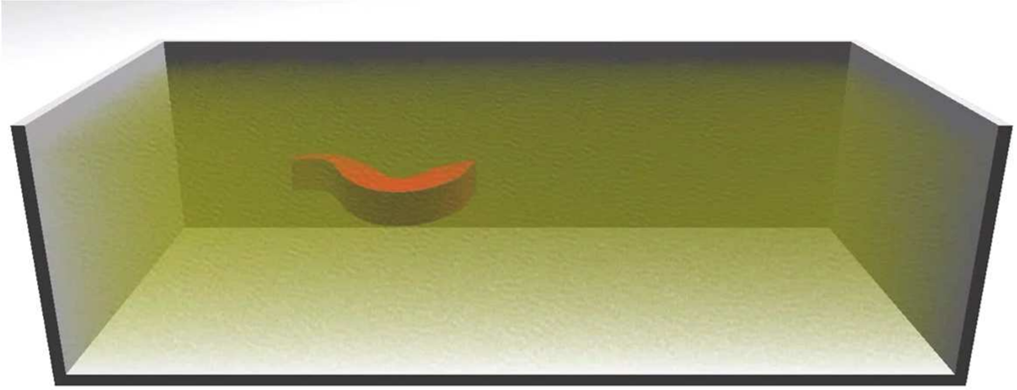
Swimming in Sand



Daniel I. Goldman

School of Physics

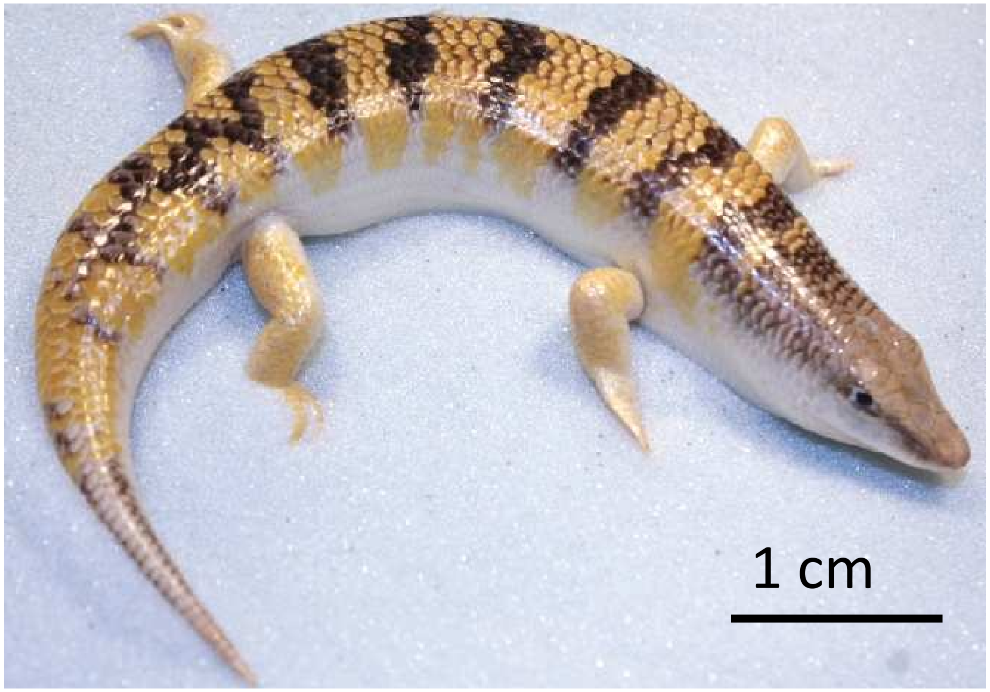
Georgia Institute of Technology

Boulder Summer School on Hydrodynamics

July 25-27

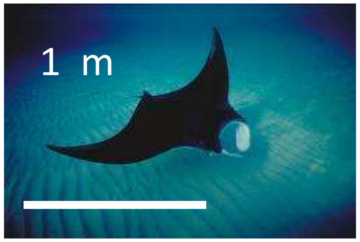
Lectures on the mechanics of interaction with granular media including biological & physics experiments, numerical, theoretical and physical robot models

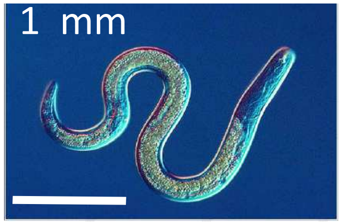
Sandfish (*Scincus scincus)*



Interaction with fluids

Aquatic





Vogel, *Life in Moving Fluids*

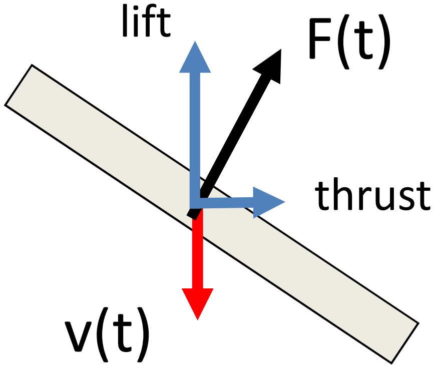
Navier-Stokes equations

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \bullet \nabla)\vec{u} = -\frac{1}{\rho}\nabla p + \frac{\mu}{\rho}\nabla^2 \vec{u}$$

$$\nabla \bullet \vec{u} = 0$$

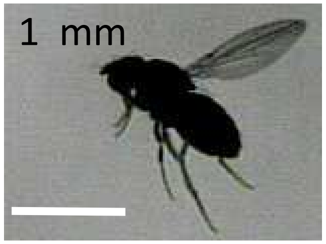
ρ=density
μ=viscosity

+ moving BC



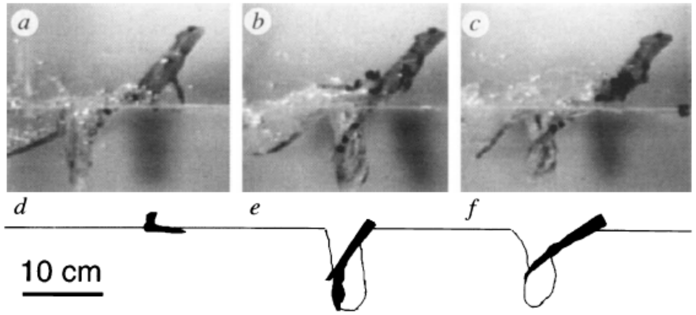
Limb (wing, fin, paddle...), body element (head, abdomen...)

Aerial

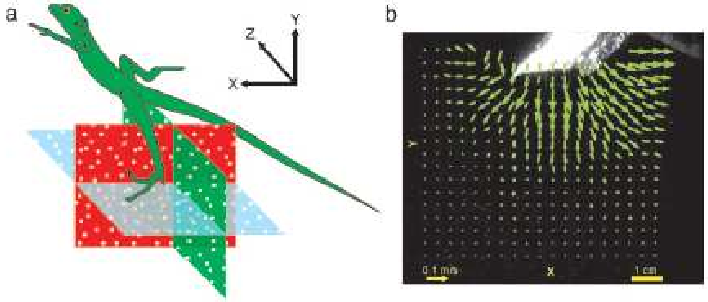


Running on water

Glasheen & McMahon, Nature, 1996



Hsieh & Lauder, PNAS, 2004



Terrestrial locomotion: diversity of substrates, diversity of solutions

Books:
Alexander 2003,
Biewener 2003,

Terrestrial Locomotion: Interaction of matter and complex media

The flowing terrestrial world

Leaf litter

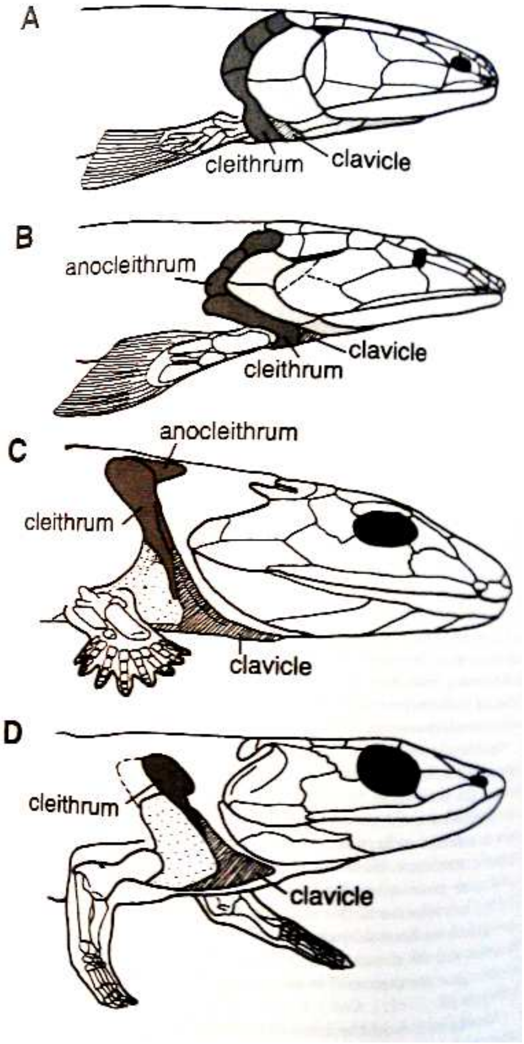
Rubble

BBC. Planet Earth, BBC. Wild Life Specials

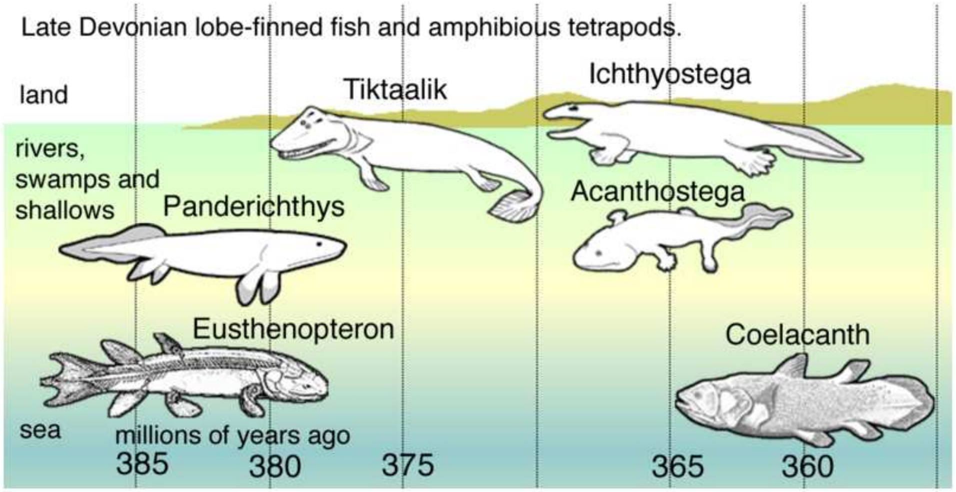
Snow

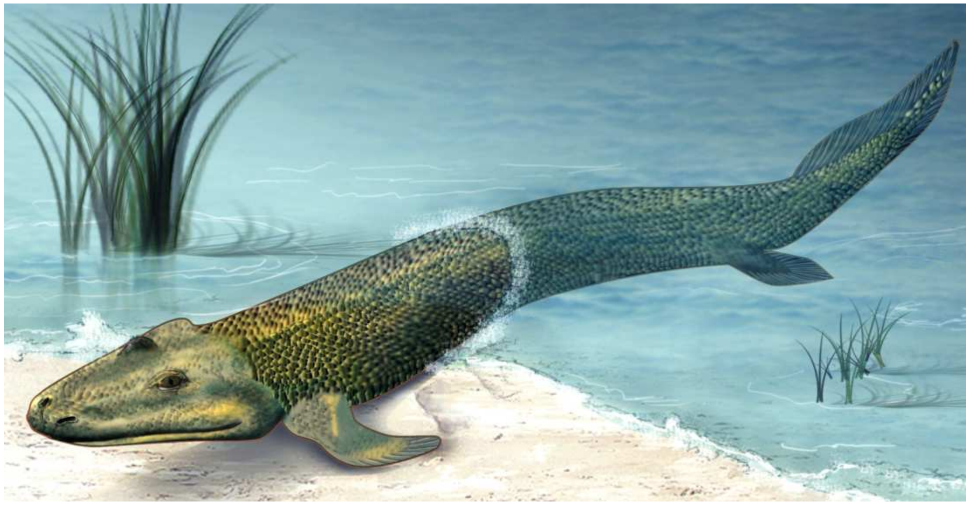
- Little known of principles of movement on this kind of ground
- Physics of interaction with such ground is poorly understood (unlike in fluids)

Early tetrapod locomotion occurred on flowing ground



From Clack 2002





http://en.wikipedia.org/wiki/Tiktaalik

Life in a granular world

Namib desert (SW coast of Africa)

In dry deserts in Africa

~10^2 species of lizards and snakes
~10^3 invertebrate species (ants, beetles, scorpions, …)

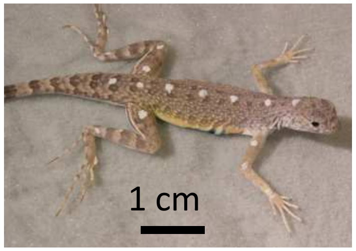
Sarah Steinmetz

Complex Rheology And Biomechanics (CRAB) Lab

Discover principles of interaction of matter and complex media

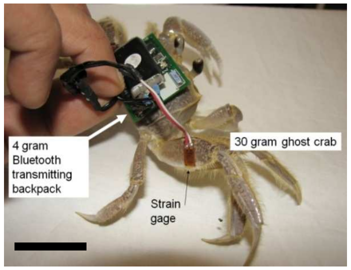
Focus: comparative studies of terrestrial locomotion on and within controlled granular media

Zebra-tailed lizard

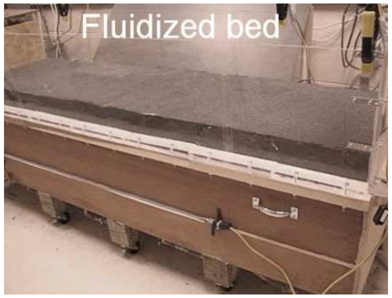


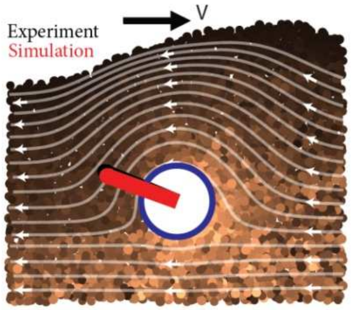
Sandfish lizard

Ghost crab

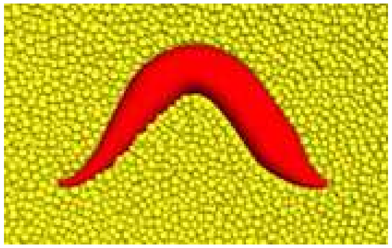


Hatchling sea turtle

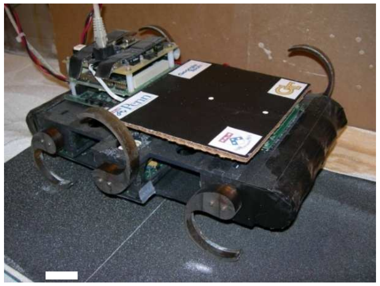




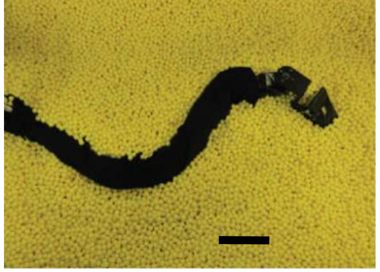
Sandfish simulation



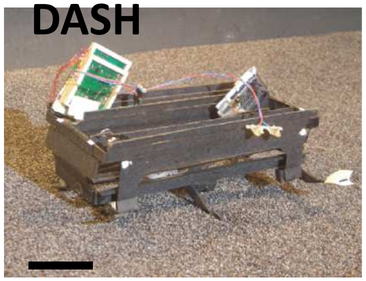
SandBot



Sandfish robot



DASH



TurtleBot

Topics in the lectures

(revised)

- General principles in terrestrial locomotion
- Intro to granular media
- Drag, lift and flow fields during localized intrusion in granular media

L1

- Modeling approaches: DEM & RFT
- Sandfish biological experiments
- Sandfish modeling: robot

L2

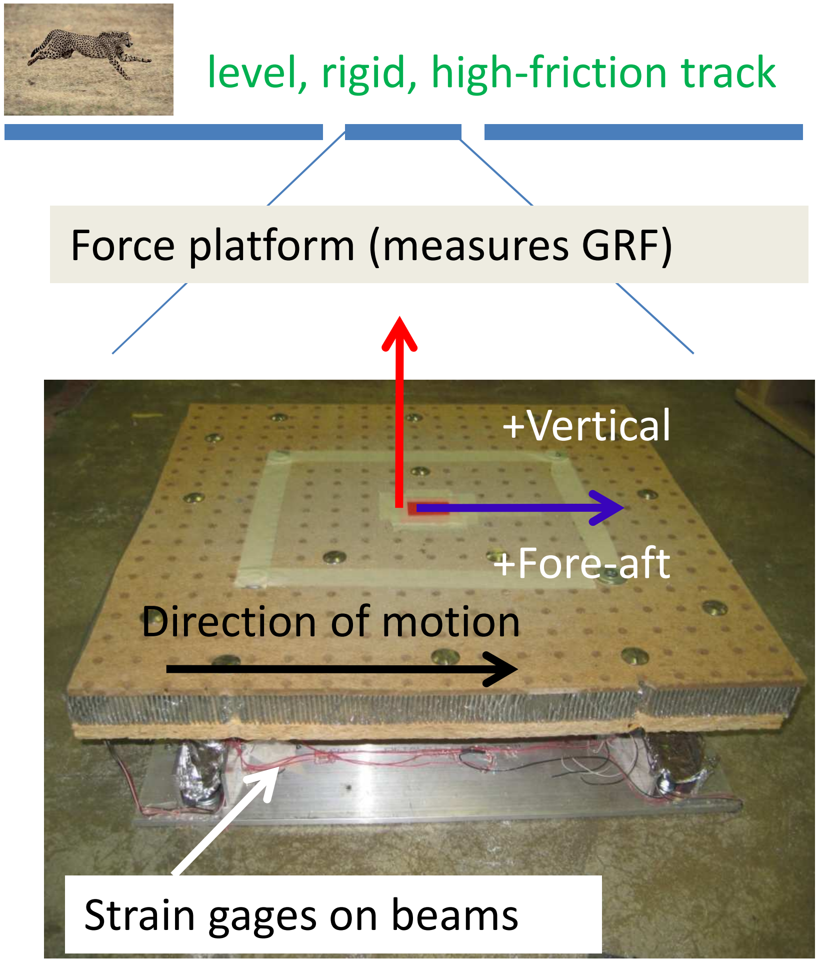
- Sandfish modeling: DEM
- Biological tests of model predictions
- RFT modeling of sand-swimming

L3

Terrestrial biomechanics

Alexander, 2005, Cavagna, 1977, McMahon 1980, Blickhan & Full 1989, ….

Principles discovered by reducing complexity of substrate interaction

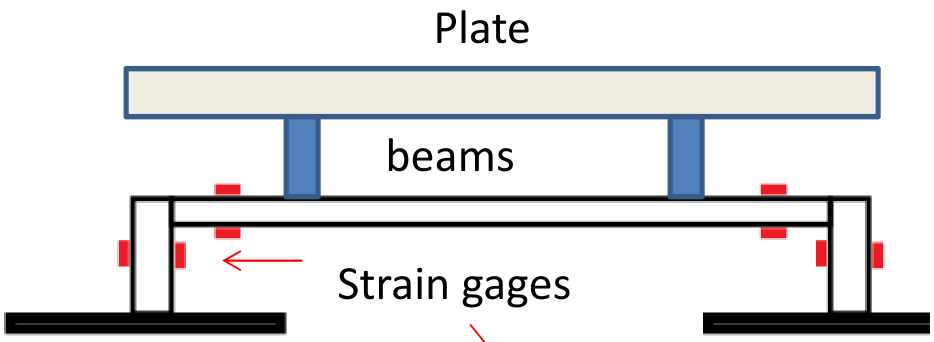


level, rigid, high-friction track

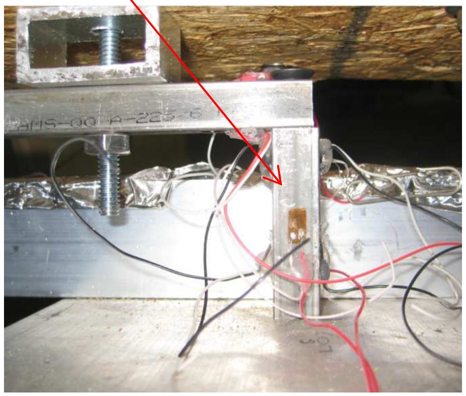
Force platform (measures GRF)

Strain gages on beams

Front view

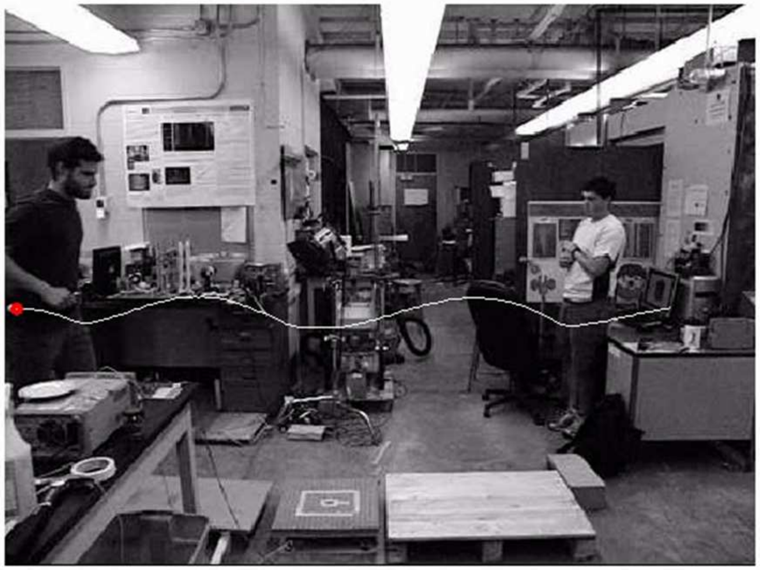


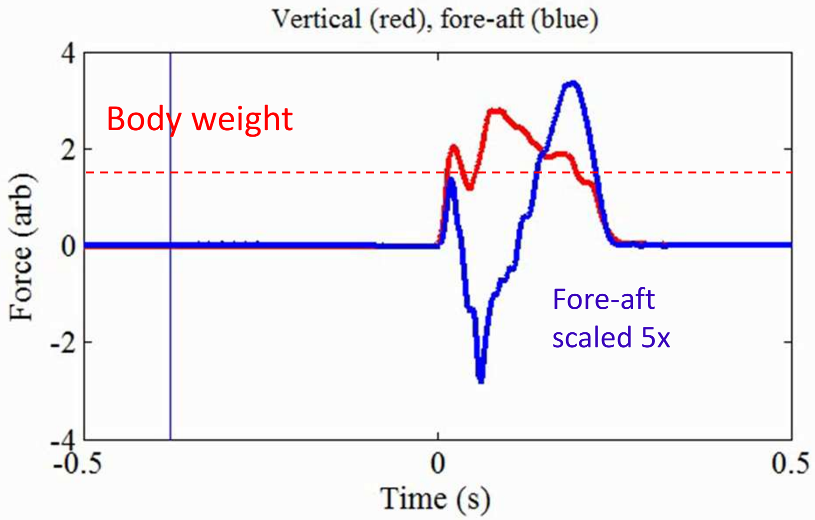
Strain gages deform as beam deflects, use bridge to meaure tiny changes in electrical resistance of gage→ ground reaction forces (GRF)



Synchronized GRF and kinematics

High speed
camera
(200 fps),
now ~$200





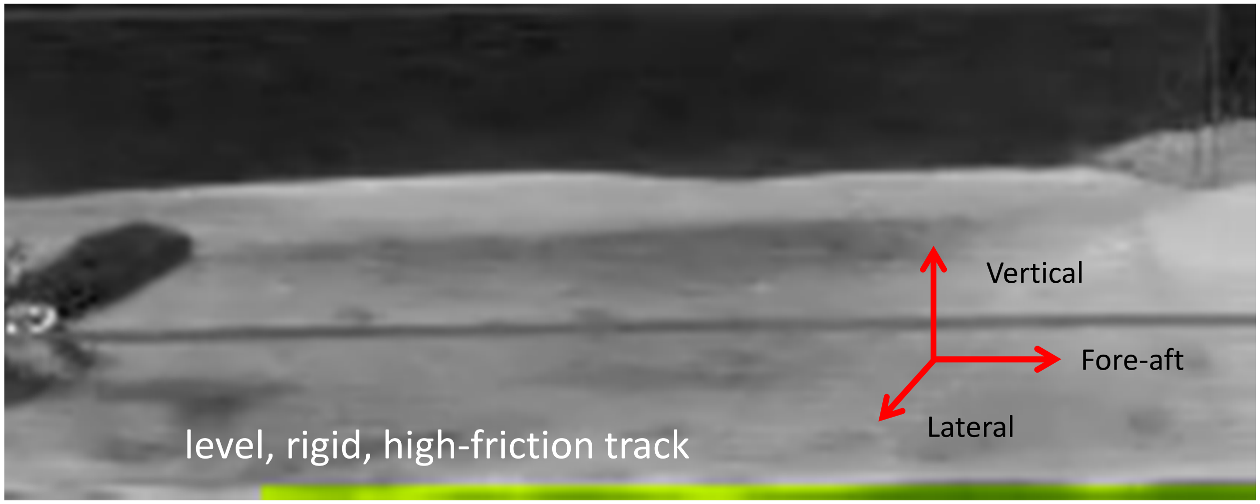
GRF during
trotting

Terrestrial biomechanics

Alexander, 2005, Cavagna, 1977, McMahon 1980, Blickhan & Full 1989, ….

