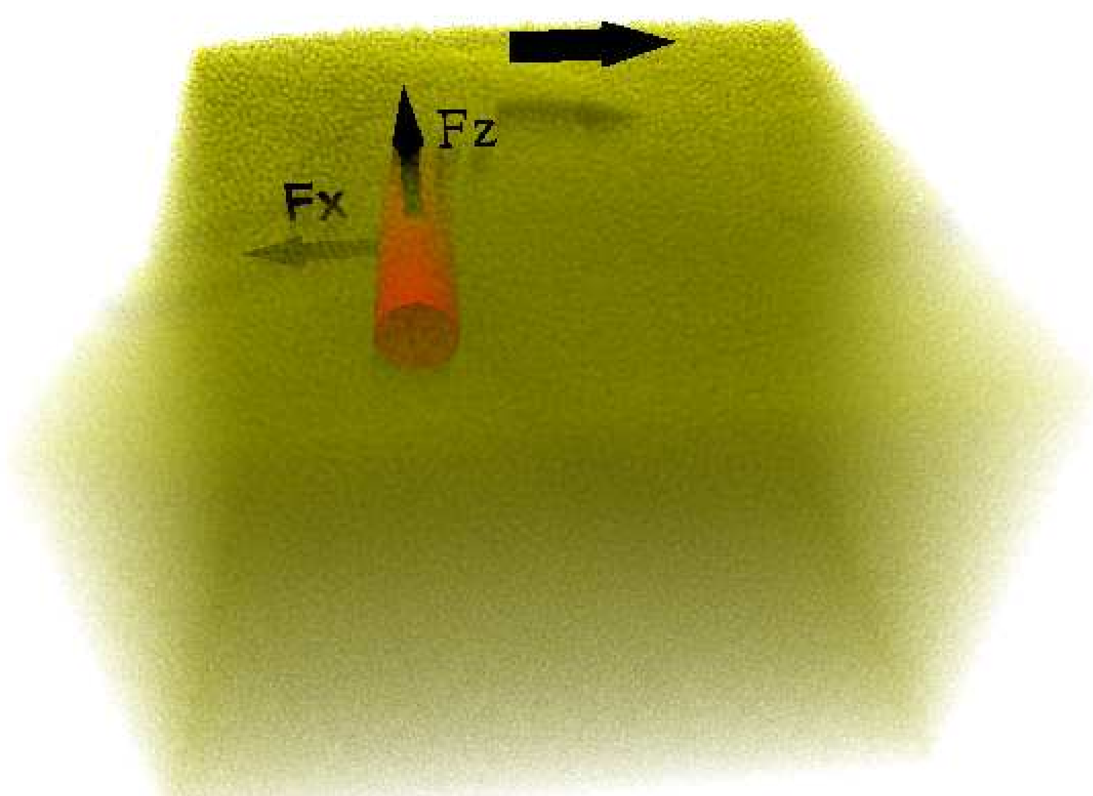


(in 0.3 mm diameter glas particles)

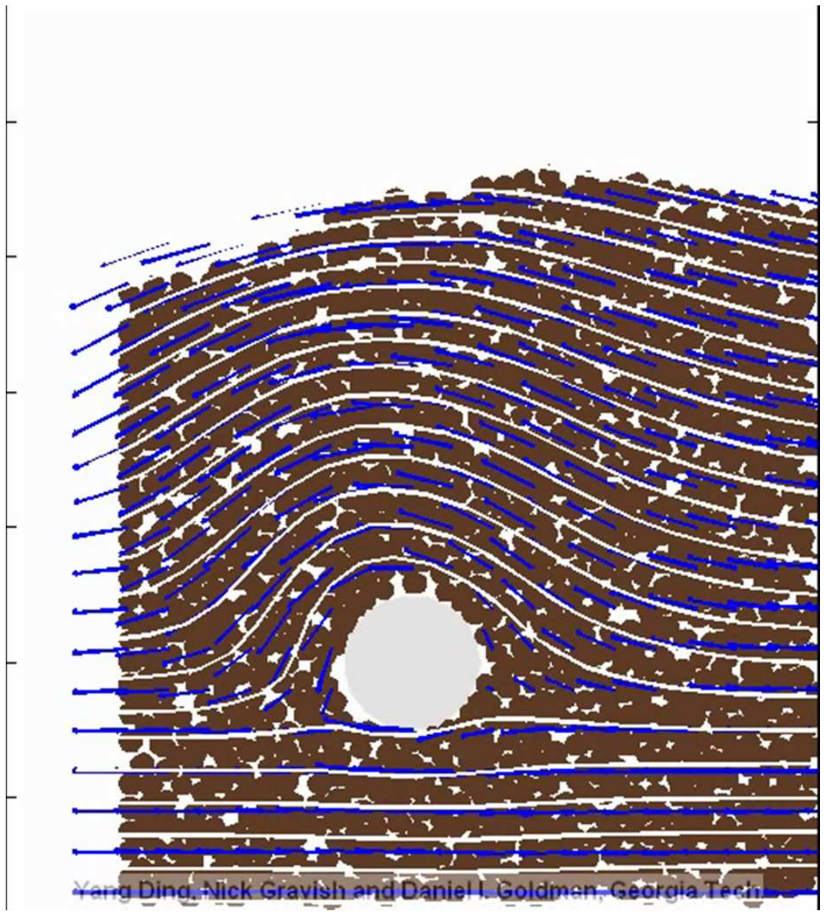
Discrete Element Method (DEM) simulation

Books:

- Rapaport, *The art of molecular dynamics simulation*, 2004
- Pöschel, *Computational granular dynamics : models and algorithms,* 2005



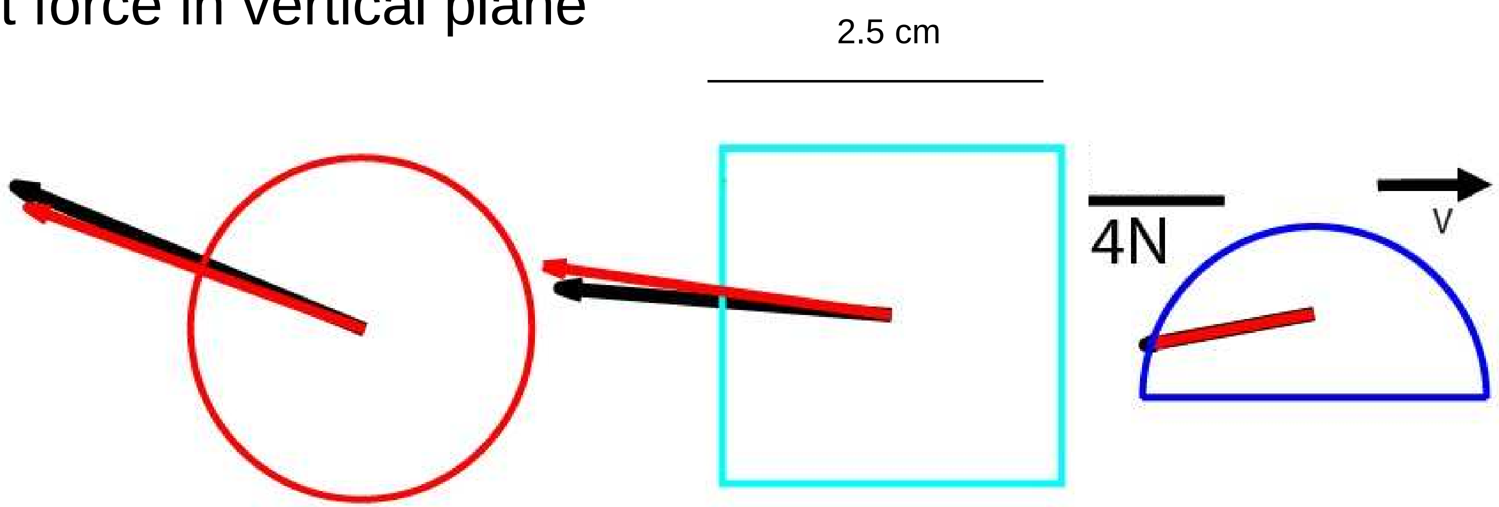
Flow field and streamlines in co-moving frame



3D simulation of 350,000 3 mm “glass” spheres (cross-section shown). Rod dragged at 10 cm/sec

Net forces on intruders

Net force in vertical plane



Positive lift force   Small lift force   Negative lift force

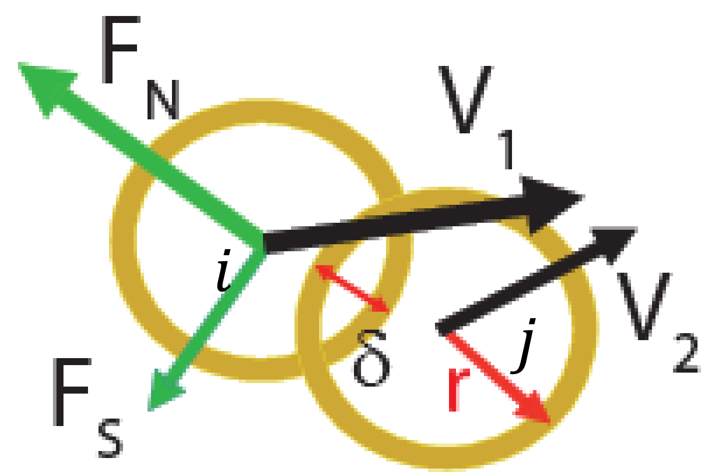
Red=simulation

Black=experiment

3 mm glass spheres, ϕ=0.62

Particle interaction force Model

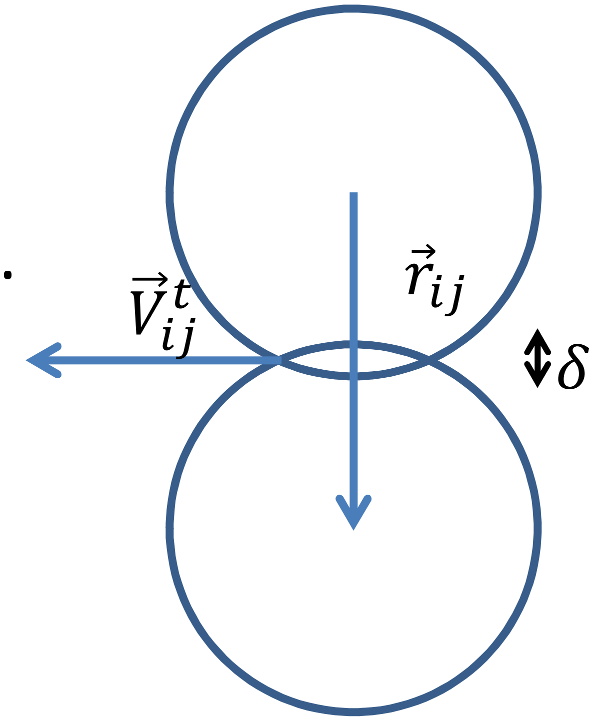
- Force is contact only, repulsive, non-conservative.
- Spherical Particles.
- Deformation treated as small overlap
- Normal force is a function of overlap and velocity
- Friction for tangential direction



Force Model (details)

- $\vec{F}_{ij} = \vec{F}_{ij}^{n} + \vec{F}_{ij}^{t}$
- $\vec{F}_{ij}^{n} = (k_n \delta^{\alpha} + G_n \dot{\delta} \delta^{\beta}) \hat{n}_{ij}$
- $\alpha = 3/2$ and $\beta = 1/2$, Hertz model*. G_n is a constant for nearly monodisperse particles.
- $\vec{F}_{ij}^{t} = -\min(k_t |\vec{\xi}_{ij}|^{\dagger}, \mu_s |\vec{F}_{ij}^{n}|) \hat{V}_{ij}^{t}$
- Slip term depends on past history:

$$\vec{\xi}_{ij}(t) = \int_{t_0}^{t} \vec{V}_{ij}^{t}(t') dt'$$



Tingnan Zhang

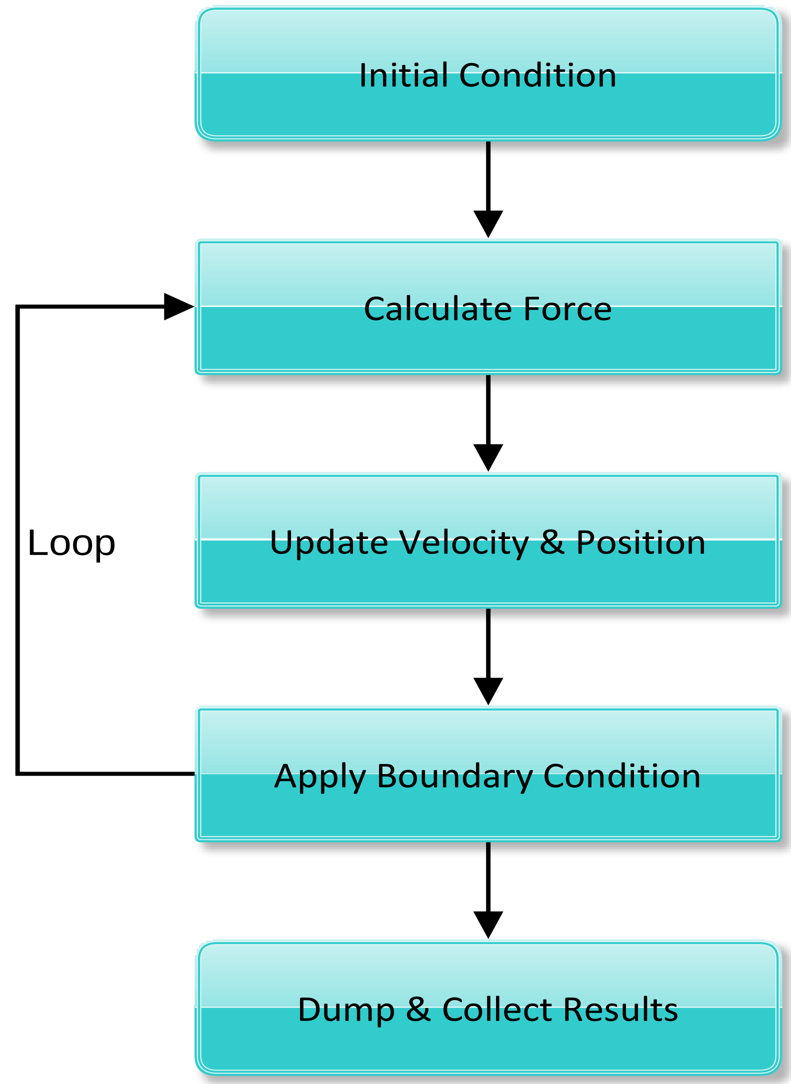
- * Nikolai V. Brilliantov , Physical Review E, 53:5382, 1996.
- † P. A. Cundall, Geotechnique, 29:47, 1979.

Computation Process

- Contact force model
- Integration method: Explicit Euler
- Set boundary conditions: hardwall, soft wall, periodic, etc.

Many open source solvers and standard techniques to make run N log N…

http://en.wikipedia.org/wiki/Discrete_element_method



Parameters

- Experimental hardness (k) is calculated using Hertz model* for 3mm glass beads using Young & Poisson modulii for glass. Simulated hardness is much smaller †but δ is always <1% radius.
- Restitution is measured by dropping one particle on another at 0.5 m/sec.
- Friction coefficients (μ) are measured by sliding block (with particles glued) on a slope with glued particles.
- Time step is set to be 1/20 collision time* and reducing it by a factor of 2 does not change measured force significantly.

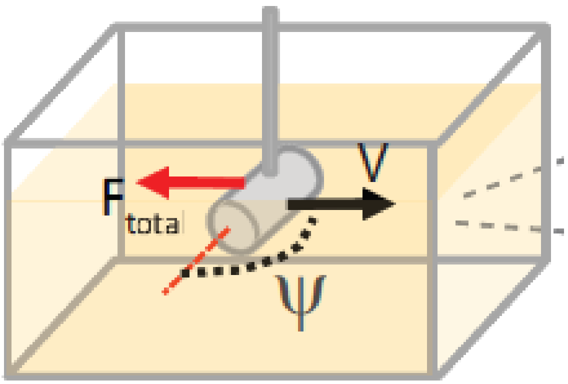
$$F_n = k\delta^{3/2} - G_n v_n \delta^{1/2}$$
$$F_s = \mu F_n$$

3 mm glass particles:

	Experiment	Simulation
Hardness (k)	5.7×10^9 kg s^{-2} m$^{-1/2}$	2×10^6 kg s^{-2} m$^{-1/2}$
Restitution coefficient	0.92 ± 0.03	0.88
G_n	15×10^2 kg m$^{-1/2}$ s^{-1}	15 kg m$^{-1/2}$ s^{-1}
$\mu_{particle-particle}$	0.10	0.10
$\mu_{particle-body}$	0.27	0.27
Density	2.47 g cm^{-3}	2.47 g cm^{-3}
Diameter	3.2 ± 0.2 mm	3.0 mm (50%) and 3.4 mm (50%)

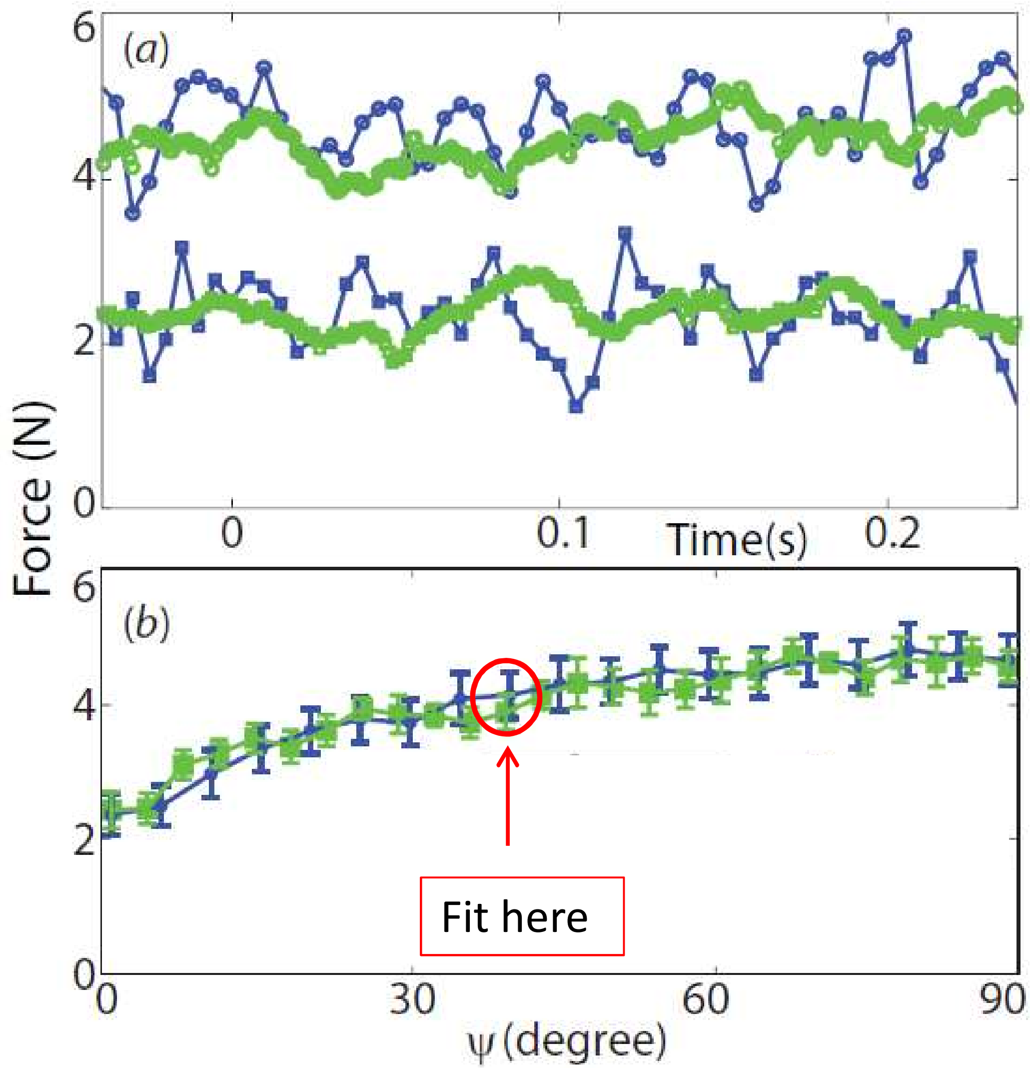
- * Nikolai V. Brilliantov , Physical Review E, 53:5382, 1996., † Y. Tsuji, Powder Technology, 77:79, 1993.

Validation: rod drag



3 mm diameter glass beads

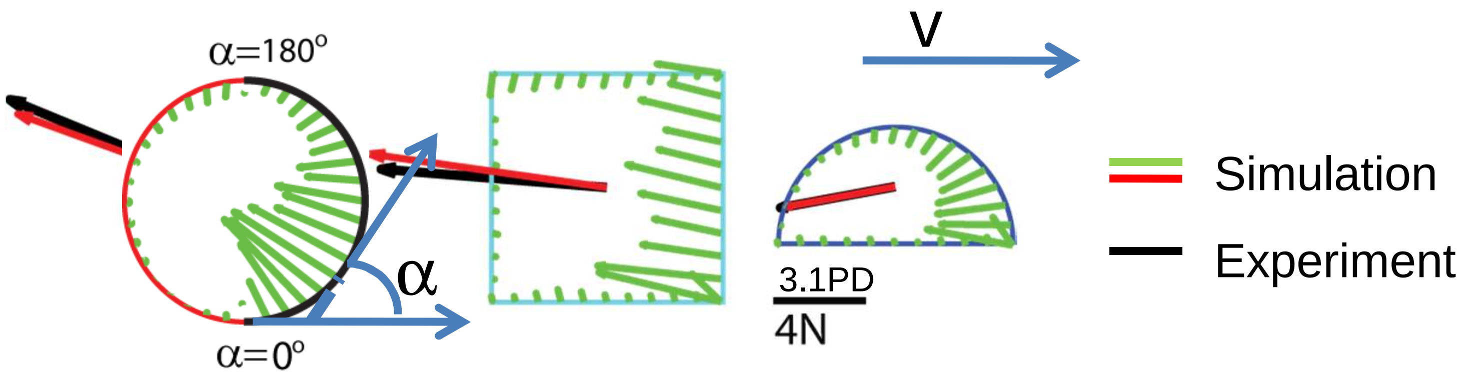
3 cm long SS cylinder



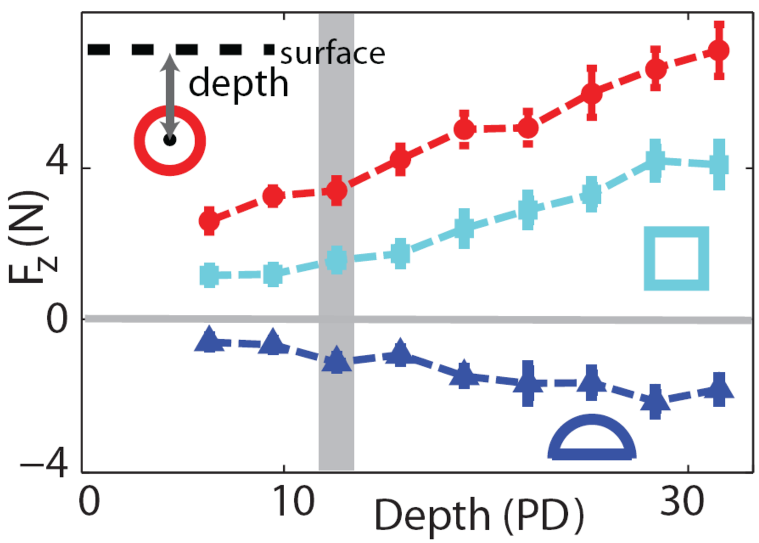
Simulation: 50:50 mix of 3.0,3.4 mm “glass spheres”

Simulation results

Forces



Depth dependence



Stress on the surface

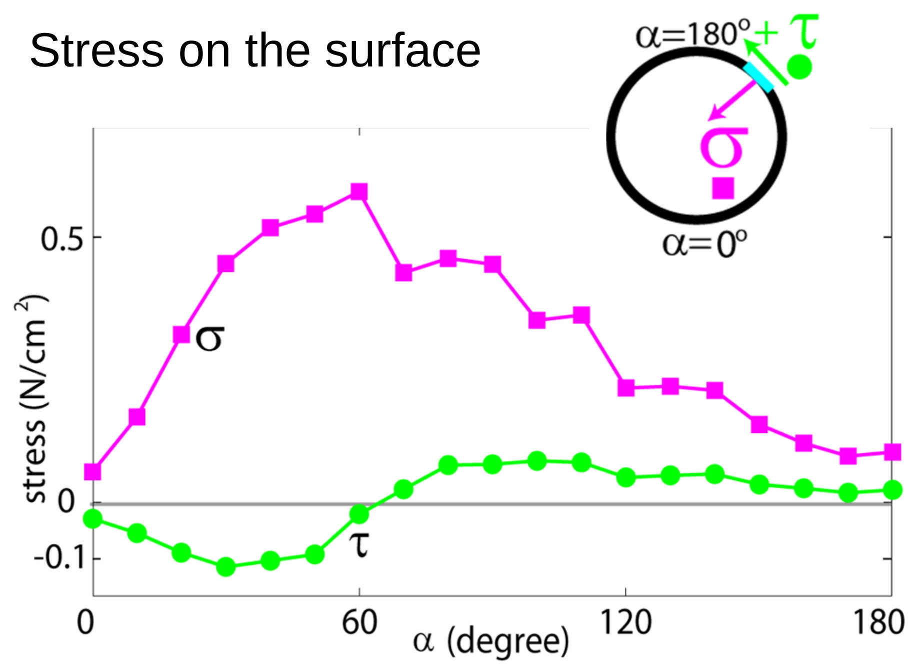


Plate as a differential element

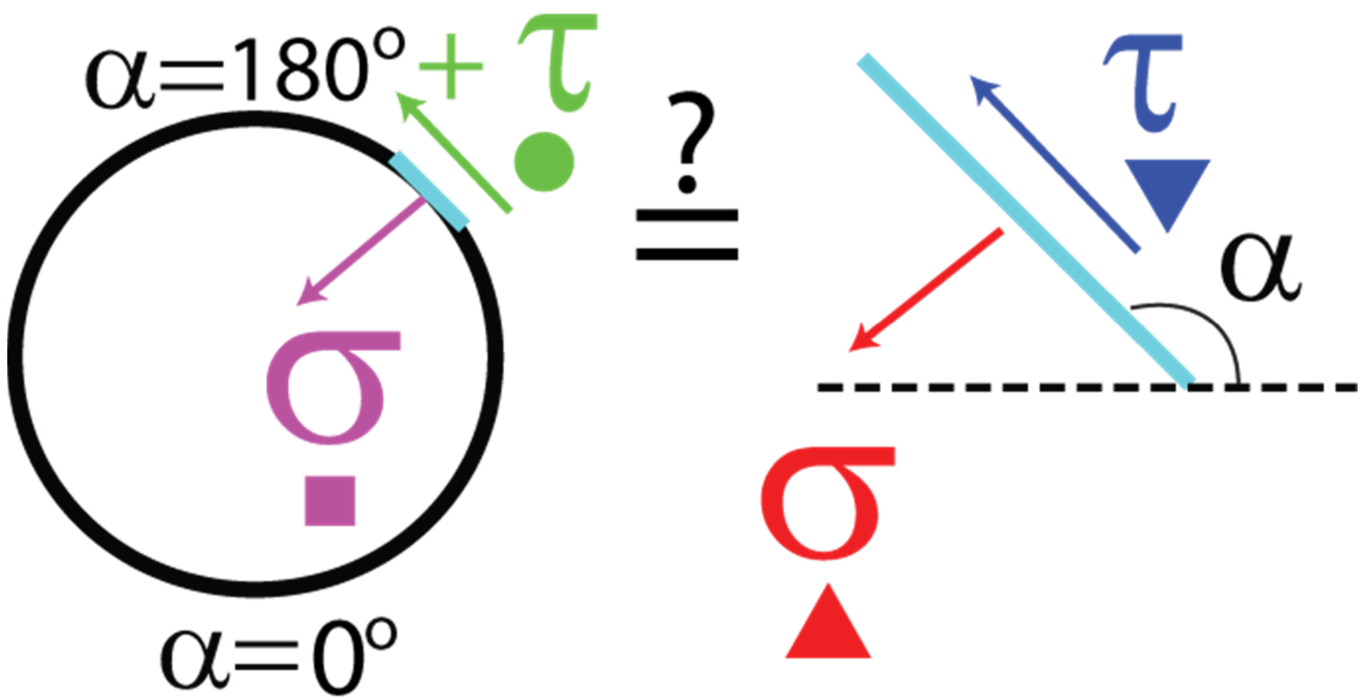
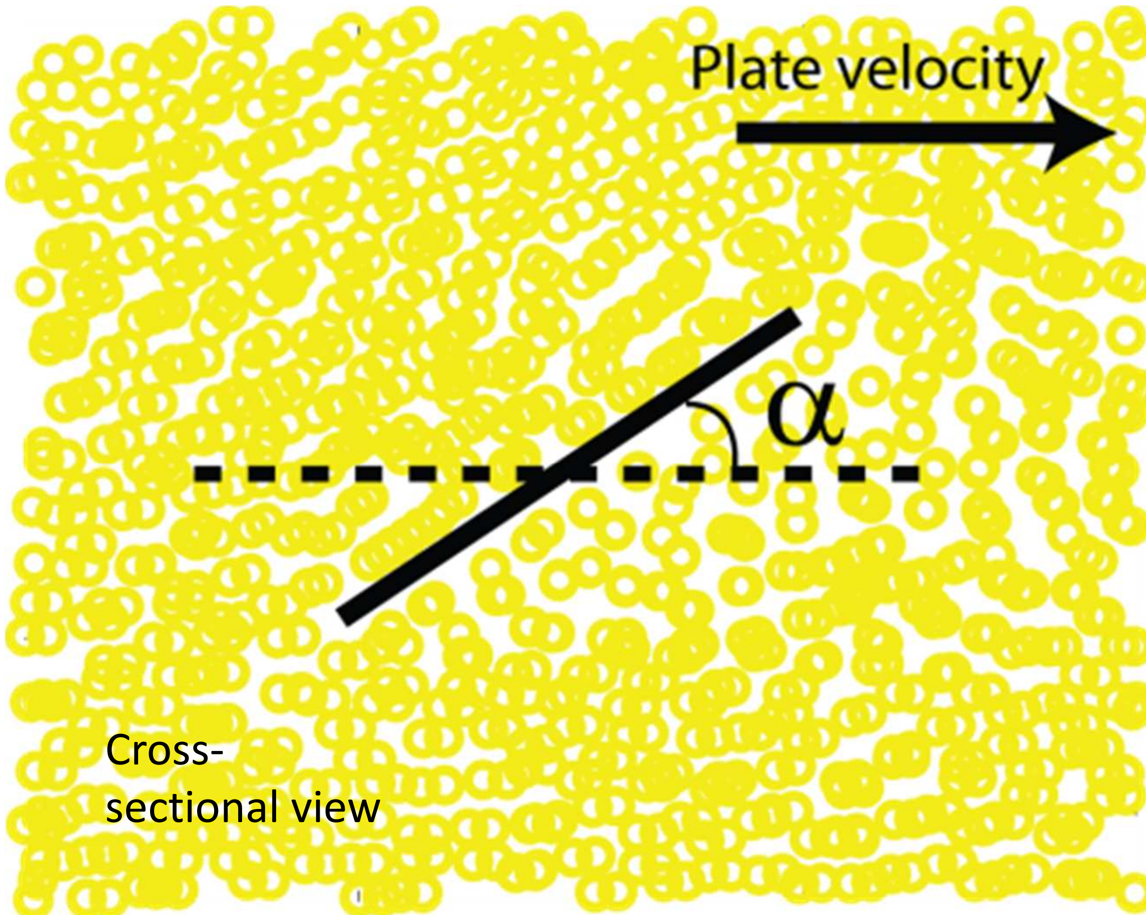


