FIG. 1. Typical magneto-optical image (size 90 × 72 μm², λ = 638.1 nm). The gray part corresponds to the surface swept by the domain wall during 111 μs at 460 Oe (T = 23 °C). The dark part is the original domain.
D = 1+1 interface (d=1, N=1)

short-range disorder

\[ \langle [u(x + L) - u(x)]^2 \rangle \propto u_c^2 \left( \frac{L}{L_c} \right)^{2\zeta} \]

thermally equilibrated

FIG. 4. Typical correlation function drawn in a ln-ln plot. The unit of \( L \) is the pixel of the CCD camera, i.e., 0.28 \( \mu \text{m} \).

FIG. 5. Wandering exponent \( 2\zeta \). Measurements on different MDW driven at \( H = 50 \text{ Oe} \) during 20–45 min and then frozen \( (T = 300 \text{ K}, \text{estimated error on} 2\zeta \text{ for a given image:} \pm 0.03) \).
FIG. 2. (a),(b): MDW velocity versus applied magnetic field at room temperature ($v$ in m/s). The dashed line in (a) is the linear fit of the high field part ($H > 0.86$ kOe) and the arrow marks its intersection with the line $v(H) = 0$. This is the definition of $H_{\text{crit}}$. 
FIG. 3. Natural logarithm of MDW velocity as a function of \((1/H)^{1/4}\) (room temperature, \(H \leq 955\) Oe).
Contact line of a fluid

E. Rolley et al. (ENS)

**Fig. 2.** Sketch of the experimental setup. Inset: photograph of the disordered substrate, the chromium defects appear as white square spots.
Fig. 1. Upper part: image of the contact line obtained with an ordinary CCD camera. Lower part: the position $\eta(x, t) \equiv y(x, t) - vt$ of the CL is defined with respect to its average position $vt$. 
Abrikosov vortex lattice (type II superconductors)

Mean-field phase diagram

vortex lattice = many parallel vortex lines aligned external field

which order into a triangular lattice
Elasticity of a lattice of vortex lines

Q: what are \( d, N \) ?

\[ E_{el} = \frac{1}{2} \int \left[ \left( C_{ll} q_{ll}^2 + C_{44} q_{ll}^2 \right) P^l_{\alpha\beta}(q_{ll}) + \left( C_{66} q_{ll}^2 + C_{44} q_{ll}^2 \right) P^T_{\alpha\beta}(q_{ll}) \right] u^\alpha u^\beta \]

\[ q = (q_x, q_z) \]
Elasticity of a lattice of vortex lines

\[ \text{line lattice} \]

\[ N=2 \]
\[ d=3 \]

\[ \nabla \times \mathbf{u} \]

\[ \partial_\alpha u_\alpha \]

\[ \text{shear } C_{11} \]

\[ \text{shear } C_{66} \]

\[ \varphi \approx u_\alpha \]

\[ \partial_\alpha u_\alpha \]

\[ \varphi \approx u_\alpha \]

\[ \text{triangular in plane} \]

\[ \text{tilt } C_{44} \]

\[ E_{el} = \frac{1}{2} \int \left[ \left( C_{\parallel} q_\parallel^2 + C_{44} q_\parallel^2 \right) P^L(q_\parallel) + \left( C_{66} q_\parallel^2 + C_{44} q_\parallel^2 \right) P^T(q_\parallel) \right] u^\alpha u^\beta - q_\parallel q_\parallel \]

\[ q = (q_\parallel, q_\parallel) \]
Vortex Lattice + Thermal fluctuations + quenched impurities

3 phases - phase diagram

A: Vortex Liquid

C: Vortex glass  strongly pinned  amorphous

B: Bragg glass  weakly pinned  - quasi-LR transl order  - no dislocations
early decoration images of vortex lattice (seen from top) + delaunay triangulation

dislocations and disclinations
Periodic object + weak disorder

Bragg Glass: No dislocation

\[ B(r) = (u(r) - u(0))^2 \]

\[ B(r) \sim |r|^{2\zeta} \ll a_0^2 \]

\[ \zeta \approx 0.22 \]

\[ d = 3, N = 2 \]
\[ B(r) \sim A_d \ln |r| \geq a_0^2 \]

divergent Bragg peaks

\[ \rho_K(x) = \rho_0 e^{iKu(x)} \]

\[ \rho_K(x)\rho_K^*(0) \sim |x|^{-\eta} \]

