

# Disordered 1D systems

T. Giamarchi

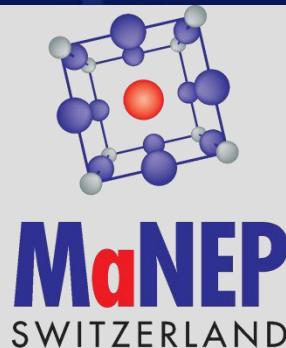
[http://dpmc.unige.ch/gr\\_giamarchi/](http://dpmc.unige.ch/gr_giamarchi/)



UNIVERSITÉ  
DE GENÈVE

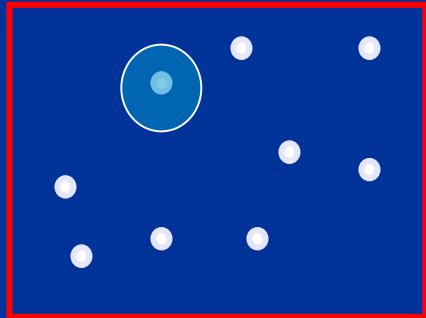


FONDS NATIONAL SUISSE  
SCHWEIZERISCHER NATIONALFONDS  
FONDO NAZIONALE SVIZZERO  
SWISS NATIONAL SCIENCE FOUNDATION



# One dimension is specially interesting

- No individual excitation can exist (only collective ones)

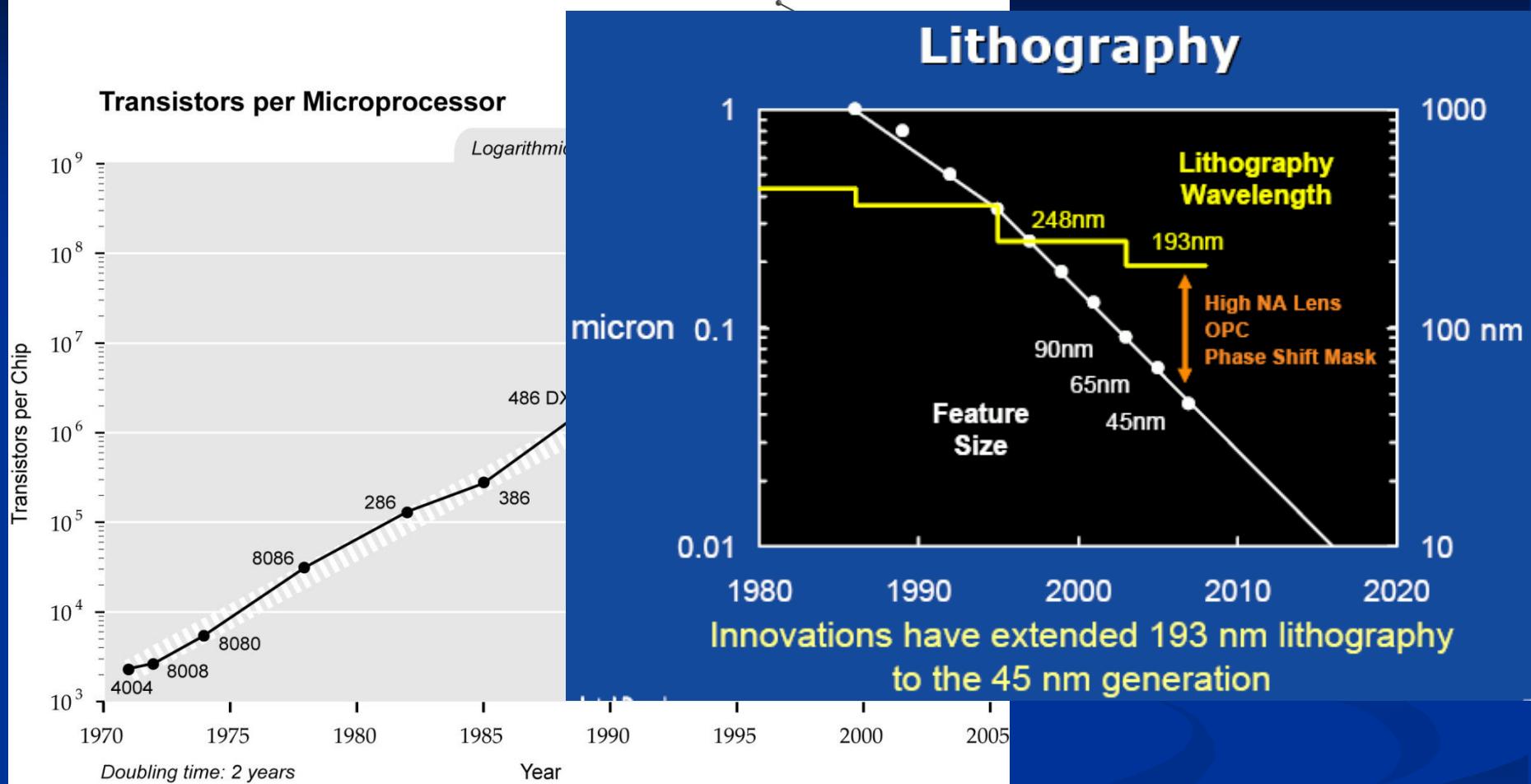


- Strong quantum fluctuations

$$\psi = |\psi| e^{i\theta}$$

Difficult to order

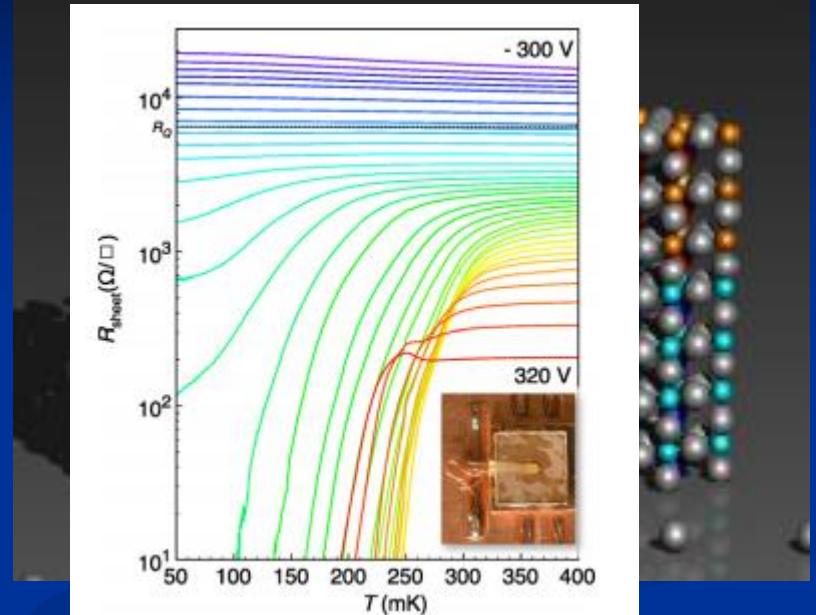
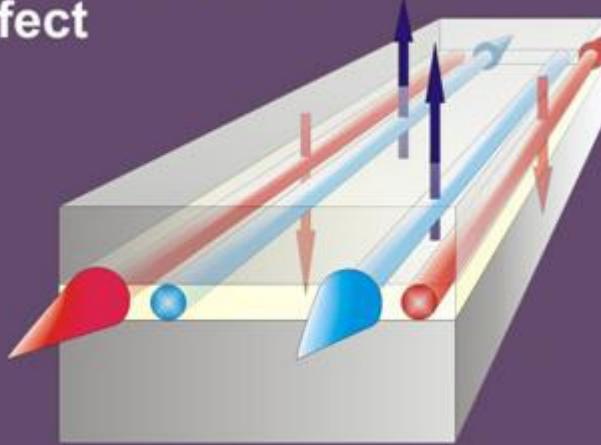
# Future electronic



Need to worry about reduced dimensionality

# Physics at the edge

Quantum Spin Hall Effect



Presence of edge  
(B. I. Halperin)



LaO/StO interface  
(JM Triscone et al.)



Quantum hall effect  
Topological insulators....

Superconductivity  
between insulators...

# A good reason to work on 1D

However, my personal reason for working on one-dimensional problems is merely that they are fun. A man grows stale if he works all the time on the insoluble and a trip to the beautiful work of one dimension will refresh his imagination better than a dose of LSD.

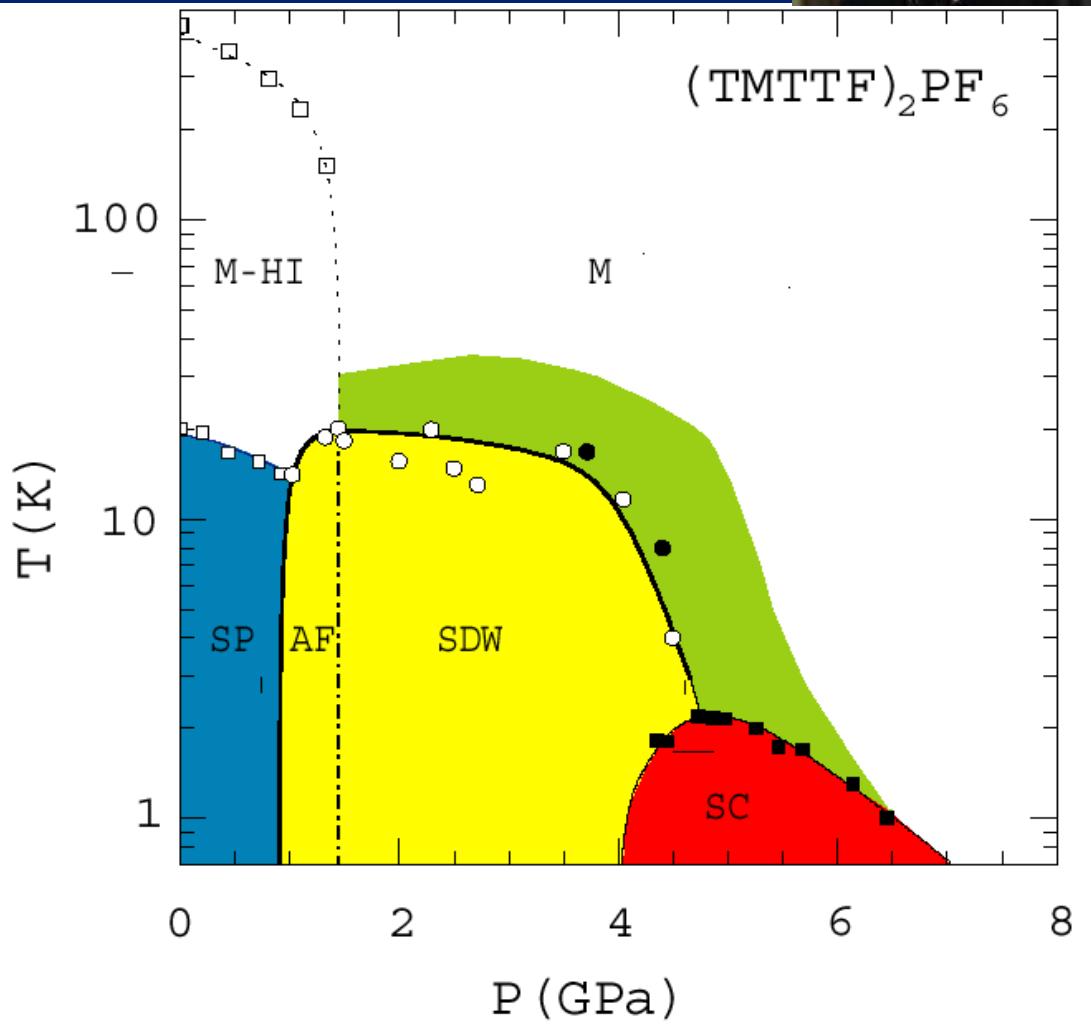
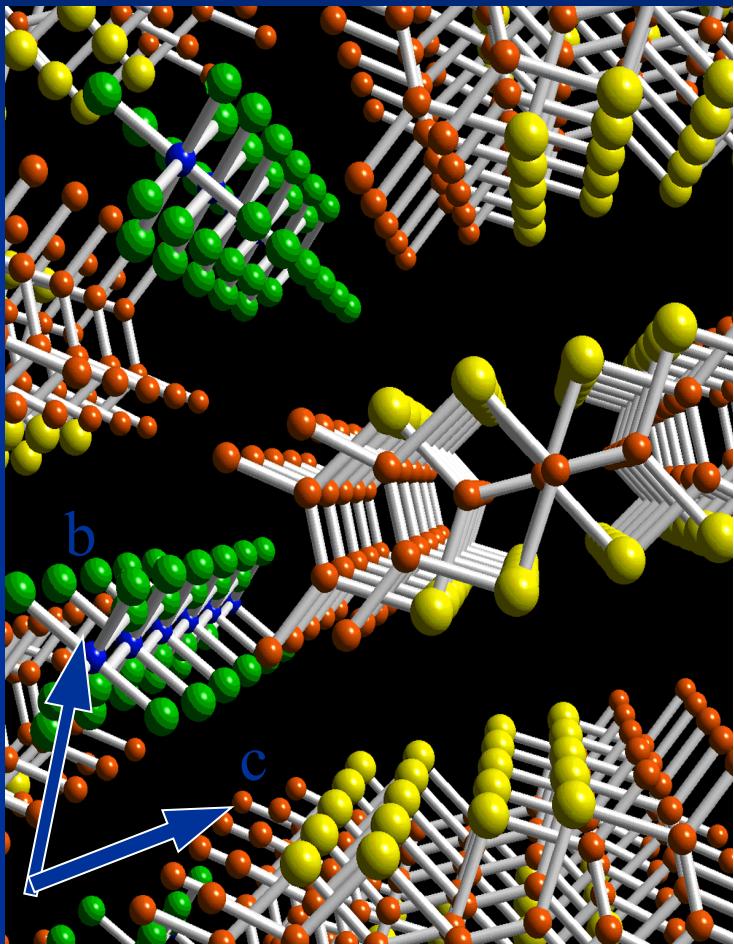
Freeman Dyson (1967)

Interesting...  
but...  
does it exist ?

