# Adventures in Phase Space: Why Boson's are Better than Qubits for Quantum Error Correction

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Disclosure: SMG is a consultant and equity holder in Quantum Circuits, Inc. and an equity holder in IBM, Inc. No matter how much progress there is in increasing coherence times, we still must contend with the fundamental law of quantum devices:

There is no such thing as too much coherence.

We need quantum error correction!

Theme: Modifying Non-Equilibrium Quantum Dynamics with a 'Maxwell Demon' to keep a qubit alive



## Take-home message:

#### Quantum error correction

&

### Quantum simulations of physical models containing bosons

#### are both vastly more efficient on hardware containing 'native' bosons



# The Quantum Error Correction Problem

I am going to give you an <u>unknown</u> quantum state.

If you measure it, it will change randomly due to state collapse ('back action').

If it develops an error, please fix it.

*Mirable dictu*: It can be done!

- No-go theorem for error correction in classical analog computation.
- Quantum machines have both analog and digital features.
- <u>Rules of the QEC game</u>:
  - Noise demon has *universal* computational power using arbitrary
     K-local (bounded Pauli weight) gates (e.g. 1- and 2-qubit (continuous) gates).
  - Noise demon has bounded speed (we hope).
  - You have *less* computational power—only non-universal Clifford gates and measurements.

• You can win!

(If you are faster than the demon and don't make too many mistakes yourself)

Quantum Error Correction for an unknown state requires storing the quantum information *non-locally* in (non-classical) *correlations* over multiple physical qubits.



Non-locality: No single physical qubit can "know" the state of the logical qubit.

Special multi-qubit measurements can tell you about errors without telling you the logical state in which the error occurred.

<u>Miracle</u>: Quantum errors are analog (i.e. continuous). Measured errors are discrete (i.e. digital). State collapse is our friend!





*N* qubits have errors *N* times faster. Maxwell demon must overcome this factor of *N* – and not introduce errors of its own! (or at least not uncorrectable errors)

# Definition of "better" (QEC Gain)



#### **Stabilizer Codes**

*N* qubits have 2<sup>*N*</sup> states. Define a 2D logical code subspace:  $C = \text{span} \{ |0_L\rangle, |1_L\rangle \}$ and logical operators  $X_L = |0_L\rangle\langle 1_L| + |1_L\rangle\langle 0_L|, \quad Z_L = |0_L\rangle\langle 0_L| - |1_L\rangle\langle 1_L|, \quad Y_L = +iX_LZ_L$ using *N*-1 stabilizers  $\{S_j; j = 1, ..., N-1\}$  and imposing *N*-1 constraints  $S_j |\psi_{\text{code}}\rangle = (+1) |\psi_{\text{code}}\rangle, \forall j.$ 

Stabilizers are mutually commuting and commute with logical operators. [So can be measured simultaneously and without affecting logical state.]

Stabilizers anti-commute with physical errors so measurement of stabilizers give error syndromes that collapse the error state without collapsing the logical state.

# Example stabilizer code

'Logical' qubit



9 qubit Shor code can correct 1 error: X,Y, or Z

3 types of errors x 9 locations = 27 possible error states + (no-error state) = 28

Code requires 8 stabilizer measurements

- Z<sub>1</sub>Z<sub>2</sub>, Z<sub>2</sub>Z<sub>3</sub>, Z<sub>4</sub>Z<sub>5</sub>, Z<sub>5</sub>Z<sub>6</sub>, Z<sub>7</sub>Z<sub>8</sub>, Z<sub>8</sub>Z<sub>9</sub>
- ightarrow Detect bit flip errors
- $X_1 X_2 X_3 X_4 X_5 X_6, X_4 X_5 X_6 X_7 X_8 X_9$

→ Detect phase flip errors

Very difficult multi-qubit measurements! [N.B. cannot measure  $Z_1$ ,  $Z_2$  separately and multiply results! Need *joint* measurements.]

#### https://quantuminstitute.yale.edu





Idea of Bosonic Codes: Don't use material objects as qubits.

Use microwave photon states stored in high-Q superconducting resonators.