Gecko (*P. bibroni*) running at 1 m/sec

Video slowed 10x



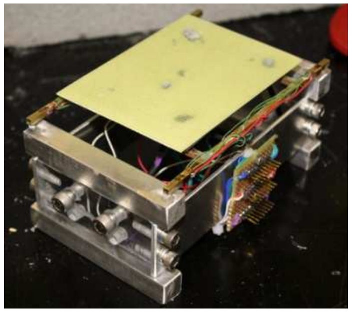
2 cm

Kinematics

Dynamics

High speed camera

(~10^3 fps)



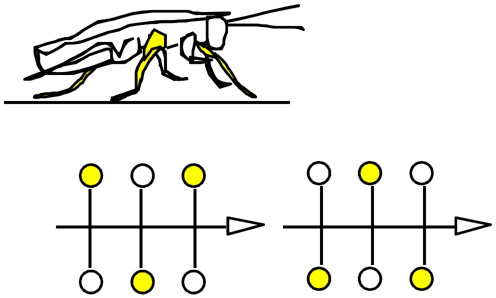
3-axis force platform

(~ 1 mN resolution)

Vertical oscillation during rapid locomotion

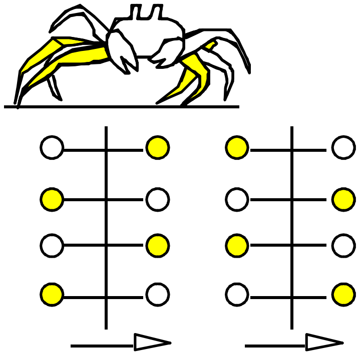
On rigid, level surface with good traction, all animals bounce when they run, trot, or hop

SIX-Legged



Cockroach

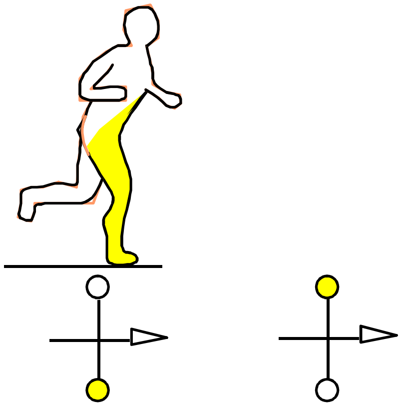
EIGHT- Legged



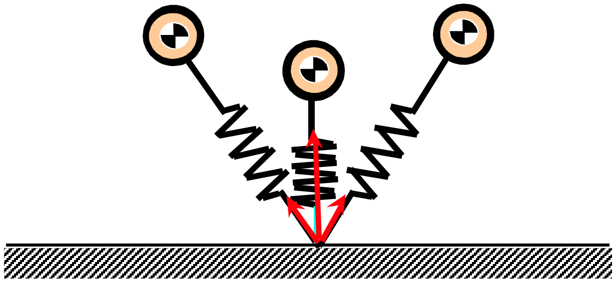
Crab

COM is lowest at mid-stance
Forward speed is maximal

TWO-Legged



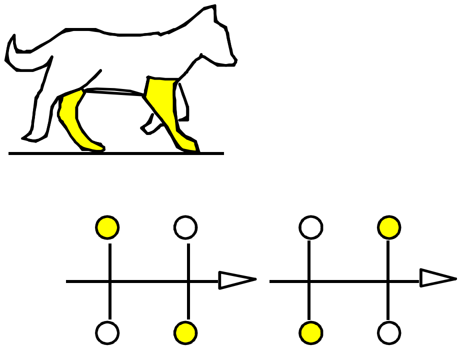
Human



Cavagna et al., 1977

Blickhan & Full, 1987

FOUR-Legged

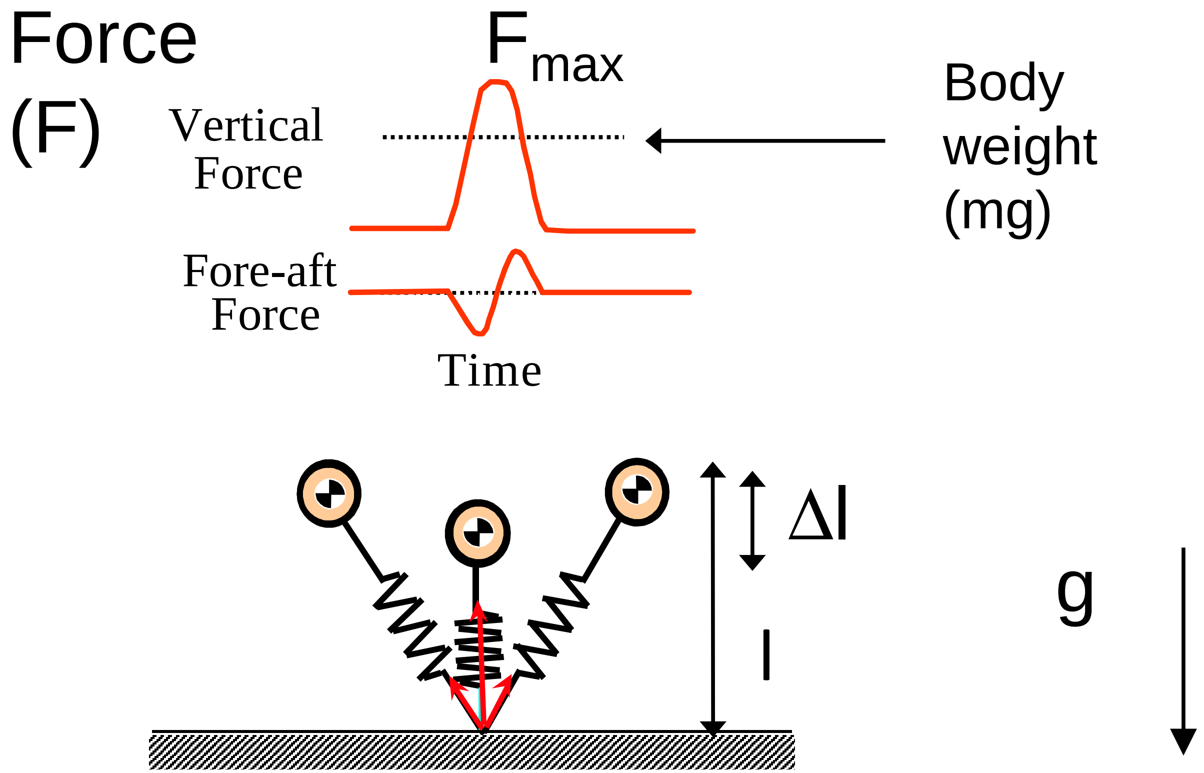


Dog

Force pattern for COM independent of morphology

Blickhan, *J. Biomechanics*, 1989
Blickhan & Full, *J. Comp. Physiol. A*, 1993

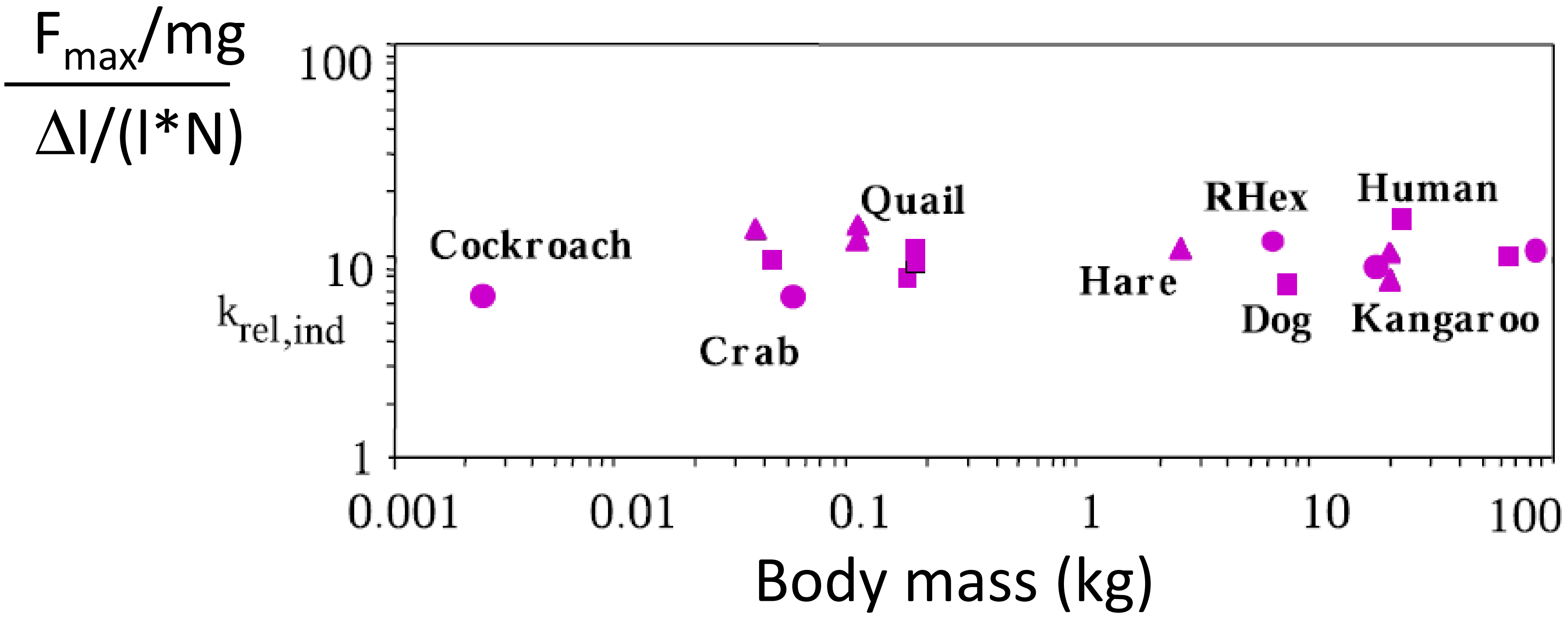
Spring
Loaded
Inverted
Pendulum
(SLIP)
model



eg, Cockroach: 0.3 mm,10 mN

Principle of terrestrial locomotion

normalized “spring” stiffness = constant



Target of control? Seems like everyone is acting like a pogo-stick (which is probably true for a kangaroo, but likely not for a cockroach)

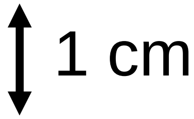
Cavagna et al., 1977

Blickhan & Full, 1987

Stability matters

Slowed 30x

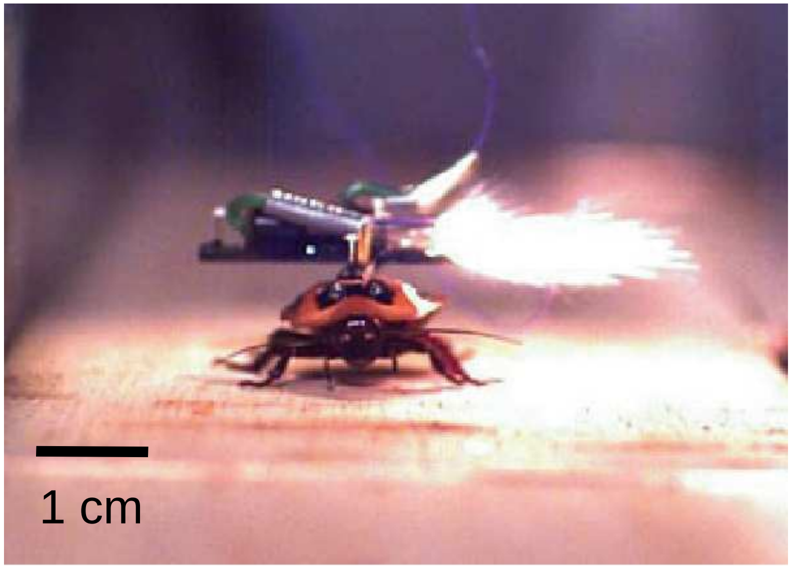
Periplaneta americana



sandpaper graphite coated stainless steel

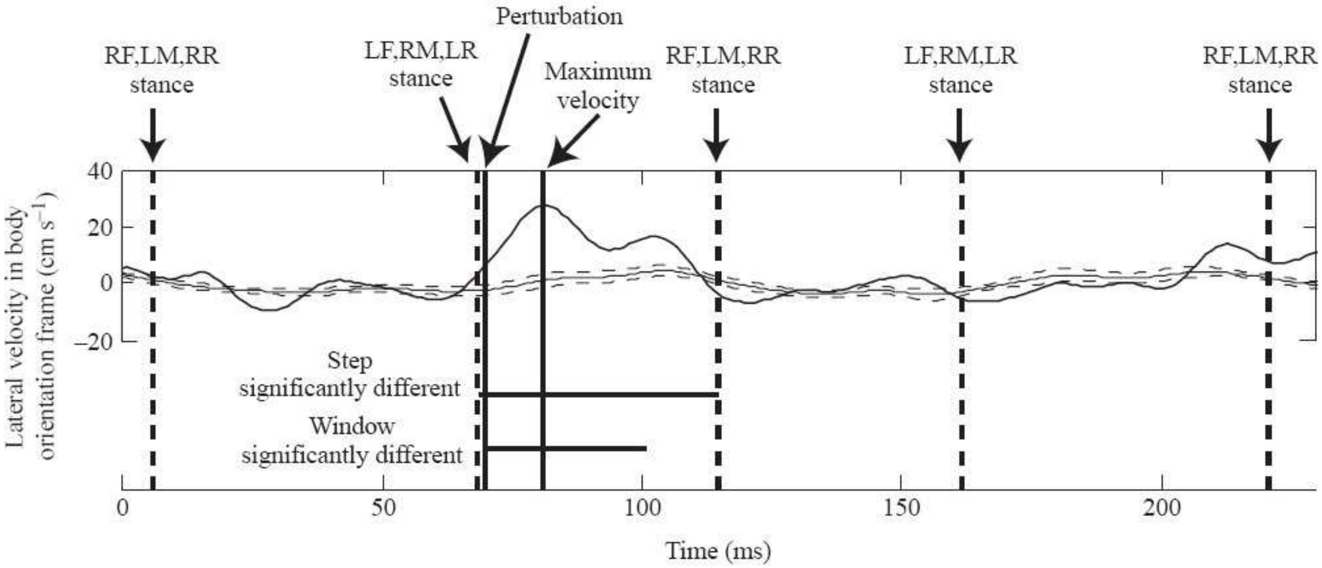
Rapid Stabilization

Jindrich, Full JEB (2002)

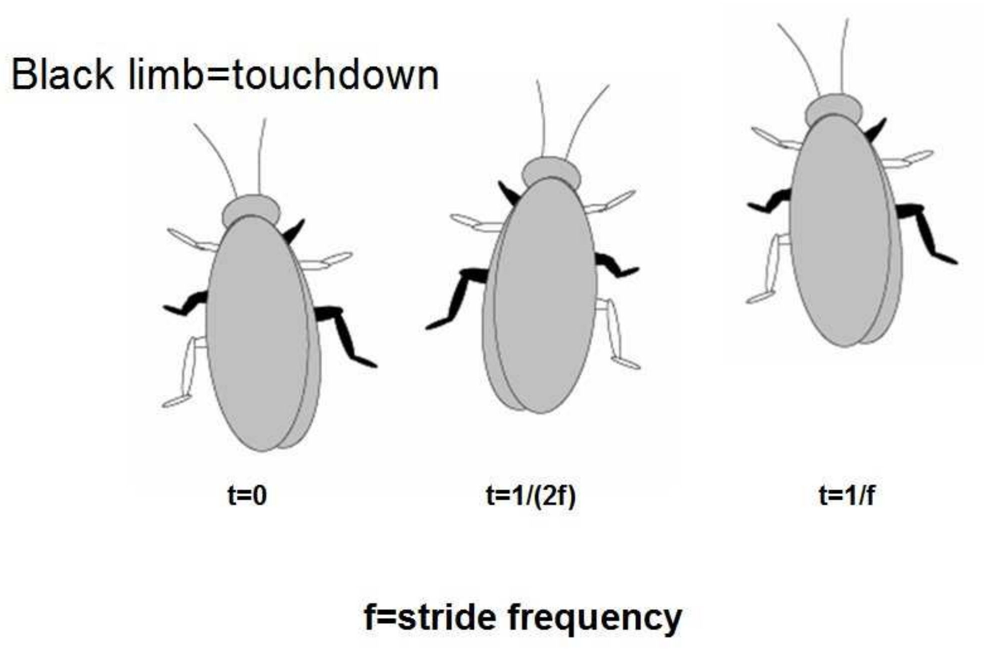


On level, rigid, no-slip ground, give large perturbation:

Recovery in less than two steps (<50 msec), challenging the fastest neural reflexes



Alternating tripod gait

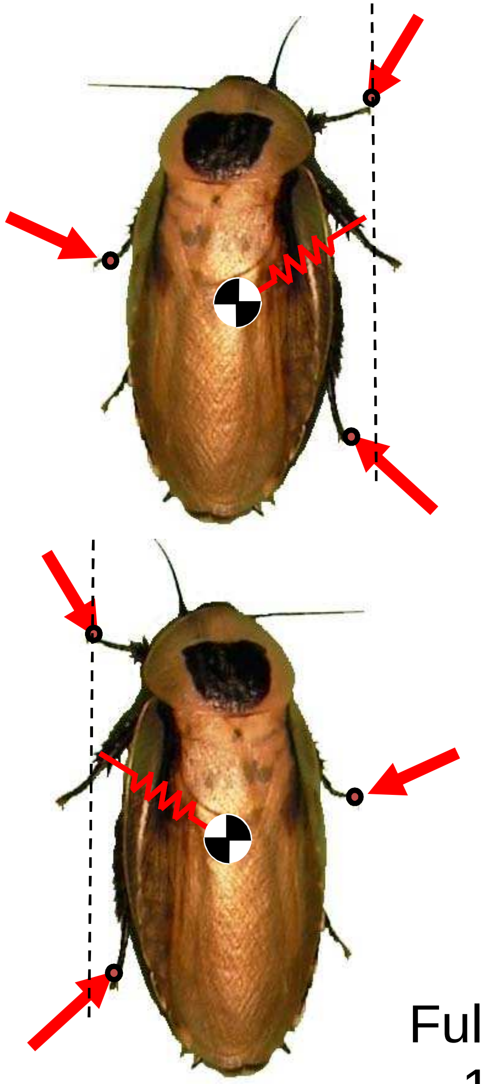


- Three legs fire in synchrony
- Used at fast speeds (>20 cm/sec)

Slowed 20x

Modeling lateral stability

Schmitt & Holmes, *Biological Cybernetics*, 2000

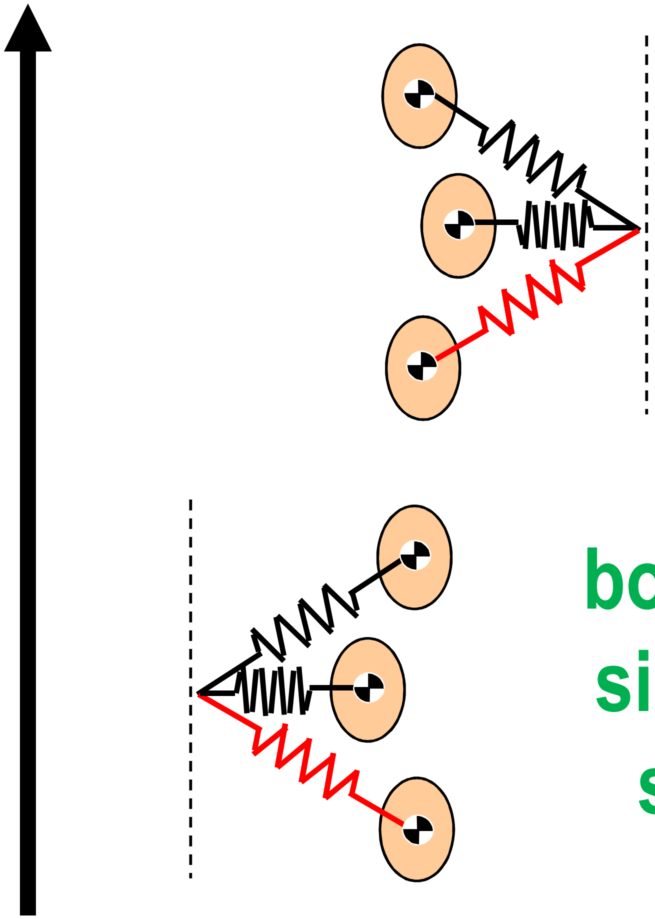


Level Running

3 Legs Acting as One

Full & Tu, 1990

Lateral Leg Spring Model



bounce side to side

Step-to-step return map

Schmitt & Holmes, *Biological Cybernetics*, 2000
Schmitt, Garcia, Razo, Holmes & Full, *Biological Cybernetics,* 2002

Equations of motion of body

$$m\ddot{r} = \mathbf{R}(\theta(t))f, \quad I\ddot{\theta} = (r_{\mathrm{F}}(t_n) - r) \times \mathbf{R}(\theta(t))f \ ,$$

With R, the rotation matrix needed to transform foot forces to body coordinates, f the leg forces, r the touchdown foot position.

