Quantum Optics with Electrical Circuits: Strong Coupling Cavity QED

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ARDA KECK FOUNDATION PACKARD FOUNDATION CRSNG NSERC

‘Circuit QED’

Blais et al.

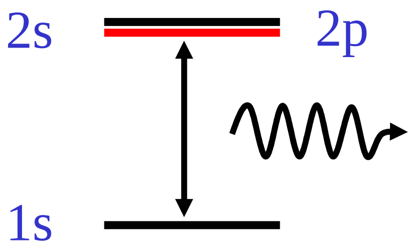
Phys. Rev. A **69**, 062320 (2004)

Wallraff et al.

[cond-mat/0407325]

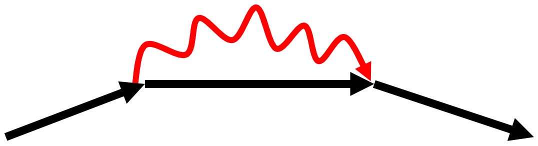
Nature (in press)

Atoms Coupled to Photons

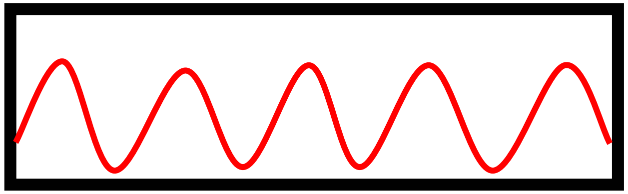


Irreversible spontaneous decay into the photon continuum:

$$2p \rightarrow 1s + \gamma \qquad T_1 \sim 1 \text{ ns}$$



Vacuum Fluctuations:
(Virtual photon emission and reabsorption)
Lamb shift lifts 1s 2p degeneracy



Cavity QED: What happens if we trap the photons as discrete modes inside a cavity?

Outline

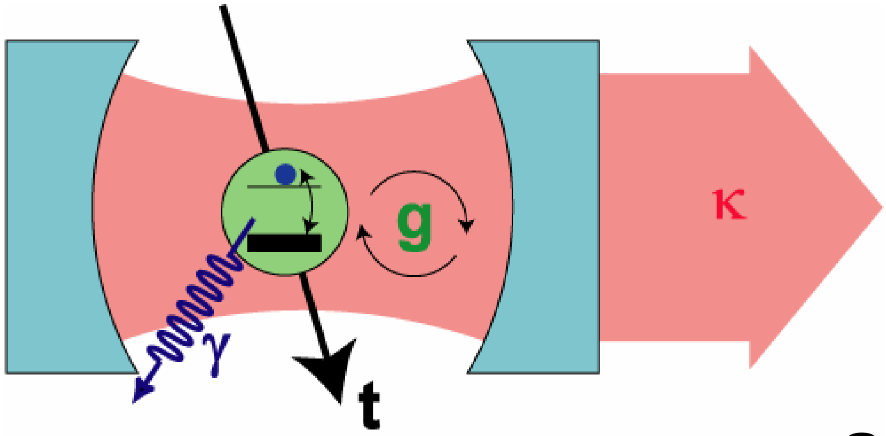
- Cavity QED in the AMO Community
  - Optical
  - Microwave

- Circuit QED: atoms with wires attached
  - What is the cavity?
  - What is the 'atom'?
  - Practical advantages

- Recent Experimental Results
  - Quantum optics with an electrical circuit

- Future Directions

Cavity Quantum Electrodynamics (cQED)



2g = vacuum Rabi freq.

κ = cavity decay rate

γ = "transverse" decay rate

t = transit time

<u>Strong Coupling</u> = **g** > κ , γ , 1/**t**

Jaynes-Cummings Hamiltonian

$$\hat{H} = \hbar\omega_r(a^\dagger a + \tfrac{1}{2}) + \frac{E_{el}}{2}\hat{\sigma}_x - \frac{E_J}{2}\hat{\sigma}_z - \hbar g(a^\dagger\sigma^- + \sigma^+ a)$$

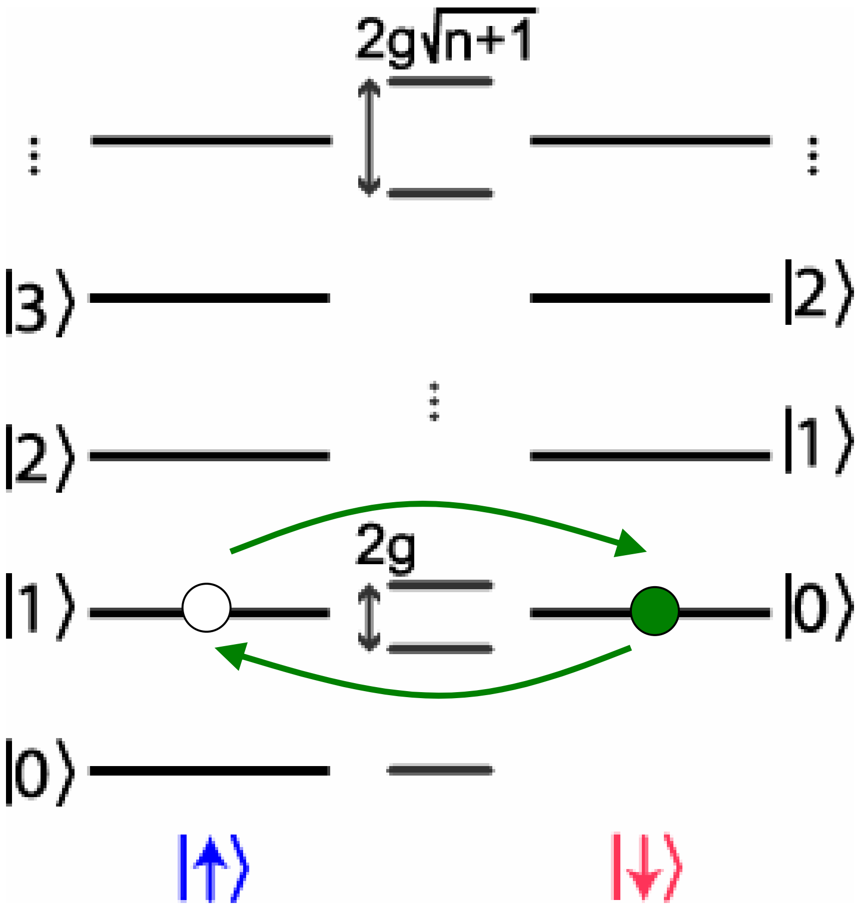
Quantized Field

2-level system

Electric dipole Interaction

Cavity QED: Resonant Case

$$\boxed{\omega_r = \omega_{01}}$$



"dressed state ladders"

with interaction eigenstates are:

$$\left|+,0\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\uparrow,1\right\rangle + \left|\downarrow,0\right\rangle\right)$$

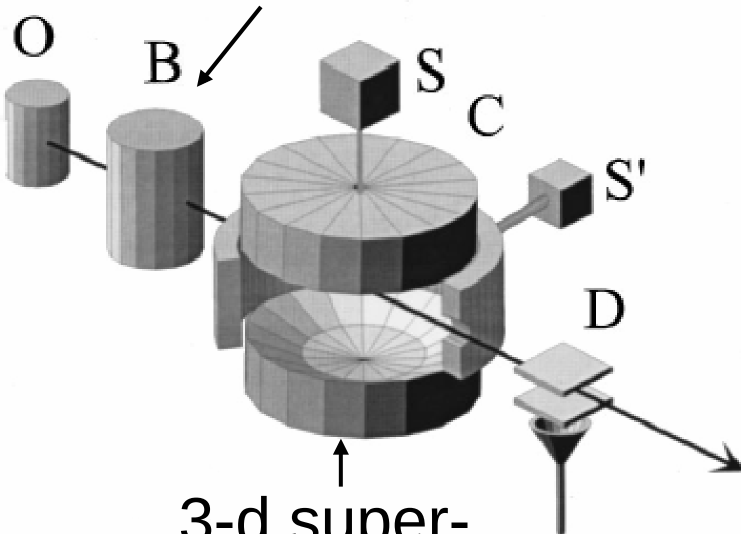
$$\left|-,0\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\uparrow,1\right\rangle - \left|\downarrow,0\right\rangle\right)$$

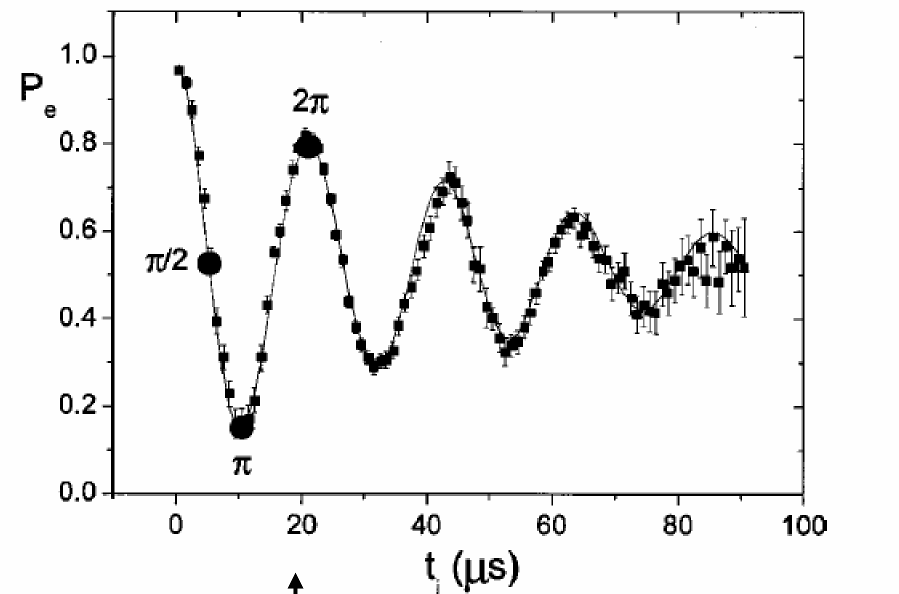
vacuum Rabi oscillations

Microwave cQED with Rydberg Atoms

beam of atoms;
prepare in |e>

vacuum Rabi oscillations





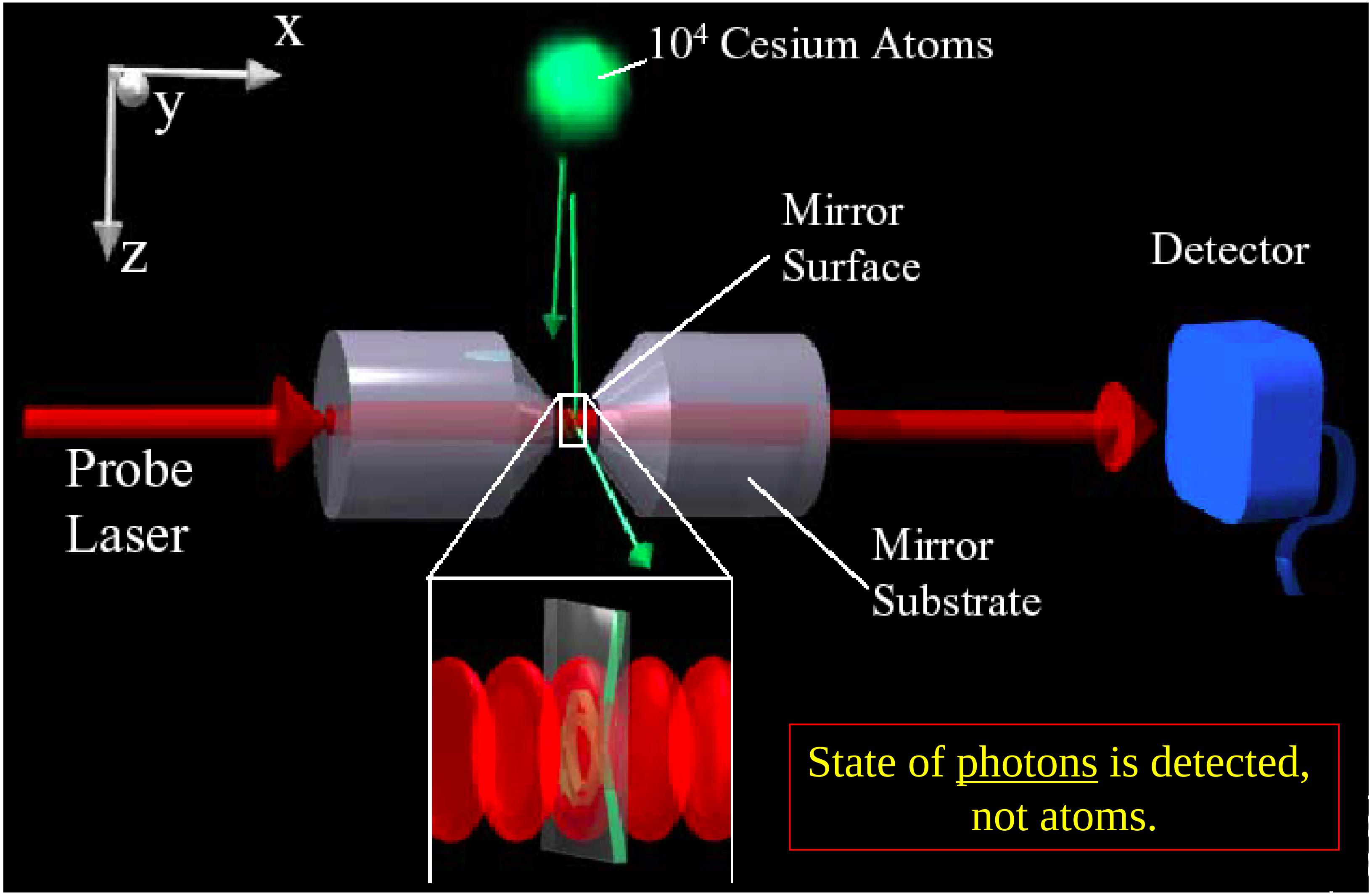
3-d super-
conducting
cavity (50 GHz)

observe dependence of atom final
state on time spent in cavity

measure atomic state, or …

Review: S. Haroche et al., *Rev. Mod. Phys.* **73** 565 (2001)

cQED at Optical Frequencies

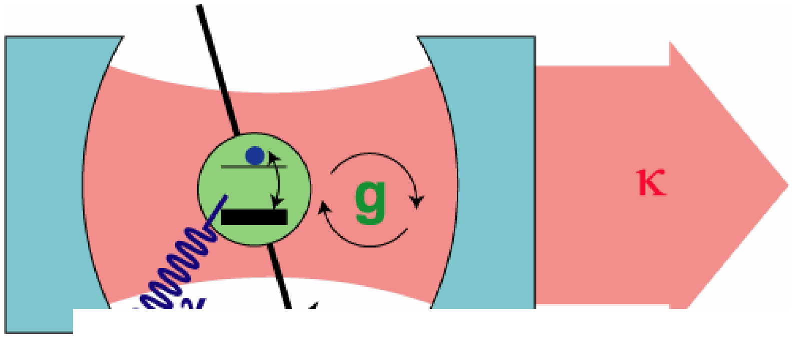


State of photons is detected, not atoms.