Calloue	(IIISt to exceed bleak-even).
	Ofek, et al., Nature <b>536</b> , 441–445 (2016)
Binomial	Code: Michael et al., <i>Phys. Rev. X</i> <b>6</b> , 031006 (2016)
	Hu et al., <i>Nature Physics</i> <b>15</b> , 503 (2019) Ni et al., <i>Nature</i> <b>616</b> , 556 (2023)
Autonom	ous Code (T4C truncated cat): Gertler et al. <i>, Nature</i> <b>590</b> , 243 (2021)
GKP Code	es:
cQED	Campagne-Ibarcq et al. Nature 584, 368 (2020)
lons	de Neeve et al. <i>, Nature Physics</i> <b>18</b> , 296 (2022) Flühman et al. <i>, Nature</i> <b>566</b> , 513–517 (2019)
Theory	Royer et al., <i>Phys. Rev. Lett.</i> <b>125</b> , 260509 (2020) <i>PRX Quantum</i> <b>3,</b> 010335 (2022)
CQED	Sivak et al., <i>Nature</i> <b>616</b> , 50 (2023)
Bosonic c	ode reviews:
	W. Cai et al., Fund. Res. 1, 50 (2021)
	A. Joshi et al., <i>Q. Sci. Tech.</i> <b>6</b> , 033001 (2021)

Cat code (first to exceed break even):

13



Single-mode microwave resonators (harmonic oscillators) are empty boxes (vacuum surrounded by superconducting walls)



"Hardware Efficiency"

Oscillators have many quantum levels so can replace multiple physical qubits without adding more 'moving parts.'



Single-mode weakly damped oscillators have a very simple error model: photon loss

Only a single mode and only <u>one</u> kind of error —photon loss — NOT *3N* errors as for qubits.

$$|\psi\rangle = \alpha |0_L\rangle + \beta |1_L\rangle$$

#### Bosonic Quantum Error Correction Codes



Harmonic oscillator has an infinite number of states. A qubit has only two states.

We need to pick out two orthogonal states to act as 'logical code words' to hold one qubit's worth of (protected) information.

$$\frac{dE}{dt} = -\kappa E \Longrightarrow \frac{d\left\langle \hat{n} \right\rangle}{dt} = -\kappa \left\langle \hat{n} \right\rangle$$

Simplest code:  $|0_L\rangle = |0\rangle$   $|1_L\rangle = |1\rangle$ 

Has smallest possible number of photons and therefore longest lifetime.

But <u>not</u> error correctable after photon loss:  $\alpha |0\rangle + \beta |1\rangle \rightarrow |0\rangle$ 

This is what we have to beat to reach break-even. 16

# Meet the sample



**Coaxial high-Q cavity** [M. Reagor *et al.,* (PRB, 2016)]; **Stripline ancilla chip geometry** [C. Axline *et al.,* (APL, 2016)]; **Tantalum transmon** [S. Ganjam *et al.,* (in preparation, 2022)], [A. Place *et al.,* (Nature Comm., 2021)].



Still keep

 $\sim 80\%$  of data

 $\approx 560 \mu s$ 

 $\tau \approx 290 \,\mu s$ 

 $\tau \approx 320 \mu s$ 

 $\tau \approx 130 \mu s$ 

 $\tau \approx 15 \mu s$ 

1.75x break even (heralded)

First code to (slightly) exceed break even: Schrödinger Cat Code



Theory: Leghtas, Mirrahimi, et al., *PRL* **111**, 120501 (2013) Experiment: Ofek et al. *Nature* **536**, 441 (2016) 'Beating the break-even point with a discrete-variable-encoded logical qubit,'

Zhongchu Ni, et al. *Nature* **616**, 56 (2023)

Binomial Code Phys. Rev. X 6, 031006 (2016) Logical code words (even parity)  $|0_{\rm L}\rangle = \frac{|0\rangle + |4\rangle}{\sqrt{2}}$   $|1_{\rm L}\rangle = |2\rangle$ 

Prior work reached 92% of break-even Luyan Sun group (Tsinghua) *Nature Phys.* **15**, 503 (2019)



# Gottesman-Kitaev-Preskill Bosonic Code and the Geometry of Phase Space



'Encoding a qubit in an oscillator'

D. Gottesman, A. Kitaev, and J. Preskill, *Phys. Rev. A* 64, 012310 (2001).

Perspective: **'Quantum Error Correction with the Gottesman-Kitaev-Preskill Code'** Arne L. Grimsmo and Shruti Puri, *PRX Quantum* **2**, 020101 (2021)