Integrate these on a step by step basis, obtain Poincare map F that takes,

$$(v_{n+1}, \delta_{n+1}, \theta_{n+1}, \omega_{n+1}) = \mathbf{F}(v_n, \delta_n, \theta_n, \omega_n) \ ,$$

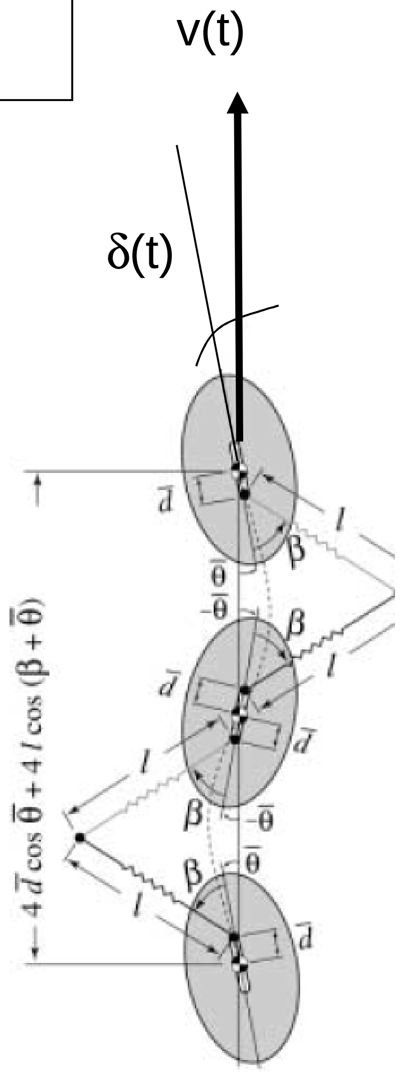
Where

v is forward velocity

δ is heading relative to velocity of COM

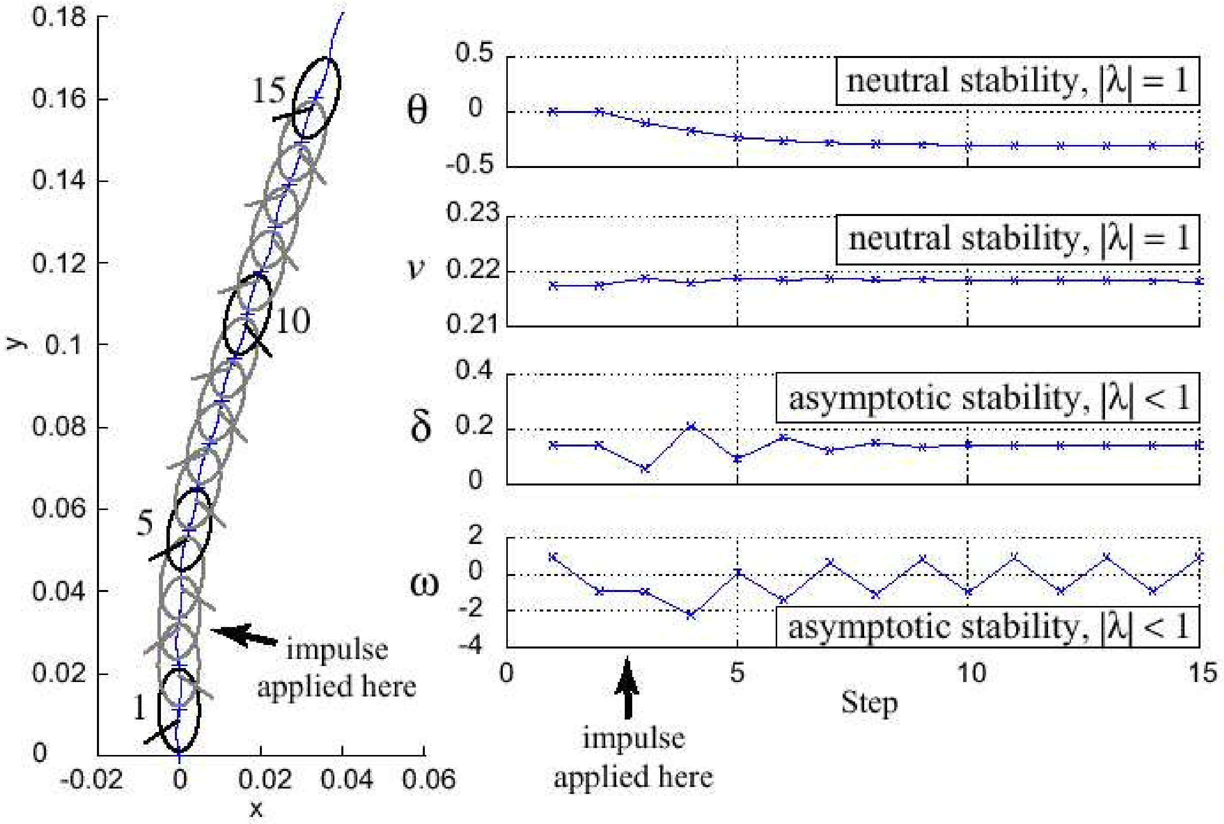
θ is angle of body in world frame

ω is $d\theta/dt$



Tuned spring leg & non-holonomic foot constraints: asymptotic stability

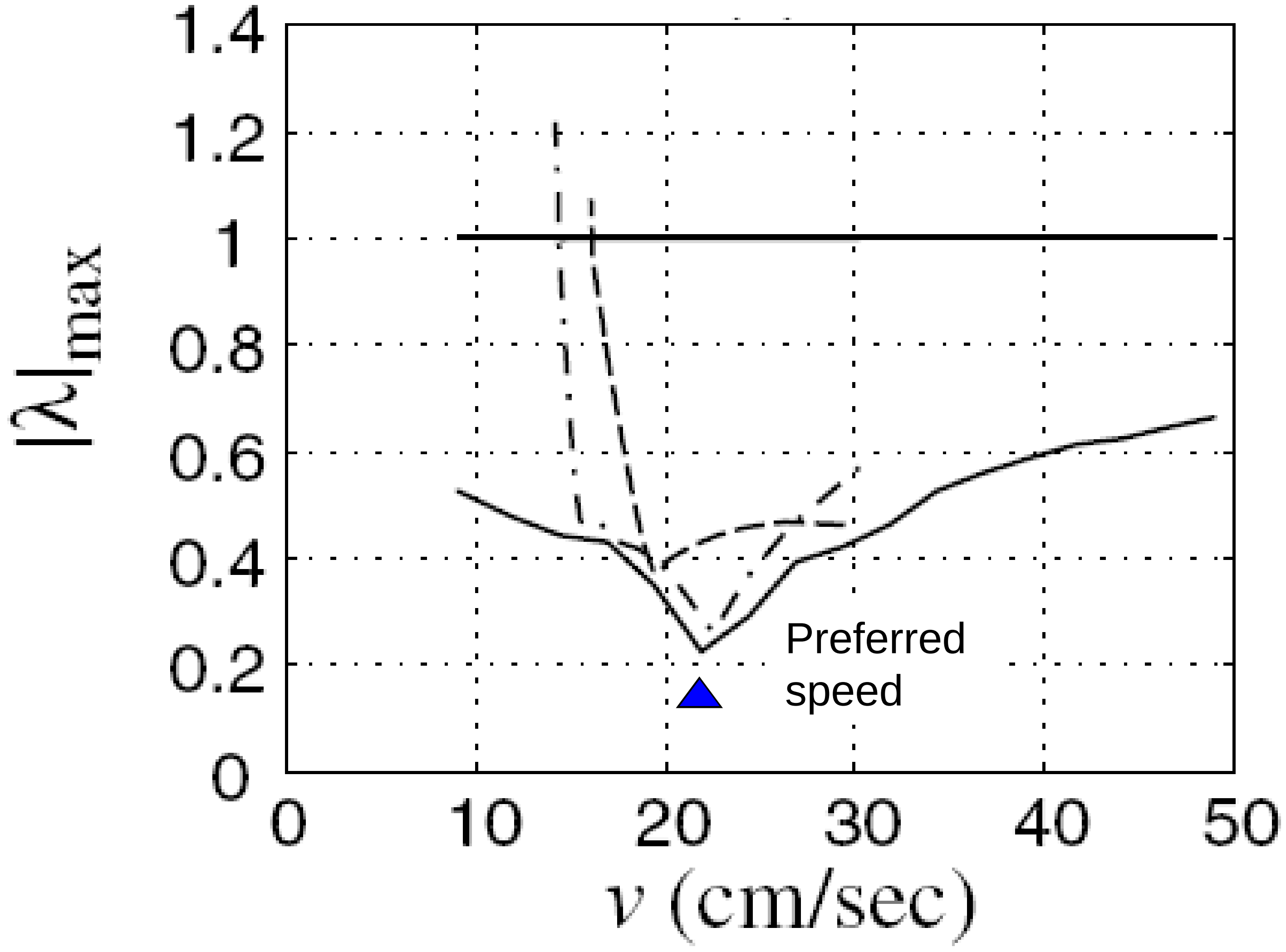
Schmitt & Holmes, *Biological Cybernetics*, 2000
Schmitt, Garcia, Razo, Holmes & Full, *Biological Cybernetics,* 2002



LLS model yields rapid stable response to perturbation-
-TURN OFF THE BRAIN?

LLS model predicts preferred speed

Schmitt, Garcia, Razo, Holmes & Full, *Biological Cybernetics,* 2002



λ are the eigenvalues of the linearized step-to-step map F

Rhex: Dynamically stable physical model

Journal paper: Saranli, Buehler & Koditschek, *Int. J. Rob. Res.*, 2001
Recent review: Holmes, Full, Koditschek, Guckenheimer *SIAM Rev.*, 2006

- Follows SLIP (on hard, flat ground)
- Control is in the hip motors—no electronic feedback on perturbations

Mass, 5 kg, **Length,** 50 cm, **Top speed** 3 m/sec

Neuromechanical modeling of locomotion

"Templates"

"Anchors…"

Full & Koditschek, *J. Exp. Biol.*, 1998, Holmes et al, *SIAM* 2006

Low-order (analytics possible, target of control)

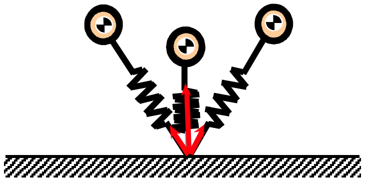
Mechanical (morphology, limb number, posture), limited control

Muscles, sensors, materials,

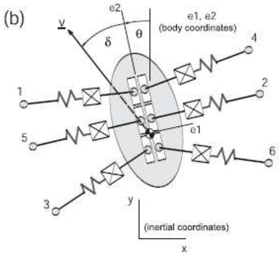
nervous system, metabolism

Models assume rigid, flat, ground with point contact interaction

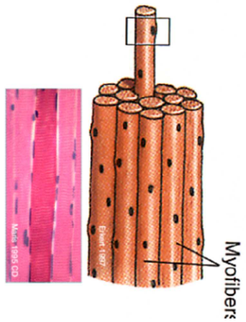
SLIP model

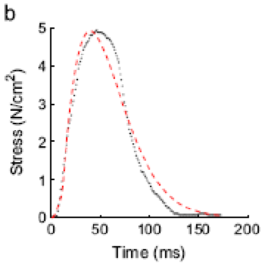


Ghigliazza et al, 2005
Seyfarth et al, 2005

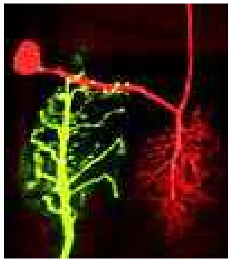


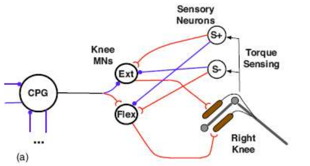
Seipel et al, 2004





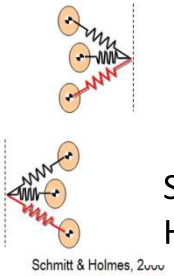
Kukillaya & Holmes, 2009





Kukillaya & Holmes, 2009

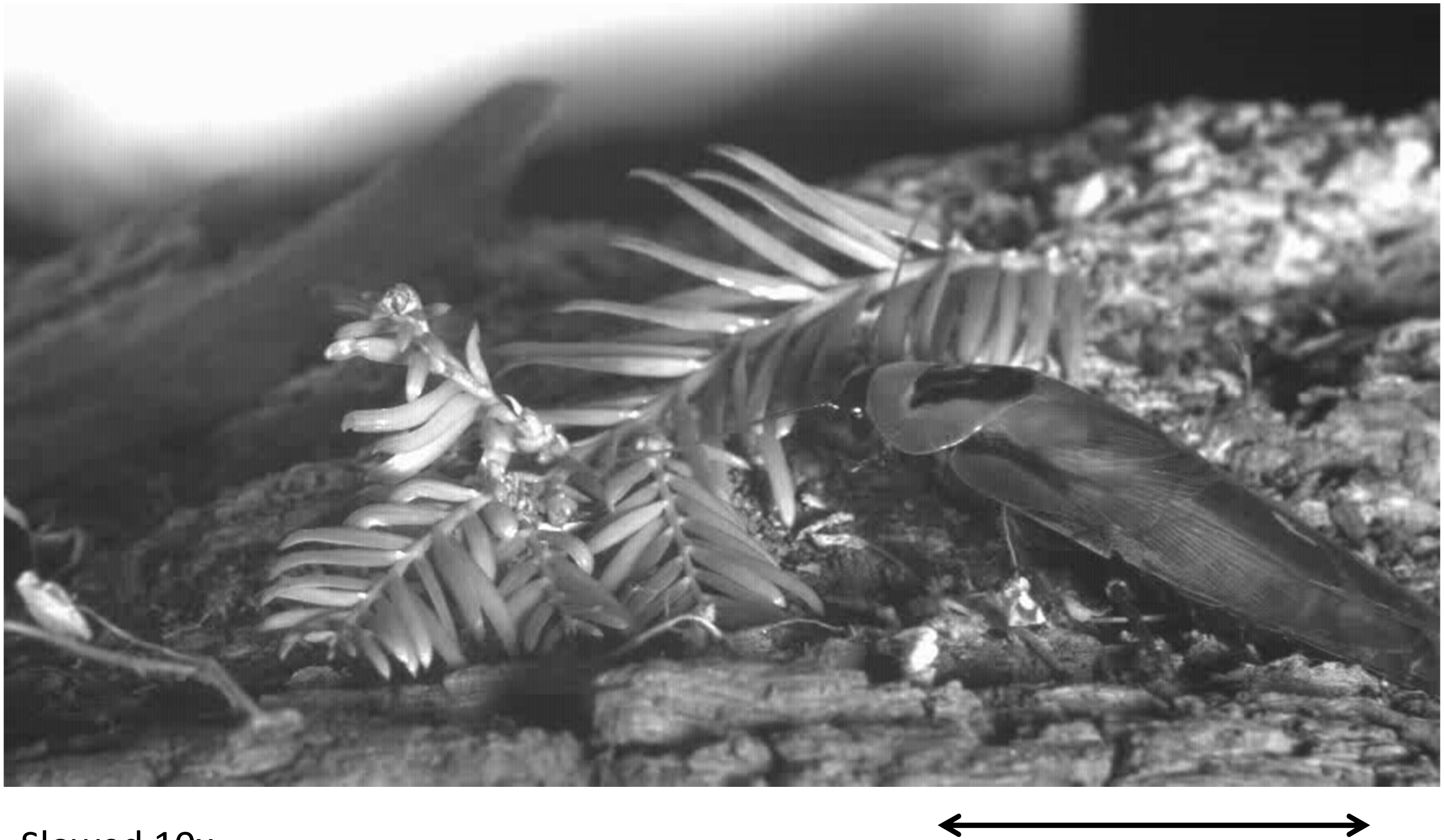
LLS model



Schmitt & Holmes, 2000

Physical model

Not point contact!



Slowed 10x

Templates?

Lack of templates is a problem

RHex, Boston Dynamics

Complex ground interaction is a feature of terrestrial environments

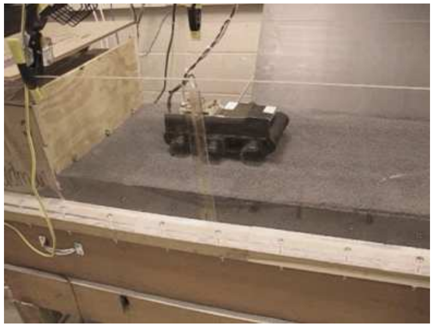
- Vertical surfaces
- Irregular footholds
- Flowing ground

Granular media: a challenging flowing terrain

~10^5 grams (185 kg), ~200 cm

~10^4 grams (16 kg), ~100 cm

~10^3 grams (2.5 kg), ~30 cm



Li, Komsuoglu, Umbanhowar, Koditschek, Goldman, PNAS, 2009, Exp Mechanics 2010

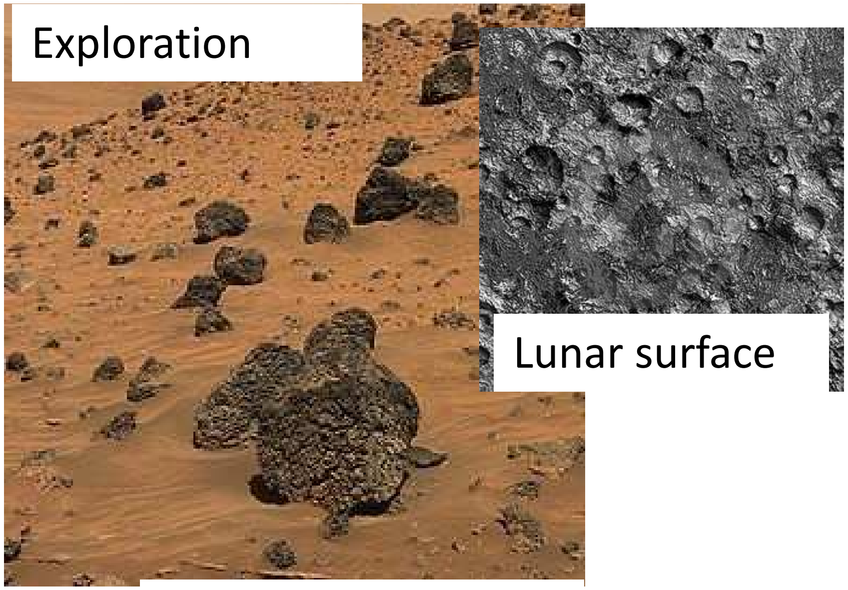
~10^1 grams (16 g), ~10 cm

Li, Hoover, Birkmeyer, Umbanhowar, Fearing, Goldman, Proc. SPIE, 2010

Applications of robots that (could) move in GM

Search and rescue

Rubble - earthquake



Exploration

Lunar surface

Martian sand

Desert IED detection

Granular materials in industry: sand, sugar, cereal, coal, cement, cosmetics, avalanches, pharmaceuticals, …

sand

Corn piles

Garlic tablets

Dry granular materials

$k_B T << mgd$

+

Interact at contact with inelastic collisions, friction

Simple description,
Complicated behavior:
display features of
solids, fluids and gases

Classification of soil particles

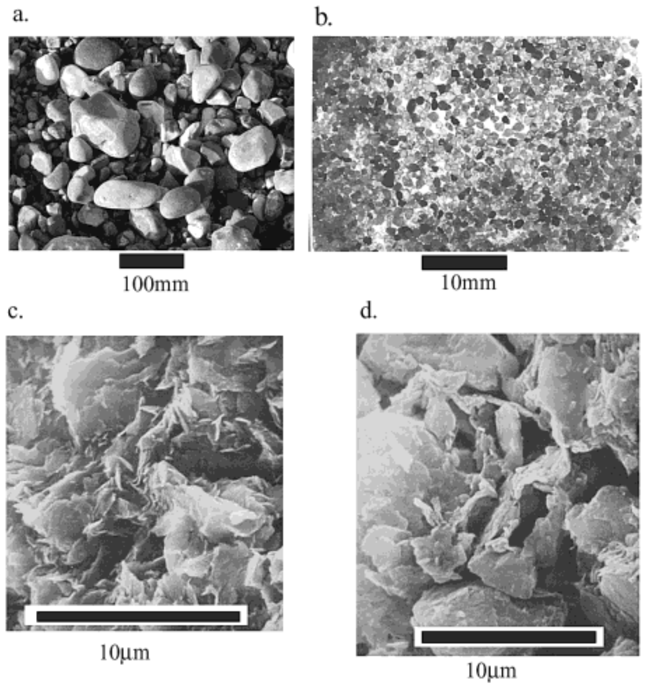
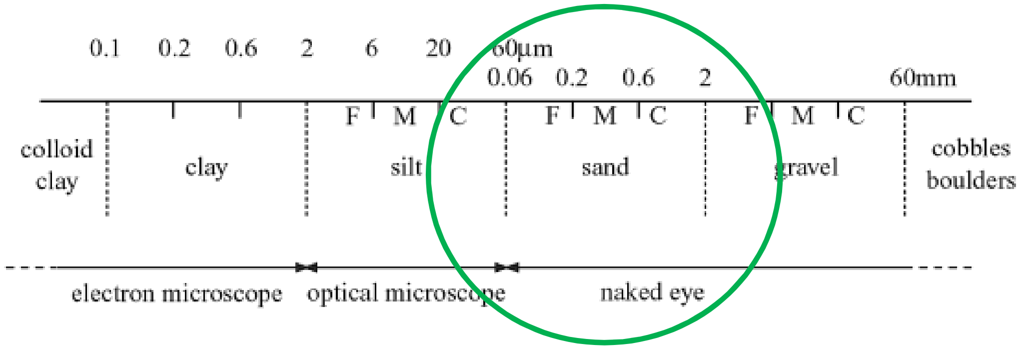
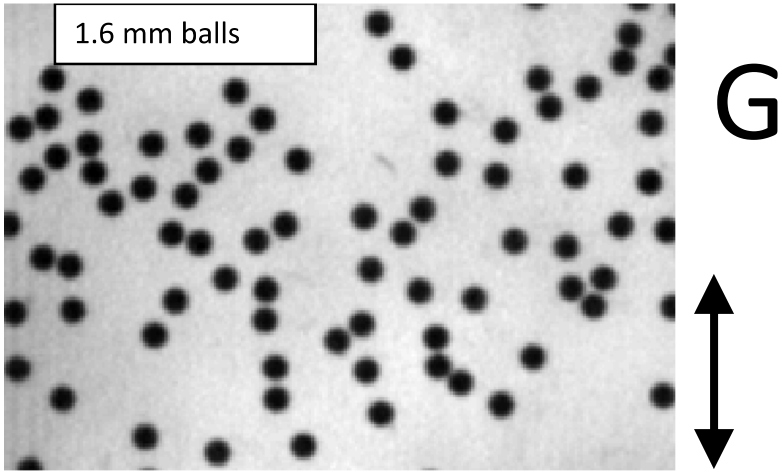


Figure 3.1. (a) Beach shingle; (b) sand; (c) scanning electron micrograph of Weald clay from south-east England (picture provided by A. Balodis); (d) scanning electron micrograph of Drammen clay from Norway (picture provided by A. Balodis).

