# Kondo effect in quantum dots

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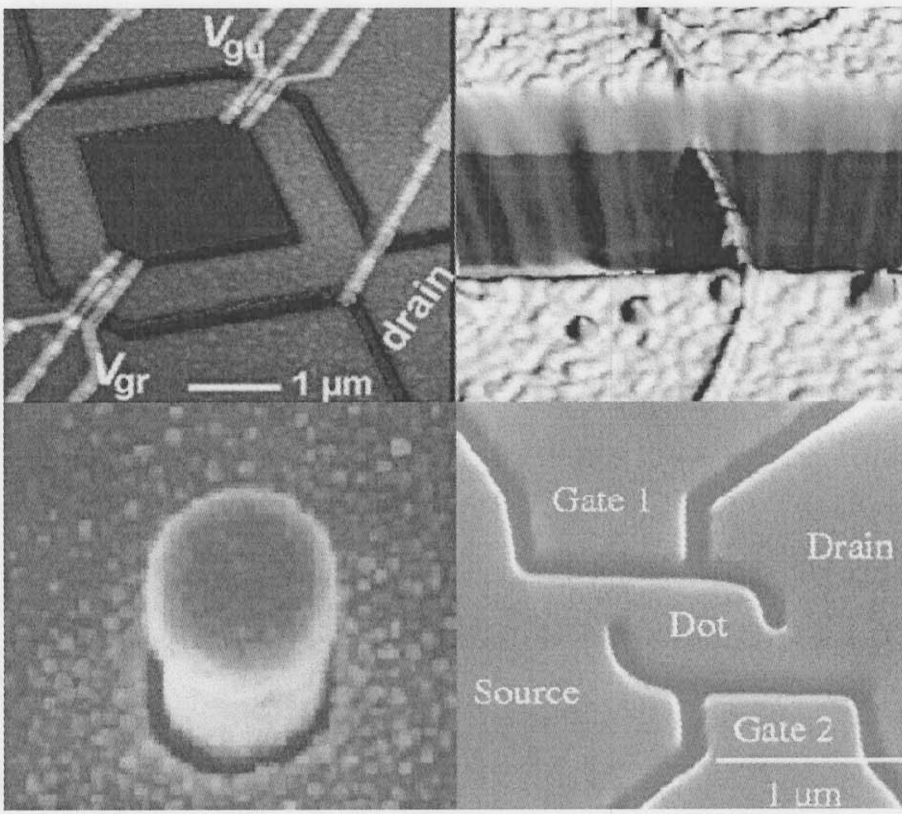
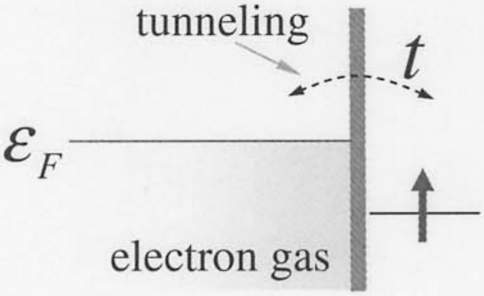


# Outline

- Anderson impurity model and exchange Hamiltonian; scattering off a localized spin
- Experiments with the odd-electron states
- Electron transport through a dot: Linear conductance
- Weak coupling, renormalization group (RG)
- Strong coupling, Fermi liquid theory
- Non-linear I-V characteristics
- Inelastic Kondo scattering
- Tuning the Kondo effect in a quantum dot

# Introduction to the Kondo effect



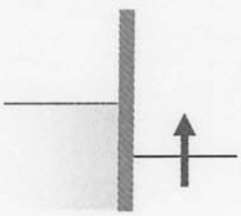

## Anderson impurity model

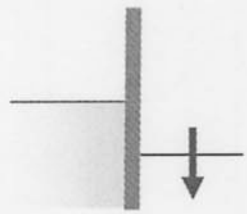
strong on-cite repulsion:

$$H_d = U\left(N-1\right)^2, \quad N = n_\uparrow + n_\downarrow$$

impurity level is **singly** occupied: $\langle N \rangle = 1$

$t = 0$: doubly degenerate ground state

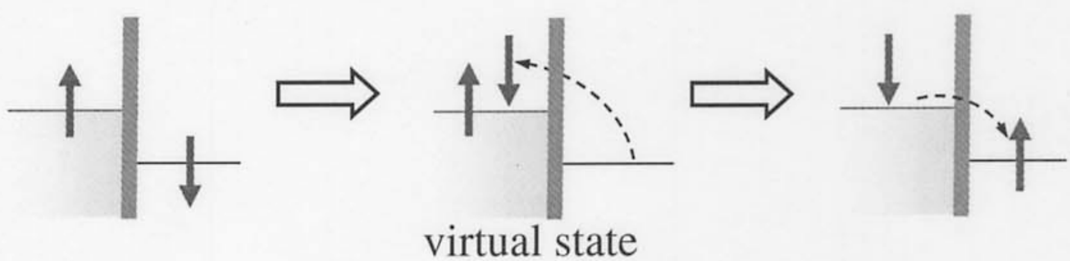




$\Psi_\uparrow = |\text{electron gas}\rangle \otimes |\uparrow\rangle$ $\quad\quad$ $\Psi_\downarrow = |\text{electron gas}\rangle \otimes |\downarrow\rangle$

$t \neq 0$: tunneling $\rightarrow$ exchange

example:




Effective exchange interaction:

$$H_{\text{exchange}} = J\left(\mathbf{s}\cdot\mathbf{S}\right) \qquad J \propto t^2/U > 0$$

conduction electrons $\quad$ impurity

# Anderson impurity model
# and exchange interaction Hamiltonian

$$
\mathcal{H} = \sum_{ks} \xi_k c^+_{ks} c_{ks} + \sum_s \varepsilon_{os} d^+_{os} d_{os} + U \hat{n}_\uparrow \hat{n}_\downarrow
$$
$$
+ \sum_{ks} \left( t\, c^+_{ks} d_s + t^* d^+_s c_{ks} \right) ; \quad \hat{n}_s = d^+_{os} d_{os} \qquad (1)
$$

$\varepsilon_F = 0$      Consider $-U < \varepsilon_0 < 0$; $U > 0$.

$- \varepsilon_0$

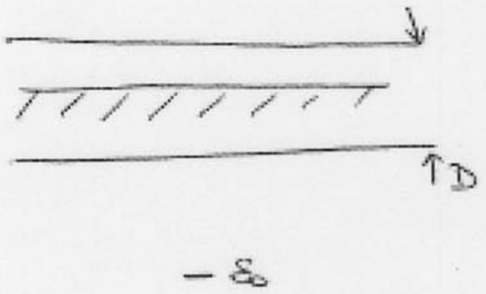
degenerate at $t=0$

$|\uparrow\rangle$, $|0\rangle$, $|\uparrow\downarrow\rangle$, $|\downarrow\rangle$

hybridize via at $t \neq 0$

Energies of $|0\rangle$, $|\uparrow\downarrow\rangle$ are high: $\varepsilon_0$ and $U - \varepsilon_0$, respectively

<u>Effective Hamiltonian</u>: acts in the low-energy subspace yielding same results as Eq (1)




Bandwidth D ; projector $P \equiv \hat{P}_D$

We are interested in the action of T-matrix in subspace D

$$T = V + V\frac{1}{E-H_0}V + \dots$$

$$\mathcal{H} = H_0 + V \;; \qquad V = \sum_{ks}\left(t\, c^{+}_{ks} d_s + t^{*} d^{+}_s c_{ks}\right)$$

Note: (1) within D: $T = PTP$

(2) $PVP = 0$; $PV(1-P) \neq 0$

$$PTP = PV(1-P)\frac{1}{E-H_0}(1-P)VP + \dots$$

Small D $\Rightarrow$ $E \to E_{gr.}$

Effective Hamiltonian of perturbation:

$$\tilde{V} = PV(1-P)\frac{1}{E_{gr}-H_0}(1-P)VP$$

(up to 2-nd order in $t$)

For the specific form of $H_0$, $V$:

$$\tilde{V} = \mathcal{H}_{exchange} + \mathcal{H}_p$$

By symmetry:

$$\mathcal{H}_{exchange} = J\,\vec{s}\cdot\vec{S}\,; \qquad \vec{s} = \sum_{\substack{kk' \\ ss'}} c^+_{ks}\,\frac{\vec{\sigma}_{ss'}}{2}\,c_{k's'}$$

$$\vec{S} = \sum_{ss'} d^+_s\,\frac{\vec{\sigma}_{ss'}}{2}\,d_{s'}$$

$$J = |t|^2\left(\frac{1}{U+\varepsilon_0} + \frac{1}{|\varepsilon_0|}\right) \quad ; \quad J>0$$

$$\mathcal{H}_p = A_p \sum_{kk's} c^+_{ks}c_{k's}\,, \quad A_p = \frac{1}{2}|t|^2\left(\frac{1}{|\varepsilon_0|} - \frac{1}{U+\varepsilon_0}\right)$$

Detailed derivation: K. Yosida, Theory of Magnetism, Springer → p. 238

Generalization for a quantum dot:

$$\mathcal{H}_{eff} = \sum_{\alpha ks} \xi_k\, c^+_{\alpha ks} c_{\alpha ks} + \sum_{\alpha\alpha'} J_{\alpha\alpha'}\,\vec{s}_{\alpha\alpha'}\cdot\vec{S}$$

lead number

$$J_{\alpha\alpha'} \propto t_{\alpha n_0} t^*_{\alpha' n_0} \Rightarrow \det\{J_{\alpha\alpha'}\} = 0\,; \quad \nu J \sim \frac{\Gamma}{E_c}$$

## Scattering off a localized spin

Consider the effect of $\mathcal{H}_{exchange}$ only.