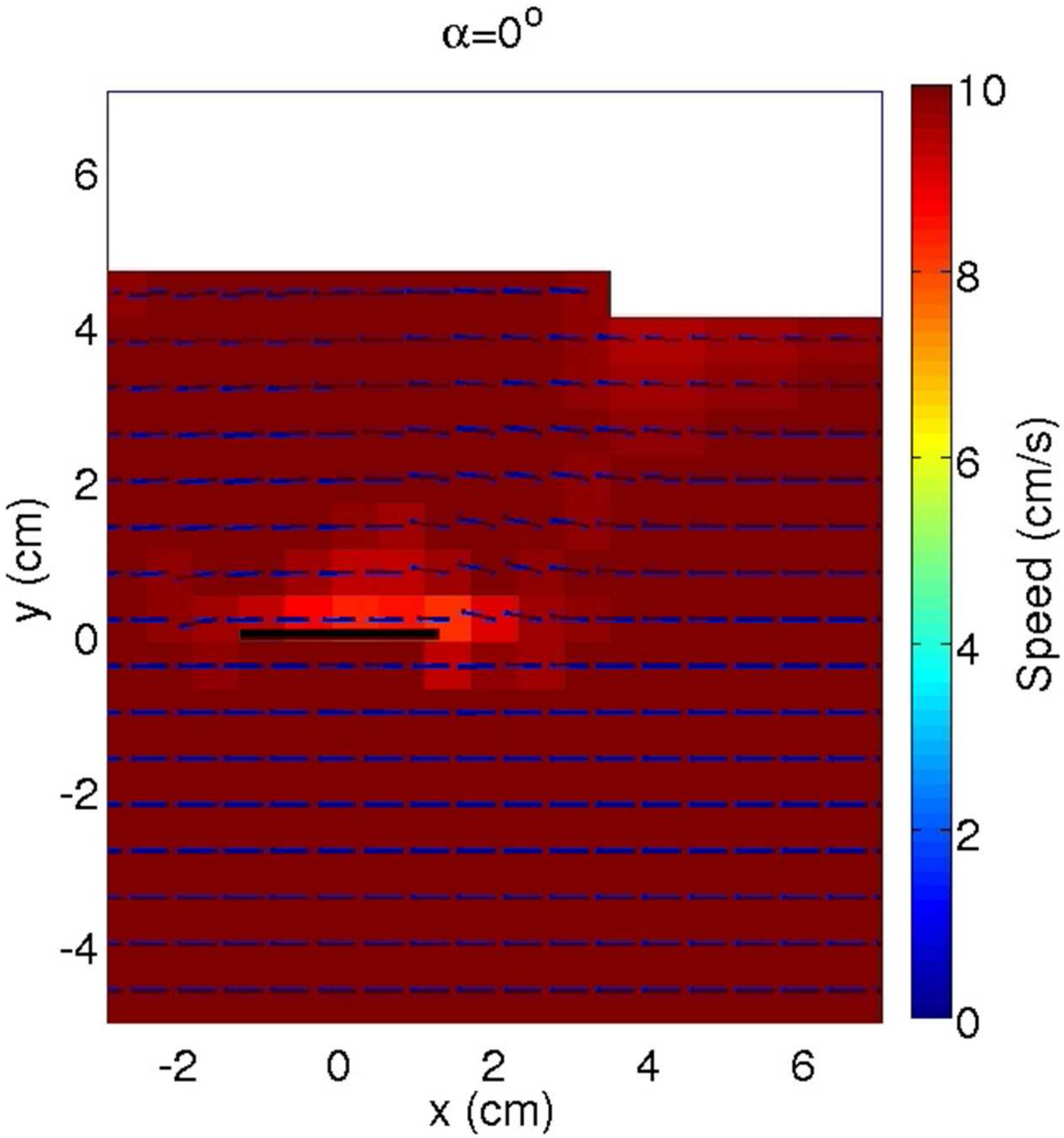
Plate drag

Depth (at
plate center)
= 3.75 cm



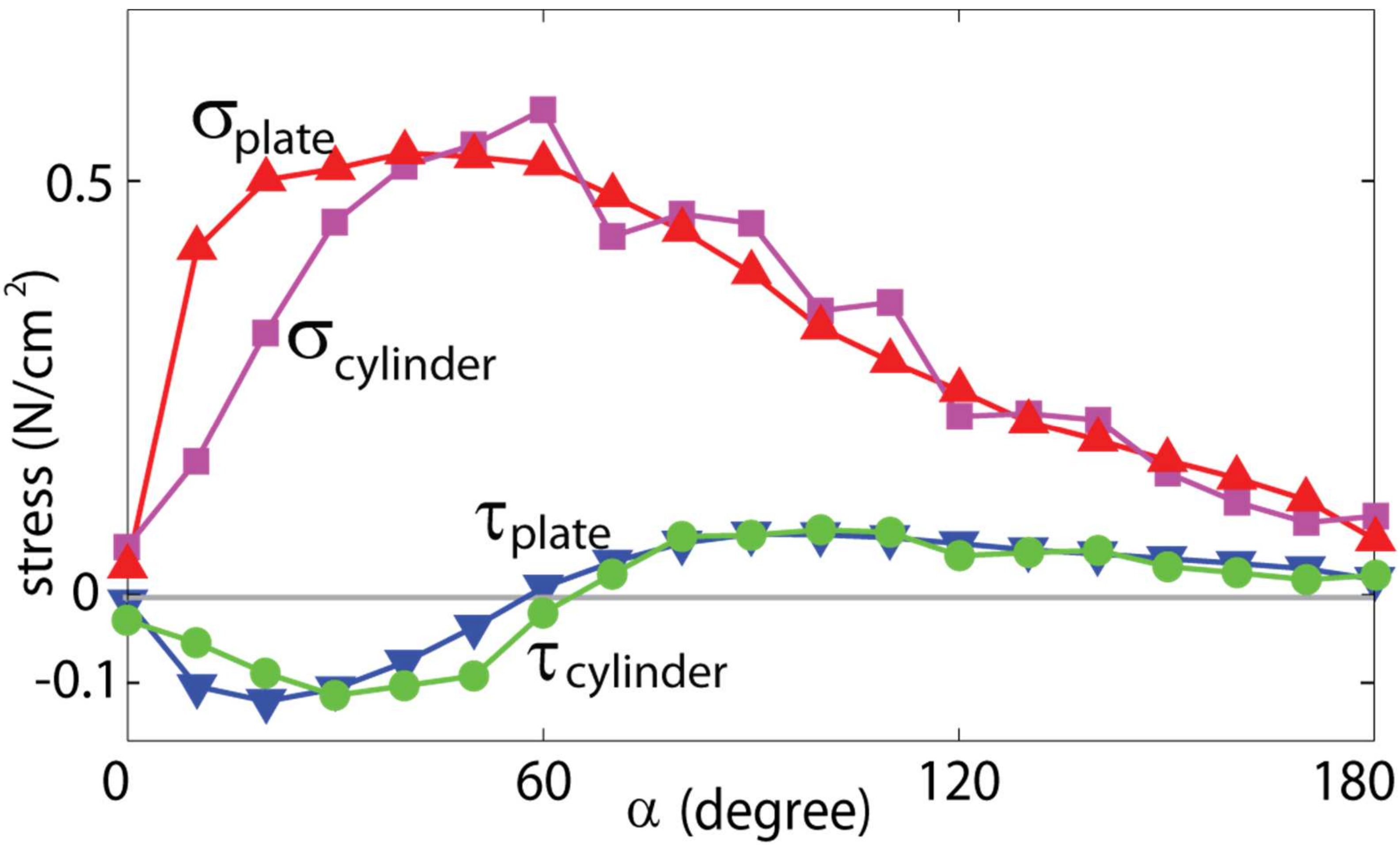
10 cm long (into page), 0.03 cm thick, 2.54 cm wide

Flow field snapshot vs plate angle (in co-moving frame)

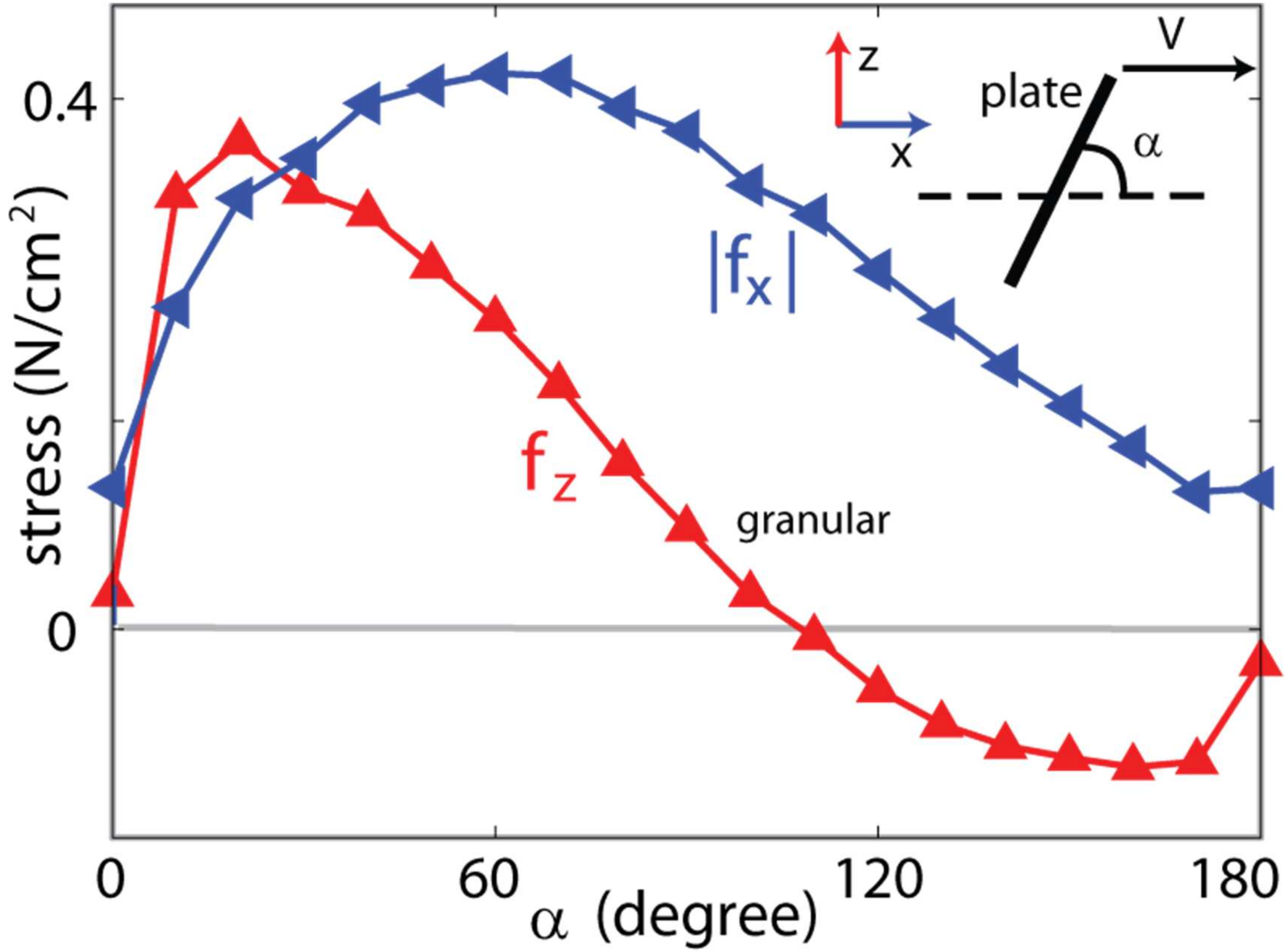


(play outside of ppt)

Local stresses are well approximated by plate elements

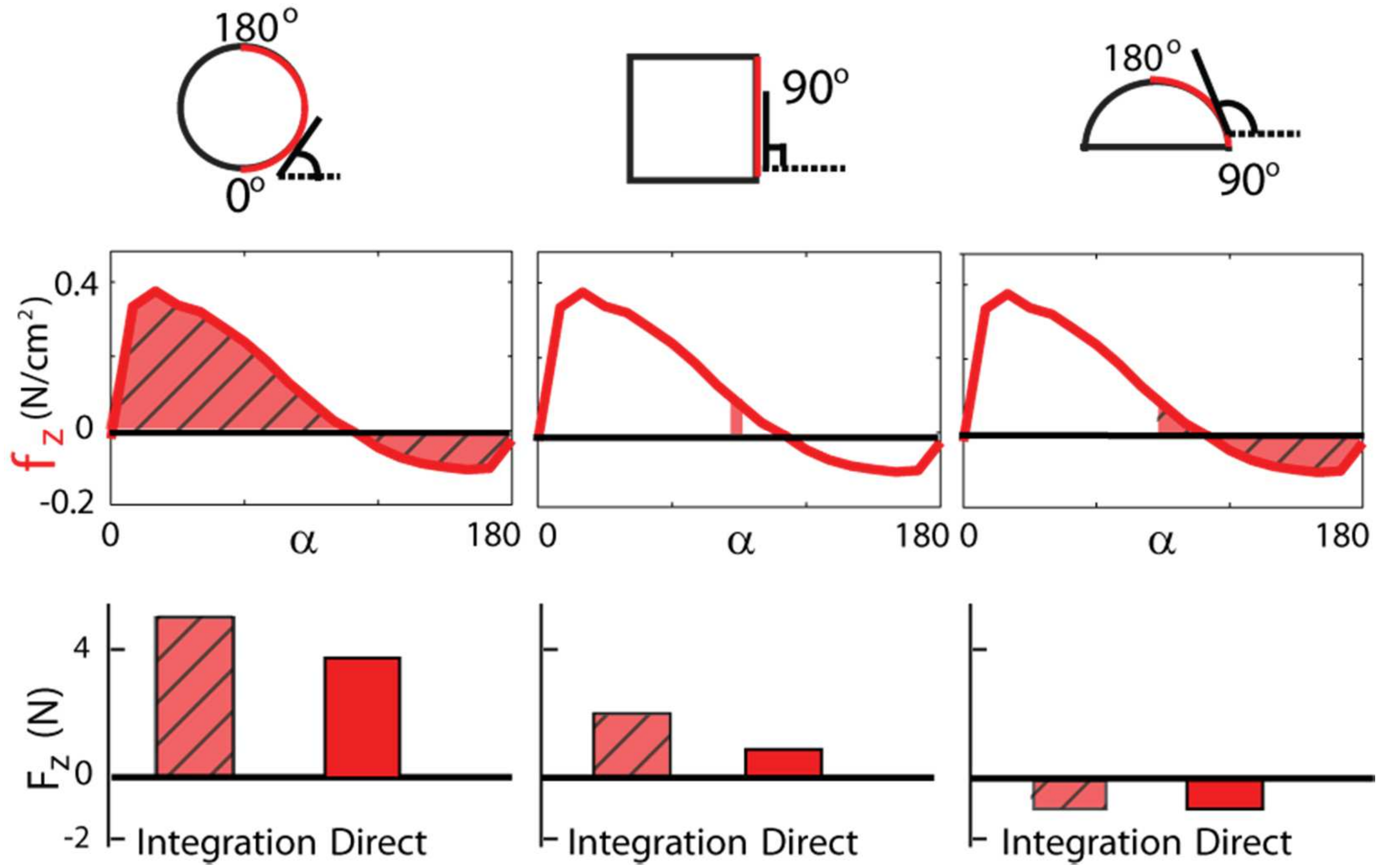


Drag and lift on a plate

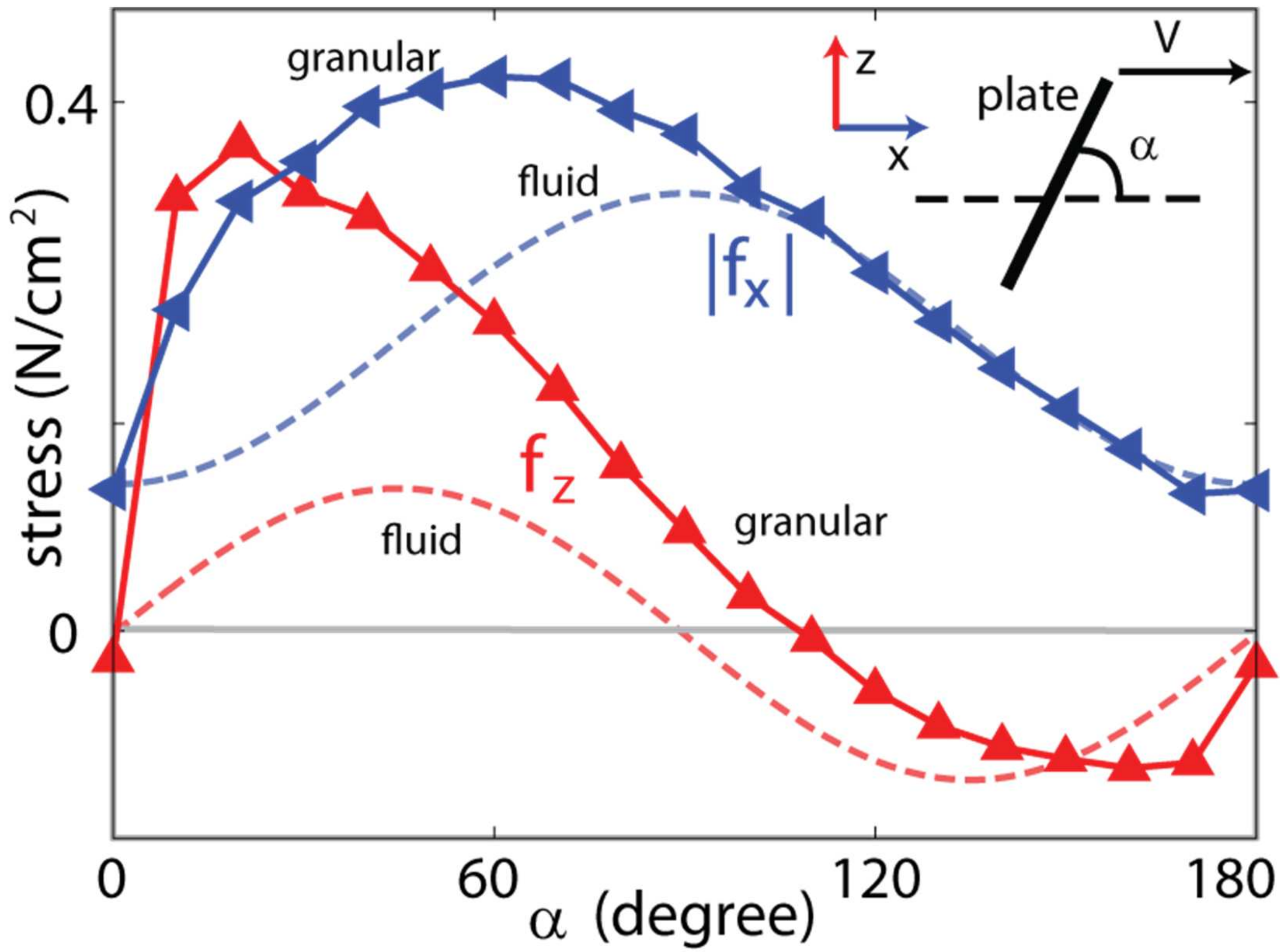


Integrate the force on the plates

$$F_z = \int f_z(\alpha)(z/d)dA$$

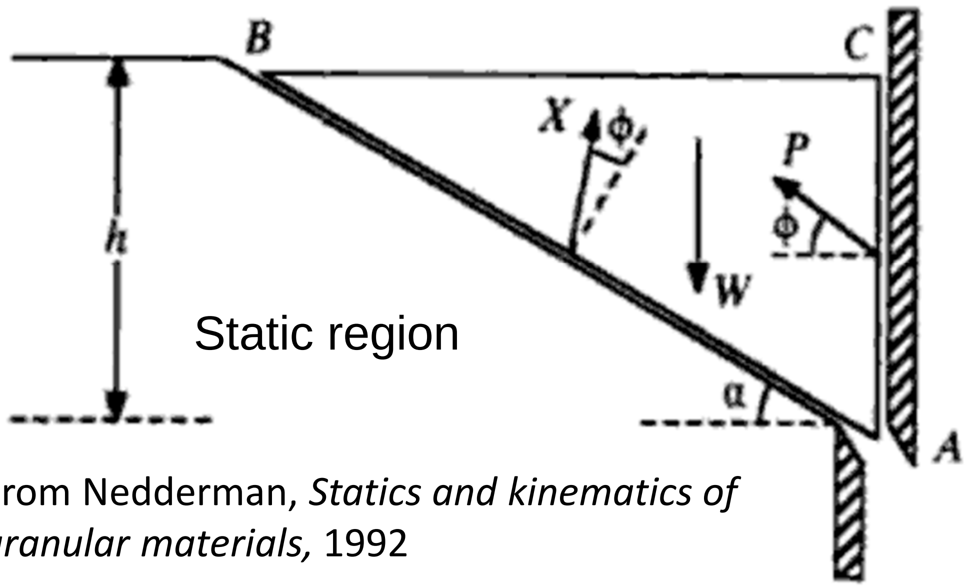


Drag and lift on a plate

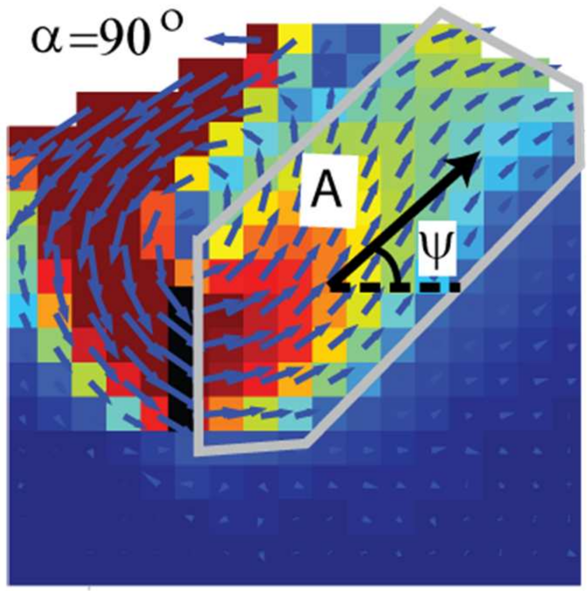


Coulomb's method

(after Wieghardt, 1975)



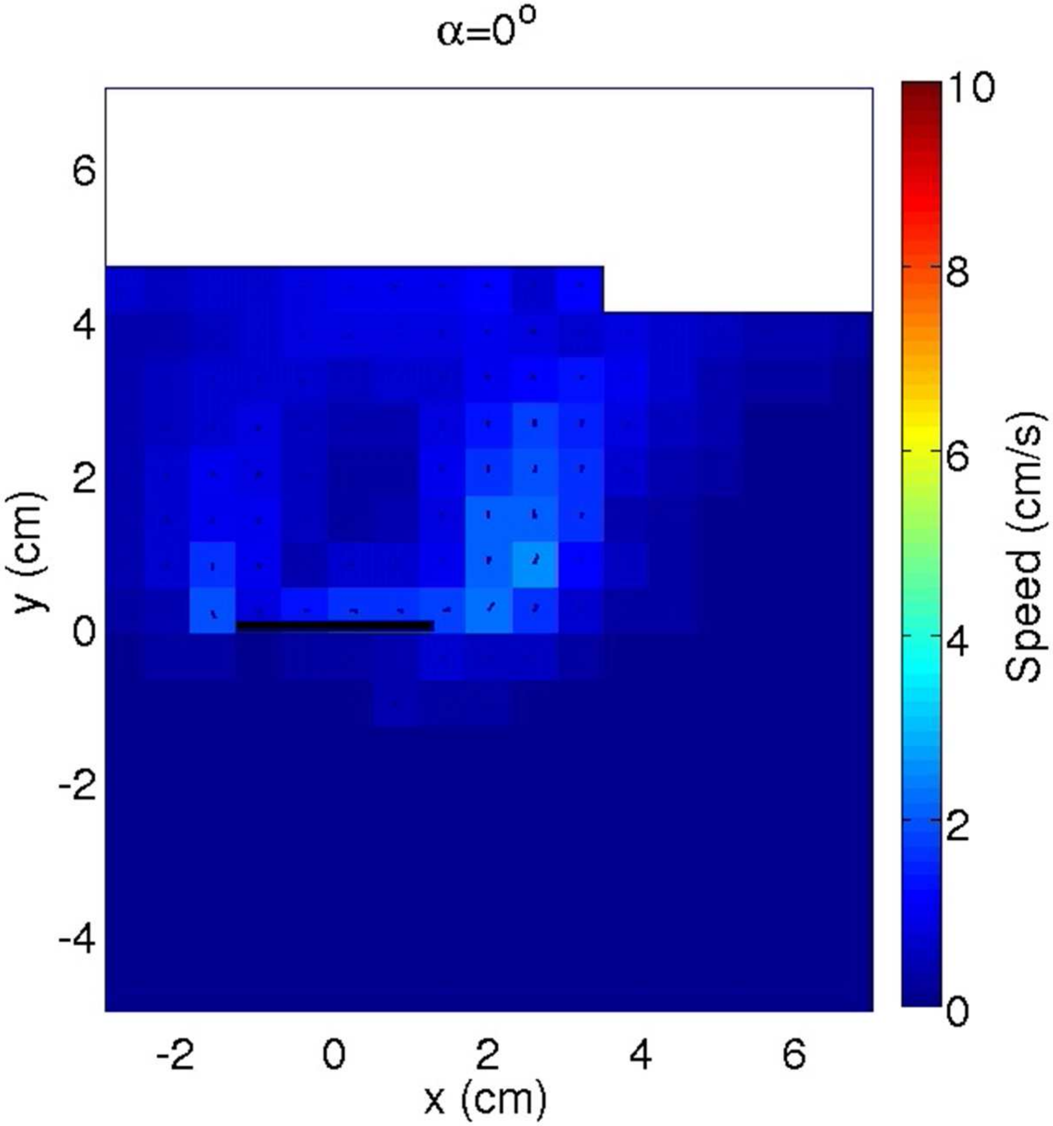
From Nedderman, *Statics and kinematics of granular materials,* 1992



1. Find the slip plane which separate flowing region and non-flow region

2. Analyze force balance on the wedge-shaped region with the plate as a boundary

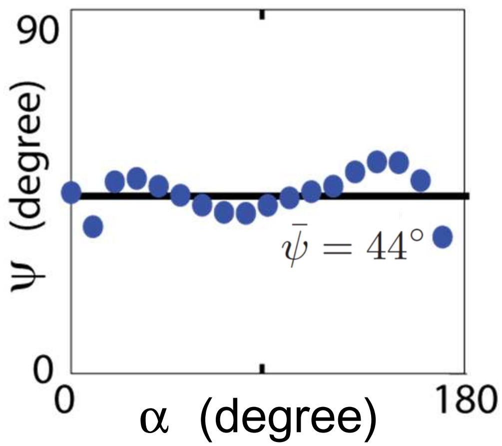
Examine flowing material near plate



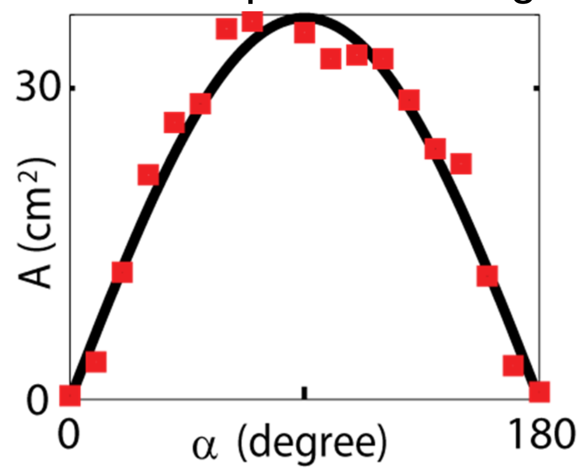
(play outside of ppt)

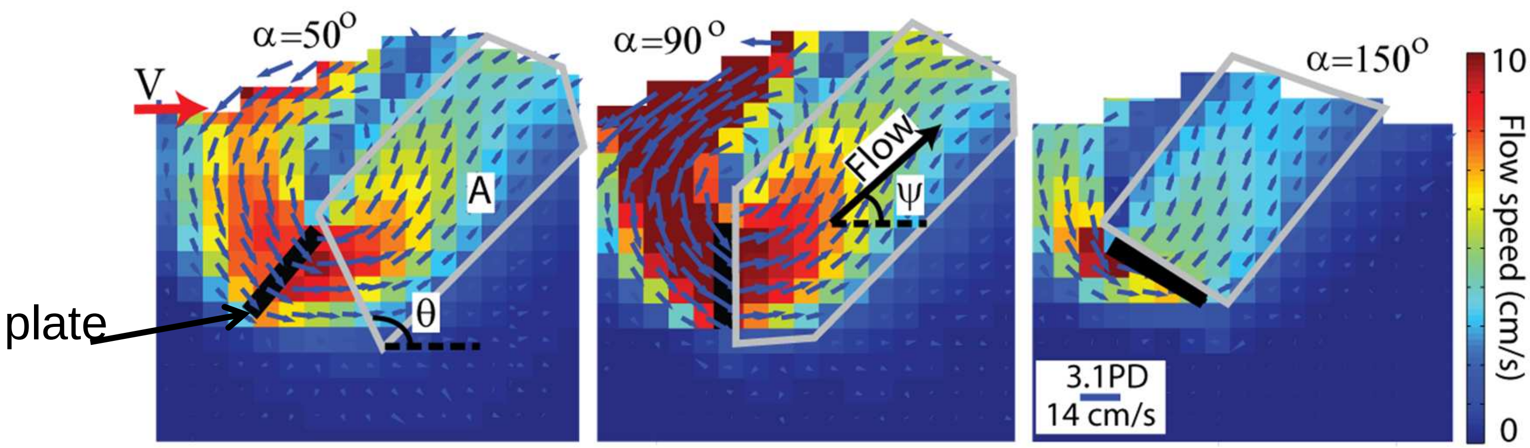
Characterize the flow field

Direction of the flow

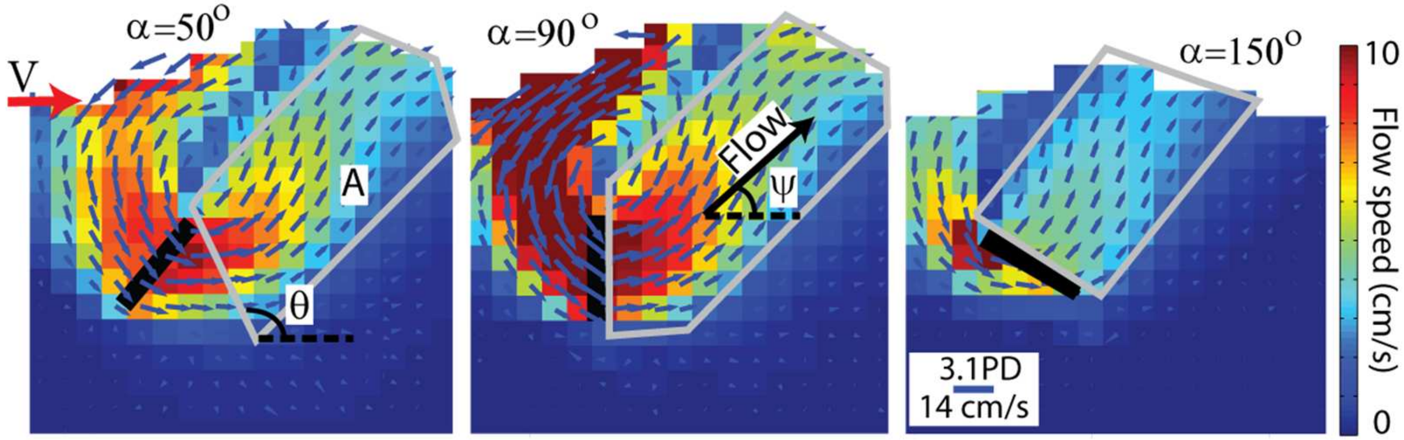


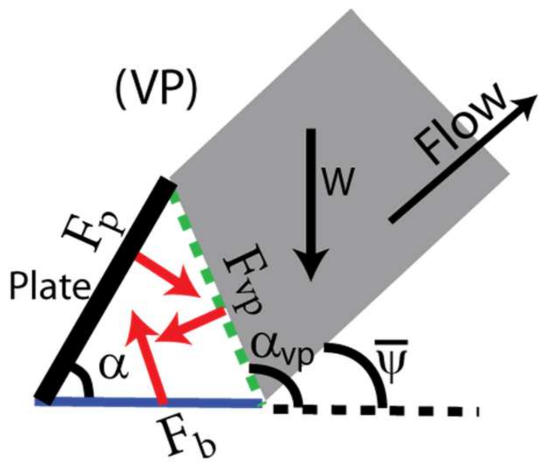
Area of the upward flowing region

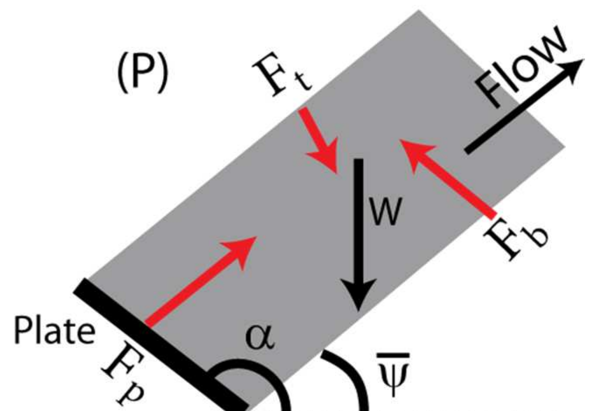




Apply Coulomb's method







$$\sigma(\alpha) = \frac{W}{lw}\,\frac{\cos\beta\,\sin(\bar{\psi}+\gamma)}{\sin(\alpha-\beta-\bar{\psi}-\gamma)}$$