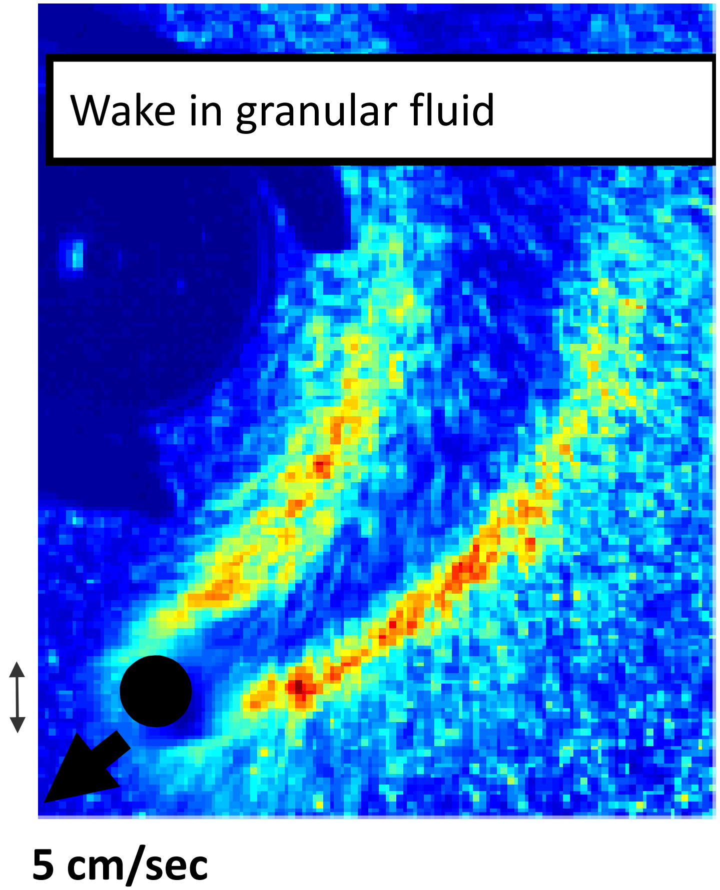


From Muir Wood, *Soil Mechanics: A One-Dimensional Introduction*



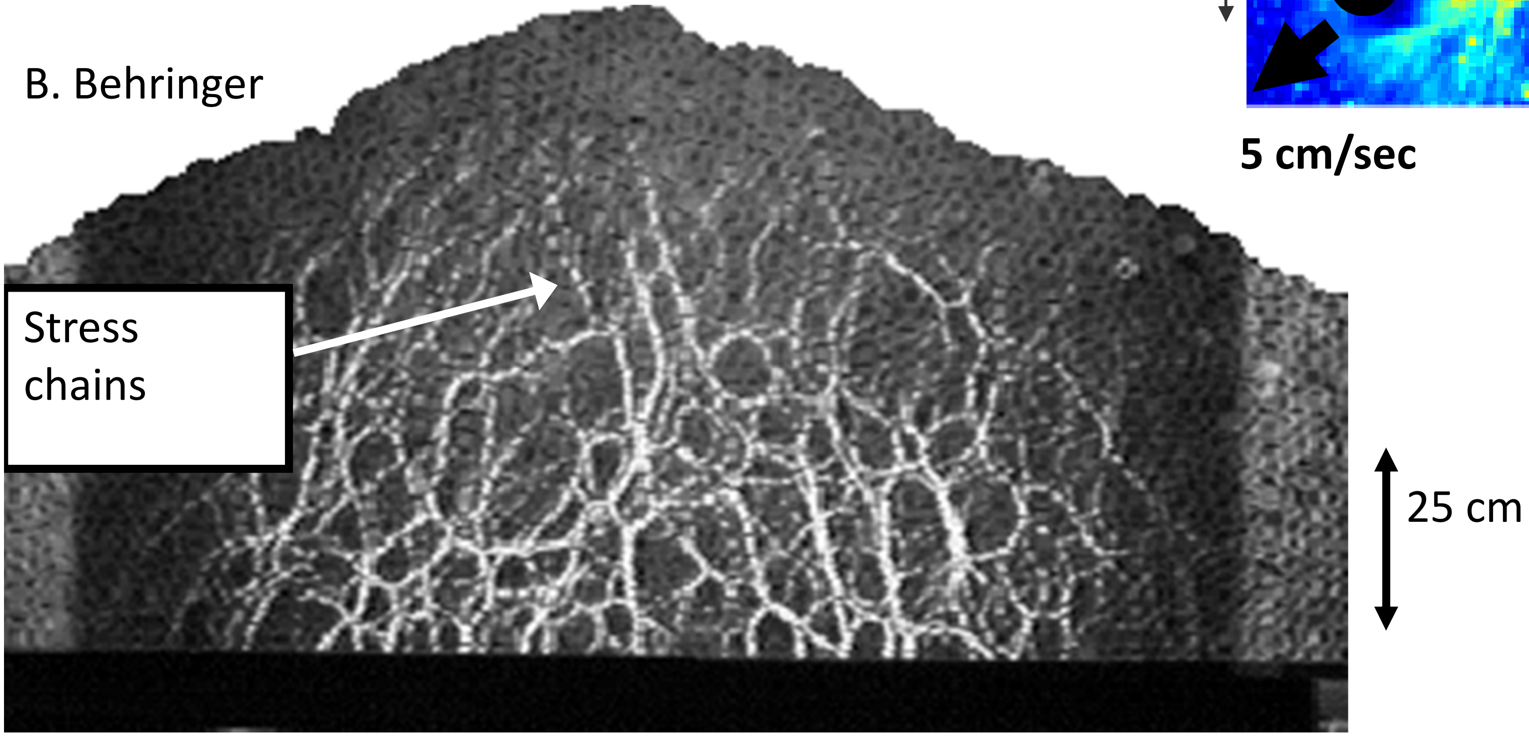
Gas

D. Miracle



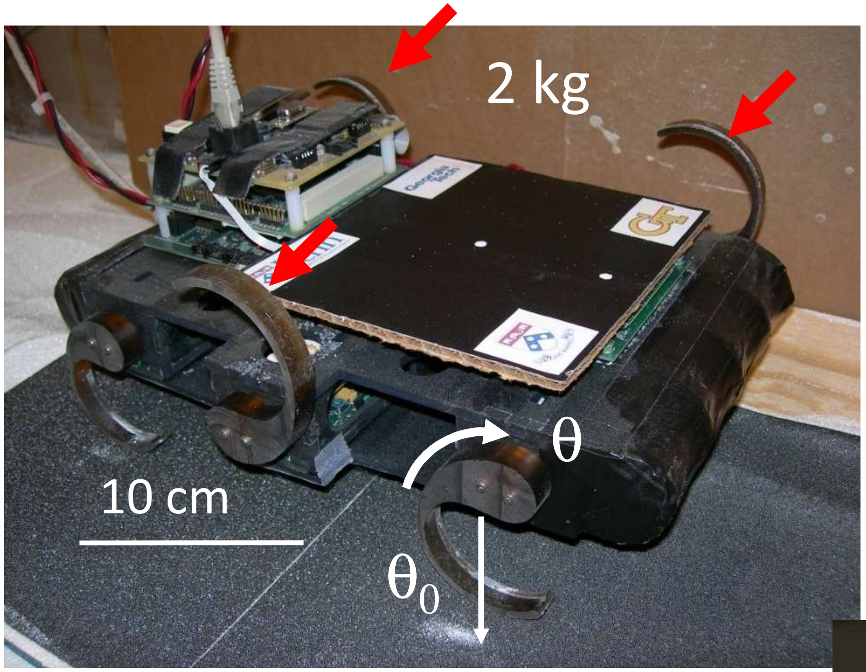
Solid

B. Behringer

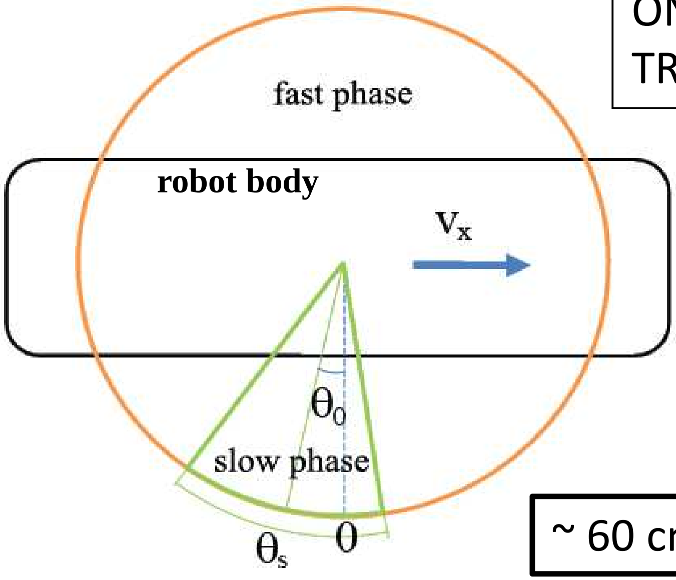


Control of gait and limb kinematics

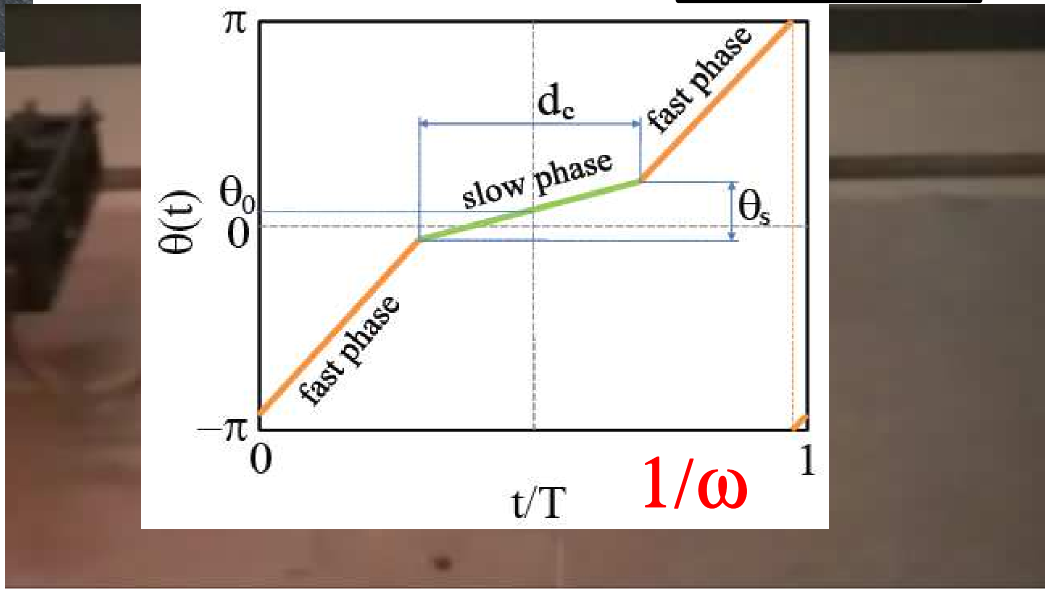
In collaboration with Dr. Hal Komsuoglu & Prof. Dan Koditschek, UPenn

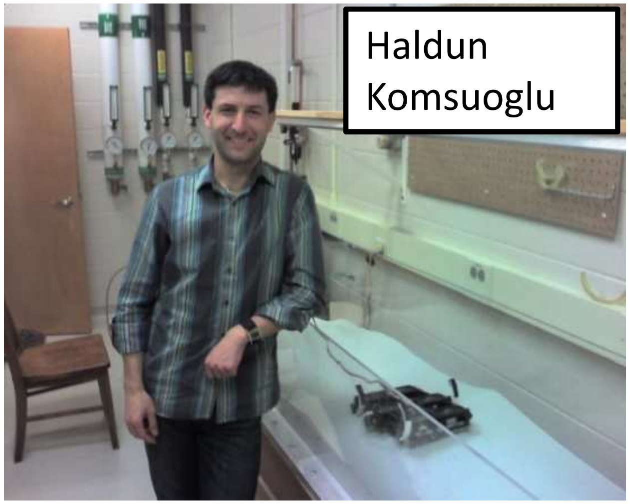


ONE TRIPOD



~ 60 cm/sec

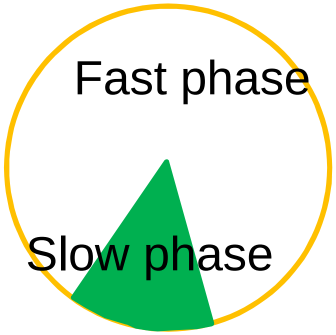



Haldun Komsuoglu

Locomotor sensitivity on GM

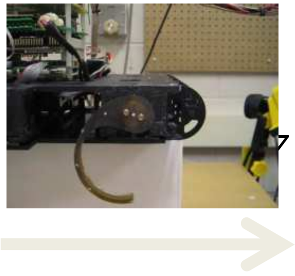
Li, Umbanhowar, Komsuoglu, Koditschek, Goldman, *PNAS*, 2009

Hard ground kinematics (**HGK**)

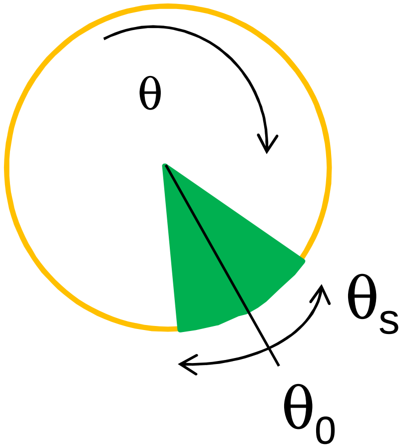

Fast phase

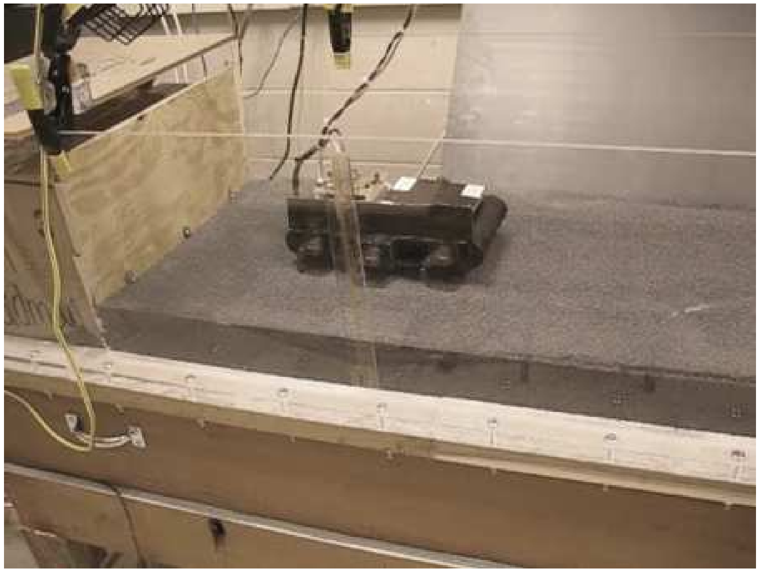
Slow phase

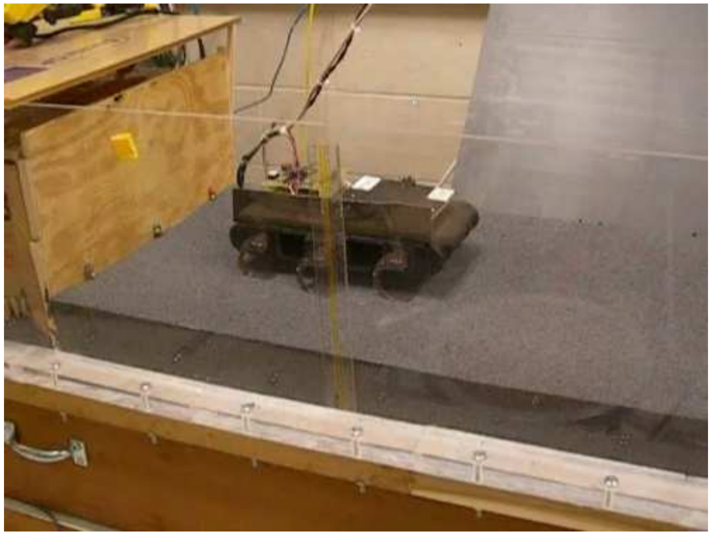
ω (rotation frequency)



Soft ground kinematics (**SGK**)







Soft Ground Kinematics optimize granular solidification

“rotary walking”

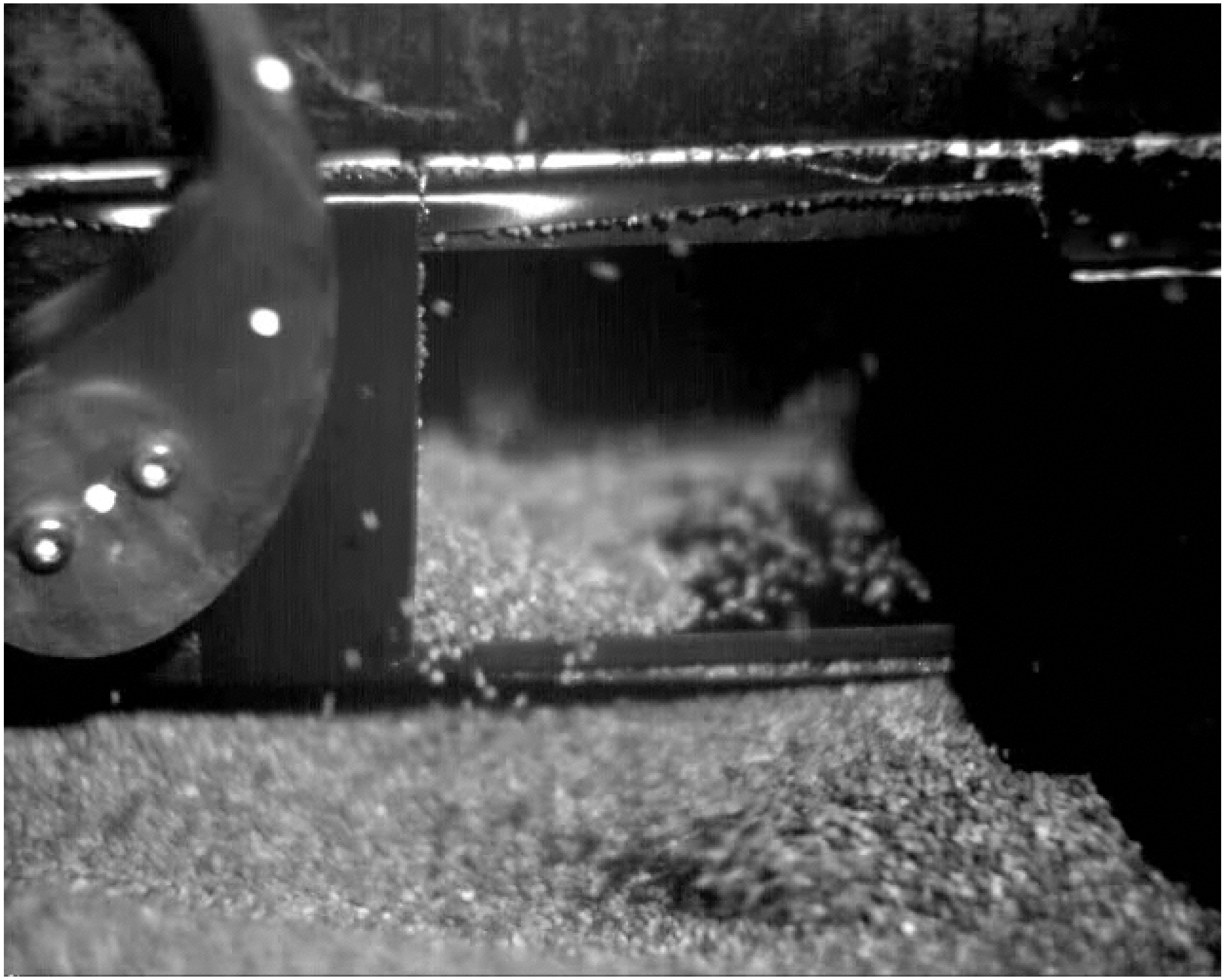
1 cm

Slowed 10x

GM=~1mm poppy seeds

Li, Umbanhowar, Komsuoglu, Koditschek, Goldman, *PNAS*, 2009, *Exp Mechanics*, 2010.

Swimming in GM

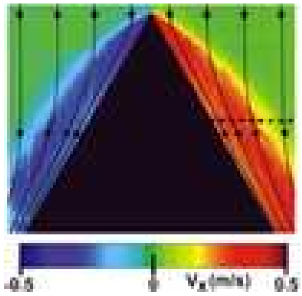


Force and flow response of granular materials

Books & reviews:
Nedderman, Muir-Wood, Terzaghi, Jaeger et al…

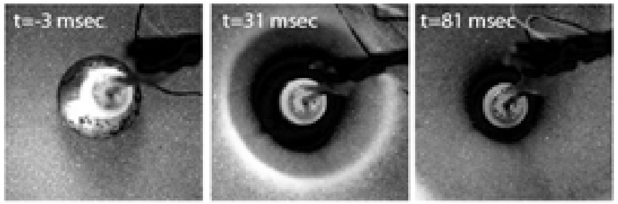
- Physics tends to focus on particle interactions & fundamental models (force chains, jamming, hydrodynamics)
- Soil mechanics/geotechnical engineering: empirical constituative stress/strain models needed to build stable structures
- Little detailed experiment & modeling of sustained/transient localized intrusion

shocks



Liquefaction

Impact

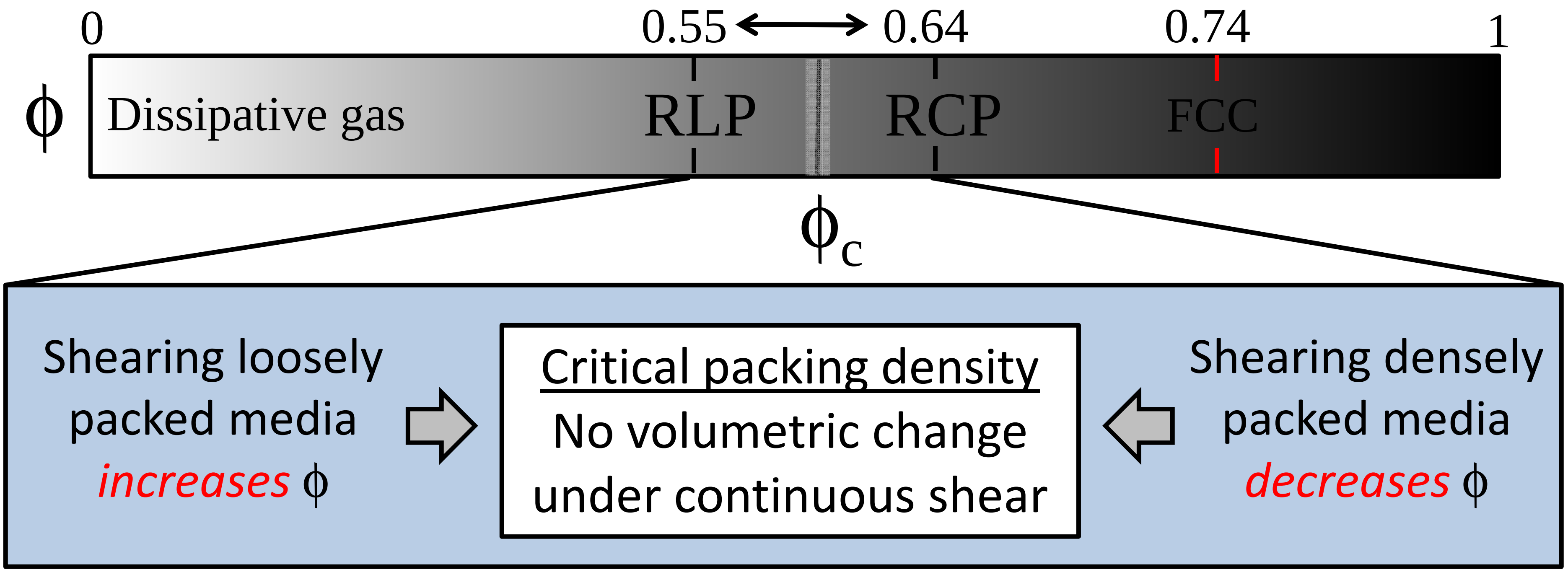


The role of volume fraction

Mechanics of shear response of a given GM affected by

- Polydispersity
- Grain shape
- Friction
- Damping
- Volume fraction

$$\phi = \frac{\text{Solid volume}}{\text{Occupied volume}}$$



Shearing loosely packed media *increases* ϕ

Critical packing density

No volumetric change under continuous shear

Shearing densely packed media *decreases* ϕ

Monodisperse, non-cohesive spheres

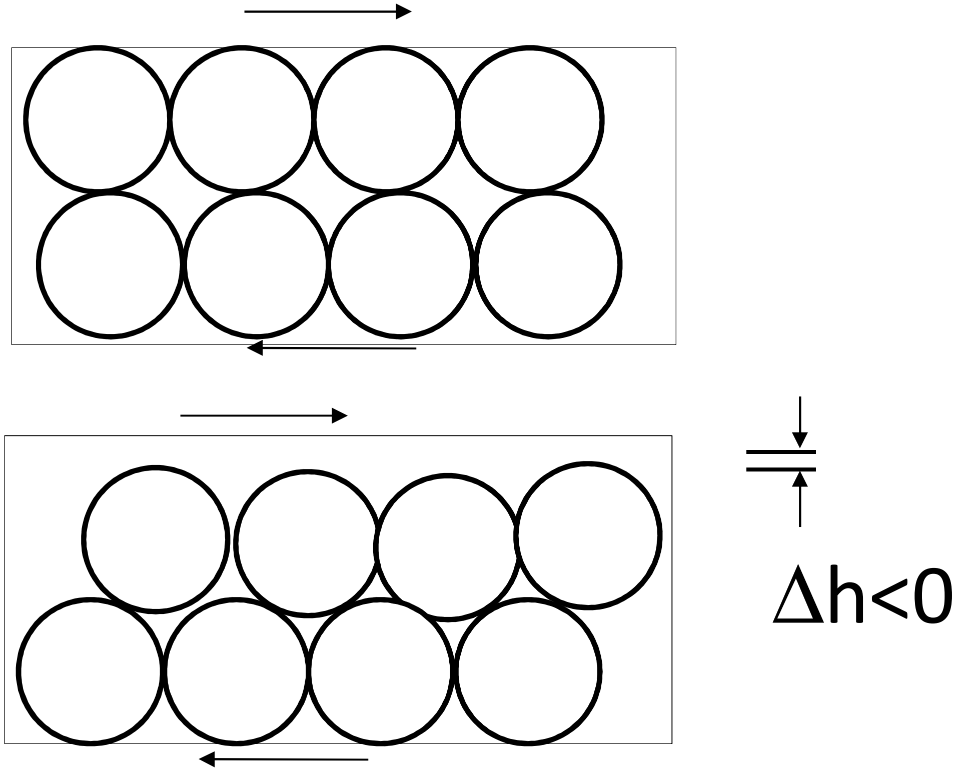
Consolidation & Dilation

Reynolds, 1885

Shear induces volume change in granular media

volume increase = dilation
volume decrease = consolidation (compaction)

Shear of idealized loosely packed GM (actual grains positions disordered)



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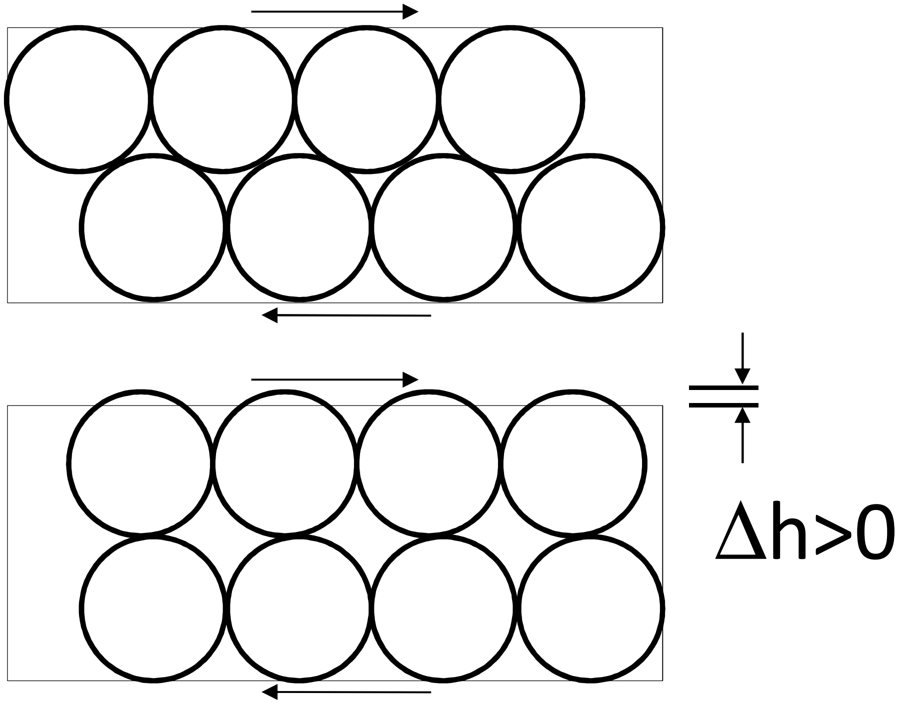
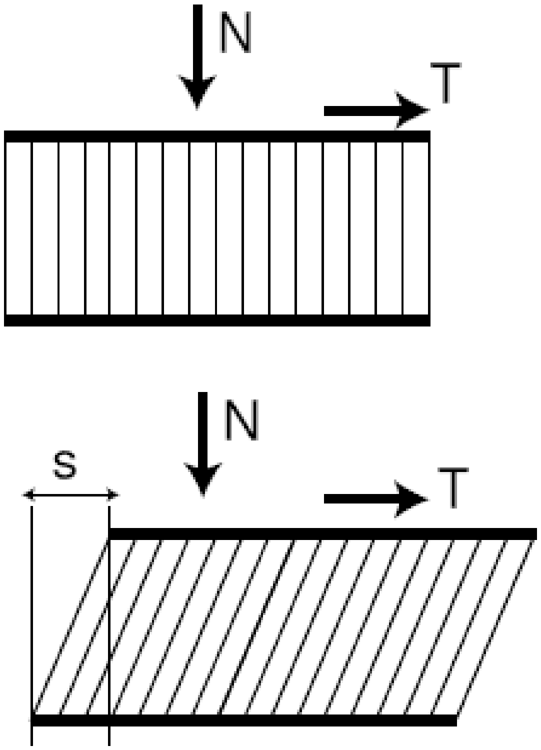


