Quantum phases of ultracold atoms in optical lattices and magnetic microtraps

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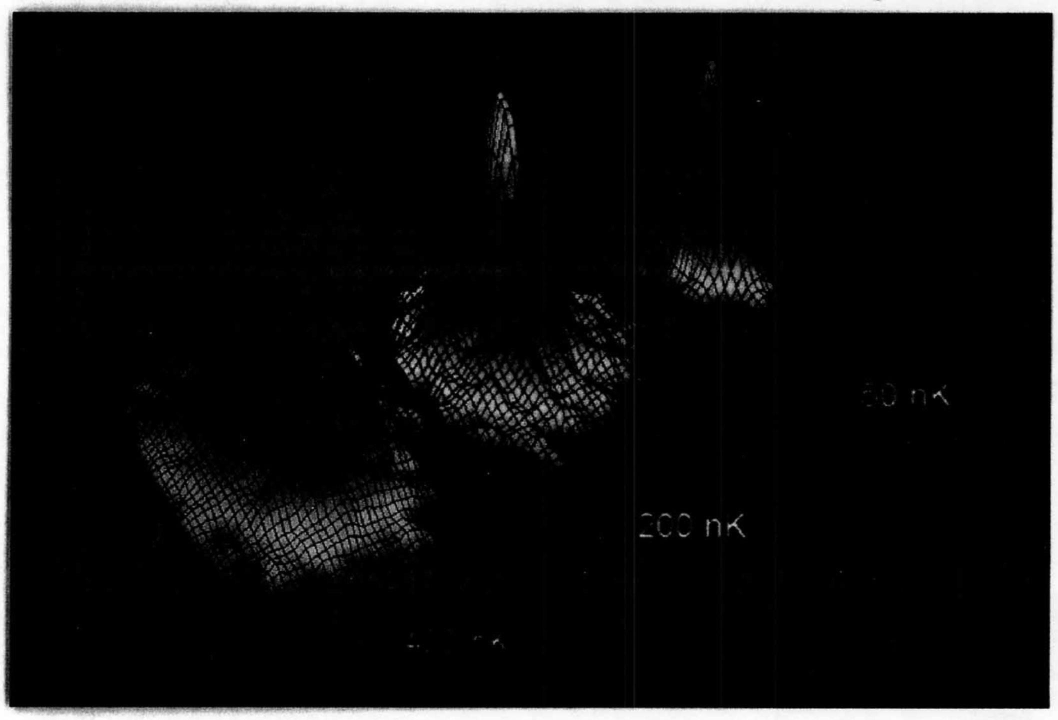
Bose-Einstein condensation of atomic gases

Anderson et.al., Science (1995)



Ultralow density condensed matter system

$$n \sim 10^{14}\ cm^{-3}$$

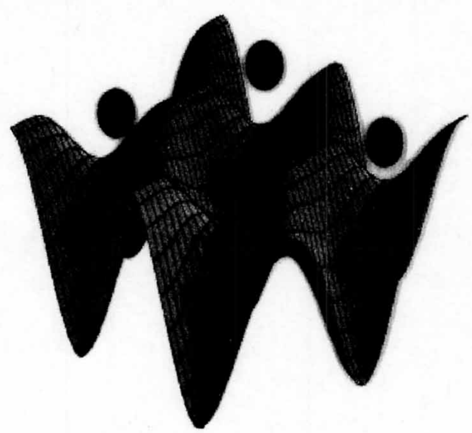
$$T_{BEC} \sim 1 \mu K$$

Interactions are weak and can be described theoretically from first principles

Strongly interacting bosons in optical lattices

C. Orzel et.al., Science (01); M.Greiner et.al., Nature (02)

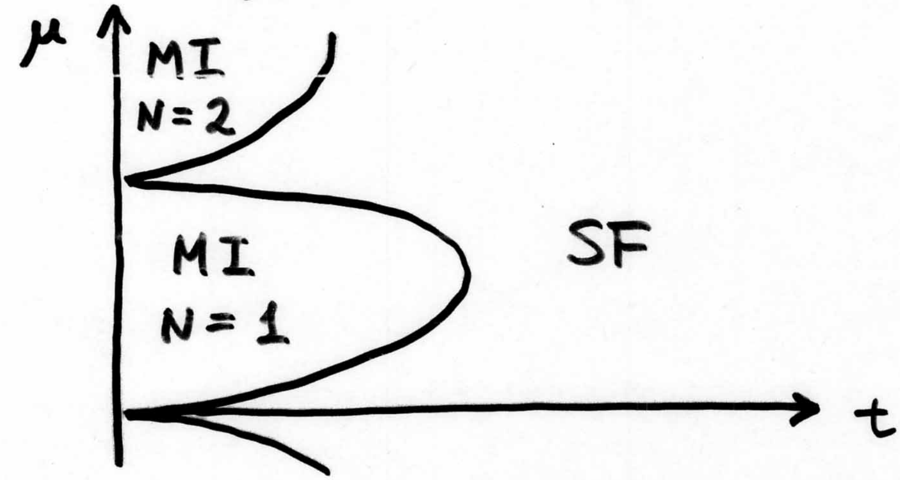
Standing wave laser fields produce a periodic potential for atoms