TG, Int J. Mod. Phys. B 26 1244004 (2012)



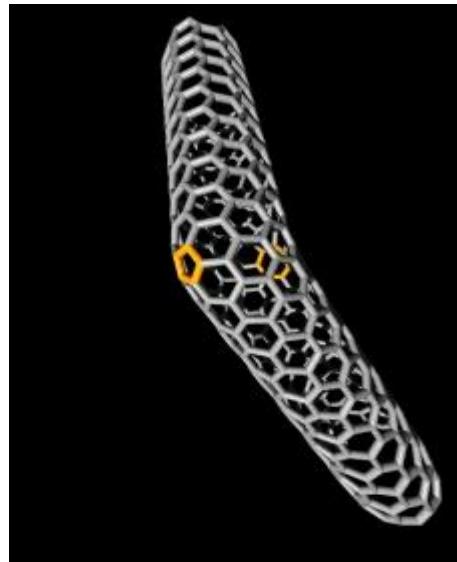
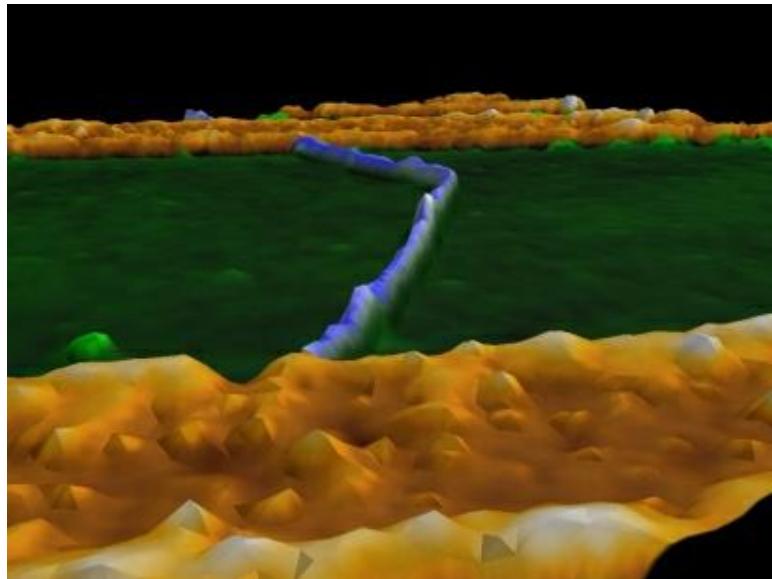
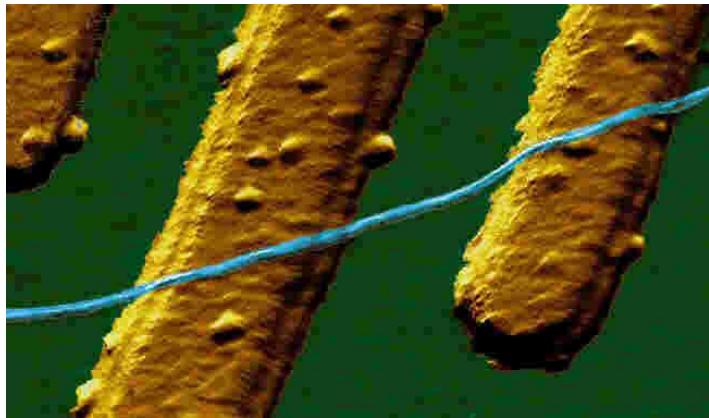
# Organic conductors

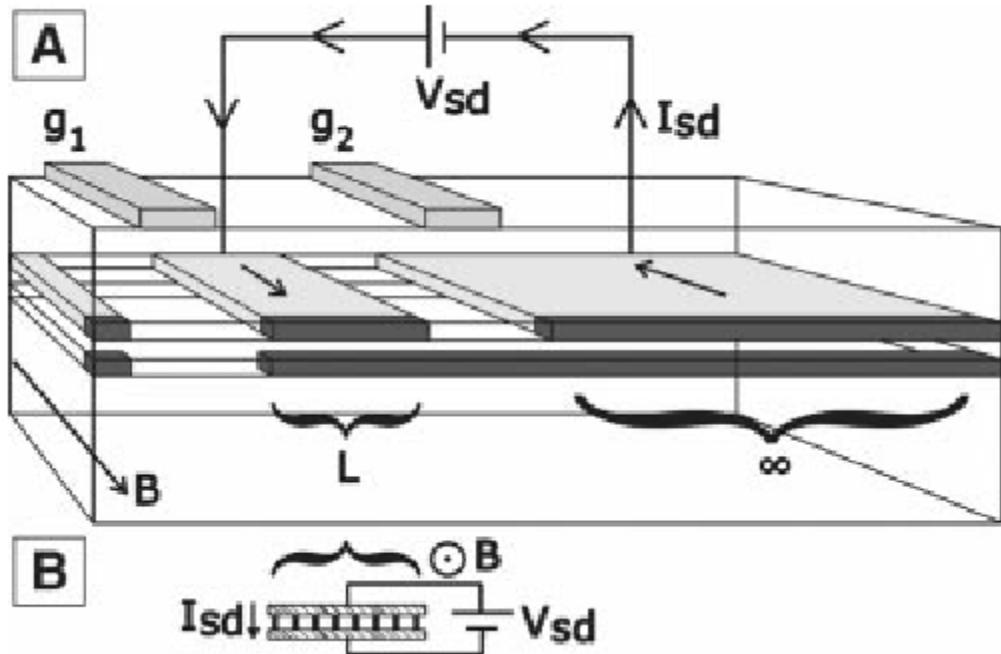


# CARBON NANOTUBES



Cees Dekker





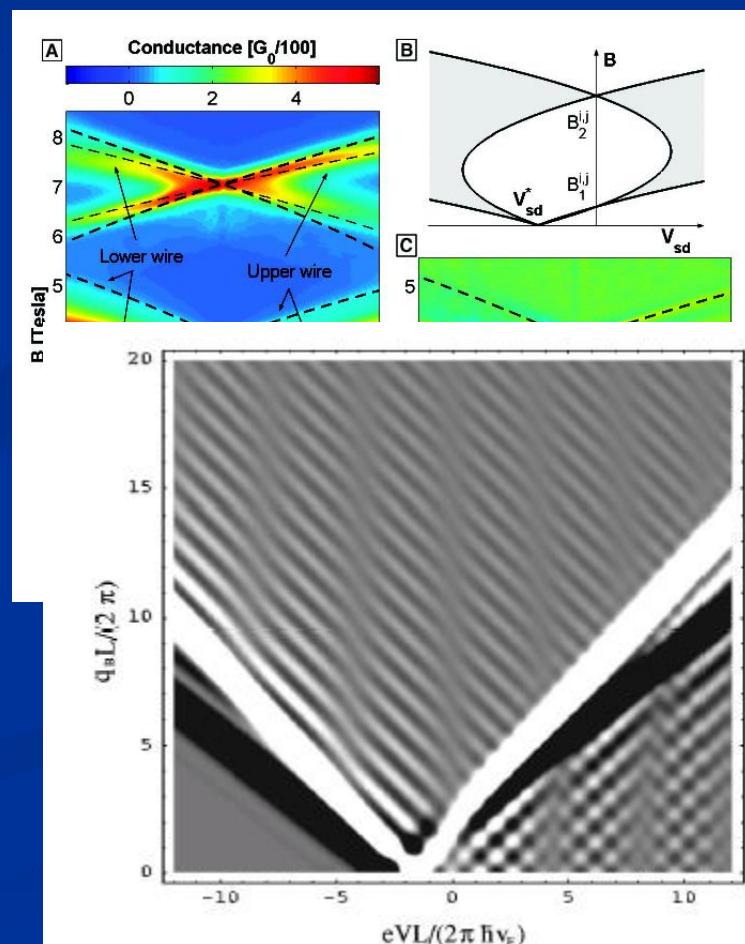
O.M Ausslander et al., Science  
298 1354 (2001)



Y. Tserkovnyak et al., PRL 89  
136805 (2002)



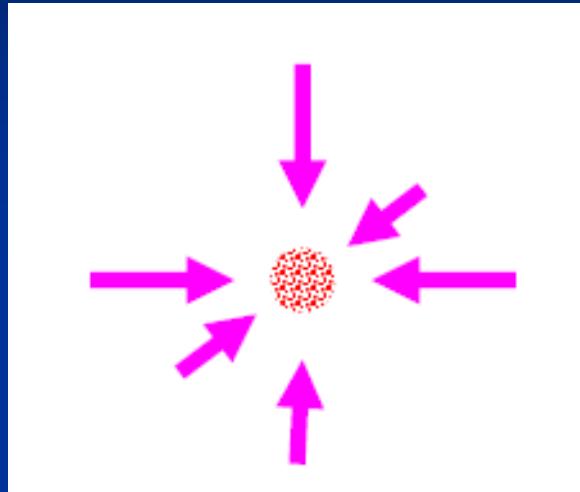
Y. Tserkovnyak et al., PRB 68  
125312 (2003)



# Cold atoms

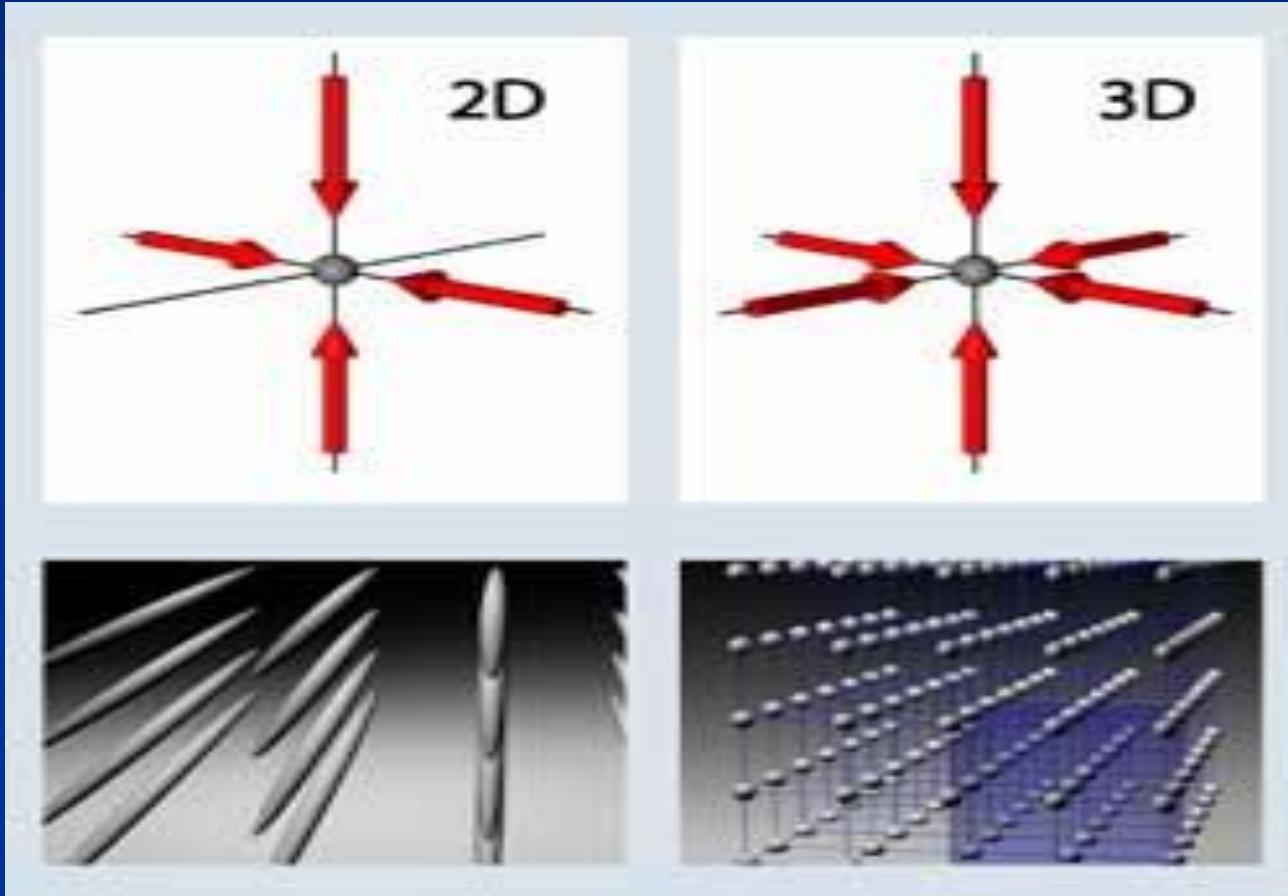


# Atom trapping



- Evaporative cooling

# Control on the dimension

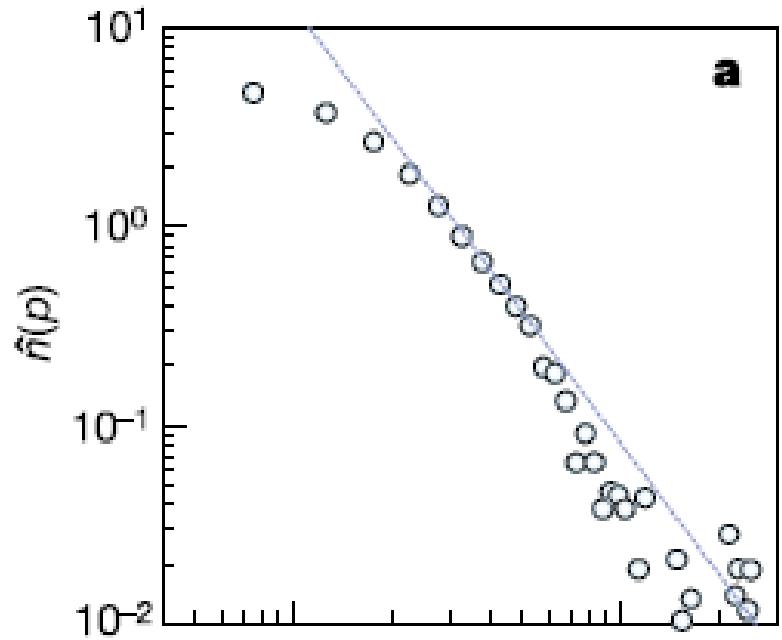
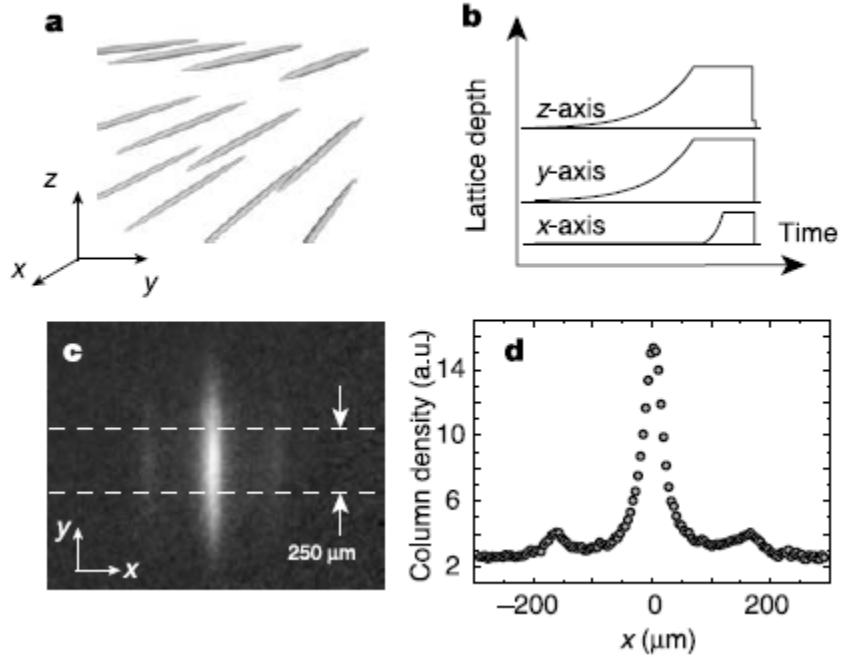