Elastic scattering: By $SU(2)$ symmetry, the amplitude

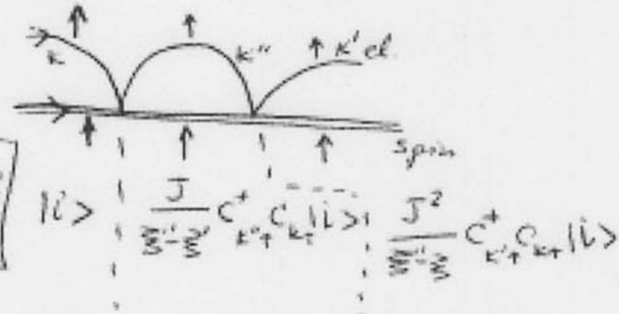
$$A_{ks\sigma\to k's'\sigma'} = \frac{1}{4}\left(\vec{\sigma}_{ss'}\cdot\vec{\sigma}_{\sigma\sigma'}\right) A_{kk'}$$

(1) Born approx.: $A^{(1)}_{kk'} \propto J$

(2) next order

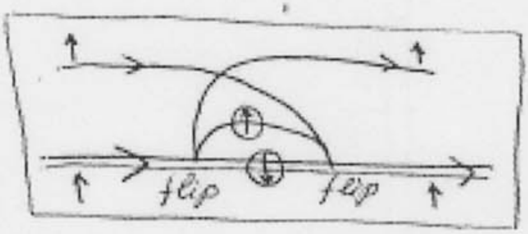
(2a) electron-like process

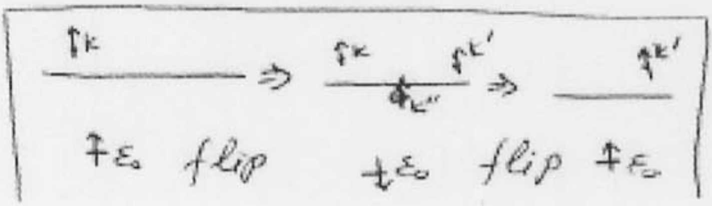
$\boxed{\text{\underline{one} set of intermediate states}}$

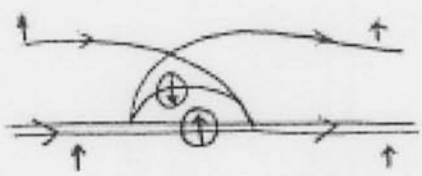



(2b) hole-like process

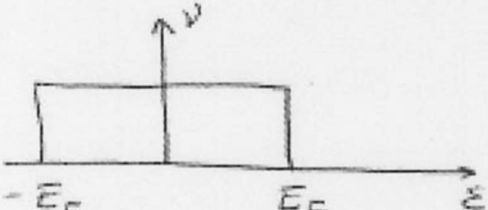
$\boxed{\text{\underline{two} sets of intermediate states}}$



Sum over $\xi''$:




$$A^{(2)}_{k\to k'} \propto 2\int_{-E_F}^{0} \frac{\nu\, d\xi''\, J^2}{\xi - \xi''} \quad \text{(hole-like)}$$

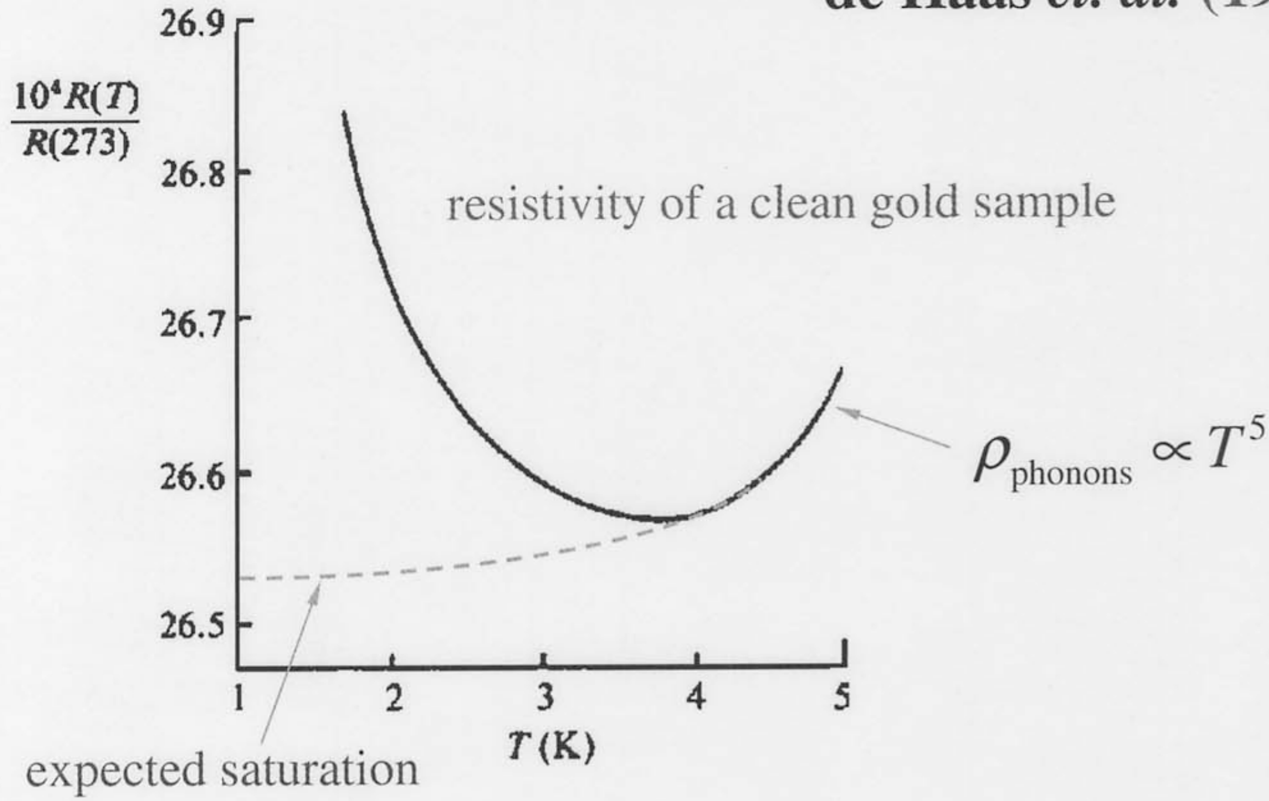
$$- \int_{0}^{E_F} \frac{\nu\, d\xi''\, J^2}{\xi'' - \xi}$$

$$A^{(2)}_{kk'} \propto \nu J^2 \ln \frac{E_F}{|\xi|} \qquad \text{(Kondo 1964)}$$

log-divergence is specific for $\mathcal{H}_{exchange}$;
does not happen for $\mathcal{H}_p$

# Kondo effect

**de Haas *et. al.* (1934)**




## J. Kondo (1964):

$$H_{\text{exchange}} = J\left(\mathbf{s}\cdot\mathbf{S}\right)$$

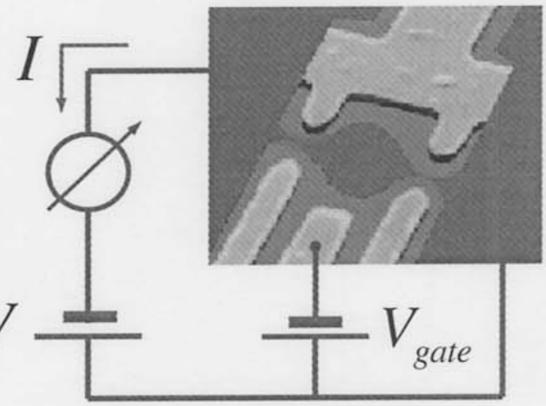
conduction electrons    magnetic impurity

$$\Delta\rho \sim J^2 + J^3 \ln\left(\varepsilon_F / T\right) \quad - \text{grows at } T \to 0$$

# Coulomb blockade of transport

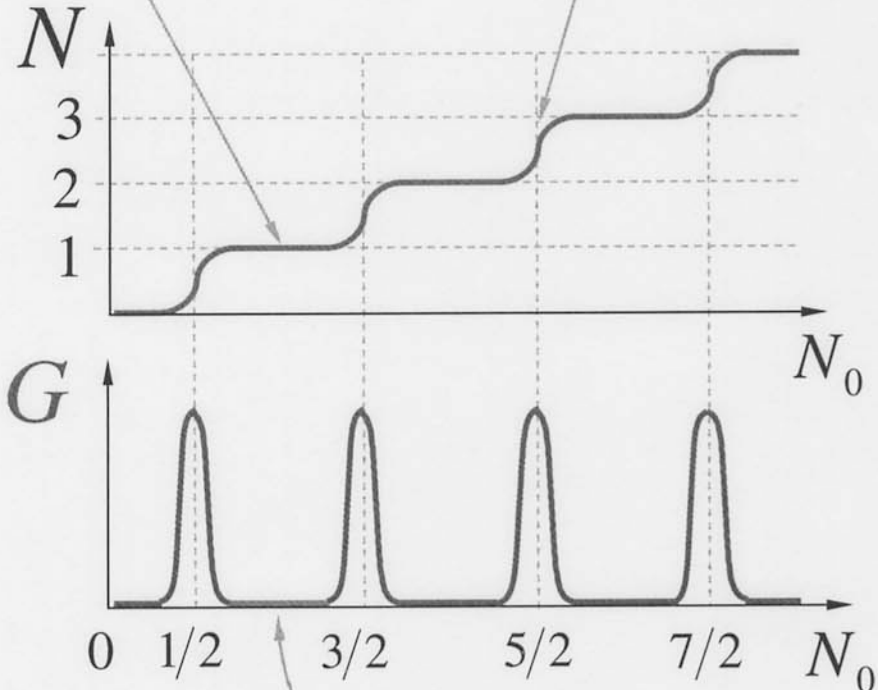
Linear conductance:

$$G = R^{-1} = \left.\frac{dI}{dV}\right|_{V \to 0}$$




**integer** $N$:
high energy cost $E \sim E_C$
blocks tunneling into the dot

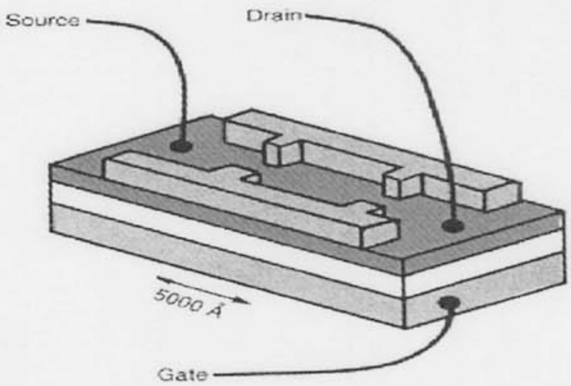
**half-integer** $N$:
charge degeneracy ⇒
tunneling costs no energy

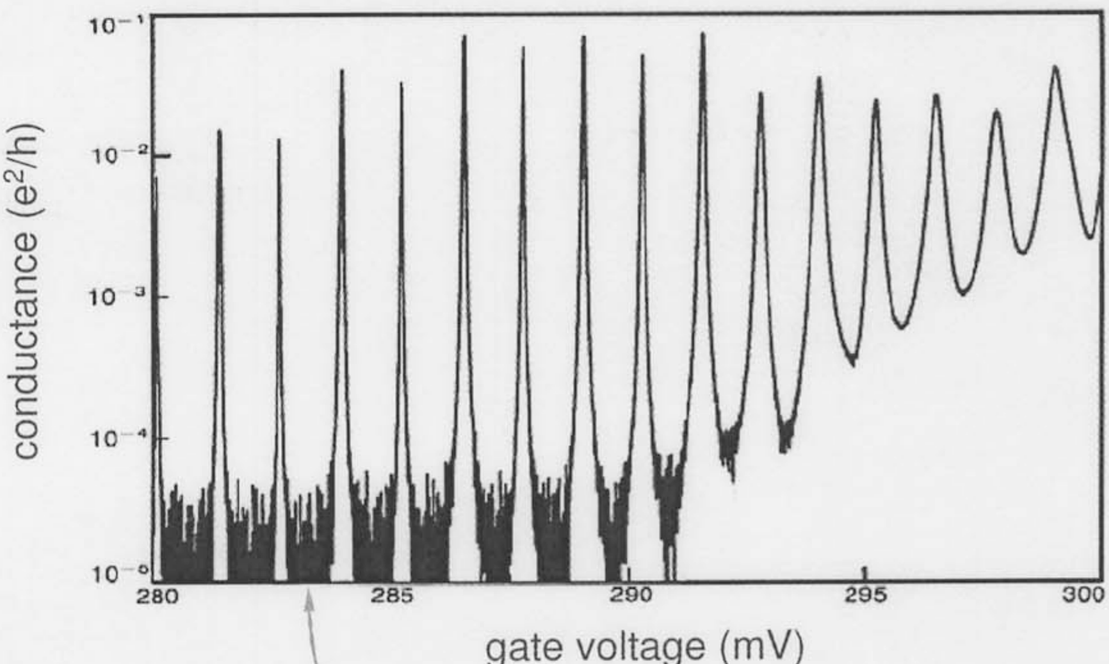



$$G \propto e^{-E_C/T}$$ (activation)