Bose Hubbard model

D. Jaksch et.al., PRL (98)

$$\mathcal{H} = -t \sum_{\langle ij \rangle} b_i^{+} b_j + U \sum_i n_i^2 - \mu \sum_i n_i$$

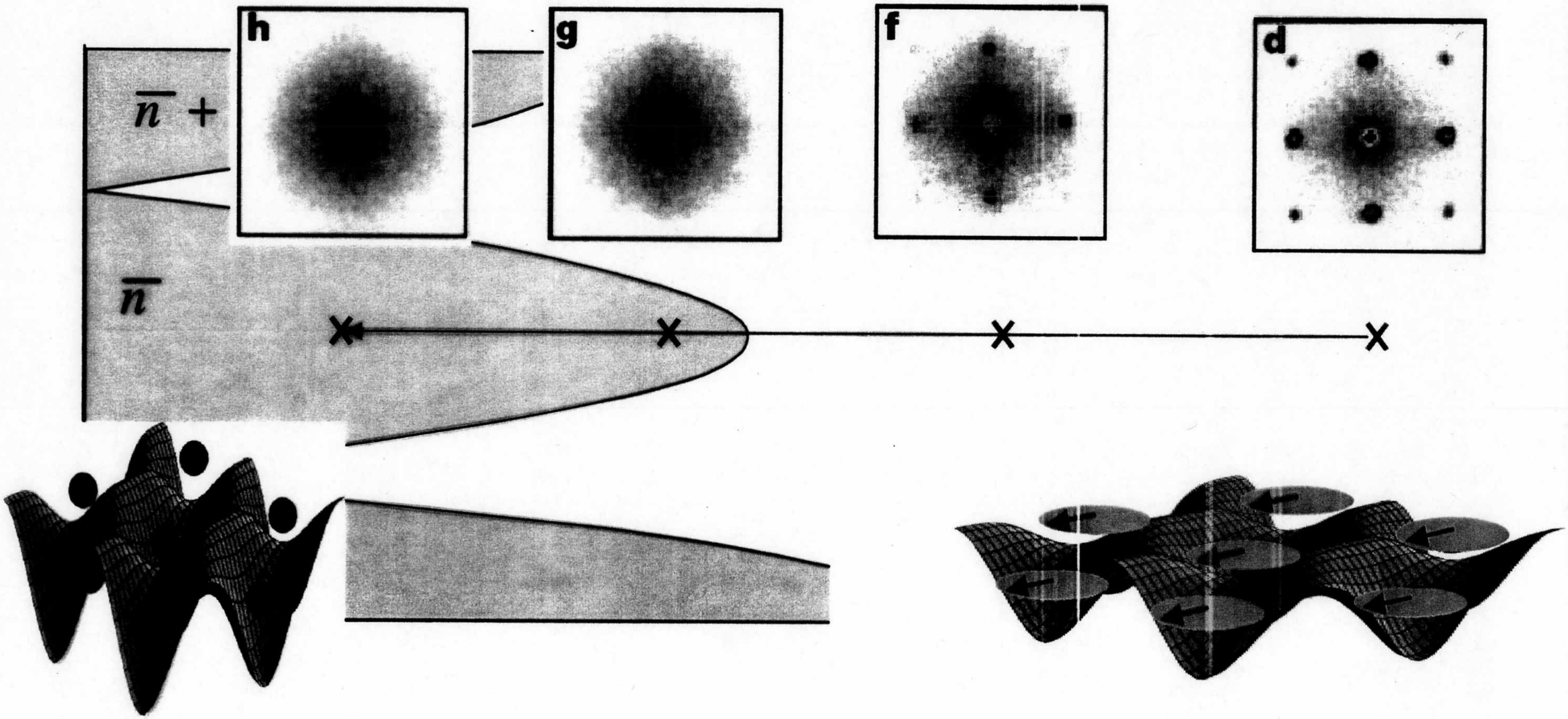


Nt >> U Superfluid phase

Nt << U Mott Insulator

Superfluid to Insulator Transition

Greiner et.al., Nature (02) following Jaksch et.al. PRL (98)



Outline

Two component Bose mixtures in optical lattices
Questions: Competition of several ordered phases.
Fractionalized phases in $d>1$ without time reversal breaking.

Spin 1 bosons in optical lattices
Questions: Exotic spin order (nematic).
Pairing in systems with repulsive interactions.

Fermions in optical lattices
Questions: Pairing of fermions with repulsive interactions. High T_c mechanism.

Boson-Fermion mixtures in 1d optical lattices
Questions: Competing orders. Polarons.

Atoms in magnetic microtraps
Questions: Interplay of disorder and interactions.
Bose glass phase.

