Lectures 1 and 2

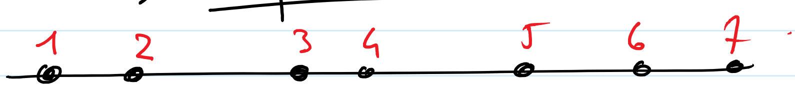
lundi 8 juillet 2013 16:43

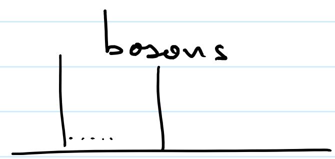
Pure 1D Systems.

I] 1D. Systems.

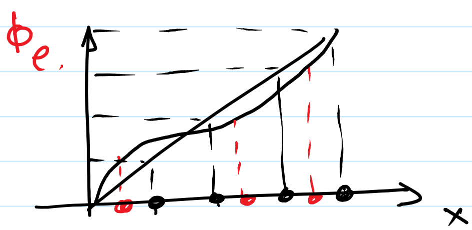
II] Bosonization technique.

1) Operators





$$\rho(x) = \sum_i \delta(x - x_i),$$



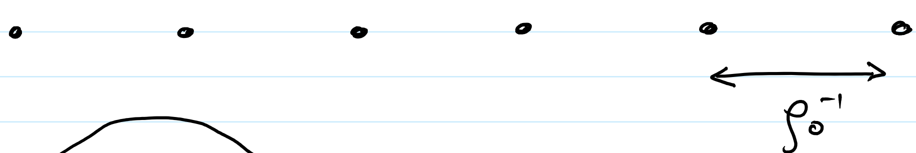
$$\phi_e(x) = 2\pi n.$$

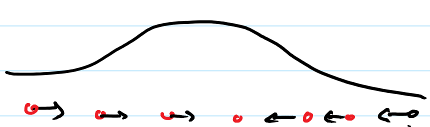
$$\rho(x) = \sum_i \delta(x - x_i) = \sum_n |\nabla\phi_e| \, \delta(\phi_e(x) - 2\pi n),$$

$$= \frac{\nabla\phi}{2\pi} \sum_p e^{i p \cdot \phi_e(x)}.$$

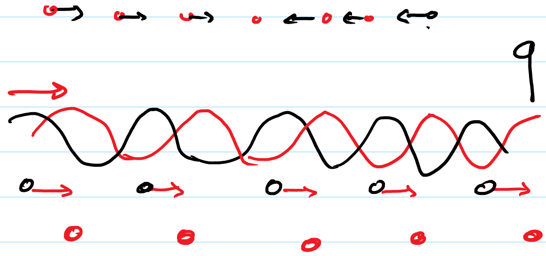
$$\phi_e(x) = 2\pi\rho_0 x - 2\phi(x)$$

$$\rho(x) = \left(\rho_0 - \frac{1}{\pi}\nabla\phi\right) \sum_p e^{i2p\left(\pi\rho_0 x - \phi(x)\right)}.$$





$$q \simeq 0 \qquad \rho \simeq \rho_0 - \frac{1}{\pi}\nabla\phi$$



$$q \simeq 0 \qquad \rho \equiv \rho_0 - \frac{1}{\pi}\nabla\phi$$

$$\rho \equiv \rho_0 \cos\left(2\pi\rho_0 x - 2\phi(x)\right).$$

$$\psi_B^{\dagger}(x) = \left[\rho(x)\right]^{1/2} e^{-i\theta(x)}$$

$$\left[\psi_B(x), \psi_B^{\dagger}(x')\right] = \delta(x-x').$$

$$\hookrightarrow \left[\phi, \frac{1}{\pi}\nabla\theta\right] = i\delta(x-x') \Rightarrow \text{Conjugate operators}$$

$$\psi_B^{\dagger}(x) = \left[\rho_0 - \frac{1}{\pi}\nabla\phi\right]^{1/2} \cdot \sum_p e^{i2p\left(\pi\rho_0 x - \phi(x)\right)} e^{-i\theta(x)}.$$