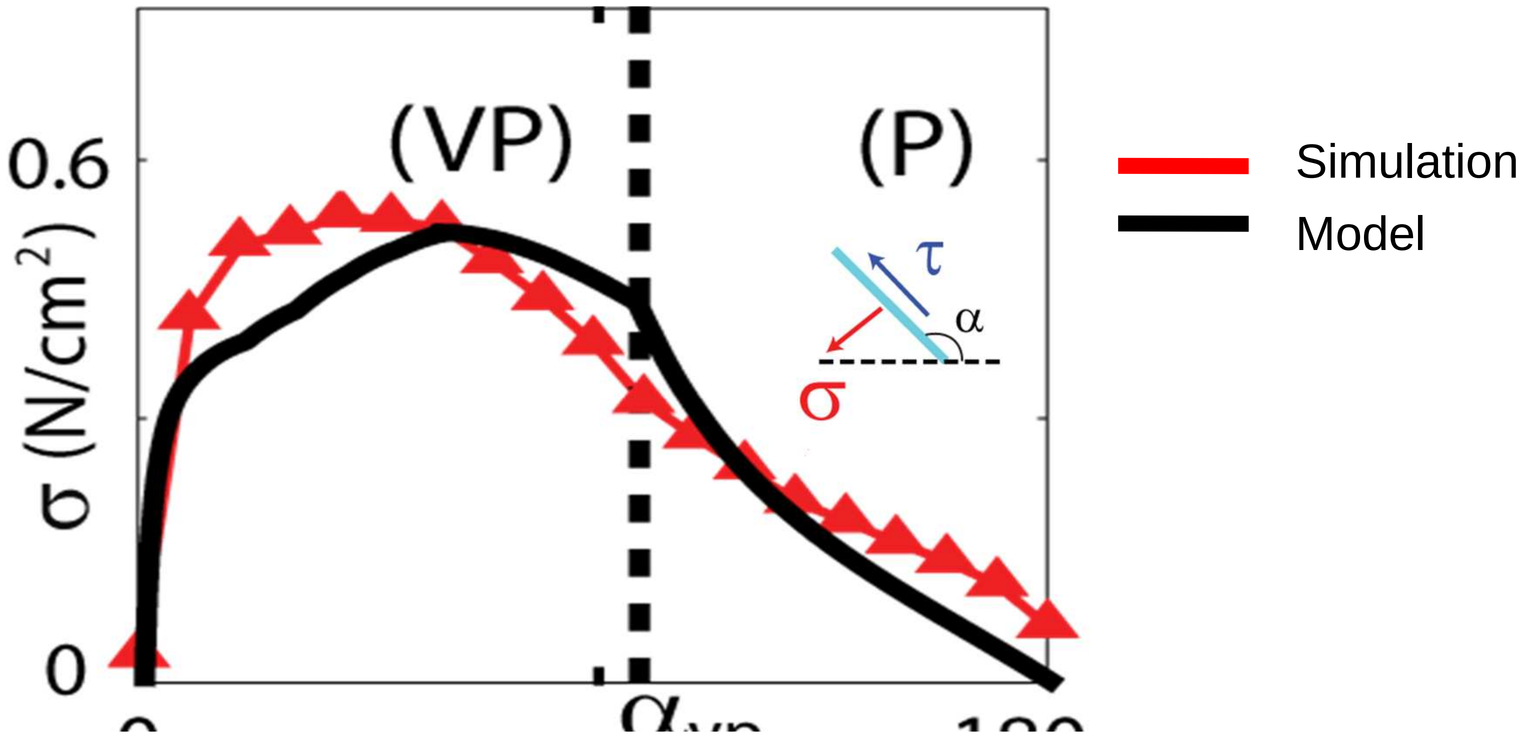
$$\sigma(\alpha) = \frac{W}{wl} \times \frac{\cos\beta\,\sin(\bar{\psi}+\gamma)}{\sin(\alpha_{vp}-\beta'-\bar{\psi}-\gamma)}\,\frac{\sin(\alpha_{vp}-\beta'-\gamma)}{\sin(\alpha-\beta-\gamma)}$$

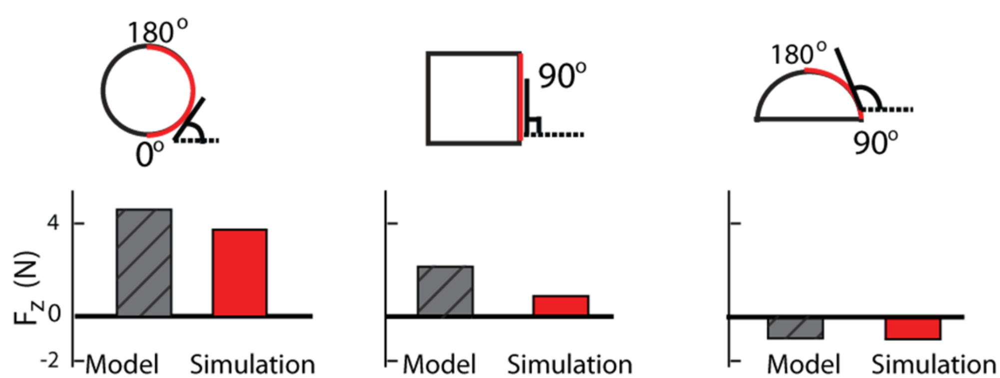
$$\beta(\alpha) \equiv \tan^{-1}\mu_{\text{peff}}$$

$$\alpha_{vp} = 97°$$

$$\beta' = \beta(\alpha_{vp})$$

Model result





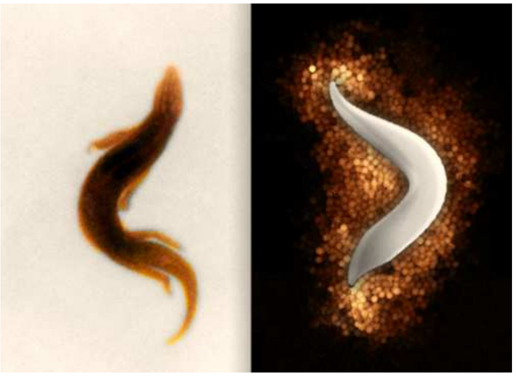
Summary

- Drag force is insensitive to shape, lift force depends on shape and increase with depth
- DEM can quantitatively model granular flows
- Drag induced lift on nonplanar intruders can be computed as the sum of lift forces from **independent** planar (plate) elements which each experience a lift force resulting from the pushing of material up a slip plane.
- “Wedge” model gives reasonable estimate based on flowing region near plate

Swimming in Sand

Papers:

Maladen et al, Science, 2009
Maladen et al, Robotics: Science & Systems conference 2010 (Best paper award)
Maladen et al, J. Royal Society Interface, 2011
Maladen et al, International Journal of Robotic Research, 2011
Maladen et al, ICRA, 2011

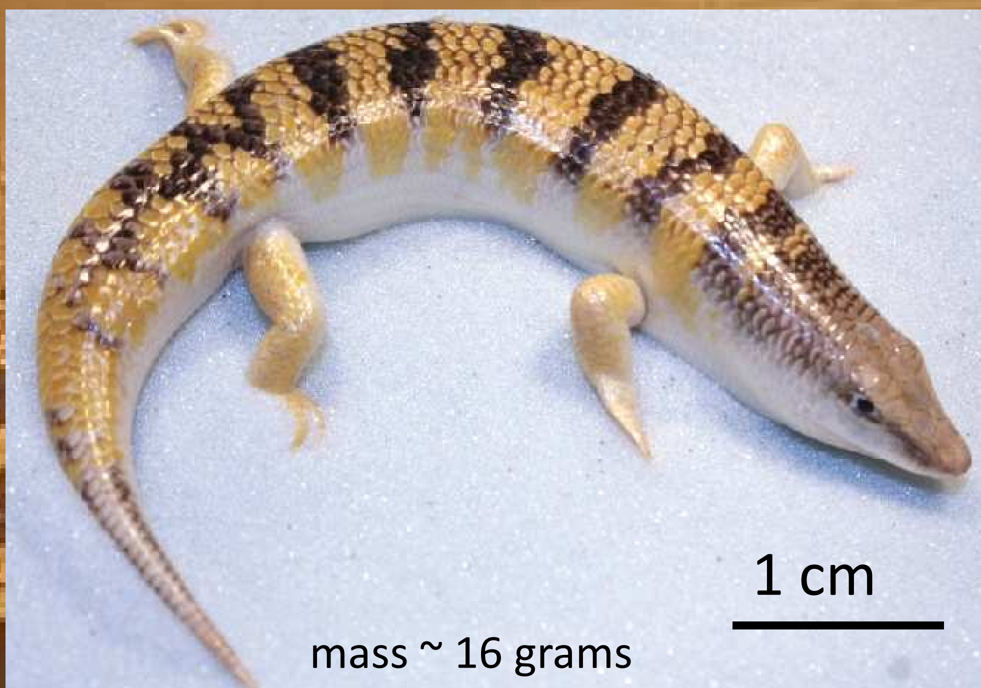


Pdfs and links to movies here:

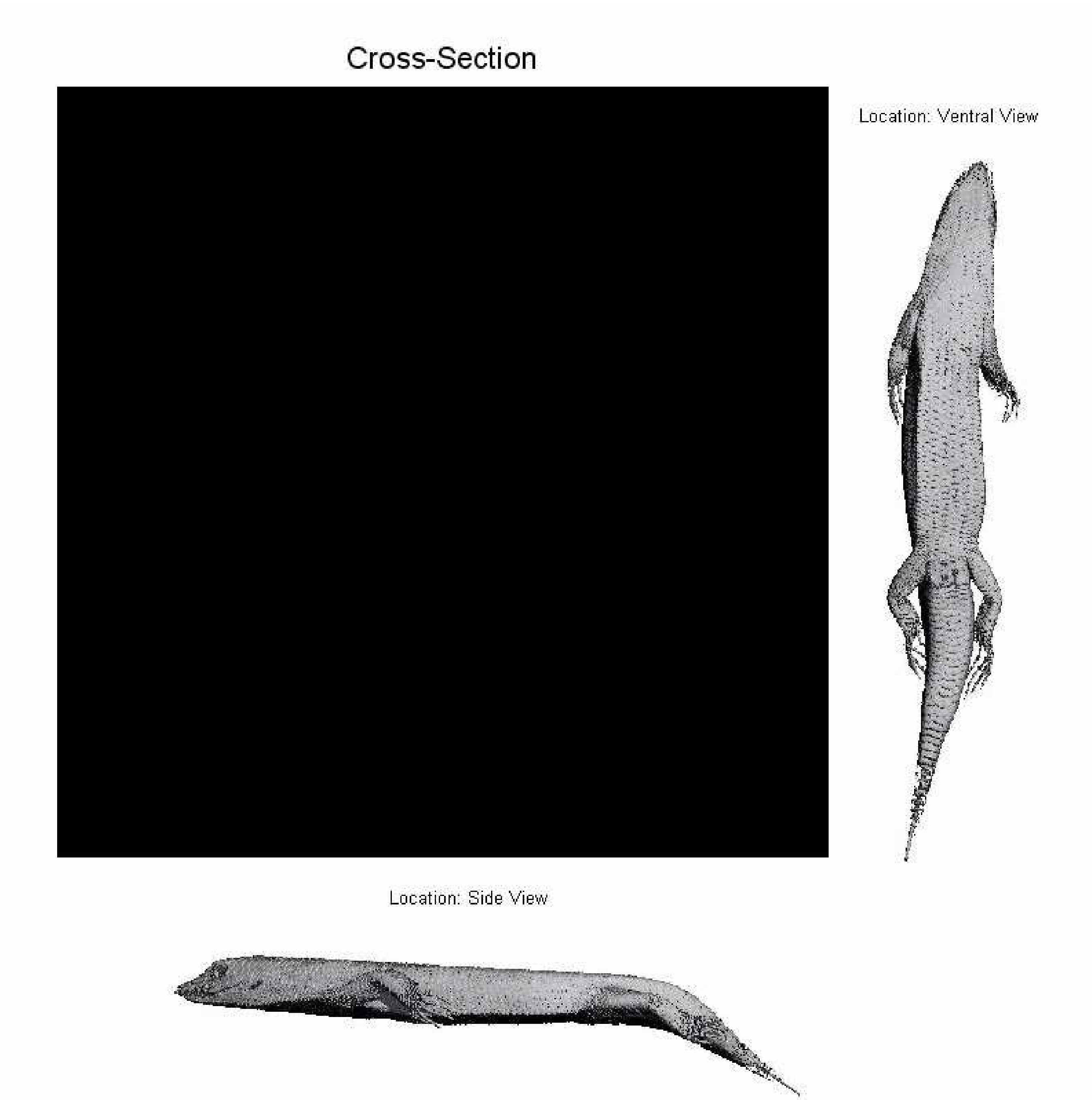
http://crablab.gatech.edu/pages/publications/index.htm

The sandfish lizard

Sandfish (*Scincus scincus)*

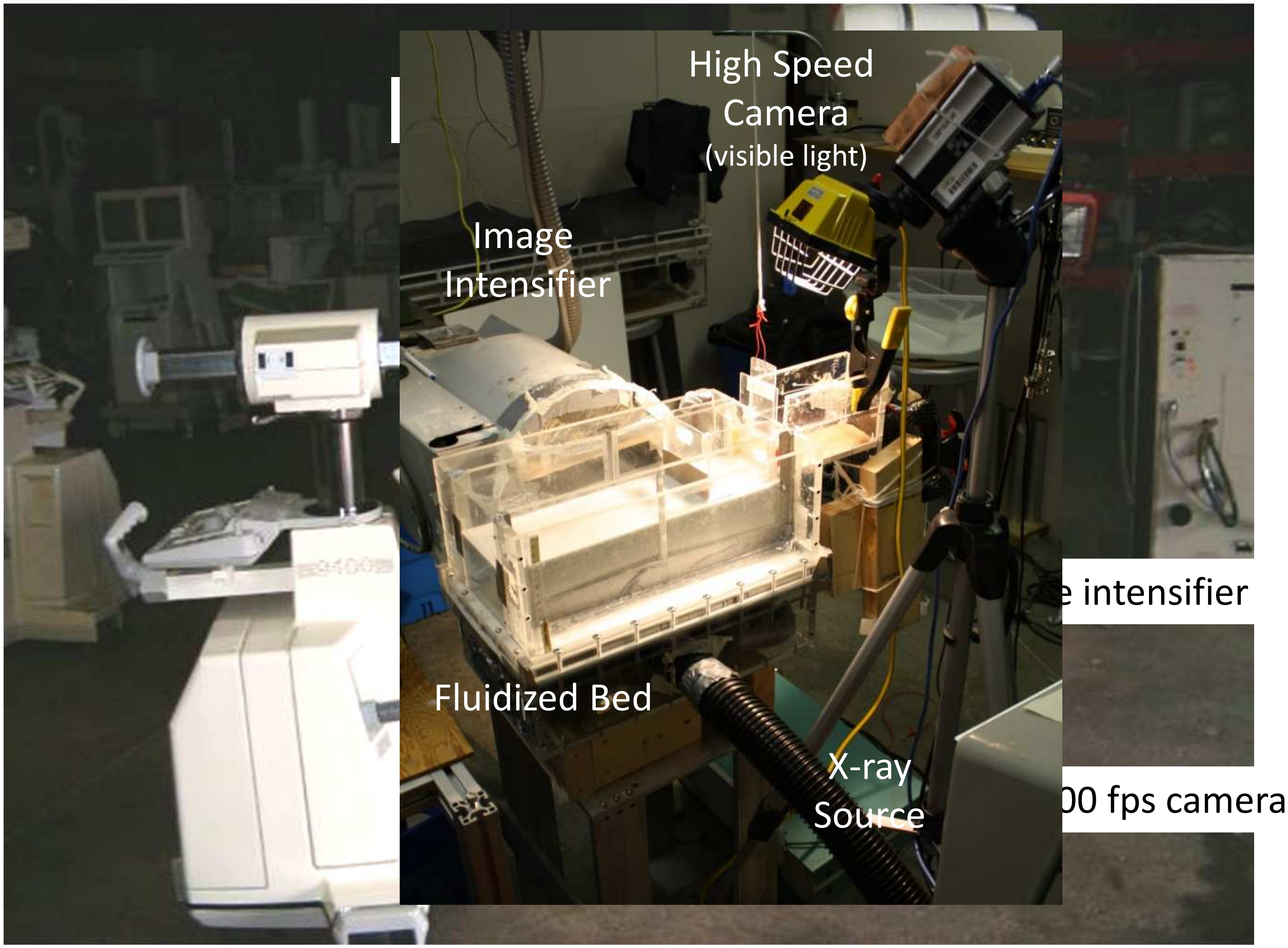


- Native to Sahara desert
- Adaptations for living in sand: countersunk jaw, fringe toes, smooth scales, flattened sidewalls
- One of ~10 species classified *subarenaceous*: "swims" within sand



Taken by Sarah Steinmetz at GT micro-CT facility, with Prof. Bob Guldberg,

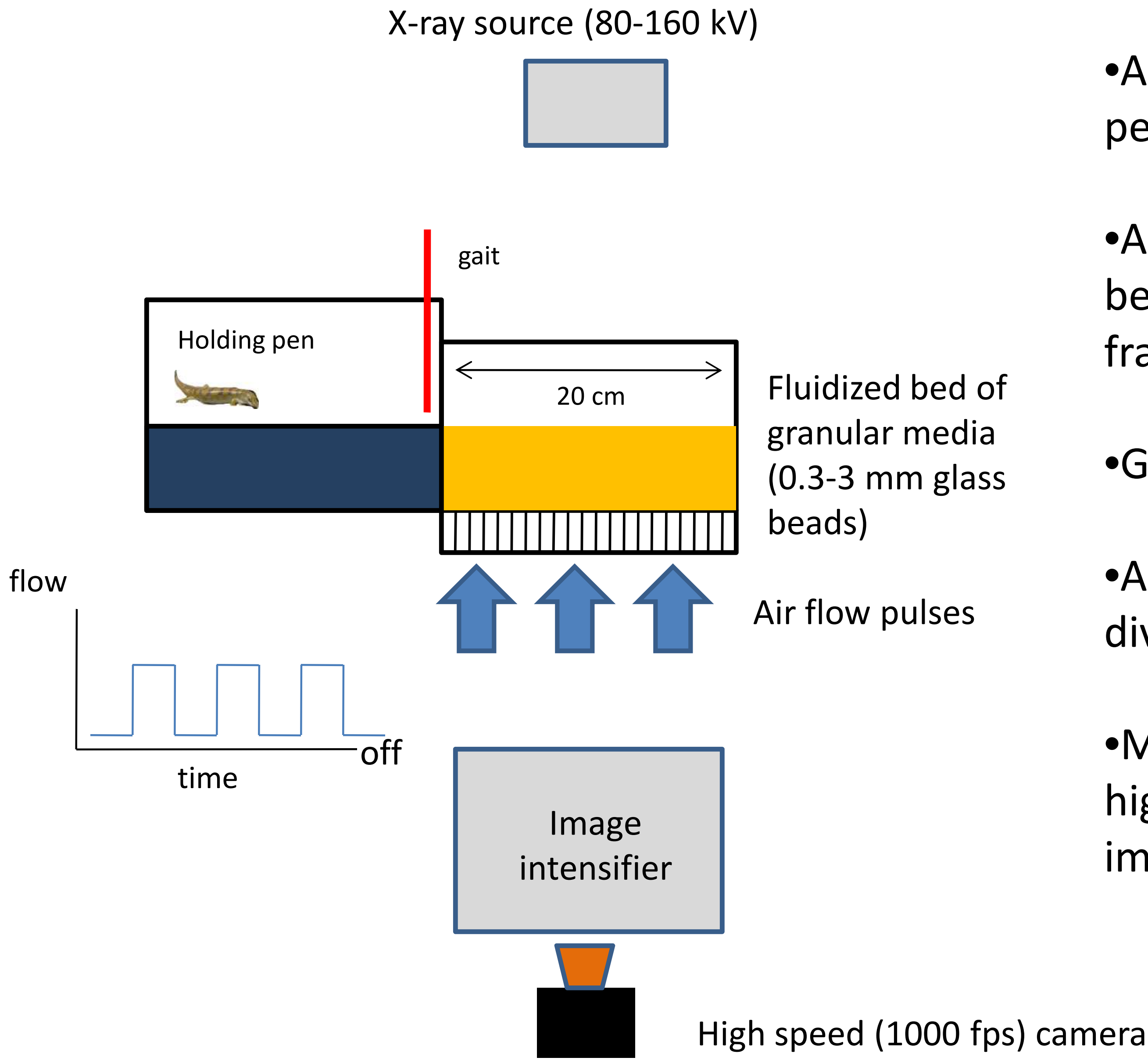
X-ray imaging to see within sand



Ryan Maladen

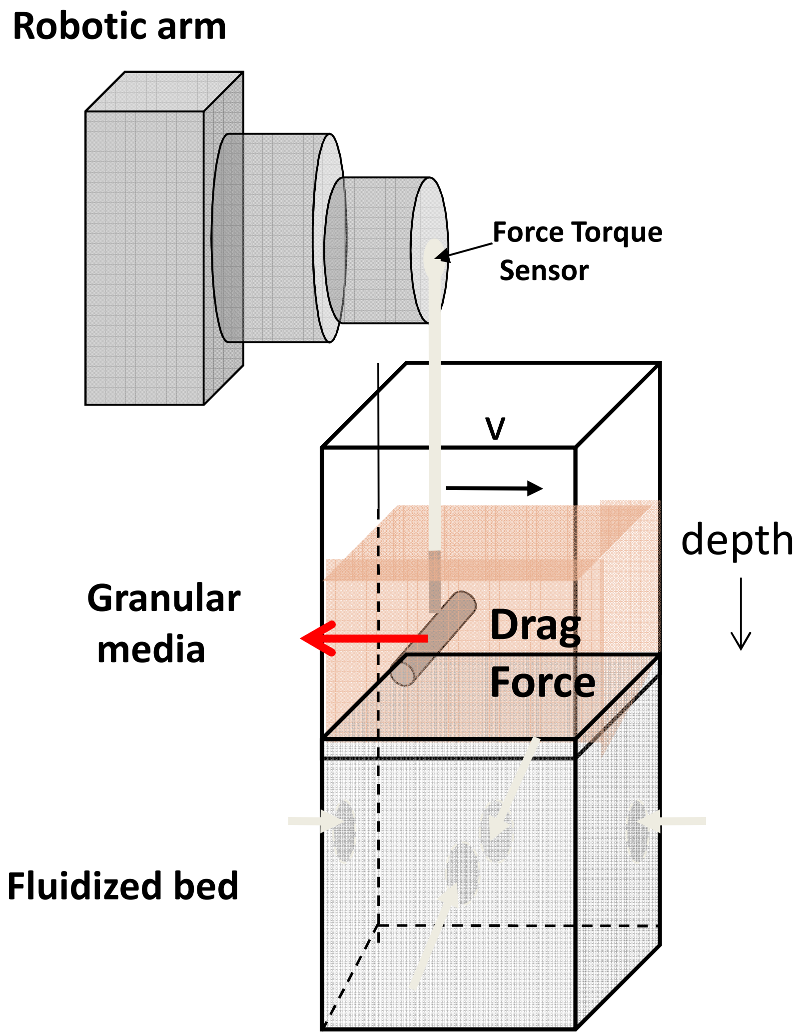
Sarah Steinmetz

Experimental apparatus



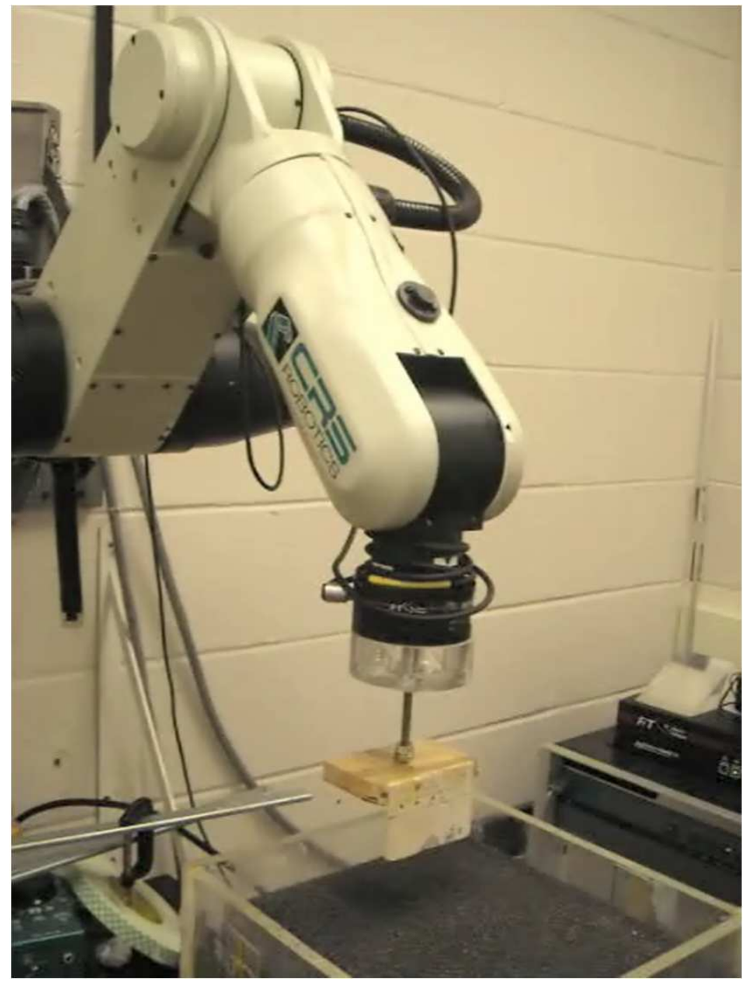
- Animal is placed in holding pen
- Air pulses to the fluidized bed sets initial volume fraction $0.58<\phi<0.63$
- Gait is pulled up
- Animal moves onto sand, dives within
- Motion is recorded with high speed visible and x-ray imagers

Probing granular media

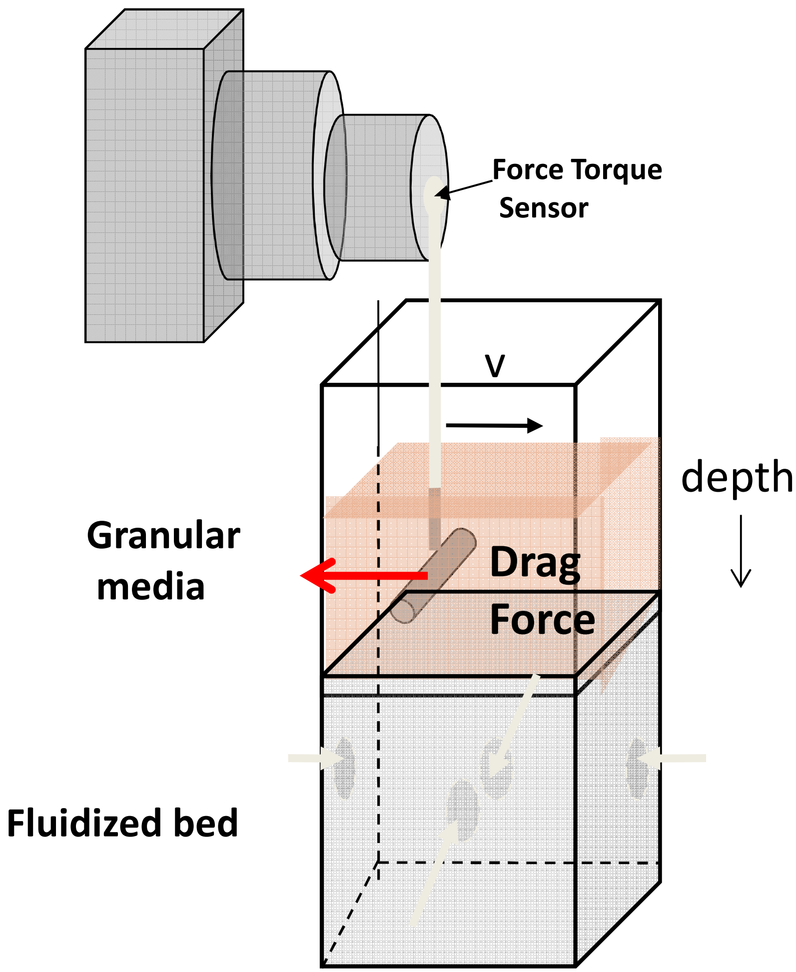


Maladen, Ding, Li, Goldman, *Science*, 2009
Gravish, Umbanhowar, Goldman, *PRL*, 2010

Robot arm with 6 axis force/torque sensor



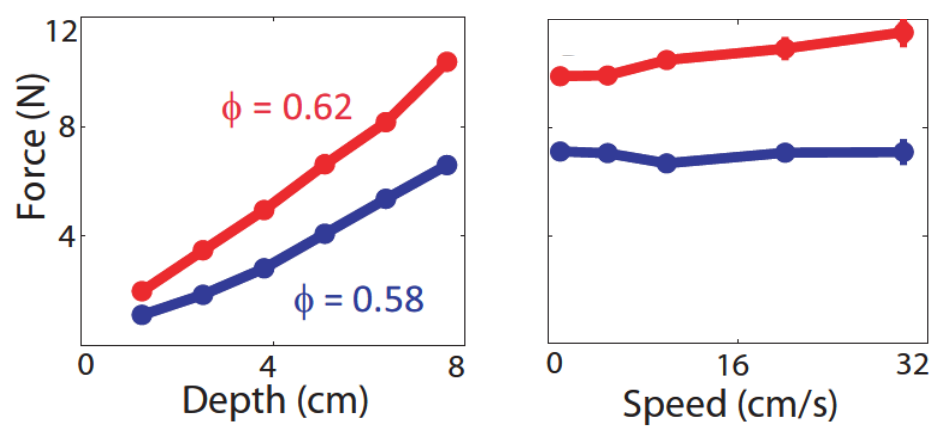
Granular media, a "frictional fluid"