… measure changes in transmission of optical cavity

(Caltech group H. J. Kimble, H. Mabuchi)

A Circuit Analog for Cavity QED



2g = vacuum Rabi freq.
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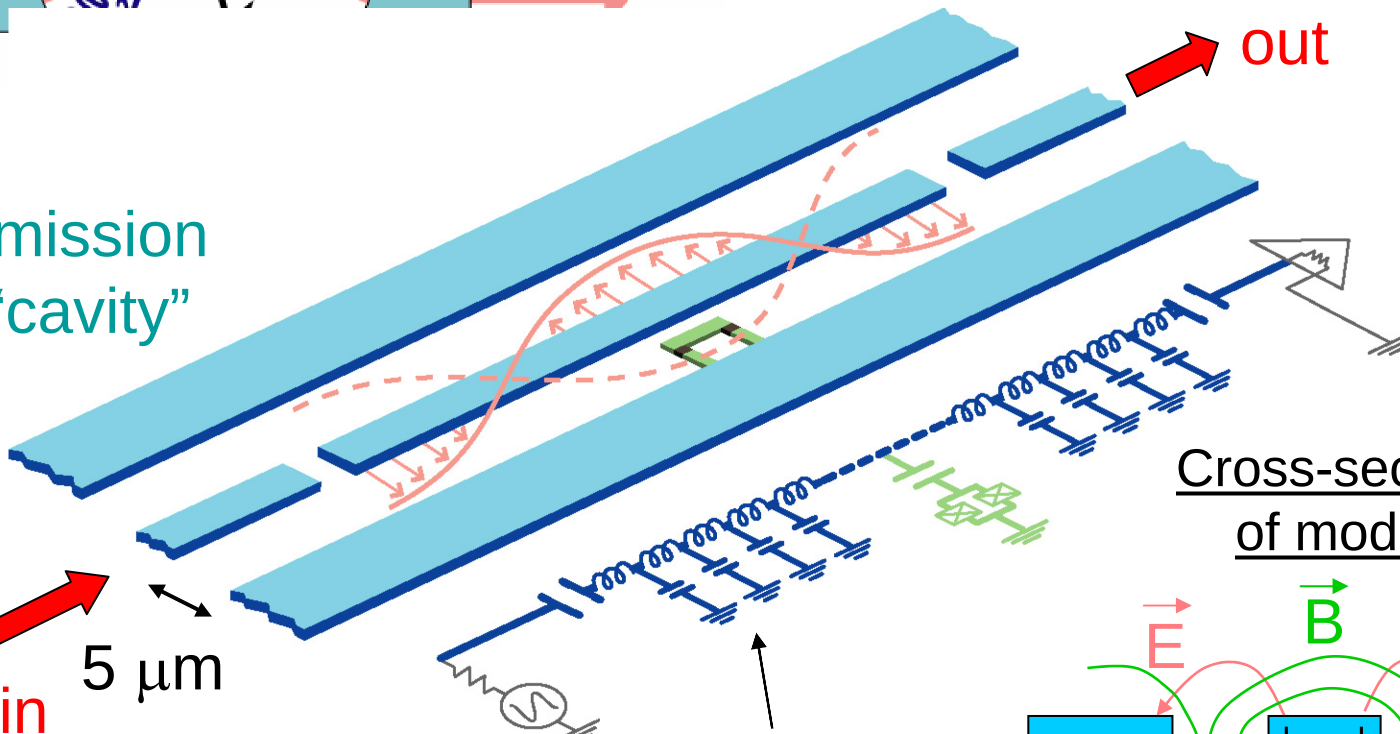
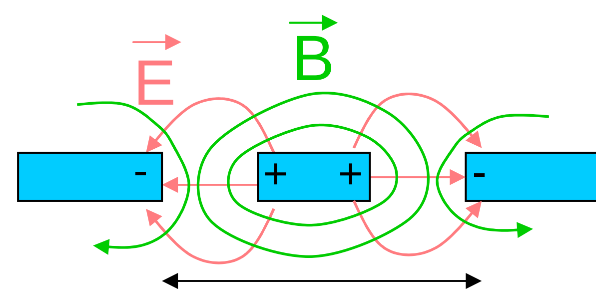


Cross-section of mode:



Blais, Huang, Wallraff, SMG & RS, PRA 2004

Advantages of 1d Cavity and Artificial Atom

$$\boxed{g = \vec{d} \bullet \vec{E} / \hbar}$$

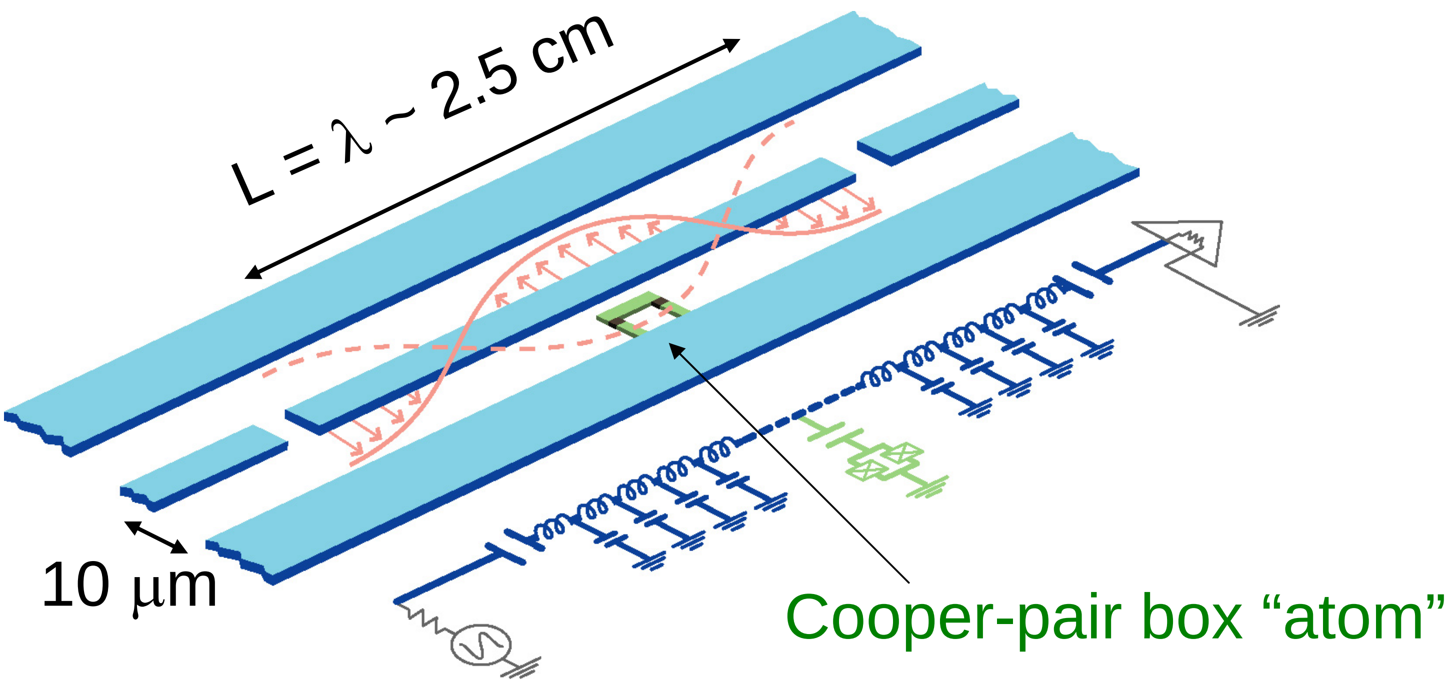
Vacuum fields:
zero-point energy confined
in < 10^{-6} cubic wavelengths

E ~ 0.25 V/m vs. ~ 1 mV/m for 3-d

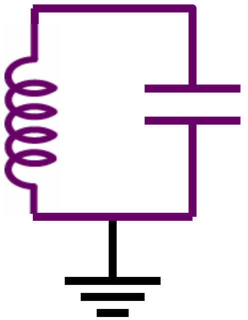
Transition dipole:

$$d \sim 40{,}000\, e a_0$$

10 x larger than
Rydberg atom



Resonator as Harmonic Oscillator



$$H = \frac{1}{2L}(LI)^2 + \frac{1}{2}CV^2$$

$\Phi \equiv LI =$ momentum

$V =$ coordinate

$$\hat{H}_{cavity} = \hbar\omega_r(a^\dagger a + 1/2)$$

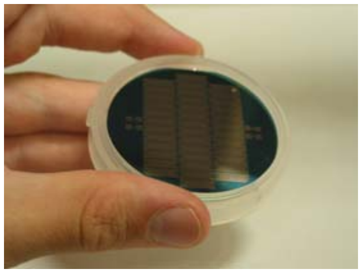
$$V = V_{\mathrm{RMS}}(a + a^\dagger)$$

$$\frac{1}{2}C\langle 0|V^2|0\rangle = \frac{1}{2}\left(\frac{1}{2}\hbar\omega\right)$$

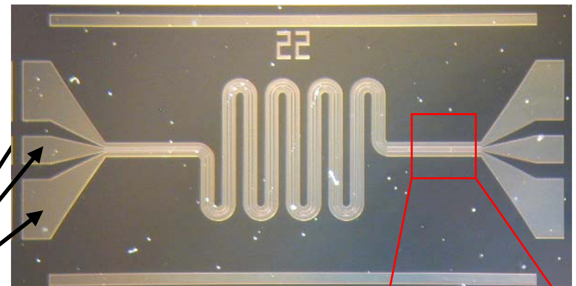
$$V_{\mathrm{RMS}} = \sqrt{\frac{\hbar\omega_r}{2C}} \sim 1-2\,\mu V$$

Implementation of Cavities for cQED

Superconducting coplanar waveguide transmission line



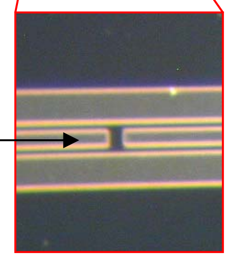
Optical lithography at Yale



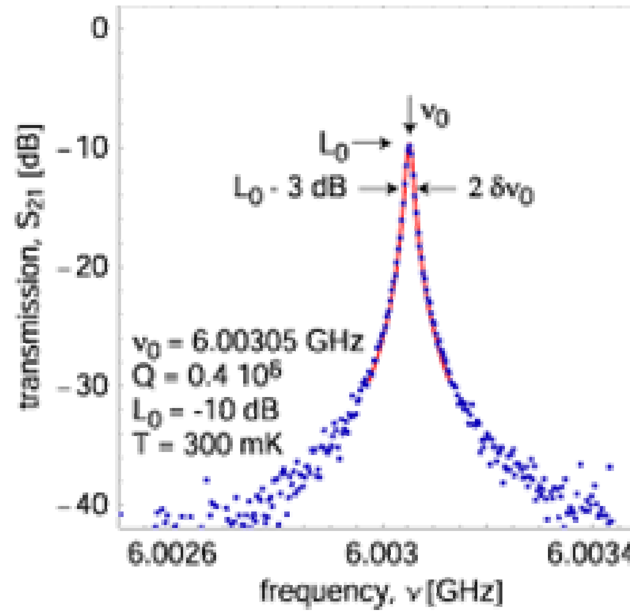
Niobium films

1 cm

gap = mirror



Q > 600,000 @ 0.025 K

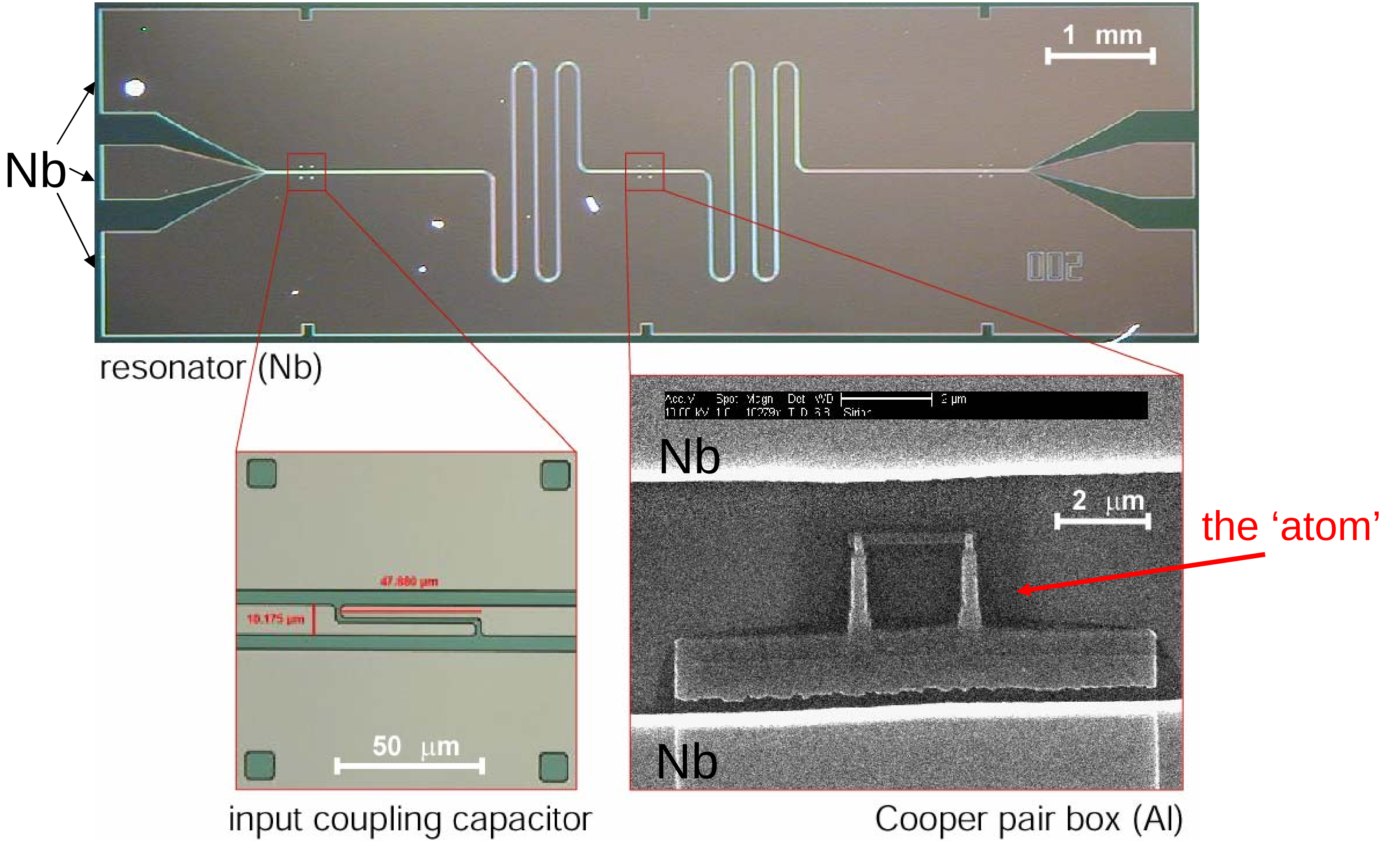


<u>6 GHz:</u>

$$\hbar\omega = 300\,mK \quad \Rightarrow \quad \langle n_\gamma \rangle \ll 1 \quad @\,20\,mK$$

- Internal losses negligible – Q dominated by coupling

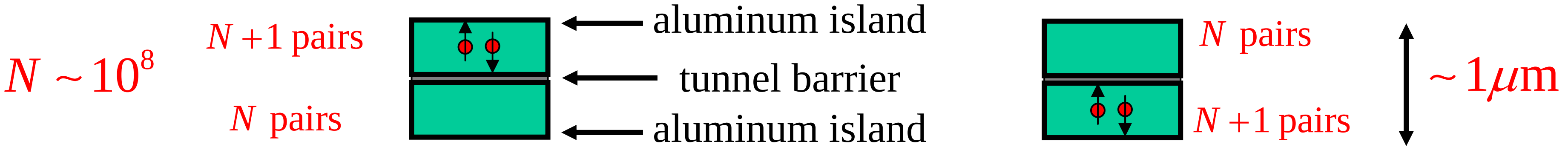
The Chip for Circuit QED



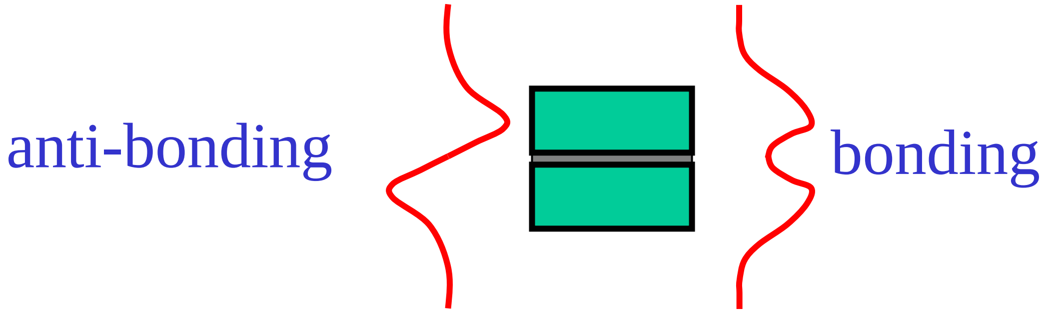
no wires attached to qubit!