# Reasonably small quantum dots

M. Kastner, Physics Today (1993)
E. B. Foxman *et. al.* PRB (1993)

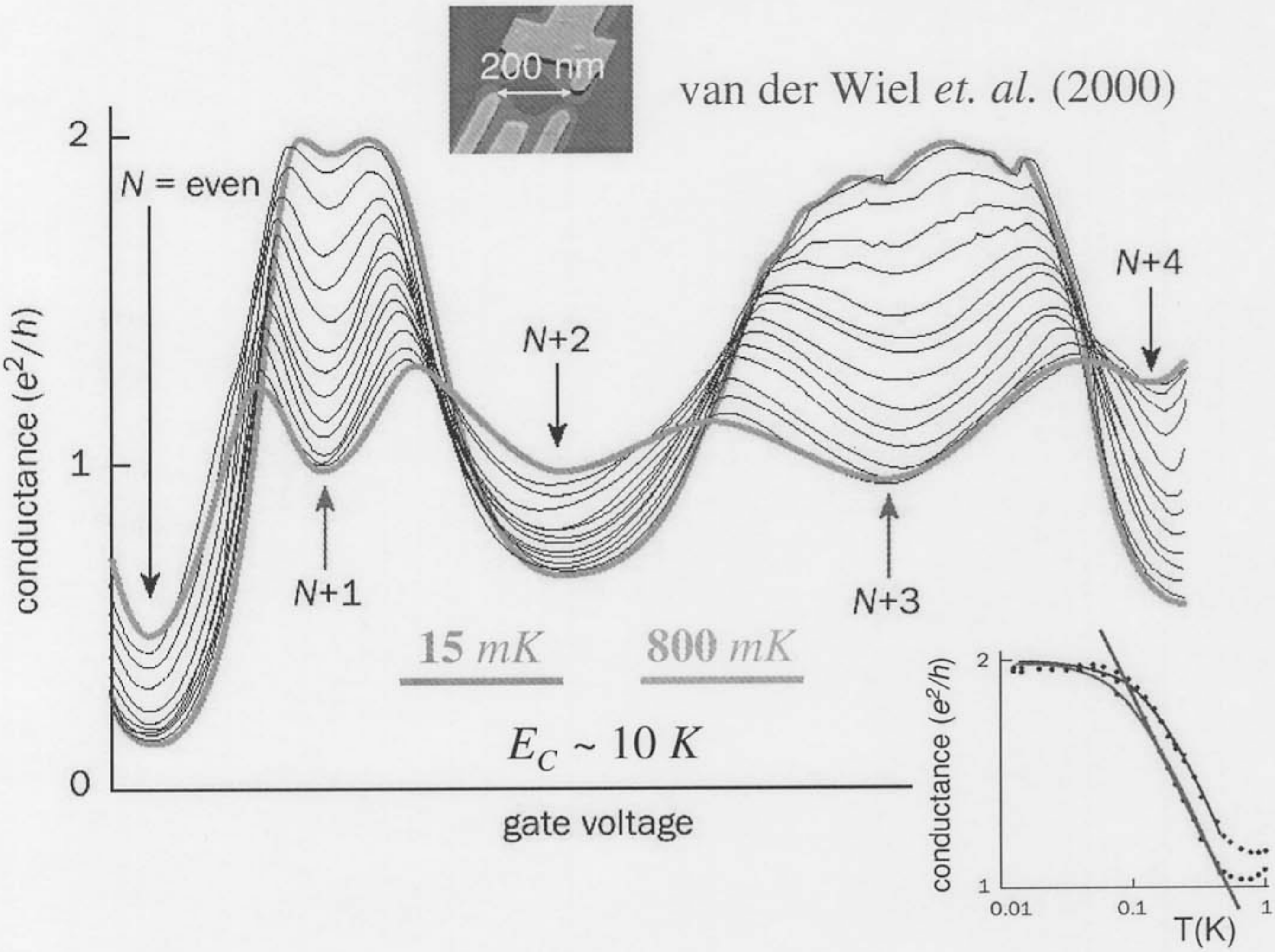



$$G \propto e^{-E_C/T}$$

# Even smaller quantum dots

**1998**
- D. Goldhaber-Gordon *et. al.* (MIT-Weizmann)
- S.M. Cronenwett *et. al.* (Delft, Kouwenhoven's group)
- J. Schmid *et. al.* (MPI @ Stuttgart)

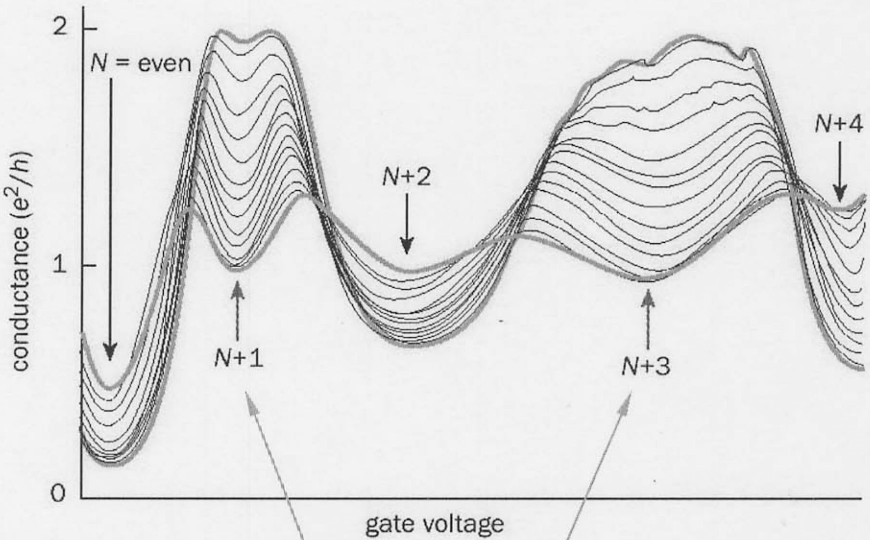



van der Wiel *et. al.* (2000)

$$\cancel{G \propto \exp(-E_C/T)} \qquad G \propto \ln(E_C/T)$$

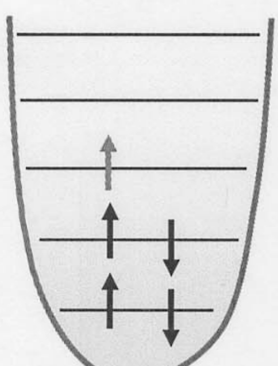
Naive theory fails qualitatively in every other valley

# Kondo physics in quantum dots




$$G \propto \ln\left(E_C/T\right)$$

$$T\text{- dependence}\ \begin{cases} N = \text{even: normal (decrease at } T \to 0) \\ N = \text{odd: anomalous (increase at } T \to 0) \end{cases}$$



for $N$ = odd all possible electronic configurations have $S \neq 0$

$$\left.\begin{array}{c} S \neq 0 \\ G \propto \ln\left(E_C/T\right) \end{array}\right\} \quad \text{signatures of Kondo physics}$$

L.G., Raikh (1988)
Ng, Lee (1988)

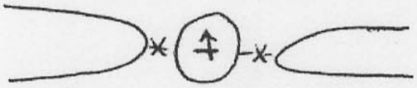
# Linear Conductance through a dot

Kubo f-la:

$$G = \lim_{\omega \to 0} \frac{1}{\hbar \omega} \int_0^\infty dt\, e^{i\omega t} \langle [\hat{I}(t), \hat{I}(0)] \rangle$$

$$\hat{I} = \frac{e}{2} \frac{d}{dt} (\hat{N}_R - \hat{N}_L), \qquad \hat{N}_\alpha = \sum_{k\alpha} c^+_{\alpha k s} c_{\alpha k s}$$

$$\alpha = L, R$$



L dot R

Recall:

$$\mathcal{H}_{eff} = \sum_{\alpha k s} \xi_k c^+_{\alpha k s} c_{\alpha k s} + \sum_{\alpha\alpha'} J_{\alpha\alpha'} \vec{s}_{\alpha\alpha'} \cdot \vec{S}$$

$$\vec{s}_{\alpha\alpha'} = \sum_{\substack{kk' \\ \alpha\alpha' \\ ss'}} c^+_{\alpha k s} \frac{\vec{\sigma}_{ss'}}{2} c_{\alpha' k' s'}$$

the exchange part may be diagonilized:

$$\begin{pmatrix} \psi_{1ks} \\ \psi_{2ks} \end{pmatrix} = \begin{pmatrix} \cos\theta_0 & \sin\theta_0 \\ -\sin\theta_0 & \cos\theta_0 \end{pmatrix} \begin{pmatrix} c_{Rks} \\ c_{Lks} \end{pmatrix}$$

$$\tan\theta_0 = \left|\frac{t_{Ln_0}}{t_{Rn_0}}\right|$$

$$\mathcal{H}_{eff} = \mathcal{H}_1 + \mathcal{H}_2 ,$$

$$\mathcal{H}_1 = \sum_{ks} \xi_k \psi^+_{1ks}\psi_{1ks} + J\,\vec{s}_i\,\vec{S}$$

$$\mathcal{H}_2 = \sum_{ks} \xi_k \psi^+_{2ks}\psi_{2ks} \quad \text{- free fermions}$$

$\mathrm{tr}\,\hat{J}_{\alpha\alpha'}$ - invariant $\Rightarrow \quad J = \mathrm{tr}\,\hat{J}_{\alpha\alpha} > 0$

$$\hat{N}_R - \hat{N}_L = \cos(2\theta_0)\,(\hat{N}_1 - \hat{N}_2) - \sin(2\theta_0)\sum_{ks}\left(\psi^+_{1ks}\psi_{2ks} + \text{h.c.}\right)$$

$$\frac{d}{dt}N_1 = \frac{d}{dt}N_2 = 0$$

$$\hat{I} \propto \sin 2\theta_0 \cdot \frac{d}{dt}\sum_{ks}\left(\psi^+_{1ks}\psi_{2ks} + \text{h.c.}\right)$$

$$\langle [I(t), I(0)] \rangle \longrightarrow \langle \psi_1^+(t)\,\psi_2(t)\,\psi_2^+(0)\,\psi_1(0) \rangle$$

structure $\mathcal{H}$ allows to <u>factorize</u> $\langle \ldots \rangle$ + use the free-electron Green function for $\langle \psi_2^+(t)\,\psi_2(0)\rangle$