I. Bloch, Nat. Phys 1, 23 (2005)

# Cold atoms (Tonks limit)

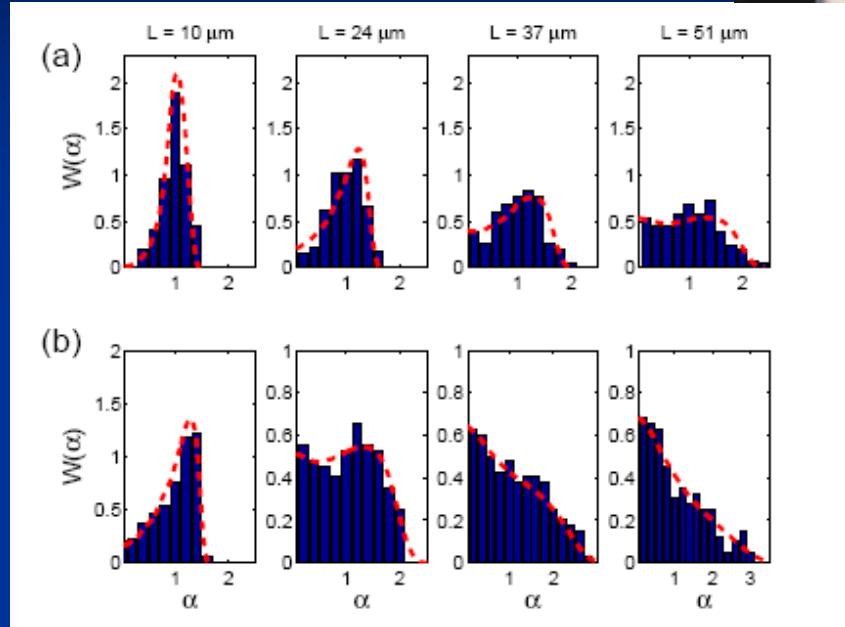
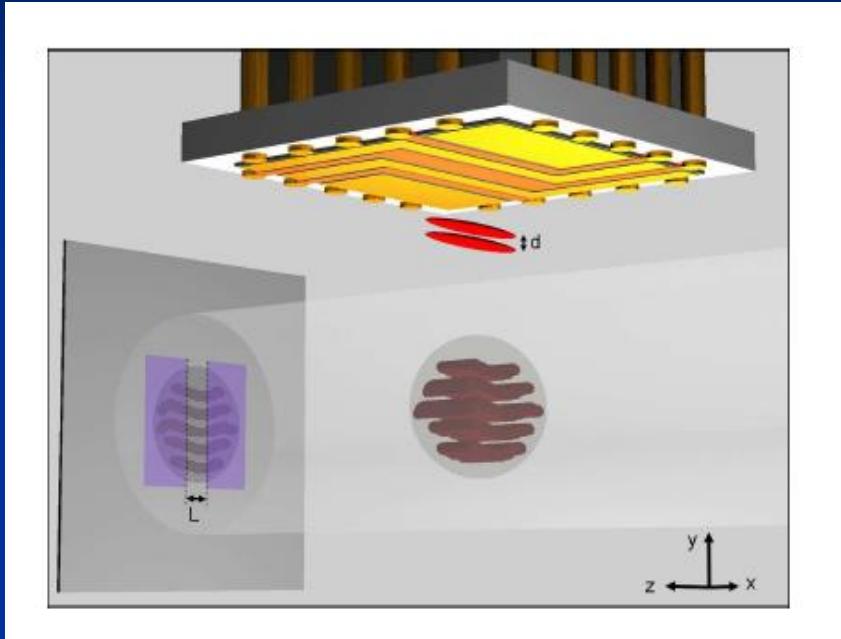


B. Paredes et al., Nature 429 277 (2004)



$$n(k) = \int dx e^{ikx} \langle \psi^\dagger(x) \psi(0) \rangle$$

# Cold atoms: Interferences



$$\int_0^L dr \langle \psi(r) \psi^\dagger(0) \rangle$$

K large (42)

S. Hofferberth et al. Nat. Phys 4  
489 (2008)

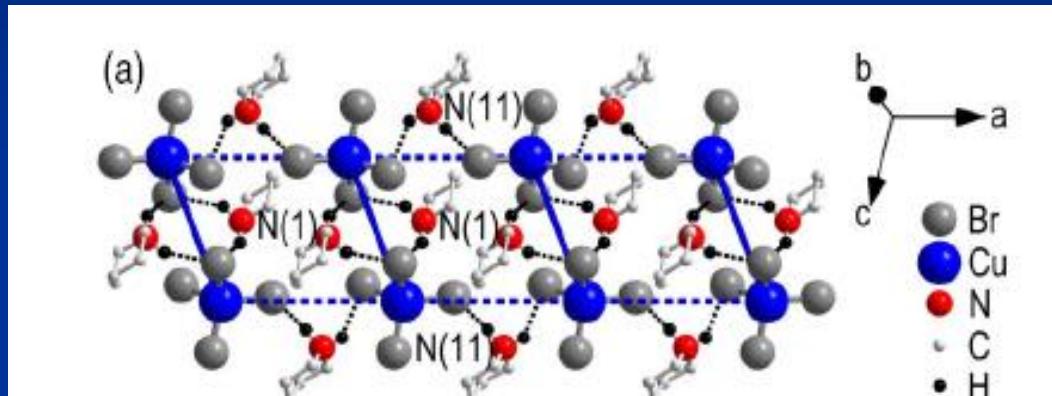


# Magnetic insulators



# Spin ladder systems

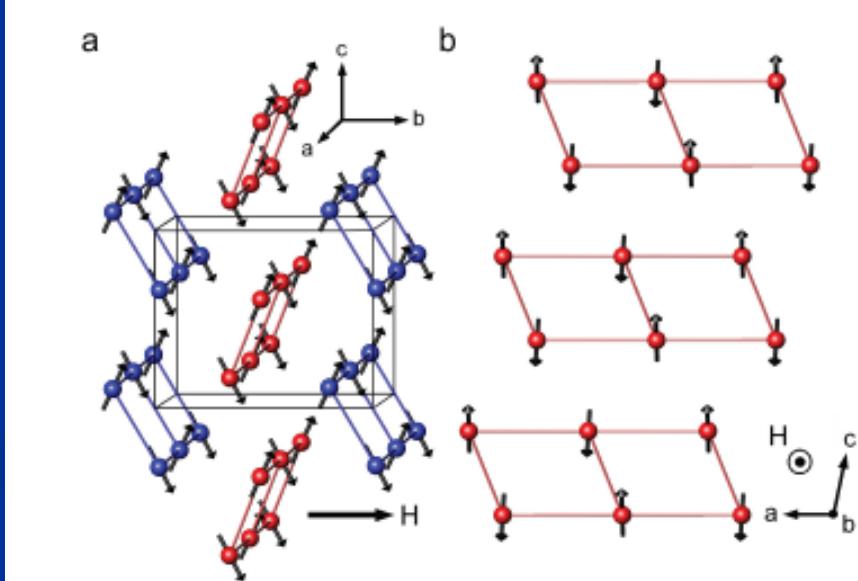
B. C. Watson et al., PRL 86 5168 (2001)



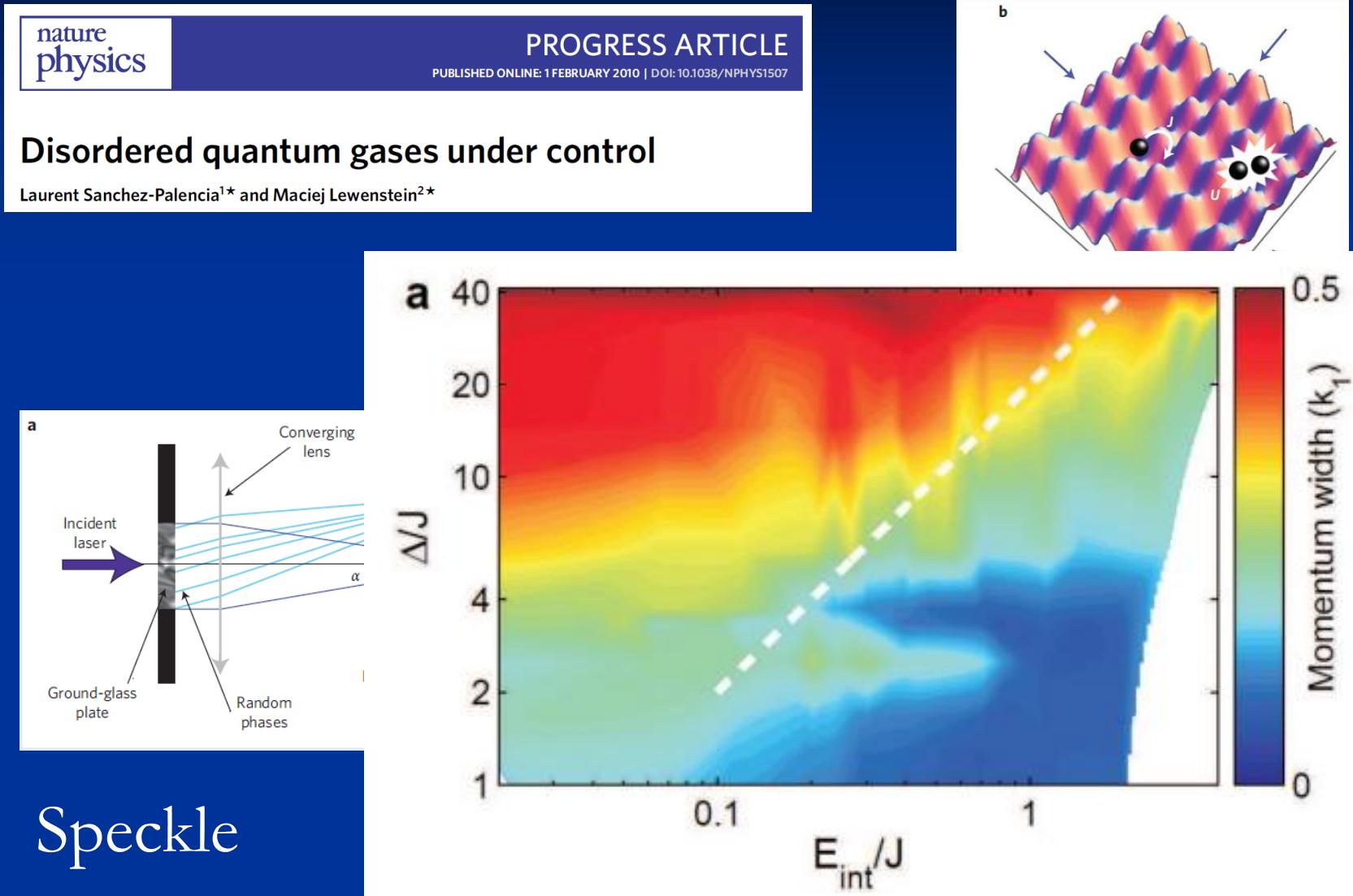
M. Klanjsek et al.,  
PRL 101 137207 (2008)

B. Thielemann et al.,

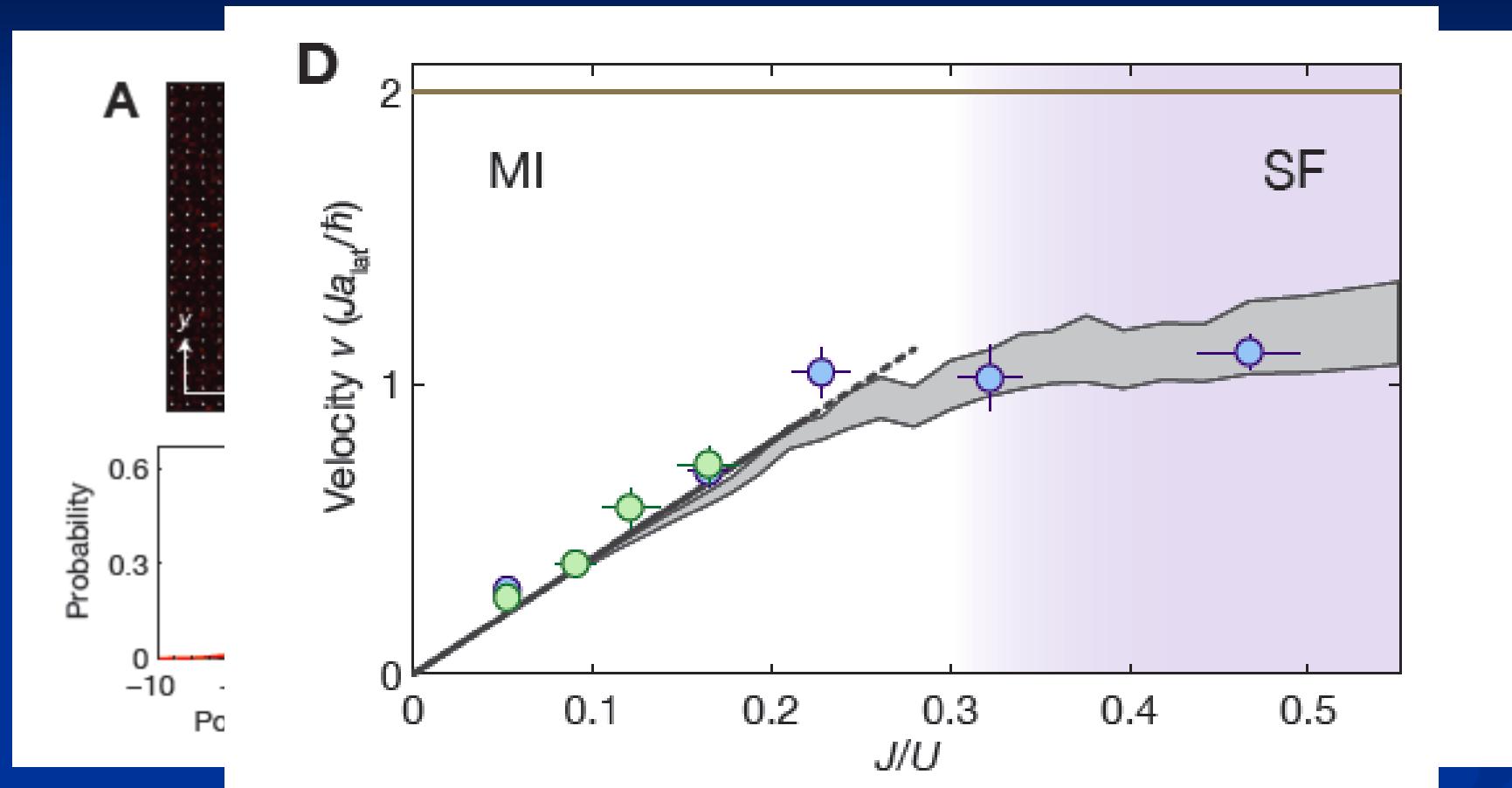
PRB 79, 020408(R) 2009



# Disorder and interactions



# Impurity in a Luttinger liquid



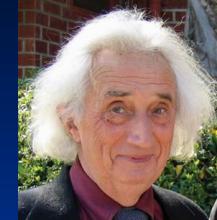
T. Fukuhara et al. Nat. Phys. (2013)



# How to treat ?



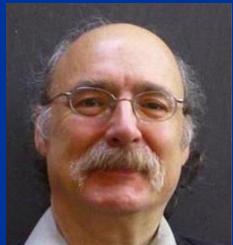
## ■ ``Standard'' many body theory



### ■ Exact Solutions (Bethe ansatz)



### ■ Field theories (bosonization, CFT)



### ■ Numerics (DMRG, MC, etc.)



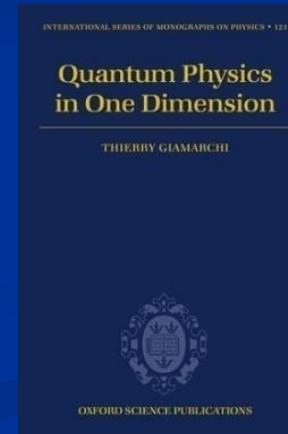
# References

TG, arXiv/0605472 (Salerno lectures)

