

## Rotating shallow water waves

Thanks to Zoe Zhu for this dedalus3 version with a doubly periodic shallow water planet.

Suggestions of numerical experiments:

- Play with the amplitude of the gravity parameter to change the trapping length scale. What is the effect of changing this parameter on the spectrum? Hint try  $g=1$  then  $g=10$ .
- Adapt the double equator configuration to a channel configuration with impermeable walls. What are the main differences with the doubly periodic case? Interpret the localization of the different modes. You may have to change the routine for mode localization.
- Compare localization of the modes and shape of the dispersion relation for different profiles of Coriolis parameter. Try for instance  $f=\beta y$  (equatorial beta-plane) or  $f=f_0 \tanh C y$  (two flat hemispheres glued together).
- Use this notebook to discuss the spectrum of shallow water waves with varying bottom topography  $H(y)$ , for a given value of Coriolis parameter  $f$ . Start with a flat bottom  $f$ -plane configuration in a channel geometry with no normal flow boundary condition along the walls.
- If you are adventurous, try other kinds of boundary conditions along the wall (reading Iga JFM 1995 may be a useful guide). -Test how the dispersion relation changes in the presence of additional terms in the equation that break the hermiticity of the wave operator. Be careful with the boundary conditions if you change the order of the equation, and with the possible appearance of an imaginary part for the eigenvalue. Try for instance viscous terms and odd viscosity terms (if you do not know odd viscosity, this is a good opportunity to learn about it).

```
# works for dedalus3
```

```
import time
import numpy as np
import matplotlib.pyplot as plt
import dedalus.public as d3
from mpi4py import MPI
CW = MPI.COMM_WORLD
import logging
logger = logging.getLogger(__name__)
import matplotlib
%matplotlib inline
import os
```

Linear shallow water equations with dimensions: 
$$\begin{pmatrix} \partial_t u \\ \partial_x h \\ \partial_t v \\ \partial_y h \end{pmatrix} = \begin{pmatrix} -f u \\ -\partial_x (H u) \\ -f v \\ \partial_y (H v) \end{pmatrix}$$
 The only spatially varying coefficient is the Coriolis parameter  $f(y)$ . We look for solutions

$$(u, v, h) = (u, v, h) e^{-i(\omega t + k_x x)}$$

The 1D eigenvalue problem to be solved below (assuming  $H$  constant) is \begin{equation} \omega \begin{pmatrix} u \\ v \\ h \end{pmatrix} = \begin{pmatrix} 0 \\ gk\_x - if \partial\_y (H k\_x - H \partial\_y) \\ h \end{pmatrix} \begin{pmatrix} u \\ v \\ h \end{pmatrix} \end{equation}

With boundary conditions (BC) at  $y = \pm L_y/2$ . Periodic boundary conditions are implemented. You could also try impermeable BC, i.e. no-normal flow condition across the boundary:  $v=0$ , or more exotic conditions.