Maladen, Ding, Li, Goldman, *Science*, 2009

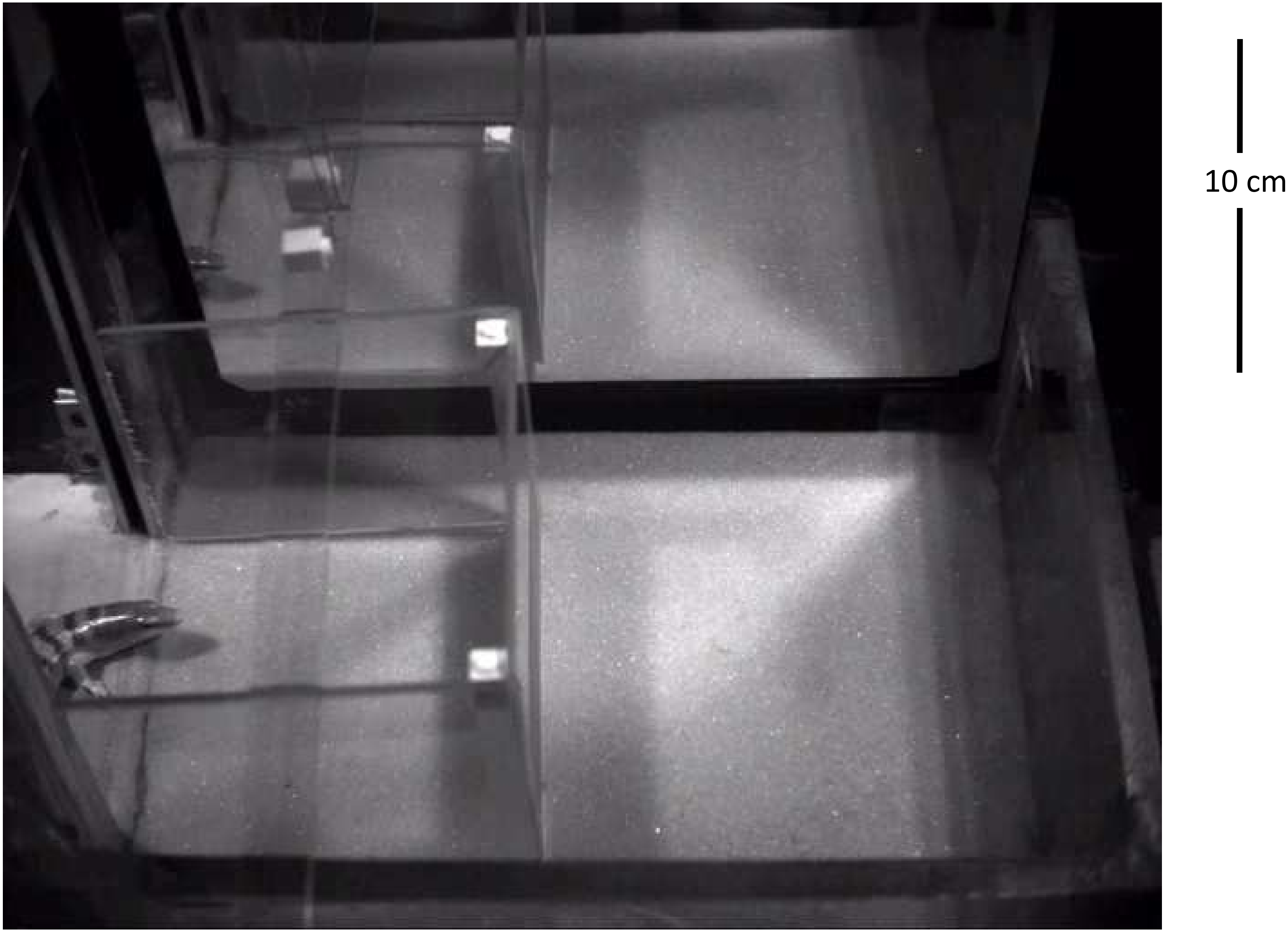
Drag forces:

1. increase with depth
2. independent of speed
3. increase with increasing compaction (volume fraction ϕ)

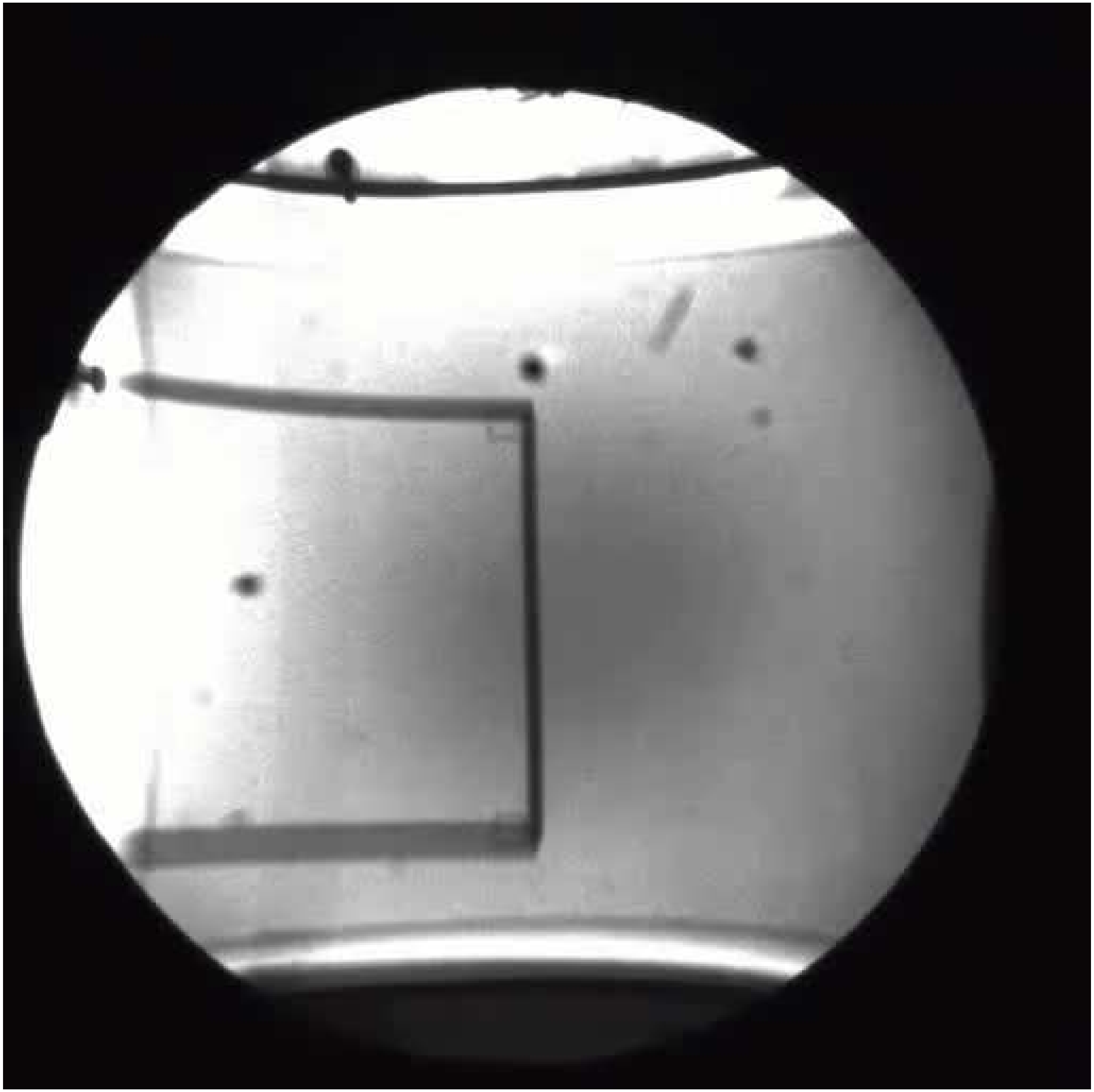


Drag experiments in 0.3 mm glass beads

0.25±0.04 mm diameter glass beads, particle density = 2.5 g/cm³ , bed depth=15 cm

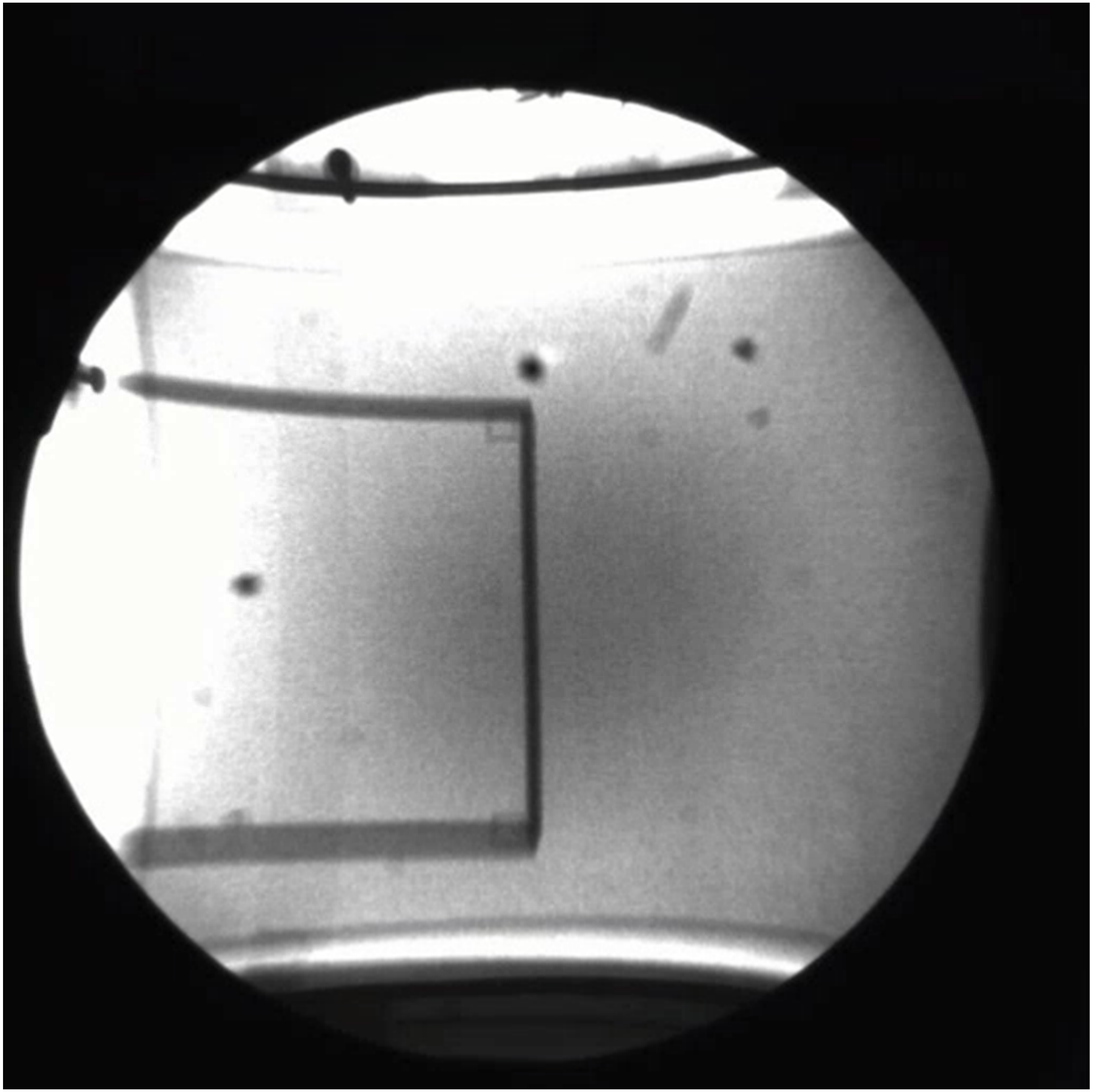


0.25±0.04 mm diameter glass beads, particle density = 2.5 g/cm^3 , bed depth=15 cm

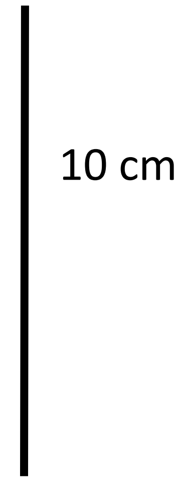


Real time

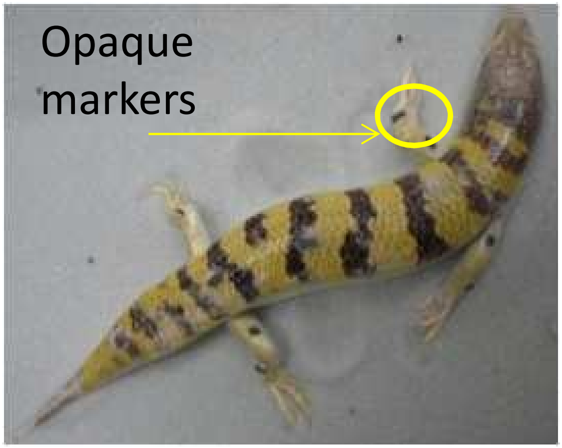
10 cm

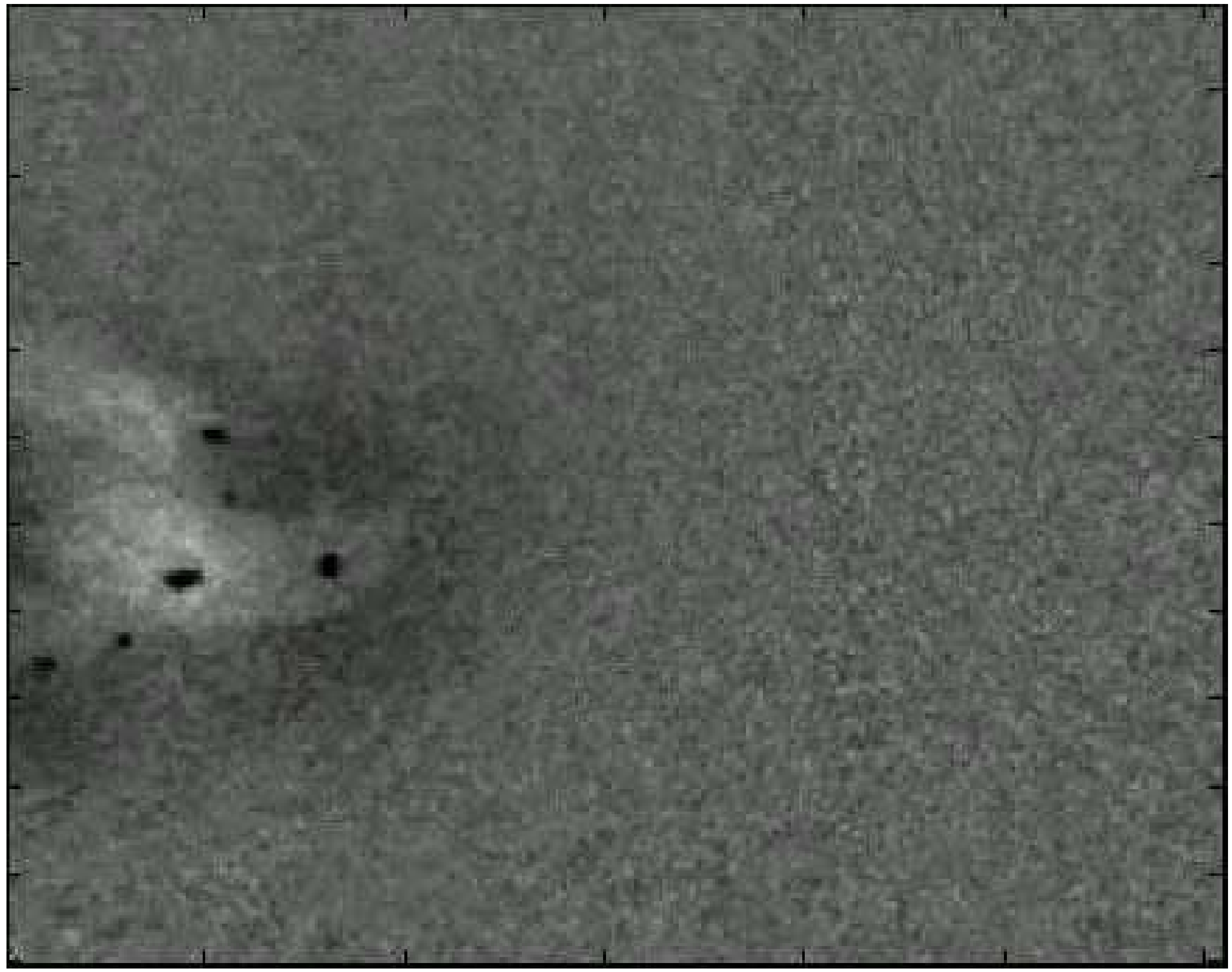


Slowed 10x

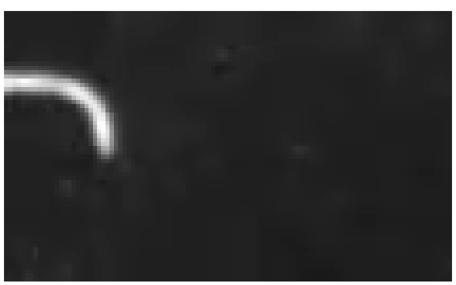


Swimming without use of limbs

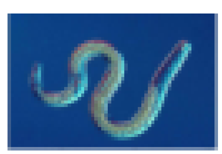




1 mm

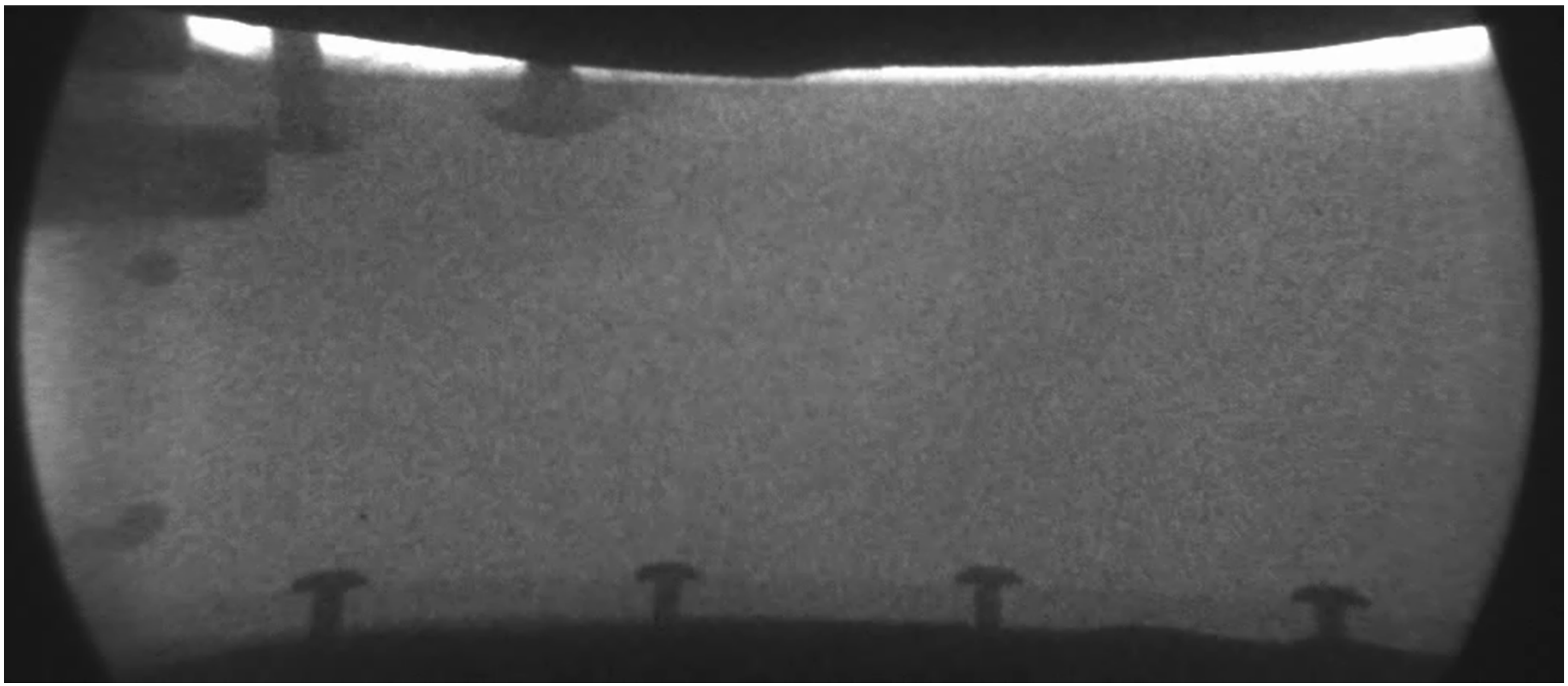


Nematode (*C. elegans*) in fluid
Hang Lu, Georgia Tech



1 cm

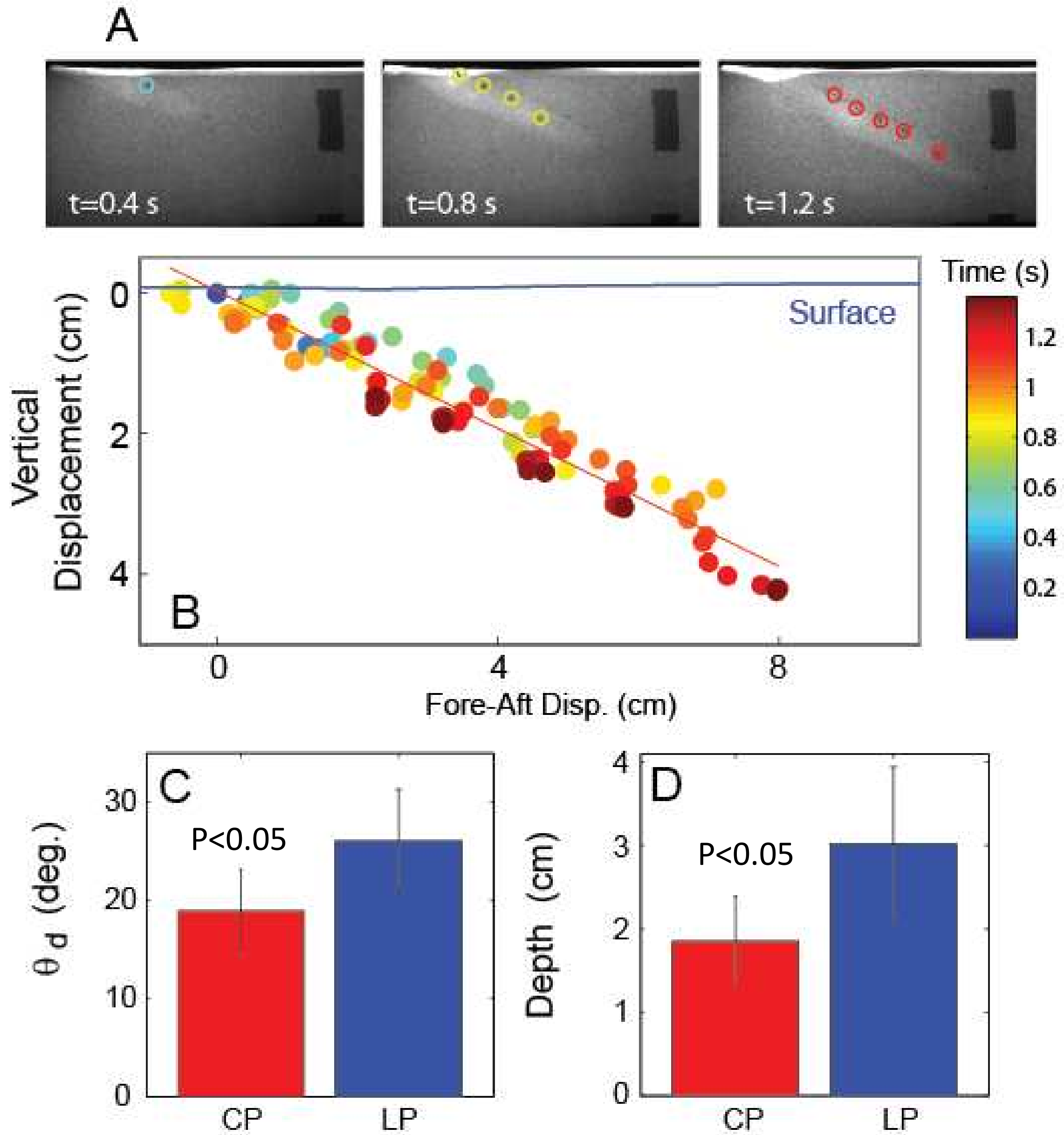
Side view



10 cm

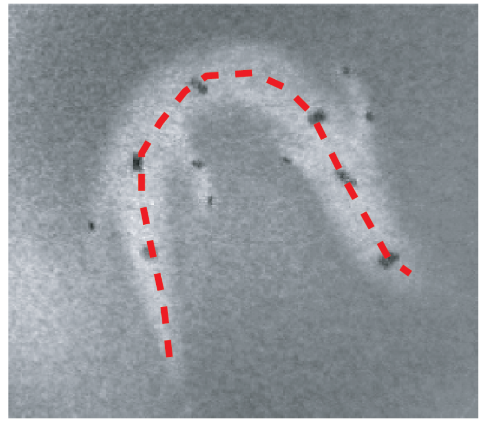
Slowed 10x

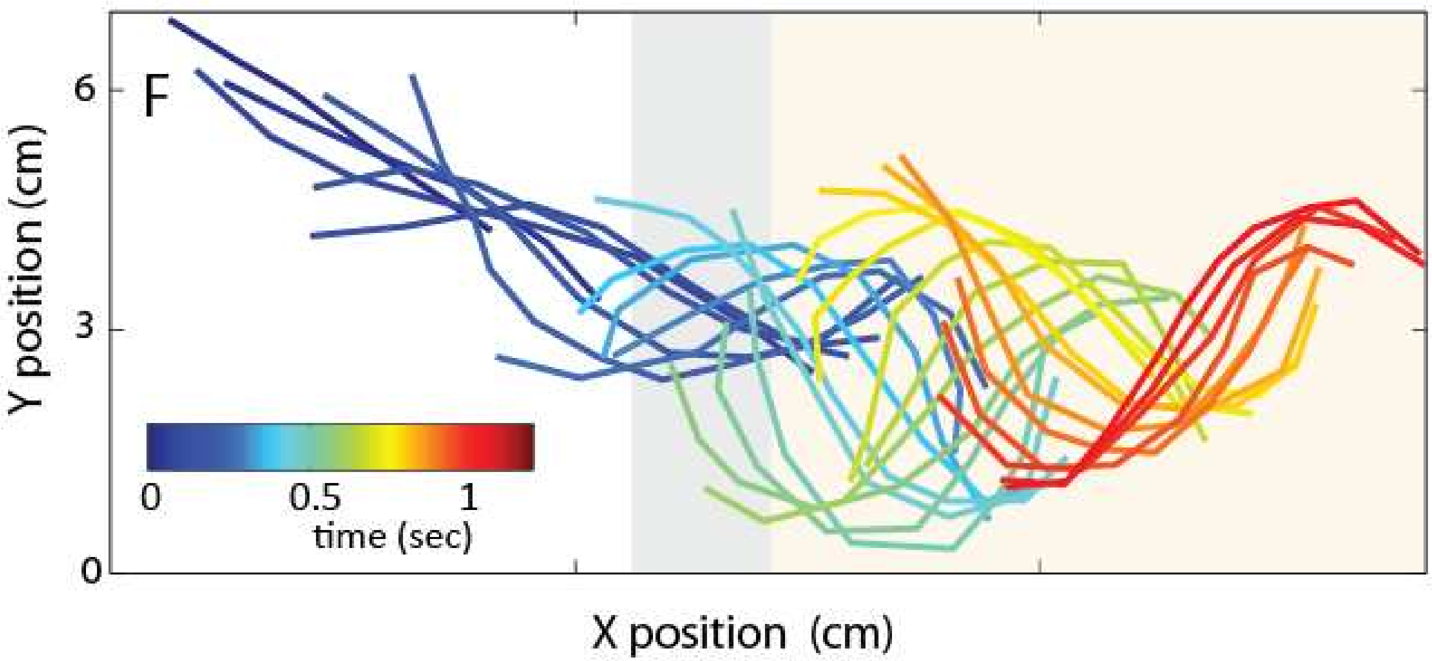
Swimming kinematics (sagittal plane)



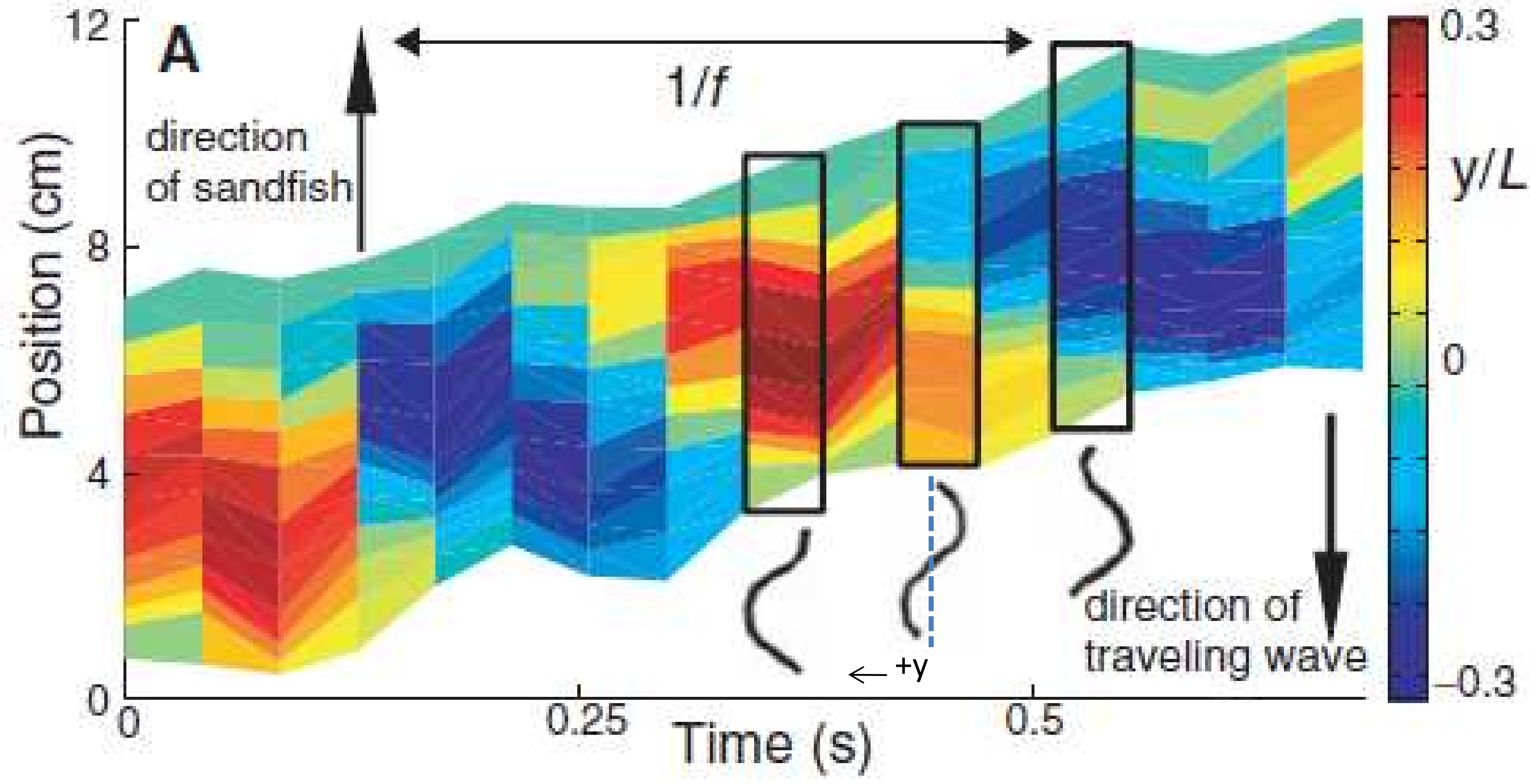
n=11 animals
mass=16.2 ± 4 g

Swimming kinematics (horizontal plane)