Superconducting Tunnel Junction as a Covalently Bonded Diatomic 'Molecule'

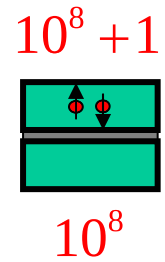
(simplified view)

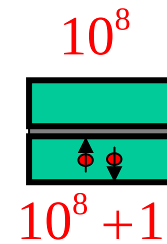


Cooper Pair Josephson Tunneling Splits the Bonding and Anti-bonding 'Molecular Orbitals'



Bonding Anti-bonding Splitting

$$|\psi_\pm\rangle = \frac{1}{\sqrt{2}}\Big(|$$


$$\rangle \pm |$$


$$\rangle\Big)$$

$$E_{\text{anti-bonding}} - E_{\text{bonding}} = E_{\text{J}} \sim 7 \text{ GHz} \sim 0.3 \text{ K}$$

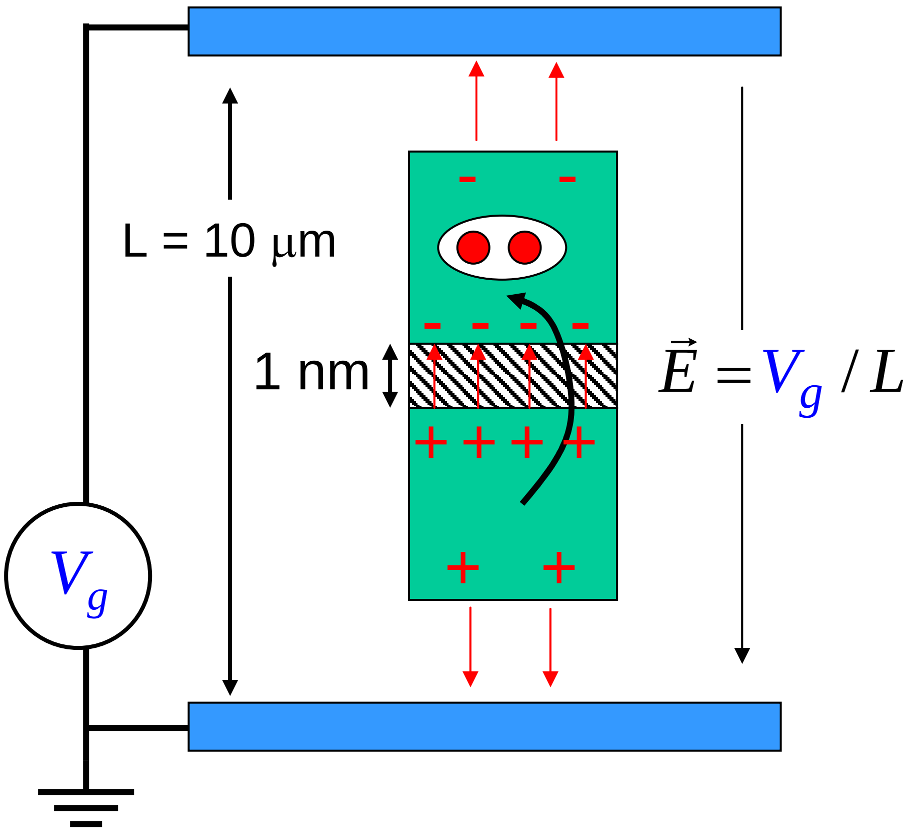
Josephson coupling

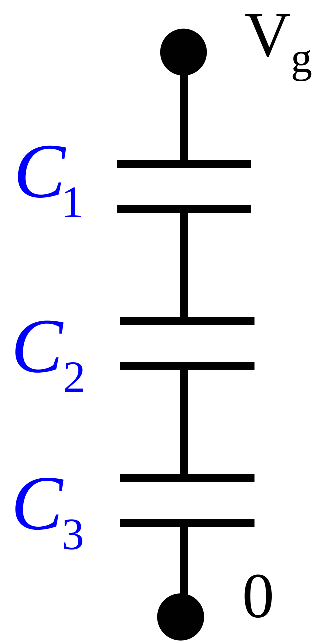
$|\uparrow\rangle =$ bonding

$|\downarrow\rangle =$ anti-bonding

$$H = -\frac{E_{\text{J}}}{2}\sigma^z$$

Dipole Moment of the Cooper-Pair Box
(determines polarizability)





$$d = (2e)L \frac{1/C_2}{1/C_1 + 1/C_2 + 1/C_3}$$

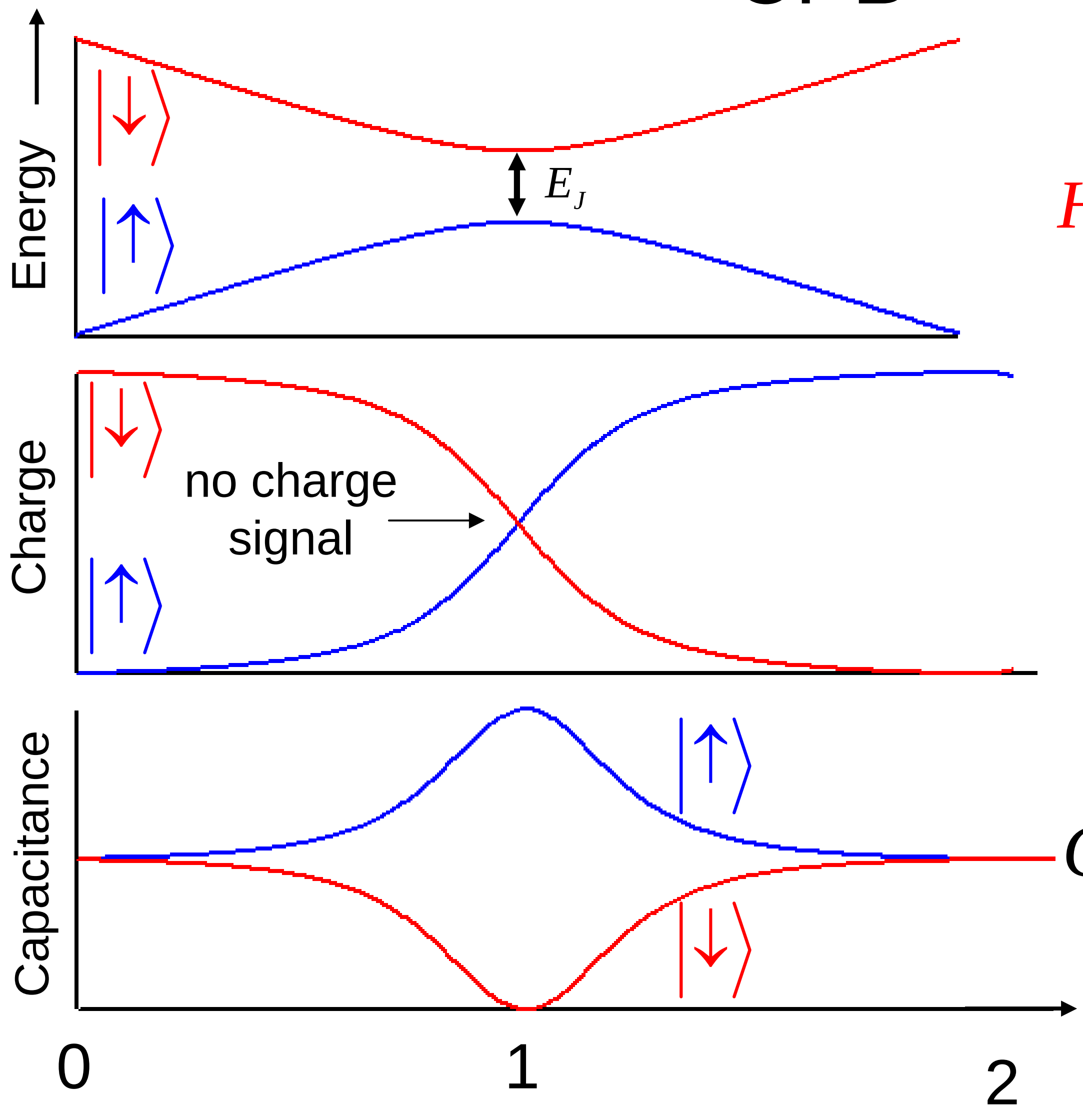
$|\uparrow\rangle =$ bonding

$|\downarrow\rangle =$ anti-bonding

$$H = -\frac{E_J}{2}\sigma^z - \frac{d}{L} V_g \sigma^x$$

$$|\vec{d}| \sim 2e\text{-}\mu\text{m}$$

Energy, Charge, and Capacitance of the CPB



$$H = -\frac{E_J}{2}\sigma^z - \frac{d}{L}V_g\sigma^x$$

$$Q = \frac{dE}{dV}$$ charge

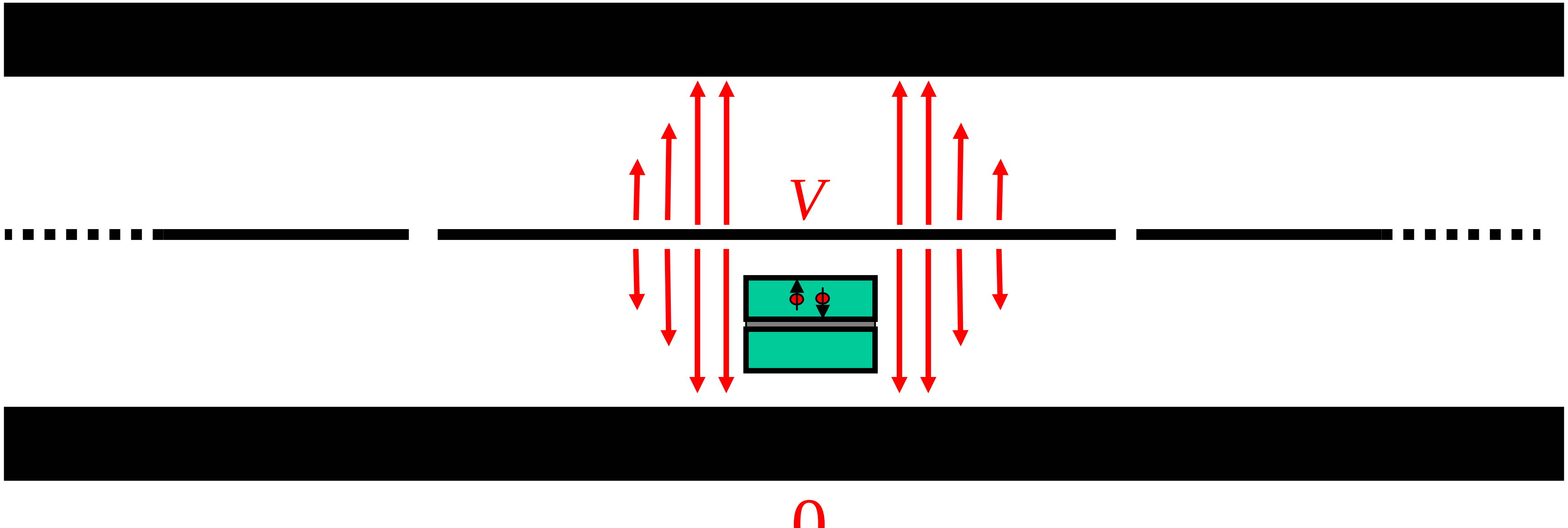
$$C = \frac{dQ}{dV}$$ polarizability is state dependent

deg. pt. = coherence sweet spot

Using the cavity to measure the state of the 'atom'

$$H = -\frac{E_J}{2}\sigma^z - \frac{d}{L}V\sigma^x \qquad V = V_{dc} + V_{RMS}(a^\dagger + a)$$

$$g = \frac{(2e)}{\hbar}\frac{1/C_2}{1/C_1 + 1/C_2 + 1/C_3}V_{RMS}$$



State dependent polarizability of 'atom' pulls the cavity frequency

Dispersive Regime

Large Detuning of Atom from Cavity

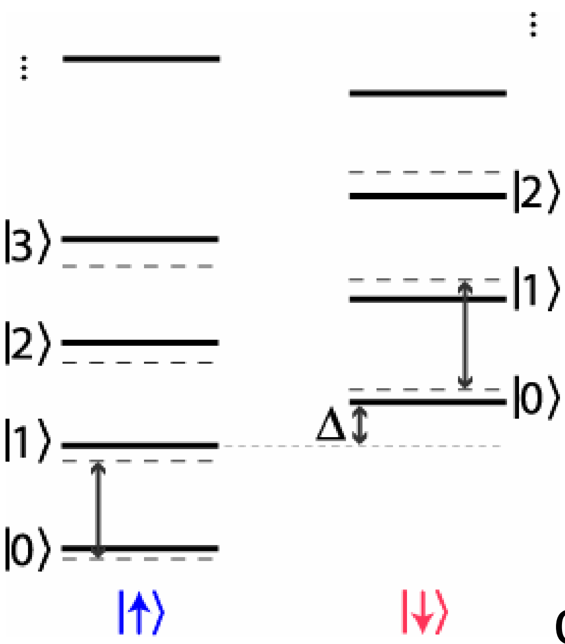
$$\Delta = \omega_{01} - \omega_r \gg g$$

$$H=-\frac{\omega_{01}}{2}\sigma^{z}+\omega_{\mathrm{R}}a^{\dagger}a+g\left(a^{\dagger}\sigma^{-}+a\sigma^{+}\right)$$

$$U=\exp\left\{\frac{g}{\Delta}\left(a\sigma^{+}-a^{\dagger}\sigma^{-}\right)\right\}$$

Large Detuning of Atom from Cavity

$$\boxed{H_{\mathrm{eff}}=UHU^{\dagger}}$$



$$\boxed{\Delta=\omega_{01}-\omega_{r}\gg g}$$

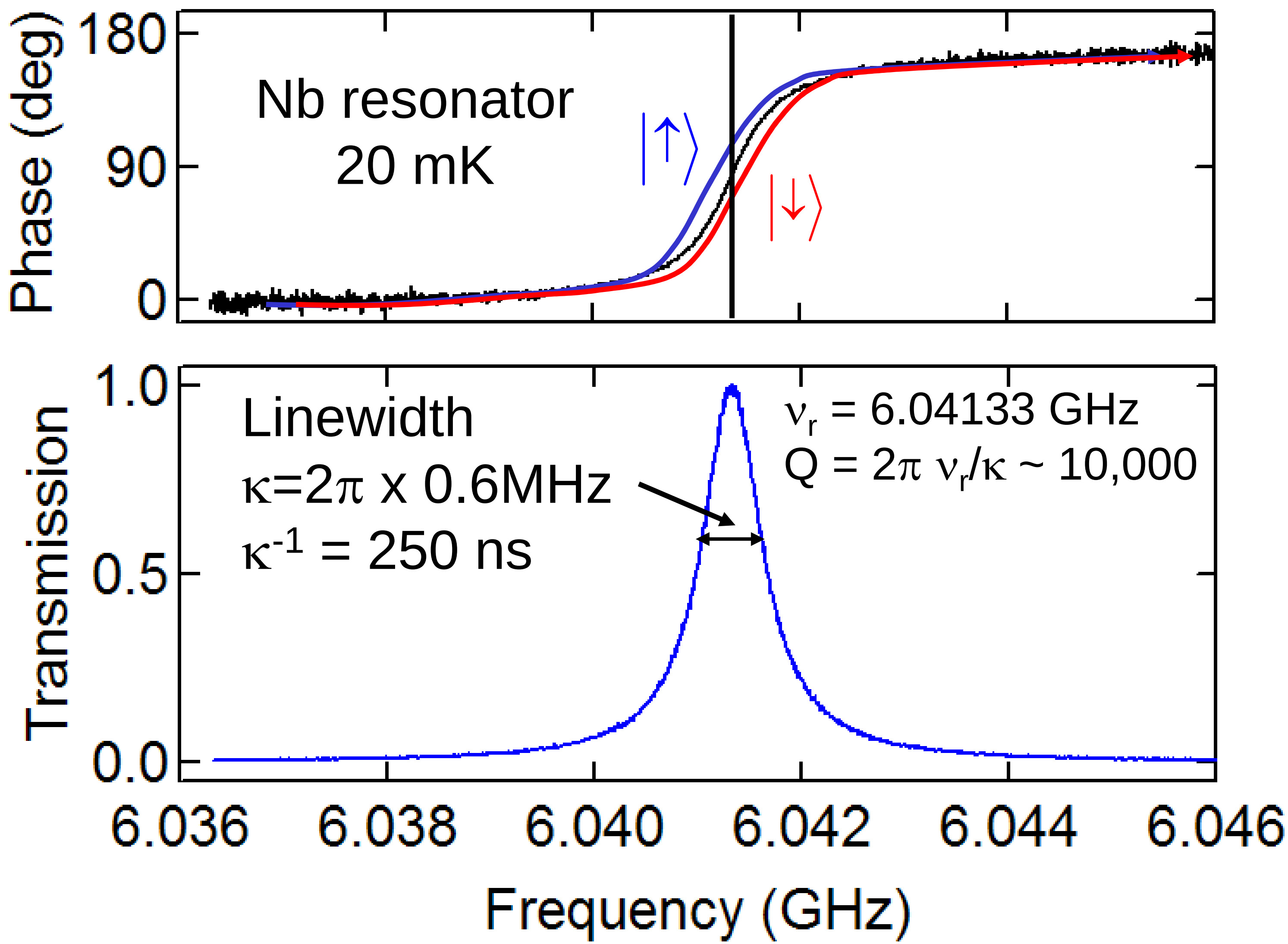
$$H_{\mathrm{eff}}\approx\left(\omega_{r}-\frac{g^{2}}{\Delta}\sigma_{z}\right)a^{\dagger}a-\frac{1}{2}\left(\omega_{01}+\frac{g^{2}}{\Delta}\right)\sigma_{z}$$