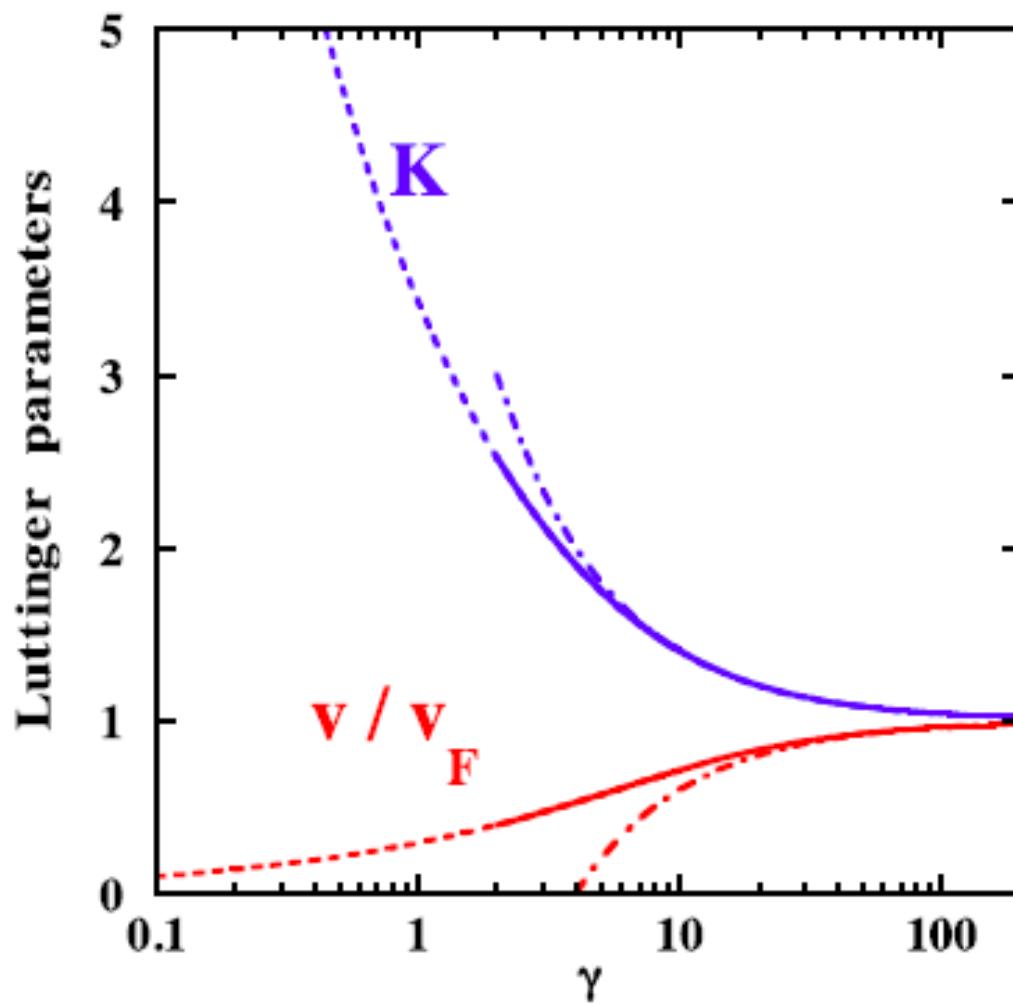
TG, Quantum physics in one dimension, Oxford (2004)

M. Cazalilla et al.,  
Rev. Mod. Phys. 83 1405 (2011)

TG, Int J. Mod. Phys. B 26 1244004 (2012)



# Tomonaga-Luttinger liquid (bosons)



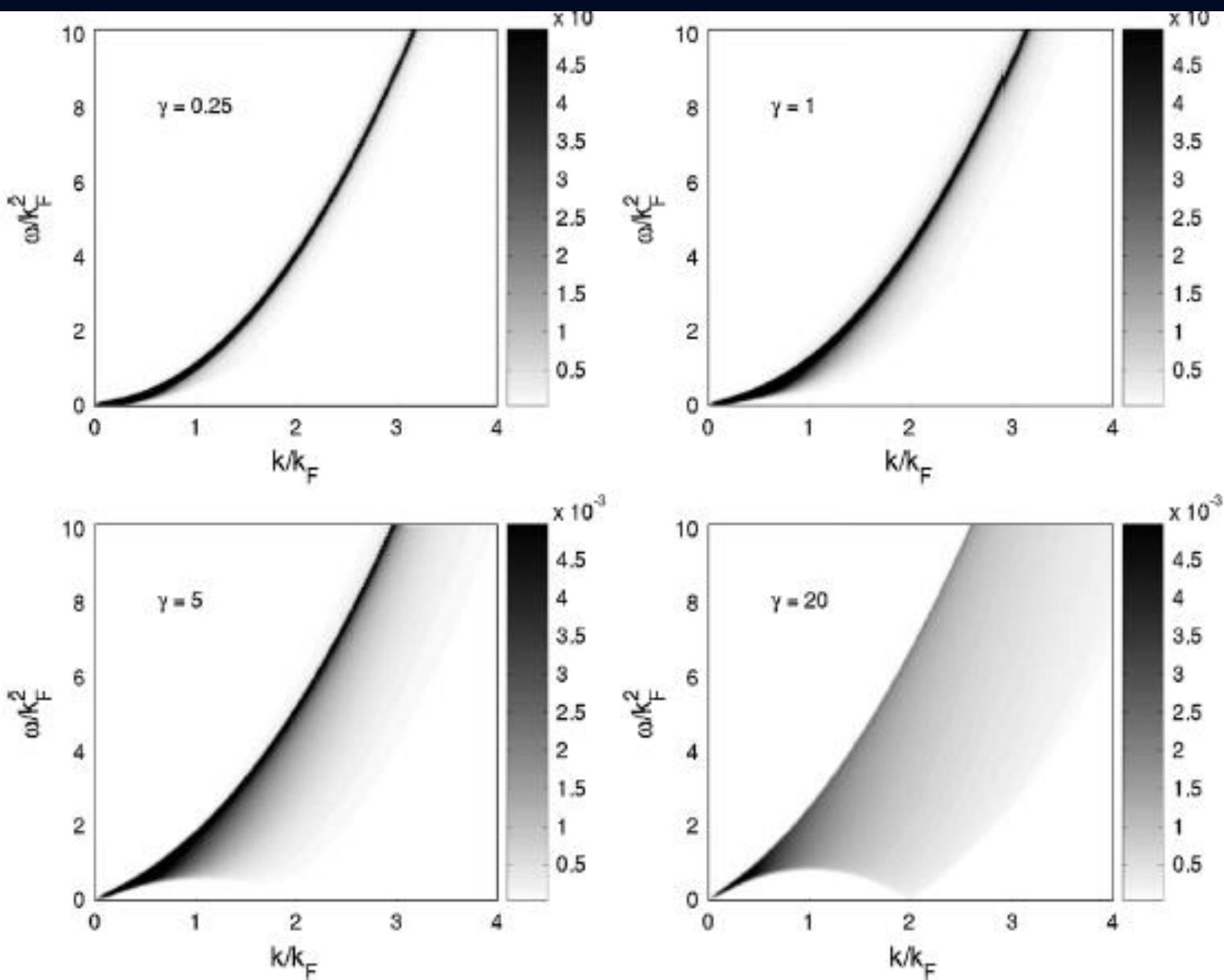


FIG. 3. Intensity plots of the dynamical structure factor [ $S(q, \omega)$ ]. Data obtained from systems of length  $L = 100$  at unit density, and  $\gamma = 0.25, 1, 5$ , and  $20$ . From Caux, and Calabrese, 2006.

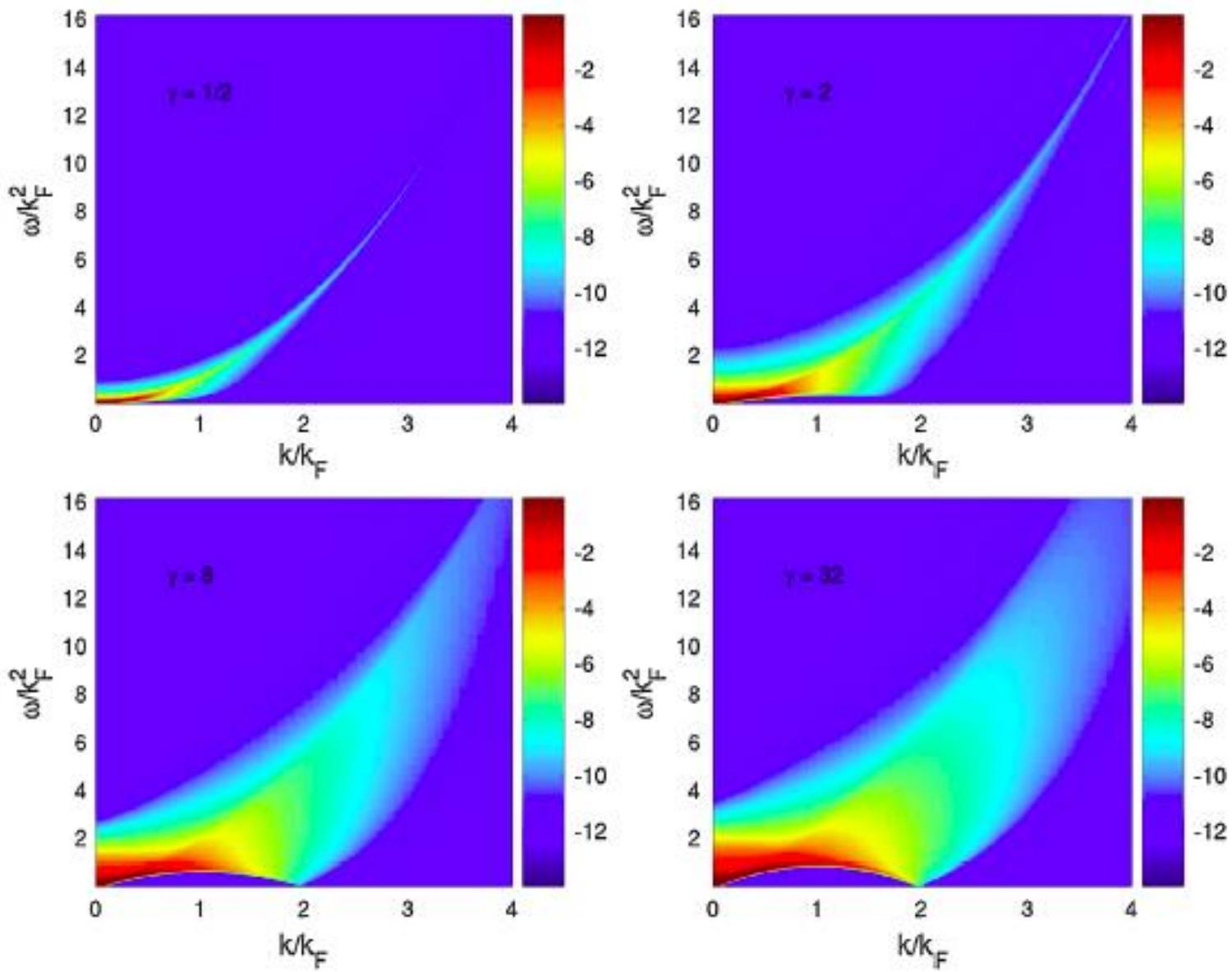
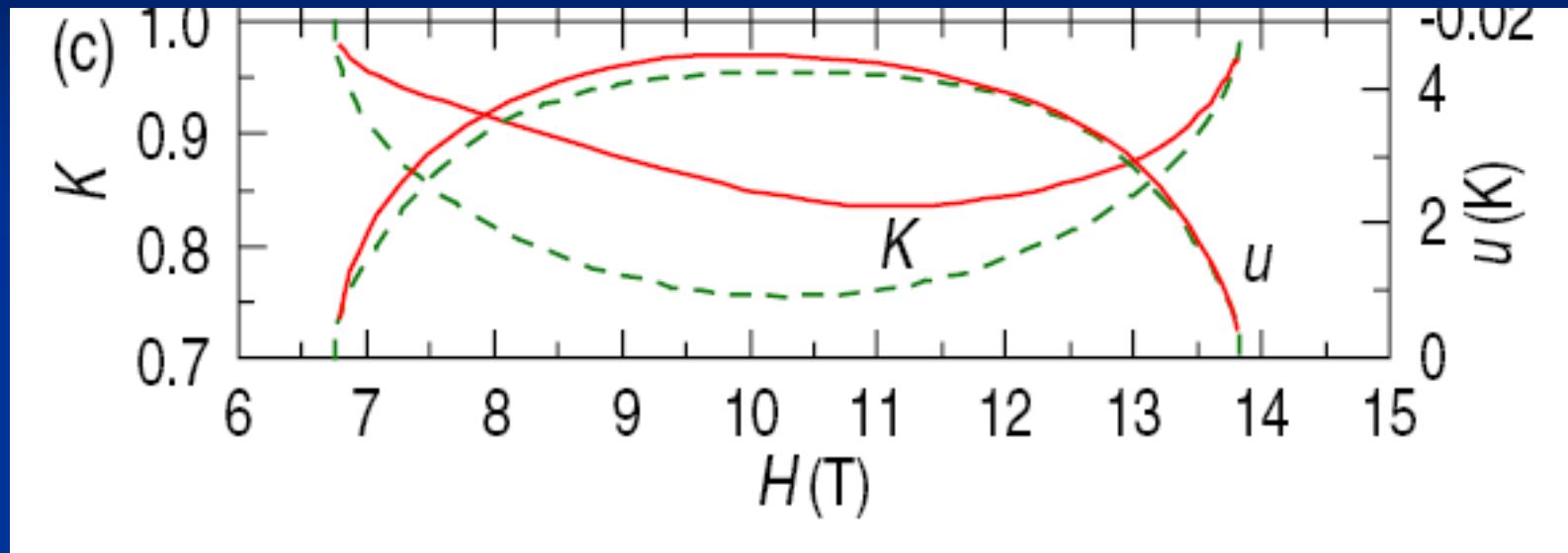


FIG. 4 (color online). Intensity plots of the logarithm of the dynamical one-particle correlation function of the Lieb-Liniger gas. Data obtained from systems of length  $L = 150$  at unit density, and  $\gamma = 0.5, 2, 8$ , and  $32$ . From J.-S. Caux, *et al.* (2007).