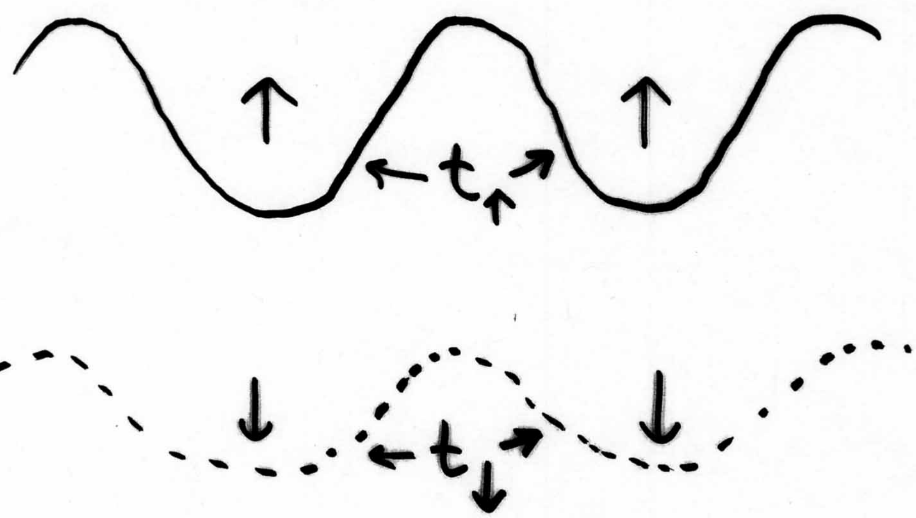
Fractional quantum Hall states of atoms in optical lattices
Questions: Charges and statistics of quasiparticles

Two component mixtures of bosonic atoms in optical lattices

Example: $\quad {}^{87}Rb \quad |\uparrow\rangle = |F=1, m_F=-1\rangle$

$|\downarrow\rangle = |F=2, m_F=-2\rangle$

Mandel et. al., Nature 2003



Two component Bose-Hubbard model

$$\mathcal{H} = -t_\uparrow \sum_{ij} a^\dagger_{i\uparrow} a_{j\uparrow} - t_\downarrow \sum a^\dagger_{i\downarrow} a_{j\downarrow}$$

$$+ U_{\uparrow\uparrow} \sum n_{i\uparrow}(n_{i\uparrow}-1) + U_{\downarrow\downarrow} \sum n_{i\downarrow}(n_{i\downarrow}-1) + U_{\uparrow\downarrow} \sum_i n_{i\uparrow} n_{i\downarrow}$$

Nature of insulating phases?

Two component bosonic mixtures in optical lattices.
Magnetic order in insulating phases

$$\mathcal{H} = -t_{\uparrow} \sum_{ij} a^{+}_{i\uparrow} a_{j\uparrow} - t_{\downarrow} \sum_{ij} a^{+}_{i\downarrow} a_{j\downarrow}$$

$$+ U_{\uparrow\uparrow} \sum_{i} n_{i\uparrow} (n_{i\uparrow} - 1) + U_{\downarrow\downarrow} \sum n_{i\downarrow} (n_{i\downarrow} - 1) + U_{\uparrow\downarrow} \sum n_{i\uparrow} n_{i\downarrow}$$

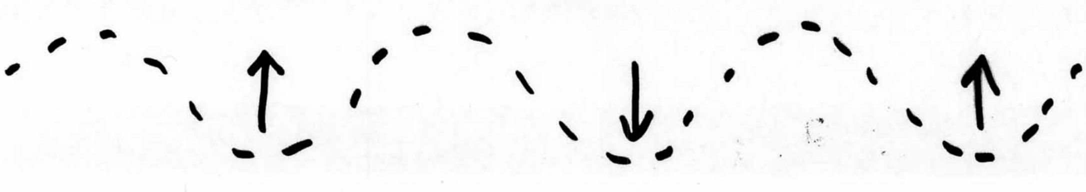
Insulating phases with $N = 1$ atom per site.
Average densities $n_{\uparrow} = n_{\downarrow} = 1/2$.

Easy plane ferromagnet

$$|\psi\rangle = \prod_{i} \left(a^{+}_{i\uparrow} + e^{i\varphi} a^{+}_{i\downarrow} \right) |0\rangle$$

Easy axis antiferromagnet

$$|\psi\rangle = \prod_{i \in A} a^{+}_{i\uparrow} \prod_{i \in B} a^{+}_{i\downarrow} |0\rangle$$



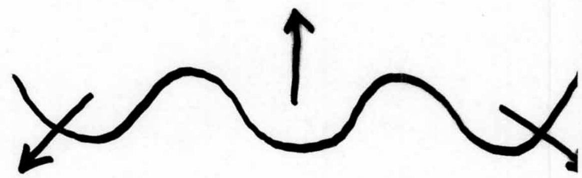
Quantum magnetism of bosons in optical lattices

