Tutorial: Time evolving block decimation (TEBD)

This exercise uses the Python Notebooks provided here: `http://go.tum.de/603150`

Part I: Imaginary time evolution

a) Try out different parameters (bond dimension χ, time step $\delta\tau$, time period T, ...) and check the convergence of the ground state energy.

b) Detect the quantum phase transition of the transverse field Ising model by plotting the entanglement entropy S for different system sizes L. What is the expected behavior of S away from the critical point?

c) Implement a function to obtain the correlation function $C_{ij} = \langle \sigma_i^z \sigma_j^z \rangle$ and use it to obtain the magnetization m^2.*

d) Replace the TEBD function with a second order Trotter decomposition.*

Hint: E.g. for `N_steps = 3`, the first order expansion evolves with

$$e^{-iH^E dt} e^{-iH^O dt} e^{-iH^E dt} e^{-iH^O dt} e^{-iH^E dt} e^{-iH^O dt}, \tag{1}$$

while the second order expansion would read

$$e^{-iH^E \frac{dt}{2}} e^{-iH^O dt} \underbrace{e^{-iH^E \frac{dt}{2}} e^{-iH^E \frac{dt}{2}}}_{=e^{-iH^E dt}} e^{-iH^O dt} \underbrace{e^{-iH^E \frac{dt}{2}} e^{-iH^E \frac{dt}{2}}}_{=e^{-iH^E dt}} e^{-iH^O dt} e^{-iH^E \frac{dt}{2}} \tag{2}$$

Part II: Real time evolution

a) Global quench: Try out different parameters (bond dimension χ, time step δt, time period T, ...) and check the convergence of the energy, magnetization, and entanglement entropy following a global quench. How does the required χ scale as time? How does the half chain entanglement scale as function of time T and systems size L?

b) Local quench: Calculate the (approximate) ground state $|\psi_0\rangle$ of a $L = 21$ chain or $g = 1.5$. Apply the local operator $\sigma^x_{n_0}$, where n_0 is the index of a site in the center of the chain, by multiplying it to the corresponding B tensor of the ground state. Perform then a real time evolution of this initial state. Measure the entropy for cuts on the different bonds. Create a color-plot showing the entropy versus time t on the y-axis and the bond of the cut n on the x-axis. How does the perturbation spread? What is the saturation value of $S(t) - S_0$ for large t and what do you expect?

c) Global quench: Modify the Hamiltonian and add a longitudinal field $h\sigma_z$ to the Hamiltonian. Start from the limit $0 < h < g \ll J$. How does it affect the entanglement growth? Try out different initial states. Compare to the results in Kormos et al., Nat. Phy. **13** 246 (2017).*

* These tasks are a bit more involved than the others.