Courtesy Nicole Mazouchova

Dilation

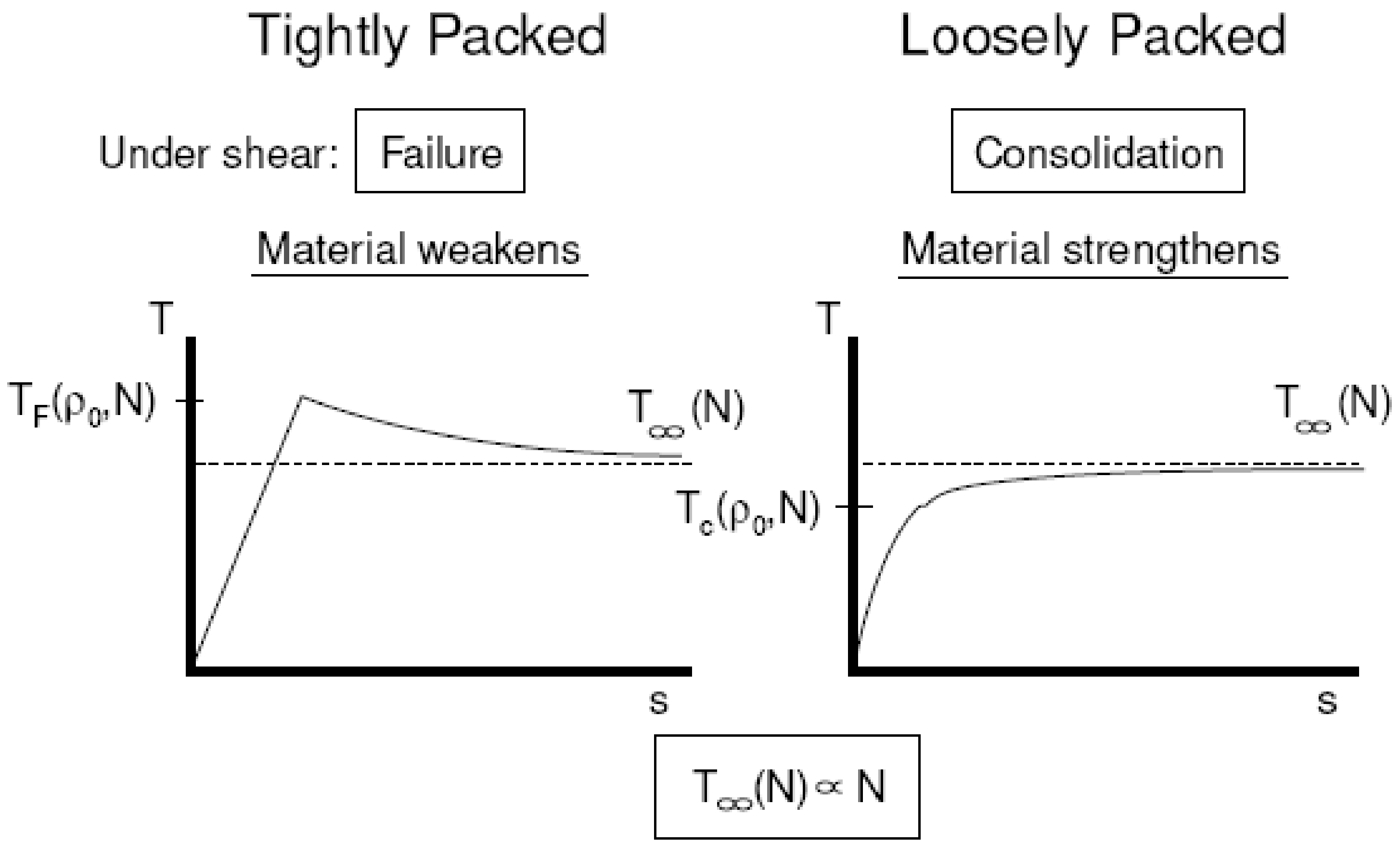
Dilation is responsible for the drying of wet sand during footsteps. Compression induces shear in the bulk which dilates the grains, pulling water into the pore volume and leaving a dry surface.

The basics of shearing GM



Apply normal force N and shear with force T, plates are free to move

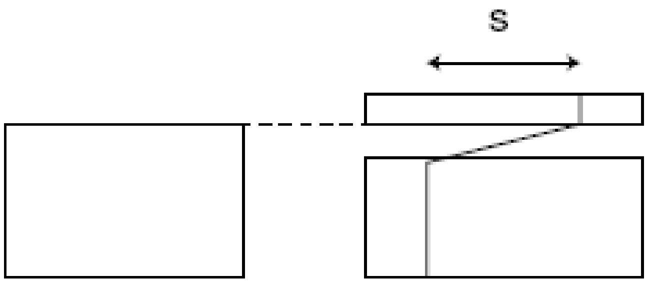
Stress strain for loose and close packed



Dilation vs. compaction

Tightly Packed

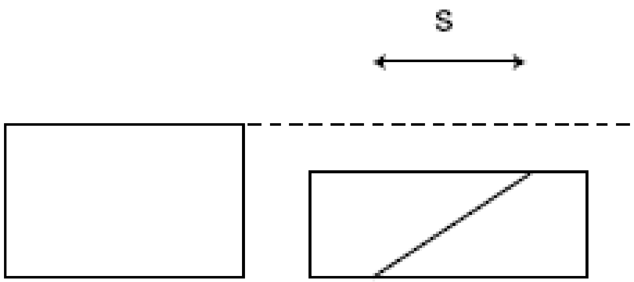
Dilation accompanies shear



Shear confined to thin region

Loosely Packed

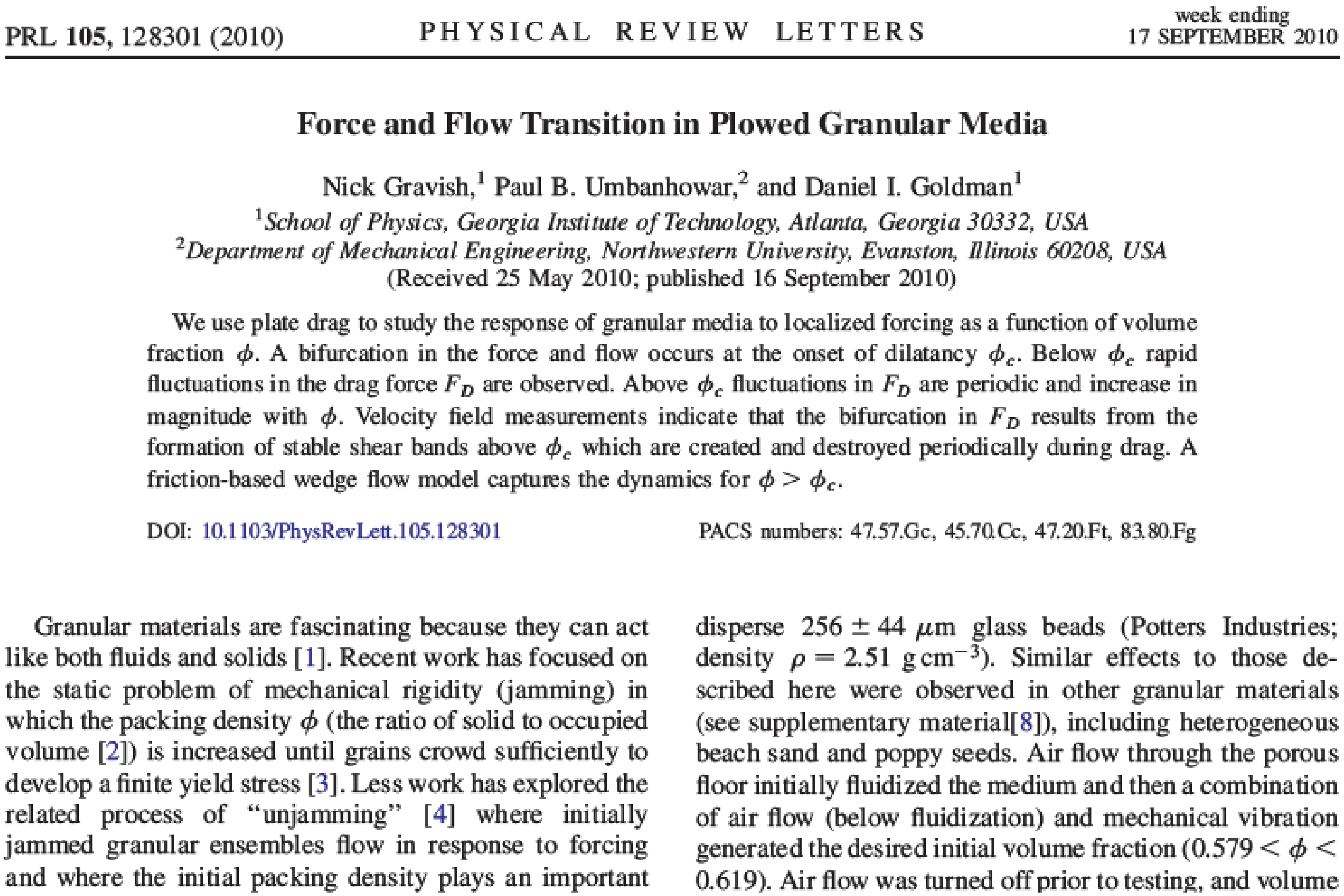
Compaction accompanies shear



Shear spreads throughout

Force and flow in plowed GM

Nick Gravish, Paul Umbanhowar DG, *PRL*, 2010



PRL **105,** 128301 (2010)
PHYSICAL REVIEW LETTERS
week ending
17 SEPTEMBER 2010

Force and Flow Transition in Plowed Granular Media

Nick Gravish,[1] Paul B. Umbanhowar,[2] and Daniel I. Goldman[1]
[1]*School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332, USA*
[2]*Department of Mechanical Engineering, Northwestern University, Evanston, Illinois 60208, USA*

(Received 25 May 2010; published 16 September 2010)

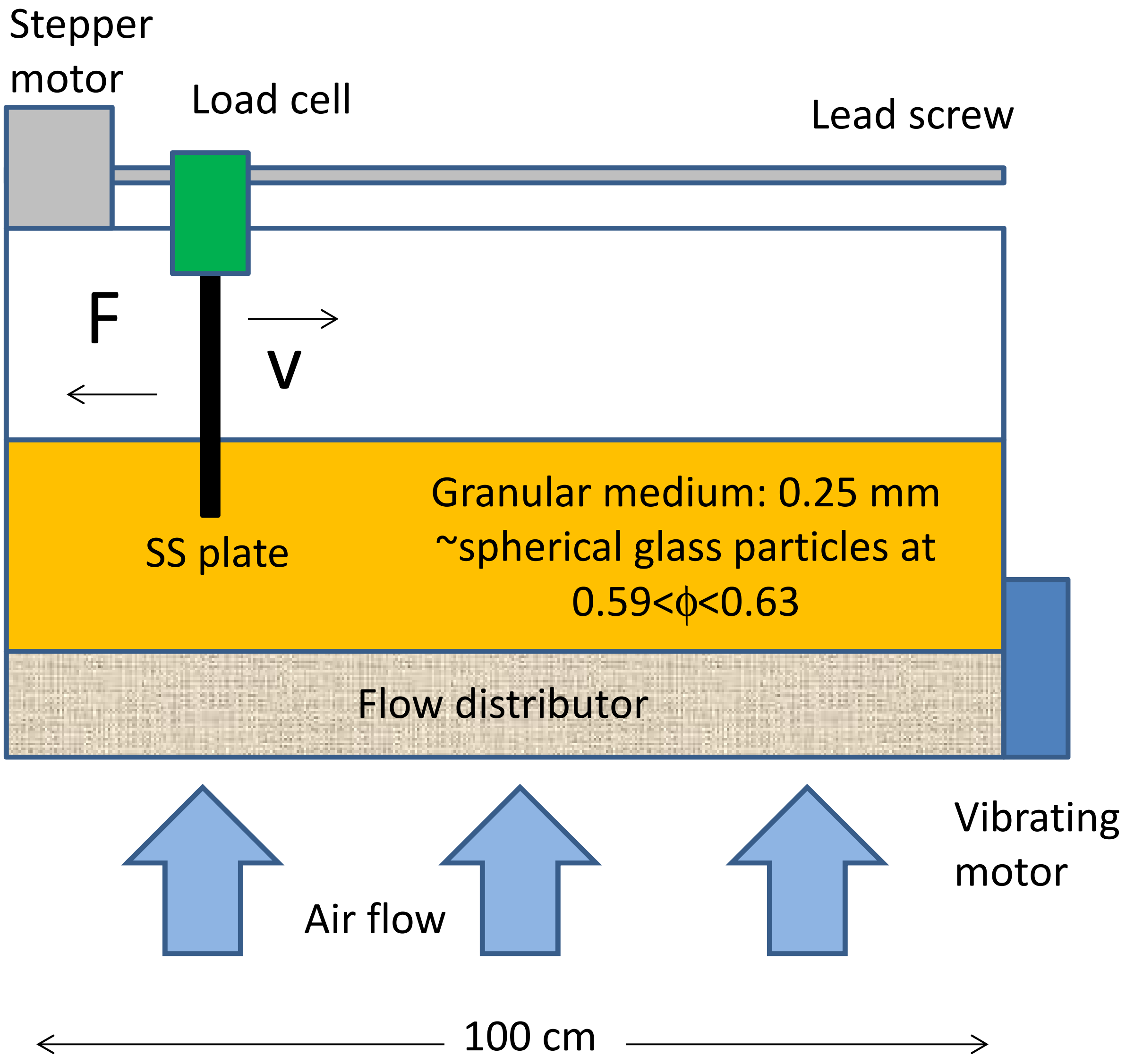
We use plate drag to study the response of granular media to localized forcing as a function of volume fraction ϕ. A bifurcation in the force and flow occurs at the onset of dilatancy ϕ_c. Below ϕ_c rapid fluctuations in the drag force F_D are observed. Above ϕ_c fluctuations in F_D are periodic and increase in magnitude with ϕ. Velocity field measurements indicate that the bifurcation in F_D results from the formation of stable shear bands above ϕ_c which are created and destroyed periodically during drag. A friction-based wedge flow model captures the dynamics for $\phi > \phi_c$.

DOI: 10.1103/PhysRevLett.105.128301
PACS numbers: 47.57.Gc, 45.70.Cc, 47.20.Ft, 83.80.Fg

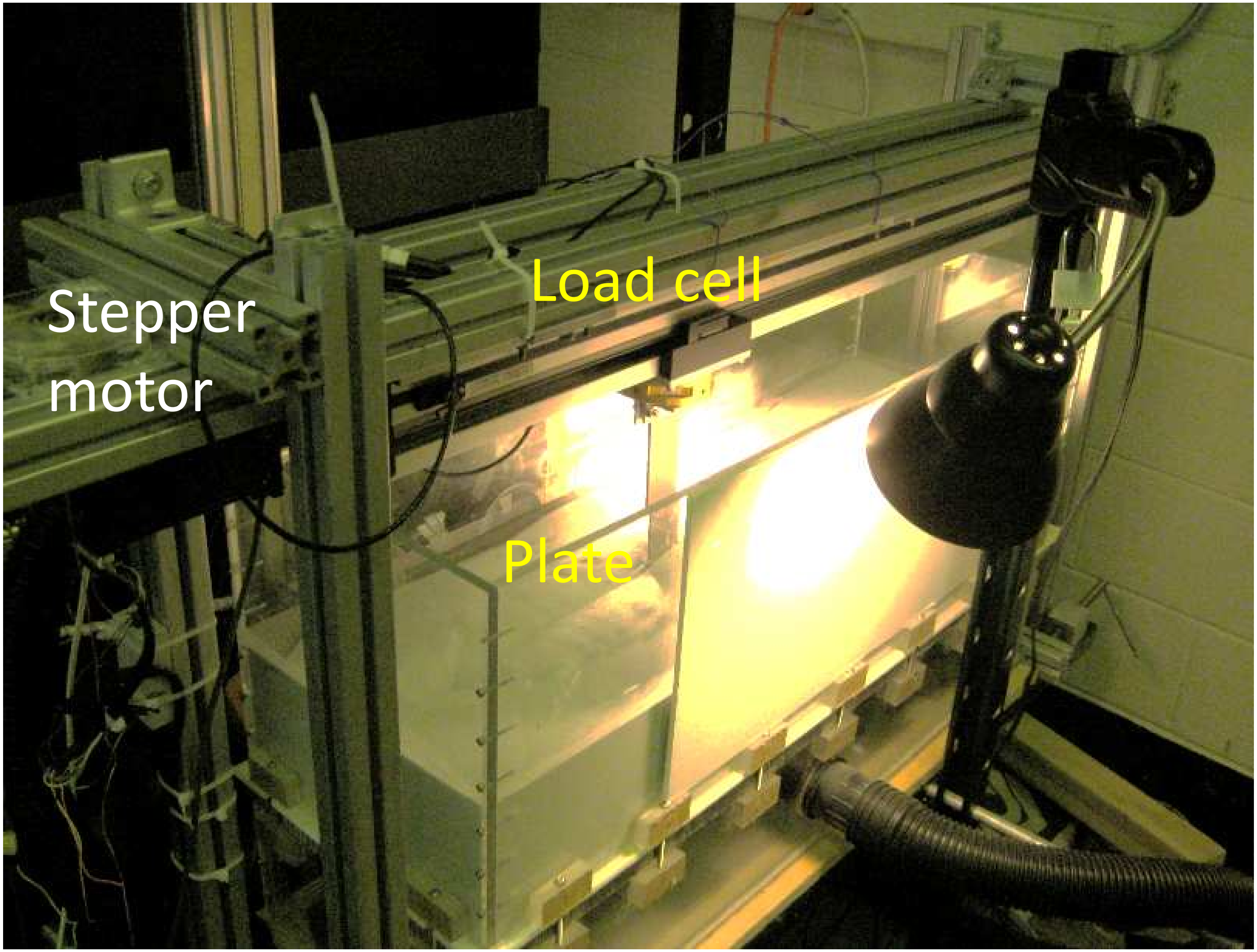
Granular materials are fascinating because they can act like both fluids and solids [1]. Recent work has focused on the static problem of mechanical rigidity (jamming) in which the packing density ϕ (the ratio of solid to occupied volume [2]) is increased until grains crowd sufficiently to develop a finite yield stress [3]. Less work has explored the related process of "unjamming" [4] where initially jammed granular ensembles flow in response to forcing and where the initial packing density plays an important

disperse 256 ± 44 μm glass beads (Potters Industries; density $\rho = 2.51$ g cm^{-3}). Similar effects to those described here were observed in other granular materials (see supplementary material[8]), including heterogeneous beach sand and poppy seeds. Air flow through the porous floor initially fluidized the medium and then a combination of air flow (below fluidization) and mechanical vibration generated the desired initial volume fraction ($0.579 < \phi <$ 0.619). Air flow was turned off prior to testing, and volume

Localized intrusion (plate drag) experiment



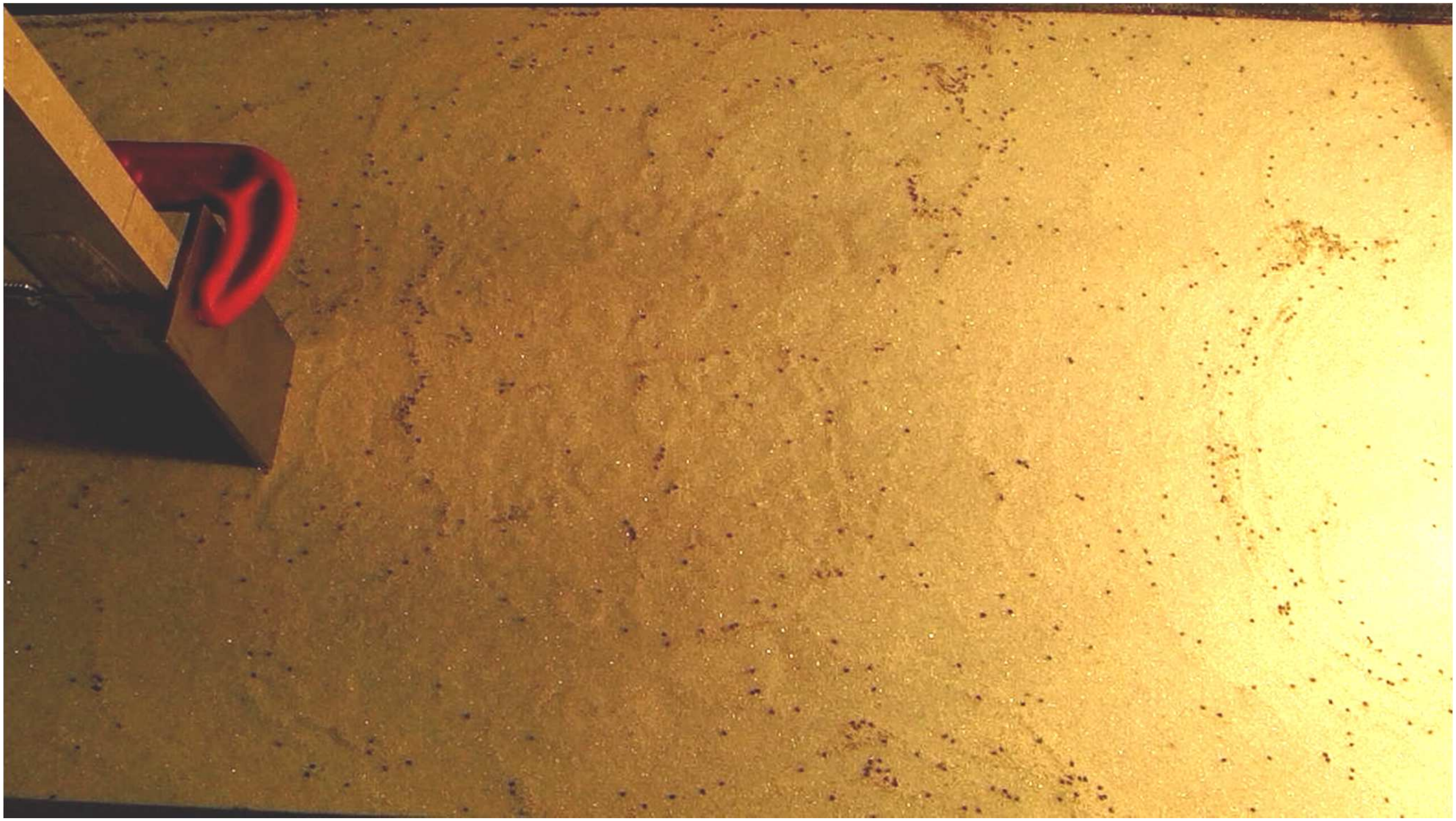
- •Prepare media through fluidization/vibration to initial packing fraction ϕ
- • 250± 44 μm polydisperse glass beads
- •Drag flat plate at 2 - 8 cm/s over 50cm (3.8cm width, 10cm depth)
- •Measure drag force @100Hz
- •Measure surface profile of granular wake