- XXZ magnetic systems with tunable interactions
  Kuklov, Svistunov, PRL (03);
  Duan, Demler, Lukin, PRL (03)

$$\mathcal{H} = J_z \sum_{ij} \sigma_i^z \sigma_j^z + J_\perp \sum_{ij} (\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y)$$

$$J_z = \frac{t_\uparrow^2 + t_\downarrow^2}{2U_{\uparrow\downarrow}} - \frac{t_\uparrow^2}{U_\uparrow} - \frac{t_\downarrow^2}{U_\downarrow}$$

$$J_\perp = -\frac{t_\uparrow t_\downarrow}{U_{\uparrow\downarrow}}$$



- By changing atomic and lattice properties we can manipulate
  - sign of interactions
    ferromagnetic $U_{\uparrow\downarrow} \gg U_{\uparrow\uparrow}, U_{\downarrow\downarrow}$
    antiferromagnetic $U_{\uparrow\downarrow} \ll U_{\uparrow\uparrow}, U_{\downarrow\downarrow}$
  - anisotropy

$$|J_z / J_\perp| > 1 \quad \text{easy axis}$$

$$|J_z / J_\perp| < 1 \quad \text{easy plane}$$

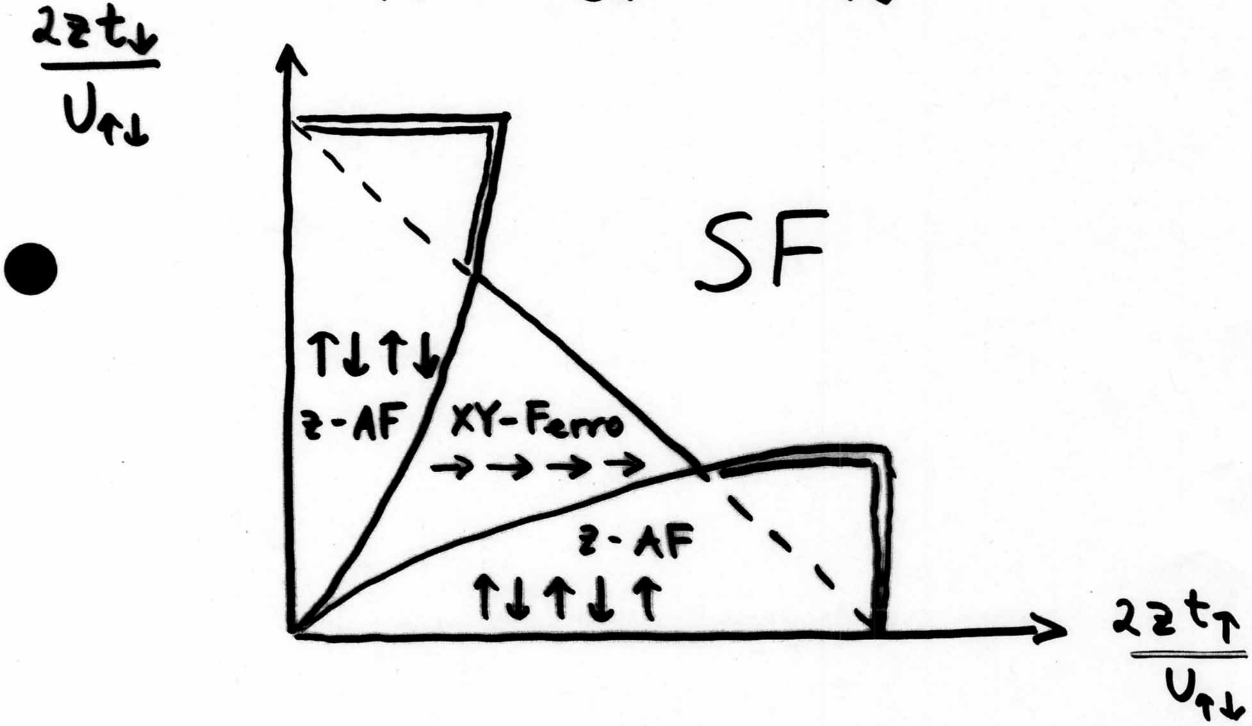
-

Two component mixture of bosonic atoms in optical lattice
Phase diagram

Altman, Hofstetter, Demler, Lukin
New J Physics (2003)

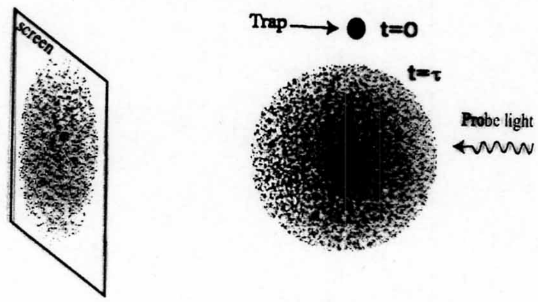
Antiferromagnetic case

$$U_{\uparrow\uparrow} = U_{\downarrow\downarrow} = 2\,U_{\uparrow\downarrow}$$



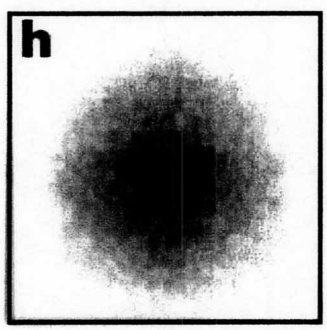
Second order coherence in the insulating state of bosons.
Hanbury - Brown - Twiss experiment for spinless bosons.