Klein et al. KBaBiO

Neutron diffraction
Vortex creep in the Bragg glass

Zeldov et al.  BSCCO

\[ R \sim v \sim e^{-U(j)/T} \]
\[ U(j) \sim j^{-\mu} \]
\[ \mu = 1/2 \]

velocity of vortex \( v \)  \( \leftrightarrow \)  electric field \( E \)

(Lorentz) force acting on vortex \( f \)  \( \leftrightarrow \)  \( j \)  super-current

creep  \( \Rightarrow \)  zero linear resistivity
= true superconductivity !
Reviews

• Pinning of elastic manifolds


  Giamarchi and PLD, in book ”Spin glasses and random fields” cond-mat/9705096

  (with more applications to superconductors)

• Functional RG


Avalanches
Avalanches


ABBM model


Nonstationary dynamics of the ABBM model

BFM and beyond mean-field

Avalanche dynamics of elastic interfaces

Size distributions of shocks and static avalanches from the FRG

Universality in the mean spatial shape of avalanches
T. Thiery, PLD, EPL 114 36003 (2016).
Fig. 2. Sketch of the experimental setup. Inset: photograph of the disordered substrate, the chromium defects appear as white square spots.
Fig. 1. Upper part: image of the contact line obtained with an ordinary CCD camera. Lower part: the position \( \eta(x, t) \equiv y(x, t) - vt \) of the CL is defined with respect to its average position \( vt \).

Friction (overdamped dynamics): \( w(t) = vt \)

\[
\eta_0 \partial_t u(x, t) = \nabla^2_x u(x, t) + m^2 (w(t) - u(x, t)) + F(u(x, t), x)
\]

- Elastic restoring force (here non-local)
- Driving
- (Quenched) substrate disorder
- Contact line: gravity (capillary length)
- Magnetic interface: demag. field
- Crack line: loading
Avalanches: reproducible

Figure 2: A contact line for the wetting of a disordered substrate by Glycerine [7]. Experimental setup (left). The disorder consists of randomly deposited islands of Chromium, appearing as bright spots (top right). Temporal evolution of the retreating contact-line (bottom right). Note the different scales parallel and perpendicular to the contact-line. Pictures courtesy of S. Moulinet, with kind permission.
$u(w) = \text{center of mass of the contact line (over 2 } L_c)$

$w - u(w)$

$w = vt$

avalanche
Barkhausen (magnetic) noise
G. Durin (Torino) F. Bohn (Brazil)

area proportional to total avalanche size $S$

FIG. 1. Experimental Barkhausen signal (voltage produced from a pickup coil around a ferromagnet subjected to a slowly varying applied field).
Here we show that collective cell migration occurs in bursts that are similar to those recorded in the propagation of cracks, fluid fronts in porous media and ferromagnetic domain walls.
Functional RG and field theory

how to measure/define it?

central object is renormalized disorder correlator \( \Delta(w) \equiv \Delta_m(w) \)

\[
\frac{(u(w) - w)(u(w') - w')}{c} = m^{-4} L^d \Delta(w - w')
\]

it obeys differential FRG equation as \( m \) is varied
Functional RG and field theory

how to measure/define it?

central object is renormalized disorder correlator \( \Delta(w) \equiv \Delta_m(w) \)

It obeys differential FRG equation as \( m \) is varied

\[
(\bar{u}(w) - w)(\bar{u}(w') - w')^c = m^{-4} L^d \Delta(w - w')
\]

\( w - \bar{u}(w) \)
Functional RG and field theory

how to measure/define it?

central object is renormalized disorder correlator \( \Delta(w) \equiv \Delta_m(w) \)

it obeys differential FRG equation as \( m \) is varied

\[
(u(w) - w)(u(w') - w')^c = m^{-4} L^d \Delta(w - w')
\]

\[
\Delta_m(w) \sim_{m \to 0} m^{\epsilon - 2\zeta} \tilde{\Delta}^*(wm^\zeta)
\]

\[
-m \partial_m \Delta = (\epsilon - 2\zeta) \Delta + \zeta \Delta' - \left( \frac{\Delta^2}{2} + \Delta \Delta(0) \right)''
\]

\[
\lambda_{stat} = -1
\]

- analytic correlator \( \Rightarrow \) Larkin

- develops a cusp at \( L_c \) (Larkin length)

\[
S_m := \frac{\langle S^2 \rangle}{2\langle S \rangle} = \frac{|\Delta'(0^+)|}{m^4}
\]