Stepper motor
Load cell
Plate

Example drag experiment

25cm drag at 2cm/s in loose packed GM

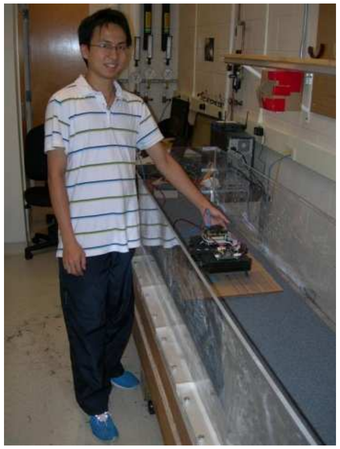
Black poppy seeds to emphasize flow

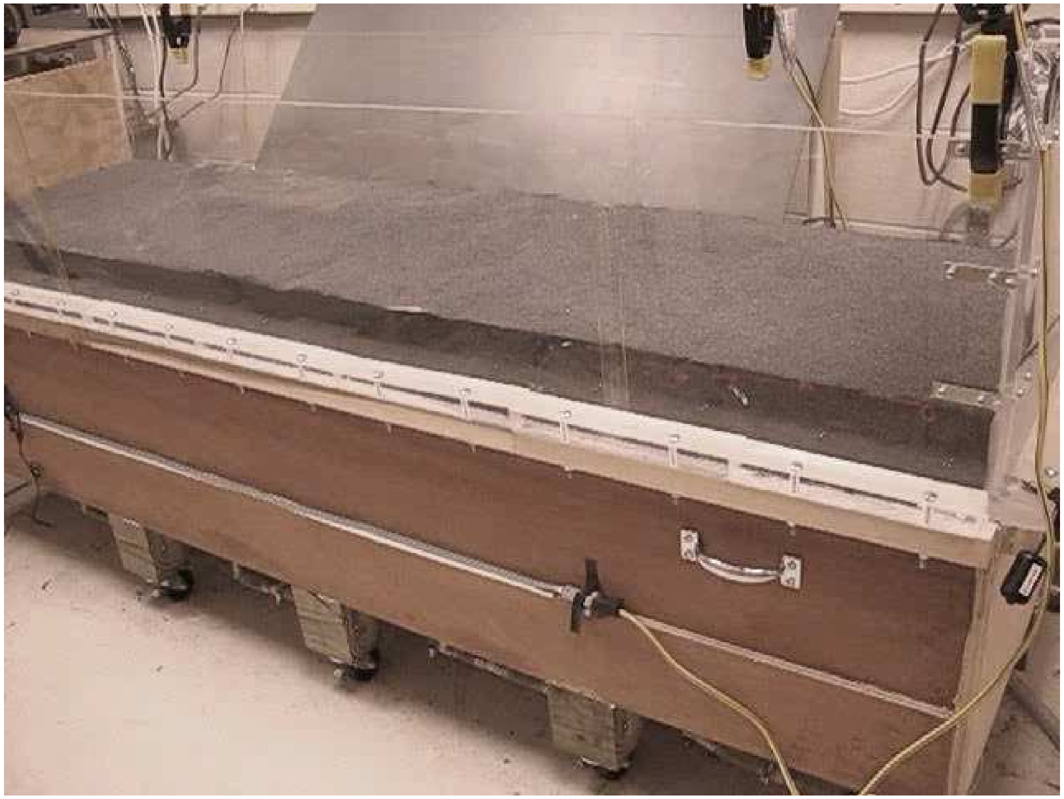


Control of GM using a fluidized bed

100 kg of
~1 mm
poppy
seeds

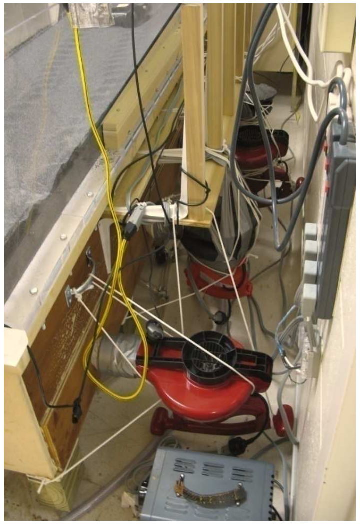
Chen Li





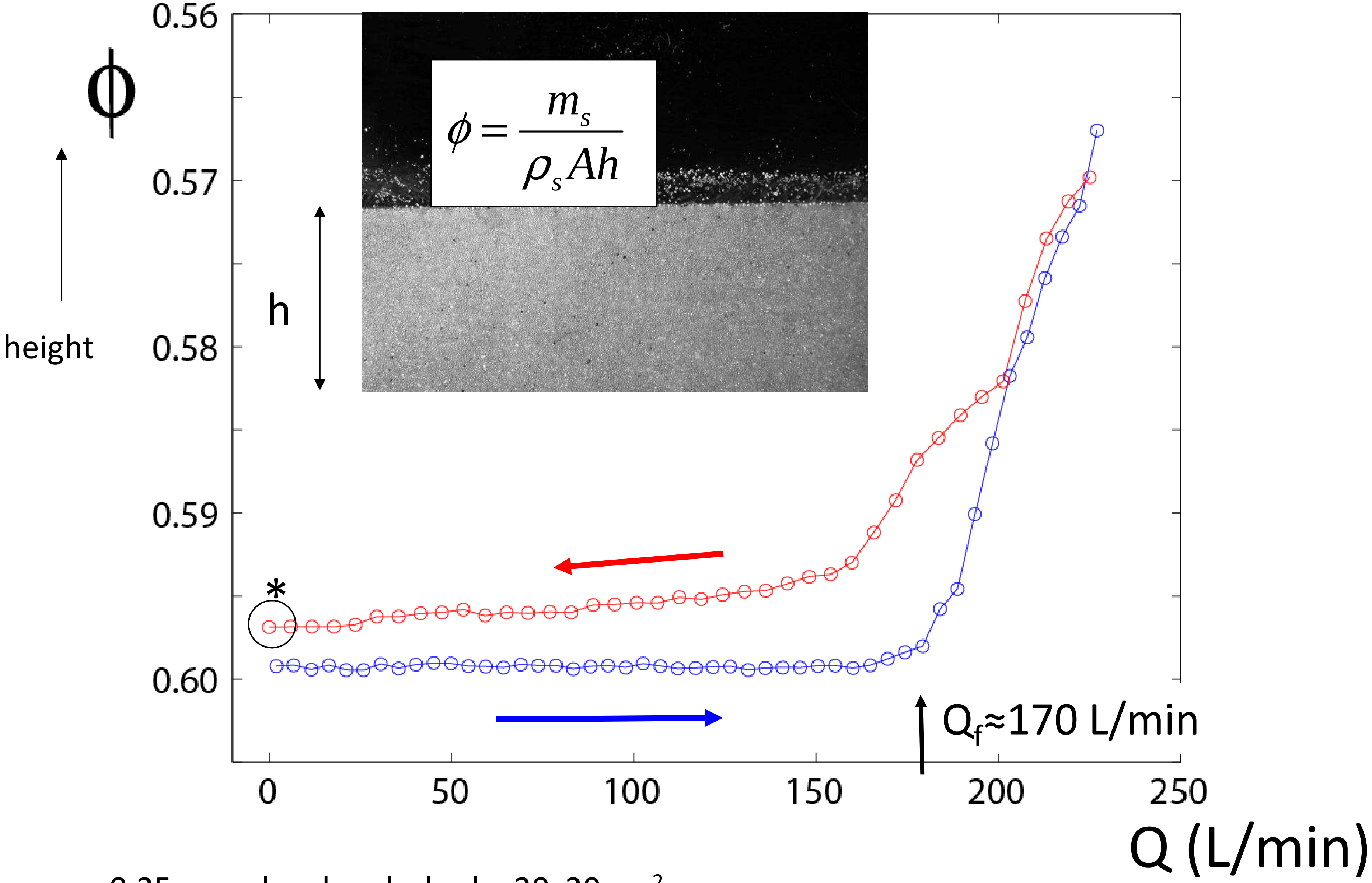
2.5 m

Leaf blowers



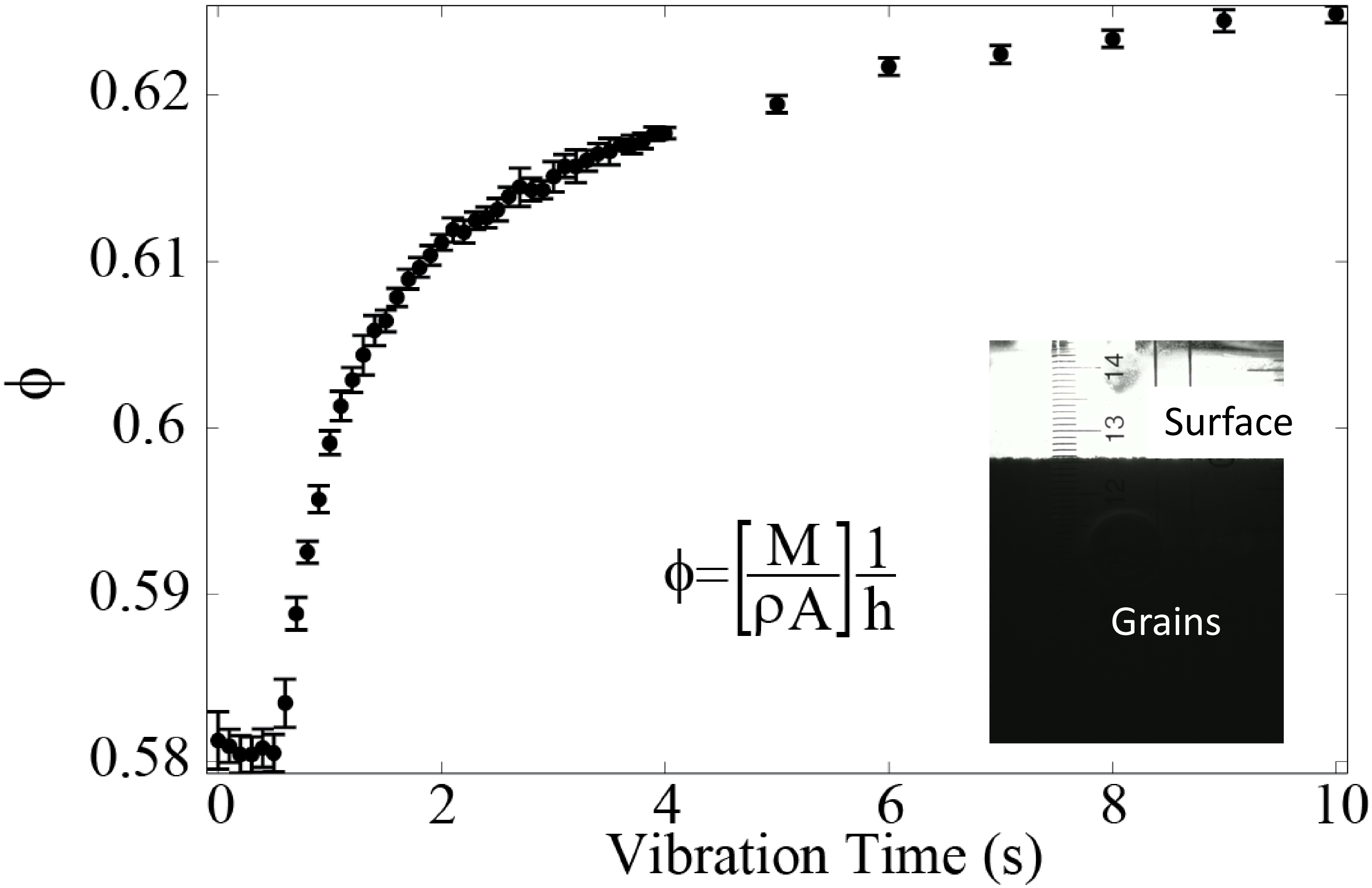
Li, Umbanhowar, Komsuoglu, Koditschek, Goldman, *PNAS*, 2009
Malden, Ding, Li, Goldman, *Science*, 2009
Umbanhowar & Goldman, PRE-R ,2010
Gravish, Umbanhowar, Goldman, *PRL* ,2010

Volume fraction, ϕ, vs. flow rate, Q



0.25 mm glass beads, bed = 20x20 cm^2

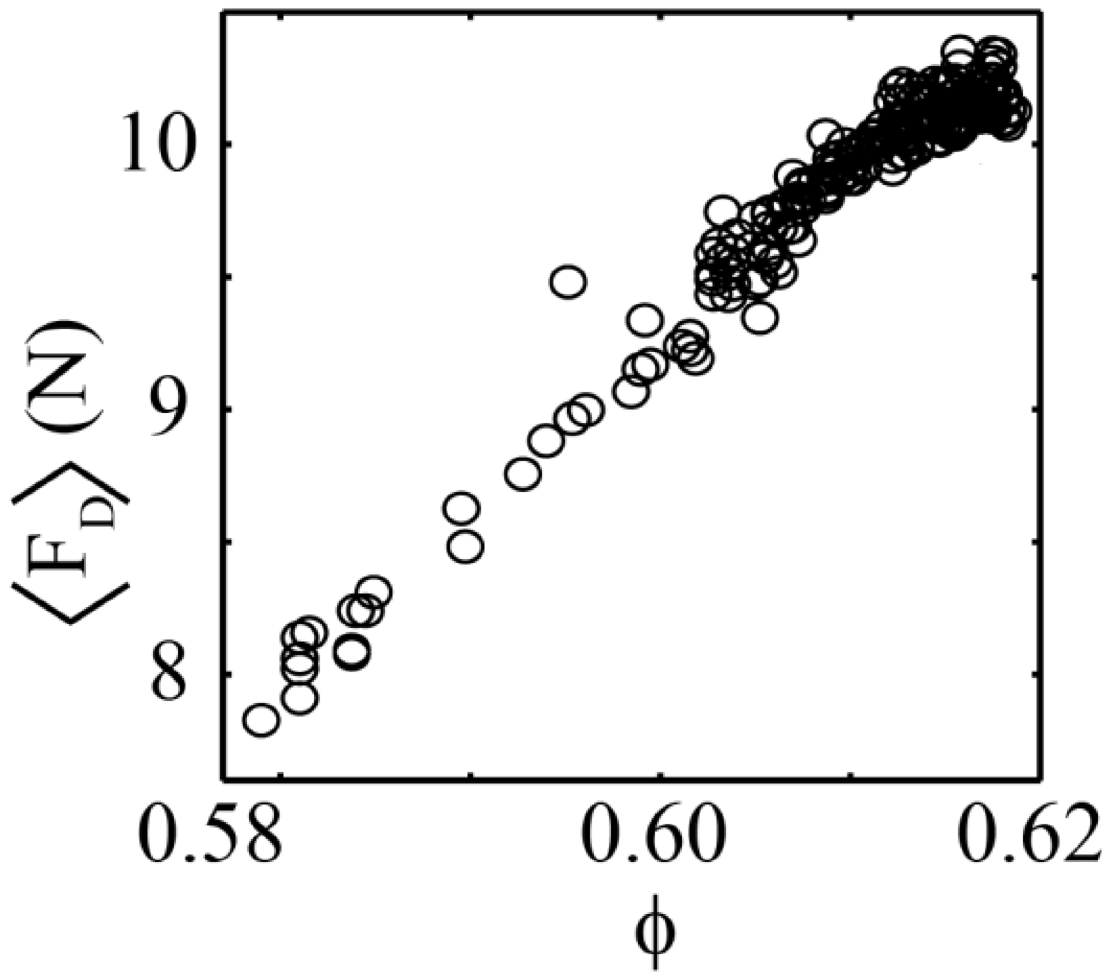
Volume fraction preparation



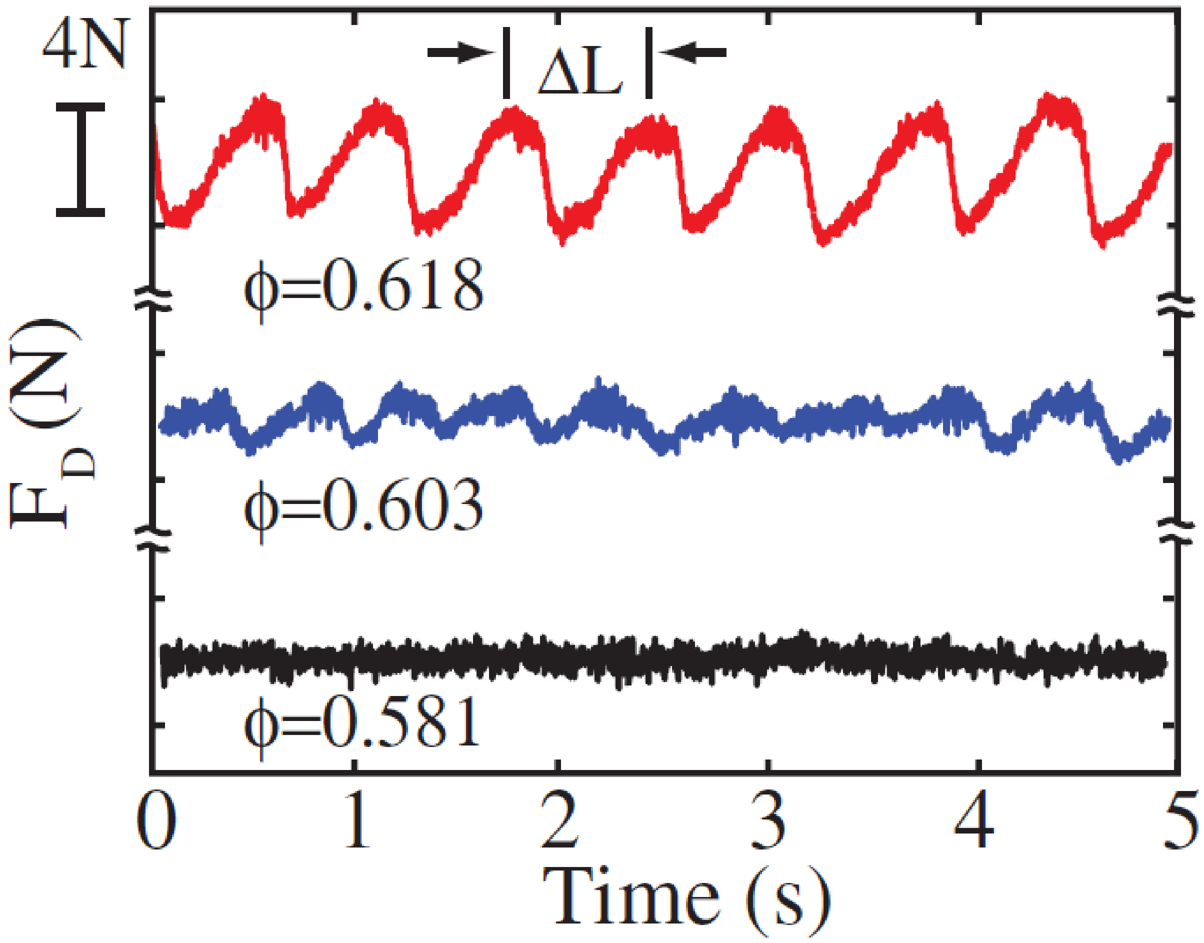
$$\phi=\left[\frac{M}{\rho A}\right]\frac{1}{h}$$

0.25 mmk glass beads

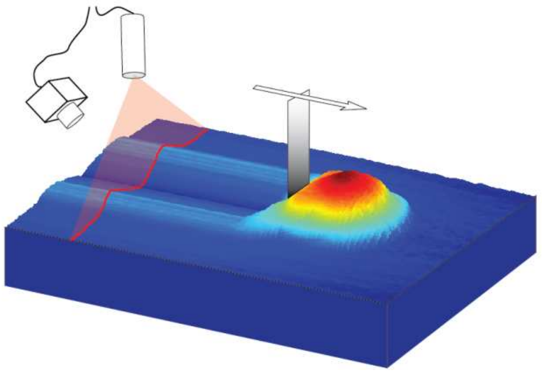
Mean drag force



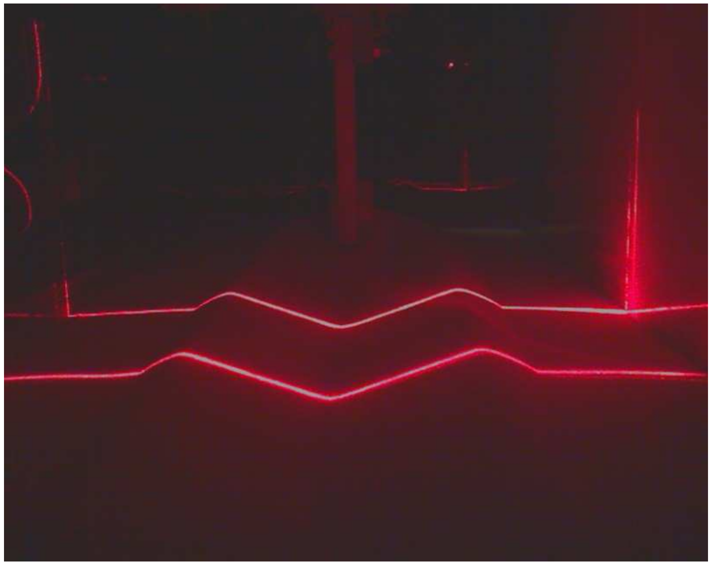
Fluctuations in drag force

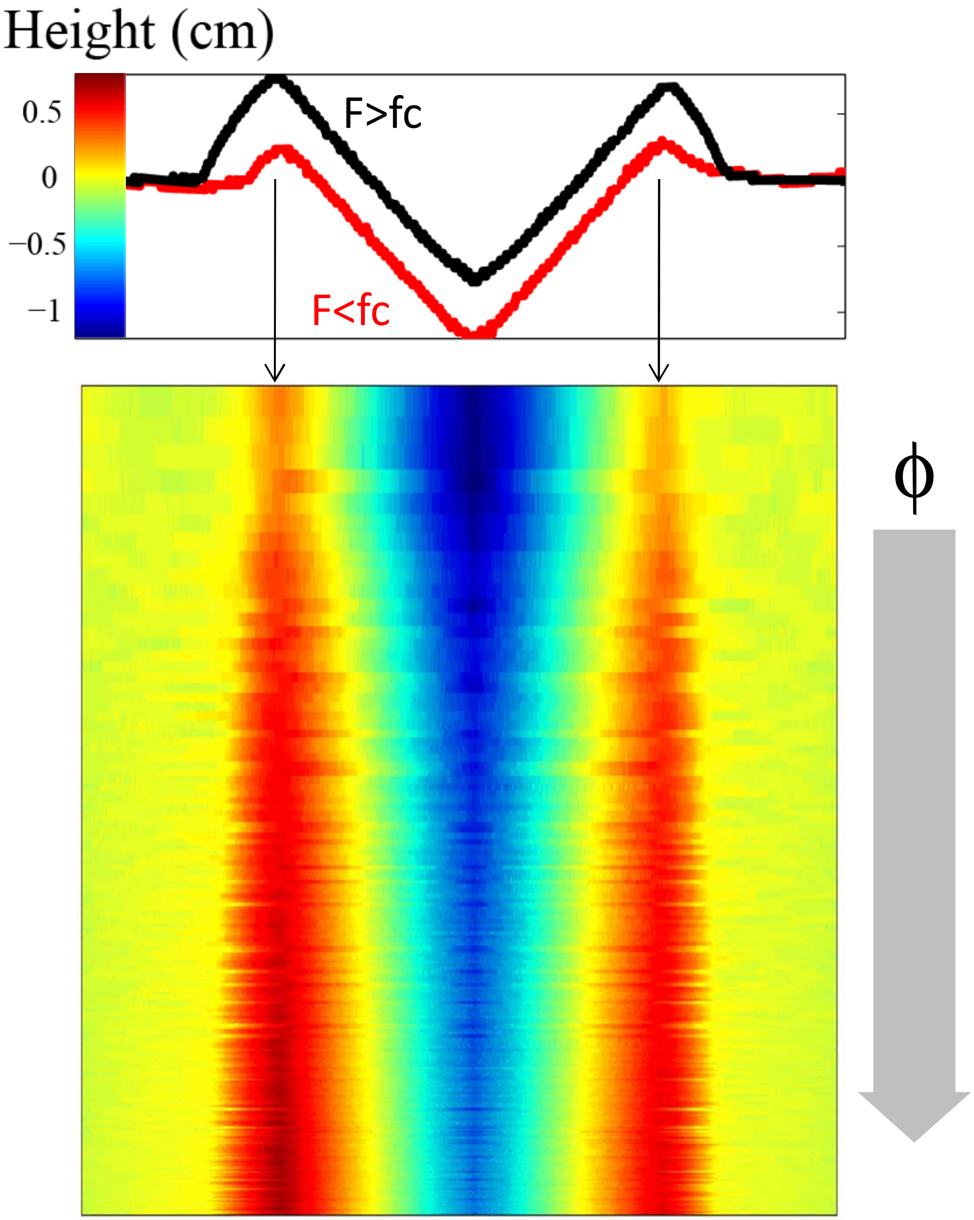


Volumetric change

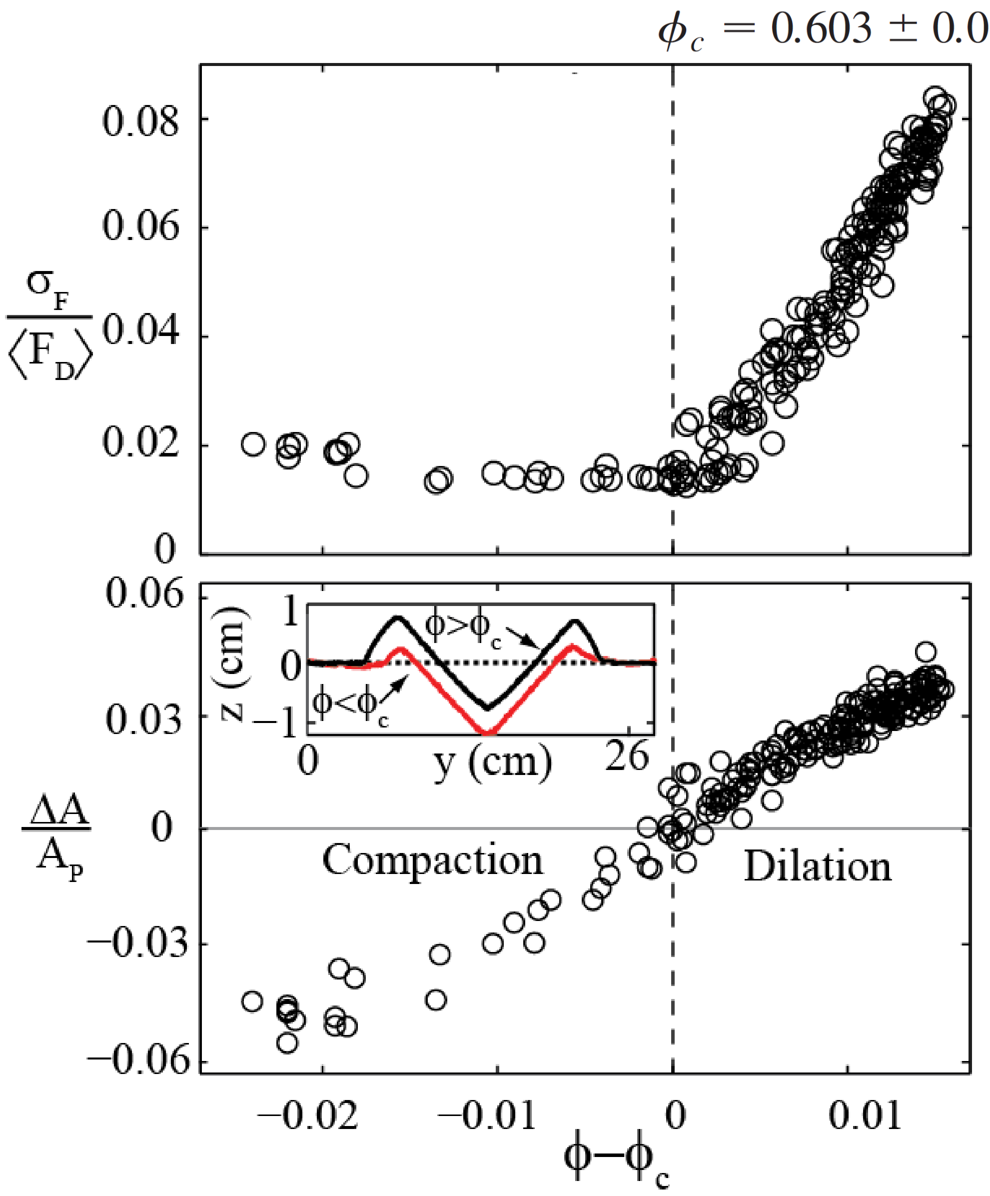


We measure the surface profile of the granular wake using laser sheets.