Note that the use of substitution in dedalus is a very convenient way to work with the second system of equations by writing just the first one, which is perhaps more familiar.

```
# Global parameters
Ny = 51 # number of points in y direction
Ly = 5*np.pi # width the domain
kmax = np.pi # horizontal wave number extrema
H=1;#
kx_global = np.linspace(-kmax,kmax,100)

ky = 0.0
H = 1#fluid layer thickness
g=1 # gravity constant
fo=1 # amplitude of Coriolis parameter
def problem_builder(kx,H,fo,g):
    # Create bases and domain
    ycoord = d3.Coordinate('y')
    dist = d3.Distributor(ycoord, dtype=np.complex128)
    ybasis = d3.Chebyshev(ycoord, size=Ny, bounds=(-Ly/2, Ly/2))
    y = dist.local_grids(ybasis) # grid

    # Fields
    u = dist.Field(name='u', bases=ybasis)
    v = dist.Field(name='v', bases=ybasis)
    h = dist.Field(name='h', bases=ybasis)
    tau_1 = dist.Field(name='tau_1')
    tau_2 = dist.Field(name='tau_2')
    omega = dist.Field(name='omega')

    # Substitution
    dy = lambda A: d3.Differentiate(A, ycoord)
    dx = lambda A: 1j*kx*A
    dt = lambda A: -1j*omega*A
    lift_basis = ybasis.derivative_basis(1)
    lift = lambda A: d3.Lift(A, lift_basis, -1)
    #lift = lambda A: d3.Lift(A, ybasis, -1)
    ky = 1

    # define non-constant coefficients
    f = dist.Field(bases=ybasis)
    f['g'] = fo*np.sin(2*np.pi*y[0]/Ly)
```

```

    problem = d3.EVP([u, v, h, tau_1, tau_2], eigenvalue=omega,
namespace=locals())

    problem.add_equation("dt(u) + g * dx(h) - f * v = 0")
    problem.add_equation("dt(v) + g * dy(h) + f * u + lift(tau_1) =
0")
    problem.add_equation("dt(h) + H * dx(u) + H * dy(v) + lift(tau_2)
= 0")
    #problem.add_equation("integ(h) = 0")
    #problem.add_equation("integ(v) = 0")

#    periodic boundary condition
    problem.add_equation("v(y='left') - v(y='right') = 0")
    problem.add_equation("u(y='left') - u(y='right') = 0")

    solver = problem.build_solver()
    return solver, ybasis, dist

#this is the function computing the spectrum for a given horizontal
wavenumber kx
def make_array_omega(kx):
    logger.info('Computing array of omega values at kx = %f' %kx)
    # Change kx parameter
    solver, ybasis, dist = problem_builder(kx,H,fo,g)
    solver.solve_dense(solver.subproblems[0],rebuild_coeffs=True)
    evals = np.sort(solver.eigenvalues)

    return evals

# compute spectrum
omega = np.array([make_array_omega(kx) for kx in kx_global])

2022-07-13 11:05:36,461 __main__ 0/1 INFO :: Computing array of omega
values at kx = -3.141593
2022-07-13 11:05:36,624 subsystems 0/1 INFO :: Building subproblem
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.1e+01/s
2022-07-13 11:05:36,673 __main__ 0/1 INFO :: Computing array of omega
values at kx = -3.078126
2022-07-13 11:05:36,761 subsystems 0/1 INFO :: Building subproblem
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s
2022-07-13 11:05:36,803 __main__ 0/1 INFO :: Computing array of omega
values at kx = -3.014660
2022-07-13 11:05:36,893 subsystems 0/1 INFO :: Building subproblem
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s
2022-07-13 11:05:36,934 __main__ 0/1 INFO :: Computing array of omega
values at kx = -2.951193
2022-07-13 11:05:37,027 subsystems 0/1 INFO :: Building subproblem
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s
2022-07-13 11:05:37,068 __main__ 0/1 INFO :: Computing array of omega

```

values at  $kx = -2.887727$   
2022-07-13 11:05:37,160 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:37,200 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -2.824260$   
2022-07-13 11:05:37,290 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:37,332 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -2.760794$   
2022-07-13 11:05:37,422 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:37,463 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -2.697327$   
2022-07-13 11:05:37,553 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s  
2022-07-13 11:05:37,593 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -2.633861$   
2022-07-13 11:05:37,684 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:37,725 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -2.570394$   
2022-07-13 11:05:37,814 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s  
2022-07-13 11:05:37,855 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -2.506927$   
2022-07-13 11:05:37,948 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:37,989 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -2.443461$   
2022-07-13 11:05:38,081 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:38,122 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -2.379994$   
2022-07-13 11:05:38,212 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:38,254 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -2.316528$   
2022-07-13 11:05:38,347 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:38,388 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -2.253061$   
2022-07-13 11:05:38,478 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:38,519 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -2.189595$   
2022-07-13 11:05:38,609 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:38,650 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -2.126128$   
2022-07-13 11:05:38,743 subsystems 0/1 INFO :: Building subproblem

matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:38,785 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -2.062662$   
2022-07-13 11:05:38,877 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:38,918 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -1.999195$   
2022-07-13 11:05:39,008 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:39,049 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -1.935729$   
2022-07-13 11:05:39,138 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s  
2022-07-13 11:05:39,179 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -1.872262$   
2022-07-13 11:05:39,270 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:39,311 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -1.808796$   
2022-07-13 11:05:39,401 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:39,443 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -1.745329$   
2022-07-13 11:05:39,534 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:39,575 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -1.681863$   
2022-07-13 11:05:39,664 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s  
2022-07-13 11:05:39,705 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -1.618396$   
2022-07-13 11:05:39,796 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:39,838 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -1.554930$   
2022-07-13 11:05:39,929 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:39,970 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -1.491463$   
2022-07-13 11:05:40,061 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:40,102 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -1.427997$   
2022-07-13 11:05:40,195 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:40,236 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -1.364530$   
2022-07-13 11:05:40,327 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:40,368 \_\_main\_\_ 0/1 INFO :: Computing array of omega

values at  $kx = -1.301064$   
2022-07-13 11:05:40,459 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:40,502 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -1.237597$   
2022-07-13 11:05:40,596 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:40,638 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -1.174131$   
2022-07-13 11:05:40,732 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:40,773 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -1.110664$   
2022-07-13 11:05:40,869 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:40,911 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -1.047198$   
2022-07-13 11:05:41,007 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:41,049 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -0.983731$   
2022-07-13 11:05:41,145 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:41,186 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -0.920265$   
2022-07-13 11:05:41,278 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:41,321 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -0.856798$   
  
2022-07-13 11:05:41,418 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:41,461 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -0.793331$   
2022-07-13 11:05:41,557 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:41,599 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -0.729865$   
2022-07-13 11:05:41,701 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.1e+01/s  
2022-07-13 11:05:41,743 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -0.666398$   
2022-07-13 11:05:41,837 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:41,879 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -0.602932$   
2022-07-13 11:05:41,973 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:42,014 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -0.539465$

2022-07-13 11:05:42,104 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s  
2022-07-13 11:05:42,145 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = -0.475999$   
2022-07-13 11:05:42,240 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:42,281 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = -0.412532$   
2022-07-13 11:05:42,380 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.1e+01/s  
2022-07-13 11:05:42,422 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = -0.349066$   
2022-07-13 11:05:42,516 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:42,558 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = -0.285599$   
2022-07-13 11:05:42,653 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:42,695 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = -0.222133$   
2022-07-13 11:05:42,789 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:42,831 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = -0.158666$   
2022-07-13 11:05:42,928 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:42,969 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = -0.095200$   
2022-07-13 11:05:43,065 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:43,107 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = -0.031733$   
2022-07-13 11:05:43,200 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:43,240 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 0.031733$   
2022-07-13 11:05:43,335 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:43,375 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 0.095200$   
2022-07-13 11:05:43,471 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:43,512 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 0.158666$   
2022-07-13 11:05:43,613 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.1e+01/s  
2022-07-13 11:05:43,655 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 0.222133$   
2022-07-13 11:05:43,749 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s

2022-07-13 11:05:43,791 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 0.285599$   
2022-07-13 11:05:43,890 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.1e+01/s  
2022-07-13 11:05:43,932 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 0.349066$   
2022-07-13 11:05:44,030 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:44,071 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 0.412532$   
2022-07-13 11:05:44,168 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:44,210 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 0.475999$   
2022-07-13 11:05:44,306 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:44,348 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 0.539465$   
2022-07-13 11:05:44,446 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:44,487 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 0.602932$   
2022-07-13 11:05:44,583 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:44,625 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 0.666398$   
2022-07-13 11:05:44,720 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:44,763 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 0.729865$   
2022-07-13 11:05:44,862 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.1e+01/s  
2022-07-13 11:05:44,904 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 0.793331$   
2022-07-13 11:05:45,003 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.1e+01/s  
2022-07-13 11:05:45,044 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 0.856798$   
2022-07-13 11:05:45,143 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.1e+01/s  
2022-07-13 11:05:45,184 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 0.920265$   
2022-07-13 11:05:45,280 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:45,322 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 0.983731$   
2022-07-13 11:05:45,420 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:45,462 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 1.047198$