cavity freq. shift

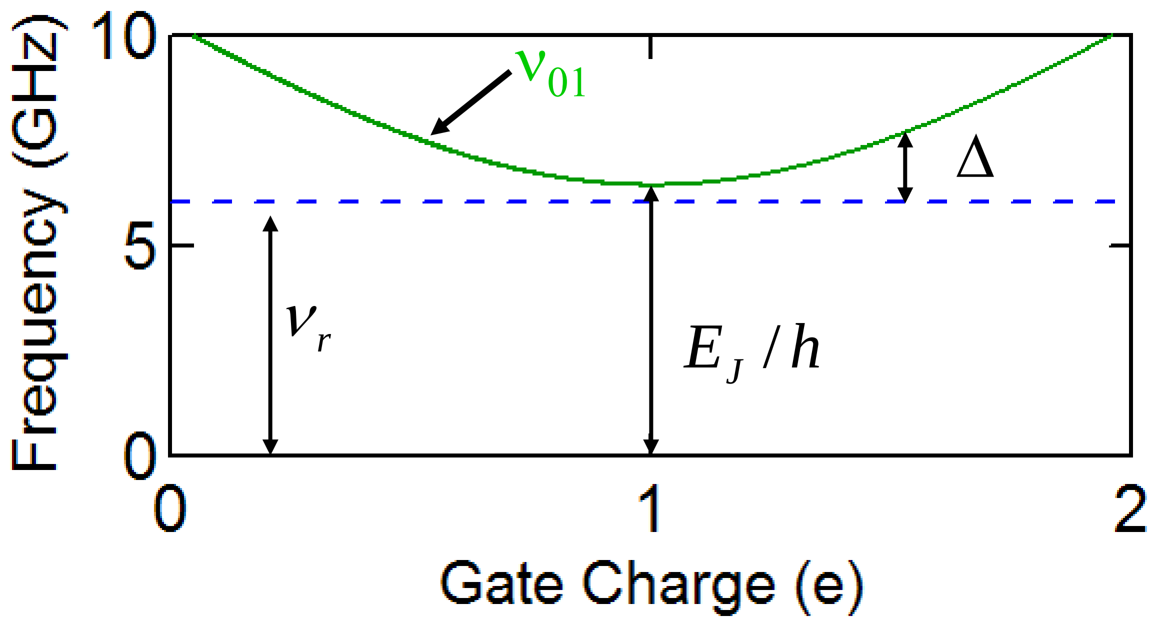
Lamb shift

$$\boxed{\text{QND}:\quad [H_{\mathrm{eff}},\sigma_{z}]=0}$$

Cavity Transmission Phase Controlled by State of Atom



QND Measurement of Qubit: Dispersive case

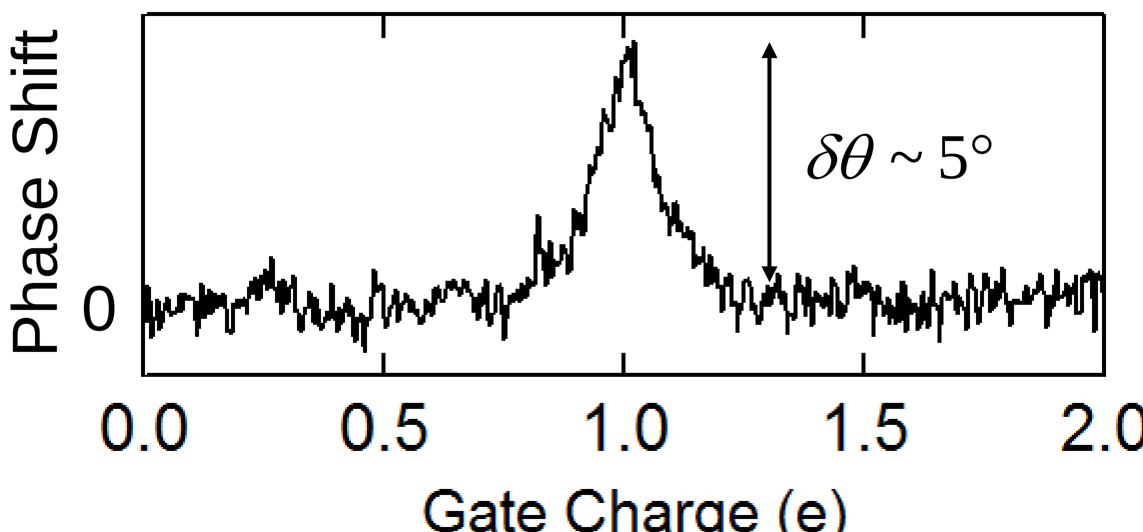


$$\Delta = 2\pi(\nu_{01} - \nu_r)$$

$$\nu_r = 6.04133\,\mathrm{GHz}$$

$$\Delta_{\min} \sim 300\ \mathrm{MHz}$$

$$(\sim 0.05\nu_r\ !)$$

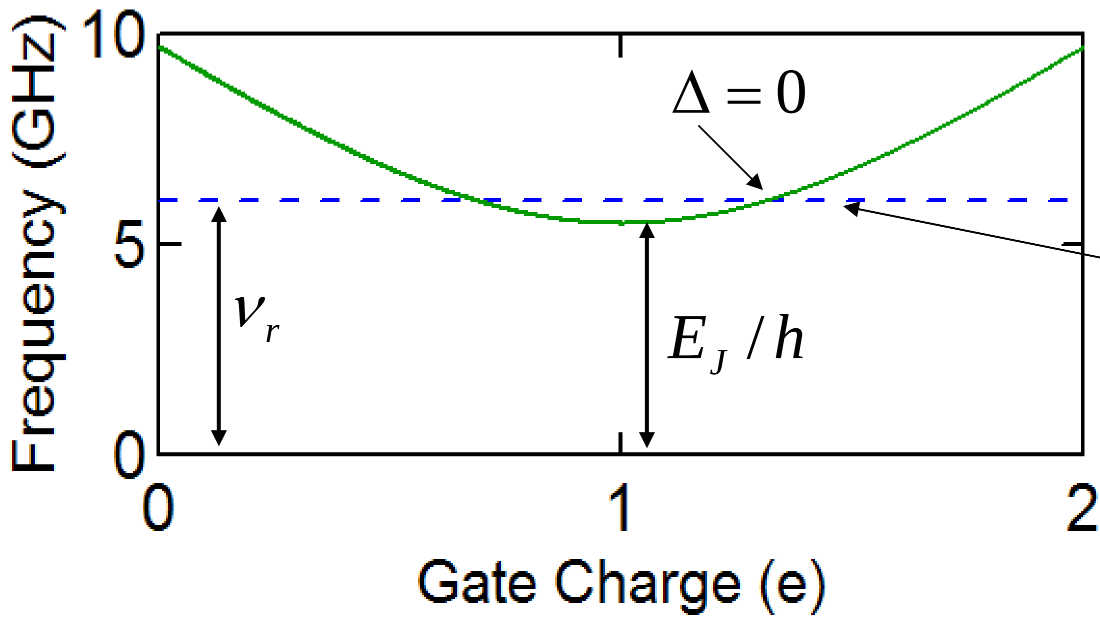


$$\delta\theta = 2g^2 / \Delta_{\min}\kappa \sim 5°$$

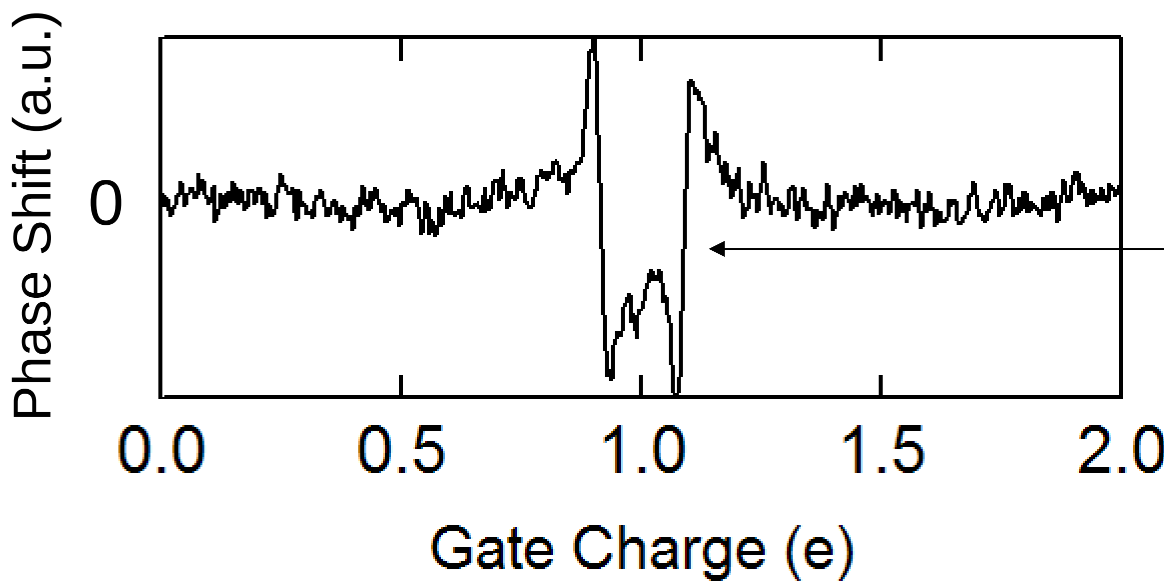
$$\boxed{g/\pi = 5\,\mathrm{MHz}}$$

vacuum Rabi frequency

Gate Sweep with Qubit Crossing Resonator



tune qubit thru resonance w/ cavity



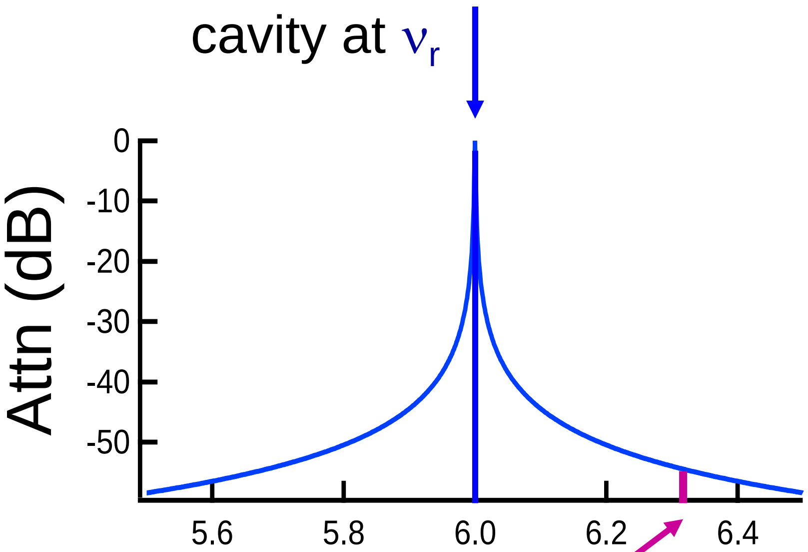
phase shift changes sign at resonance

Spectroscopy of Qubit in Cavity

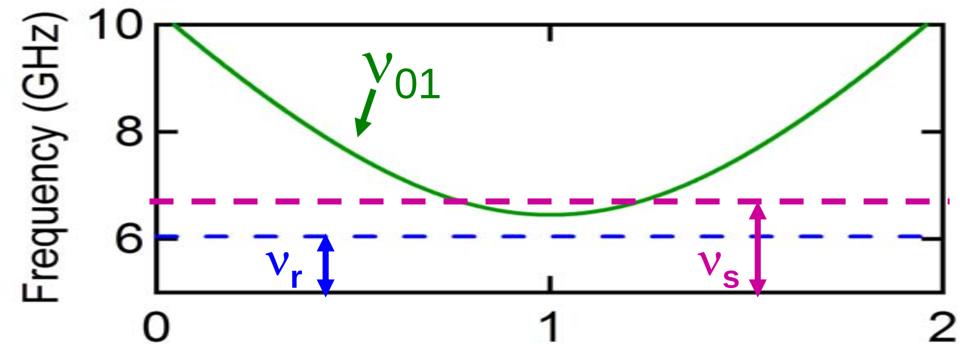
Send in 2 frequencies

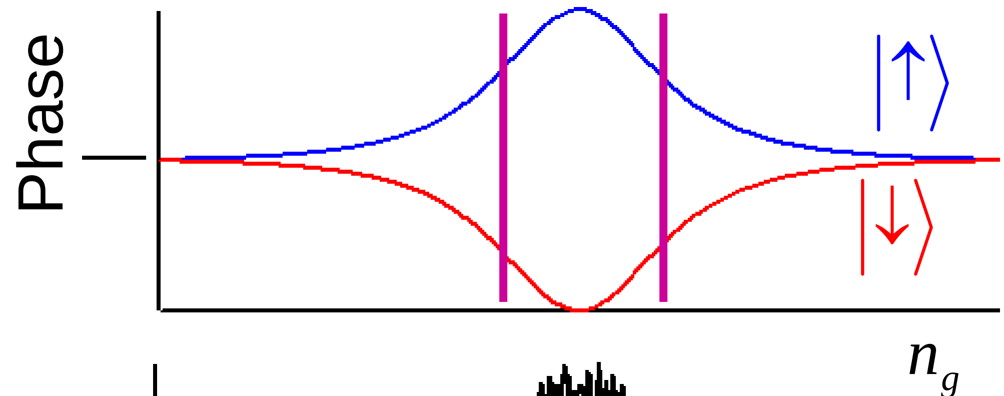
- Readout
- Spectroscopy

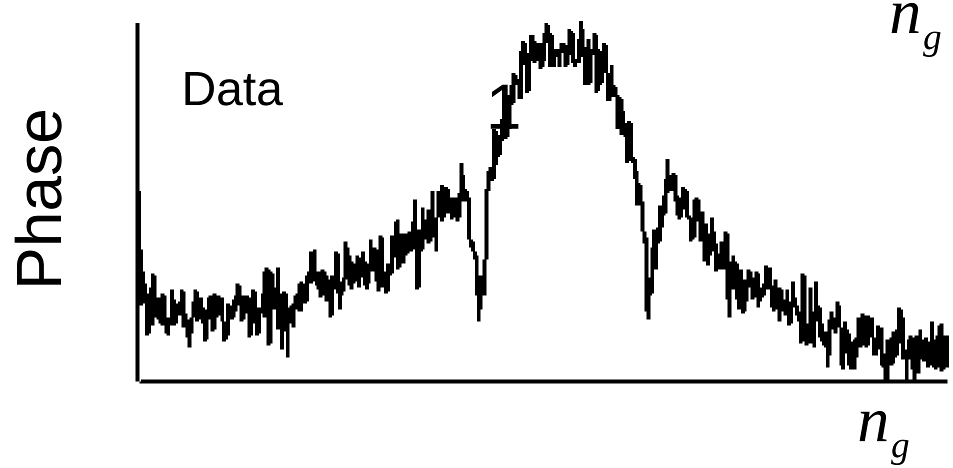
Probe (CW) cavity at ν_r



Spectroscopy (CW) at 6.3 GHz near ν_{01}

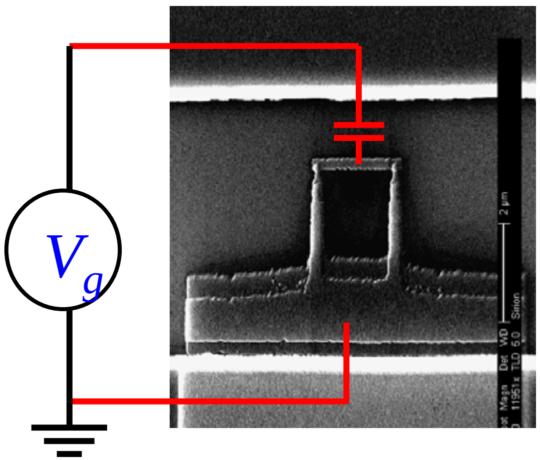


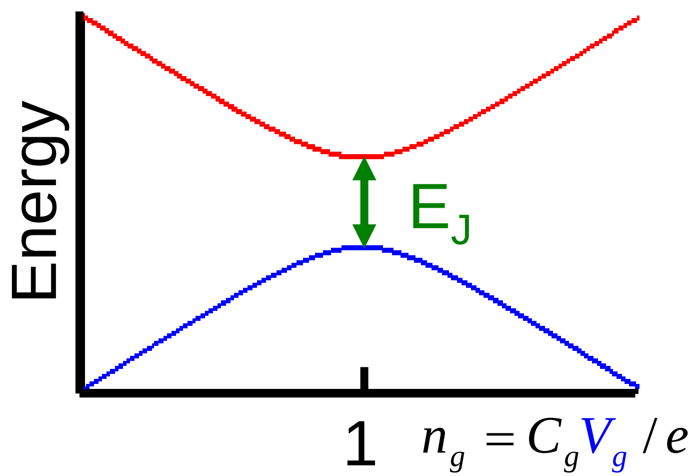


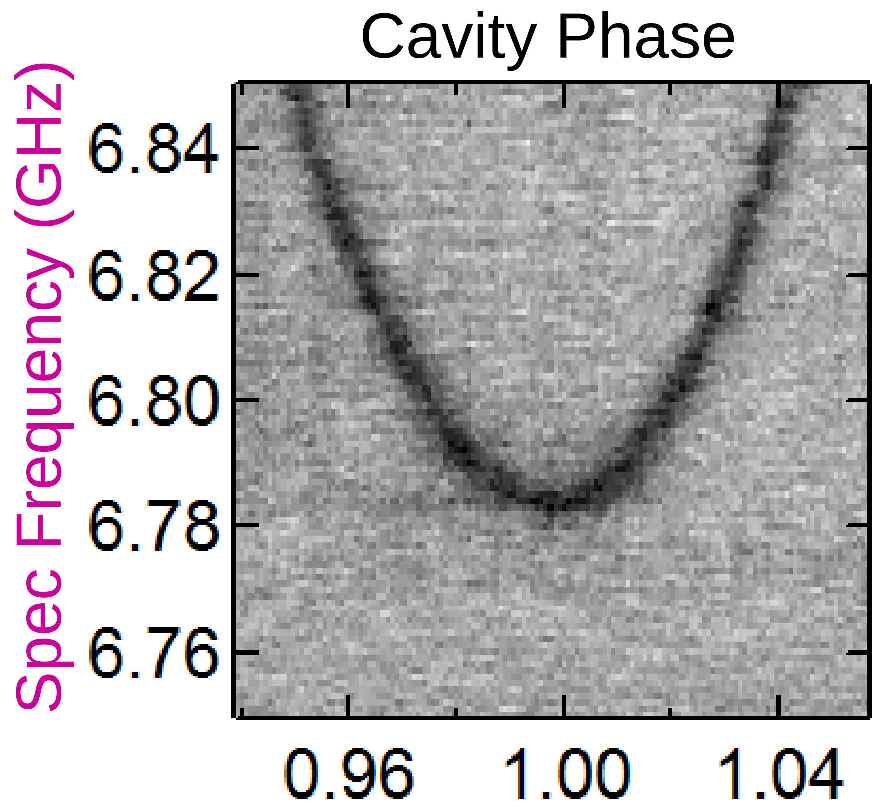


Spectrum of Qubit

$$H = -\frac{E_J}{2}\sigma^z - \frac{d}{L}V_g\sigma^x$$





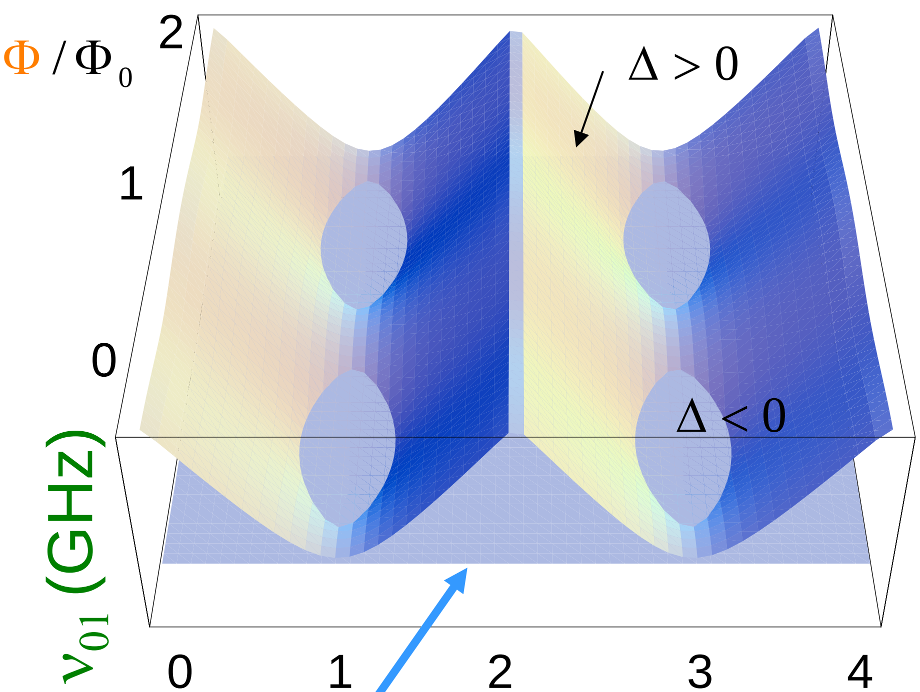


$$n_g = \frac{C_g V_g}{e}$$

Using Cavity to Map Qubit Parameter Space

$$\boxed{\Delta = \omega_{01} - \omega_r}$$

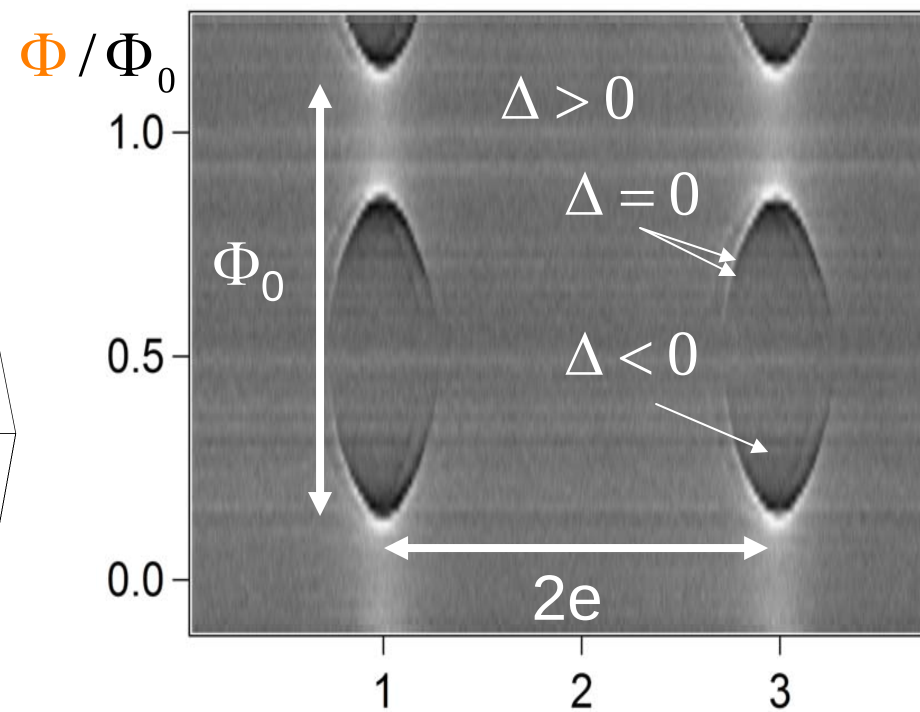
Transition frequency of qubit



Slice at Δ=0

$$n_g = \frac{C_g V_g}{e}$$

Cavity phase shift



$$n_g = \frac{C_g V_g}{e}$$

$$\boxed{E_J^{\max} \sim 6.7\,\mathrm{GHz} \quad E_C \sim 5.25\,\mathrm{GHz}}$$