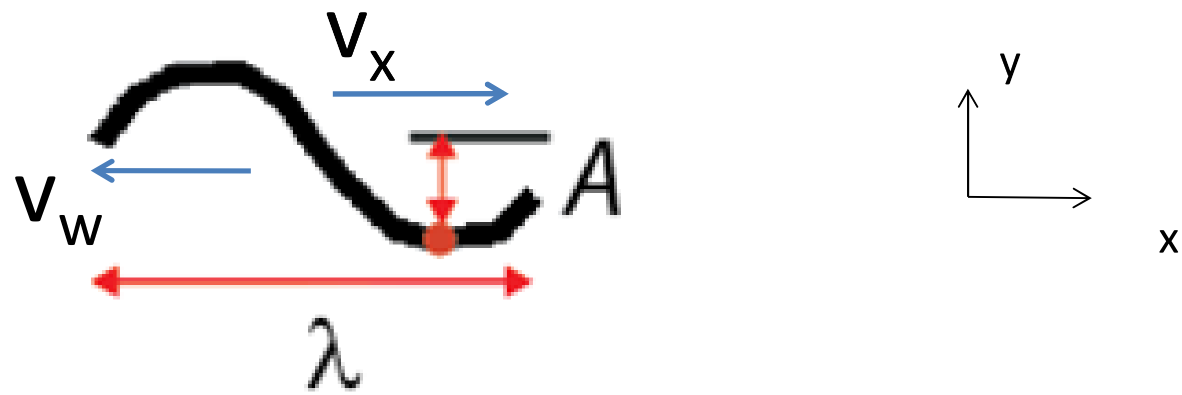




Traveling wave, head to tail



Kinematics during steady swimming



fit

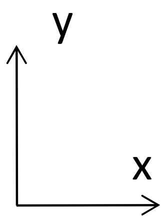
$$y = A\sin\frac{2\pi}{\lambda}(x + v_w t) \qquad v_w = \lambda f$$

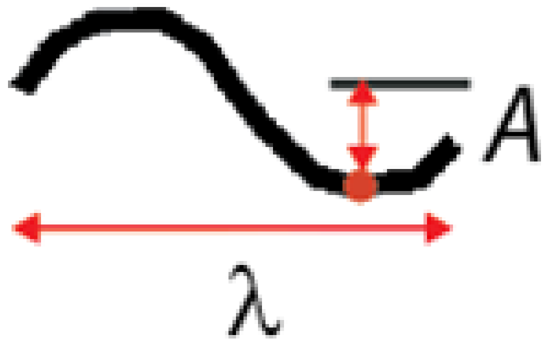
$R^2 > 0.95$ at all phases in cycle

Single period sinusoidal wave, traveling head to tail

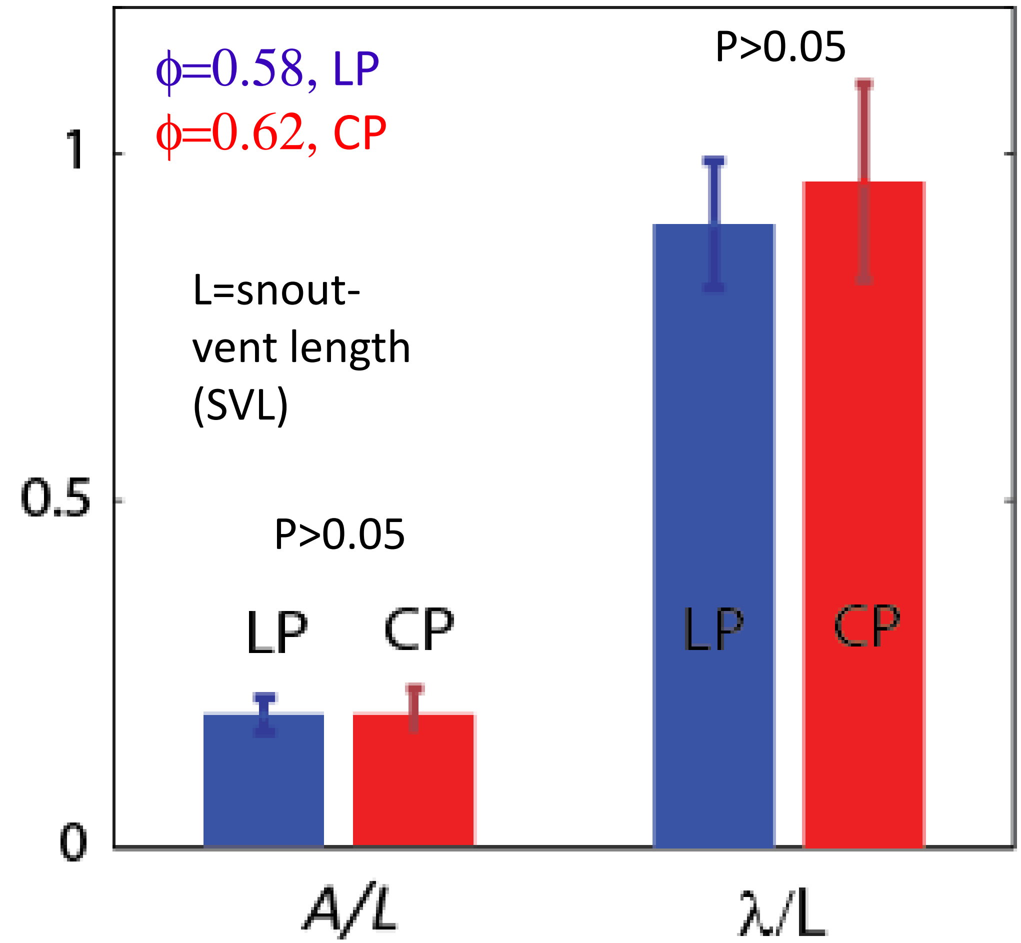
n=11 animals
mass=16.2 ± 4 g

Swimming kinematics





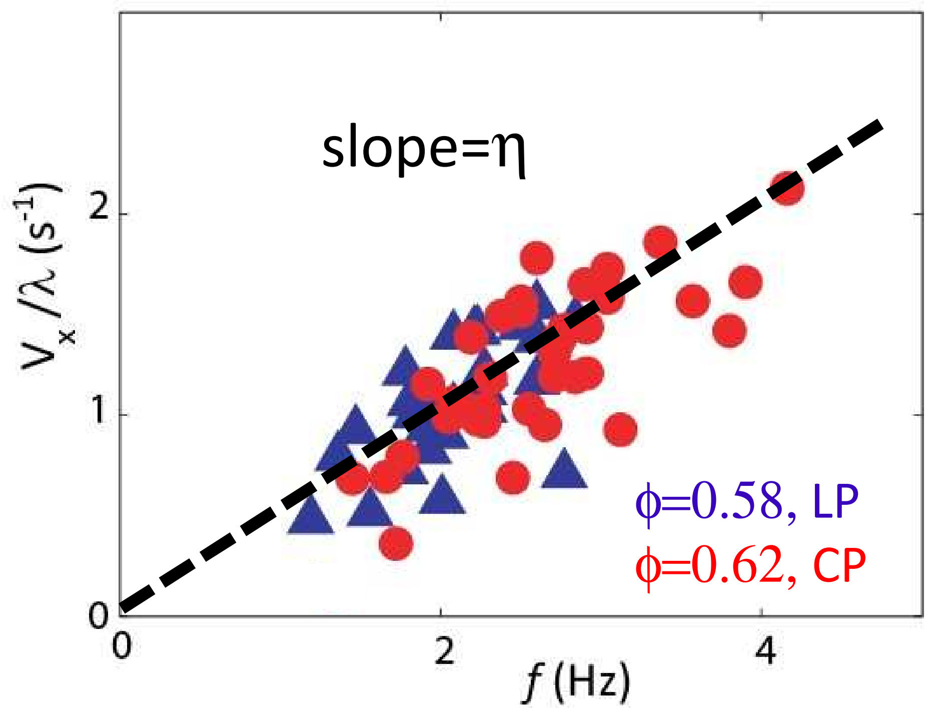
Travelling sinusoidal wave, kinematics independent of ϕ



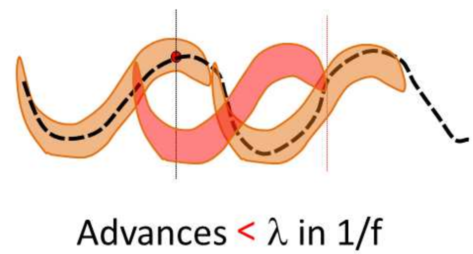
A/λ = 0.20 ± 0.04 LP
0.22 ± 0.06 CP

n=11 animals
mass=16.2 ± 4 g

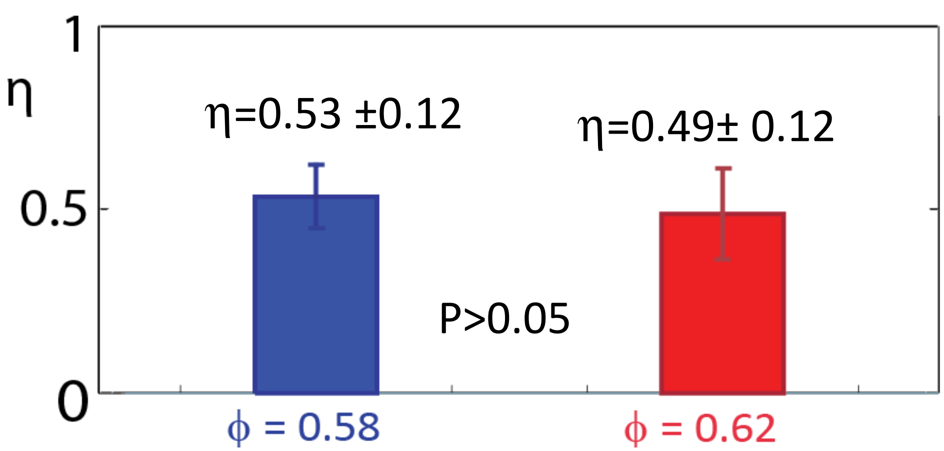
Swimming speed vs frequency & wave efficiency



$$\eta = \frac{v_x}{v_w} = \frac{v_x}{f\lambda} = \frac{v_x/\lambda}{f}$$



Measures amount of “slip” relative to movement in a tube

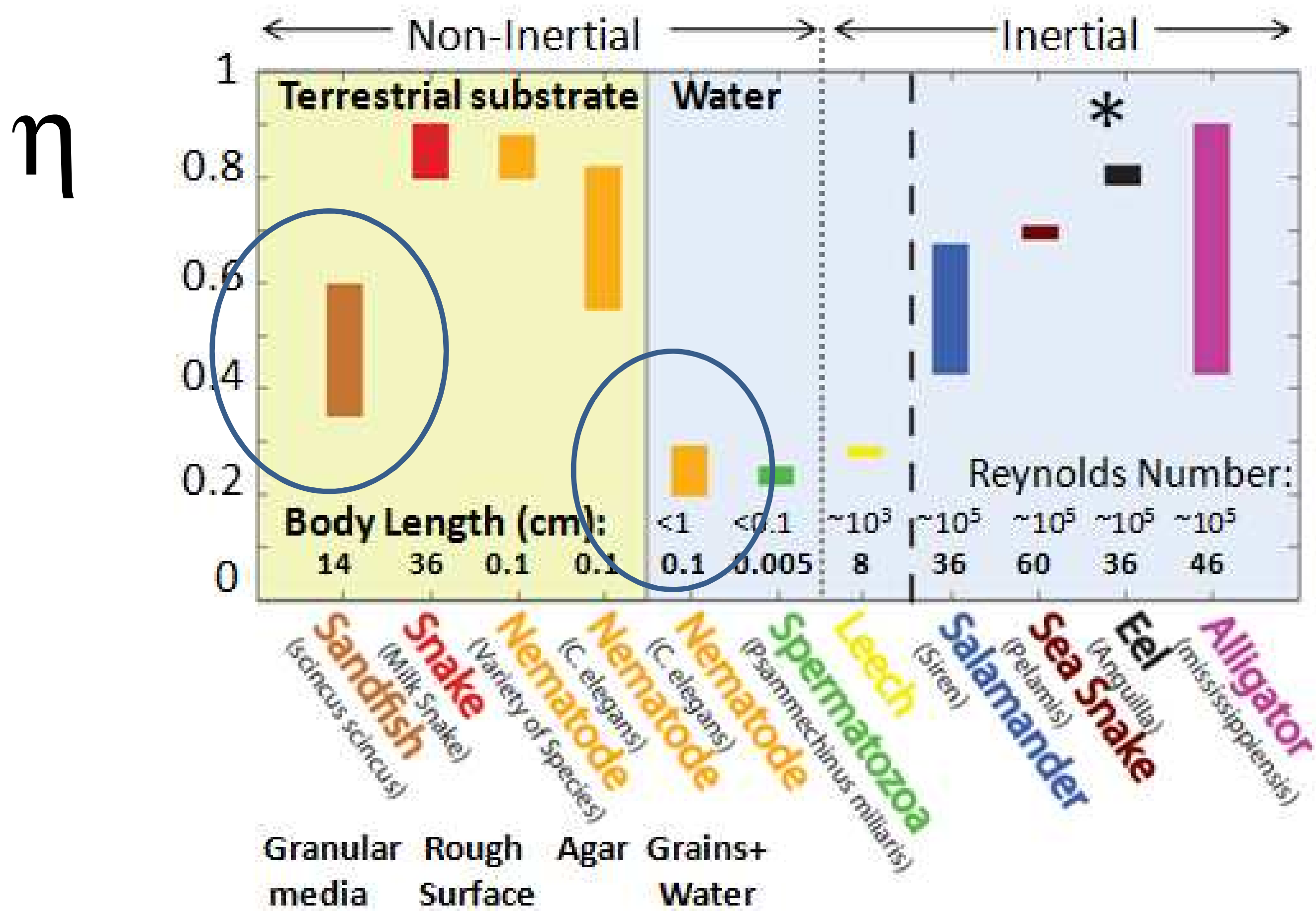


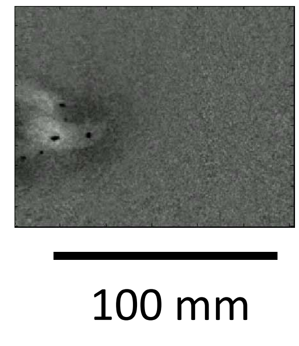
• Wave efficiency (η~0.5) is independent of ϕ

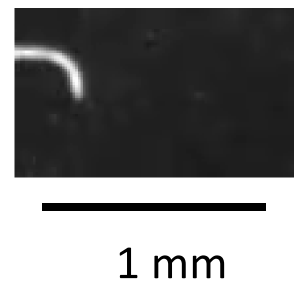
n=11 animals
mass=16.2 ± 4 g

Wave efficiencies of undulatory swimmers

(see Alexander, Vogel, Gray & Hancock, Lighthill, etc..)



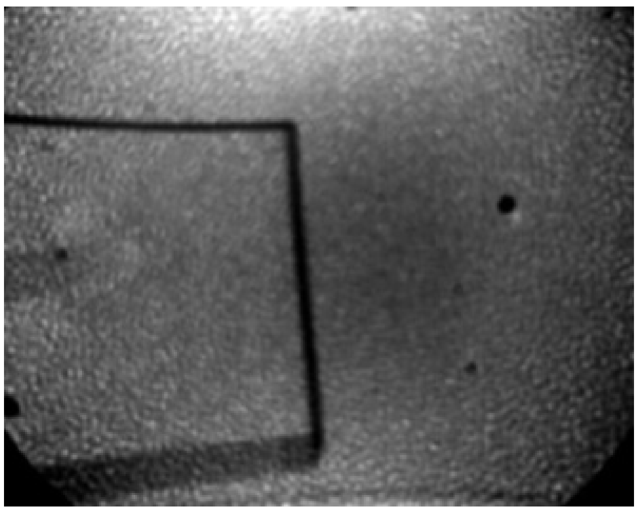




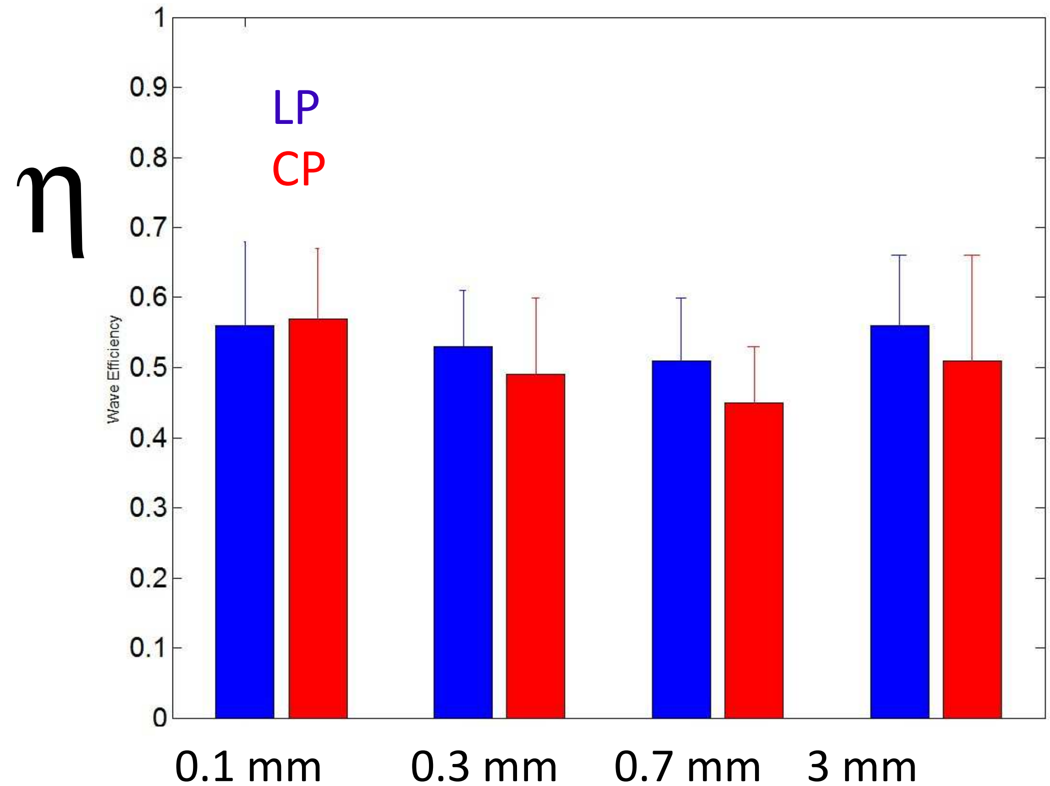
Sarah Steinmetz

Maladen, et. al (2009), Hu (2010), Jung(2010),Gray and Lissman (1964),Gray and Hancock (1955),Gillis(1996),Fish (1984)

Particle size has little effect on swimming



3 mm glass particles



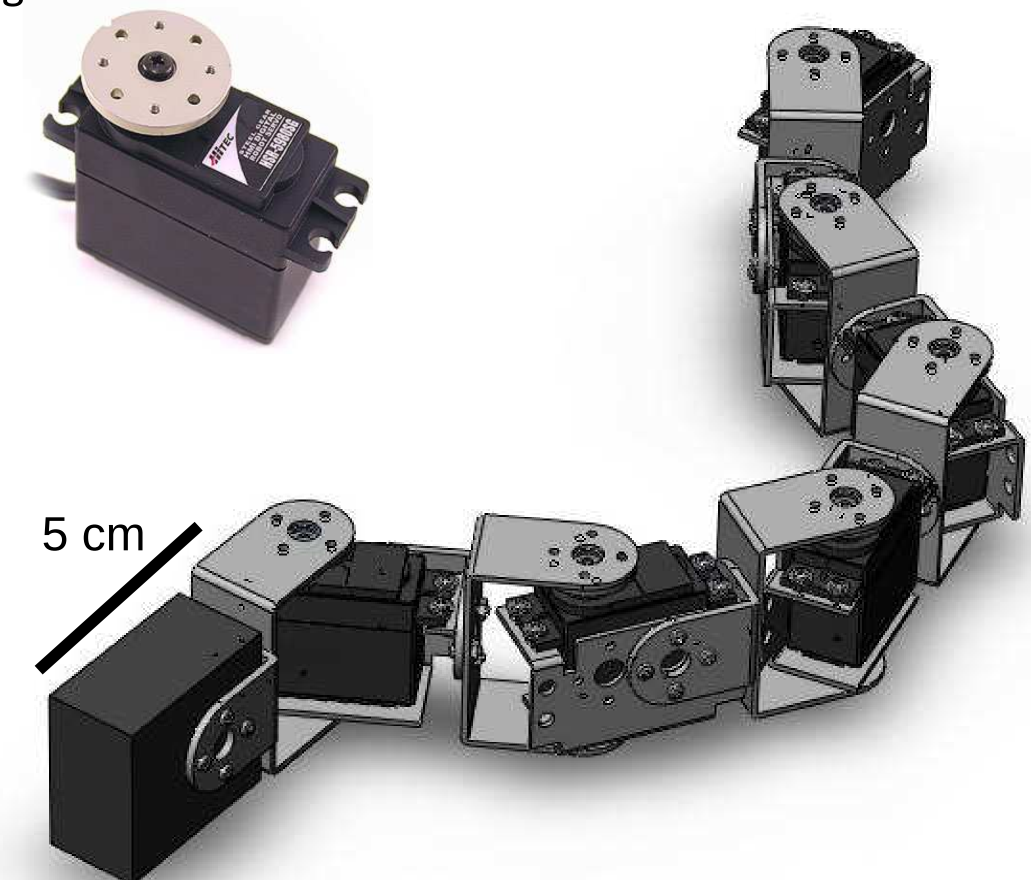
Glass beads with ±15% polydispersity

$A/\lambda \approx 0.2$, independent of particle size too…

…a template? (Full & Koditschek, JEB, 1999)

Sand swimming physical model design

HSR 5980SG
Digital standard servo



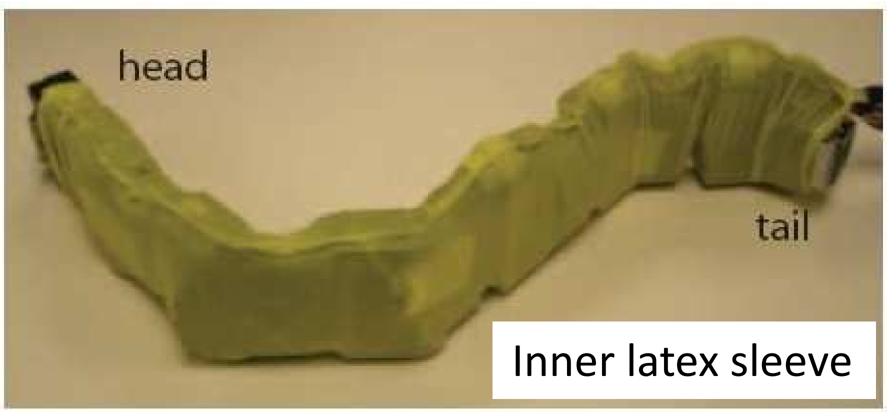
7 segment,
6 motor robot

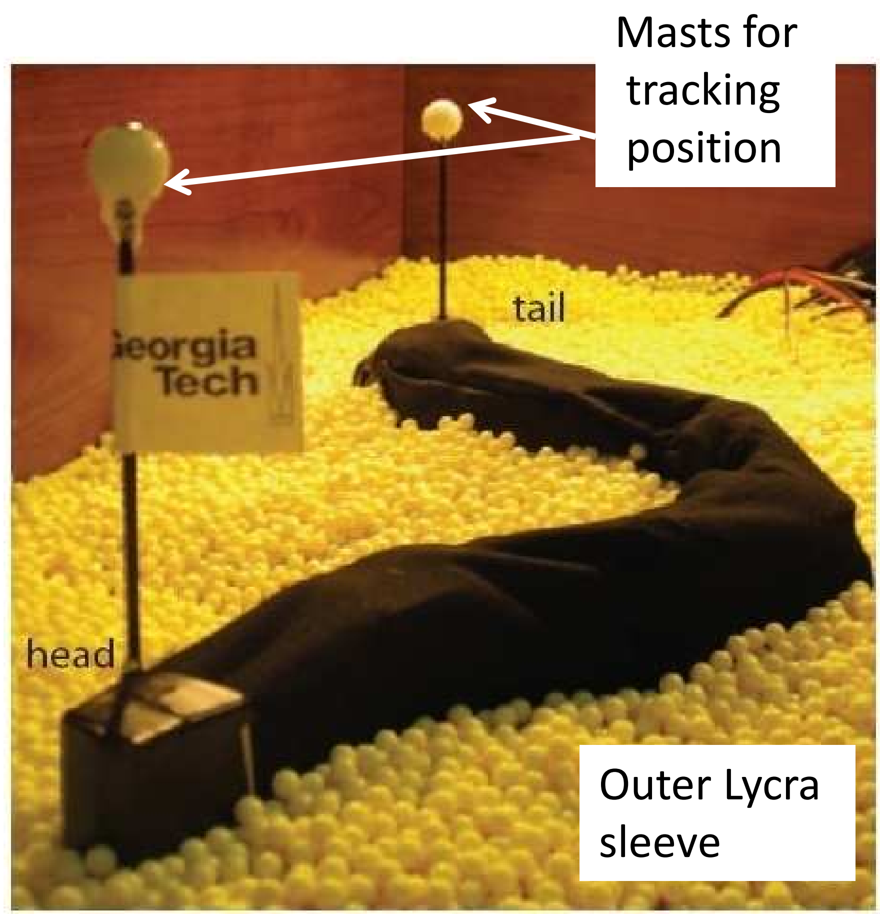
5.87 ± 0.06 mm diameter
plastic spheres,
particle density = 1 g/cm^3

Andrew Masse

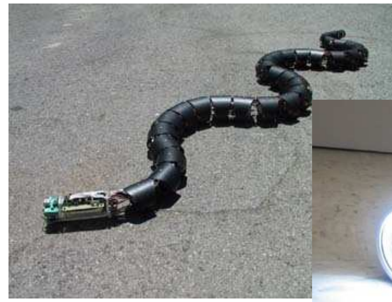
Maladen et al, J. Royal Society Interface, 2011
Maladen et al., Int. Journal of Robotics Research *(in press)*
Maladen et al, Proc. of Robotics Science and Systems (2010); Best Paper Award

Sand swimming robot design

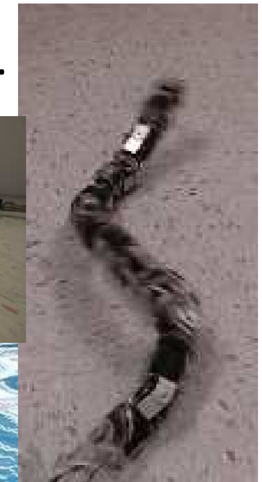




Limbless robots



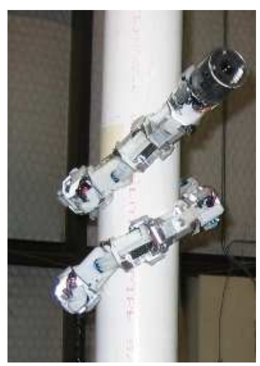
Choset et al.



Applications of these robots

Kuka snake arm

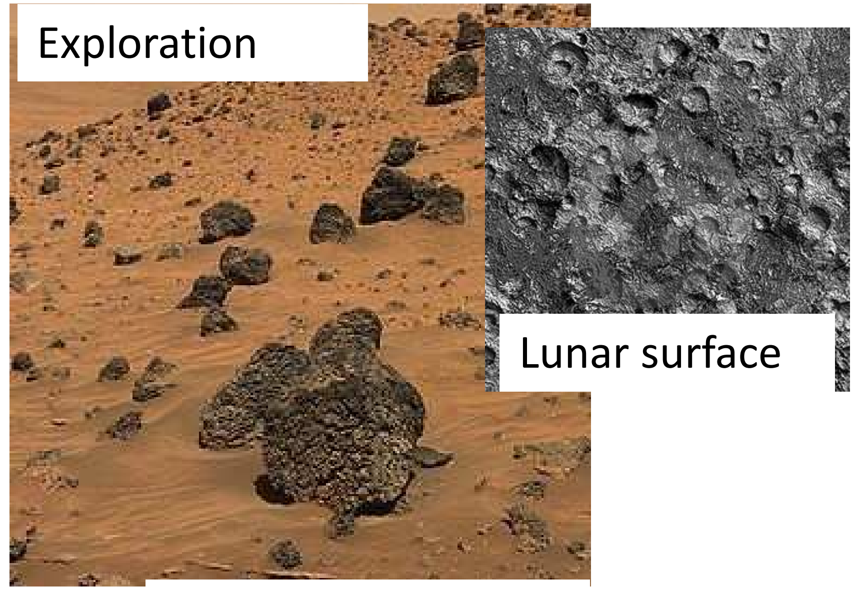
Surgery robot, JHU



Choset et al.

Choset et al.