Functional RG and field theory

central object is renormalized disorder correlator \( \Delta(w) \equiv \Delta_m(w) \)


it obeys differential FRG equation as \( m \) is varied

\[
(u(w) - w)(u(w') - w')^c = m^{-4} L^d \Delta(w - w')
\]

FRG fixed point:
\[
\Delta_m(w) \sim_{m \to 0} m^{\epsilon - 2\zeta} \tilde{\Delta}^*(wm^{\zeta}) \quad \epsilon = d_{uc} - d
\]

\[
\tilde{\Delta}^*(u) = \epsilon d_1(u) + \epsilon^2 d_2(u) + ..
\]

All universal observables can be obtained in perturbation in \( \tilde{\Delta}^*(u) \) i.e. in \( \epsilon \)

Allows to calculate depinning critical exponents: two independent exponents

\[
u \sim x^\zeta \quad \zeta = \frac{\epsilon}{3} (1 + 0.1433\epsilon) \quad \epsilon = 4 - d \quad \text{SR}
\]

\[
x \sim t^z \quad \zeta = 1.250 \pm 0.005 \quad \epsilon = 2 - d \quad \text{SR d=1}
\]

\[
z = 2 - \frac{2}{9}\epsilon - 0.0432\epsilon^2 - 0.1133\epsilon^2 \quad \zeta = 0.39.. \quad \text{LR}
\]

predicts 0.4 confirmed numerics

fracture: Ponson, Santucci, ..
FRG fixed point at depinning: numerics and experiments

cusp!

\[ \Delta'(0^+) < 0 \]

numerics: interface driven quasi-statically by quadratic well

A. Rosso, PLD, KW
condmat/0610821

cusp!

\[ \Delta'(0^+) < 0 \]

contact line experiment:

E. Rolley, S. Moulinet, PLD, KW
EPL, 87 (2009) 56001

\[ \Delta(w - w') := \langle \overline{h}_l(w)\overline{h}_l(w') \rangle \]

\[ \overline{h}_l(t) := \frac{1}{t} \int_0^t h(x, t) \, dx \]

\[ h(x, w) := h(x, w/v) - h_0 \]

Fig. 6: Inset: The disorder correlator \( \Delta(w) \) for iso/Si at \( v = 1 \mu m/s \) up to \( w = 35 \mu m \), and then at \( v = 10 \mu m/s \) for \( w > 35 \mu m \), with error-bars as estimated from the experiment. Main plot: The rescaled disorder correlator \( \Delta(w)/\Delta(0) \) (green/solid) with error bars (red). The dashed line is the 1-loop result from equation (6).
Quantitative scaling of magnetic avalanches

G. Durin,¹,² F. Bohn,³ M. A. Corrêa,³ R. L. Sommer,⁴ P. Le Doussal,⁵ and K. J. Wiese⁵

LR elasticity samples, comparison with MF

LR polycrystalline Ni₈₁Fe₁₉ Permalloy (Py) 200 nm thick

\[ \tau_m = 39 \mu s \] is ONLY parameter!

\[ 2 \hat{T} \coth(\hat{T}/2) = 4 \]

\[ \langle \dot{u}(t) \rangle_S = \frac{S}{\tau_m} \left( \frac{S}{S_m} \right)^{-\frac{1}{\gamma}} f \left( \frac{t}{\tau_m} \left( \frac{S_m}{S} \right)^{\frac{1}{\gamma}} \right) \]

\[ S_m := \frac{\langle S^2 \rangle}{2\langle S \rangle} \]

\[ f_0(t) = 2te^{-t^2} \]

\[ \gamma = 2 \]

Figure 1. Normalized average size \( \langle S \rangle_T / S_m \) of Barkhausen avalanches in the FeSi ribbon (blue dots) and the Py thin film (red dots) as a function of the normalized duration \( \hat{T} = T/\tau_m \). The continuous line is the theoretical prediction \( g_1^{MF} \) of Eq. (4). For the ribbon, the deviation at large durations is more evident due to effect of the eddy currents. The inset shows the second moment \( \langle S^2 \rangle_T / S_m^2 \) compared to the prediction \( g_2^{MF} \) of Eq. (4).

\[ m = k0 \text{ demag. field} \]