Probe Beam at Cavity Frequency Induces ‘Light Shift’ of Atom Frequency

$$H_{\text{eff}} \approx \left(\omega_r - \frac{g^2}{\Delta}\sigma_z \right) a^\dagger a - \frac{1}{2}\left(\omega_{01} + \frac{g^2}{\Delta} \right)\sigma_z$$

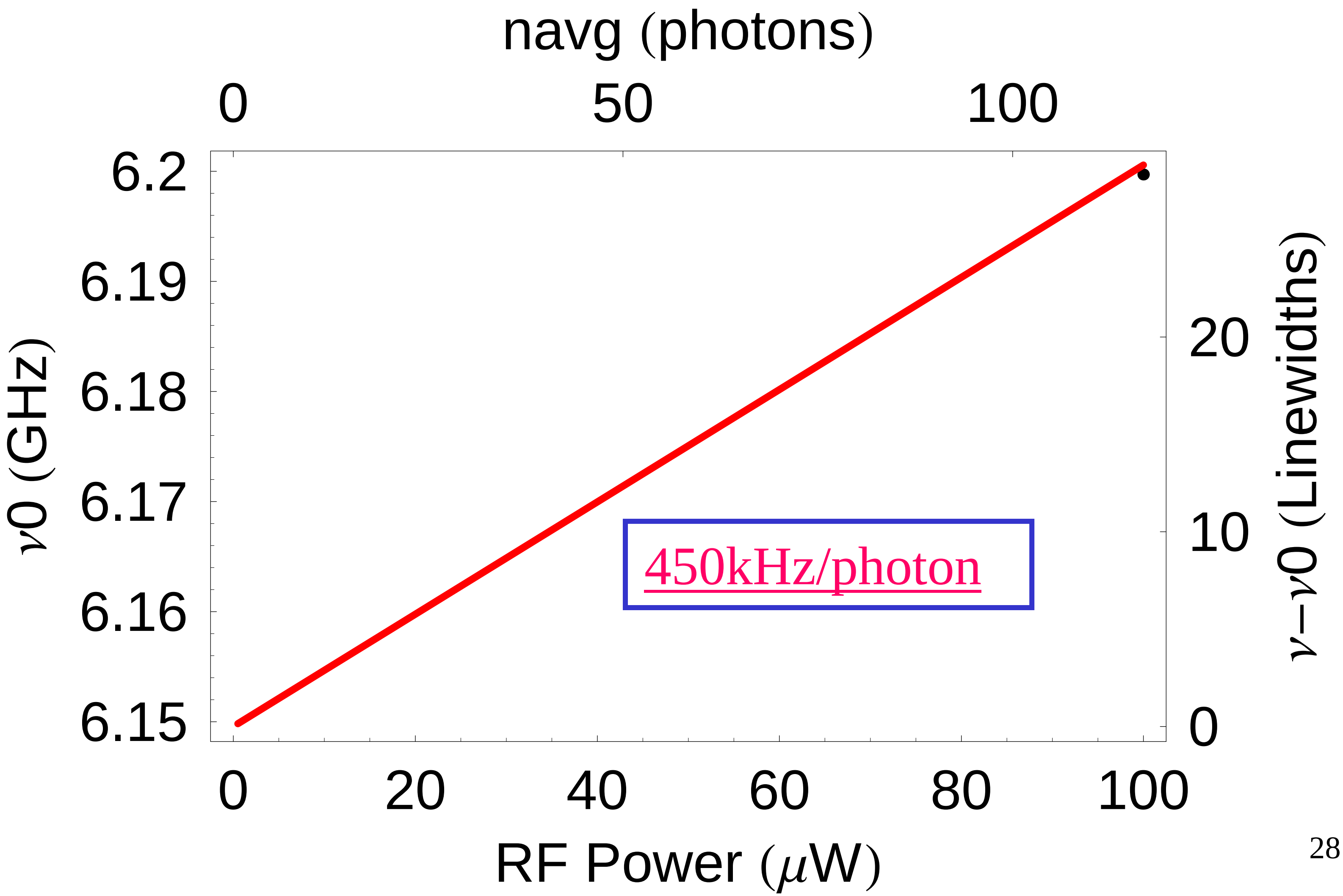
cavity freq. shift
atom ac Stark shift
(light shift)
$= 2\bar{n} \times$ cavity pull

Lamb shift

vacuum ac Stark shift

$$H_{\text{eff}} \approx \omega_r a^\dagger a - \frac{1}{2}\left(\omega_{01} + 2\frac{g^2}{\Delta}\left[a^\dagger a + \frac{1}{2} \right] \right)\sigma_z$$

Atom ac Stark Shift (Light Shift) Induced by Cavity Photons

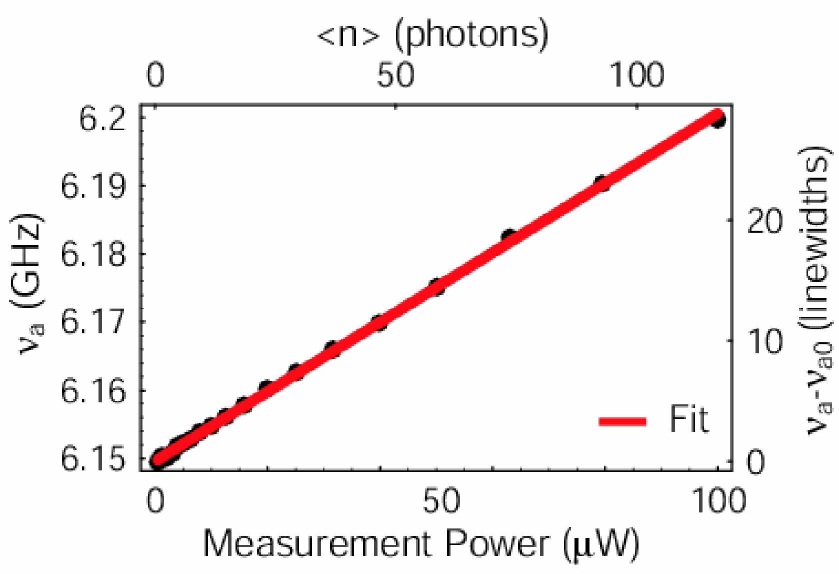


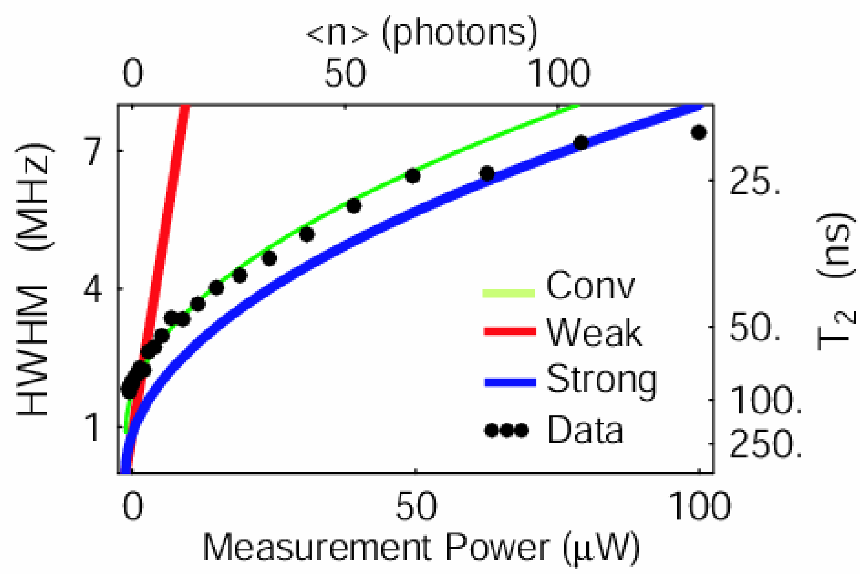
Measurement Induced Dephasing: back action = quantum noise in the light Shift

$$H_{\mathrm{eff}} \approx \omega_{\mathrm{r}} a^\dagger a - \frac{1}{2}\left(\omega_{01} + 2\frac{g^2}{\Delta}\left[a^\dagger a + \frac{1}{2}\right]\right)\sigma_z$$

$\sqrt{n}$ fluctuations in photon number

$$\left\langle \delta\hat{n}(\tau)\,\delta\hat{n} \right\rangle = \overline{n} e^{-\frac{\kappa}{2}|\tau|}$$





Measurement Back Action: Quantum Noise in ac Stark Shift

$$H_{\mathrm{eff}} \approx \omega_{\mathrm{r}} a^{\dagger} a + \frac{1}{2}\left(\omega_{01} + 2\frac{g^2}{\Delta}\left[a^{\dagger} a + \frac{1}{2}\right]\right)\sigma_z$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\downarrow\rangle + e^{-i[\omega_{01} t + \varphi(t)]}|\uparrow\rangle\right)$$

$$\varphi(t) = \frac{2g^2}{\Delta}\left[\underbrace{\bar{n}}_{\text{light shift}} + \underbrace{\int_0^t d\tau\, \delta\hat{n}(\tau)}_{\text{random dephasing}}\right]$$