2022-07-13 11:05:45,559 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:45,601 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 1.110664$   
2022-07-13 11:05:45,699 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.1e+01/s  
2022-07-13 11:05:45,741 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 1.174131$   
2022-07-13 11:05:45,835 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:45,876 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 1.237597$   
2022-07-13 11:05:45,971 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:46,014 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 1.301064$   
2022-07-13 11:05:46,113 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.1e+01/s  
2022-07-13 11:05:46,156 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 1.364530$   
2022-07-13 11:05:46,254 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:46,296 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 1.427997$   
2022-07-13 11:05:46,393 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
  
2022-07-13 11:05:46,435 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 1.491463$   
2022-07-13 11:05:46,529 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:46,570 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 1.554930$   
2022-07-13 11:05:46,668 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:46,710 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 1.618396$   
2022-07-13 11:05:46,839 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:46,880 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 1.681863$   
2022-07-13 11:05:46,977 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:47,019 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 1.745329$   
2022-07-13 11:05:47,120 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.1e+01/s  
2022-07-13 11:05:47,162 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 1.808796$   
2022-07-13 11:05:47,258 subsystems 0/1 INFO :: Building subproblem

matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:47,300 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = 1.872262$   
2022-07-13 11:05:47,394 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:47,436 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = 1.935729$   
2022-07-13 11:05:47,533 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:47,575 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = 1.999195$   
2022-07-13 11:05:47,669 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:47,710 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = 2.062662$   
2022-07-13 11:05:47,805 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:47,846 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = 2.126128$   
2022-07-13 11:05:47,942 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:47,984 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = 2.189595$   
2022-07-13 11:05:48,079 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:48,121 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = 2.253061$   
2022-07-13 11:05:48,221 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.1e+01/s  
2022-07-13 11:05:48,263 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = 2.316528$   
2022-07-13 11:05:48,359 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:48,401 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = 2.379994$   
2022-07-13 11:05:48,497 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:48,538 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = 2.443461$   
2022-07-13 11:05:48,634 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:48,675 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = 2.506927$   
2022-07-13 11:05:48,769 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:48,810 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = 2.570394$   
2022-07-13 11:05:48,904 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:48,945 \_\_main\_\_ 0/1 INFO :: Computing array of omega

```
values at kx = 2.633861
2022-07-13 11:05:49,041 subsystems 0/1 INFO :: Building subproblem
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s
2022-07-13 11:05:49,084 __main__ 0/1 INFO :: Computing array of omega
values at kx = 2.697327
2022-07-13 11:05:49,177 subsystems 0/1 INFO :: Building subproblem
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s
2022-07-13 11:05:49,219 __main__ 0/1 INFO :: Computing array of omega
values at kx = 2.760794
2022-07-13 11:05:49,312 subsystems 0/1 INFO :: Building subproblem
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s
2022-07-13 11:05:49,355 __main__ 0/1 INFO :: Computing array of omega
values at kx = 2.824260
2022-07-13 11:05:49,448 subsystems 0/1 INFO :: Building subproblem
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s
2022-07-13 11:05:49,490 __main__ 0/1 INFO :: Computing array of omega
values at kx = 2.887727
2022-07-13 11:05:49,584 subsystems 0/1 INFO :: Building subproblem
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s
2022-07-13 11:05:49,626 __main__ 0/1 INFO :: Computing array of omega
values at kx = 2.951193
2022-07-13 11:05:49,721 subsystems 0/1 INFO :: Building subproblem
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s
2022-07-13 11:05:49,761 __main__ 0/1 INFO :: Computing array of omega
values at kx = 3.014660
2022-07-13 11:05:49,854 subsystems 0/1 INFO :: Building subproblem
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s
2022-07-13 11:05:49,895 __main__ 0/1 INFO :: Computing array of omega
values at kx = 3.078126
2022-07-13 11:05:49,987 subsystems 0/1 INFO :: Building subproblem
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s
2022-07-13 11:05:50,028 __main__ 0/1 INFO :: Computing array of omega
values at kx = 3.141593
2022-07-13 11:05:50,118 subsystems 0/1 INFO :: Building subproblem
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s
```

```
# compute the localization
```

```
def make_loc(kx):
```

```
    logger.info('Computing array of omega values at kx = %f' %kx)
```

```
    # Change kx parameter
```

```
    solver, ybasis, dist = problem_builder(kx,H,fo,g)
```

```
    y = ybasis.local_grids()[0]
```

```
    solver.solve_dense(solver.subproblems[0],rebuild_coeffs=True)
```

```
    order = np.argsort(solver.eigenvalues.real)
```

```
    evals = np.sort(solver.eigenvalues)
```

```
    solver.eigenvalues = evals
```

```
    solver.eigenvectors = solver.eigenvectors[:, order]
```

```
# define new fields for localization based on local energy
```

```

(kinetic plus potential)
    loc=np.zeros(order.size)
    for iorder in np.arange(order.size):
        solver.set_state(iorder,solver.subsystems[0])
        umode = solver.state[0]['g'] # find the first field
        vmode = solver.state[1]['g']
        hmode = solver.state[2]['g']
        norm = umode * umode.conjugate() + vmode * vmode.conjugate() +
g * hmode * hmode.conjugate()
        normf = dist.Field(name='normf', bases=ybasis)
        normf['g'] = norm.real
        normy = np.cos(2*np.pi*dist.local_grids(ybasis)[0]/Ly) * norm
        normyf = dist.Field(name='normyf', bases=ybasis)
        normyf['g'] = normy.real
        loc[iorder]= d3.Integrate(normyf).evaluate()['g']
[0].real/d3.Integrate(normf).evaluate()['g'][0].real

    return loc