# Conductance

$$\boxed{G = G_0 \int d\omega \left(-\frac{\partial f}{\partial \omega}\right) \cdot \frac{1}{2}\sum_s (-\pi\nu)\, \mathrm{Im}\, T_s(\omega)}$$

$$G_0 = \frac{e^2}{\pi\hbar}\sin^2 2\theta_0 = \frac{e^2}{\pi\hbar}\frac{4\,|t_{Ln_0} t_{Rn_0}|^2}{\left(|t_{Ln_0}|^2 + |t_{Rn_0}|^2\right)^2}$$

$$f(\omega) = \frac{1}{e^{\omega/T}+1}$$

def $T_s(\omega)$:

$$G_{ks,k's}(\omega) = G_k^{(0)}(\omega) + G_k^{(0)}(\omega)\, T_s(\omega)\, G_{k'}^{(0)}(\omega)$$

Notes:

(1) $G_{ks,k's}(\omega)$ is a FT of $G_{ks,k's} = i\theta(t)\langle\{\psi_{ks}(t)\psi^\dagger_{k's}\}\rangle$

(2) Local perturbation $\Rightarrow$ $T_s$ is independent of $k, k'$

(3) Spin conservation $\Rightarrow$ $G_{ks,k's'} = \delta_{ss'}\, G_{ks,k's}$

(4) valid at any temperature

Details: Pustilnik, L.G. J. Phys. Cond. Mat. 16 R513 (2004)
+ cond-mat/0501007

# Perturbation theory for $\mathrm{Im}\, T_s(\omega)$

In the first two orders, we saw that $\propto J$ and $\propto J^2 \nu \ln \frac{E_F}{|\xi|}$ terms in the scattering amplitude correspond to <u>elastic</u> processes

Elastic transitions $\Rightarrow$ optical theorem holds within the single-particle ~~spin~~ sector ($\omega = \xi_k$)

$$\boxed{-\pi\nu\, \mathrm{Im}\, T_s = \frac{1}{2} \sum_{\sigma} \sum_{s'\sigma'} |\pi\nu A^2_{ks\sigma \to k's'\sigma'}|^2}$$

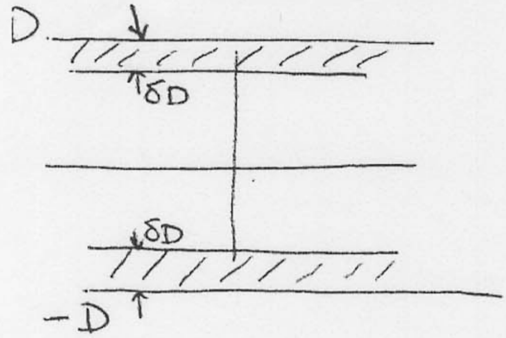
probabilities of two initial directions of the localized spin

$$A_{ks\sigma \to k's'\sigma'} = \frac{1}{4}\left(\sigma_{s's} \cdot \sigma_{\sigma'\sigma}\right) A(\omega) \; ; \quad \omega = \xi_k$$

$$-\pi\nu\, \mathrm{Im}\, T_s(\omega) = \frac{3\pi^2}{16} \nu^2 |A(\omega)|^2$$

## The idea of the renormalization group (RG)

## Poor-man scaling (P.W. Anderson 1970)




Temporary notations:

$$T = V + V G_0 T$$

$$G_0 = \frac{1}{E - H_0} \; ; \quad P = P_{\delta D}$$

$$T = V + VPG_0 T + V(1-P)G_0 T$$

$$T = V + VPG_0 \left(V + VPG_0 T\right) + V(1-P)G_0 T$$

$$= V + VPG_0 V + V(1-P)G_0 T + VPG_0 V(1-P)G_0 T$$
$$+ VPG_0 VPG_0 T \Leftarrow \underset{\text{small}}{O(V^2P^2)},$$

In the band $D - \delta D$, define $\boxed{V' = V + VP\frac{1}{E-H_0}V}$. Then

$$T = V' + V' G_0 T$$

(accuracy: linear in $V'$) Transformation $V \to V'$ upon band width reduction

$V = J \vec{S} \cdot \vec{s}$ preserves its form

$$J(D - \delta D) = J(D) + \nu J^2(D)\frac{\delta D}{D}$$

$(E \ll D)$

Introduce new variable: $\zeta = \ln\left(\frac{D_0}{D}\right)$ ; $D_0 = \cancel{8E} E_F$

$$J(D-\delta D) - J(D) = \nu J^2(D)\,\frac{\delta D}{D} \Rightarrow \boxed{\frac{dJ}{d\zeta} = \nu J^2}$$

RG equation

$J(\zeta=0) \equiv J$ (bare value)

$$\frac{1}{\nu J(\zeta=0)} - \frac{1}{\nu J(\zeta)} = \zeta \Rightarrow \nu J(\zeta) = \frac{1}{\frac{1}{\nu J_0} - \zeta} = \frac{1}{\ln\left(\frac{D}{T_k}\right)},$$

$T_k = E_F \exp\left(-\frac{1}{\nu J_0}\right)$

Band $D$ must accomodate state with energy $\omega$

$\Rightarrow$ stop RG at $\omega \sim D$

$$\nu J(\omega) = \frac{1}{\ln \frac{\omega}{T_k}}$$

Procedure OK as long as $\nu J(\omega) \ll 1$

(but maybe exceeding greatly the <u>bare</u> $\nu J$ value)

$\nu J(\omega) \ll 1 \Rightarrow$ use Born approx for scattering amplitude, $A(\omega) \approx \nu J(\omega)$

Anderson imp. → dot (change of the band width)

$$A^{(1)} = J$$

$$A^{(2)} = \nu J^2 \ln \frac{E_F}{\omega} \rightarrow A^{(2)} = \nu J^2 \ln \frac{\delta E}{\omega}$$

(Anderson imp.) (dot)

Leading-log. terms (≡ Poor-man RG result)

$$\nu A^{(n)}(\omega) = (\nu J)^n \left[\ln \frac{\delta E}{\omega}\right]^{n-1} \qquad \text{(Abrikosov 1965)}$$

$$\nu A(\omega) = \frac{\nu J}{1 - \nu J \ln\left(\frac{\delta E}{\omega}\right)} \equiv \frac{1}{\ln\left(\frac{\omega}{T_k}\right)}$$

def.: $\boxed{T_k = \delta E \exp\left\{-\frac{1}{\nu J}\right\}}$

$$\ln \frac{\delta E}{T_k} \sim \frac{1}{\nu J} \sim \frac{E_C}{\Gamma} \sim \underbrace{\frac{E_C}{\delta E}}_{\text{large}} \cdot \underbrace{\frac{e^2/h}{G_L + G_R}}_{\text{large}}$$

parameters

$T \gg T_k$: $\boxed{G = G_0 \frac{3\pi^2/16}{\ln^2 T/T_k}}$, $G_0 = \frac{e^2}{\pi\hbar} \frac{4\Gamma_L \Gamma_R}{(\Gamma_L + \Gamma_R)^2}$

# Kondo exchange

**Cartoon:**

Fermi Sea replaced by a single spin **s**

$$H_{\text{exchange}} = J\left(\mathbf{s}\cdot\mathbf{S}\right)$$

$$E_{S=1} - E_{S=0} = J$$

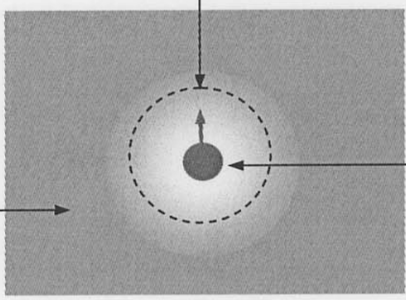
$\uparrow\uparrow$ $S=1$ $\downarrow\uparrow$ $S=0$

Ground state: singlet for $J>0$, triplet for $J<0$

## Exchange with electron gas:

$$H_{\text{exchange}} = J\left(\mathbf{s}\cdot\mathbf{S}\right),\ J>0 \quad \text{favors} \quad \uparrow\downarrow$$

small interaction radius



magnetic impurity

electron gas

but conduction electrons are delocalized!

# Kondo singlet

conjecture (Anderson, 1960s):
ground state is a <u>singlet</u>

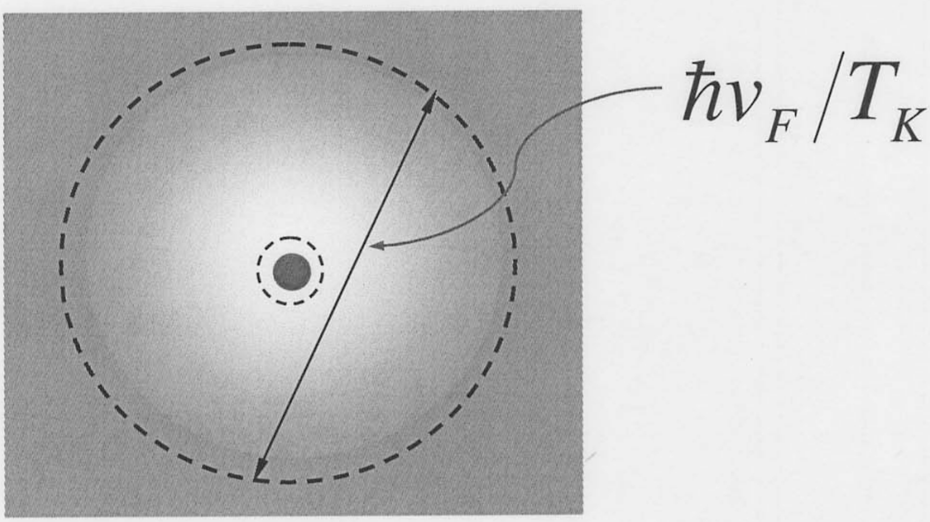
proof by numerical renormalization group:
Wilson, 1974

characteristic energy:

$$\cancel{J} \qquad T_K = U \exp\left(-1/\nu J\right) \quad \text{- Kondo temperature}$$

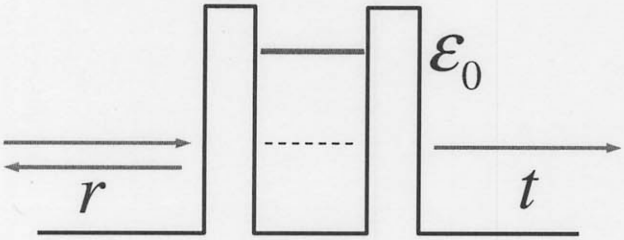
$$\Psi_{\text{Kondo}} = \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$

screening cloud




Kondo effect = lifting the ground state degeneracy

# Digression: Resonant scattering

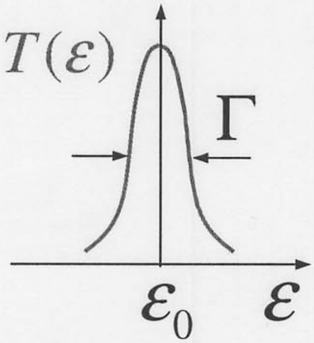



$$T(\varepsilon) = |t|^2 = \frac{\Gamma^2}{(\varepsilon - \varepsilon_0)^2 + \Gamma^2}$$