Measurement Back Action: Quantum Noise in ac Stark Shift

$$\delta\varphi(t) = \frac{2g^2}{\Delta}\int_0^t d\tau\, \delta\hat{n}(\tau)$$

$$\left\langle e^{i\delta\varphi(t)} \right\rangle \approx e^{-\frac{1}{2}\left\langle \delta\varphi^2(t) \right\rangle}$$

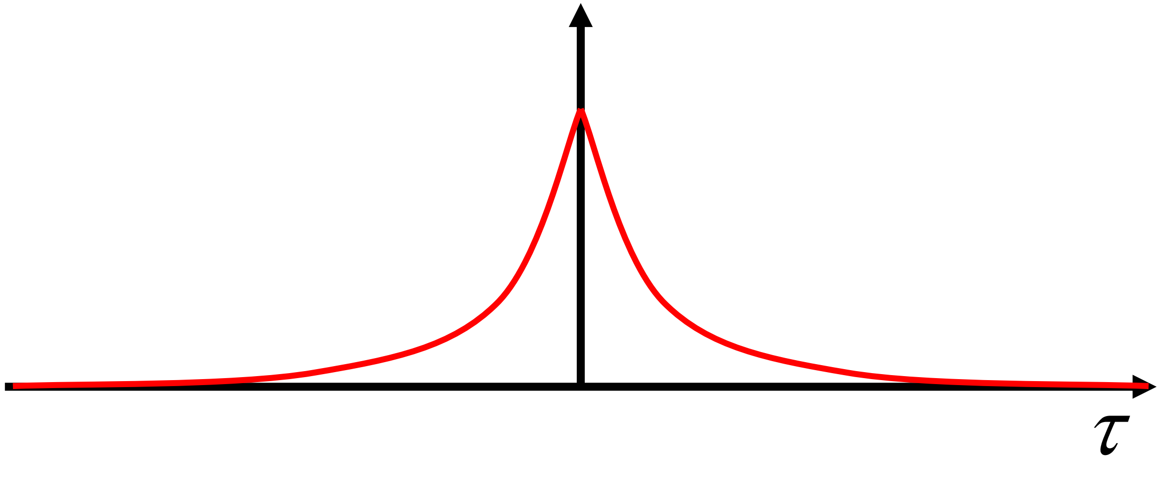
Assuming gaussian fluctuations

$$\left\langle \delta\varphi^2(t) \right\rangle = \left(\frac{2g^2}{\Delta}\right)^2 \int_0^t d\tau \int_0^t d\tau' \left\langle \delta\hat{n}(\tau)\delta\hat{n}(\tau') \right\rangle$$

Measurement Back Action: Quantum Noise in ac Stark Shift

Coherent state in driven cavity with damping rate κ

$$\left\langle \delta\hat{n}(\tau)\,\delta\hat{n}(0)\right\rangle = \bar{n}e^{-\frac{\kappa}{2}|\tau|}$$



Measurement Back Action: Quantum Noise in ac Stark Shift

$$\left\langle \delta\varphi^2(t) \right\rangle = \left(\frac{2g^2}{\Delta} \right)^2 \int_0^t d\tau \int_0^t d\tau' \,\bar{n} e^{-\frac{\kappa}{2}|\tau-\tau'|}$$

$$\approx \left(\frac{2g^2}{\Delta} \right)^2 \bar{n}\, t^2 \qquad \kappa t \ll 1$$

(Gaussian inhomogeneous broadening)

$$\approx \left(\frac{2g^2}{\Delta} \right)^2 \bar{n} \frac{4}{\kappa} t \qquad \kappa t \gg 1$$

(phase random walks--phase diffusion)

(Lorentzian homogeneous broadening)

Qubit Phase Diffusion (weak measurement)

$$\left\langle \delta\varphi^2(t) \right\rangle \approx \left(\frac{2g^2}{\Delta} \right)^2 \bar{n} \frac{4}{\kappa} t$$

$$\left\langle e^{-i\varphi(t)} \right\rangle = e^{-\frac{1}{2}\left\langle \varphi^2(t) \right\rangle} = \exp\left[-2\left(\frac{2g^2}{\kappa\Delta} \right)^2 \bar{n}\kappa\, t \right] = e^{-\Gamma_\varphi t}$$

$$S(\omega) = -\frac{1}{\pi} \mathrm{Im}\left\{ \text{-i}\int_0^\infty \mathrm{dt}\, \mathrm{e}^{\mathrm{i}\omega \mathrm{t}} e^{-\Gamma_\varphi t} \right\} = \frac{1}{\pi} \frac{\Gamma_\varphi}{(\omega-\omega_0)^2 + \Gamma_\varphi{}^2}$$

$$\boxed{\Gamma_\varphi \propto \bar{n}}$$

valid for $\Gamma_\varphi \ll \kappa$

Measurement induced dephasing rate

Qubit Inhomogeneous Broadening (strong measurement)

$$\left\langle e^{-i\varphi(t)} \right\rangle = e^{-\frac{1}{2}\left\langle \varphi^2(t) \right\rangle} = \exp\left[-\frac{1}{2}\left(\frac{2g^2}{\Delta} \right)^2 \bar{n}\, t^2 \right] = e^{-\frac{1}{2}(\Gamma_\varphi t)^2}$$

$$\Gamma_\varphi \propto \sqrt{\bar{n}}$$

$$S(\omega) = -\frac{1}{\pi}\,\mathrm{Im}\left\{ -\mathrm{i}\int_0^\infty \mathrm{dt}\, \mathrm{e}^{\mathrm{i}\omega \mathrm{t}} e^{-\frac{1}{2}(\Gamma_\varphi t)^2} \right\} = \frac{1}{\sqrt{2\pi}\,\Gamma_\varphi} e^{-\frac{(\omega-\omega_0)^2}{2\Gamma_\varphi{}^2}}$$

$$\boxed{\Gamma_\varphi \propto \sqrt{\bar{n}}}$$

valid for $\Gamma_\varphi \gg \kappa$

Measurement Induced Dephasing: back action = quantum noise in the light Shift

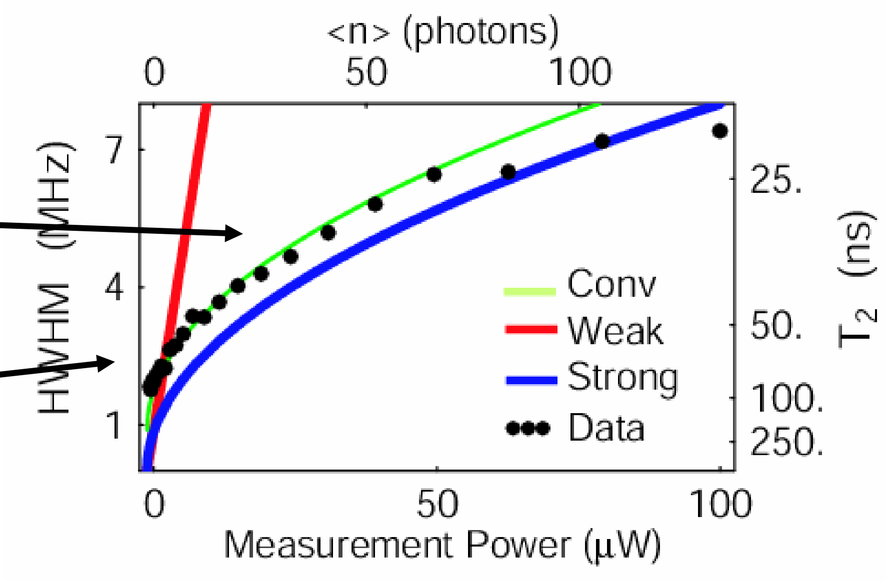
$$H_{\mathrm{eff}} \approx \omega_{\mathrm{r}} a^{\dagger} a - \frac{1}{2}\left(\omega_{01} + 2\frac{g^2}{\Delta}\left[a^{\dagger} a + \frac{1}{2}\right]\right)\sigma_z$$

$$\left\langle \delta\hat{n}(\tau)\,\delta\hat{n}\right\rangle = \bar{n} e^{-\frac{\kappa}{2}|\tau|}$$

$\sqrt{n}$ fluctuations in photon number

$$\boxed{\Gamma_{\varphi} \propto \sqrt{\bar{n}}}$$ Gaussian

$$\boxed{\Gamma_{\varphi} \propto \bar{n}}$$ Lorentzian



Summary of Dispersive Regime Results

Every thing works as predicted except the cavity enhanced lifetime has not been observed.

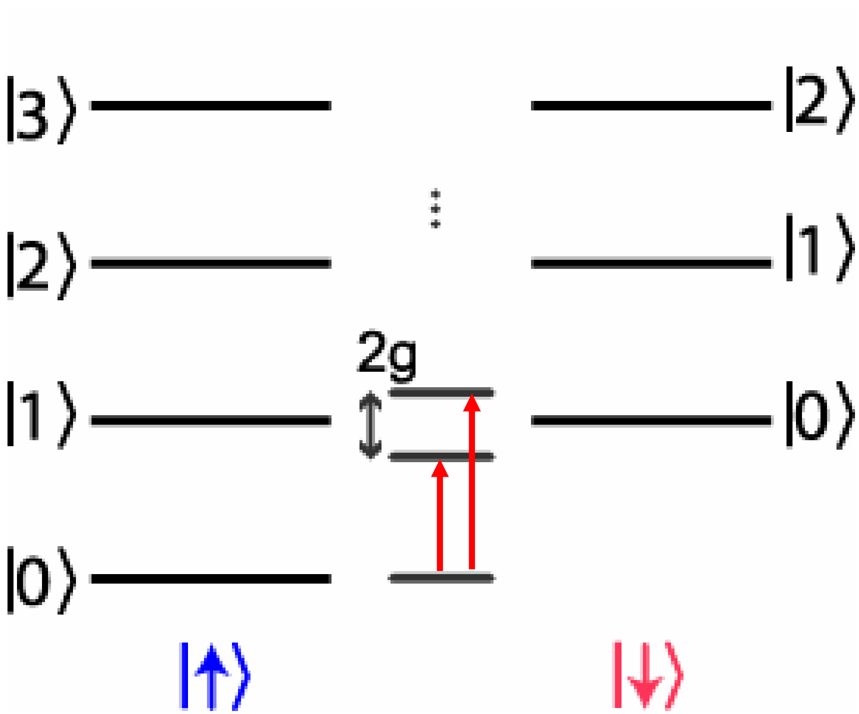
Non-radiative decay channels?

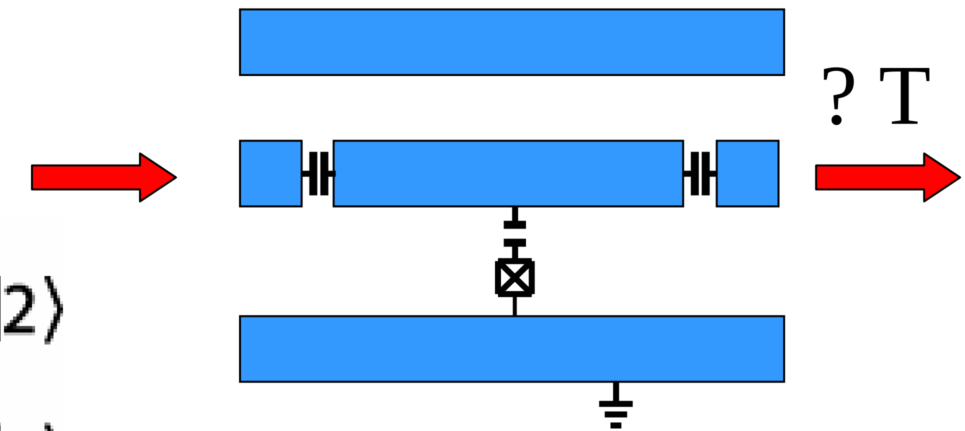
-glassy losses in oxide barriers loss tangent $\frac{\varepsilon_2}{\varepsilon_1} \sim 10^{-4}$

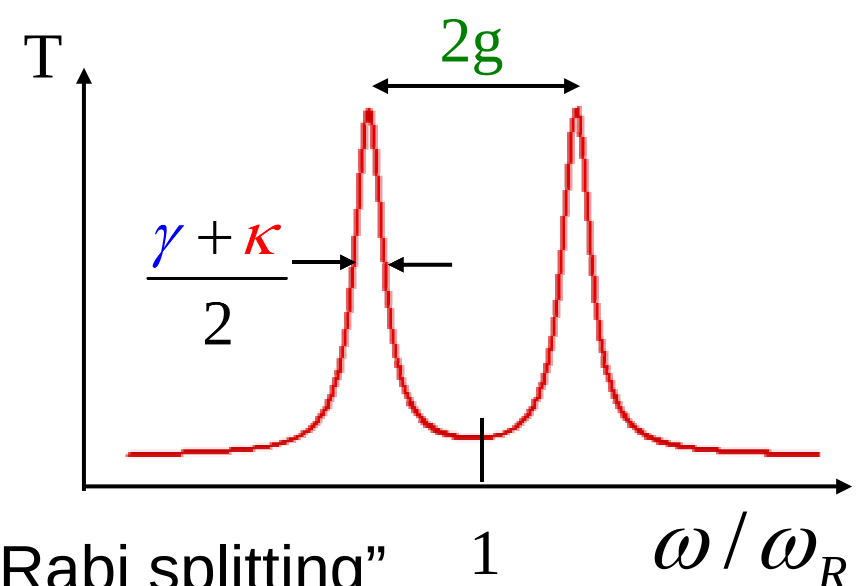
-electroacoustic coupling to phonons? (Ioffe, Blatter)

Dressed Artificial Atom: Resonant Case

$$\omega_{01} = \omega_R$$







Fourier transform of Haroche Rabi flopping expt.

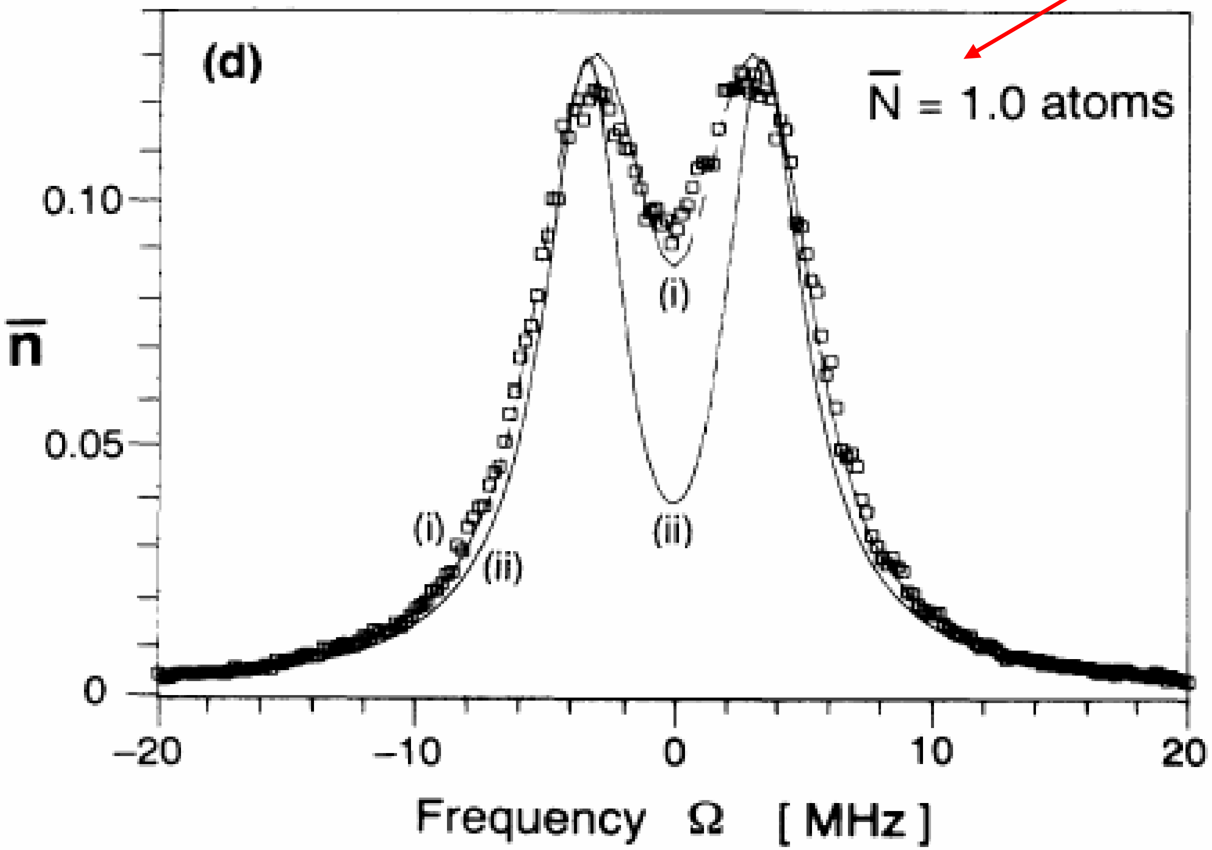
"vacuum Rabi splitting"