# Luttinger parameters



M. Klanjsek et al., PRL 101 137207 (2008)

Red : Ladder (DMRG)

Green: Strong coupling ( $J_r \rightarrow \infty$ ) (BA)

# Correlation functions

M. Klanjsek et al., PRL 101 137207 (2008)

R. Chitra, TG PRB 55 5816 (97); TG, AM Tsvelik PRB 59 11398 (99)

## ■ NMR relaxation rate:

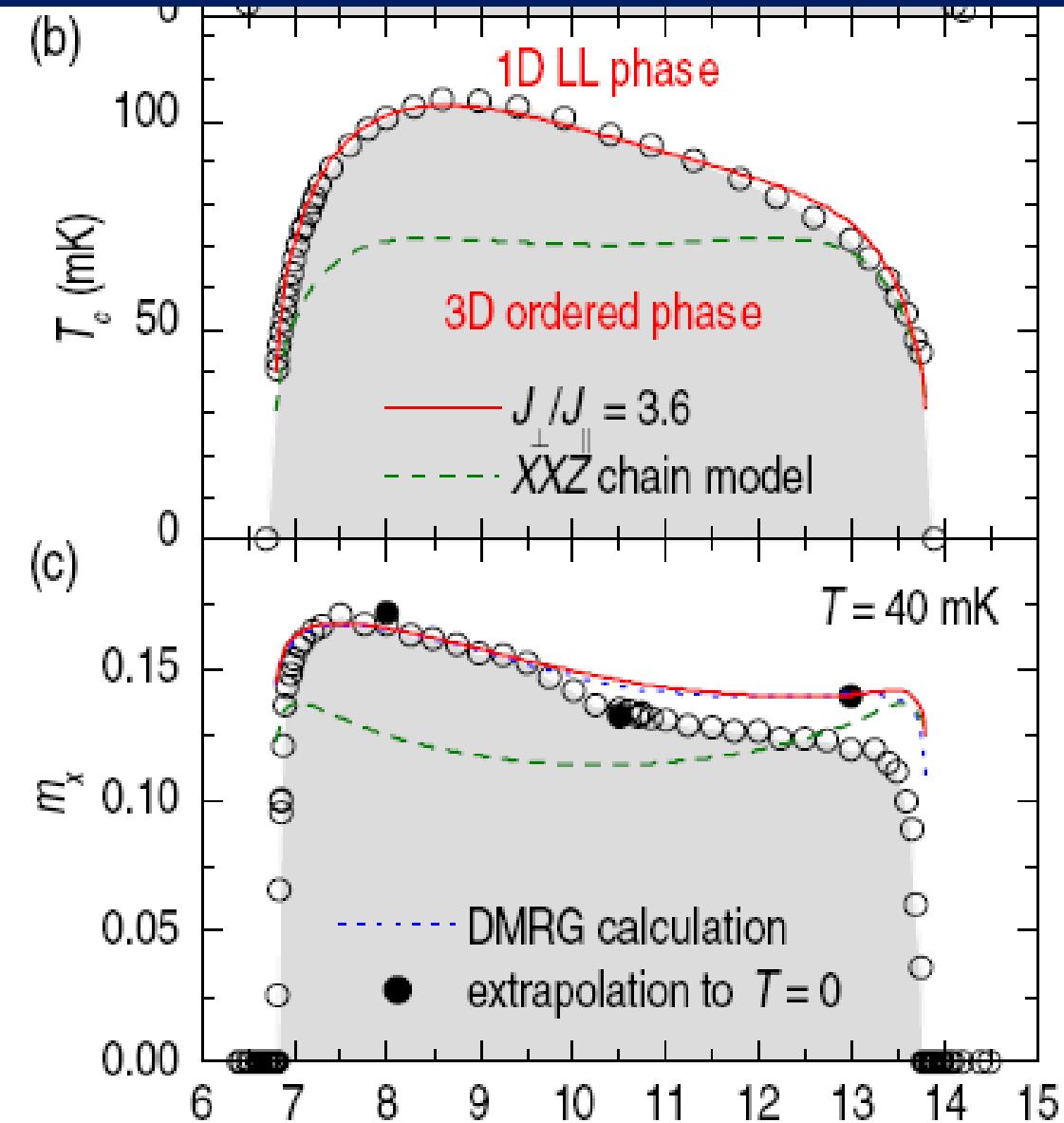
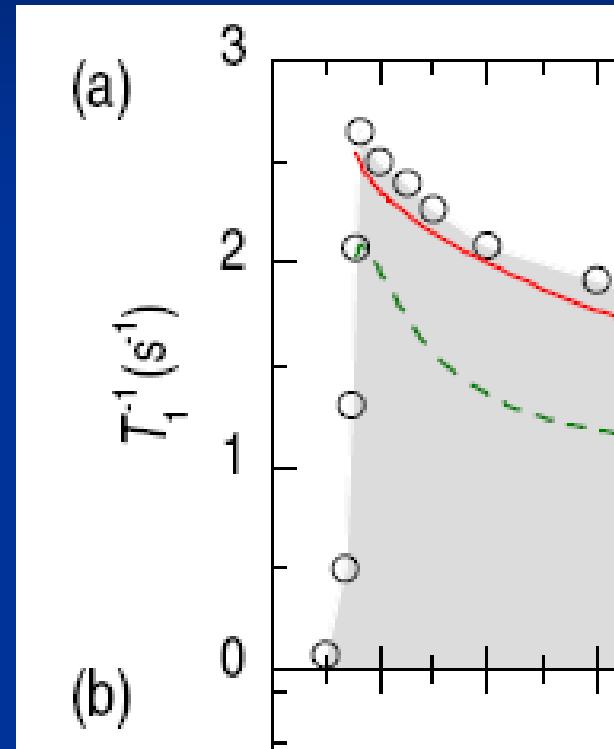
$$T_1^{-1} = \frac{\hbar\gamma^2 A_{\perp}^2 A_0^x}{k_B u} \cos\left(\frac{\pi}{4K}\right) B\left(\frac{1}{4K}, 1 - \frac{1}{2K}\right) \left(\frac{2\pi T}{u}\right)^{(1/2K)-1},$$

## ■ Tc to ordered phase: $1/J' = \chi_{1D}(T_c)$

$$T_c = \frac{u}{2\pi} \left[ \sin\left(\frac{\pi}{4K}\right) B^2\left(\frac{1}{8K}, 1 - \frac{1}{4K}\right) \frac{zJ'A_0^x}{2u} \right]^{2K/(4K-1)}.$$



# NMR



M. Klanjsek et al.,

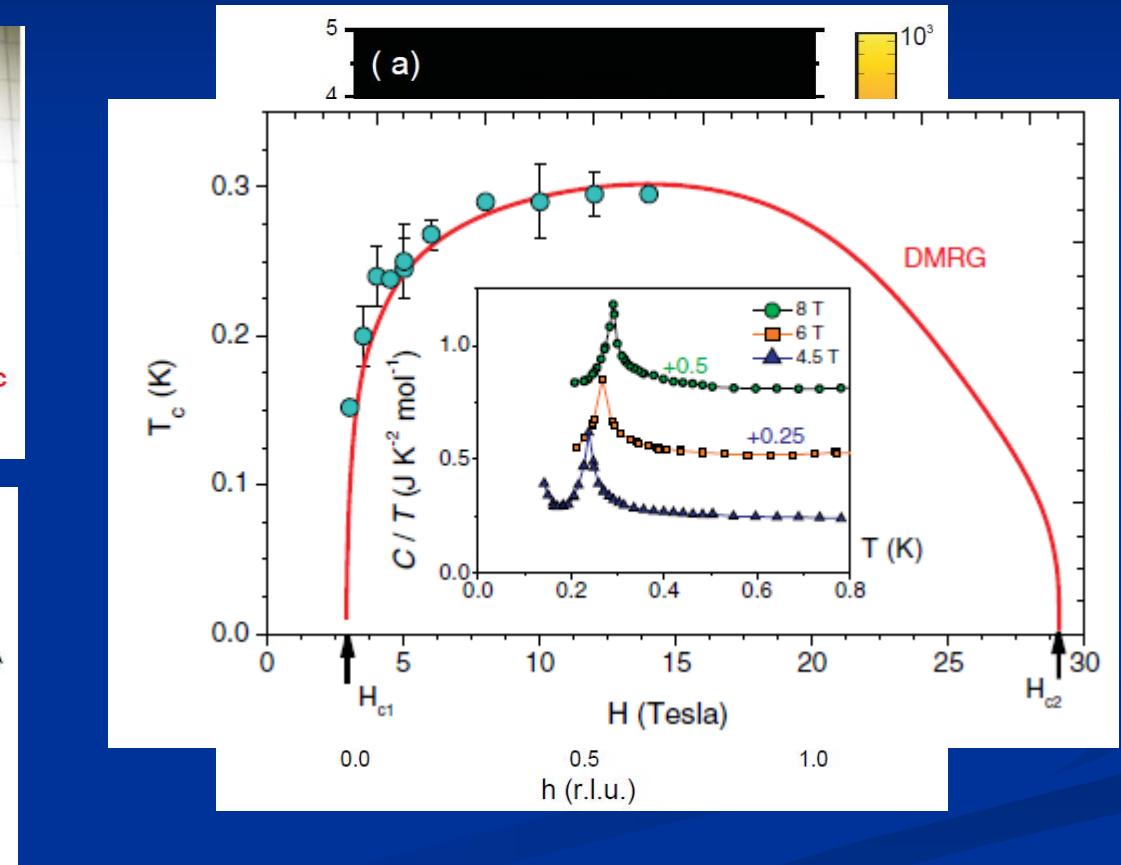
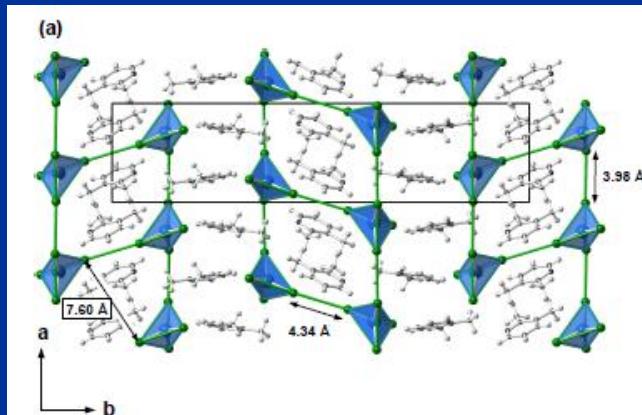
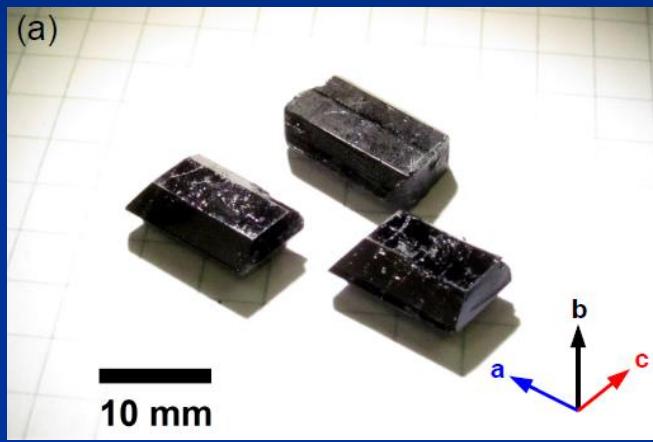
PRL 101 137207 (2008)



# Ab initio reconstruction



D. Schmidiger et al. PRL 108 167201 (2012)



$$\mathcal{H} = J_{\text{leg}} \sum_{l,j} S_{l,j} \cdot S_{l+1,j} + J_{\text{rung}} \sum_l S_{l,1} \cdot S_{l,2} - g\mu_B H \sum_{l,j} S_{l,j}^z.$$

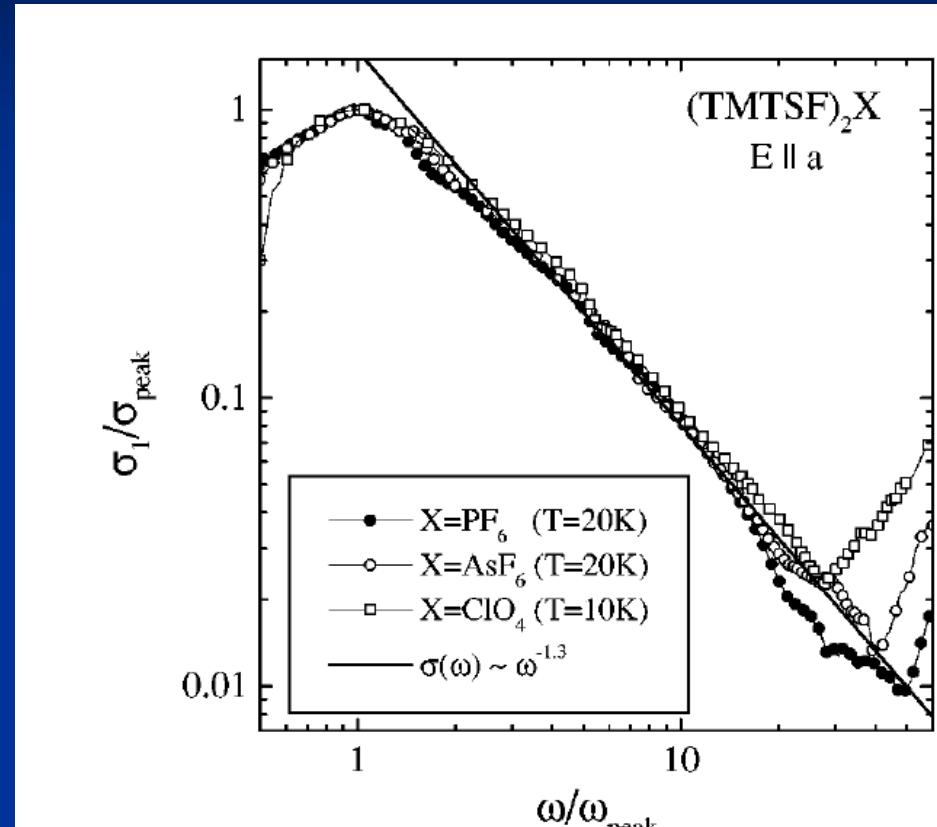
# Optical conductivity



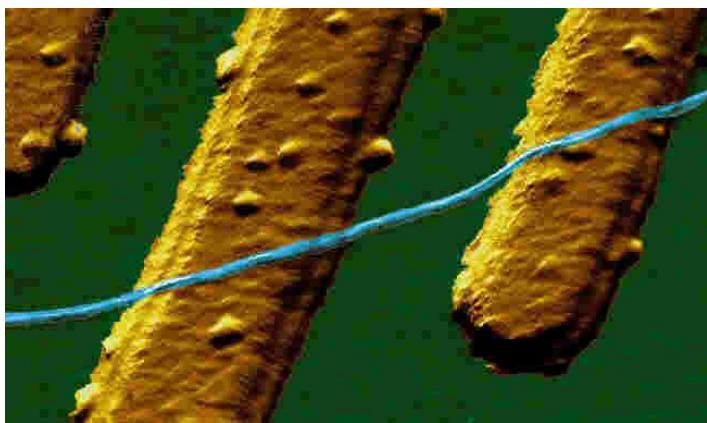
$$\sigma(\omega) \propto \omega^\nu$$

TG PRB (91) :  
Physica B 230 (1996)

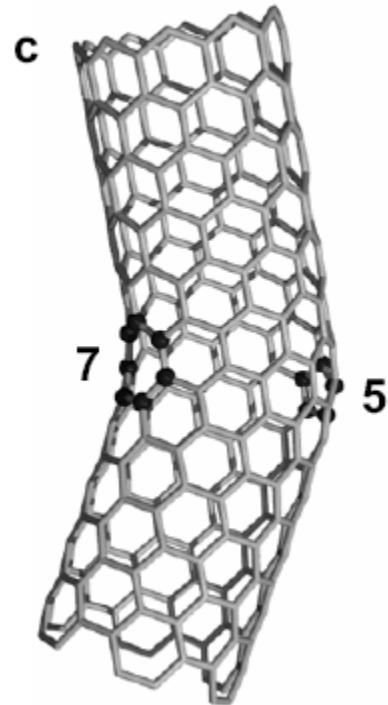
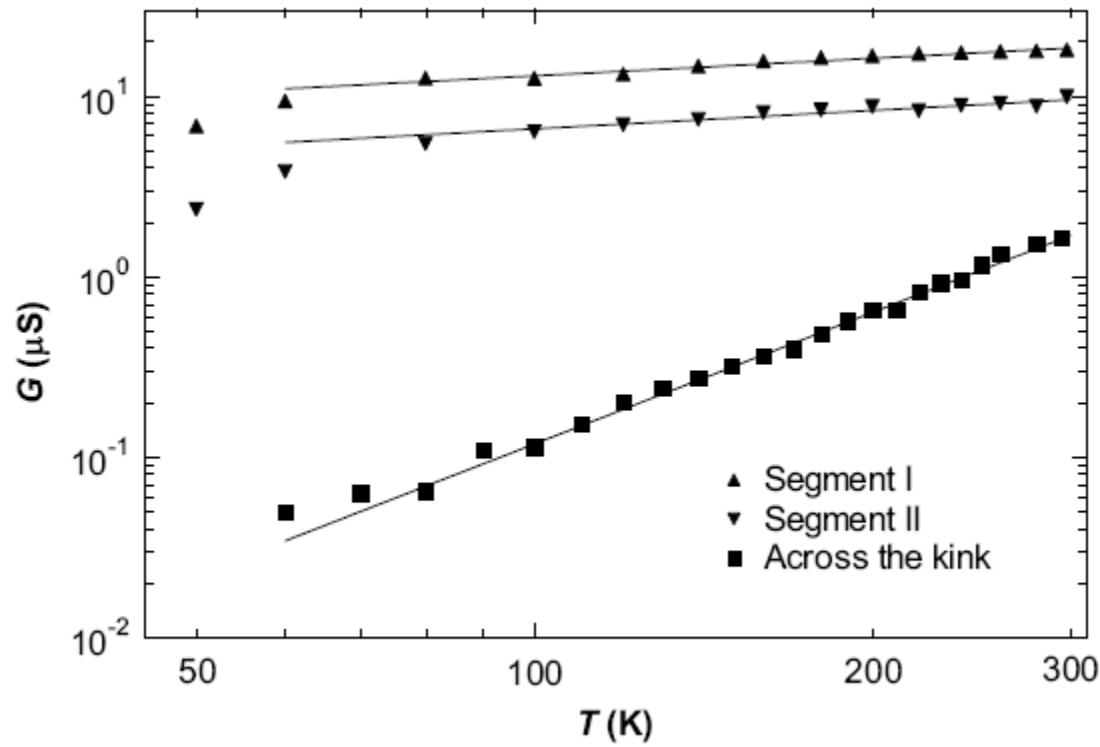
First observation of LL !!