Applications of granular swimmers



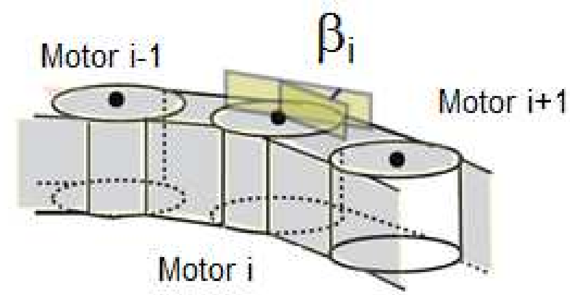
Martian sand

Control of the motors

Angle between adjacent segments modulated using:

Angular approximation of a sinusoidal traveling wave

$$\beta_i = \beta_0 \xi \sin(2\pi\xi i/6 - 2\pi f t)$$



$\beta(i,t)$ - motor angle of the i^{th} motor at time t, (i=1-6)

β_o - maximum angular amplitude, determines A/λ

ξ- number of wavelengths along the body (period)

f=undulation frequency

Swimming by the sandfish inspired robot

ξ=1,
A/λ=0.2
f=1 Hz

10 cm

Robot on the surface

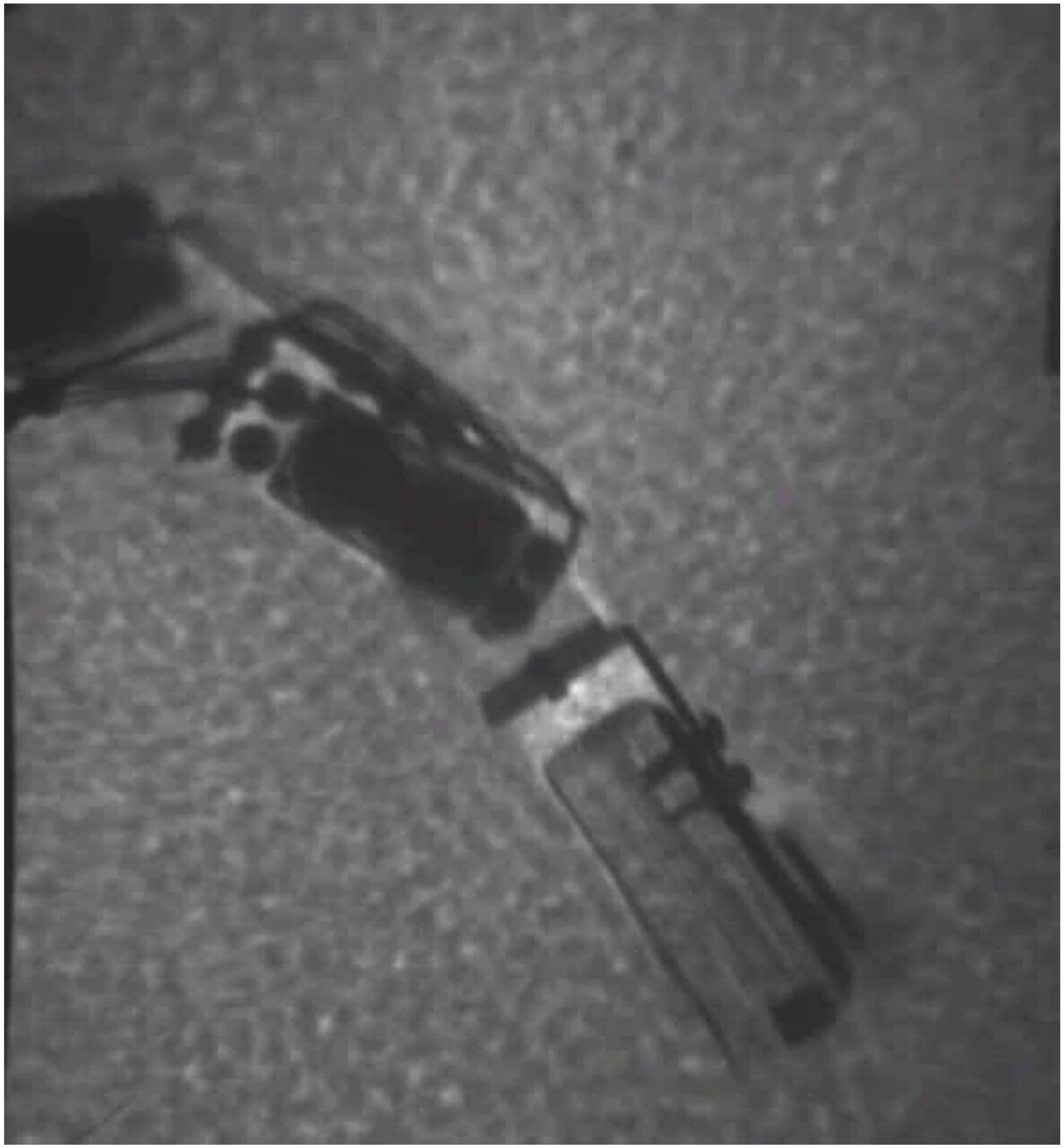
ξ=1,
A/λ=0.2
f=0.25 Hz

Submerge robot to a depth of 4 cm in closely packed bed

Robot sub-surface

Real time

Robot swimming subsurface: x-ray video



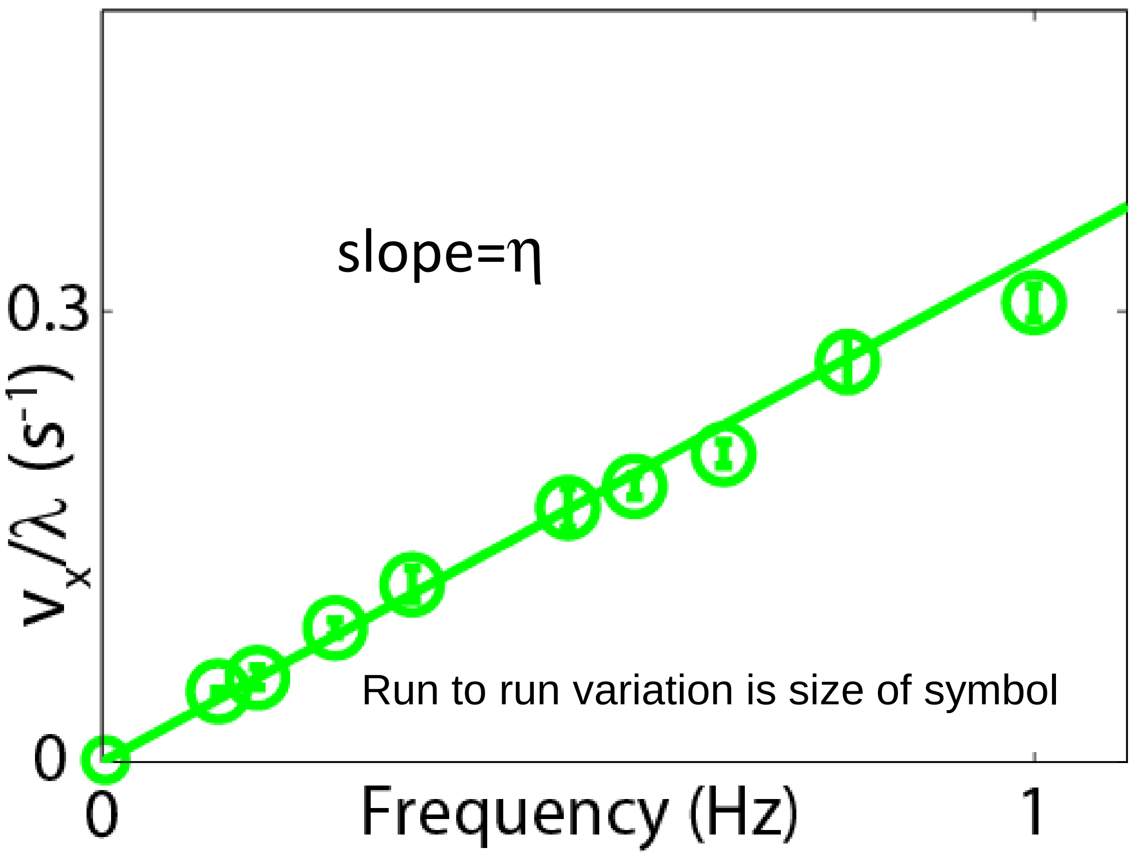
Buried 4 cm deep.

ξ=1,
A/λ=0.2
f=0.25 Hz

5 cm

Comparison of robot model and sandfish

Set A/λ = 0.2, ξ=1 (from animal experiment)

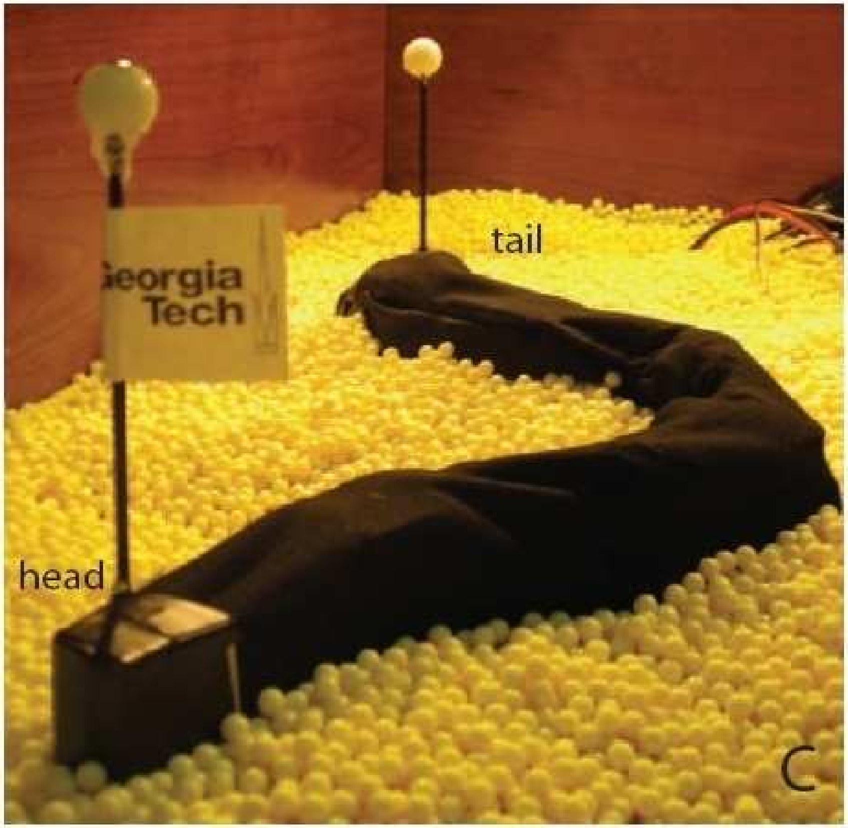


- v_x increases linearly with f (like sandfish)

$$\eta = \frac{v_x}{v_w} = \frac{v_x}{f\lambda} = \frac{v_x/\lambda}{f}$$

- $\eta = 0.33 \pm 0.03$ (unlike sandfish $\eta \approx 0.5$)

Why is the performance different ?

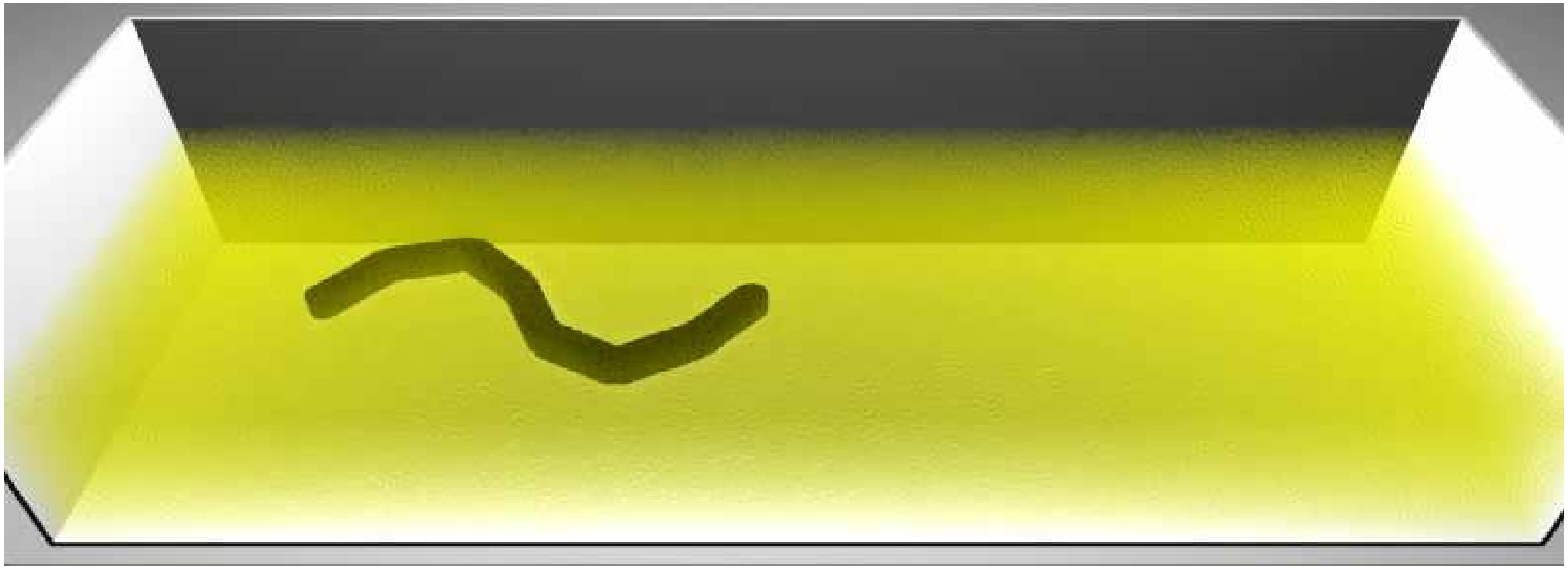


Some potential reasons:
Scaling, smoothness, friction, body morphology, GM properties…

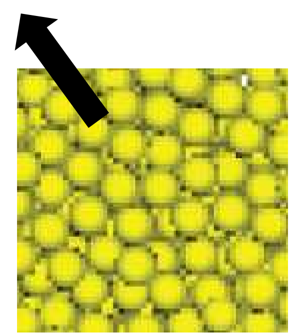
Need insight into locomotor-medium interaction at particle level and a tool that we can vary the above

Sand swimming robot simulation

10 cm



Box dimensions: : 108cm x 40cm x 15cm
Number of particles:3 x 10^5
Particle size : 0.6cm



6mm spherical “plastic particles”

Maladen et al., J. Roy Soc. Interface *2011*
Maladen et al., Int. Journal of Robotics Research
Maladen et al, Proc. of Robotics Science and Systems (2010) : Best Paper Award

Part I: Simulating and validating media

Discrete Element Method (DEM) simulation

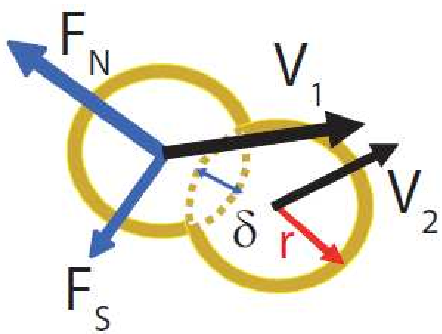
(e.g., see book by Rapaport)

3 parameter collision contact model:

normal: elastic & dissipative

+

tangential: friction



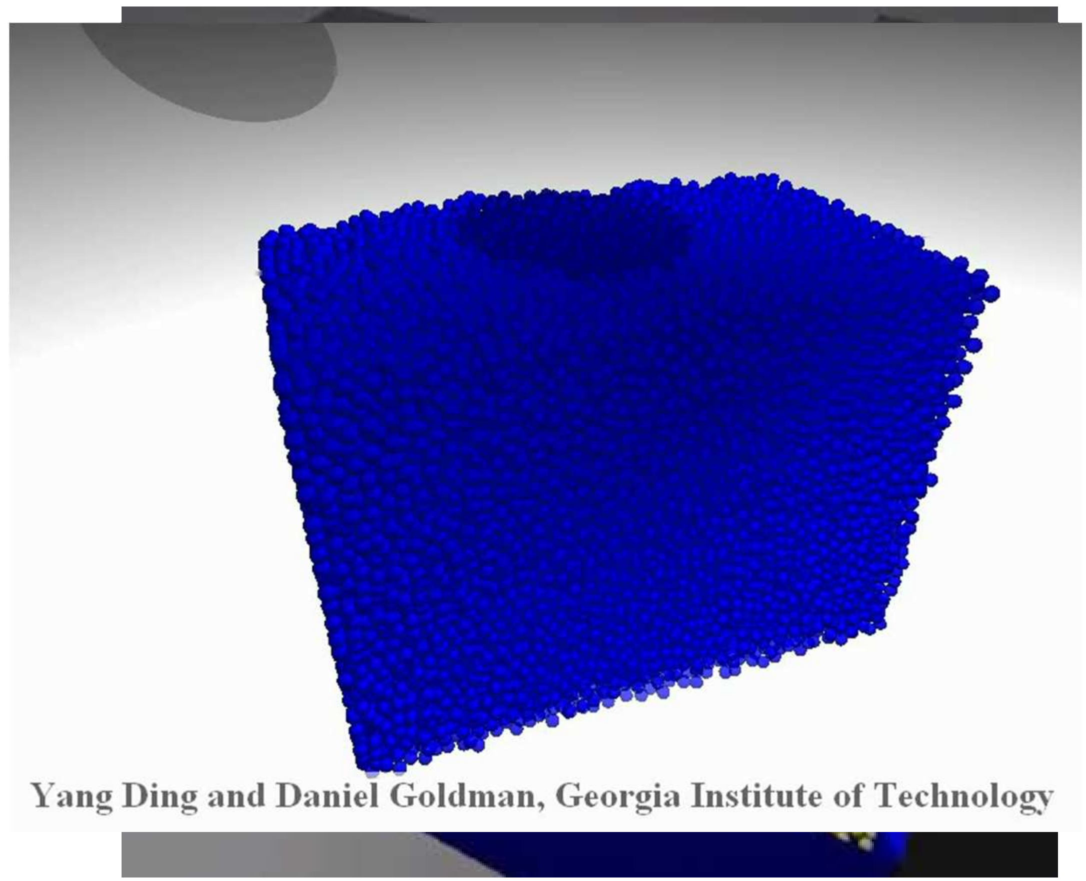
$$F_n = k\delta^{3/2} - G_n v_n \delta^{1/2}$$

$$F_s = \mu F_n$$

$k = 2 \times 10^5$ kg s^{-2} $m^{-1/2}$

G_n = 5 kg s^{-1} $m^{-1/2}$

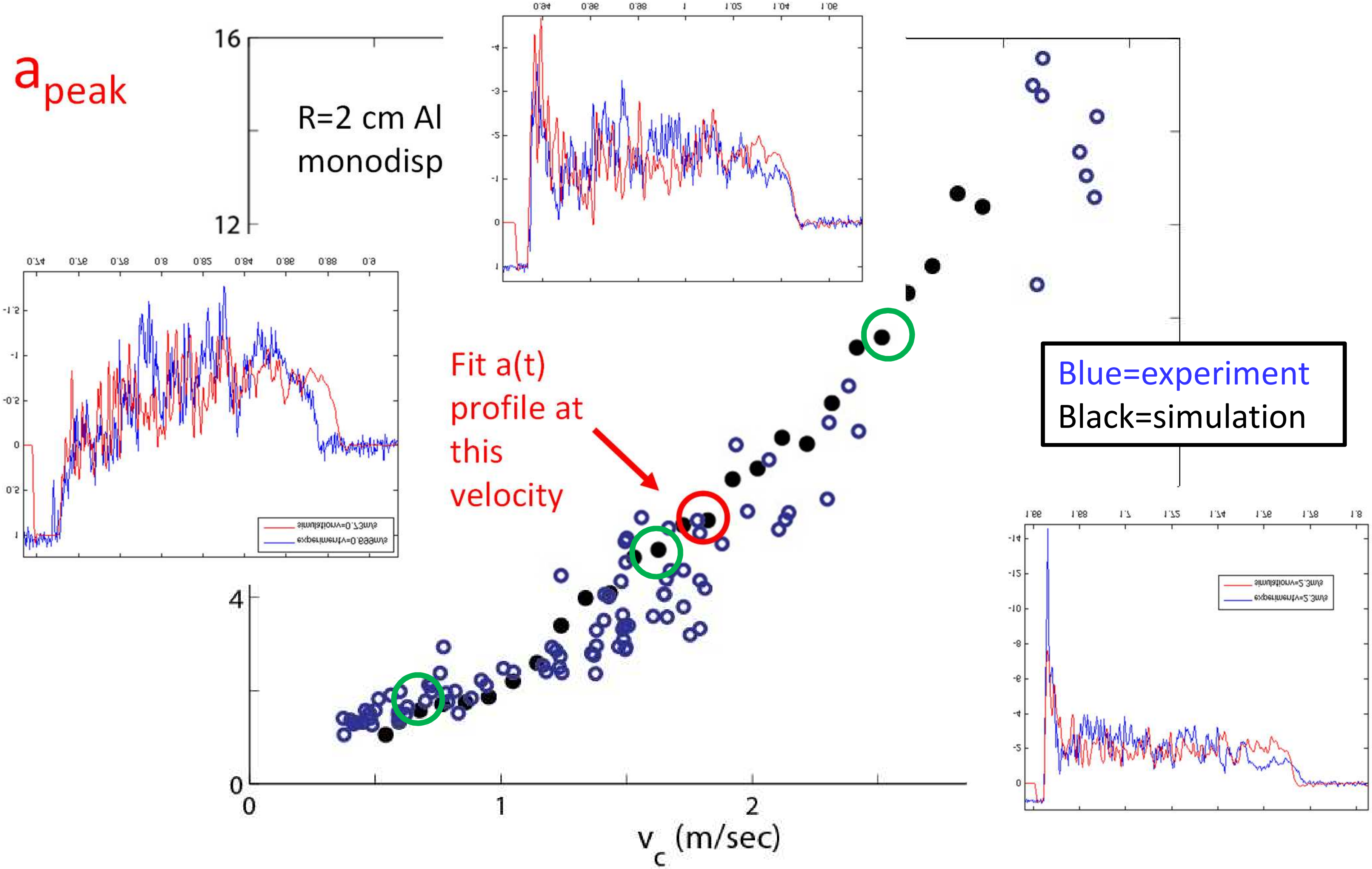
μ_{pp} = 0.1



50:50 mix of 5.81, 5.93 mm "plastic" spheres, particle density = 1 g/cm^3

0^5 particles

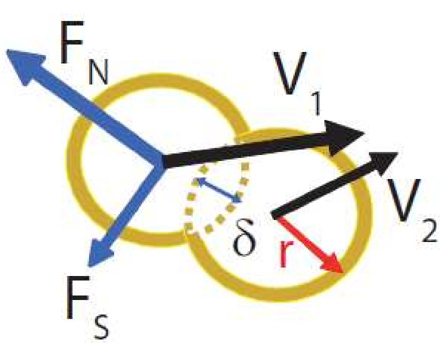
DEM simulation has predictive power



Parameters

6 mm plastic particles:

	Experiment	Simulation
Hardness (k)	1.7×10^8 kg s^{-2} m$^{-1/2}$	2×10^5 kg s^{-2} m$^{-1/2}$
Restitution coefficient	0.96	0.88
G_n	1×10^2 kg m$^{-1/2}$ s^{-1}	5 kg m$^{-1/2}$ s^{-1}
$\mu_{particle-particle}$ (μ_{pp})	0.073	0.080
$\mu_{body-particle}$ (μ_{bp})	0.27	0.27
Density	1.03 ± 0.04 g cm^{-3}	1.06 g cm^{-3}
Diameter	5.87 ± 0.06 mm	5.81 mm (50%) and 5.93 mm (50%)
Granular volume	188 PD $\times$ 62 PD $\times$ 35 PD	188 PD $\times$ 62 PD $\times$ 24 PD



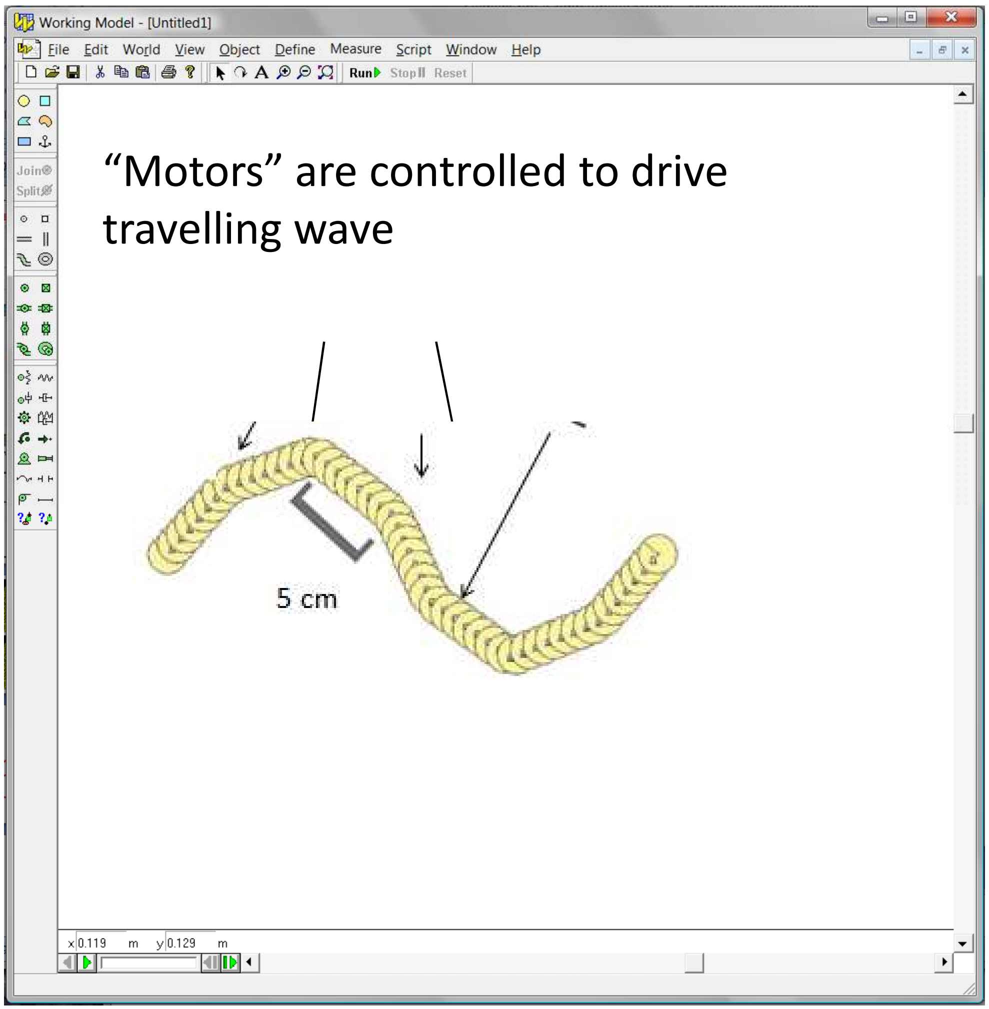
$$F_n = k\delta^{3/2} - G_n v_n \delta^{1/2}$$

$$F_s = \mu F_n$$

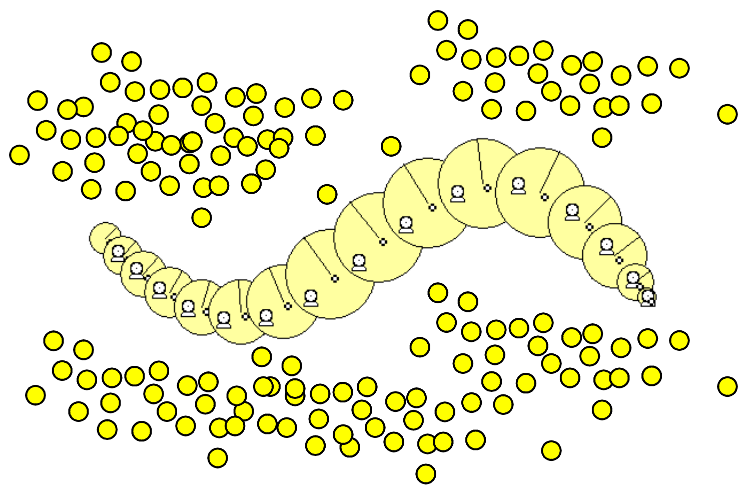
Integrated numerical simulation

Maladen, Ding, Kamor, Umbanhowar, Goldman, in prep, 2010

Multi-body solver + DEM simulation

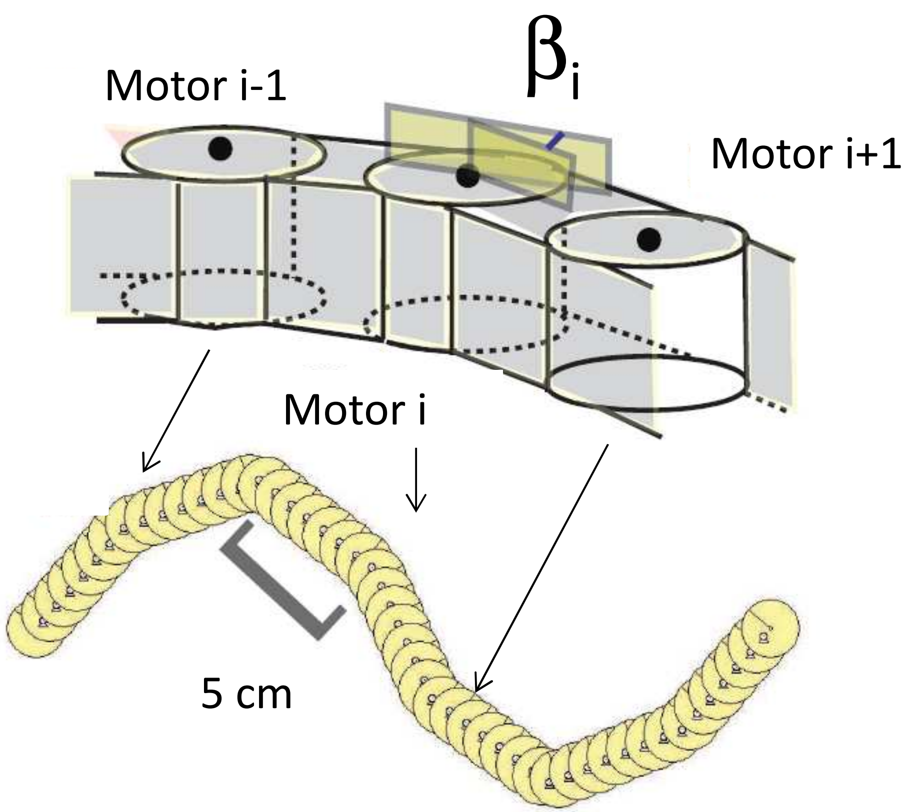


DEM code computes forces from segment collisions with grains and grain/grain collision



Part II: Simulating the robot

Multi-body simulator Working model (**WM**) 2D



Angular approximation of sinusoidal traveling wave

$$\beta_i = \beta_0 \xi \sin(2\pi\xi i/6 - 2\pi f t)$$

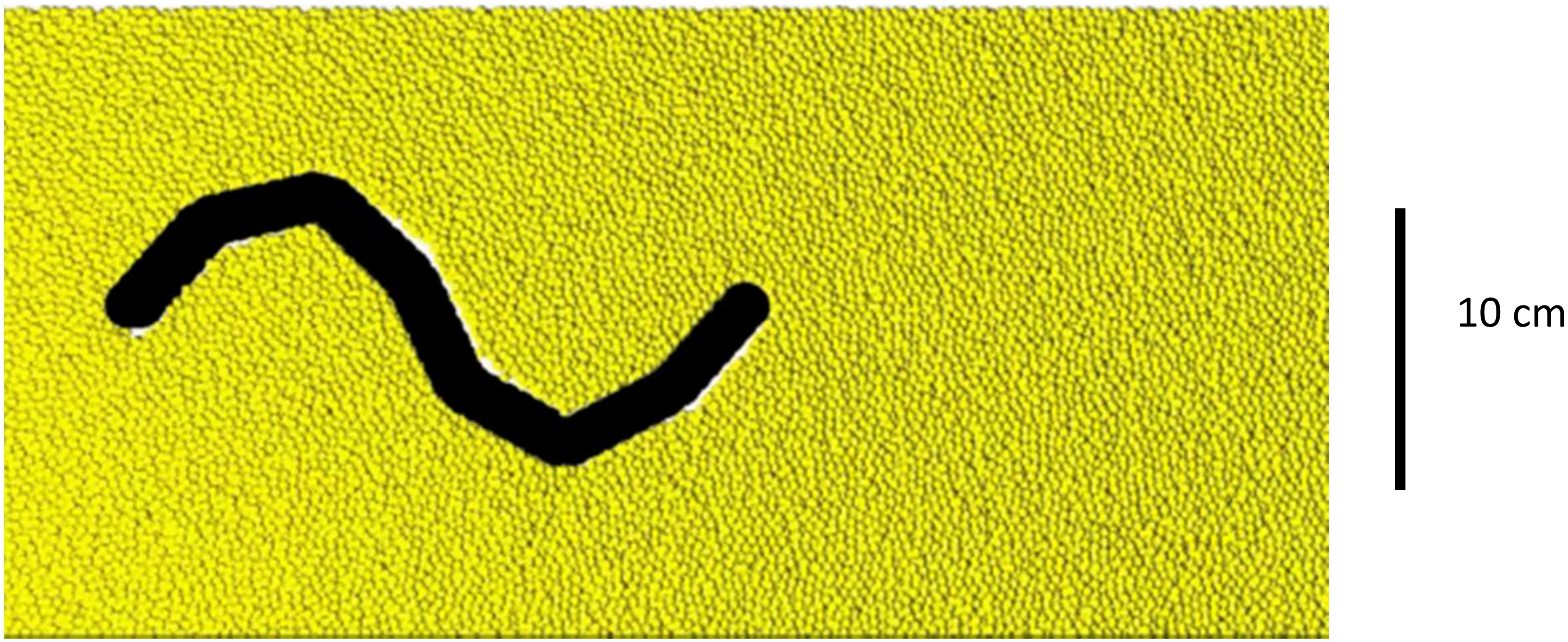
(like in experiment)