Drag fluctuations and volumetric

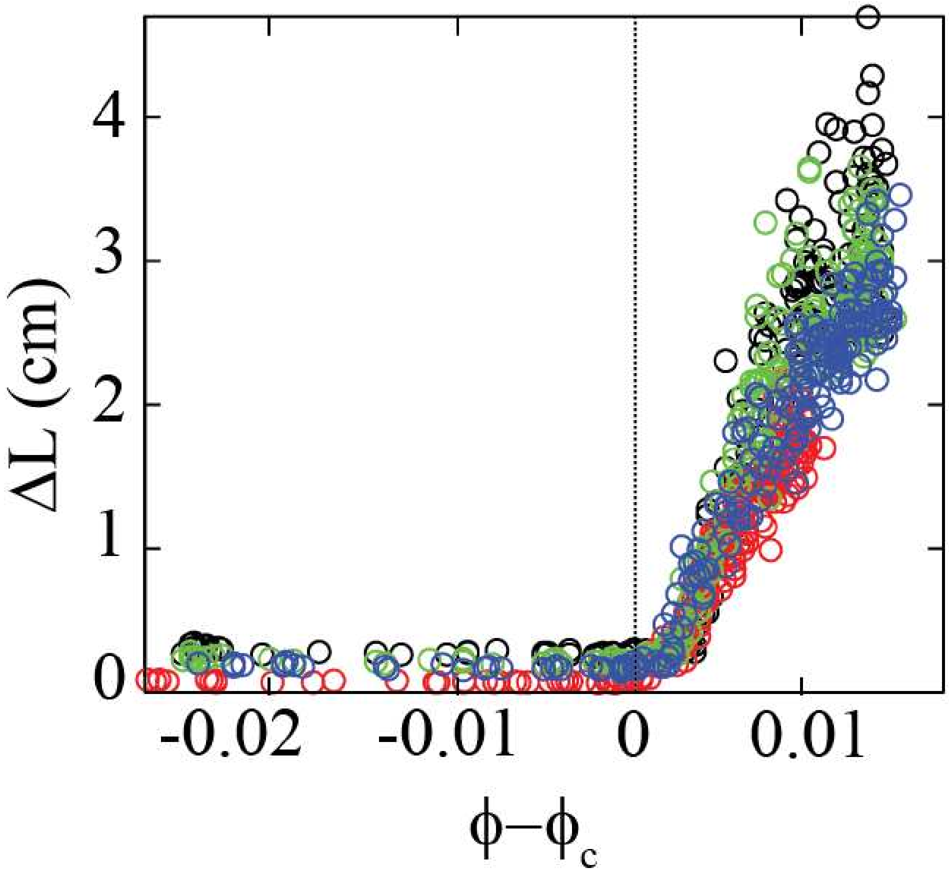


Quantify fluctuations in F_D as RMS of drag force σ_F

Volume change from area of wake pattern

Results

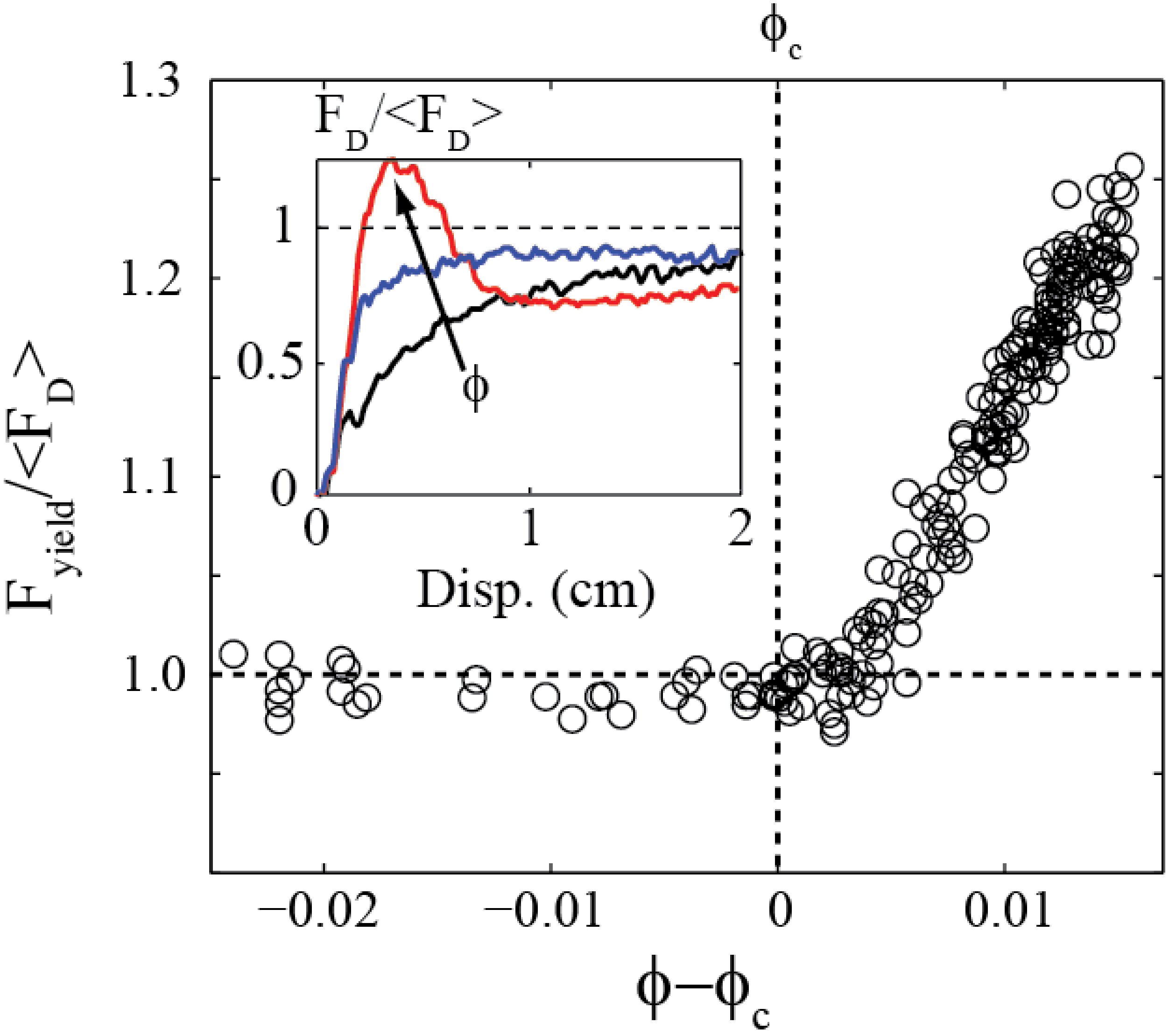
v = 2,4,6,8 cm s^{-1} in red, blue, green, black



Lack of v dependence implies that ΔL is a length scale.

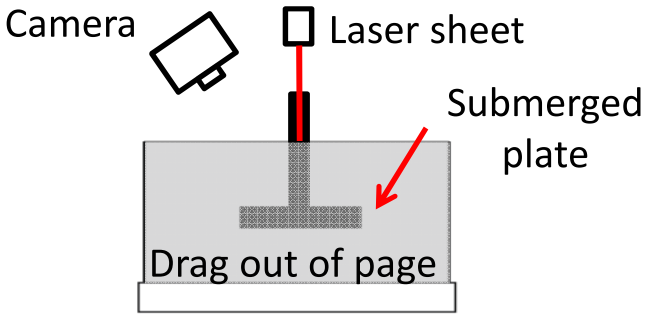
We now study the granular flow to understand oscillations in F_D

Yield force



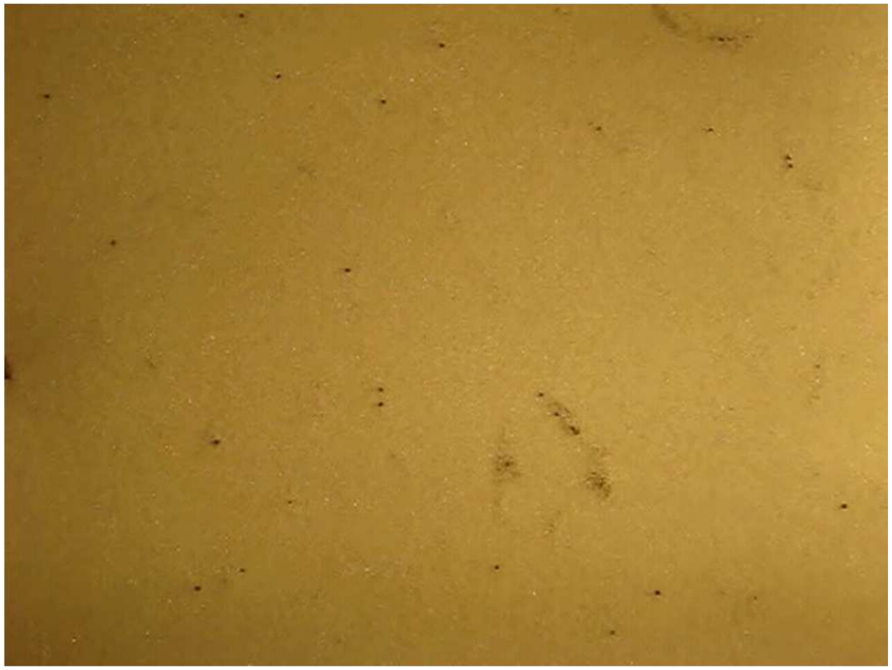
ϕ_c
$F_D/\langle F_D \rangle$
1
0.5
0
0
1
2
Disp. (cm)
ϕ
1.3
1.2
1.1
1.0
$F_{yield}/\langle F_D \rangle$
−0.02
−0.01
0
0.01
$\phi-\phi_c$

Surface flow



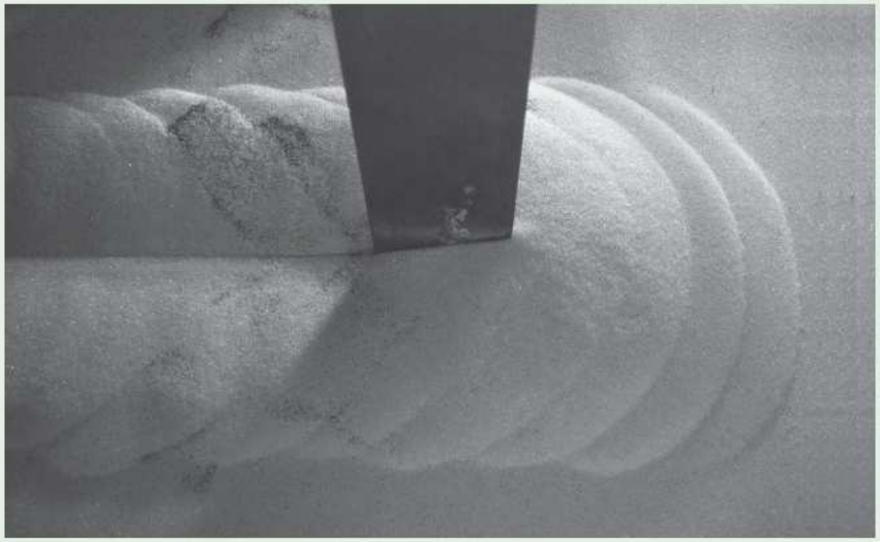
Loose pack ($\phi < \phi_c$)

Close pack ($\phi > \phi_c$)



PHYSICAL REVIEW LETTERS

Articles published week ending 17 SEPTEMBER 2010

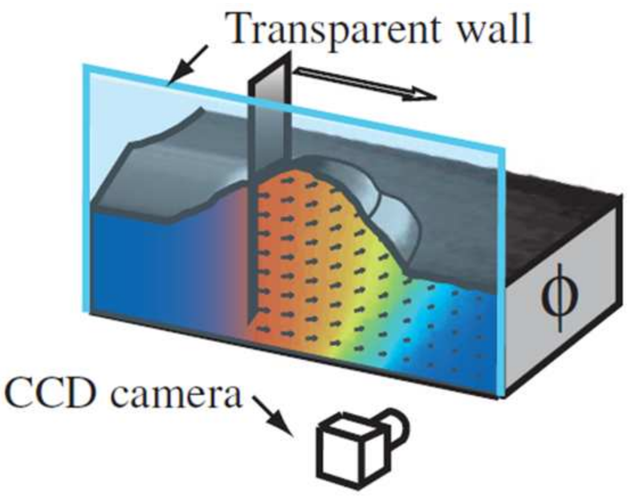


Force and Flow Transition in Plowed Granular Media
Nick Gravish, Paul B. Umbanhowar, and Daniel I. Goldman

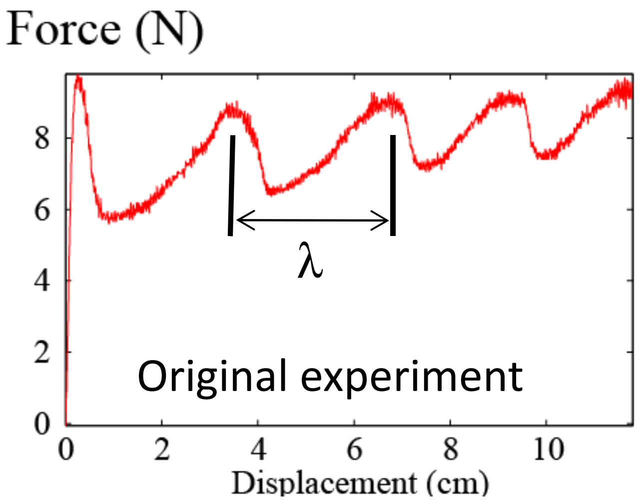
Published by the
American Physical Society

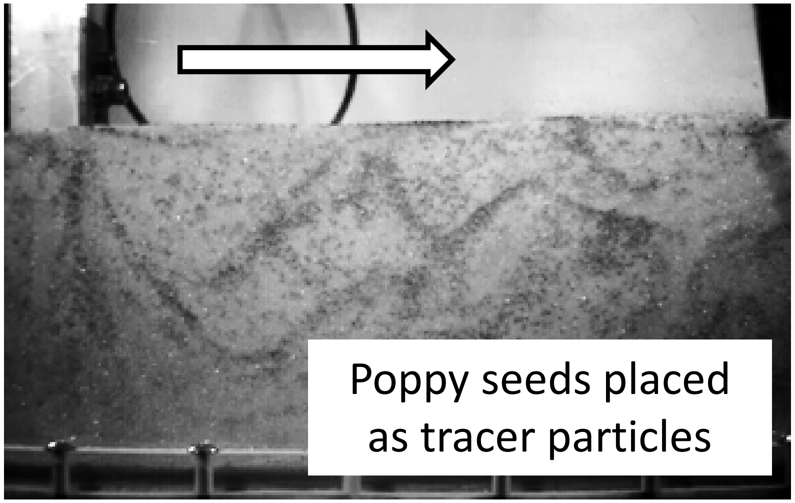
Volume 105, Number 12

Wall drag experiments

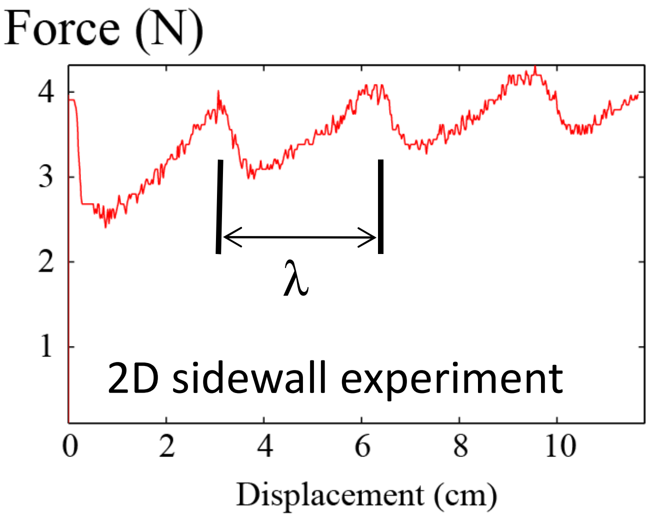


Wall bounded experiment



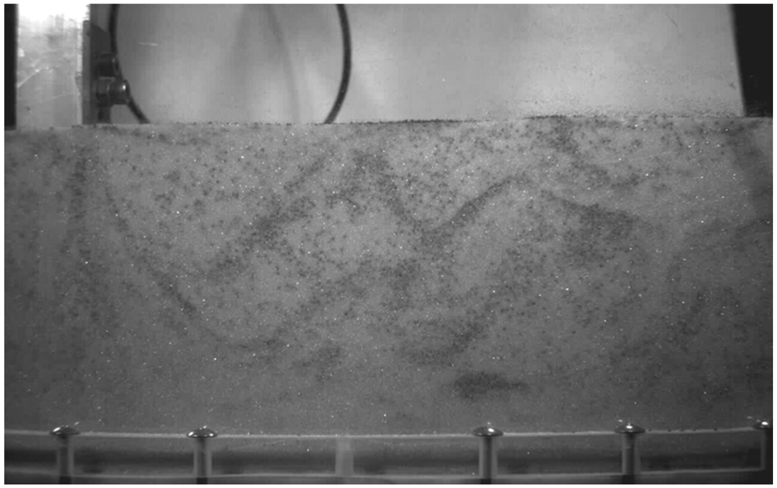


Poppy seeds placed as tracer particles

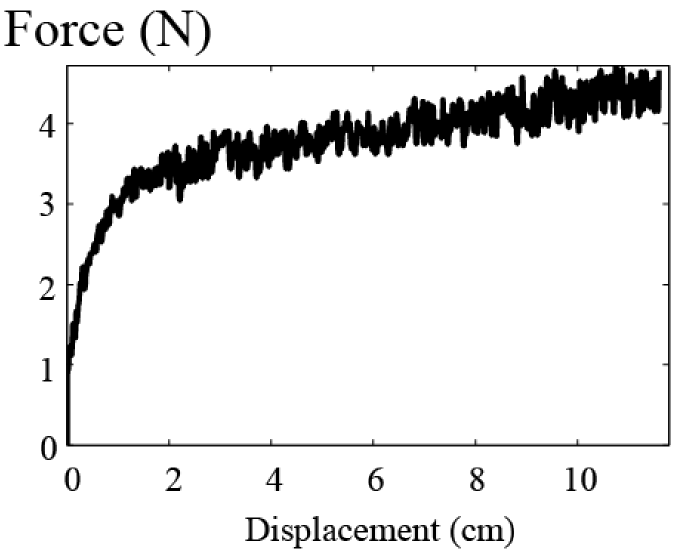


Visualizing granular flow during drag

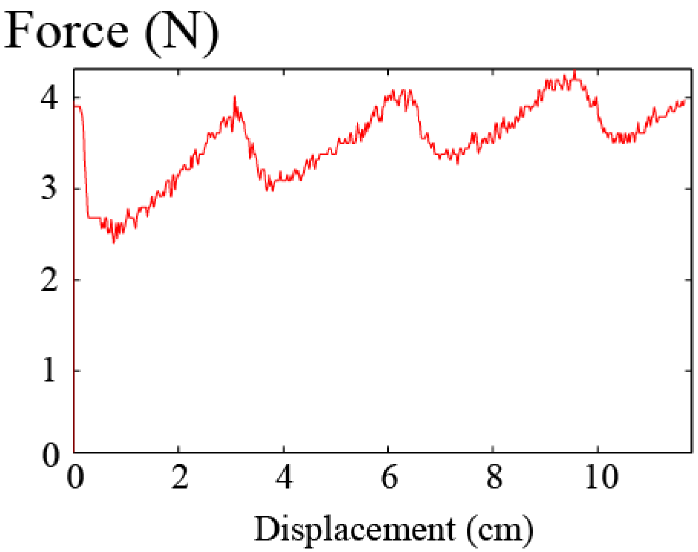
Loose pack ($\phi < \phi_c$)



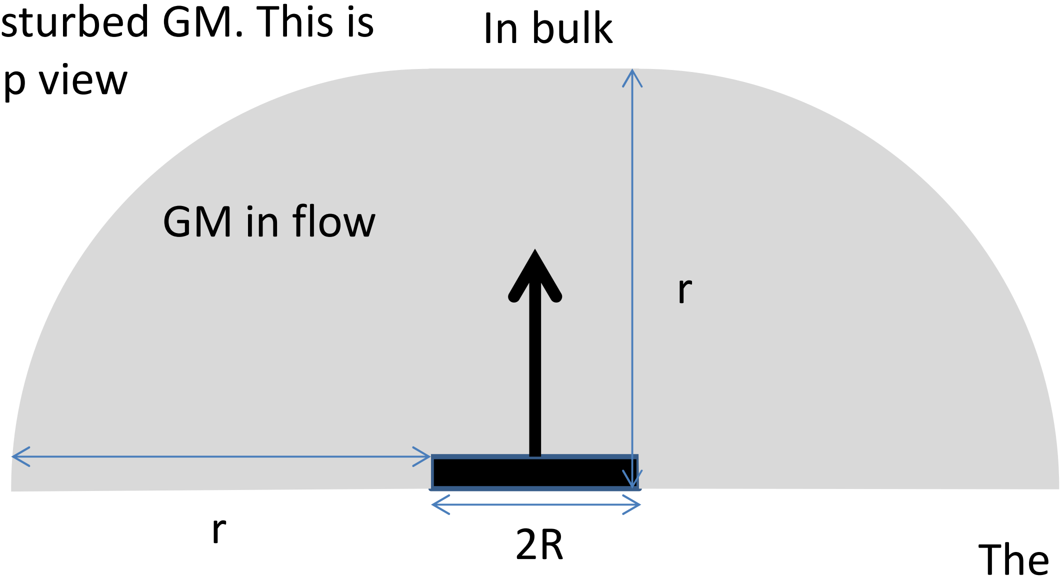
4 cm



Close pack ($\phi > \phi_c$)

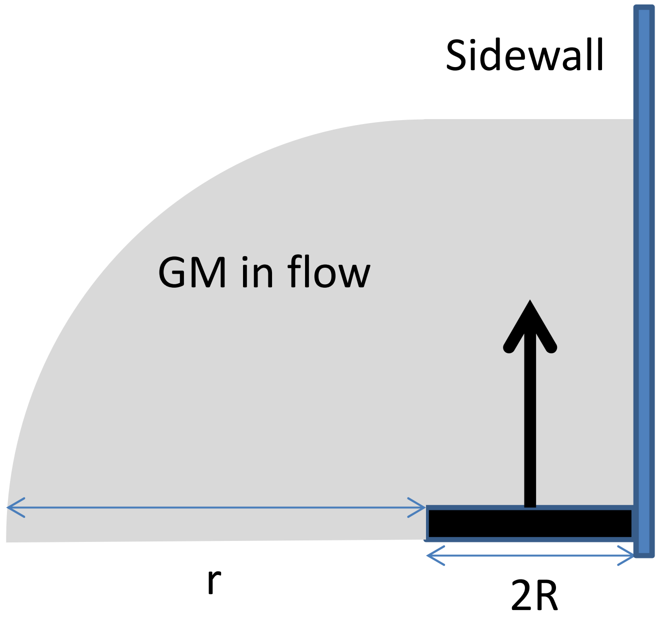


Black is drag plate, gray
Is disturbed GM. This is
A top view

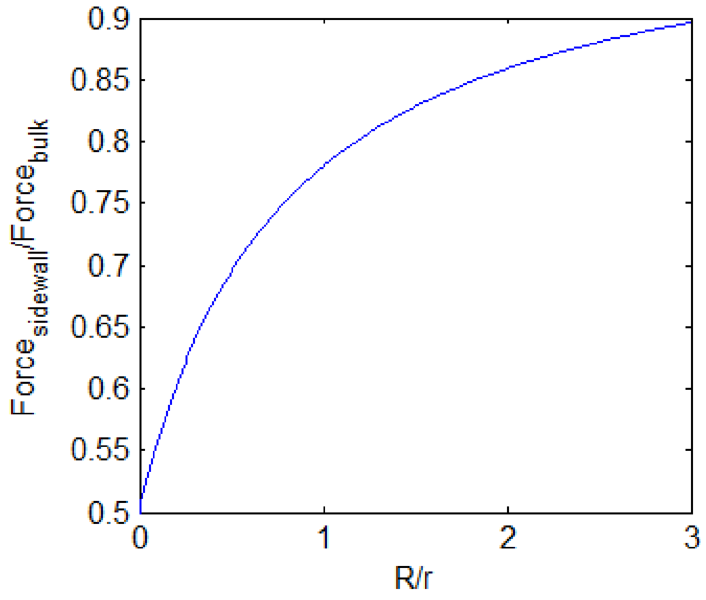


Total disturbed region in bulk
A_b = (1/2) * pi * r^2 + 2*R*r

Total disturbed region in sidewall
A_s = (1/4) * pi * r^2 + 2*R*r



The ratio of bulk and sidewall force versus R/r. For r>R (expected in drag expt) we see that force should decrease by 0.5. As r becomes bigger force approaches 1