$$\psi_B^{\dagger}(x), \rho(x) \longleftrightarrow \phi(x), \theta(x).$$

periodic BC. : $\phi(x+L) = \phi(x) + \pi N.$

$\theta(x+L) = \theta(x) + \pi J.$

2) <u>Hamiltonian.</u>

$$H = \frac{1}{2m}\int dx\, \nabla\psi_B^{\dagger}\nabla\psi_B + \frac{1}{2}\int dx\,dx'\, V(x-x')\rho(x)\rho(x').$$

$$V(x-x') = V_0\,\delta(x-x')$$

Lieb Lininger model.

$$\psi_B^{\dagger} \approx \rho_0^{1/2} e^{-i\theta}.$$

$$H_{Kin} = \frac{\rho_0}{2m} \int dx \, (\nabla\theta)^2$$

$$H_{int} = \frac{1}{2} V_0 \int dx \, [\rho(x) - \rho_0]^2 = \frac{V_0}{2\pi^2} \int dx \, (\nabla\phi)^2$$

$$H = \frac{\rho_0}{2m} \int dx \, [\nabla\theta]^2 + \frac{V_0}{2\pi^2} \int dx \, (\nabla\phi)^2$$

$$\Pi_\phi = \frac{1}{\pi} \nabla\theta \qquad = \frac{\rho_0}{2m} \int dx \, (\pi \Pi_\phi)^2 + \frac{V_0}{2\pi^2} \int dx (\nabla\phi)^2$$

$$H = \frac{1}{2\pi} \int dx \left[(uK)(\pi \Pi_\phi)^2 + \frac{u}{K} (\nabla\phi)^2 \right]$$

$$S = \frac{1}{2\pi K} \int dx \, d\tau \left[\frac{1}{u} (\partial_\tau \phi)^2 + u (\nabla\phi)^2 \right]$$

$$\frac{1}{u} \omega_n^2 + u q^2 \qquad \boxed{\omega \simeq u q}$$

$$i\omega_n \to \omega$$

$$\langle \rho(x) \rho(0) \rangle \qquad \langle \nabla\phi_{x\tau} \nabla\phi_{00} \rangle$$

$$\langle \quad \rangle = \frac{1}{Z} \int \mathcal{D}\phi \; e^{-S(\phi)} \quad \ldots\ldots$$

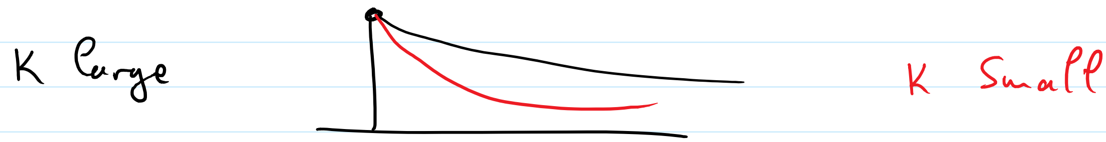
$$Z = \int \mathcal{D}\phi \; e^{-S(\phi)} \qquad \langle e^{i(2\phi - \theta)_x} e^{-i(2\phi-\theta)_0} \rangle$$

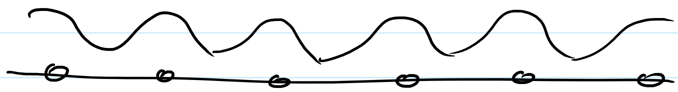
$$\langle \psi_B(x) \psi_B^\dagger(0) \rangle = A \left(\frac{\alpha}{x}\right)^{\frac{1}{2K}} + A_3 \cos(2\pi\rho_0 x) \left(\frac{\alpha}{x}\right)^{f(K) = \frac{1}{2K} + 2K}$$

$$\langle \rho(x) \rho(0) \rangle = \rho_0^2 - \frac{K}{2\pi^2} \frac{1}{x^2} + A_2 \cos(2\pi\rho_0 x)$$