A. Schwartz et al. PRB 58  
1261 (1998)

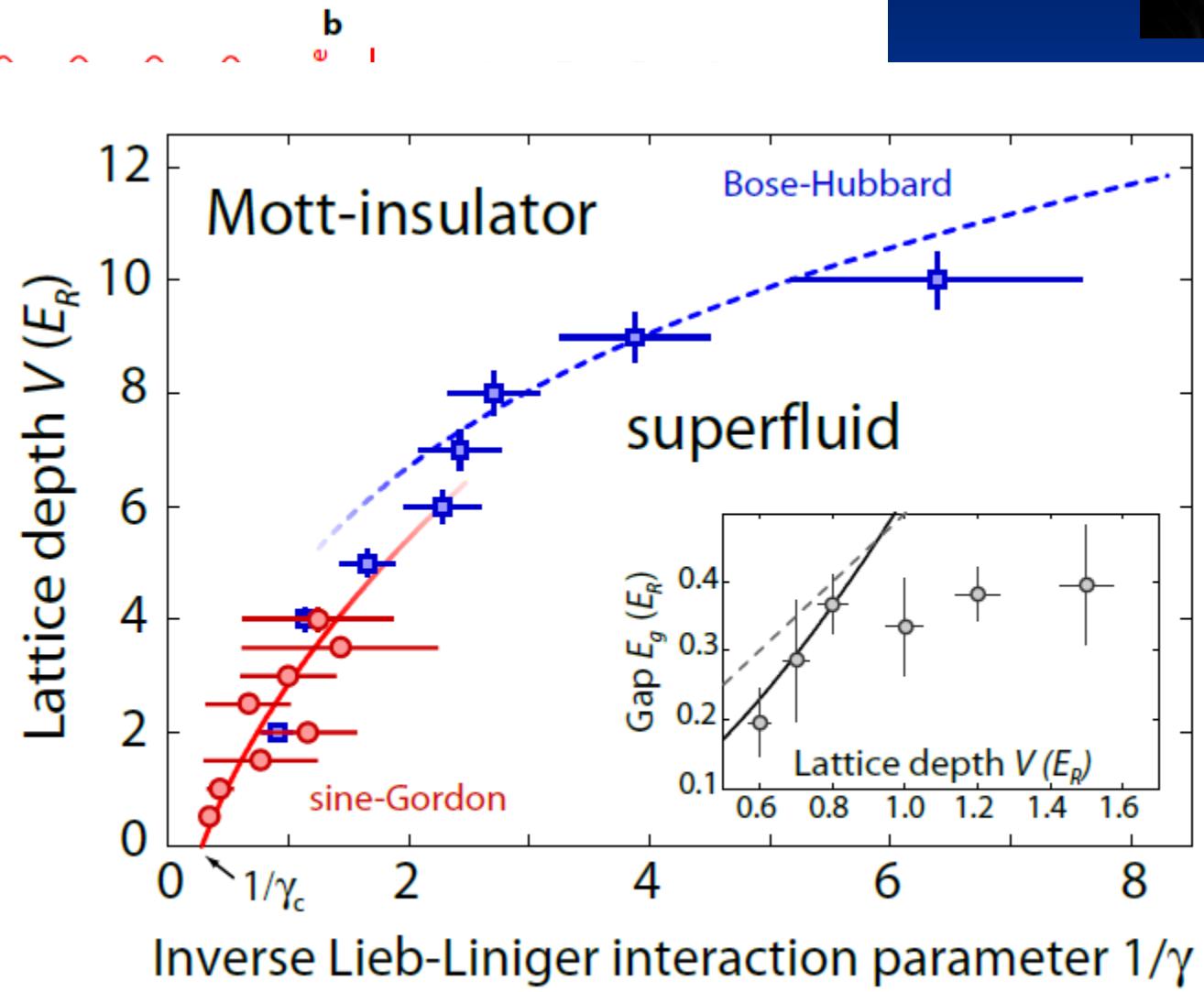
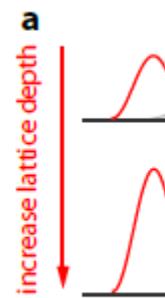


Z. Yao et al. Nature 402  
273 (1999)



# External potentials





# Disorder



# Bose glass phase

1D : TG + H. J. Schulz PRB 37 325 (1988)

Superfluid – Localized (Bose glass) transition  
for  $K < 3/2$

BKT like transition

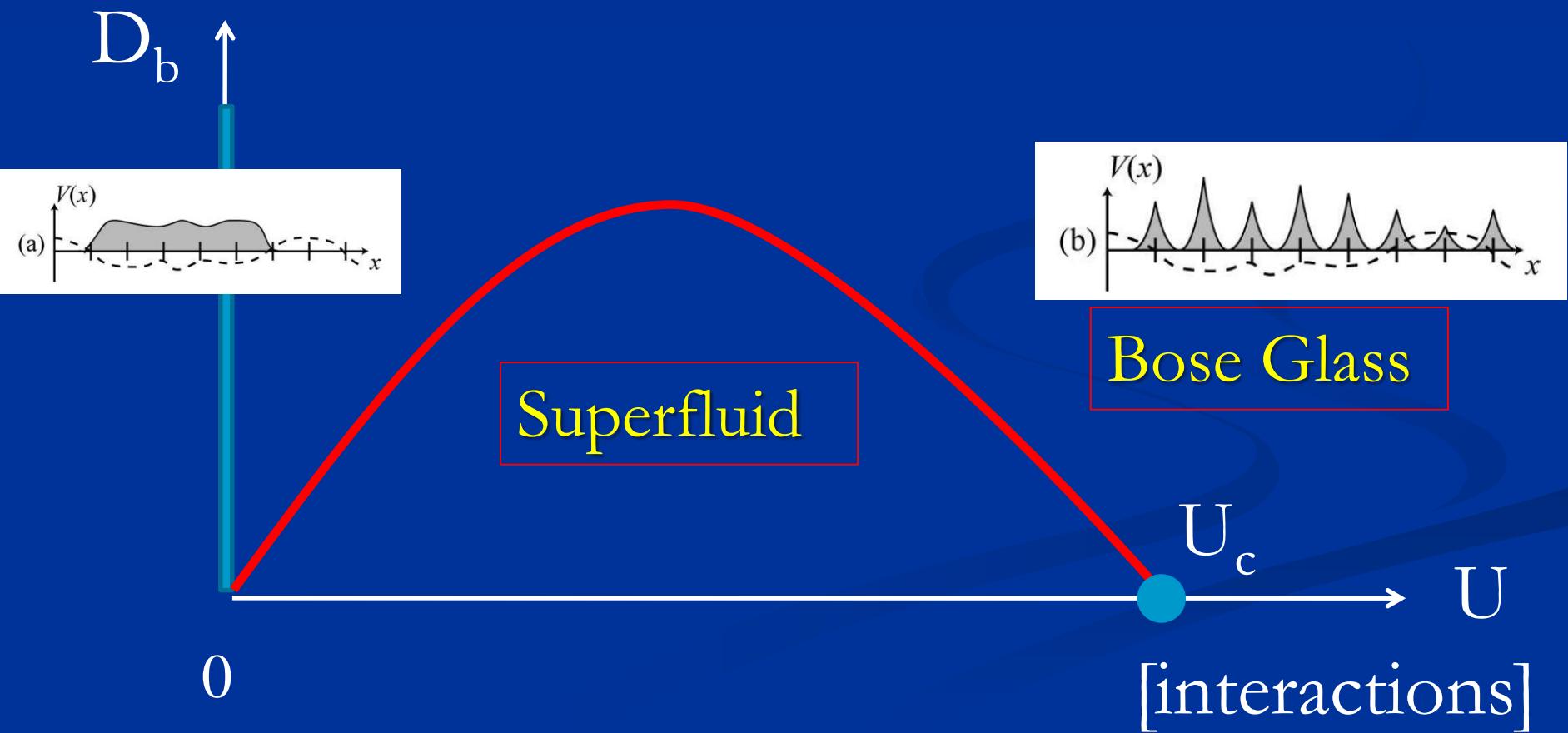
Higher dimensions: M.P.A. Fisher et al. PRB 40 546 (1989)

Bose glass also exists (scaling theory)

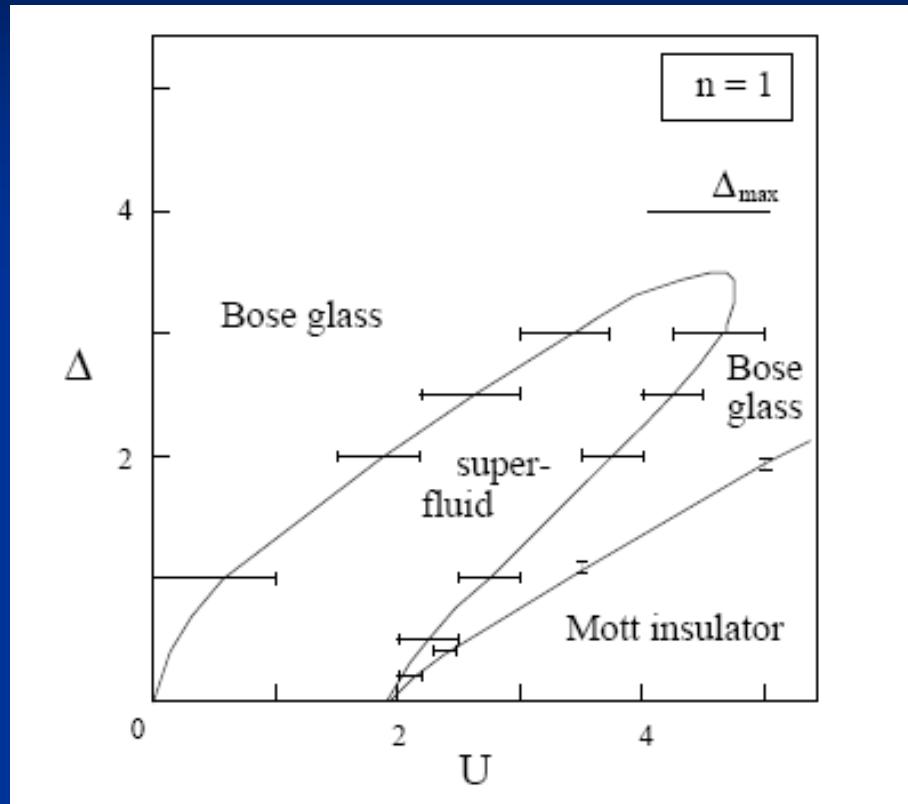
continuous transition

# Bose glass phase

TG + H. J. Schulz EPL 3 1287 (87); PRB 37 325 (1988);  
M.P.A. Fisher et al. PRB 40 546 (1989)



# Numerics



S. Rapsch, U. Schollwoeck,  
W. Zwerger EPL 46 559  
(1999);

G. Batrouni et al. PRL 65  
1765 (90);  
N. Prokofev et al. PRL 92  
015703 (04);  
O. Nohadani et al. PRL 95,  
227201 (05)  
K. G. Balabanyan et al. PRL  
95, 055701 (05);  
L. Pollet et al. PRL 103,  
140402 (2009)

.....

# Strong disorder, weak interactions

PHYSICAL REVIEW B 81, 174528 (2010)

## Superfluid-insulator transition of disordered bosons in one dimension

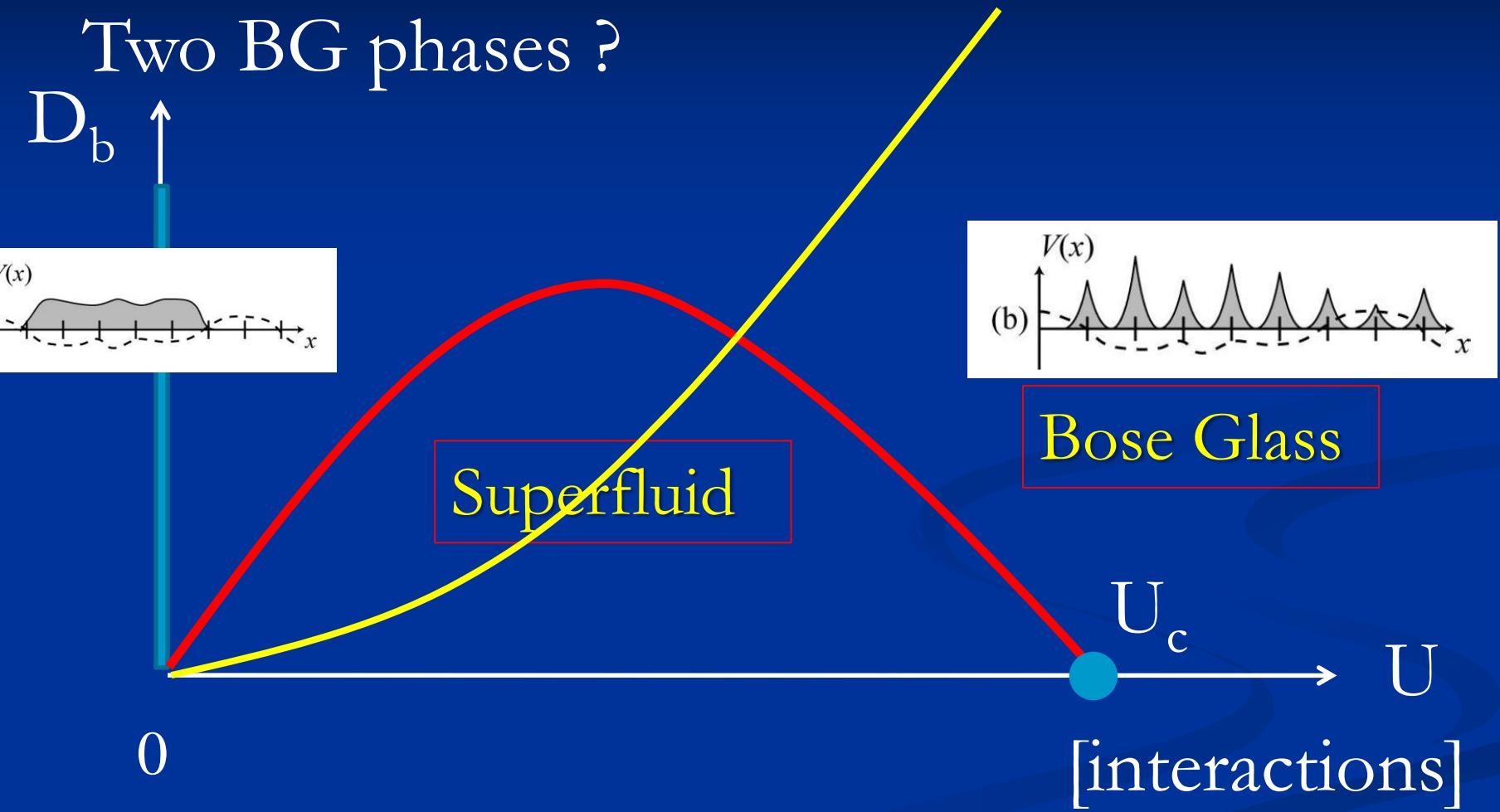
Ehud Altman,<sup>1</sup> Yariv Kafri,<sup>2</sup> Anatoli Polkovnikov,<sup>3</sup> and Gil Refael<sup>4</sup>

phase diagram. We show that the superfluid-insulator transition is always Kosterlitz-Thouless like in the way that length and time scales diverge at the critical point. Interestingly however, we find that the transition at strong disorder occurs at a nonuniversal value of the Luttinger parameter, which depends on the disorder strength. This result places the transition in a universality class different from the weak disorder transition first analyzed by Giamarchi and Schulz [Europhys. Lett. 3, 1287 (1987)]. While the details of the disorder potential

How to connect the two results ?