Lycra skin – particle friction estimated experimentally $\mu_{particle-robot}$: 0.27

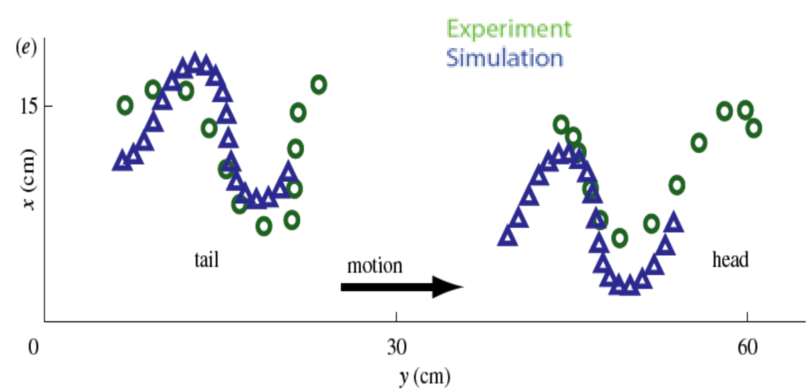
Maladen et al., J. Roy Soc. Interface, 2011

Integrating WM with DEM simulation

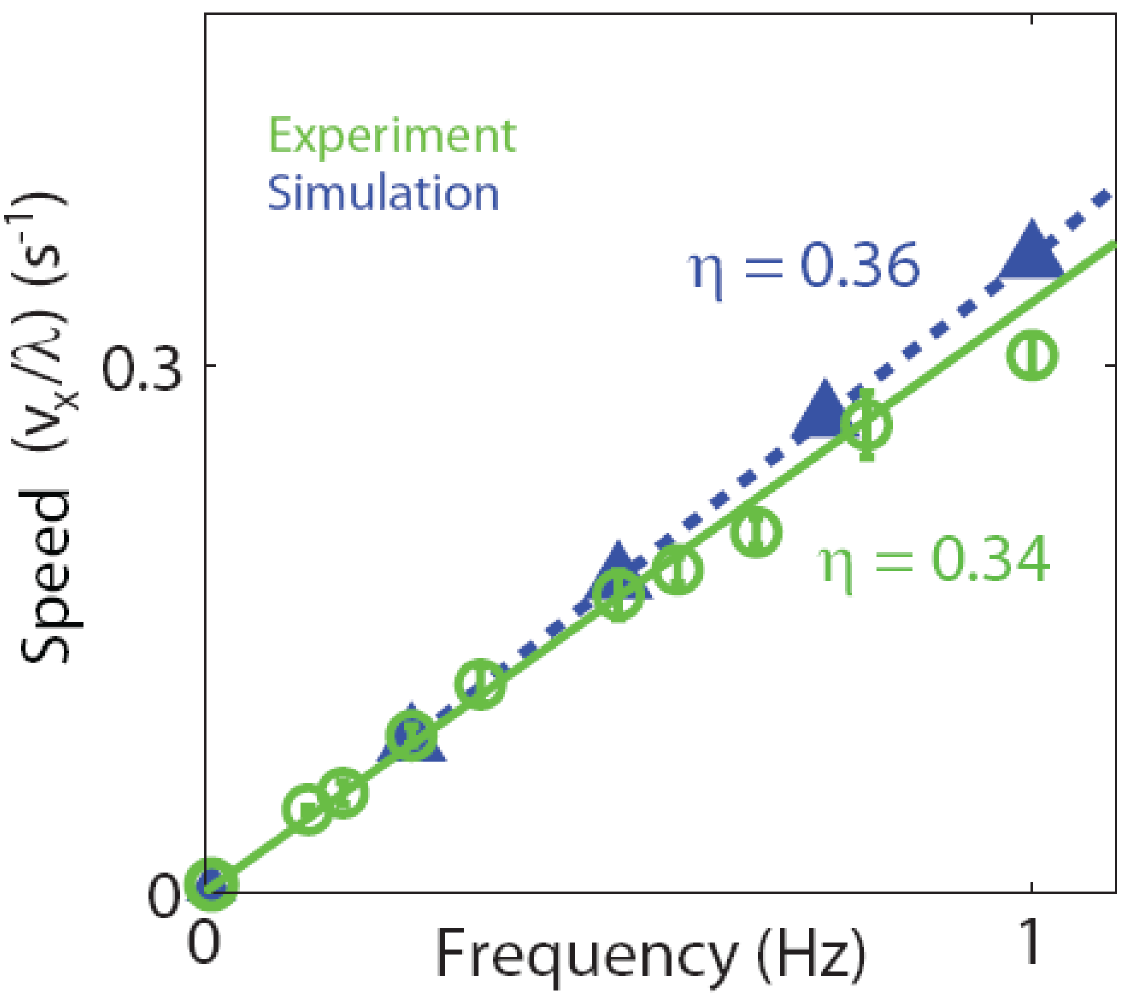
Particles above the robot rendered transparent



Box dimensions: 108cm x 40cm x 15cm
Number of particles: 3e5
Particle size : 0.6cm

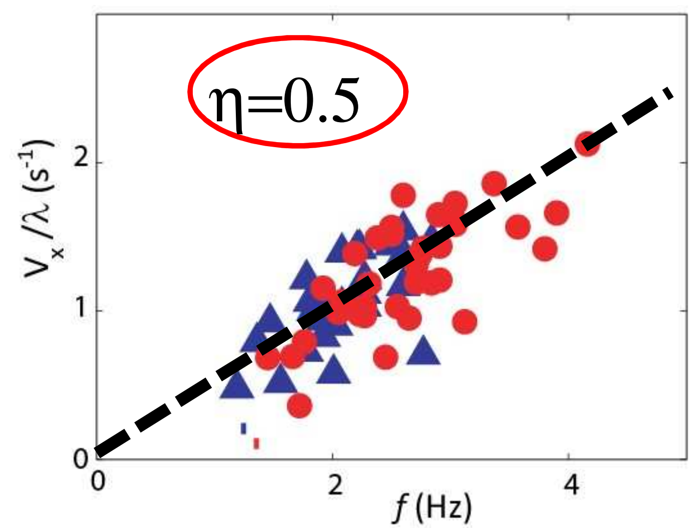


Simulated robot vs. physical robot



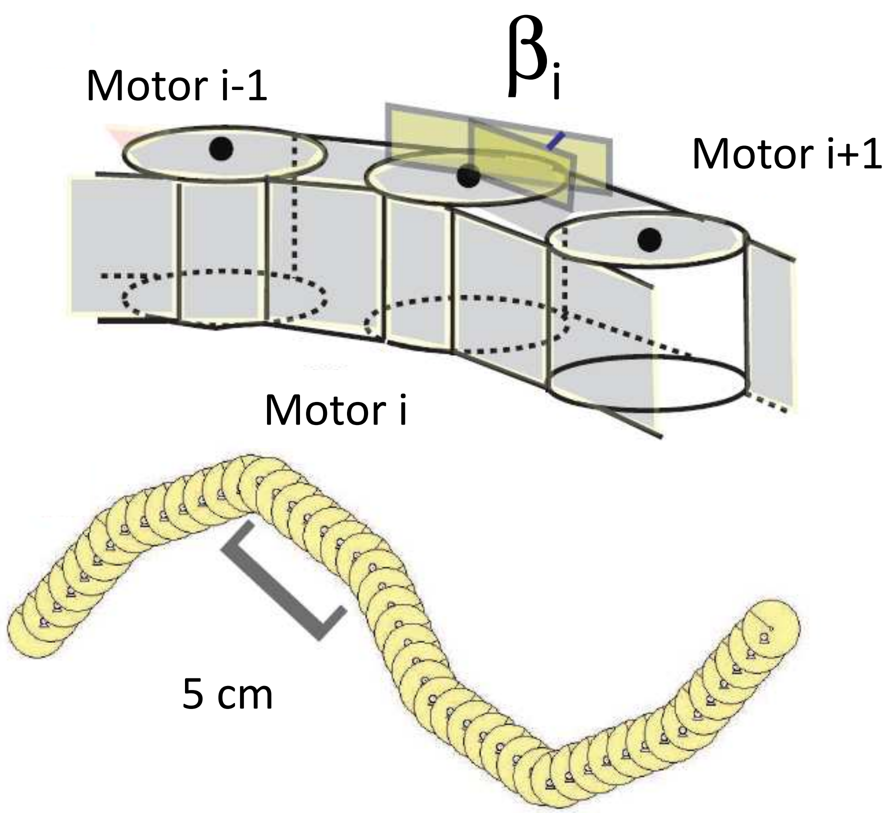
Simulated and physical robot swimming speeds agree!

vs. sandfish

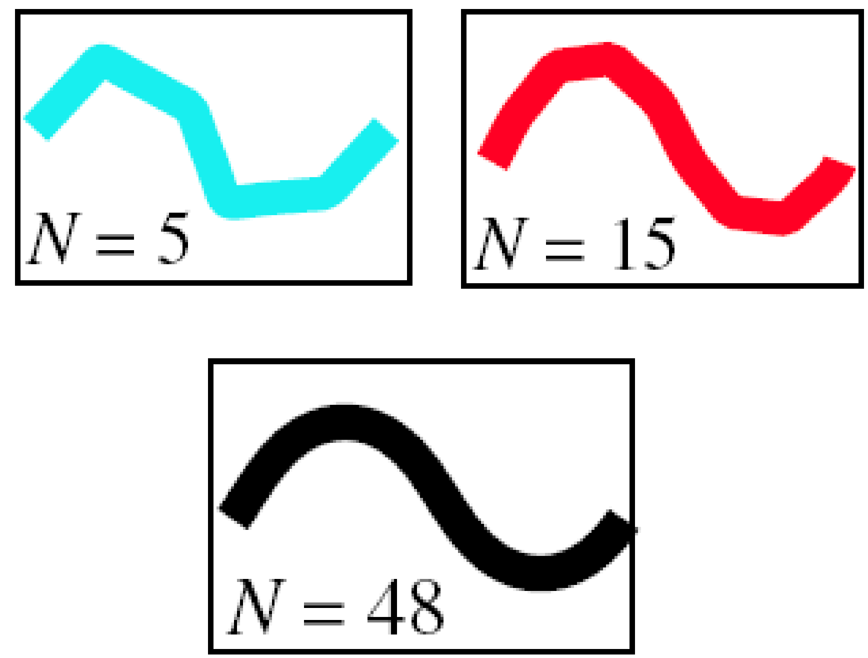


Changing smoothness of wave

activate different numbers of motors

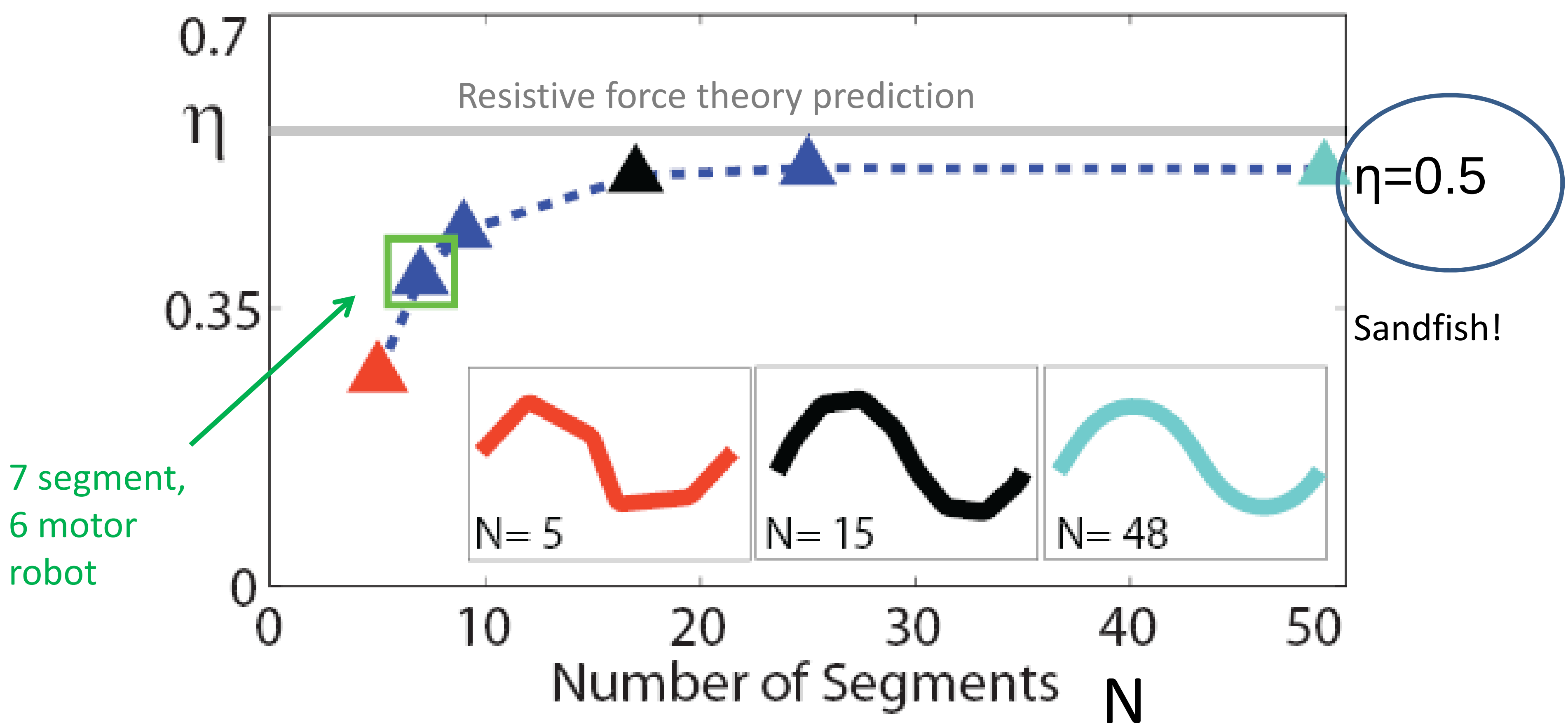


$$\beta_i = \beta_0 \xi \sin(2\pi \xi i / N + 2\pi f t)$$



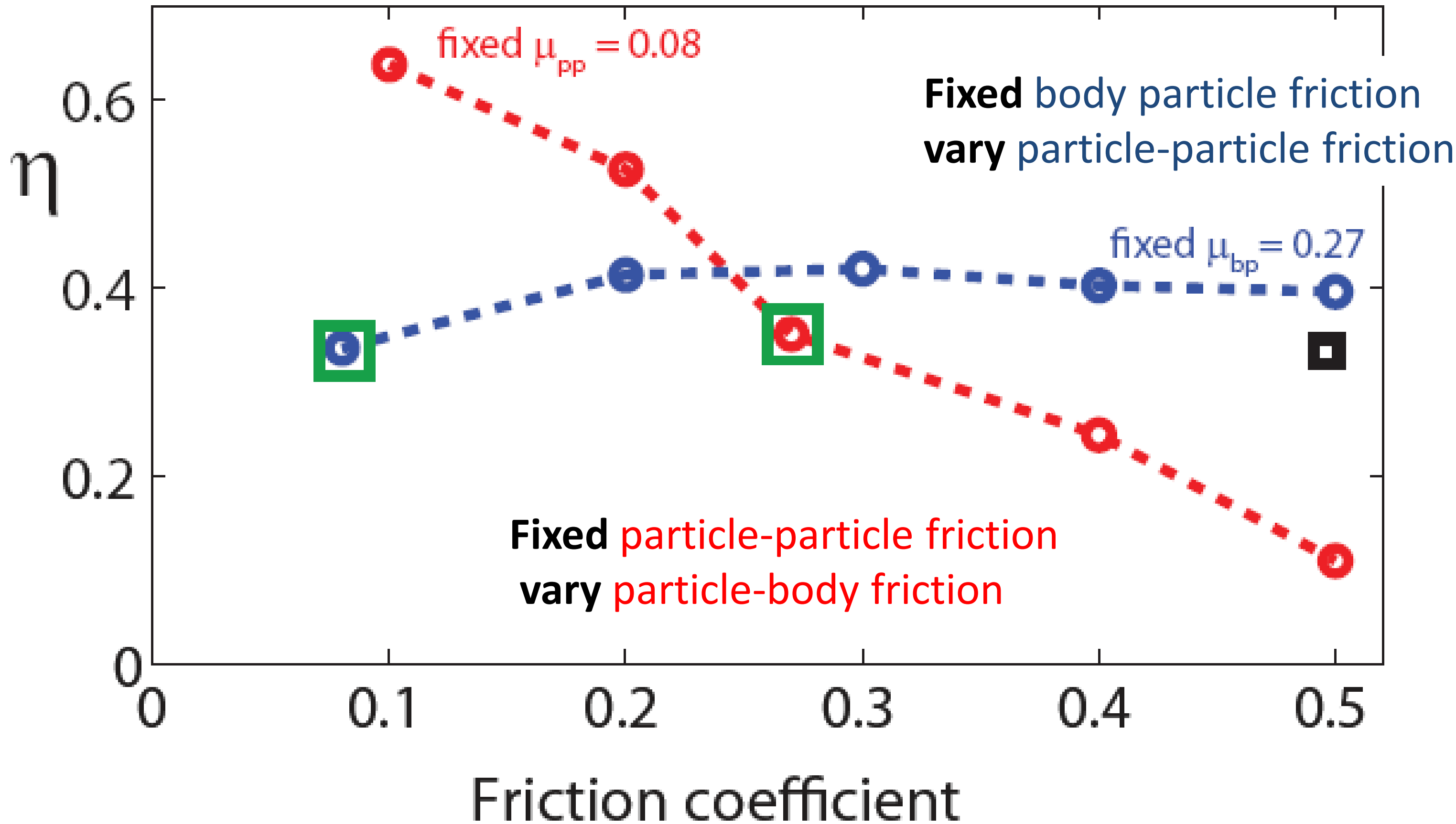
Maladen et al., J. Roy Soc. Interface, 2011

Wave efficiency vs # of segments



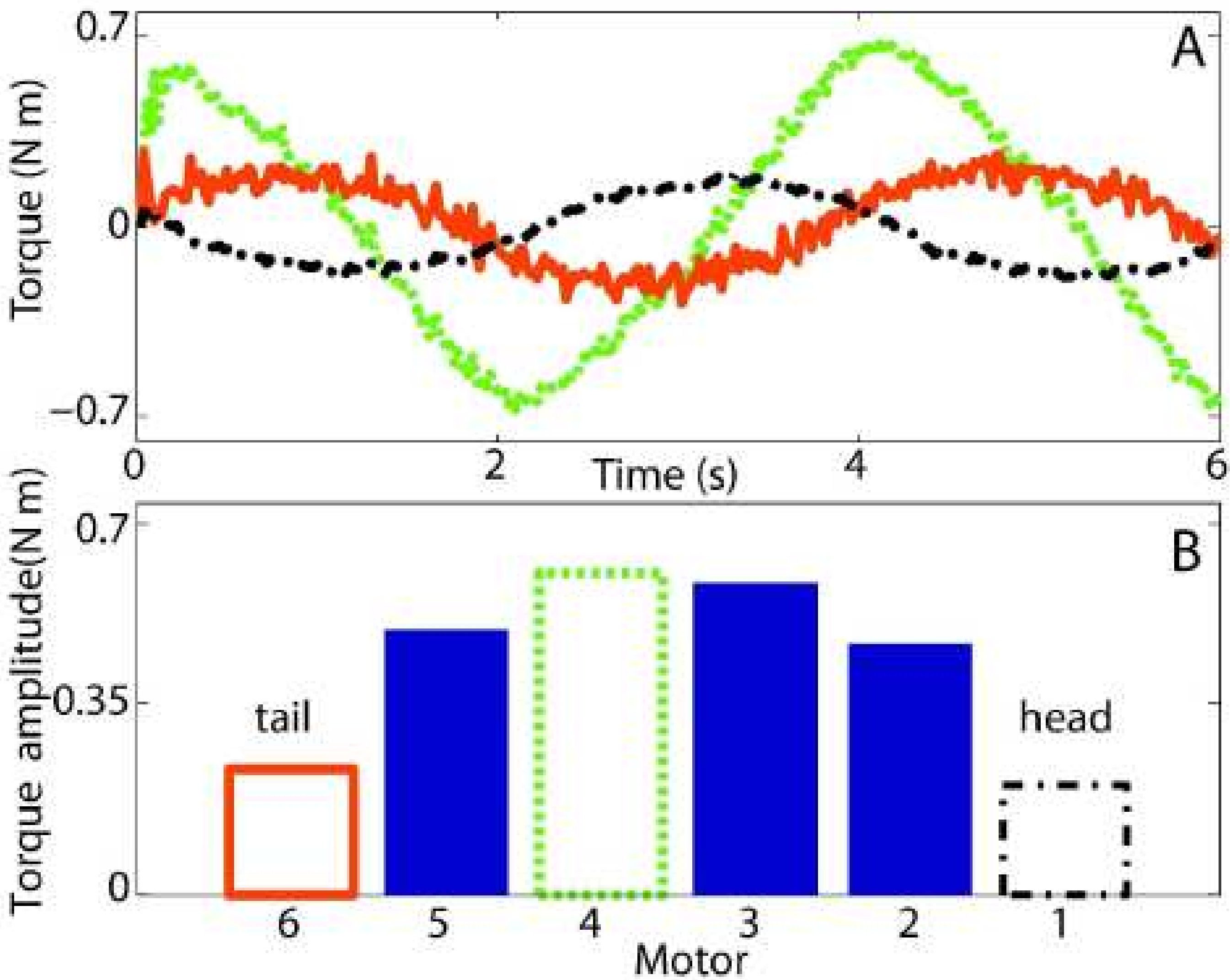
Changing friction

7 segment, 6 motor robot

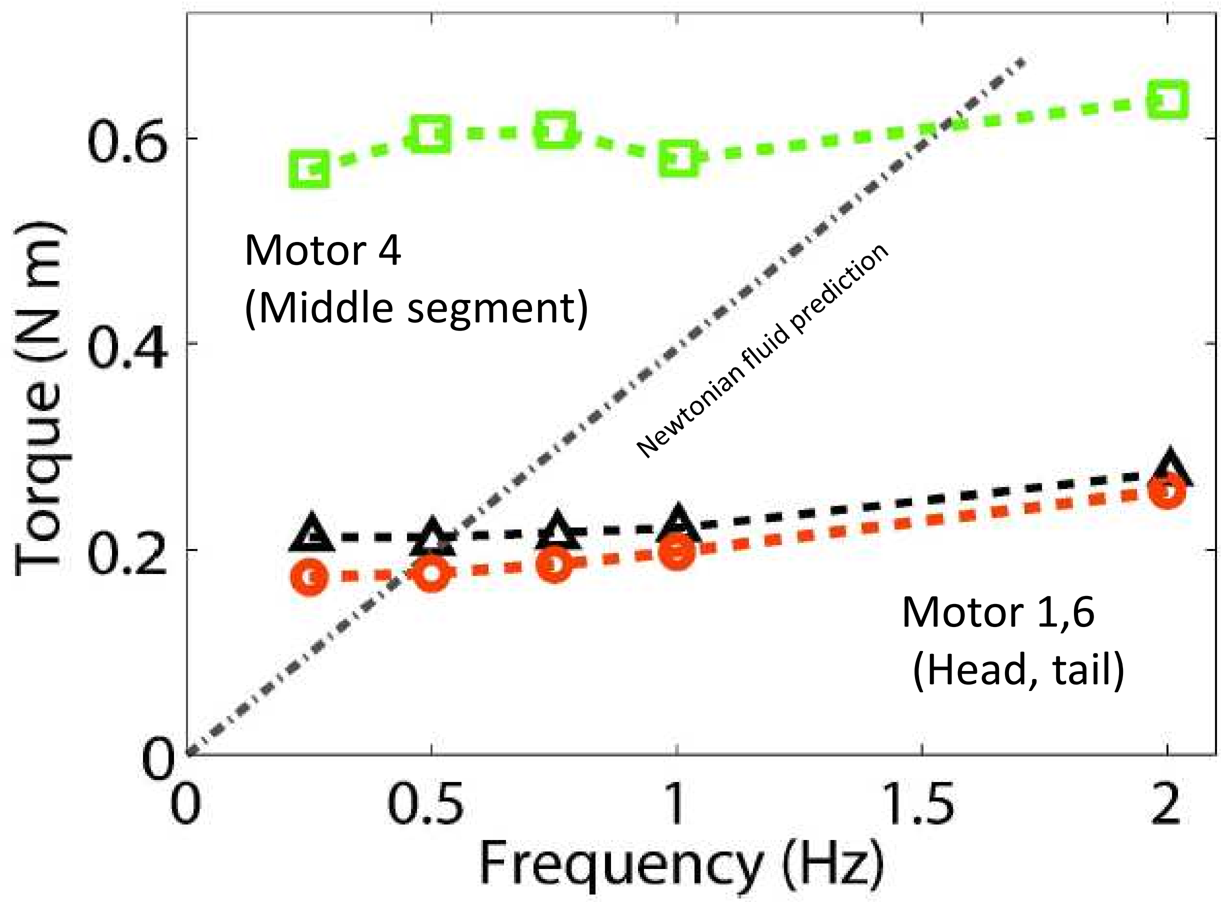


Body-particle friction dominates

Motor torque vs. time



Motor torque vs. frequency

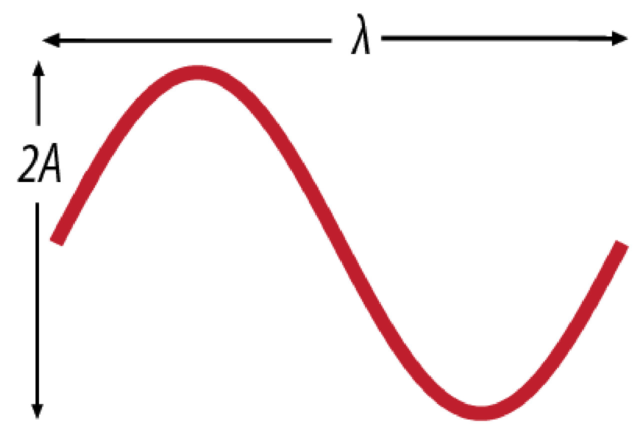


Swims in a "frictional fluid"—friction dominates all forces

7 segment, 6 motor robot

Use physical model to test for template

Hypothesis: Sandfish kinematics are adapted to rapidly swim within sand → sinusoidal wave of A/λ=0.2 is a template for this behavior