$$\langle \rho(x)\,\rho(0)\rangle = \rho_0^2 - \frac{K}{2\pi^2}\,\frac{1}{x^2} + A_2 \cos(2\pi\rho_0 x)\left(\frac{\alpha}{x}\right)^{2K}$$

$$\lim_{x\to\infty} \langle \psi(x)\,\psi^\dagger(0)\rangle = |\psi|^2 = 0$$

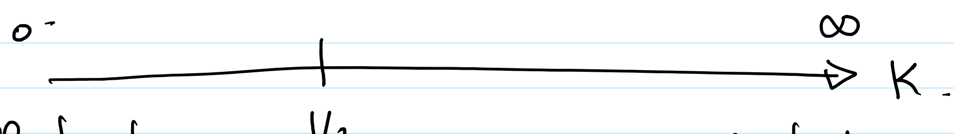




Tomonaga Luttinger Liquid.

u : velocity of sound.

K : TLL parameter



CDW fluctuations. SU fluctuations

$$uK \propto \frac{\rho_0}{2m} \qquad \frac{u}{K} \propto V_0$$

$$u \sim \sqrt{V_0} \qquad K \sim \frac{1}{\sqrt{V_0}}$$

<u>Periodic BC</u> : $\phi(x+L) = \phi(x) + \pi N$

$\theta(x+L) = \theta(x) + \pi J$ $\quad J$ even number

3) <u>TLL fixed point</u>:

$\to$ Structure of H <u>and</u> correlation functions is correct.

→ u, K have to be computed precisely.

$$C_v \sim \frac{T}{u}.$$

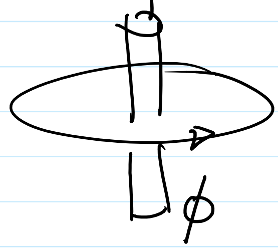
$$H_B = -t \sum_{\langle ij \rangle} \left(b_i^\dagger b_j + b_j^\dagger b_i \right) + \frac{U}{2} \sum_i n_i (n_i - 1).$$

Bose Hubbard model.

↳ Extract u, K from $\begin{cases} \text{BA (thermodynamics).} \\ \text{Numerics.} \end{cases}$

BA:

K compressibility $\quad \frac{\partial N}{\partial \mu} \to \frac{u}{K}$.



$$\frac{\partial^2 E_0}{\partial \Phi^2} = \mathcal{D} = uK. \qquad \sigma(\omega) = \mathcal{D}\,\delta(\omega) + \sigma_{reg}(\omega)$$

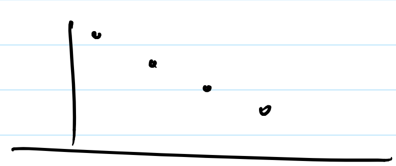
Numerics

ED → $K, \mathcal{D}$

DMRG → $K \Rightarrow \frac{u}{K}$ $\qquad \langle \psi_x \psi_0^\dagger \rangle$ → extract K

$$b^\dagger_{x=r_i} \to \sqrt{A}\, e^{-i\Theta}$$

↳ extract from numerics.

$\langle b_j b_0^\dagger \rangle \sim$ 

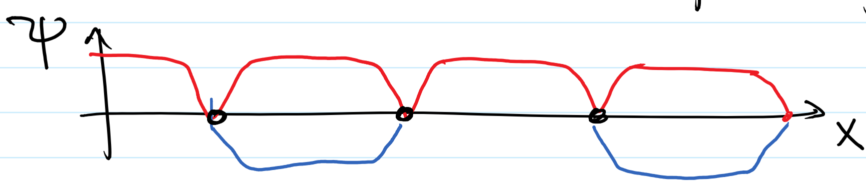
$$\langle e^{i\Theta(r_j)} e^{-i\Theta(0)} \rangle \sim \left(\frac{\alpha}{r_j} \right)^{\frac{1}{2K}}.$$

Bosons:

TLL. bosons. ?