```

```

loc_real = np.array([make_loc(kx) for kx in kx_global])
kx_real =np.array([kx*np.ones(omega.shape[1]) for kx in kx_global])

```

```

2022-07-13 11:05:54,569 __main__ 0/1 INFO :: Computing array of omega
values at kx = -3.141593

```

```

2022-07-13 11:05:54,677 subsystems 0/1 INFO :: Building subproblem
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.1e+01/s

```

```

2022-07-13 11:05:54,817 __main__ 0/1 INFO :: Computing array of omega
values at kx = -3.078126

```

```

2022-07-13 11:05:54,909 subsystems 0/1 INFO :: Building subproblem
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s

```

```

/var/folders/0f/8fb706zw8xj7p2001s77wknr0000gp/T/

```

```

ipykernel_68360/3749403121.py:27: RuntimeWarning: invalid value
encountered in double_scalars

```

```

    loc[iorder]=
d3.Integrate(normyf).evaluate()['g'][0].real/d3.Integrate(normf).evalu
ate()['g'][0].real

```

```

2022-07-13 11:05:55,046 __main__ 0/1 INFO :: Computing array of omega
values at kx = -3.014660

```

```

2022-07-13 11:05:55,138 subsystems 0/1 INFO :: Building subproblem
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s

```

```

2022-07-13 11:05:55,274 __main__ 0/1 INFO :: Computing array of omega
values at kx = -2.951193

```

```

2022-07-13 11:05:55,366 subsystems 0/1 INFO :: Building subproblem
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s

```

```

2022-07-13 11:05:55,500 __main__ 0/1 INFO :: Computing array of omega
values at kx = -2.887727

```

```

2022-07-13 11:05:55,590 subsystems 0/1 INFO :: Building subproblem
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s