# Phase diagram

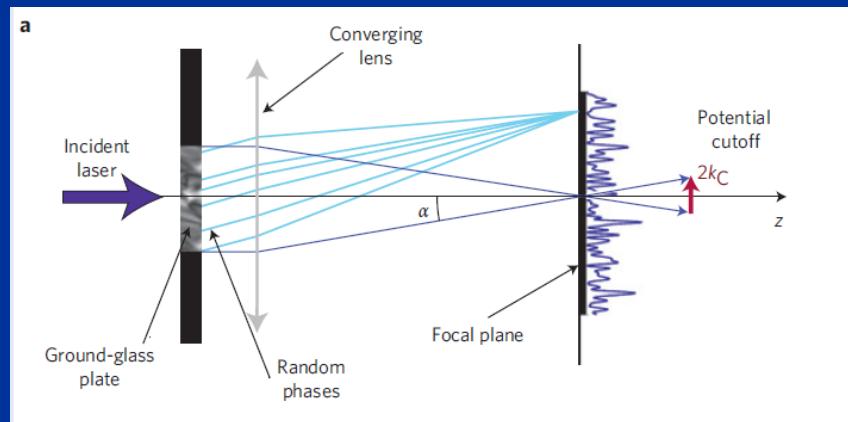
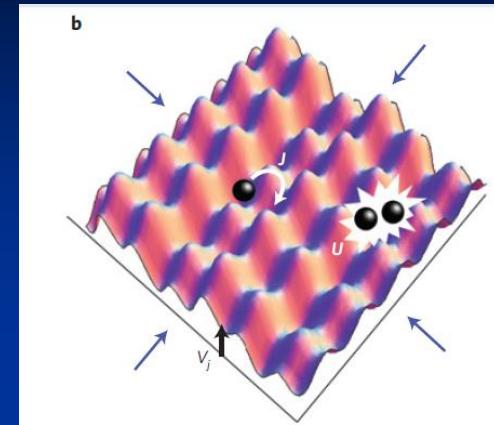
Two BG phases ?



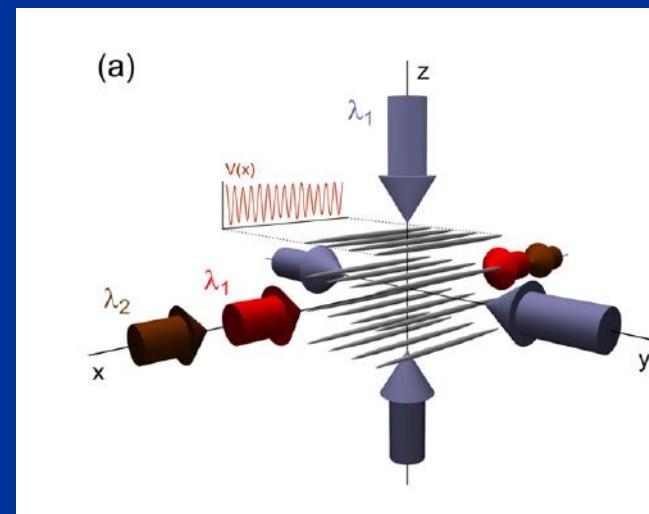
Order parameter ? Moments of distribution ?

# Experiments

# Cold atomic gases



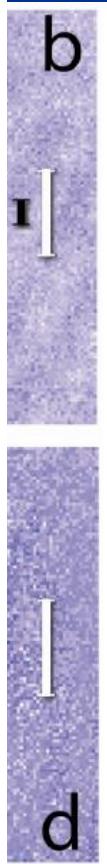
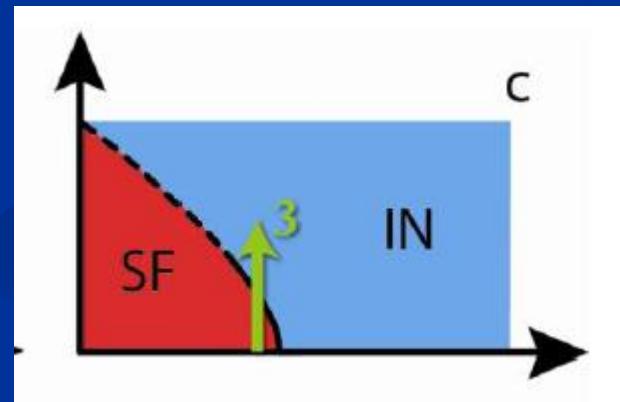
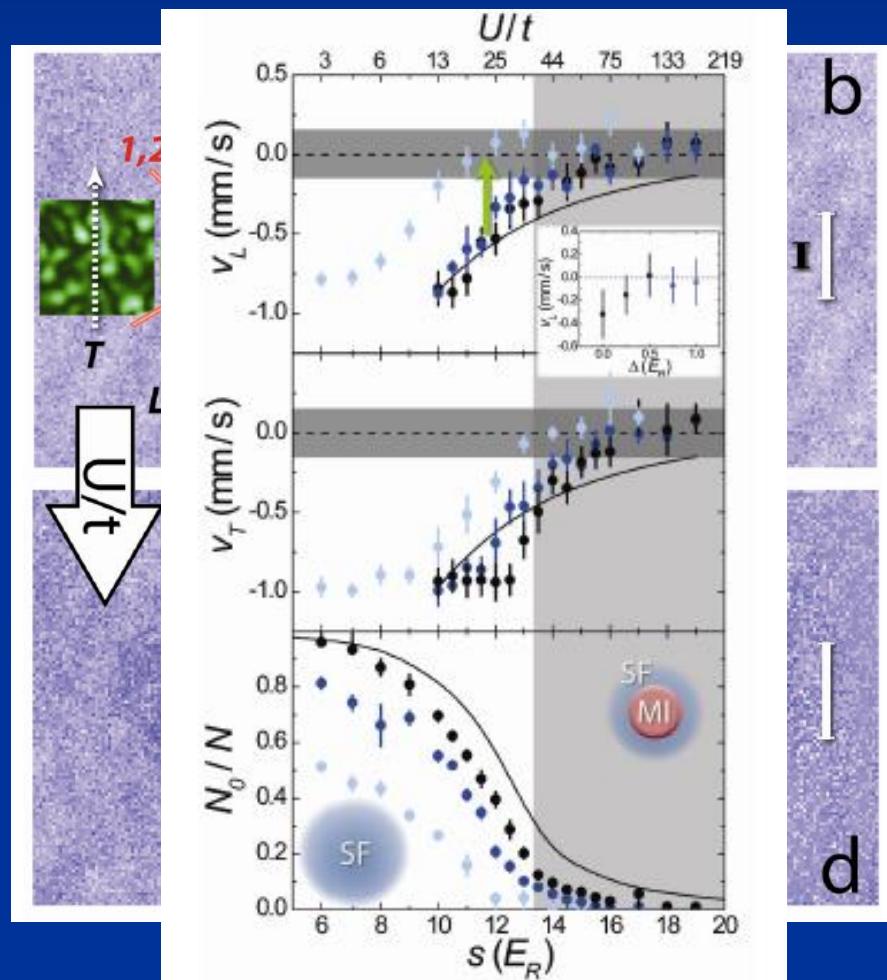
Speckle



Biperiodic lattices

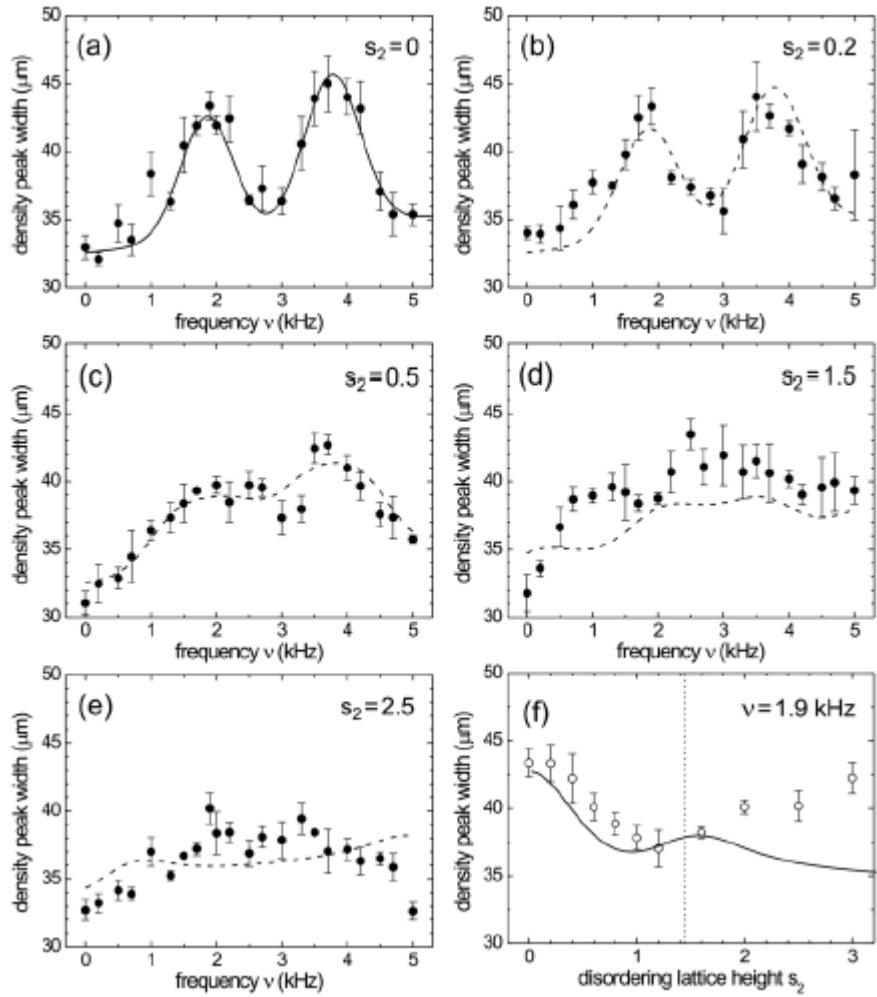
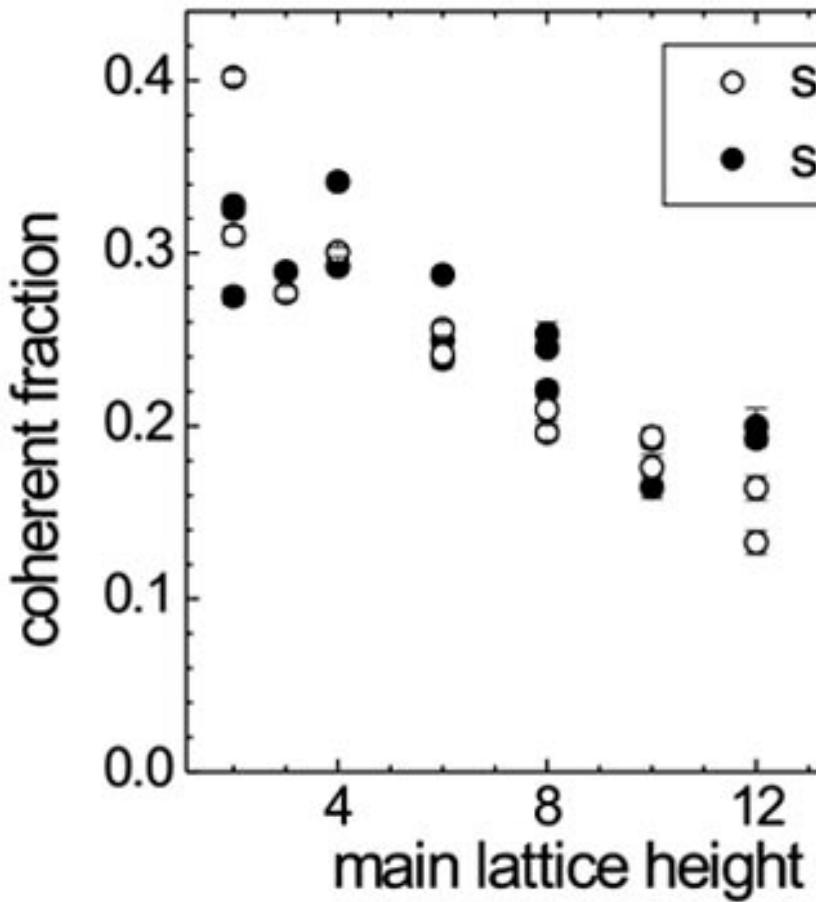
# Speckle

Pasienski et al. Nat. Phys 6 677 (2010)



# Quasi-periodic

Quasiperiodic (1D):