$V_0 \to \infty$. Tonks–Girardeau limit

$\hookrightarrow$ Bosons $\longleftrightarrow$ Spinless fermions.

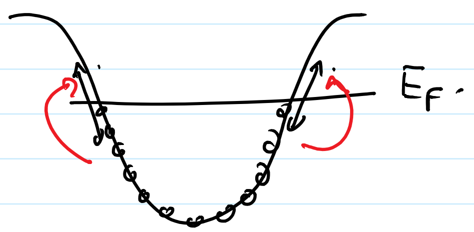


ψ_B symmetric.
ψ_F antisymmetric.

$V_0 = \infty$ bosons $\longleftrightarrow$ non-interacting spinless Fermions.

$$\langle \psi_B(x)\, \psi_B^\dagger(0) \rangle \;\cancel{\longleftrightarrow}\; \langle \psi_F(x)\, \psi_F^\dagger(0) \rangle$$

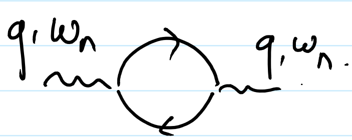
$$\langle \rho_B(x)\, \rho_B(0) \rangle = \langle \rho_F(x)\, \rho_F(0) \rangle$$



$$\varepsilon(k) \to \frac{k^2}{2m}. \quad -2t\cos(k)$$

$$\langle \rho(x)\, \rho(0) \rangle \sim \frac{1}{x^2}. + \cos(2k_F x) \frac{1}{x^2}.$$

$$2\pi\rho_0.$$



$$\langle \rho_B(x)\, \rho_B(0) \rangle \sim \frac{1}{x^2} + \cos(2\pi\rho_0 x) \frac{1}{x^{2K}}.$$

$K \to 1.$

$$\langle \psi_B(x)\, \psi_B^\dagger(0) \rangle \sim \frac{1}{x^{1/2K}} \sim \frac{1}{\sqrt{x}}.$$

$$\langle \rho(x)\,\rho(0)\rangle \sim \left(\frac{1}{x-ut}\right)^{2} + \cos(2\pi\rho_0 x)\,\frac{1}{(x^2+\tau^2)^K}$$

▷ Extension 1:

$$\psi^\dagger = \sqrt{\rho}\; e^{-i\Theta} \qquad \rho \simeq \rho_0 - \frac{1}{\pi}\nabla\phi \;\ldots$$

$$\nabla\psi^\dagger = -i\nabla\Theta\,\sqrt{\rho}\; e^{-i\Theta} + \frac{-\frac{1}{\pi}\nabla^2\phi}{2\sqrt{\rho}}\; e^{-i\Theta}$$

$$\frac{1}{2m}\nabla\psi^\dagger\nabla\psi = \frac{\rho_0}{2m}(\nabla\Theta)^2 + \frac{1}{2\rho_0}(\nabla^2\phi)^2$$

$$H = \frac{\rho_0}{2m}(\partial_\tau\phi)^2 + \frac{1}{2\rho_0}(\nabla^2\phi)^2 + \circ\, V_0(\nabla\phi)^2$$

$$\omega^2 = \circ\, k^2 + \circ\, k^4$$

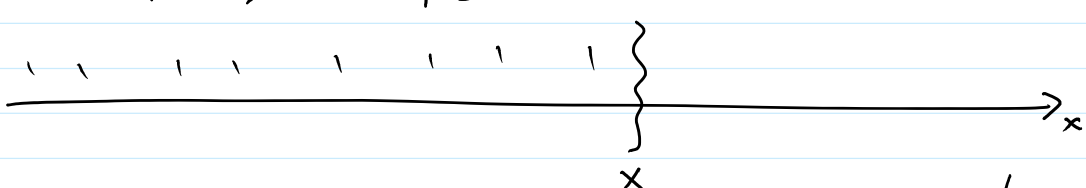
▷ Extension 2

$$\rho_F = \left(\rho_0 - \frac{1}{\pi}\nabla\phi\right)\sum_p e^{i2p(\pi\rho_0 x - \phi(x))}$$

$$\{\psi_F(x), \psi_F^\dagger(x')\} = \delta(x-x')$$

$$\psi_B(x) = \sqrt{\rho(x)}\; e^{-i\Theta(x)} \qquad [\psi_B(x), \psi_B^\dagger(x')] = \delta(x-x')$$