```

```

2022-07-13 11:05:55,727 __main__ 0/1 INFO :: Computing array of omega

```

values at  $kx = -2.824260$   
2022-07-13 11:05:55,819 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:55,958 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -2.760794$   
2022-07-13 11:05:56,053 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:56,190 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -2.697327$   
2022-07-13 11:05:56,282 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:56,418 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -2.633861$   
2022-07-13 11:05:56,509 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:56,645 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -2.570394$   
2022-07-13 11:05:56,736 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:56,872 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -2.506927$   
2022-07-13 11:05:56,963 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:57,099 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -2.443461$   
2022-07-13 11:05:57,191 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:57,327 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -2.379994$   
2022-07-13 11:05:57,419 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:57,555 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -2.316528$   
2022-07-13 11:05:57,643 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s  
2022-07-13 11:05:57,779 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -2.253061$   
2022-07-13 11:05:57,869 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s  
2022-07-13 11:05:58,004 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -2.189595$   
2022-07-13 11:05:58,095 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:58,229 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -2.126128$   
2022-07-13 11:05:58,319 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:58,454 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -2.062662$   
2022-07-13 11:05:58,545 subsystems 0/1 INFO :: Building subproblem

matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:58,679 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -1.999195$   
2022-07-13 11:05:58,770 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:58,905 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -1.935729$   
2022-07-13 11:05:58,993 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s  
2022-07-13 11:05:59,128 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -1.872262$   
2022-07-13 11:05:59,217 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s  
2022-07-13 11:05:59,352 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -1.808796$   
2022-07-13 11:05:59,443 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:59,578 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -1.745329$   
2022-07-13 11:05:59,669 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:05:59,805 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -1.681863$   
2022-07-13 11:05:59,894 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s  
2022-07-13 11:06:00,029 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -1.618396$   
2022-07-13 11:06:00,119 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:06:00,260 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -1.554930$   
2022-07-13 11:06:00,358 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:06:00,504 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -1.491463$   
2022-07-13 11:06:00,601 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:06:00,740 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -1.427997$   
2022-07-13 11:06:00,831 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:06:00,969 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -1.364530$   
2022-07-13 11:06:01,065 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:06:01,210 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = -1.301064$   
2022-07-13 11:06:01,307 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:06:01,451 \_\_main\_\_ 0/1 INFO :: Computing array of omega

values at  $kx = -1.237597$   
2022-07-13 11:06:01,550 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.1e+01/s  
2022-07-13 11:06:01,691 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = -1.174131$   
2022-07-13 11:06:01,789 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.1e+01/s  
2022-07-13 11:06:01,930 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = -1.110664$   
2022-07-13 11:06:02,024 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:06:02,171 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = -1.047198$   
2022-07-13 11:06:02,270 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:06:02,414 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = -0.983731$   
2022-07-13 11:06:02,508 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:06:02,650 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = -0.920265$   
2022-07-13 11:06:02,748 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.1e+01/s  
2022-07-13 11:06:02,895 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = -0.856798$   
2022-07-13 11:06:02,993 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:06:03,139 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = -0.793331$   
2022-07-13 11:06:03,241 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.1e+01/s  
2022-07-13 11:06:03,385 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = -0.729865$   
  
2022-07-13 11:06:03,481 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:06:03,625 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = -0.666398$   
2022-07-13 11:06:03,723 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.1e+01/s  
2022-07-13 11:06:03,861 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = -0.602932$   
2022-07-13 11:06:03,952 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:06:04,089 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = -0.539465$   
2022-07-13 11:06:04,178 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s  
2022-07-13 11:06:04,313 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = -0.475999$