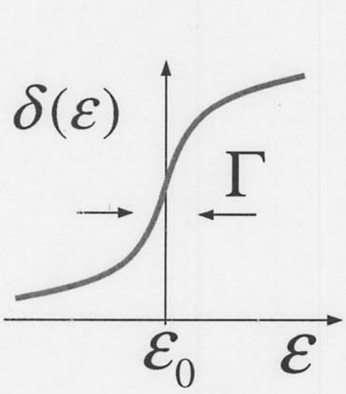
(Breit - Wigner)

transmission coefficient: $T(\varepsilon) = \sin^2 \delta(\varepsilon)$

scattering phase shift

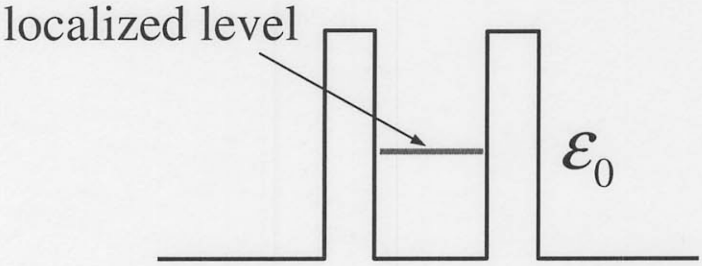


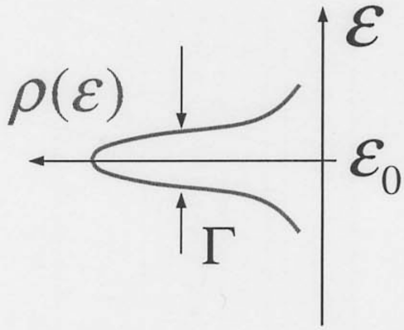

$$\boxed{T(\varepsilon) = 1 \leftrightarrow \delta(\varepsilon) = \pi/2}$$



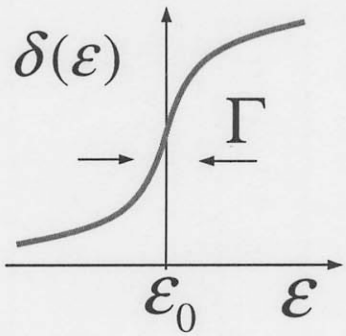
# Resonant scattering

density of states:

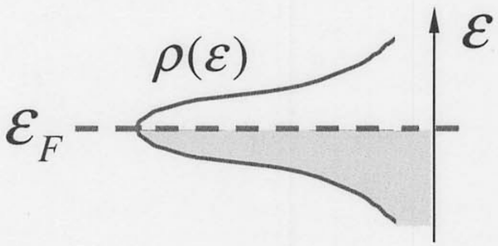



$$\rho(\varepsilon) = \frac{1}{\pi}\frac{\partial\delta(\varepsilon)}{\partial\varepsilon}$$




resonance at $\varepsilon = \varepsilon_F \Rightarrow$ localized level is half-occupied




$$N = \int_{-\infty}^{\varepsilon_F} \rho(\omega)\,d\omega = \frac{1}{2}$$

ground state expectation value

$$T(\varepsilon_F) = 1 \;\Rightarrow\; \delta(\varepsilon_F) = \frac{\pi}{2} \;\Rightarrow\; \rho(\varepsilon_F) = \max \Rightarrow N = 1/2$$

# Friedel sum rule

$N$ and $\delta(\varepsilon_F)$ are related!

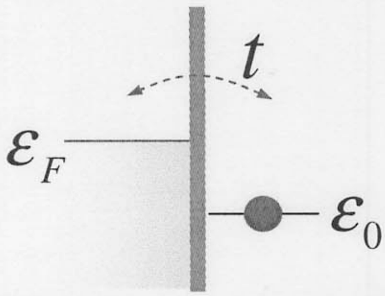
Friedel sum rule:

$$N = \frac{1}{\pi}\delta(\varepsilon_F) \qquad \text{(spinless fermions)}$$

$$\boxed{N = \frac{1}{\pi}\left[\delta_\uparrow(\varepsilon_F) + \delta_\downarrow(\varepsilon_F)\right]} \qquad \text{(electrons)}$$

$$N = 2 \;\Rightarrow\; \delta_\downarrow(\varepsilon_F) = \delta_\uparrow(\varepsilon_F) = \pi \qquad \text{no resonance}$$

**Anderson impurity:**




$$\varepsilon_0 < \varepsilon_F \;\text{ but }\; \varepsilon_F - \varepsilon_0 < U$$

$$\Downarrow$$

impurity level is singly occupied: $N = 1$

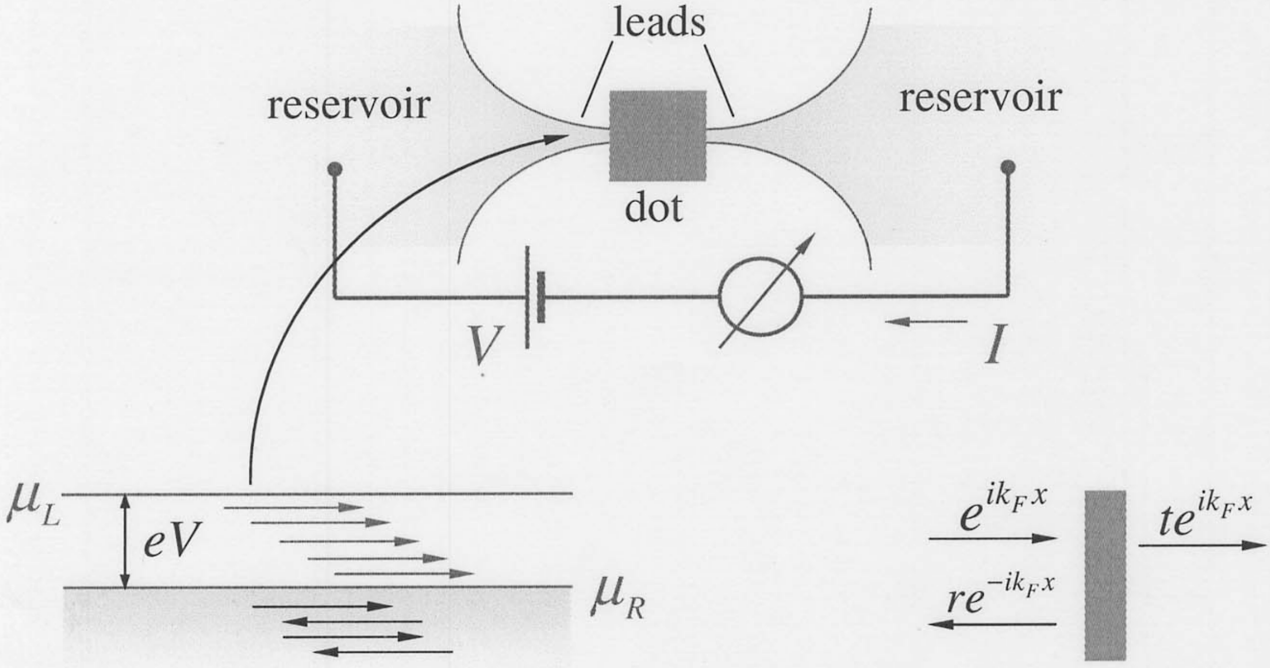
$$\Rightarrow\; \delta_\uparrow(\varepsilon_F) + \delta_\downarrow(\varepsilon_F) = \pi$$

singlet ground state $\Rightarrow\; \delta_\uparrow(\varepsilon_F) = \delta_\downarrow(\varepsilon_F) = \pi/2$

$$T(\varepsilon_F) = \sin^2\delta(\varepsilon_F) = 1$$

**resonance!**

# From scattering to transport




transmission coefficient

$$I = e \int_{\mu_R < \varepsilon_p < \mu_L} \frac{dp}{h} \left( \frac{\partial \varepsilon_p}{\partial p} \right) T\left(\varepsilon_p\right) = \frac{e}{h} \int_{\mu_R}^{\mu_L} d\varepsilon \; T\left(\varepsilon\right) = \frac{e}{h} T\left(\varepsilon_F\right) eV$$

velocity

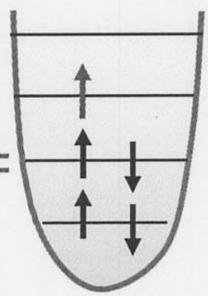
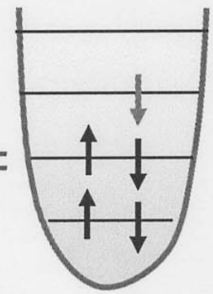
$$\Rightarrow \boxed{\textbf{Landauer formula:} \quad G = \frac{e^2}{h}\left(T_\uparrow + T_\downarrow\right)}$$

$T_\uparrow = T_\downarrow = 1$ (resonance) $\Rightarrow G = 2e^2/h$

conductance quantum: $e^2/h \approx \left(25\ k\Omega\right)^{-1}$

# Transport in the Kondo regime, T=0

Isolated dot: doubly-degenerate ground state

$|\uparrow\rangle =$  $|\downarrow\rangle =$ 

Dot in contact with leads: Kondo singlet

$$\Psi_{\text{Kondo}} = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

Conductance: $G = \frac{e^2}{h}\left(T_\uparrow + T_\downarrow\right) = \frac{e^2}{h}\left(\sin^2\delta_\uparrow + \sin^2\delta_\uparrow\right)$

scattering phase shifts

Friedel sum rule:

$$\delta_\uparrow = \pi N_\uparrow, \qquad \delta_\downarrow = \pi N_\downarrow$$

ground state expectation values

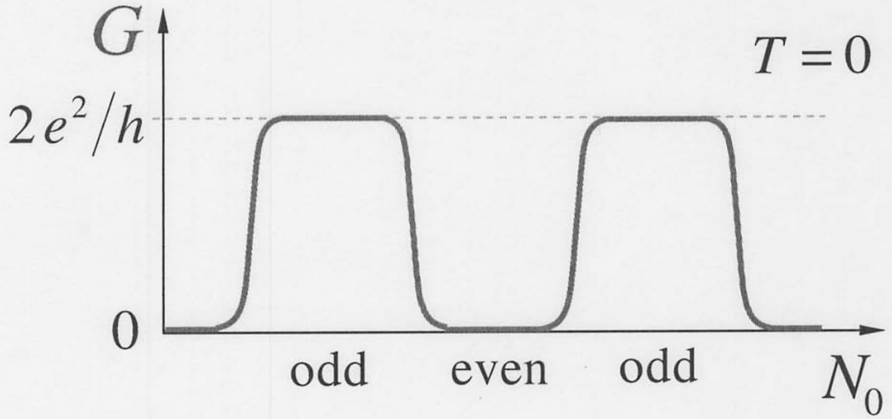
$$N_\uparrow = \langle \Psi_{\text{Kondo}} | N_\uparrow | \Psi_{\text{Kondo}} \rangle = N_\downarrow = N/2$$