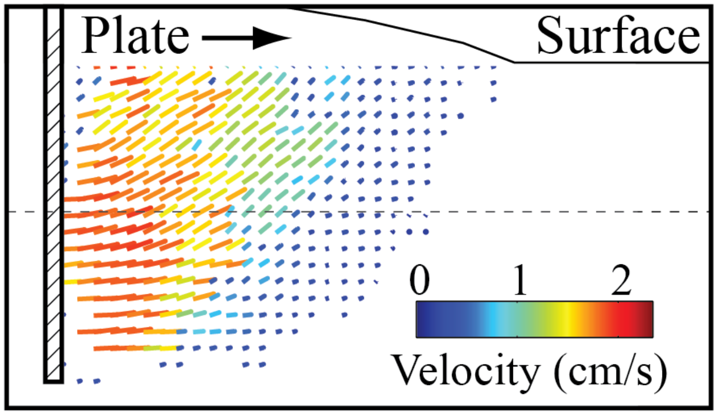
Visualizing granular flow during drag

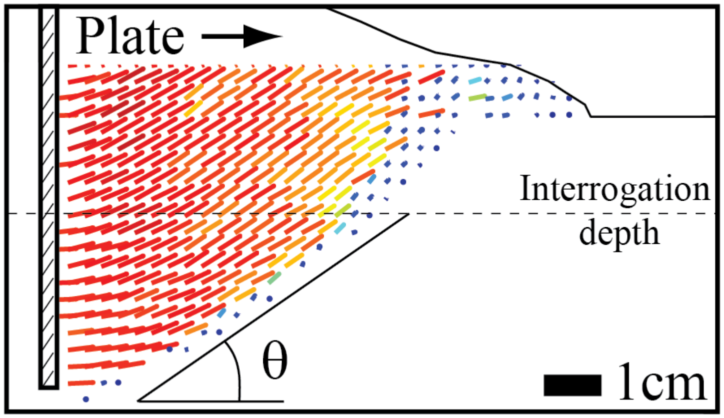
Loose pack ($\phi < \phi_c$)

2 cm/s

4 cm

Close pack ($\phi > \phi_c$)

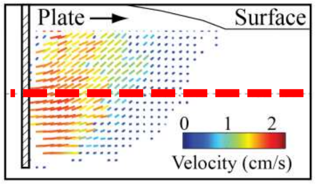


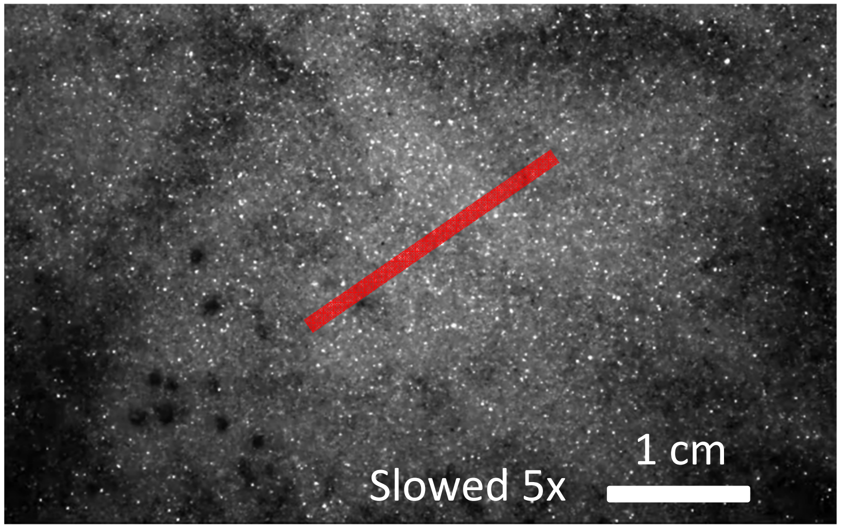


Open source PIV software (OSIV; Least-squares cross-corr., http://osiv.sourceforge.net/)

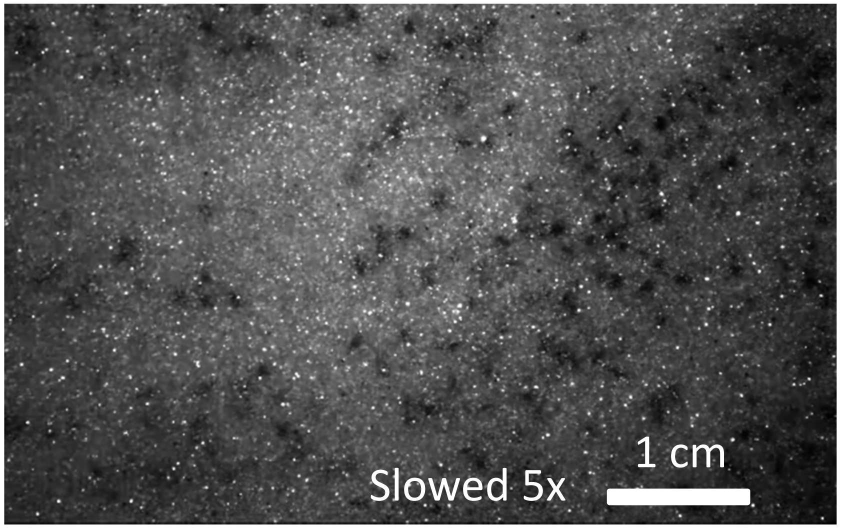
Thanks Mike Shatz

Visualizing granular flow during drag





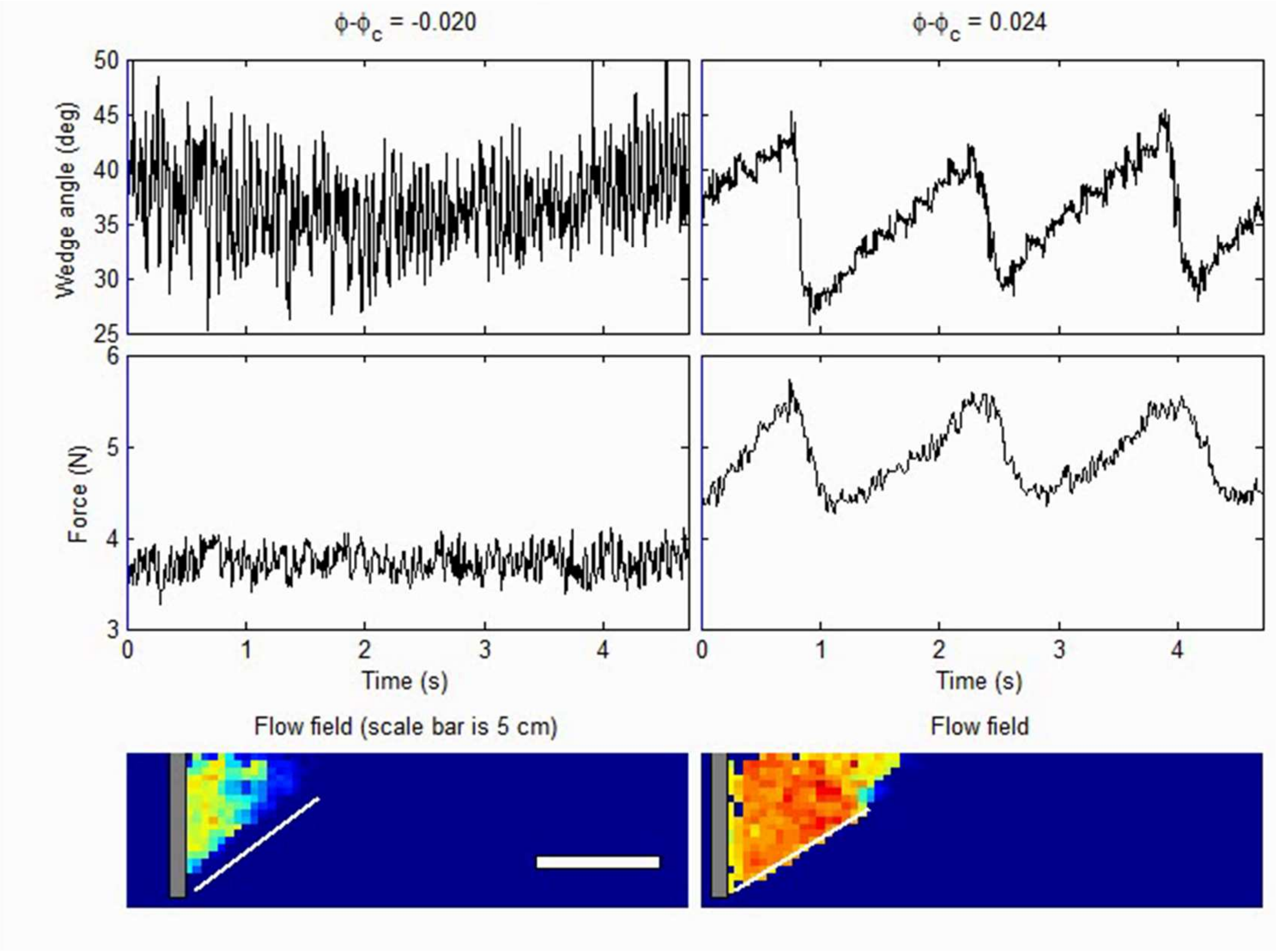
Solid like wedge flows up a solid like base separated by stationary shear band.



Rapid fluctuations in the velocity field ahead of flowing regions which moves with plate

Force and flow correlation

At close pack wedge angle is correlated with drag force



A model to understand force fluctuations

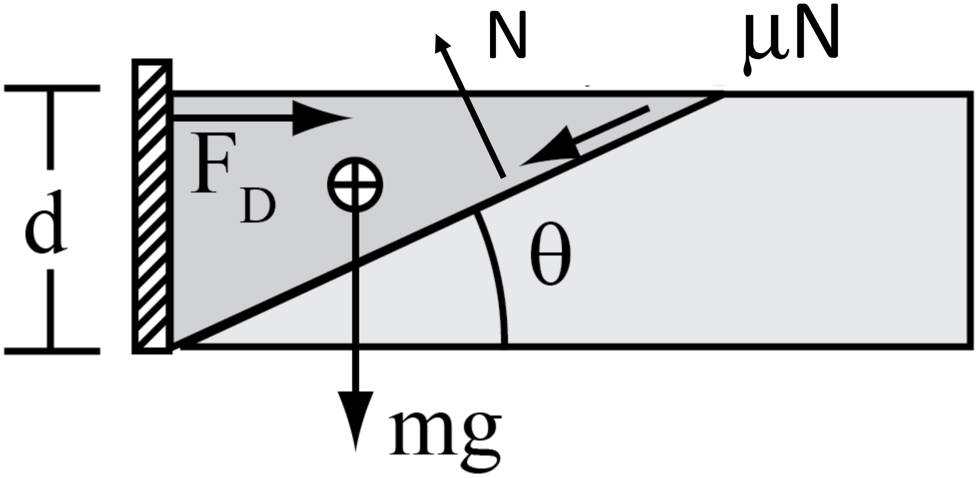
Consider the force required to push solid block of mass m up an inclined plane of angle θ.

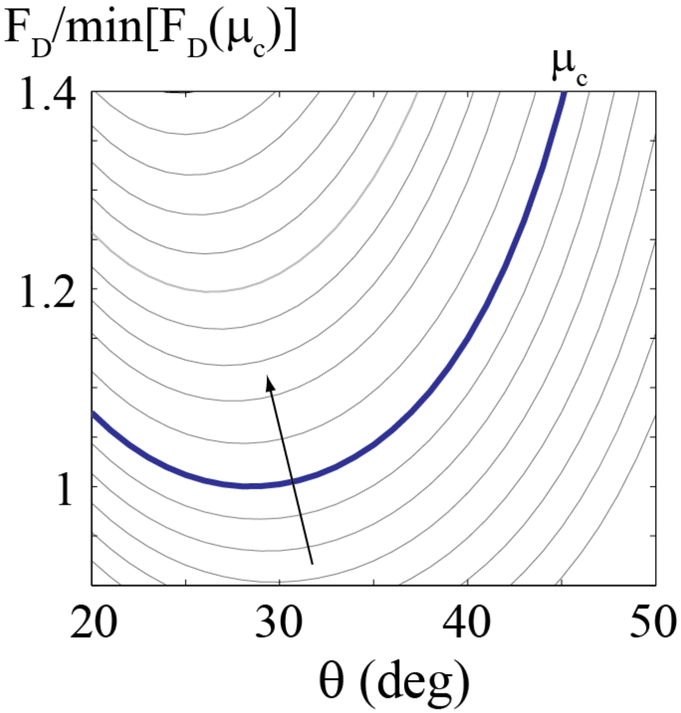
W=plow width

$$m = \frac{\rho W d^2}{2\tan\theta}$$

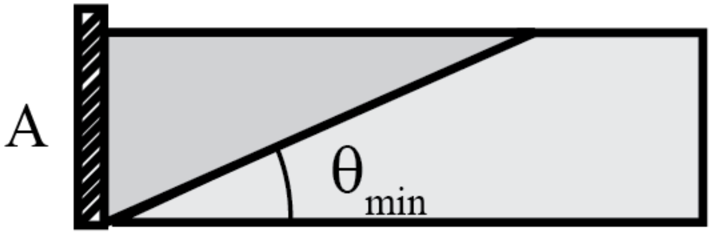
$$F_D(\theta, \mu) = \frac{\rho w d^2}{2}\frac{1+\mu/\tan(\theta)}{1-\mu\tan(\theta)}\,g$$

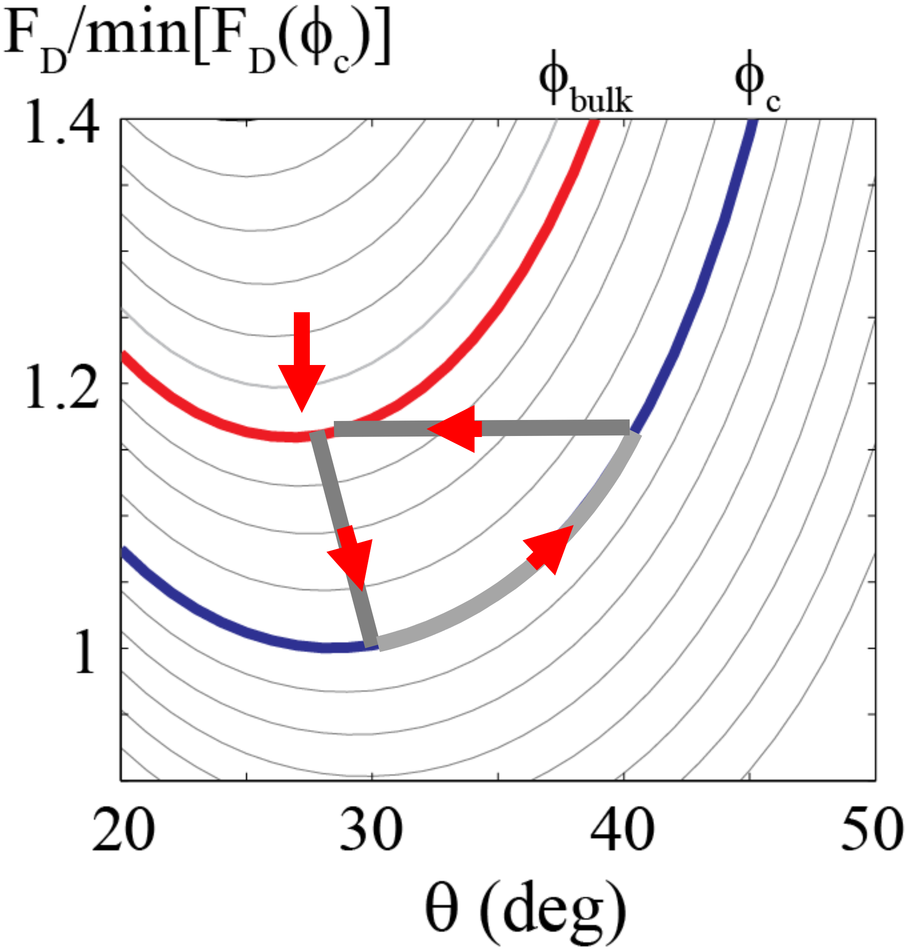
Assumptions for $\phi>\phi_c$:
– Sheared GM dilates to ϕ_c
– Force is balanced and minimized
–μ increases with ϕ.





A model to understand force fluctuations





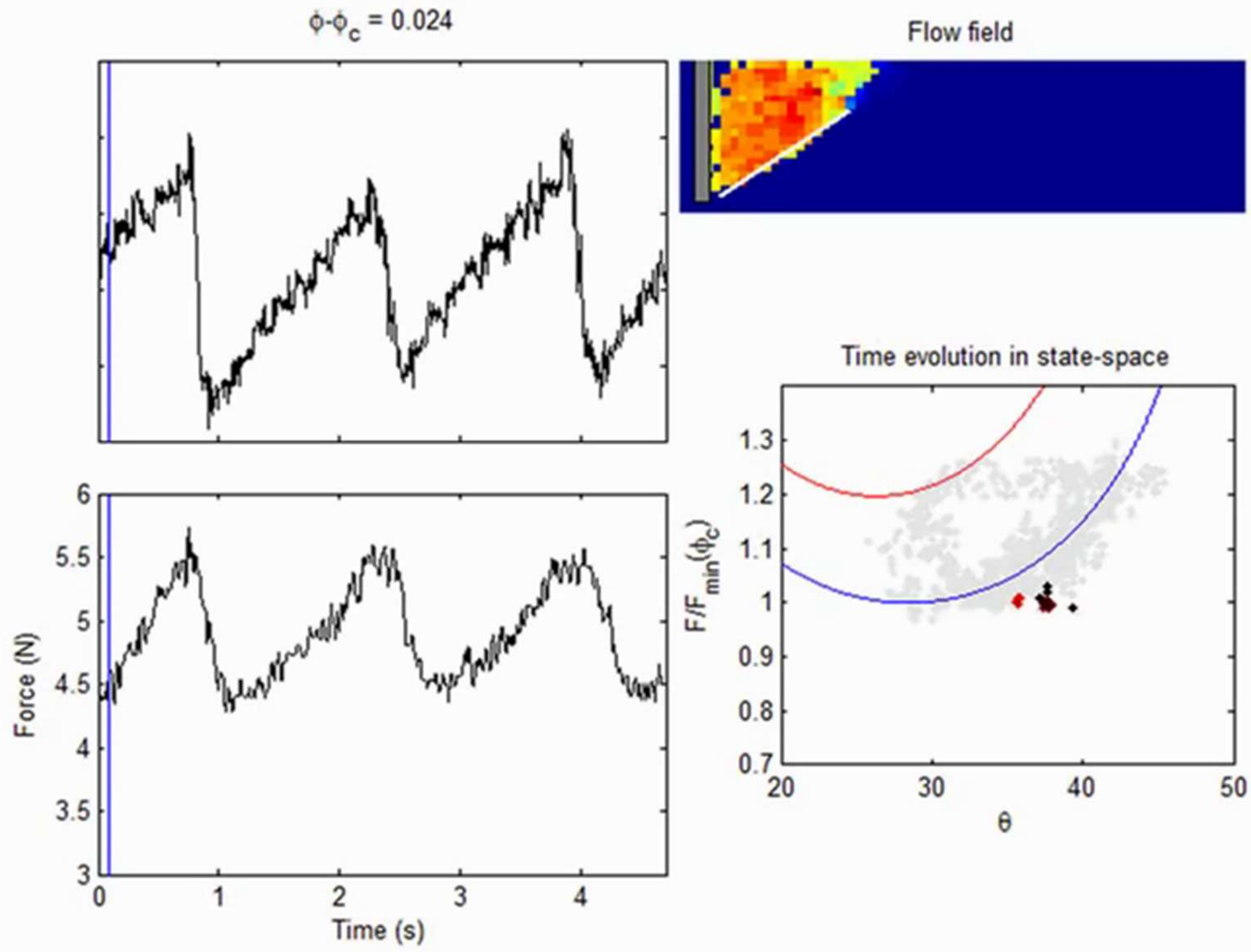
Model evolution in experiment

40

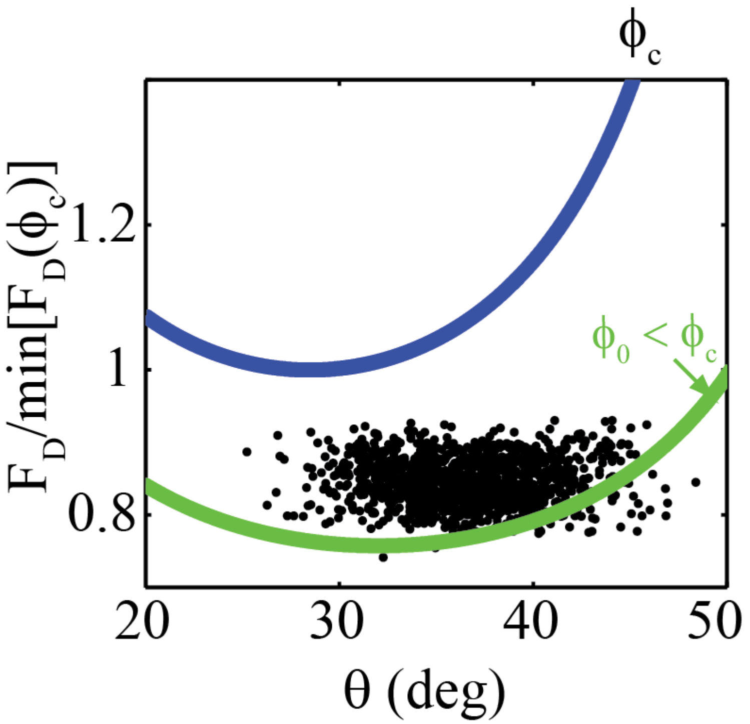
Angle (deg)

30

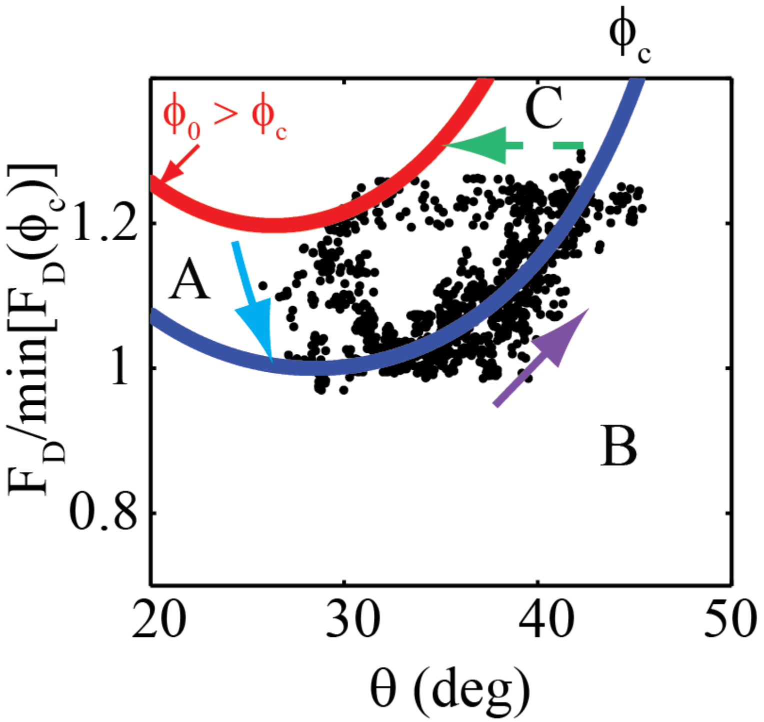
Force (N)



Model agreement at close pack



Loose pack strengthens and compacts under shear.



Close pack weakens and dilates under sheared.

Drag in granular media: conclusions

- Dilation transition in granular media controls flow and force response in drag

- Loose packed GM strengthens and compacts under shear

- Close packed GM weakens and dilates
  - Weakened shear planes are stable and their periodic nucleation gives rise to large force fluctuations → non hydrodynamic behaviors

- Must understand heterogeneous evolution of ϕ during localized perturbation.

END