Altman, Demler, Lukin, c-m/0306226



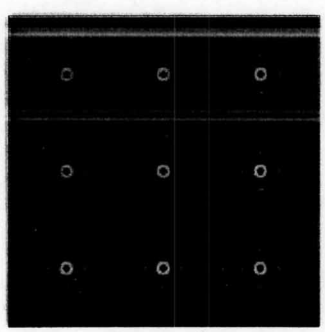
Time of flight imaging

First order coherence, $n(r)$



Second order coherence $G(r,r') = \langle n(r)\, n(r')\rangle - \langle n(r)\rangle \langle n($

$$G(r,r') = A(t) \sum_{G} \delta\left(r - r' + \frac{\hbar t}{m} G\right)$$

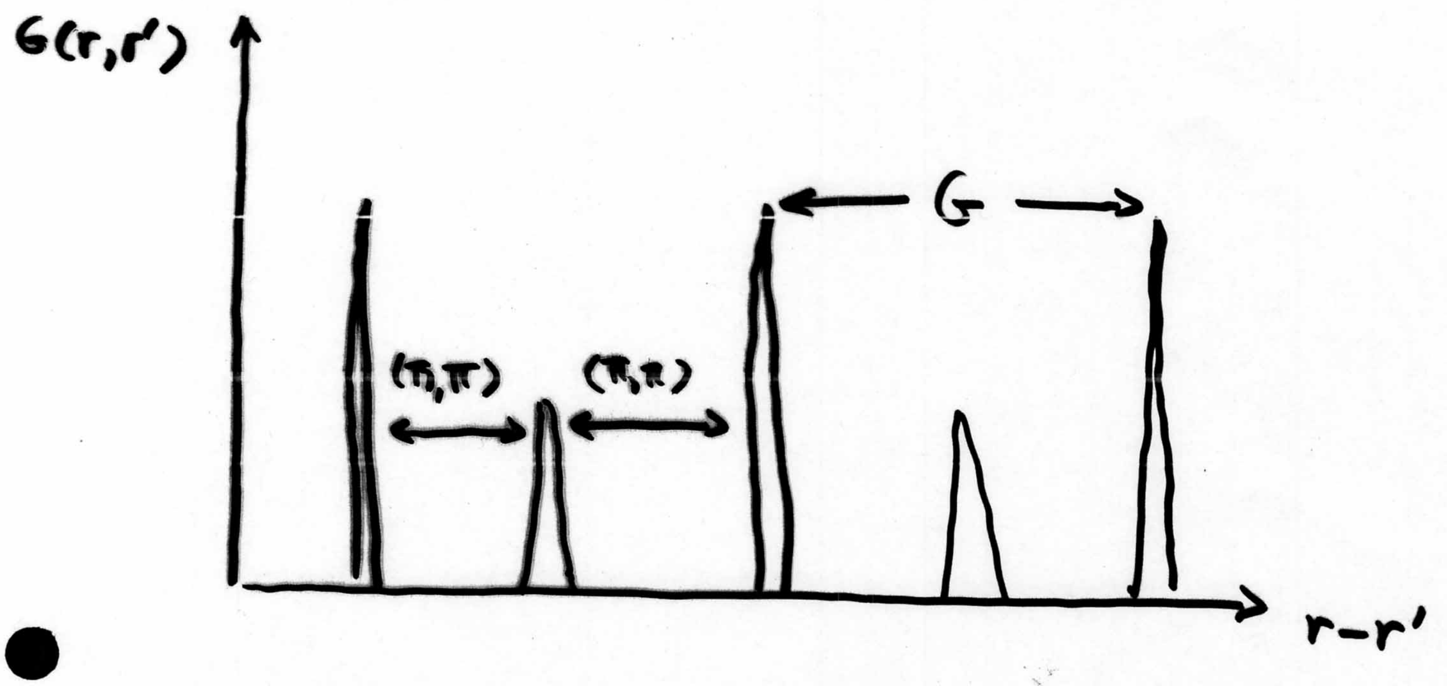


Probing spin order of bosons

Antiferromagnetic insulating state of spin-$\frac{1}{2}$ bosons

$$|\psi\rangle = \prod_{i\in A} a_{i\uparrow}^{+} \prod_{j\in B} a_{j\downarrow}^{+} |0\rangle$$

$$G(r,r') = \langle n(r)\, n(r')\rangle - \langle n(r)\rangle\langle n(r')\rangle$$