Figure 4. Scaling collapse of the average shapes at fixed avalanche sizes \( \langle \dot{u}(t) \rangle_S \), according to Eq. (7), in the Py thin film. The continuous line is the mean-field universal scaling function in Eq. (8).
avalanche shape at fixed size: beyond mean-field

samples with SR elasticity

SR amorphous Fe$_{75}$Si$_{15}$B$_{10}$ (FeSiB) alloy 1000 nm

comparison with theory with epsilon=2

\[ \tau_m = 38 \mu s \]

Figure 5. Scaling collapse of the average shape at fixed avalanche sizes \( \langle u(t) \rangle_S \), according to Eq. (7), in the FeSiB thin film. The continuous line is the prediction for the universal SR scaling function of Eq. (9). The insets show comparisons of the tails of the data with the predicted asymptotic behaviors of Eqs. (10) and (11), setting \( \epsilon = 2 \), with \( A = 1.094, A' = 1.1, \beta = 0.89, C = 1.15 \), and \( \delta = 2.22 \).
Avalanche size distribution beyond mean-field

recall: \( d = d_{uc} \) \quad \rho_{MF}(s) = \frac{1}{2\sqrt{\pi}s^{3/2}}e^{-s/4} \n
\[ d = 4 - \epsilon \]

\[ p(s) = \frac{A}{2\sqrt{\pi}} \frac{1}{s^\tau}e^{Cs^{1/2} - \frac{B}{4}s^\delta} \]

\[ \gamma_E = 0.577216 \]
\[ A = 1 - \frac{2 - 3\gamma_E}{36}\epsilon \]
\[ B = 1 + \frac{2}{9}(1 + \frac{\gamma_E}{4})\epsilon \]
\[ C = \frac{\sqrt{\pi}}{9}\epsilon \]

avalanche exponent:
\[ \tau = \frac{3}{2} - \frac{\epsilon}{12} \]
\[ \delta = 1 + \frac{\epsilon}{6} \]

agrees to \( O(\epsilon) \) with Narayan-Fisher conjecture

\[ \tau_{conj} = 2 - \frac{2}{d + \zeta} \]
\[ \tau_{num}^{d=1} = 1.08 \pm 0.02 \]

NF = 1.11
Additional topics
Experimental evidence for three universality classes for reaction fronts in disordered flows

Séverine Atis,1 Awadhesh Kumar Dubey,1 Dominique Salin,1 Laurent Talon,1 Pierre Le Doussal,2 and Kay Jörg Wiese2

Front velocity

\[ \dot{v} = \frac{V_f}{V_\chi} \]

\[ F = \frac{(\bar{U} + V_\chi)}{V_\chi} + 0.38 \]
Fig. 3. Roughness $W$ as a function of distance $L$ for different drift velocities. The upper (respectively, lower) graph corresponds to data obtained with water (respectively, water-glycerol mixture). For both graphs, the data $o$ have been obtained with a larger magnification (resolution 2.1 $\mu$m) than the others (resolution 6.1 $\mu$m).
**Fig. 7.** Probability $P(L)$ of occurrence of an avalanche of length $L$ for different drift velocities and different viscosities. $P$ is the number of avalanches divided by the effective area swept by the CL and by the effective pixel size. This curves are obtained with the same magnification of the microscope; other magnifications lead to the same curves. The solid line is the power law dependence expected from numerical simulation.
Q: is there translational order in the moving lattice?
Q: is there translational order in the moving lattice?

“Moving glass effect”

In lattice moving along principal axis direction $x$

the Fourier modes $(0, K_y)$ of the disorder are NOT averaged out by motion!

$\Rightarrow$ transverse displacements $u_y$ still see static disorder!

Moving glass equation

$$\eta \partial_t u^y(x, t) + \eta v \cdot \nabla u^y(x, t) = c \nabla^2 u^y(x, t) + U(x) \rho_0 \sum_{K_y} iK_y e^{iK_y(y - u^y(x, t))}$$

is a pinning equation with additional convective term

- upper critical dimension is $d_{uc}=3$ instead of 4
- there is a transverse pinning $f_c$
Moving Bragg glass and moving smectic

Fig. 18. Left: Decoration in motion images in NbSe$_2$ at 4 K from Ref. 170. Right column: Real space image (Fourier filtered) which shows the static channels along direction of motion. Left column: Its Fourier transform, which shows transverse order (two peaks) or full triangular lattice order (six peaks). Top: Moving smectic (low-field, higher-velocity). Bottom: Moving Bragg glass (high-field, lower-velocity). Right: (top) Moving Bragg glass on static channels, (bottom) moving smectic, channels are decoupled by dislocations (black square).