#### **GKP** wave functions

Bosonic QEC with (idealized) GKP states of an oscillator

Stabilizers define code space:

$$\psi_0(q) = \langle q \mid 0_L \rangle$$

$$-6\sqrt{\pi} -4\sqrt{\pi} -2\sqrt{\pi} \quad 0 \quad +2\sqrt{\pi} +4\sqrt{\pi} +6\sqrt{\pi} \quad q$$



Code space is stabilized by:

$$egin{aligned} S_p &= e^{i2\sqrt{\pi}\hat{q}}\ S_q &= e^{i2\sqrt{\pi}\hat{p}} \end{aligned}$$

N.B. Unlike ordinary qubit stabilizers, these have a continuum of eigenvalues on the unit circle corresponding to continuous drift of position or momentum.

$$S_{p} \left| \Psi_{\delta} \right\rangle = e^{i2\sqrt{\pi}\delta} \left| \Psi_{\delta} \right\rangle$$

Continuous stabilizer eigenvalue on the unit circle in the complex plane.



Code space is stabilized by:

$$S_p = e^{i2\sqrt{\pi}\hat{q}}$$
  
 $S_q = e^{i2\sqrt{\pi}\hat{p}}$ 

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Continuous stabilizer eigenvalue on the unit circle in the complex plane.

ONLY 2 STABILIZERS NEEDED TO REDUCE INFINITE STATE SPACE DOWN TO 2 LOGICAL STATES!



Code space is stabilized by:

$$S_p = e^{i2\sqrt{\pi}\hat{q}}$$
  
 $S_q = e^{i2\sqrt{\pi}\hat{p}}$ 

N.B. Unlike ordinary qubit stabilizers, these have a continuum of eigenvalues on the unit circle corresponding to continuous drift of position or momentum. Stabilization against drift errors in *position q* 

*Measure stabilizer to detect error:* 

$$\operatorname{Im}\left\langle S_{p}\right\rangle = \left\langle \sin[2\sqrt{\pi}\hat{q}]\right\rangle$$
$$= \int dq \sin[2\sqrt{\pi}q] |\psi(q)|^{2} = \sin[2\sqrt{\pi}\delta]$$

and feedback to correct.



Geometry of phase space....



But recall that a crystal lattice produces sharp Bragg peaks in x-ray diffraction.



Gottesman, Kitaev and Preskill, Phys. Rev. A 64, 012310 (2001)

Proposed encoding a logical qubit in oscillator 'grid' states.

How can the points in this phase space grid be smaller than the minimum uncertainty wave packet?

They seem to be squeezed in both position AND momentum!?

This is possible for special choices of lattice unit cell areas.

 $[\hat{q}, \hat{p}] = +i \implies$  translations in phase space do not commute

$$\mathcal{D}(\Delta_q)\Psi(q) = e^{-i\Delta_q \hat{p}}\Psi(q) = \Psi(q - \Delta_q)$$
$$\mathcal{D}(i\Delta_p)\Psi(q) = e^{i\Delta_p q}\Psi(q)$$

$$\mathcal{D}(\Delta_q)\mathcal{D}(i\Delta_p) = e^{i\Delta_p\Delta_q}\mathcal{D}(i\Delta_p)\mathcal{D}(\Delta_q)$$
area

p  $\Delta_p$   $\Delta_q$  q

Harmonic Oscillator Phase Space

Inside the code space: *X,Y,Z* translations obey Pauli group

GKP code space is stabilized by special translations that <u>do</u> commute

$$S_p = e^{i2\sqrt{\pi}\hat{q}}$$
  
 $S_q = e^{i2\sqrt{\pi}\hat{p}}$ 

$$S_{q}S_{p} = e^{i4\pi}S_{p}S_{q}$$
$$S|0_{L}\rangle = (+1)|0_{L}\rangle$$
$$S|1_{L}\rangle = (+1)|1_{L}\rangle$$

$$2\sqrt{\pi}$$

$$S_{p} \pi \pi \pi$$

$$Y = 2\sqrt{\pi}$$

$$Z \frac{\pi}{2} \pi$$

$$X = S_{q}$$

$$Area 4\pi = 2 \text{ states}$$

$$S_{p} = S_{q} = 1$$
$$X^{2} = S_{q} = 1$$
$$Z^{2} = S_{p} = 1$$
$$ZX = e^{i\pi}XZ = -XZ$$
$$ZX = e^{i\pi/2}Y = iY$$

Stabilizers and Pauli (and Clifford) operations are translations in phase space.