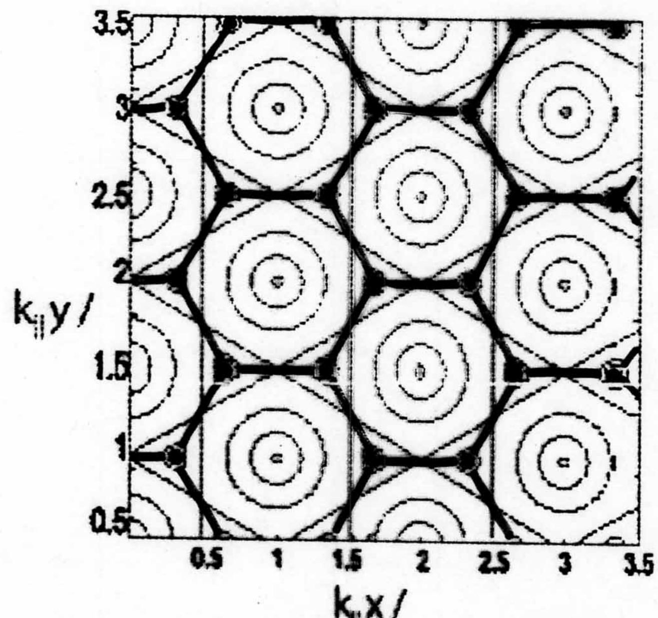


Designing exotic phases

Optical lattice in 2 or 3 dimensions: polarization and frequencies may be different for different directions

Exactly solvable lattice model on a honeycomb lattice by Kitaev

$$\mathcal{H} = J_x \sum_{ij\in x} \sigma_i^x \sigma_j^x + J_y \sum_{ij\in y} \sigma_i^y \sigma_j^y + J_z \sum_{ij\in z} \sigma_i^z \sigma_j^z$$



- Can be created with 3 sets of standing wave light beams
- Has non-trivial topological order, anyons, ...

Spin F=1 atoms in optical lattices

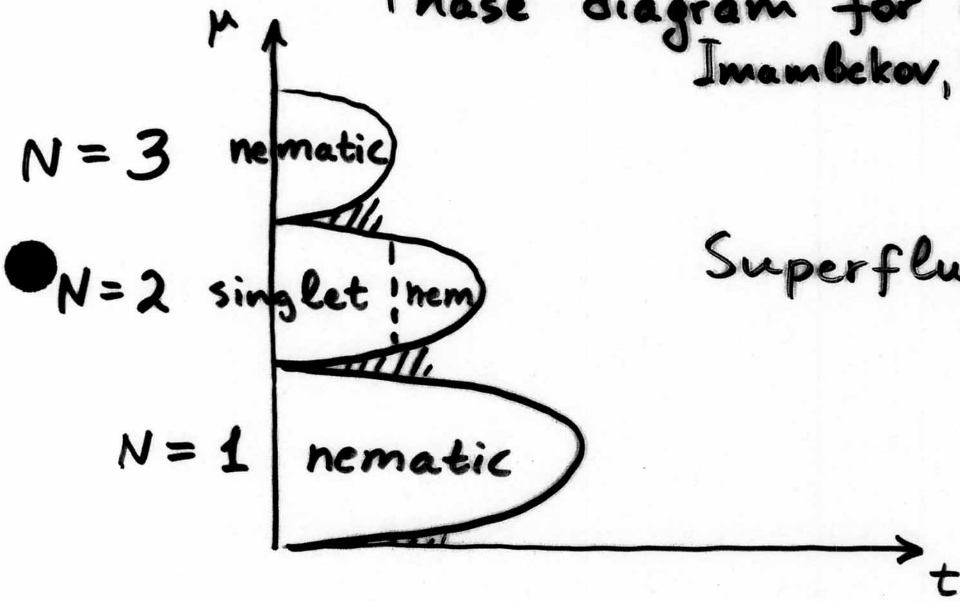
Hubbard Hamiltonian Demler, Zhou, PRL (01)

$$\mathcal{H} = -t \sum_{ijm} a_{im}^{+} a_{jm} + U_0 \sum_i n_i^2 + U_2 \sum_i \vec{S}_i^{\,2} - \mu \sum_i n_i$$

Symmetry constraints: $N_i + S_i = \text{even}$ $S_i \le N_i$

Phase diagram for $d > 1$

Imambekov, Lukin, Demler, PRA (03)



~~Insulating~~ phases

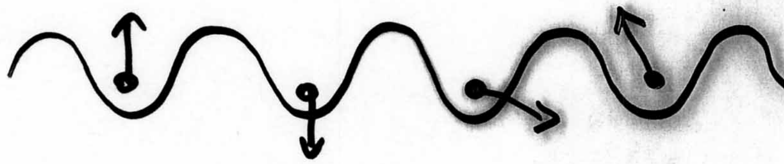
Nematic phase breaks spin rotational symmetry but not time reversal symmetry. $\langle \vec{S} \rangle = 0$ $\langle S_a S_b \rangle \neq 0$

$$|N\rangle = \prod_i (n_x a_{ix}^{+} + n_y a_{iy}^{+} + n_z a_{iz}^{+})^N |0\rangle$$