# Same as true disorder ?

# Renormalization treatment

J. Vidal, D. Mouhanna, TG PRL 83 3908 (1999); PRB 65 014201 (2001)

$$\frac{dK}{dl} = -K^2 \Xi(l),$$

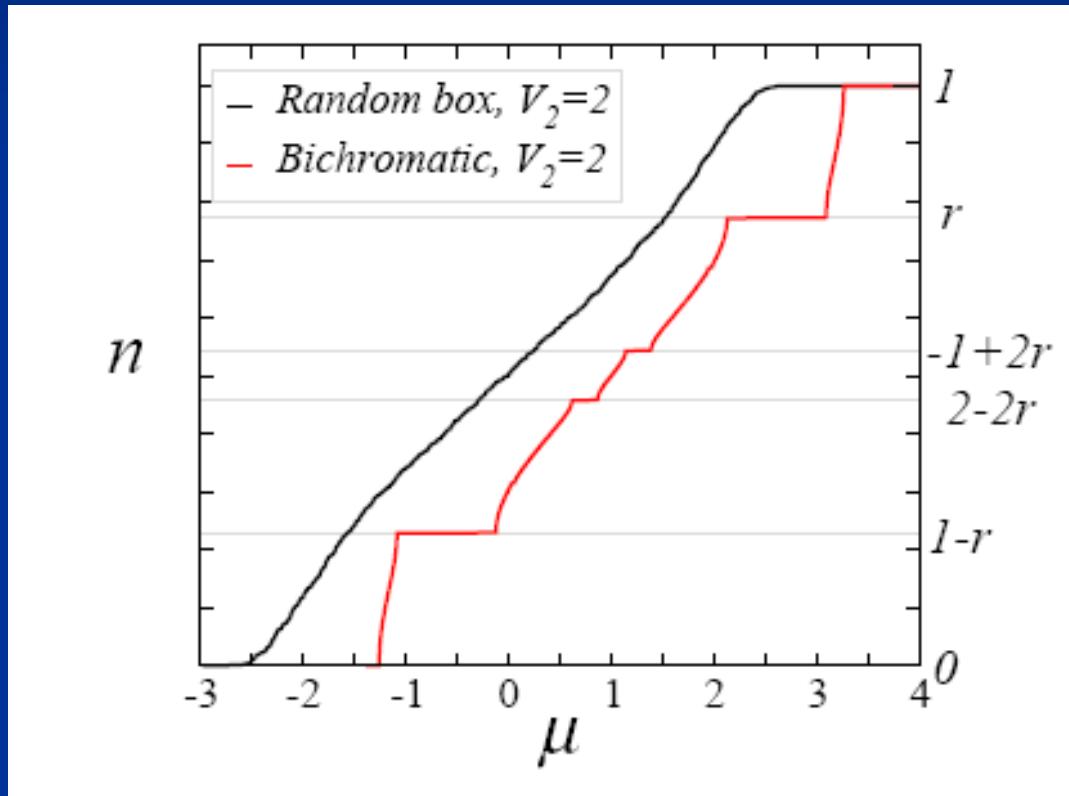
$$\frac{dy_Q}{dl} = (2 - K)y_Q,$$

$y_Q$  : Fourier components  
of potential

$$\Xi(l) = \frac{1}{2} \sum_Q y_Q^2 [J(Q^+ \alpha(l)) + J(Q^- \alpha(l))],$$

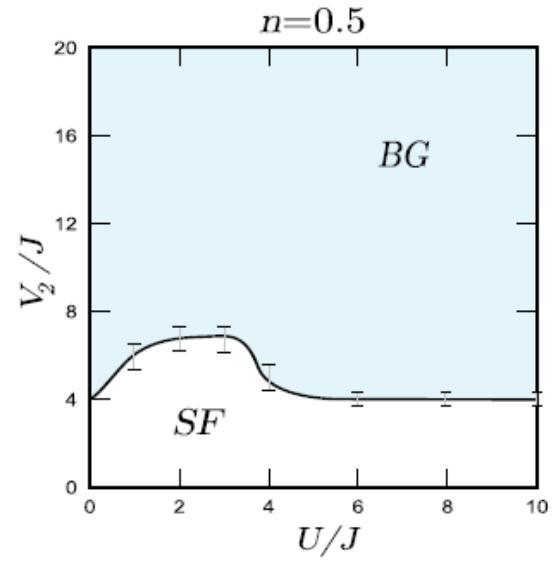
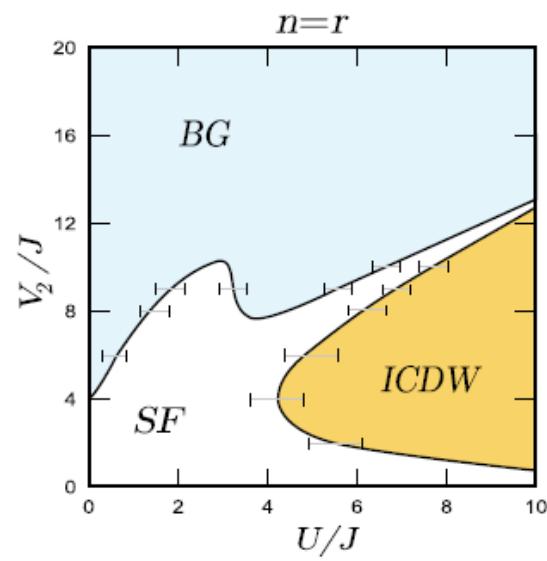
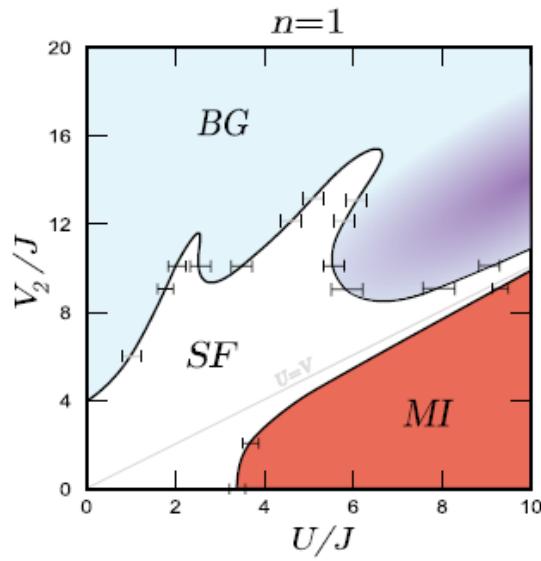
# Numerics: DMRG

G. Roux et al. PRA 78 023628 (2008)

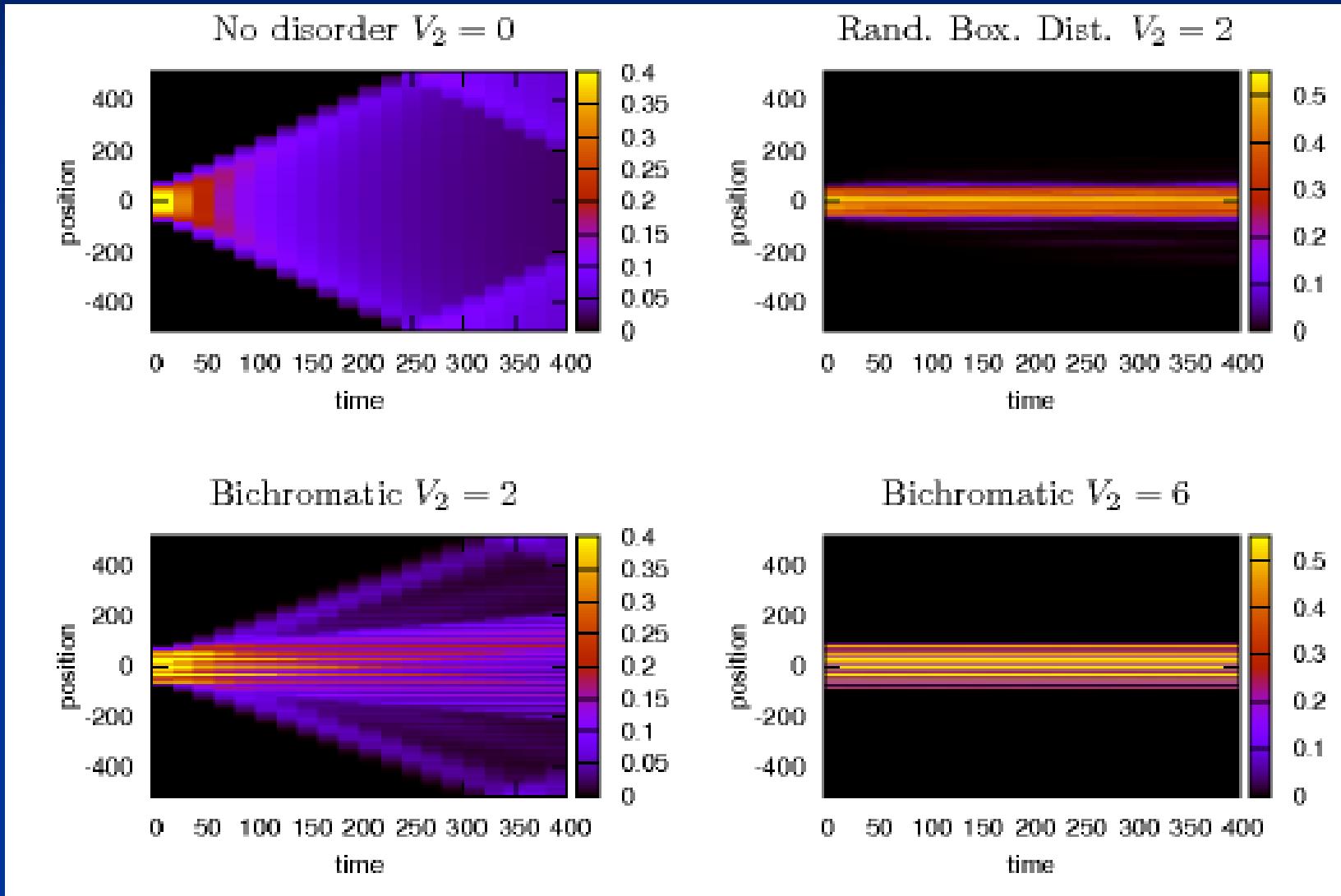


Related works: T. Roscilde, Phys. Rev. A 77, 063605 2008;  
X. Deng et al PRA 78, 013625 (2008)

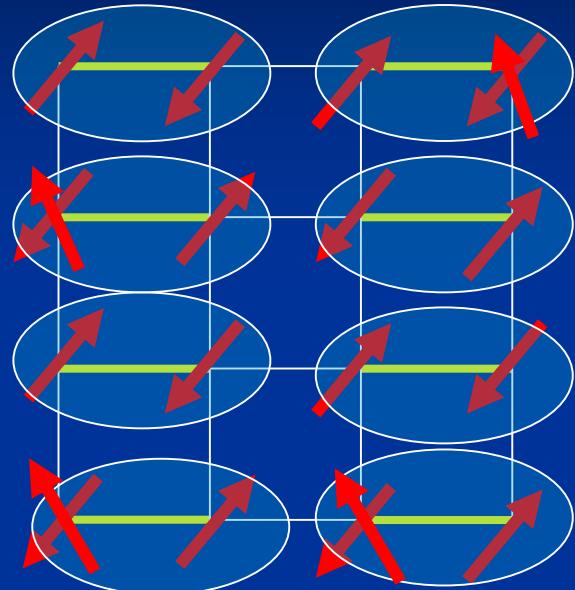
# Phase diagram



# Expansion



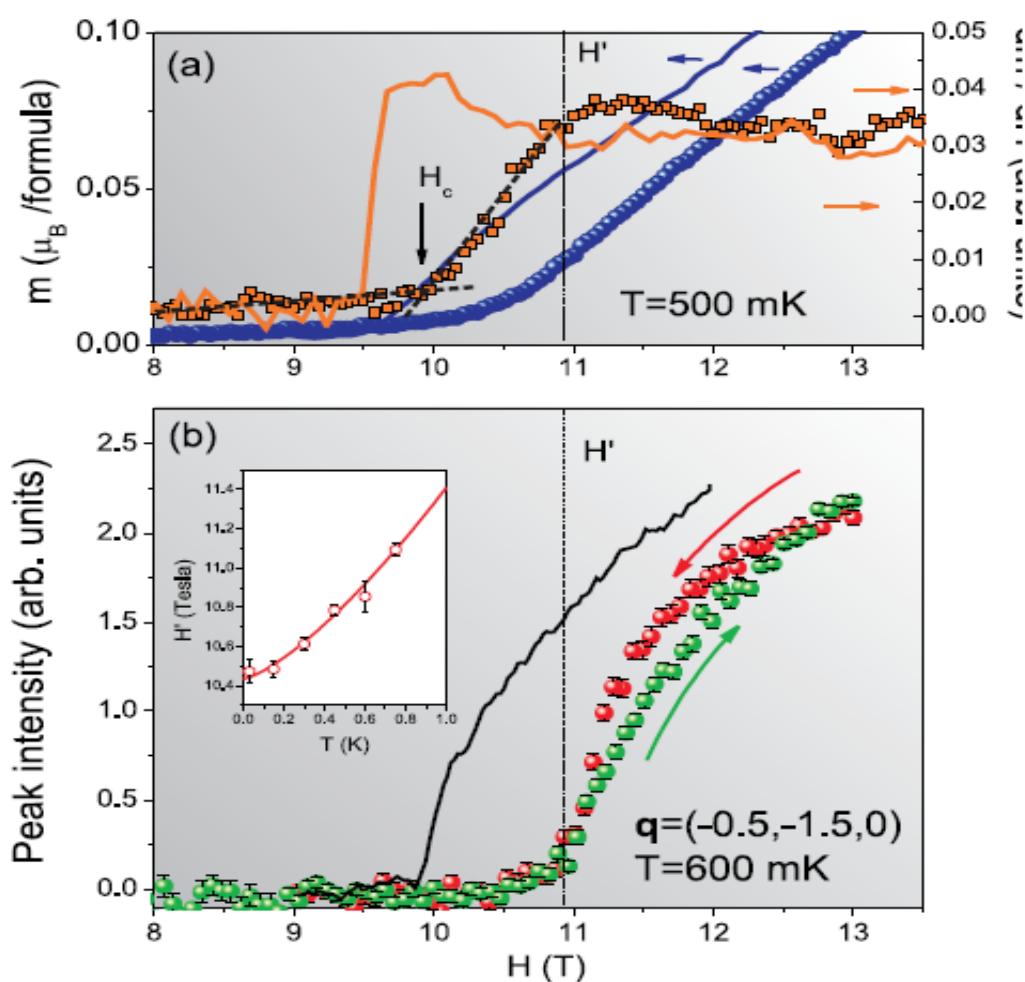
# Disordered spin dimers



$$|S\rangle, |T\rangle \rightarrow |0\rangle, b^\dagger |0\rangle$$

$d m / d h =$  compressibility

$\langle S_x \rangle = \langle \tilde{A} \rangle$  superfluid  
order parameter

Evidence of a magnetic Bose glass in  $(\text{CH}_3)_2\text{CHNH}_3\text{Cu}(\text{Cl}_{0.95}\text{Br}_{0.05})_3$  from neutron diffractionTao Hong,<sup>1</sup> A. Zheludev,<sup>2,3,\*</sup> H. Manaka,<sup>4</sup> and L.-P. Regnault<sup>5</sup>

$$\frac{dm}{dh} \neq 0$$

$$\langle \psi \rangle = 0$$

Bose Glass  
phase

# DTN

## Bose glass and Mott glass of quasiparticles in a doped quantum magnet

arXiv:1109.4403v2

Rong Yu,<sup>1</sup> Liang Yin,<sup>2</sup> Neil S. Sullivan,<sup>2</sup> J. S. Xia,<sup>2</sup> Chao Huan,<sup>2</sup> Armando Paduan-Filho,<sup>3</sup> Nei F. Oliveira Jr.,<sup>3</sup> Stephan Haas,<sup>4</sup> Alexander Steppke,<sup>5</sup> Corneliu F. Miclea,<sup>6</sup> Franziska Weickert,<sup>6</sup> Roman Movshovich,<sup>6</sup> Eun-Deok Mun,<sup>6</sup> Vivien S. Zapf,<sup>6</sup> and Tommaso Roscilde<sup>7</sup>