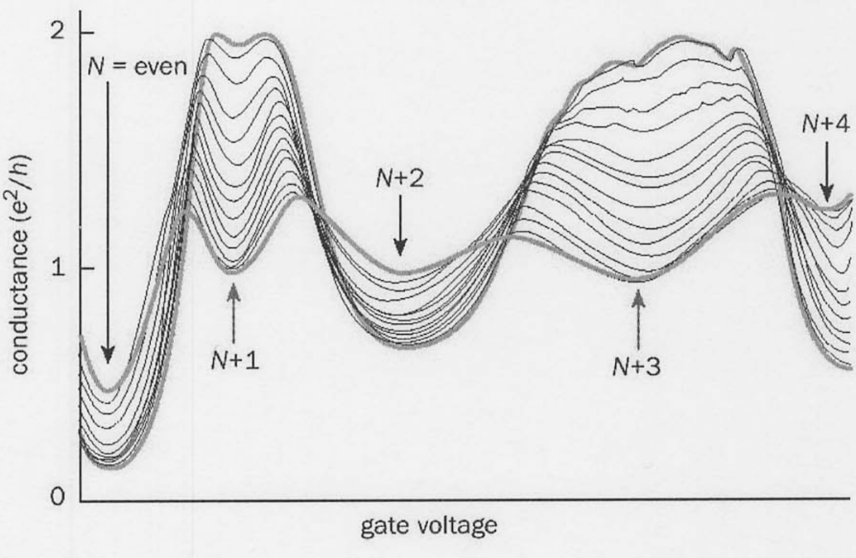
number of electrons on the dot

# Transport in the Kondo regime, T=0

$$\rightarrow \quad \boxed{G = \frac{2e^2}{h}\sin^2(\pi N/2)}$$

**odd N**: $G = 2e^2/h$ - perfect transmission
**even N**: $G = 0$ - perfect blockade

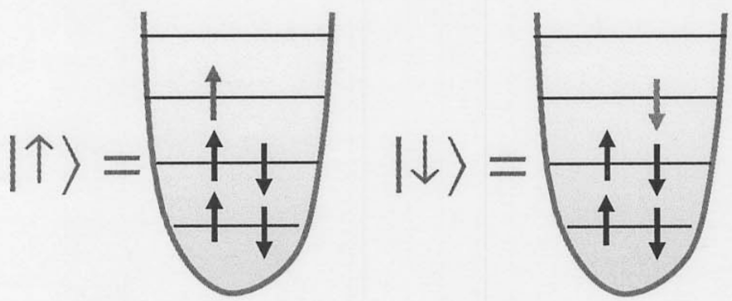


# Effect of a magnetic field

What is necessary for the Kondo effect to occur?

- ☞ interaction
- ☞ degeneracy ← lifted by a magnetic field
- ☞ electron gas

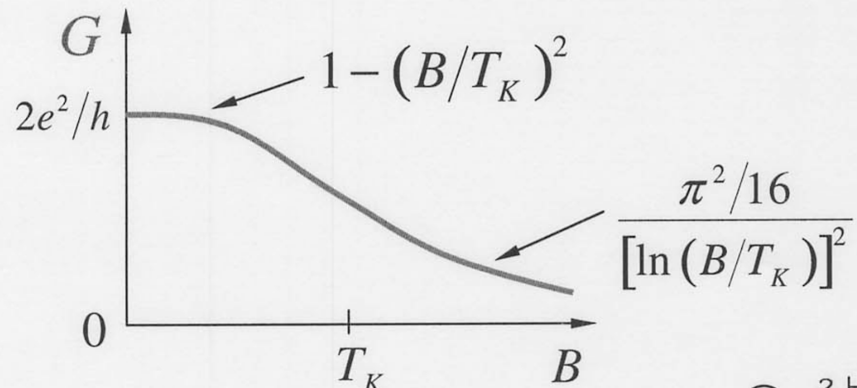



Zeeman energy:

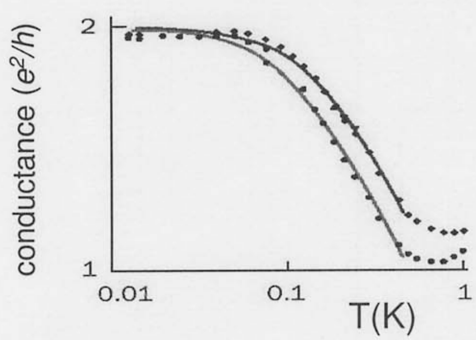
$$E_{|\uparrow\rangle} - E_{|\downarrow\rangle} = B = g\mu_B H$$

$$N_\uparrow \neq N_\downarrow$$

Controlling parameter: $B/T_K$




Thermal fluctuations have similar effect:



# Effect of a bias

$$G_L \ll G_R$$

$$\frac{dI}{dV} = G_0 \frac{1}{2} \sum_s \left[-\pi\nu \operatorname{Im} T_s(eV)\right]$$

$$\frac{1}{G_0}\frac{dI}{dV} = \begin{cases} 1 - \frac{3}{2}\left(\frac{eV}{T_K}\right)^2, \\ \frac{3\pi^2/16}{\ln^2(eV/T_K)}, \end{cases}$$

<u>Peak splits in the presence of magnetic field</u>

Born approximation (elastic and inelastic co-tunneling):
$\frac{dI}{dV} \propto a + b\theta(|eV| - g\mu B)$, $g\mu B$ is the spin-flip energy

Next order:

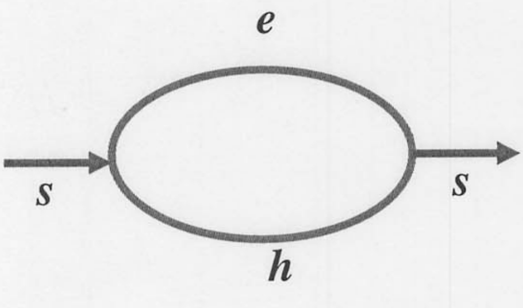
$$\frac{dI}{dV} \propto \ln\left(\frac{g\mu B}{max(T, ||eV| - g\mu B|)}\right) / \ln^3\left(\frac{g\mu B}{T_K}\right)$$

# Effect of a bias and magnetic field

*T=0*: the log-divergence is cut-off by the Korringa relaxation ra

$$\Gamma_K(B) = 2\pi(J\nu)^2 \int d\epsilon_1 \int d\epsilon_2 n(\epsilon_1)$$

$$\times[1 - n(\epsilon_2)]\delta(\epsilon_1 + g\mu B - \epsilon_2)$$

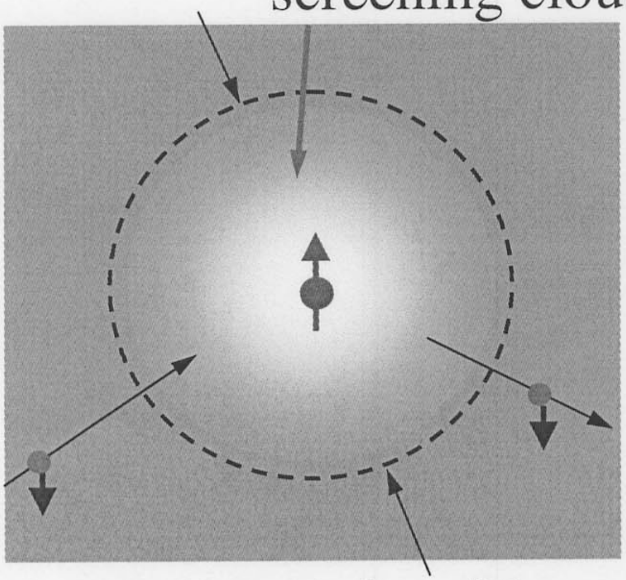



$$\Gamma_K(B) \sim \frac{B}{\ln^2(B/T_K)}$$

The split peaks are **perturbative** even at *T=0*:

$$\frac{G(eV = \pm g\mu B)}{G(V=0)} \sim \frac{\ln[\ln^2(g\mu B/T_K)]}{\ln(g\mu B/T_K)}$$

# Inelastic Scattering at Low Energies




Nozieres (1974)

$$H_{\text{fixed point}} = \sum_{ks} \varepsilon_k \varphi^\dagger_{ks} \varphi^\dagger_{ks} - \underline{\sum_{kk's} \frac{\varepsilon_k + \varepsilon_{k'}}{2\pi\nu T_K} \varphi^\dagger_{ks} \varphi^\dagger_{k's}}$$

$$+ \frac{1}{\pi\nu^2 T_K} \underline{\sum_{kk'pp'} : \varphi^\dagger_{k\uparrow} \varphi_{k'\uparrow} \varphi^\dagger_{p\downarrow} \varphi_{p'\downarrow} :}$$

$\varepsilon \ll T_K$

$$\sigma_{\text{inel}}(\varepsilon) = \sigma_{\text{tot}}(\varepsilon) - \sigma_{\text{el}}(\varepsilon) \sim \lambda_F^2 \left( \frac{\varepsilon}{T_K} \right)^2$$

$$\sigma_{\text{inel}}(\varepsilon \sim T_K) \sim \sigma_{\text{el}}(\varepsilon \sim T_K) \sim \lambda_F^2$$

# Inelastic scattering: perturbation theory

1. Simplest inelastic process in a toy model

$$\mathcal{H} = \mathcal{H}_0 + \hat{V}_{\text{toy}} \qquad\qquad \mathcal{H}_0 = \sum_{\mathbf{p}\sigma} \xi_{\mathbf{p}} c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma}$$

$$\hat{V}_{\text{toy}} = J_0 \sum_{\mathbf{p}_1\mathbf{p}_2} \left( S^+\sigma^- c^\dagger_{\mathbf{p}_2\downarrow} c_{\mathbf{p}_1\uparrow} + S^-\sigma^+ c^\dagger_{\mathbf{p}_2\uparrow} c_{\mathbf{p}_1\downarrow} \right)$$

| el.1 | el.2 | imp |
|---|---|---|

only two electrons in the band, $\Psi_{\text{in}} = |\mathbf{p}'\uparrow,\, \mathbf{p}\downarrow,\, \uparrow\rangle$

$$\Psi_{\text{out}} \equiv \hat{T}\Psi_{\text{in}} \qquad \text{T-matrix:} \quad \hat{T} = \underbrace{\hat{V}}_{\boxed{\text{Born}}} + \underbrace{\hat{V}\frac{1}{\varepsilon - \mathcal{H}_0}\hat{V}}_{\boxed{\text{2nd order}}} + \dots$$

$$\Psi^{(2)}_{\text{out}} \propto J_0^2 \times$$

$$\sum_{\mathbf{p}_1\mathbf{p}_2} S^+\sigma^- c^\dagger_{\mathbf{p}_1\downarrow} c_{\mathbf{p}_2\uparrow} \frac{1}{\varepsilon - \mathcal{H}_0} \sum_{\mathbf{p}_3\mathbf{p}_4} S^-\sigma^+ c^\dagger_{\mathbf{p}_3\uparrow} c_{\mathbf{p}_4\downarrow} |\mathbf{p}'\uparrow,\, \mathbf{p}\downarrow,\, \uparrow\rangle$$

$$= \frac{J_0^2}{\xi_{\mathbf{p}} - \xi_{\mathbf{p}_3}} |\mathbf{p}_1\downarrow,\, \mathbf{p}_3\uparrow,\, \uparrow\rangle$$