Spin singlet phase

$$|S\rangle = \prod_i (a_{ix}^{+2} + a_{iy}^{+2} + a_{iz}^{+2})^{N/2} |0\rangle$$

- Nematic insulating phase for $N=1$

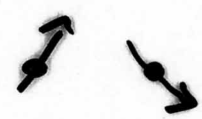


Effective $S=1$ spin model

$$\mathcal{H} = -J_1 \sum_{\langle ij \rangle} \vec{S}_i \vec{S}_j - J_2 \sum_{\langle ij \rangle} (\vec{S}_i \vec{S}_j)^2$$

$$J_1 = \frac{2t^2}{U_0+U_2} \qquad J_2 = \frac{2t^2}{3(U_0+U_2)} + \frac{4t^2}{3(U_0-2U_2)}$$

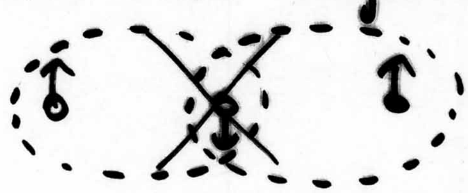
- Two site problem



S_{tot}	$\vec{S}_1\vec{S}_2$	$(\vec{S}_1\vec{S}_2)^2$
2	1	1
0	-2	4

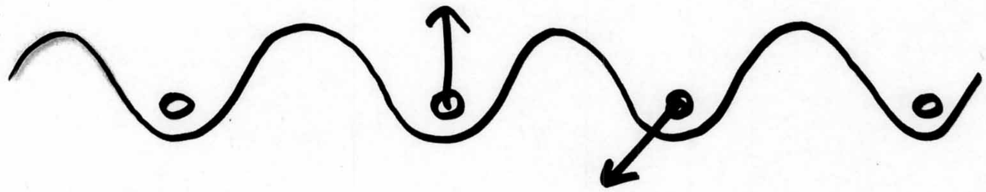
Singlet state is favored when $J_2 > J_1$

Can not have singlets on neighboring bonds



- Classical nematic state is a superposition of $S_{tot}=0$ and $S_{tot}=2$ on each bond

- Singlet and nematic insulating phases for $N=2$



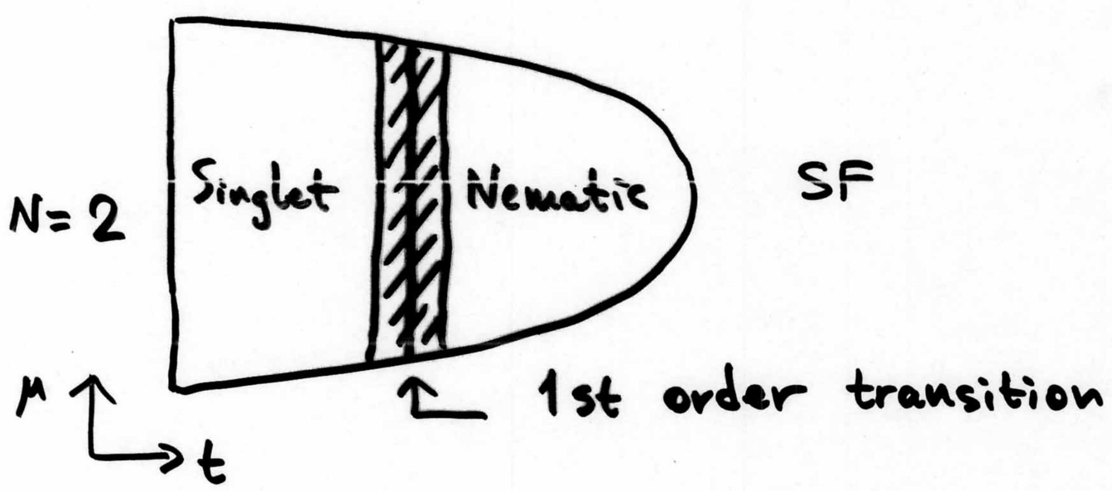
$S_i = 0$ and $S_i = 2$ states are allowed

$U_2 S_i^2$ favors $S_i = 0$

Spin exchange ($\sim t^2/U_0$) allows "scattering"