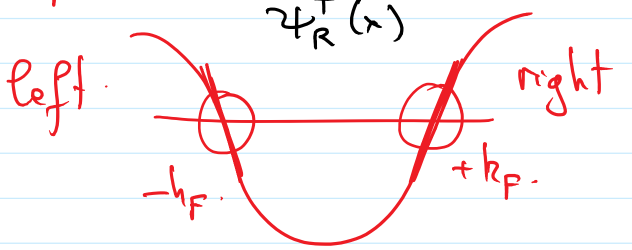
$$\psi_F^\dagger(x) = \psi_B^\dagger(x)\; e^{i\frac{1}{2}\phi_\rho(x)}$$



$$\psi_F^\dagger(x) = \left[\rho_0 - \frac{1}{\pi}\nabla\phi\right]^{1/2}\sum_p e^{i(2p+1)(\pi\rho_0 x - \phi(x))}\; e^{-i\Theta(x)}$$

$$\psi_F(x) = [\rho_0 - \tfrac{1}{\pi}\nabla\phi] \quad \rho \quad e \qquad e$$

$$e^{i\pi\underbrace{\rho_0 x}_{k_F}} \underbrace{e^{-i\phi(x)} e^{-i\theta(x)}}_{\psi_R^+(x)} \quad \text{or} \quad e^{-i\pi\underbrace{\rho_0 x}_{-k_F}} \underbrace{e^{i[\phi(x) - \theta(x)]}}_{\psi_L^+(x)}$$

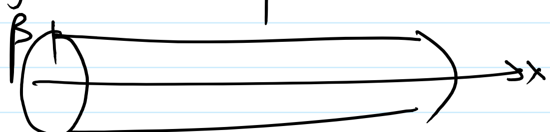


$$e^{i\pi \sum_{j<i} c_i^\dagger c_i} \qquad \int_{-\infty}^{x} \qquad \rho(x) \simeq \rho_0 - \frac{1}{\pi}\nabla\phi$$

$$\pi x \rho_0 - \phi(x)$$

▷ Extension 3

finite temperature



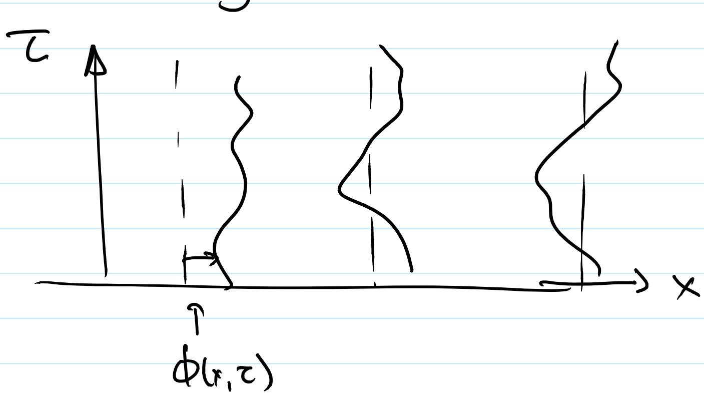
$$G(r) \sim \left(\frac{1}{r}\right)^K \to e^{-|x| K/\beta u}$$

$$r = \sqrt{x^2 + \tau^2}$$

$$\xi \simeq \frac{\beta u}{\pi K}$$

▷ Extension 4

$$S = \int (\partial_\tau \phi)^2 + (\partial_x \phi)^2$$



TG + P. Le Doussal

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