# Inelastic scattering off a magnetic impurity

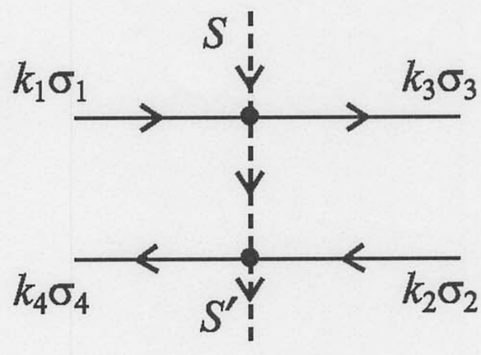
$$A^{(2)}_{\mathbf{p},\mathbf{p}'\to\mathbf{p}_1,\mathbf{p}_3} \sim \frac{J_0^2}{\xi_{\mathbf{p}} - \xi_{\mathbf{p}_3}}$$

Energy transferred in the collision: $\xi_{\mathbf{p}} - \xi_{\mathbf{p}_3} \equiv \omega$

Scattering cross-section:

$$\left| A^{(2)}_{\mathbf{p},\mathbf{p}'\to\mathbf{p}_1,\mathbf{p}_3} \right|^2 \sim \frac{J_0^4}{E^2}\delta(\xi_{\mathbf{p}} + \xi_{\mathbf{p}'} - \xi_{\mathbf{p}_1} - \xi_{\mathbf{p}_3})$$

2. Full 2nd order perturbation theory result



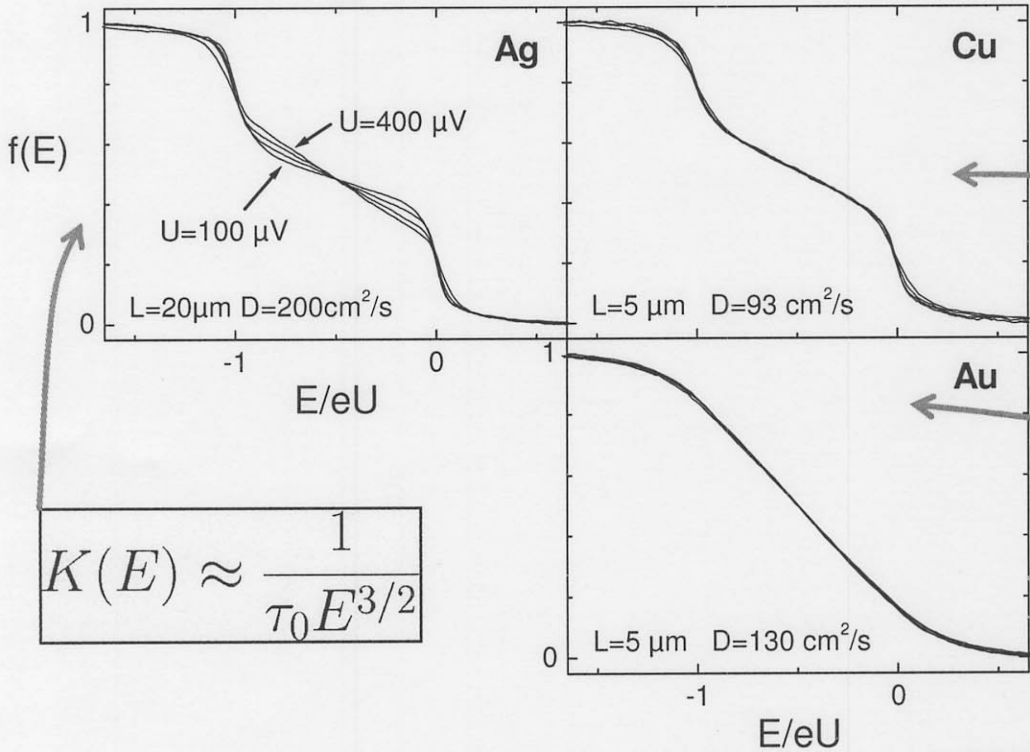

Total cross-section $\varepsilon, \varepsilon' \to \varepsilon - \omega, \varepsilon' + \omega$

averaged over $\mathbf{S}$: $\boxed{K(\omega) = \frac{\pi}{2}\frac{n_{\mathrm{s}}}{\nu}(J\nu)^4 S(S+1)\frac{1}{\omega^2}}$

$\sigma_{\mathrm{inel}}(\varepsilon \gg T_K) \sim 1/\ln^2(\varepsilon/T_K)$ $\sigma_{\mathrm{el}}(\varepsilon) = 0$

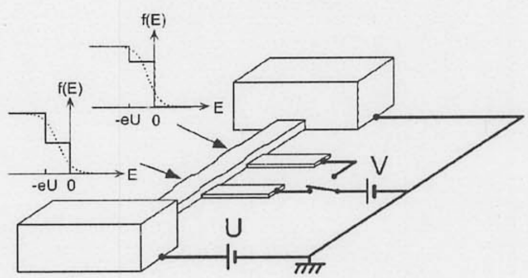
# Energy Relaxation in Ag, Cu, and Au wires

F. Pierre et al., JLTP 118, 437 (2000) and NATO Proceedings (cond-mat/0012038)




$$\boxed{K(E) \approx \frac{1}{\tau_0 E^{3/2}}}$$

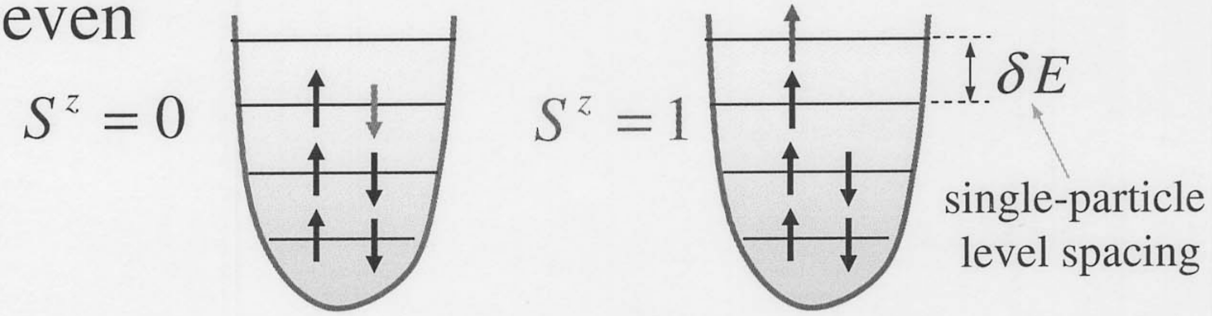
$$\boxed{K(E) \approx \frac{1}{\tau_0 E^{2}}}$$

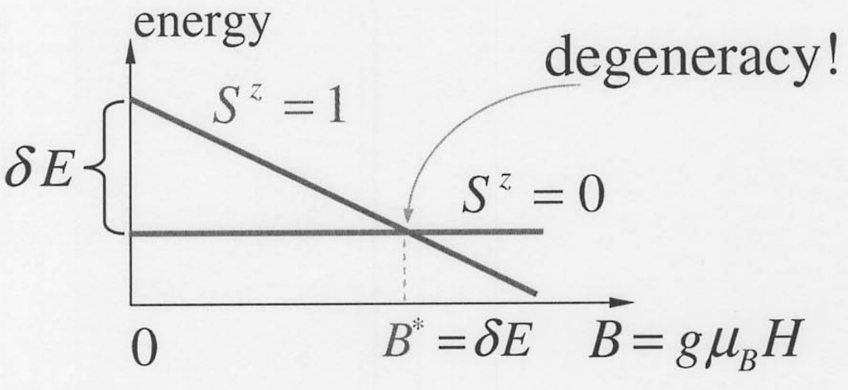


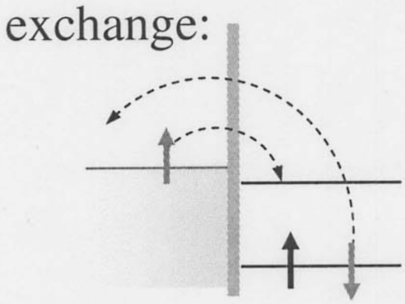
# Tuning Kondo effect by Zeeman splitting

M. Pustilnik *et. al.*, PRL **84**, 1756 (2000)

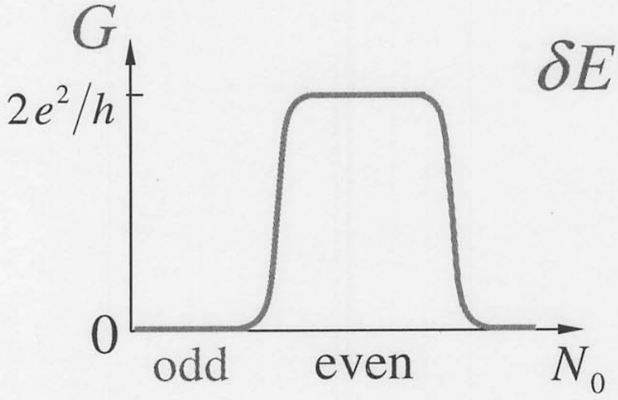
$N =$ even




Resurrection of the Kondo effect at $B = \delta E$ :