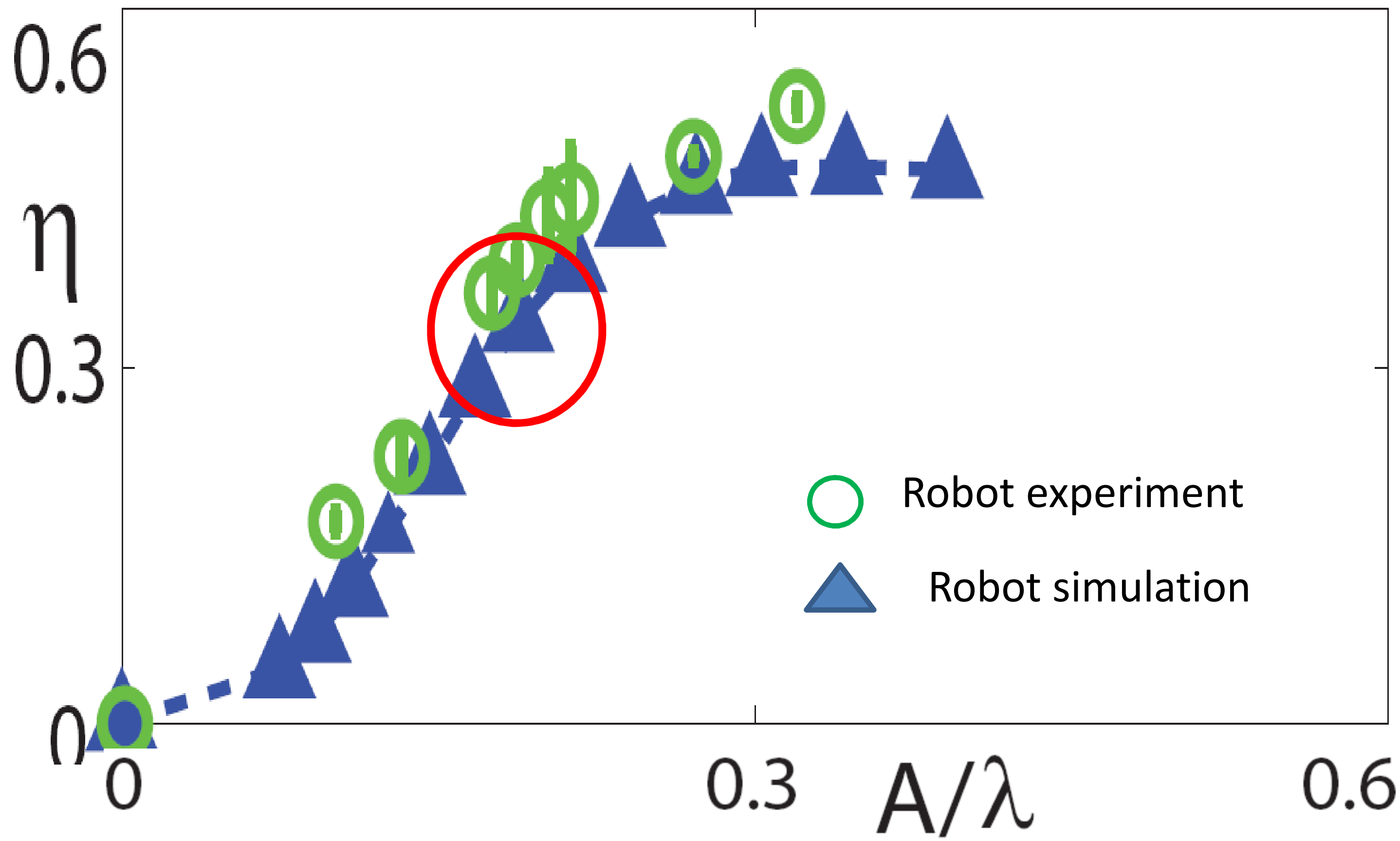


A/λ ≈ 0.2, single period

Test effect of A/λ on performance

Vary sand swimming kinematics

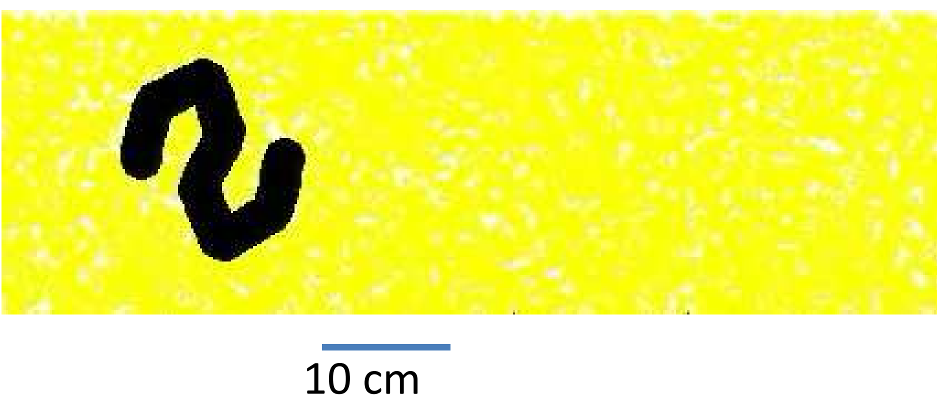
Vary A/λ for a single period wave



Vary sand swimming kinematics

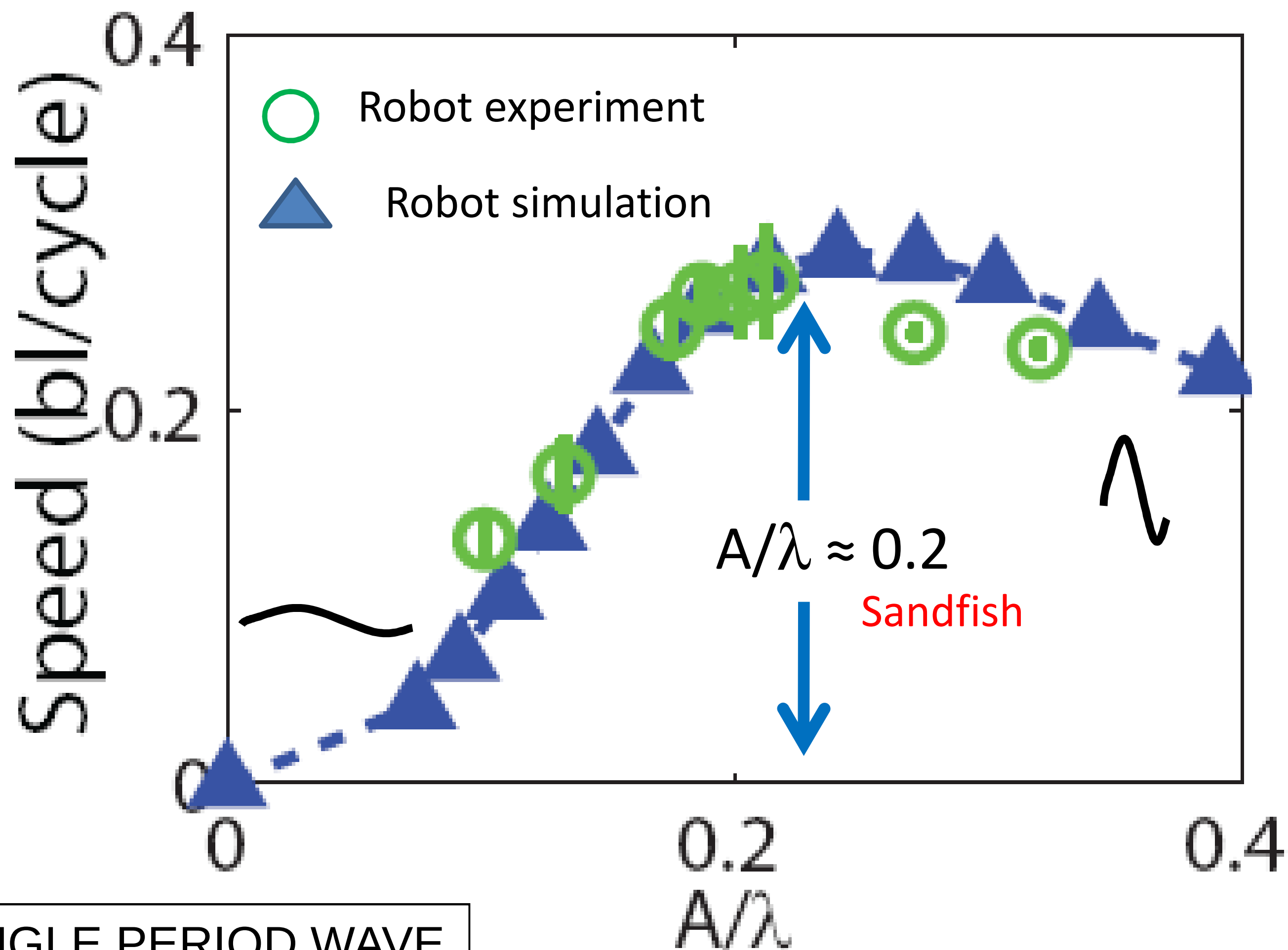
$A/\lambda = 0.05$
λ –High / η-Low

$A/\lambda = 0.55$
λ –low / η- High



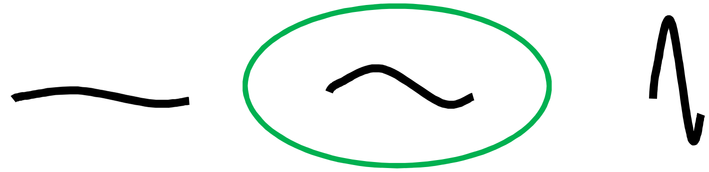
Highest performance gait→ robot advances most body-lengths per cycle

Maximum performance of the physical model



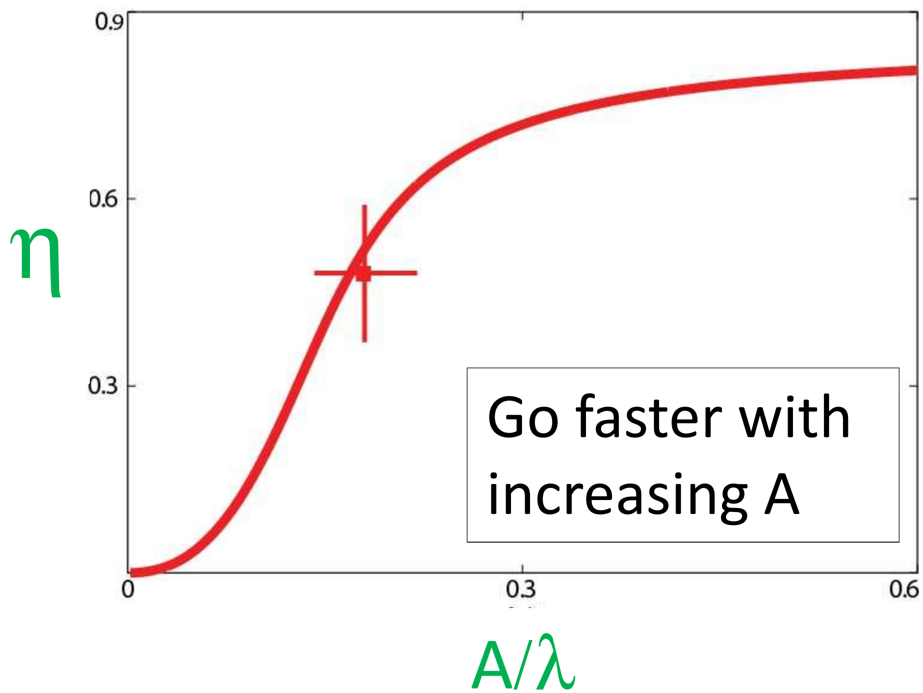
SINGLE PERIOD WAVE

Competition of effects leads to maximum

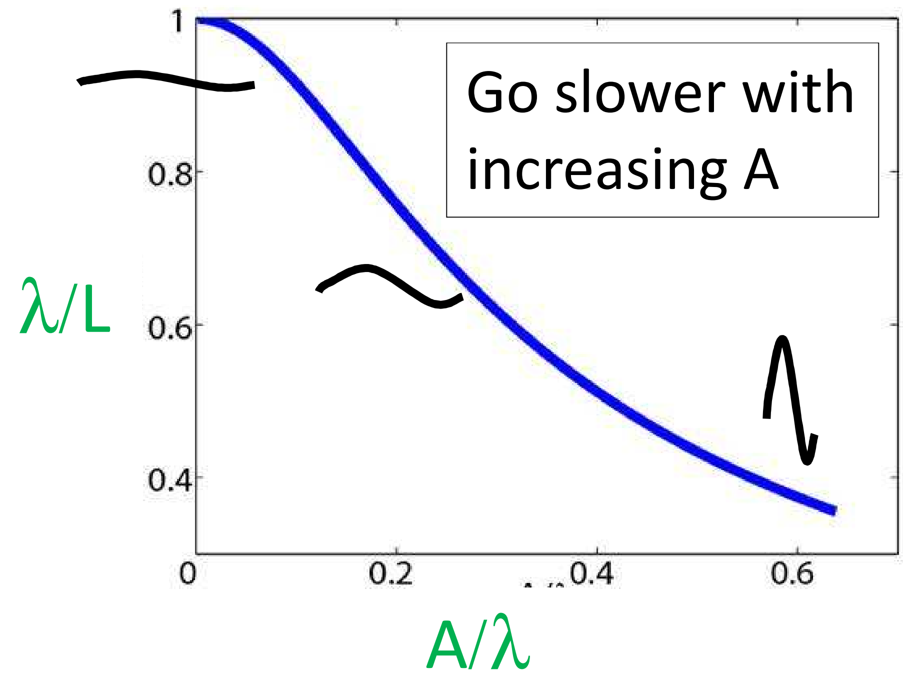


Body lengths/cycle= $\frac{v_x}{fL} = \frac{\eta \lambda f}{fL} = \eta \times \frac{\lambda}{L}$

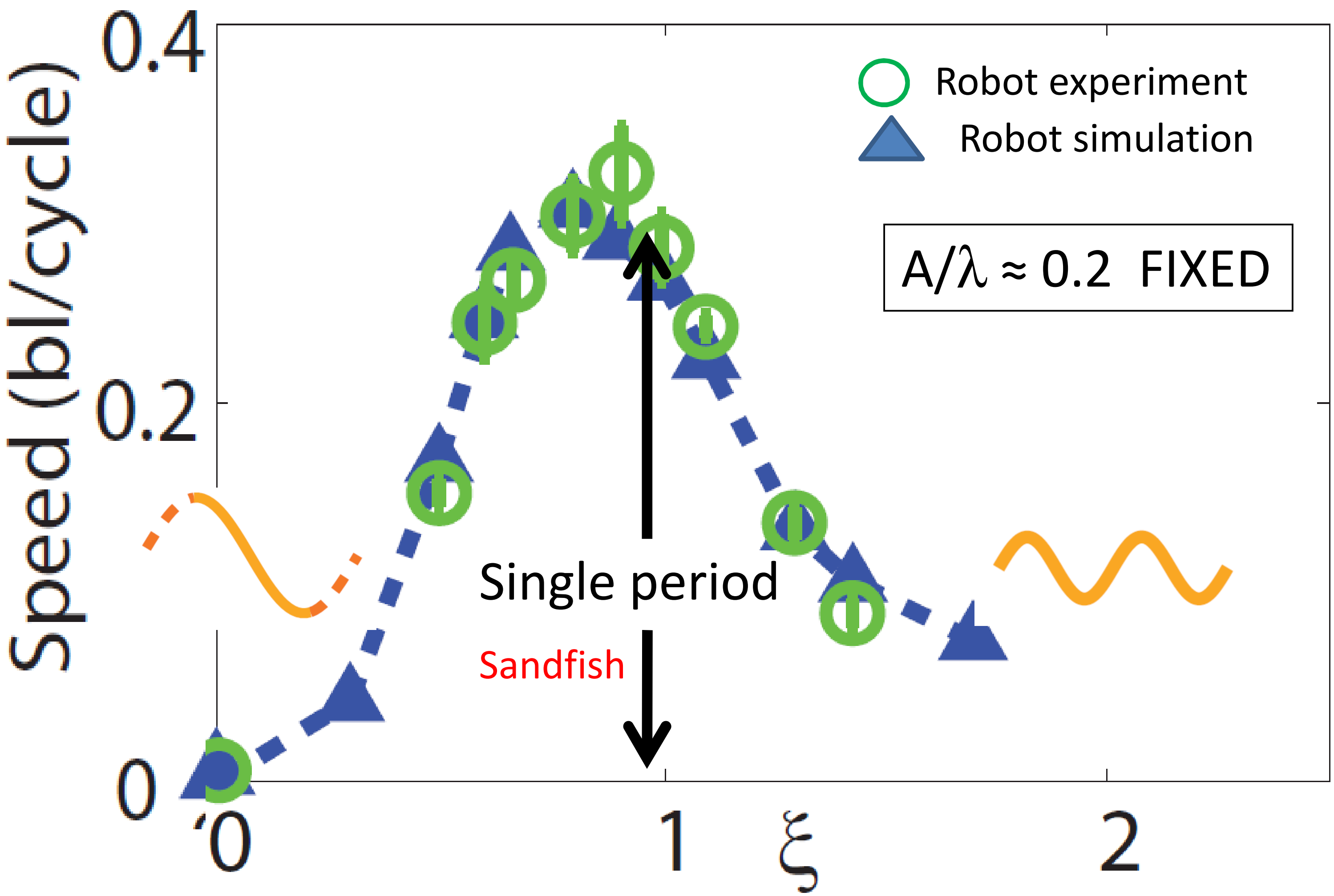
MEDIUM EFFECT



GEOMETRY EFFECT



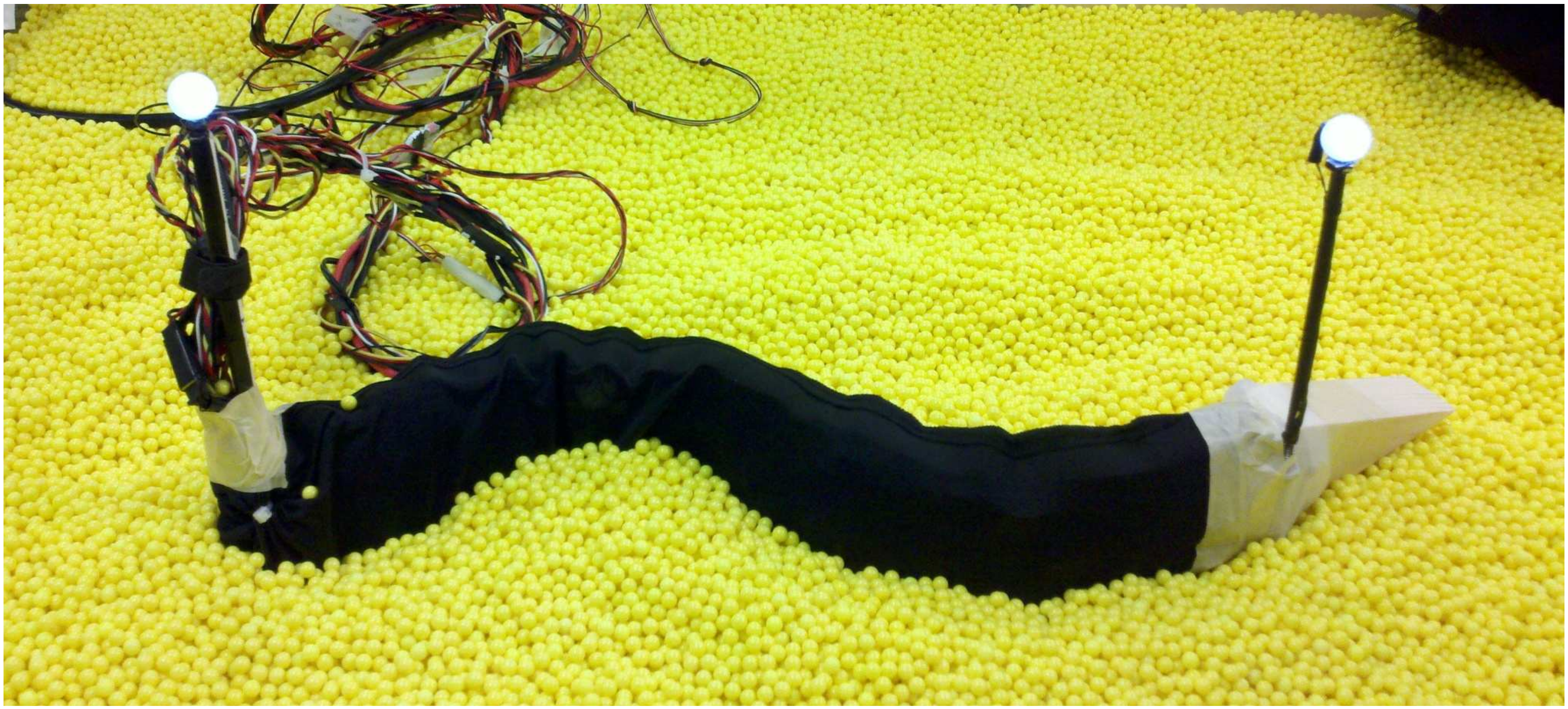
Varying the number of periods, ξ



Sandfish kinematics maximize robot speed

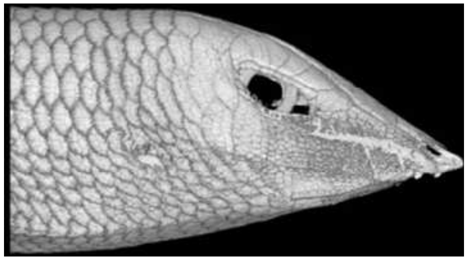
Vertical control surface?

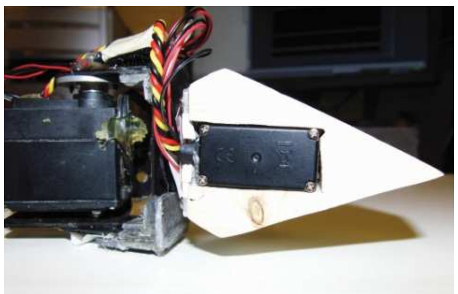
Robot with tiltable head and masts for subsurface tracking



Andrew Masse

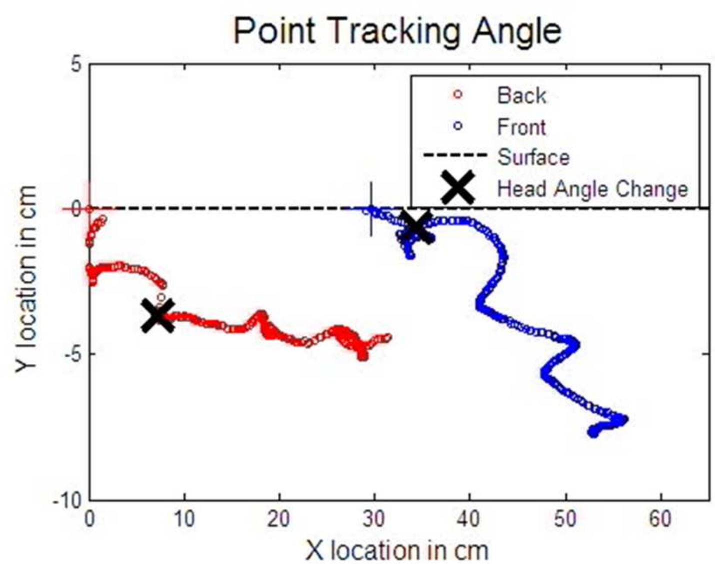
Active head to control vertical position





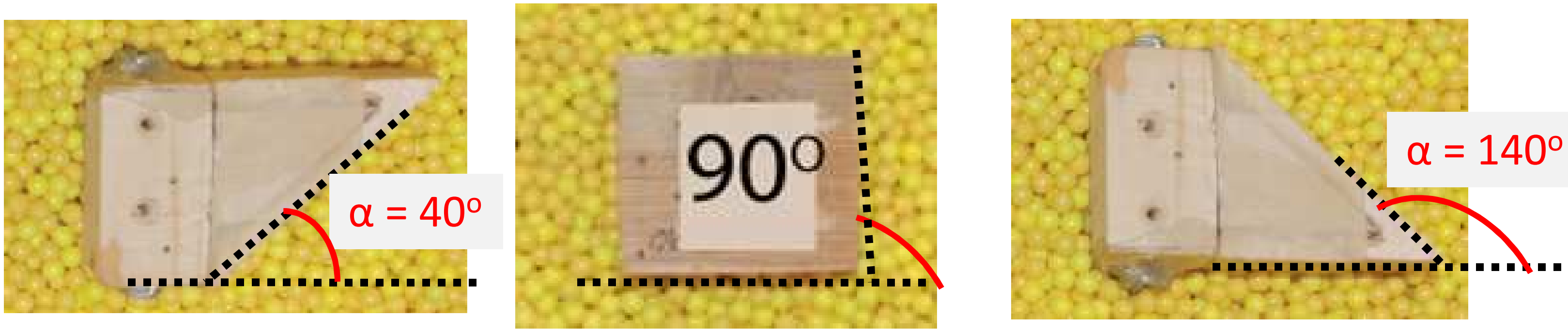
- Pitch control of wedge-shaped head (-30° to 30°) using a single servo-motor

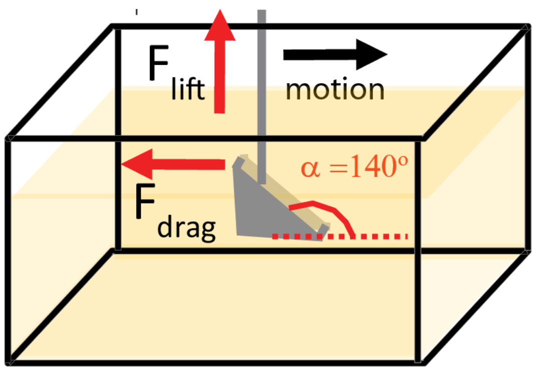
- Embedded tilt sensor: accelerometer & gyro

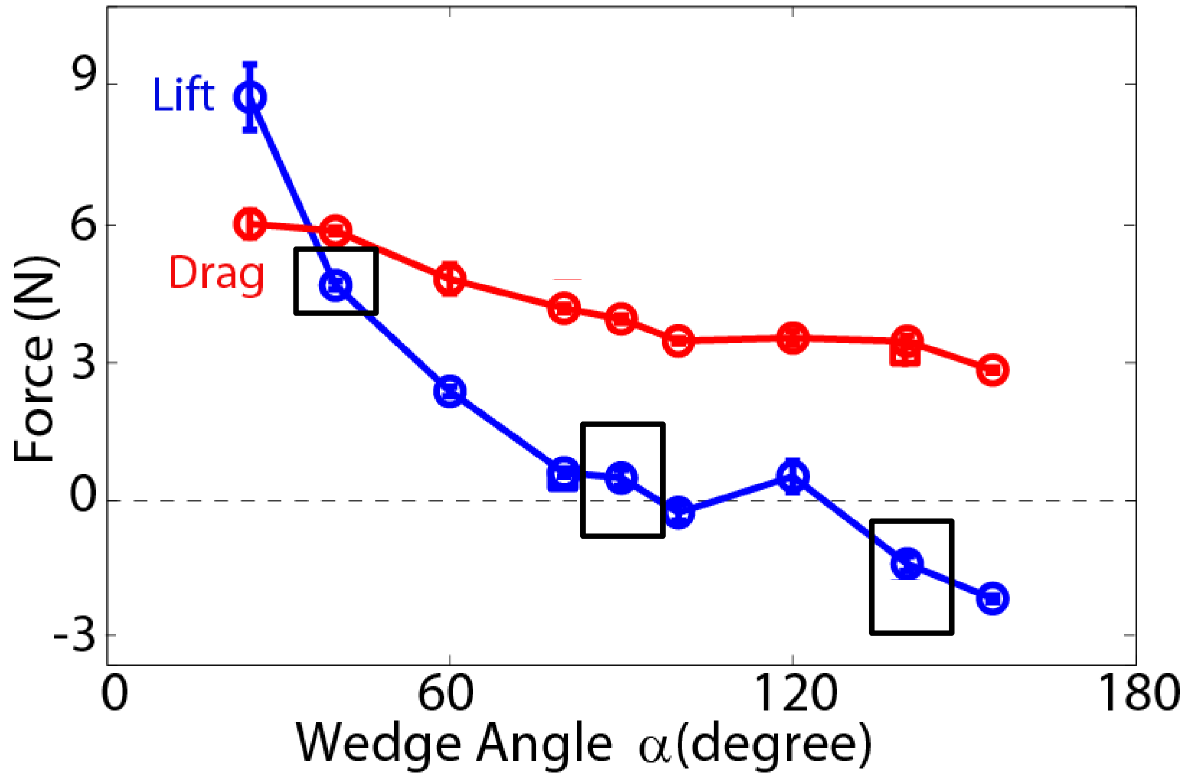


Drag and lift on wedge-like shapes

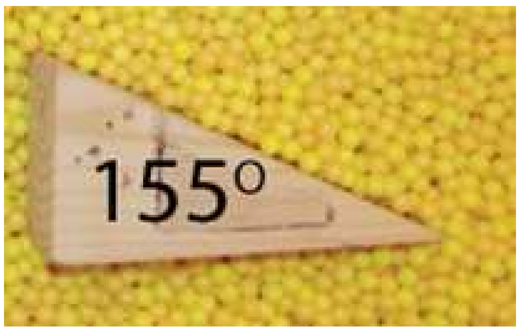
6mm plastic particles

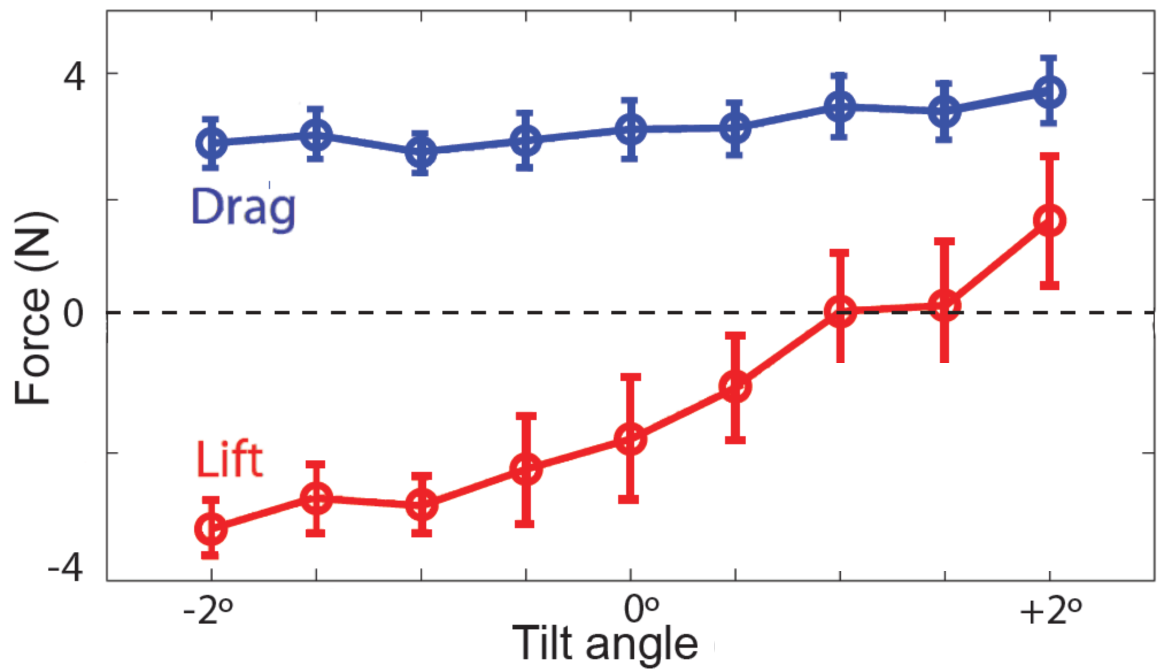


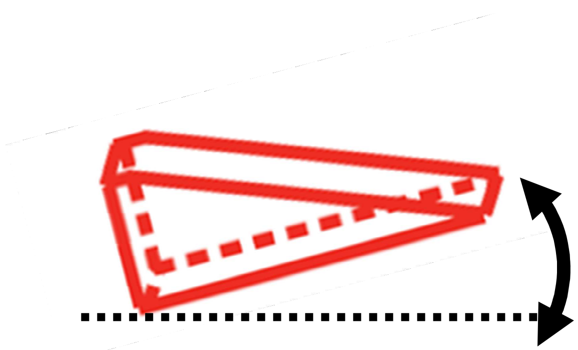




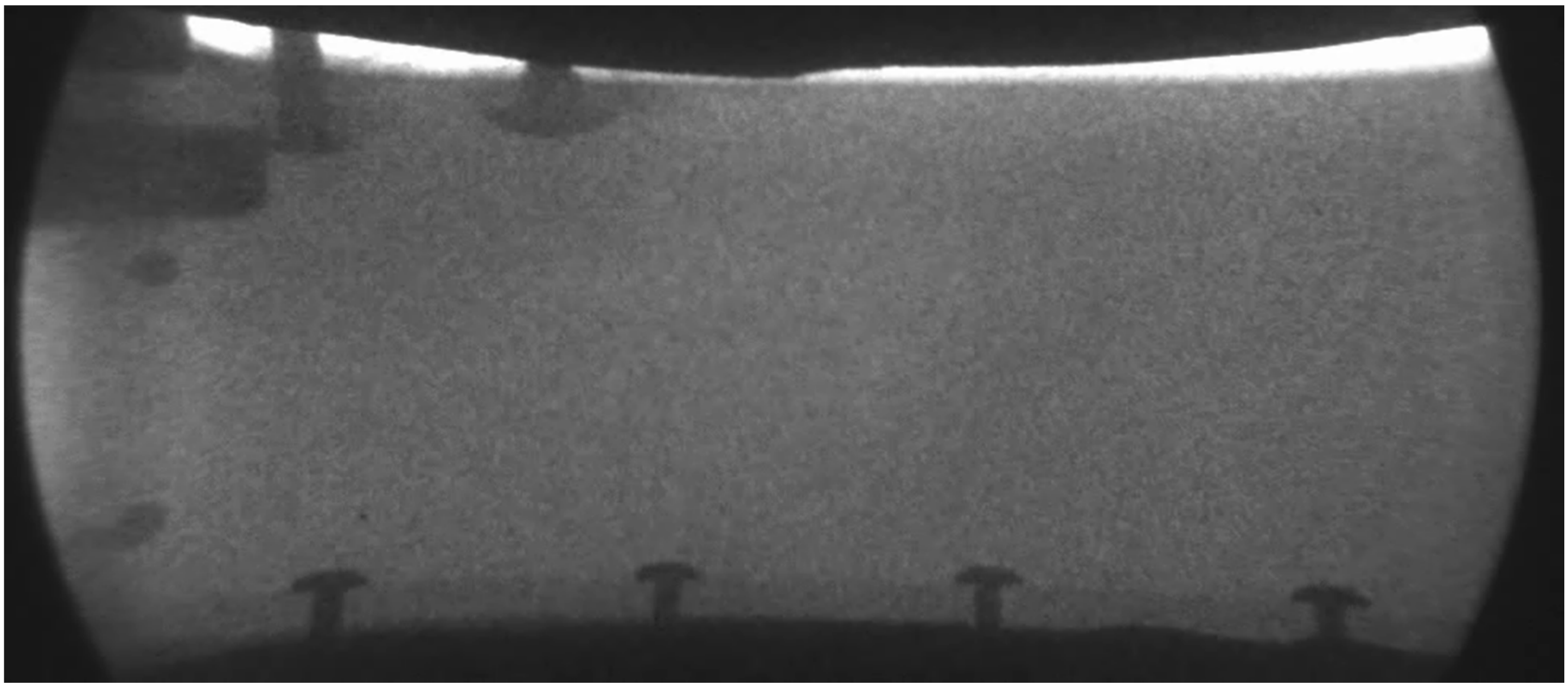
Sensitive dependence of lift force on tilt angle







Head movement?



10 cm

Slowed 10x

END