First Observation of Vacuum Rabi Splitting for a Single Atom

(on average)

Cs atom in an optical cavity

photons



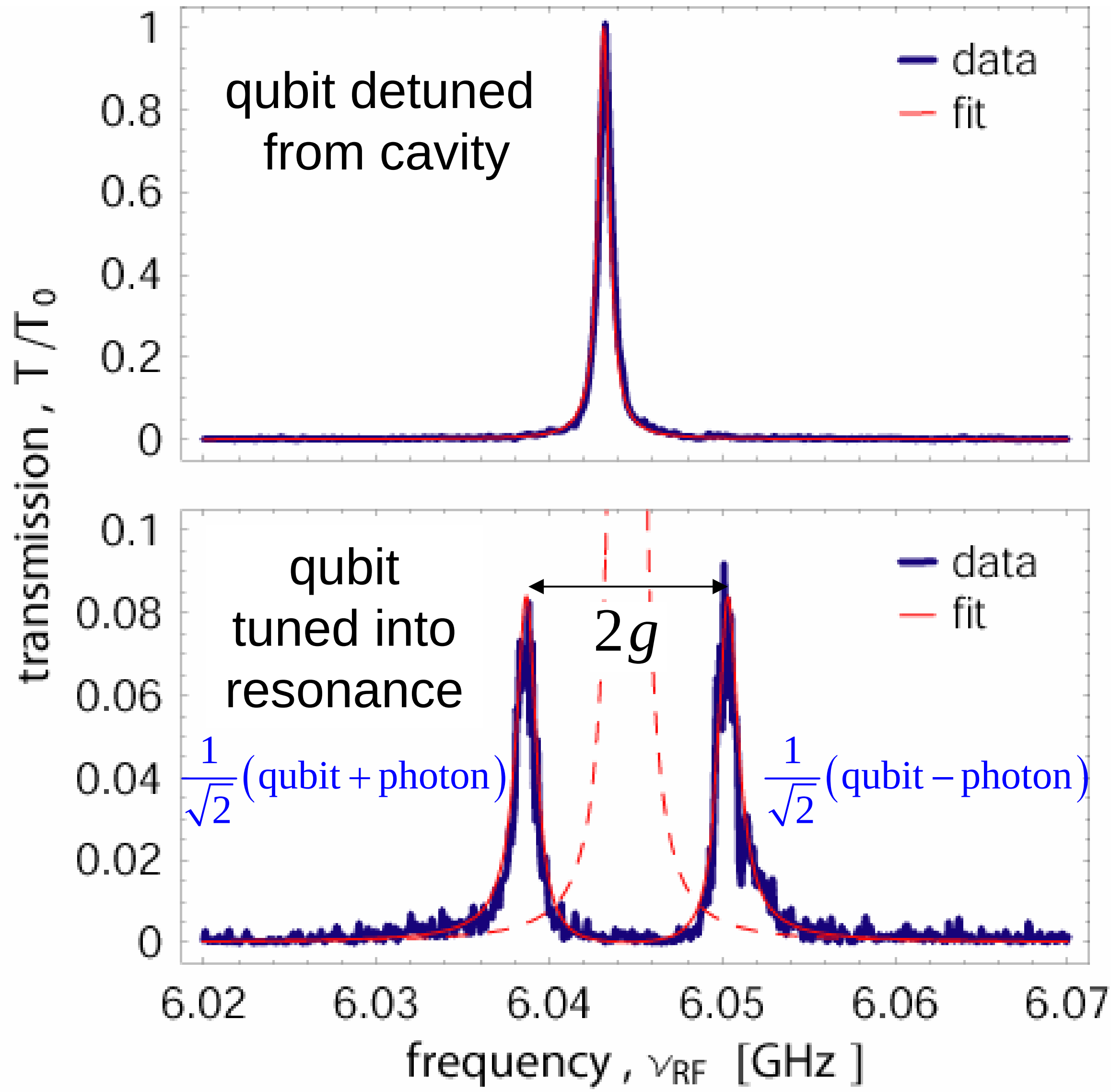
Thompson, Rempe, & Kimble 1992

First Observation of Vacuum Rabi Splitting in a Superconducting Circuit

$$P_{probe} = -140\ \text{dBm}$$
$$= 10^{-17}\ \text{W}$$
$$= \langle n \rangle \hbar \omega_r \kappa / 2$$

$$\Rightarrow \langle n \rangle \leq 1$$

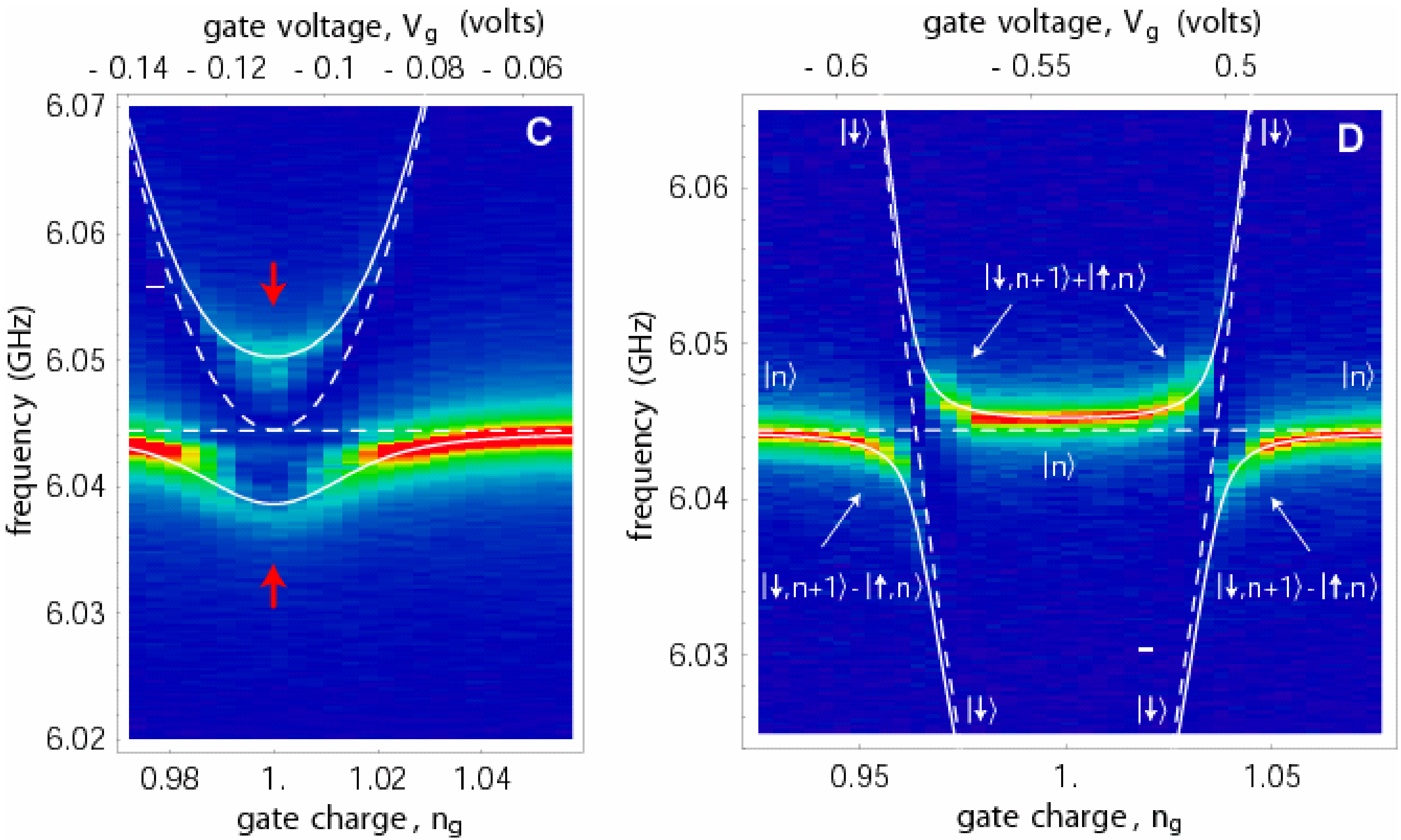
$$2g / 2\pi = 12\ \text{MHz}$$
$$\kappa / 2\pi = 0.6\ \text{MHz}$$
$$\gamma / 2\pi = 1\ \text{MHz}$$



Observing the Avoided Crossing of "Atom" & "Photon"

$E_J = \omega_r$

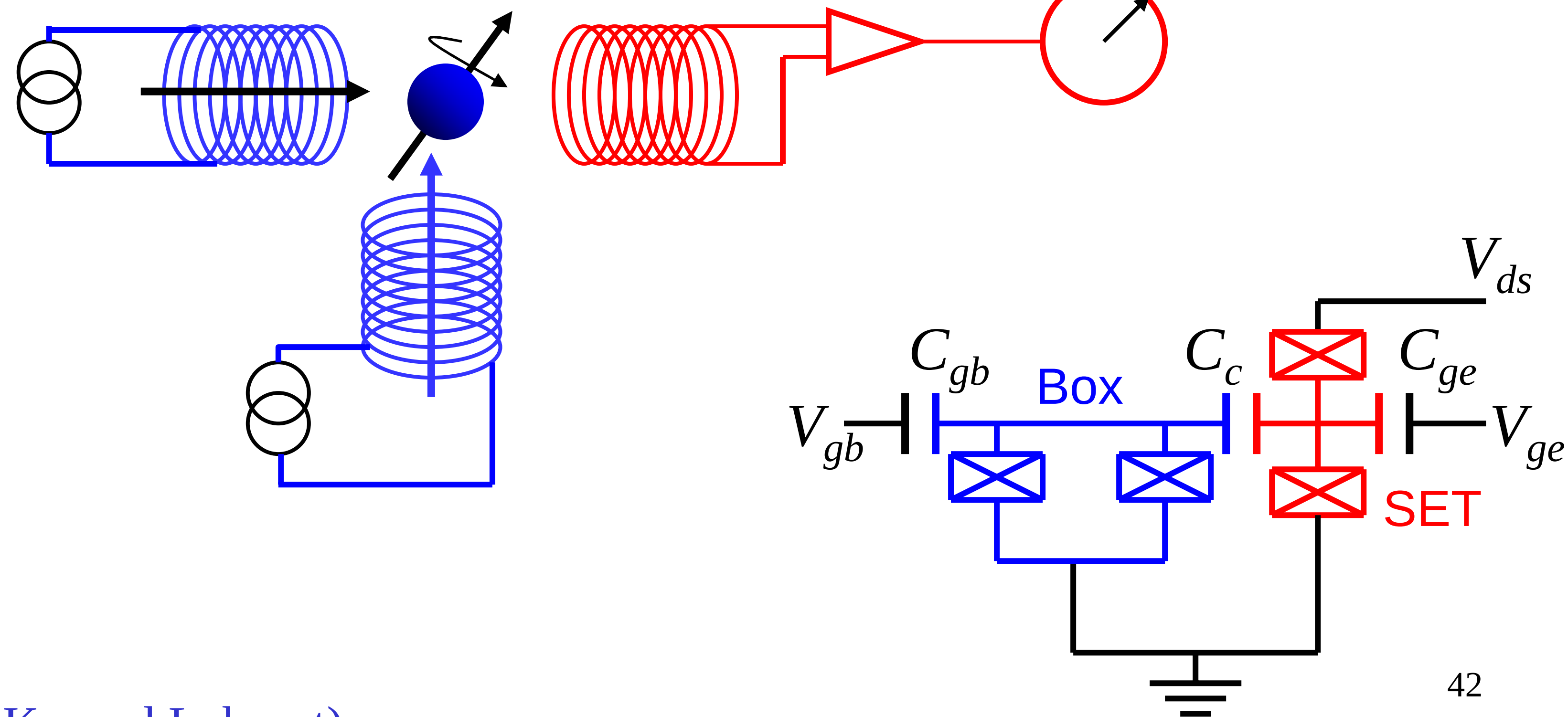
$E_J < \omega_r$



Quantum Computation and NMR of a Single 'Spin'

Single Spin ½

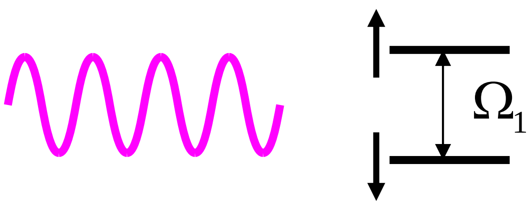
Quantum Measurement

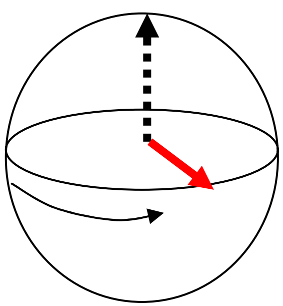


(After Konrad Lehnert)

NMR language

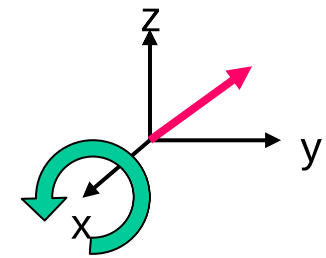
microwave pulse

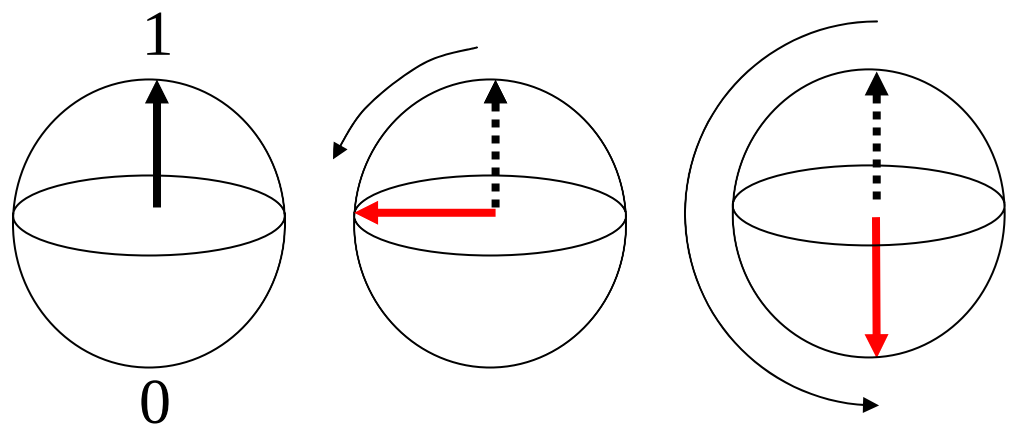




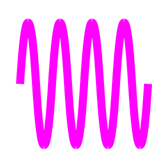
free evolution (analogous to gyroscopic precession)