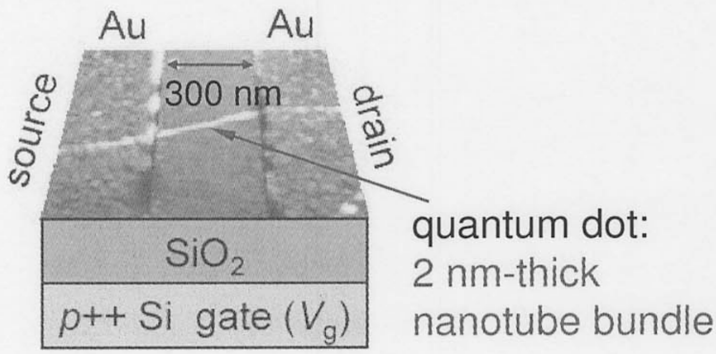


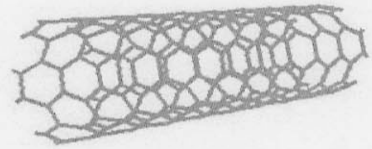

$$\delta E \sim 0.1\,meV \rightarrow H^* \sim \text{few Tesla}$$

$$T_K^{\text{even}} \sim T_K^{\text{odd}}$$

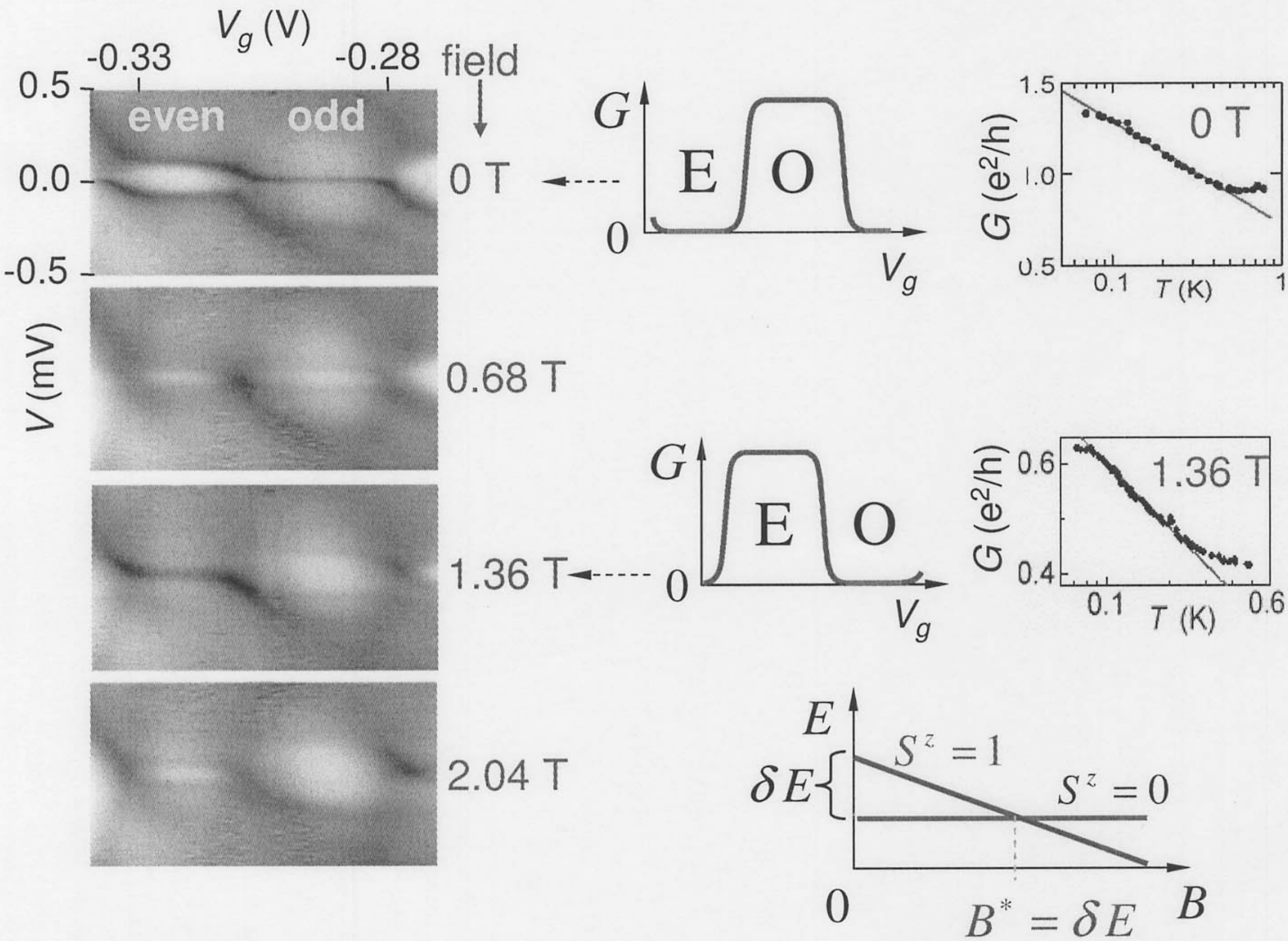
# Experimental test:
## Kondo effect in carbon nanotube

J. Nygård, D.H. Cobden, P.E. Lindelof, Nature **408**, 342 (2000)
review: M. Pustilnik *et. al.* cond-mat/0010336; LNP **579**, 3 (2001)



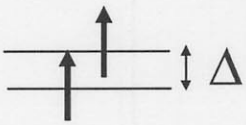
$g \approx 2$



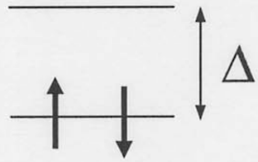
# Singlet-triplet transition

M. Pustilnik and L. Glazman, PRL **85**, 2993 (2000)

2 electrons, 2 levels:

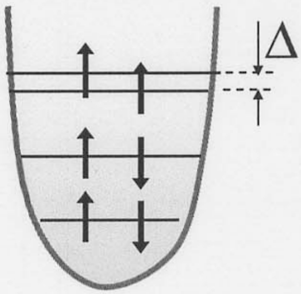



small Δ: triplet (Hund's rule)

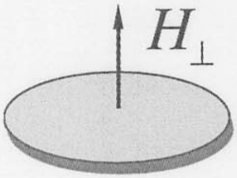



large Δ: singlet

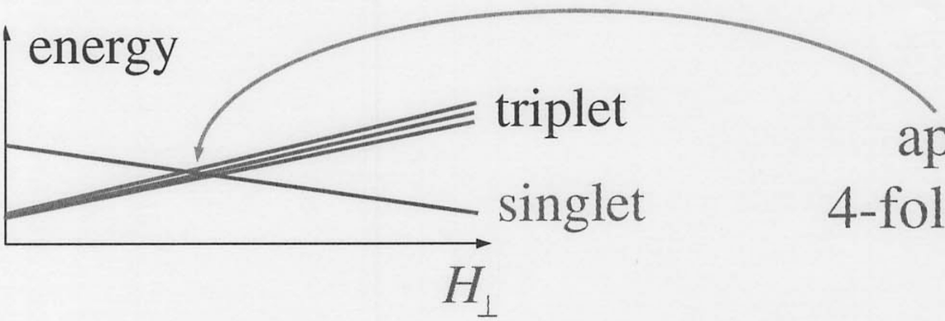
Quantum dots: Δ can be small (e.g. by accident)




← $S = 1$ (triplet)




Δ is affected by magnetic field

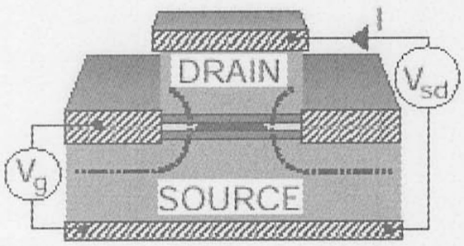


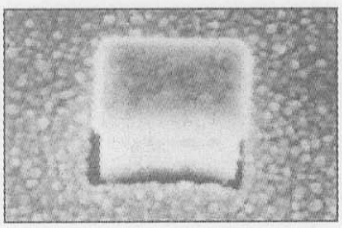

approximate
4-fold degeneracy

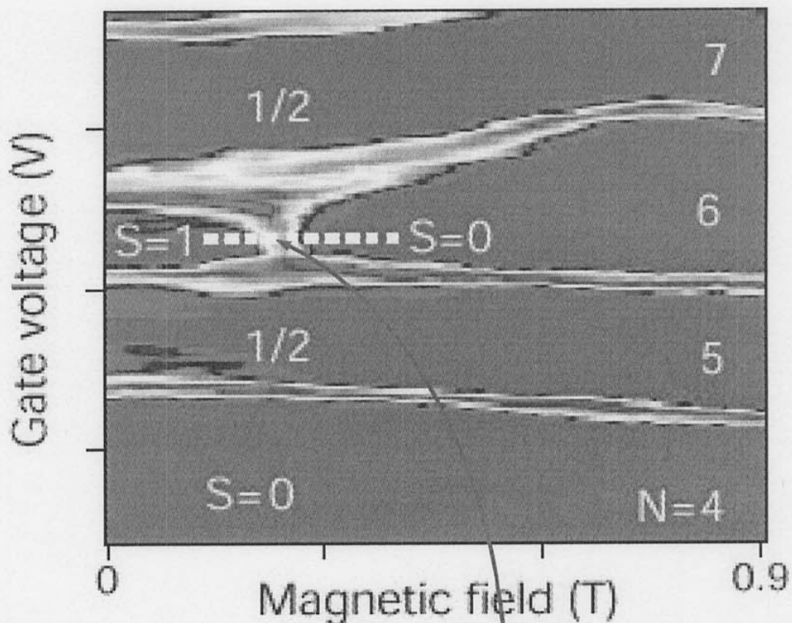
⇒ "Kondoesque" behavior

# Singlet-triplet transition in a vertical dot

S. Sasaki *et. al.*, Nature **405**, 764 (2000)
review: M. Pustilnik *et. al.* cond-mat/0010336; LNP **579**, 3 (2001)

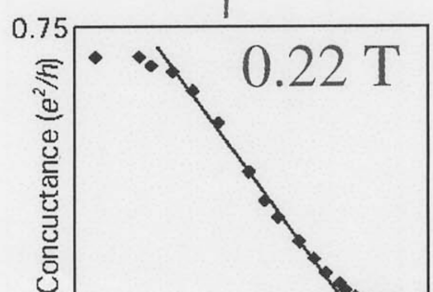


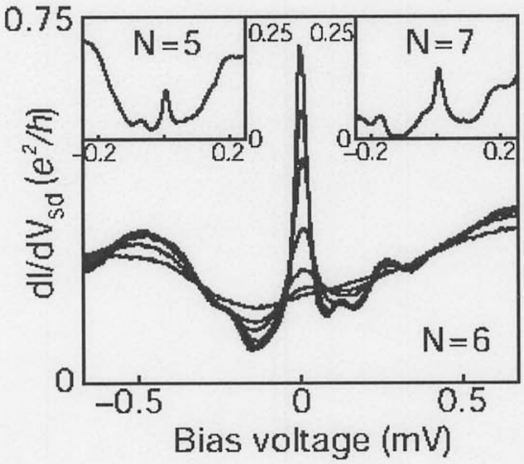



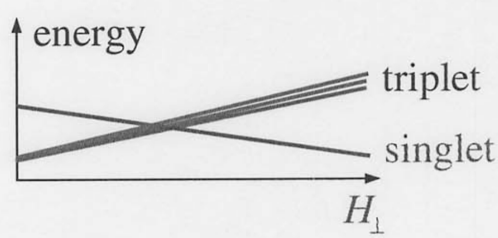



<u>GaAs:</u>

$m^* \approx 0.067\, m_e \Rightarrow$ strong orbital effect
$g \approx -0.44 \Rightarrow$ small Zeeman energy $B$



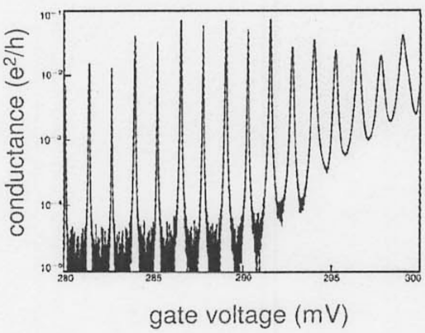



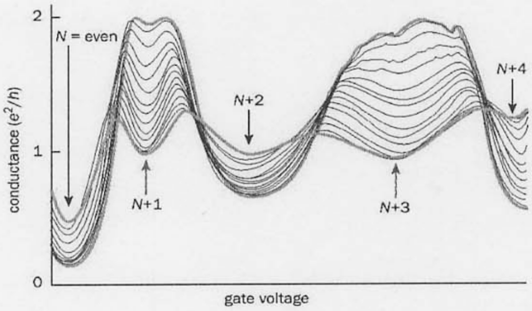



# Kondo effect in quantum dots

## Strong effect:

lifting of the Coulomb blockade at low $T$




## Ubiquity in nanostructures:

always present
- ☞ interaction
- ☞ degeneracy
- ☞ electron gas

can be tuned