2022-07-13 11:06:04,404 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:06:04,537 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = -0.412532$   
2022-07-13 11:06:04,624 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s  
2022-07-13 11:06:04,758 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = -0.349066$   
2022-07-13 11:06:04,845 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s  
2022-07-13 11:06:04,984 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = -0.285599$   
2022-07-13 11:06:05,075 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:06:05,214 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = -0.222133$   
2022-07-13 11:06:05,305 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:06:05,448 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = -0.158666$   
2022-07-13 11:06:05,541 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:06:05,678 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = -0.095200$   
2022-07-13 11:06:05,769 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:06:05,903 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = -0.031733$   
2022-07-13 11:06:05,995 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:06:06,132 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 0.031733$   
2022-07-13 11:06:06,221 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s  
2022-07-13 11:06:06,354 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 0.095200$   
2022-07-13 11:06:06,442 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s  
2022-07-13 11:06:06,576 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 0.158666$   
2022-07-13 11:06:06,664 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s  
2022-07-13 11:06:06,798 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 0.222133$   
2022-07-13 11:06:06,884 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s  
2022-07-13 11:06:07,019 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 0.285599$   
2022-07-13 11:06:07,141 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s



2022-07-13 11:06:07,274 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 0.349066$   
2022-07-13 11:06:07,362 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s  
2022-07-13 11:06:07,496 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 0.412532$   
2022-07-13 11:06:07,586 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:06:07,720 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 0.475999$   
2022-07-13 11:06:07,809 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s  
2022-07-13 11:06:07,945 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 0.539465$   
2022-07-13 11:06:08,035 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:06:08,170 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 0.602932$   
2022-07-13 11:06:08,258 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s  
2022-07-13 11:06:08,393 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 0.666398$   
2022-07-13 11:06:08,483 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:06:08,618 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 0.729865$   
2022-07-13 11:06:08,706 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s  
2022-07-13 11:06:08,842 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 0.793331$   
2022-07-13 11:06:08,932 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:06:09,067 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 0.856798$   
2022-07-13 11:06:09,156 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s  
2022-07-13 11:06:09,291 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 0.920265$   
2022-07-13 11:06:09,379 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s  
2022-07-13 11:06:09,514 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 0.983731$   
2022-07-13 11:06:09,601 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s  
2022-07-13 11:06:09,737 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 1.047198$   
2022-07-13 11:06:09,827 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:06:09,963 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 1.110664$

2022-07-13 11:06:10,054 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:06:10,189 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 1.174131$   
2022-07-13 11:06:10,278 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s  
2022-07-13 11:06:10,412 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 1.237597$   
2022-07-13 11:06:10,501 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s  
2022-07-13 11:06:10,636 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 1.301064$   
2022-07-13 11:06:10,728 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:06:10,862 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 1.364530$   
2022-07-13 11:06:10,952 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:06:11,087 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 1.427997$   
2022-07-13 11:06:11,179 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:06:11,313 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 1.491463$   
2022-07-13 11:06:11,401 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s  
2022-07-13 11:06:11,536 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 1.554930$   
2022-07-13 11:06:11,625 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s  
  
2022-07-13 11:06:11,759 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 1.618396$   
2022-07-13 11:06:11,846 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s  
2022-07-13 11:06:11,981 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 1.681863$   
2022-07-13 11:06:12,068 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s  
2022-07-13 11:06:12,202 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 1.745329$   
2022-07-13 11:06:12,290 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s  
2022-07-13 11:06:12,425 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 1.808796$   
2022-07-13 11:06:12,515 subsystems 0/1 INFO :: Building subproblem matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:06:12,651 \_\_main\_\_ 0/1 INFO :: Computing array of omega values at  $kx = 1.872262$   
2022-07-13 11:06:12,741 subsystems 0/1 INFO :: Building subproblem

matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:06:12,876 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = 1.935729$   
2022-07-13 11:06:12,968 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:06:13,101 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = 1.999195$   
2022-07-13 11:06:13,188 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s  
2022-07-13 11:06:13,322 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = 2.062662$   
2022-07-13 11:06:13,411 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s  
2022-07-13 11:06:13,546 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = 2.126128$   
2022-07-13 11:06:13,636 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:06:13,770 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = 2.189595$   
2022-07-13 11:06:13,860 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:06:13,994 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = 2.253061$   
2022-07-13 11:06:14,083 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s  
2022-07-13 11:06:14,217 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = 2.316528$   
2022-07-13 11:06:14,307 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:06:14,441 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = 2.379994$   
2022-07-13 11:06:14,532 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:06:14,667 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = 2.443461$   
2022-07-13 11:06:14,757 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s  
2022-07-13 11:06:14,891 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = 2.506927$   
2022-07-13 11:06:14,979 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s  
2022-07-13 11:06:15,113 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = 2.570394$   
2022-07-13 11:06:15,202 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s  
2022-07-13 11:06:15,336 \_\_main\_\_ 0/1 INFO :: Computing array of omega  
values at  $kx = 2.633861$   
2022-07-13 11:06:15,422 subsystems 0/1 INFO :: Building subproblem  
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s  
2022-07-13 11:06:15,557 \_\_main\_\_ 0/1 INFO :: Computing array of omega

```
values at kx = 2.697327
2022-07-13 11:06:15,647 subsystems 0/1 INFO :: Building subproblem
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s
2022-07-13 11:06:15,782 __main__ 0/1 INFO :: Computing array of omega
values at kx = 2.760794
2022-07-13 11:06:15,869 subsystems 0/1 INFO :: Building subproblem
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s
2022-07-13 11:06:16,003 __main__ 0/1 INFO :: Computing array of omega
values at kx = 2.824260
2022-07-13 11:06:16,093 subsystems 0/1 INFO :: Building subproblem
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s
2022-07-13 11:06:16,228 __main__ 0/1 INFO :: Computing array of omega
values at kx = 2.887727
2022-07-13 11:06:16,317 subsystems 0/1 INFO :: Building subproblem
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s
2022-07-13 11:06:16,451 __main__ 0/1 INFO :: Computing array of omega
values at kx = 2.951193
2022-07-13 11:06:16,540 subsystems 0/1 INFO :: Building subproblem
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s
2022-07-13 11:06:16,674 __main__ 0/1 INFO :: Computing array of omega
values at kx = 3.014660
2022-07-13 11:06:16,765 subsystems 0/1 INFO :: Building subproblem
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s
2022-07-13 11:06:16,899 __main__ 0/1 INFO :: Computing array of omega
values at kx = 3.078126
2022-07-13 11:06:16,986 subsystems 0/1 INFO :: Building subproblem
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.3e+01/s
2022-07-13 11:06:17,120 __main__ 0/1 INFO :: Computing array of omega
values at kx = 3.141593
2022-07-13 11:06:17,211 subsystems 0/1 INFO :: Building subproblem
matrices 1/1 (~100%) Elapsed: 0s, Remaining: 0s, Rate: 1.2e+01/s
```

```
# plot the spectrum
```

```
fig, ax = plt.subplots(figsize=(10, 5))
```

```
yylim = np.array([-3., 3.])
```

```
plt.yylim(yylim)
```

```
# Plot the analytical shallow water spectrum for f = 1
```

```
kx=kx_global
```

```
om = ( kx**2 + 1)**0.5
```

```
im = ax.plot(kx_global,om,color='black')
```

```
im = ax.plot(kx_global,-om,color='black')
```

```
plt.xlabel('$k_x$')
```

```
plt.ylabel('$\omega$')
```

```
# plot the real part of the spectrum
```

```
im = ax.scatter(kx_real, omega.real, c=loc_real, cmap=plt.cm.coolwarm,  
vmin=-1, vmax=1)
```

```
cbar=fig.colorbar(im, ax=ax, ticks=[-1.0,0.0,1.0])
```

```

cbar.ax.tick_params(labelsize=20)
cbar.set_label('$y^*$', size=20)

ax.set_xticks([-3, -2, -1, 0, 1, 2, 3,])
ax.set_yticks([-1.5, 0, 1.5])

for item in ([ax.title, ax.xaxis.label, ax.yaxis.label] +
             ax.get_xticklabels() + ax.get_yticklabels()):
    item.set_fontsize(20)
plt.tight_layout()
plt.xlim([np.min(kx_global), np.max(kx_global)])
(-3.141592653589793, 3.141592653589793)

```