Quantum control of qubits



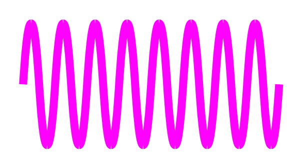


$\pi/2$ pulse



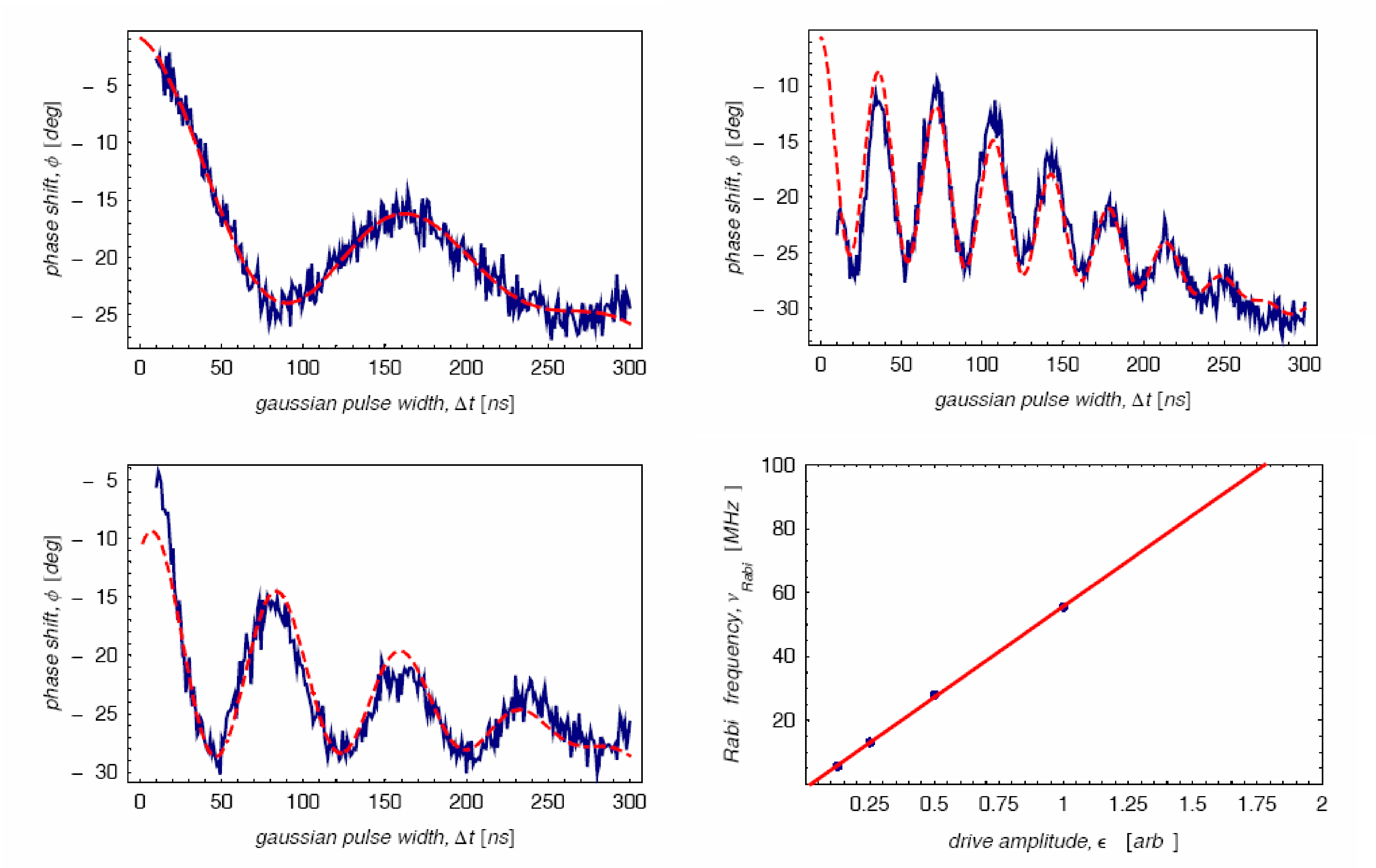
$\sqrt{\text{NOT}}$

π pulse



NOT

Rabi Flopping of Qubit Under Continuous Measurement



FUTURE DIRECTIONS

- strongly non-linear devices for microwave quantum optics
    - single atom optical bistability
    - photon `blockade'
- single photon microwave detectors
- single photon microwave sources
- quantum computation
    - QND dispersive readout of qubit state via cavity
    - resonator as 'bus' coupling many qubits
    - cavity enhanced qubit lifetime

SUMMARY

Cavity Quantum Electrodynamics

$\Downarrow$

cQED

$\Downarrow$

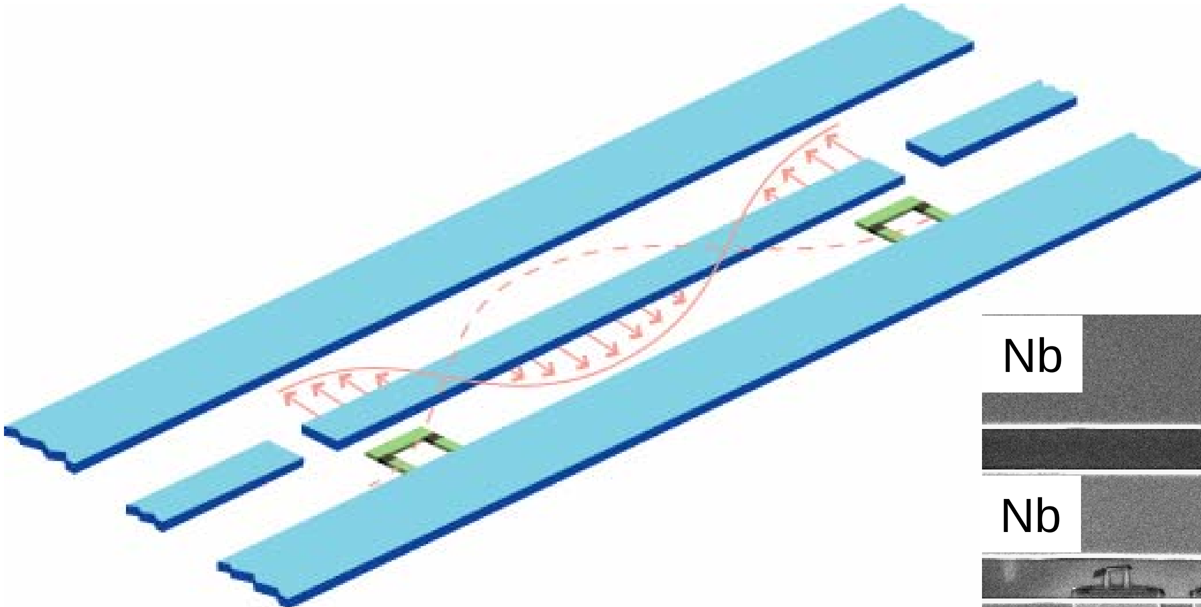
"circuit QED"

$\Downarrow$

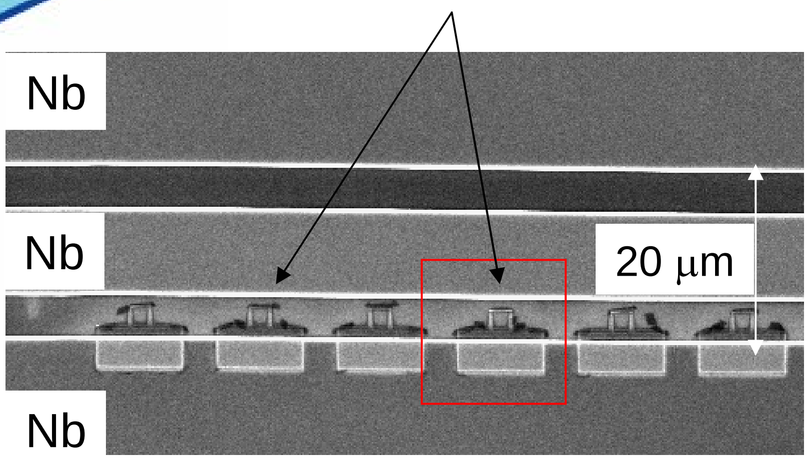
Coupling a Superconducting Qubit to a Single Photon

-first observation of vacuum Rabi splitting
-initial quantum control results

Coupling Qubits via Cavity Mode

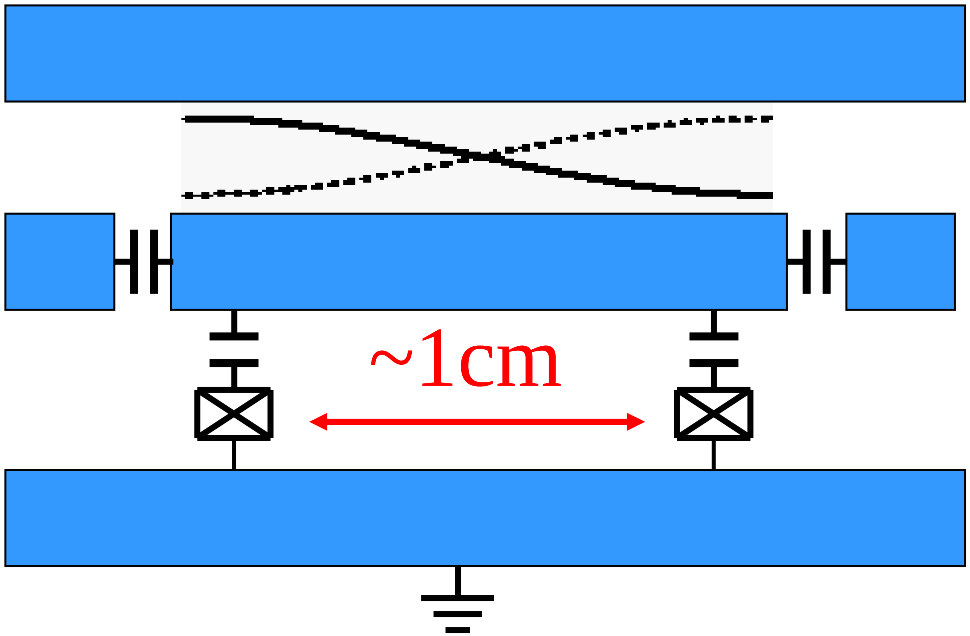


multiple CPB
qubits in a cavity



can integrate multiple qubits in a single cavity,
with no additional fabrication complexity

Entanglement via Resonator "Bus"



Qubit coupling via virtual photon exchange:

$$\boxed{J_{12} \sim g^2 / \Delta}$$

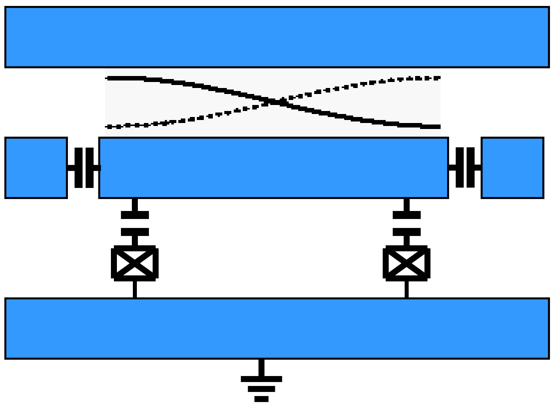
Room for many qubits in single resonator

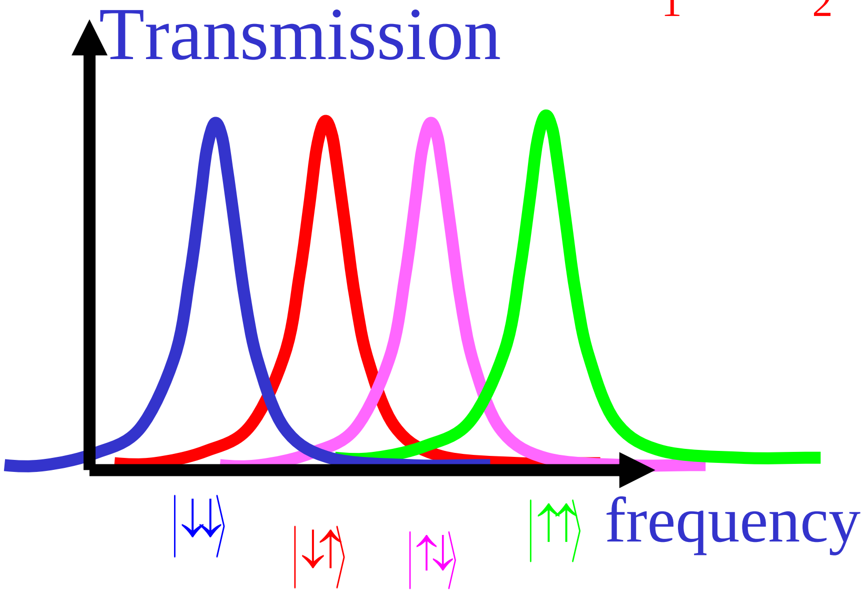
Operation rate: $\Gamma_{op} \sim g^2 / \Delta$ (t_{op}~10-100 ns)

$$\text{Number of Ops} \sim \Gamma_{op} \Big/ \max\left[\gamma_{\mathrm{NR}}, \kappa\left(g/\Delta\right)^2\right] \sim 40 - 1200$$

Multi-qubit readout: multiple cavity pulls

$$\pm\frac{g_1^2}{\Delta_1}\pm\frac{g_2^2}{\Delta_2}$$





$\Rightarrow$ Single readout line, 2 bits of information: Two qubit readout without extra wires

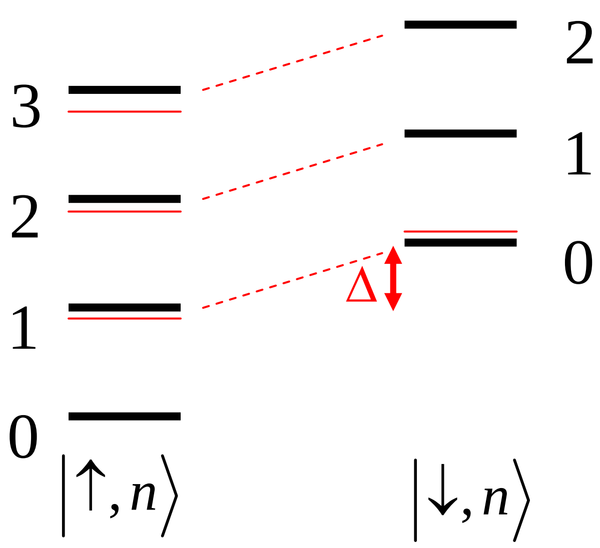
Permits selective projection of 2 bit states

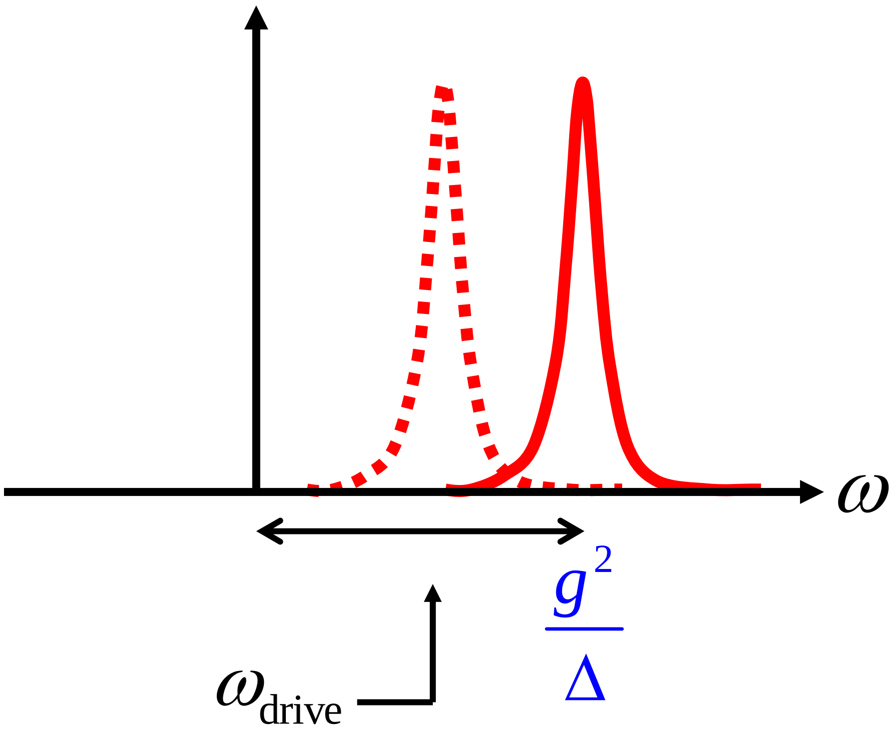
Single Atom Optical Bistability

$$\bar{\omega}_{\mathrm{r}} = \omega_{\mathrm{r}} + \frac{g^2}{\Delta}\frac{1}{\sqrt{1+n/n_{\mathrm{c}}}}$$

$n_{\mathrm{c}} \approx 250$ photons

$\mathrm{P} = \kappa n_{\mathrm{c}} \sim 10^{-15}\,\mathrm{W}$





Comparison of cQED with Atoms and Circuits

Parameter	Symbol	Optical cQED with Cs atoms	Microwave cQED/ Rydberg atoms	Super-conducting circuit QED
Dipole moment	d/ea_o	1	1,000	20,000
Vacuum Rabi frequency	g/π	220 MHz	47 kHz	100 MHz
Cavity lifetime	$1/\kappa$; Q	1 ns; 3×10^7	1 ms; 3×10^8	160 ns; 10^4
Atom lifetime	$1/\gamma$	60 ns	30 ms	> 2 μs
Atom transit time	$t_{transit}$	> 50 μs	100 μs	Infinite
Critical atom #	$N_0=2\gamma\kappa/g^2$	6×10^{-3}	3×10^{-6}	6×10^{-5}
Critical photon #	$m_0=\gamma^2/2g^2$	3×10^{-4}	3×10^{-8}	1×10^{-6}
# of vacuum Rabi oscillations	$n_{Rabi}=2g/(\kappa+\gamma)$	10	5	100