GKP Syndrome measurements are similar but with larger displacements.



Experimental Calibration of Controlled Displacements Non-Commutativity (Devoret Group)



#### EXP. DATA: BERRY PHASE OSC.



Campagne-Ibarcq, Eickbusch, Touzard, et al Nature 584, 368 (2020)

Measured 'phase kickback' of controlled unitary

#### **Co-Design Center for Quantum Advantage https://bnl.gov/quantumcenter**

Microwave cavity holds GKP bosonic code



Oscillator phase-space 'image' (characteristic function) of GKP code words 'GKP code': Continuous variable qubit in an Oscillator Gottesman, Kitaev, Preskill (2001)

Squeezed state lattice in phase space ("cat in 35 places at once")

Logical gates and code stabilizers are simple phase-space translations.

Clifford group operations are all Gaussian linear optics operations

Sivak et al. [Devoret group], *Nature* **616**, 50 (2023)

# QEC circuit for the grid code



Improved (autonomous) Syndrome mapping unitary [B. Royer et al., (PRL, 2020)]

Multi-layered parametrized circuit [A. Eickbusch et al., (Nature Physics, 2022)]

Real-time processing with FPGA [N. Ofek et al., (Nature, 2016)]

Circuit training with reinforcement learning [V. Sivak et al., (PRX, 2022)]

This circuit implements a rank-4 channel:  $\{K_{qq}, K_{qe}, K_{eq}, K_{eq}, K_{ee}\}$ 

#### **Co-Design Center for Quantum Advantage https://bnl.gov/quantumcenter**

Microwave cavity holds GKP bosonic code



Oscillator phase-space 'image' (characteristic function) of GKP code words Model-free <u>Reinforcement Learning Agent</u> tunes up ~45 variables in the experimental QEC protocol.

[RL agent chose to slow down the readout to improve QEC fidelity.]

Logical gates and code stabilizers are simple phase-space translations.

Clifford group operations are all Gaussian linear optics operations

Sivak et al. [Devoret group], Nature **616**, 50 (2023)

# Wigner tomography (experiment)



# **Comparison of logical vs. physical qubits**



Average channel fidelity  
M. Nielsen, Phys. Lett. A (2002)  
$$ar{\mathcal{F}}(\delta t) = 1 - rac{1}{2}\Gamma\,\delta t$$

{|**0**>, |**1**>} **qubit** Amplitude damping + Dephasing  $\Gamma_{\{01\}} = \frac{2\gamma_2^c + \gamma_1^c}{3}$ 



# QEC: state of play



# Outlook: short term



| Ancilla  $T_1$ ,  $T_{2E}$  | Oscillator  $T_1$ ,  $T_2$  | Grid code  $T_Z$ ,  $T_Y$  |

# Outlook: long term

# Possible roadmap for future hardware-efficient quantum error correction



Example 2<sup>nd</sup> Layer QEC

Bosonic QEC code in each oscillator

Lattice of oscillator logical qubits encoded into surface code as second layer d+1

Highly hardware efficient if  $p_L \sim$  Bosonic QEC layer gets you well below next layer threshold

Microwave

resonator

 $\left( rac{p_{ ext{bosonic code}}}{p_{ ext{surface code}}^{ ext{threshold}}} 
ight)$ 

2

# Yale GKP Team

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Stabilizers and Pauli (and Clifford) operations are translations in phase space.

How do we do a continuous Pauli rotation? Requires superposition of translation and no translation:

$$e^{-i\frac{\theta}{2}Z_{\rm L}} = \cos\frac{\theta}{2}\hat{I} - i\sin\frac{\theta}{2}Z_{\rm L}$$



Small-Big-Small (SBS) protocol (autonomous and tuned for finite-energy approximate GKP states)



B. Terhal et al., (PRA, 2016); P. Campagne-Ibarcq et al., (Nature, 2020).

# Bosonic Qiskit



Qiskit SDK extension co-design tool to simulate hybrid hardware containing both qubits and bosonic modes

#### **Details**

- Open-Source code available at: <u>https://github.com/C2QA/bosonic-qiskit</u>
- Includes tutorials, example use cases and visualization tools
- Experimentally realized gate sets and measurement protocols have been implemented

#### Reference <a href="https://arxiv.org/abs/2209.11153">https://arxiv.org/abs/2209.11153</a>