- $S_i = 0 \;\; S_j = 0 \implies S_i = 2 \;\; S_j = 2 \;\; S_i + S_j = 0$

First order singlet-nematic transition $\frac{zt^2}{U_0 U_2} \simeq \frac{1}{2}$



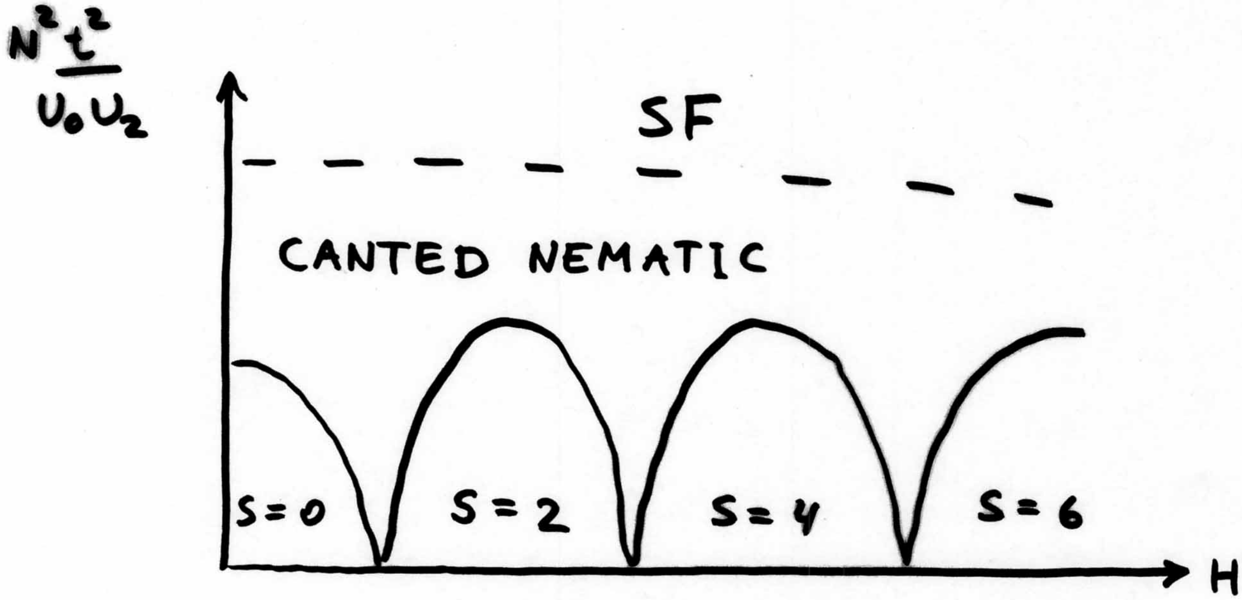
- For even filling factors S-N transition

$$\frac{zN^2t^2}{U_0 U_2} \simeq 9$$

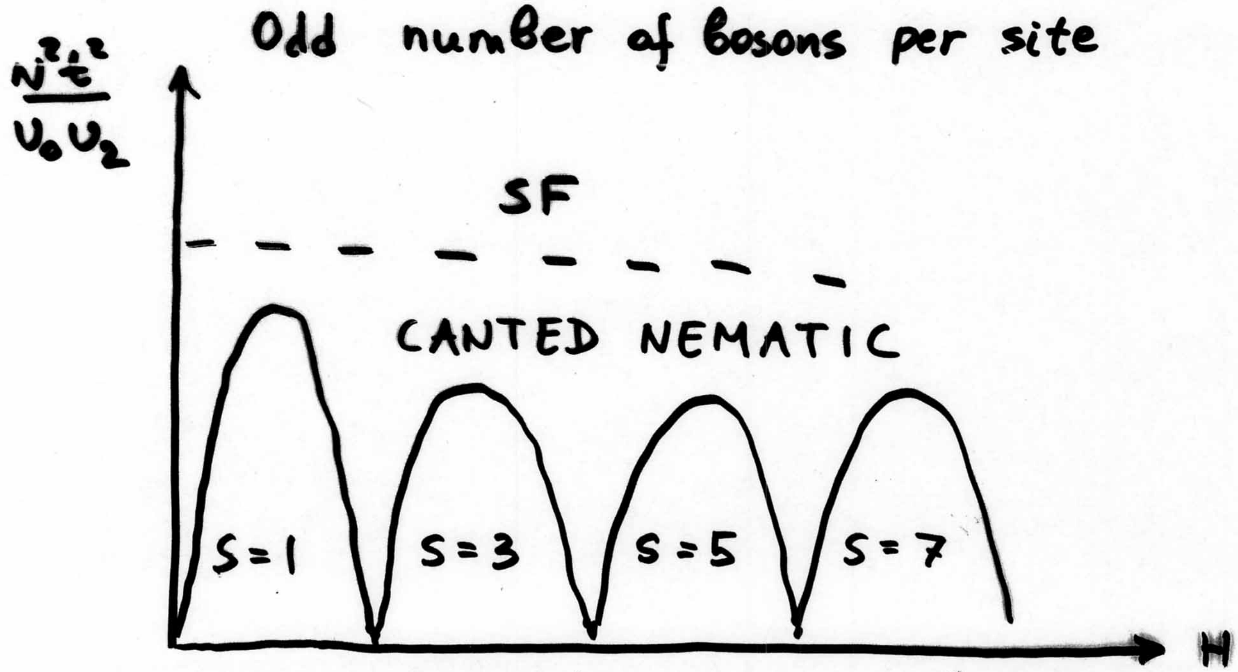
S=1 Bosons in optical lattice. Insulating phases in magnetic field

A. Imambekov, M. Lukin, E. Demler, cond-mat/0401526

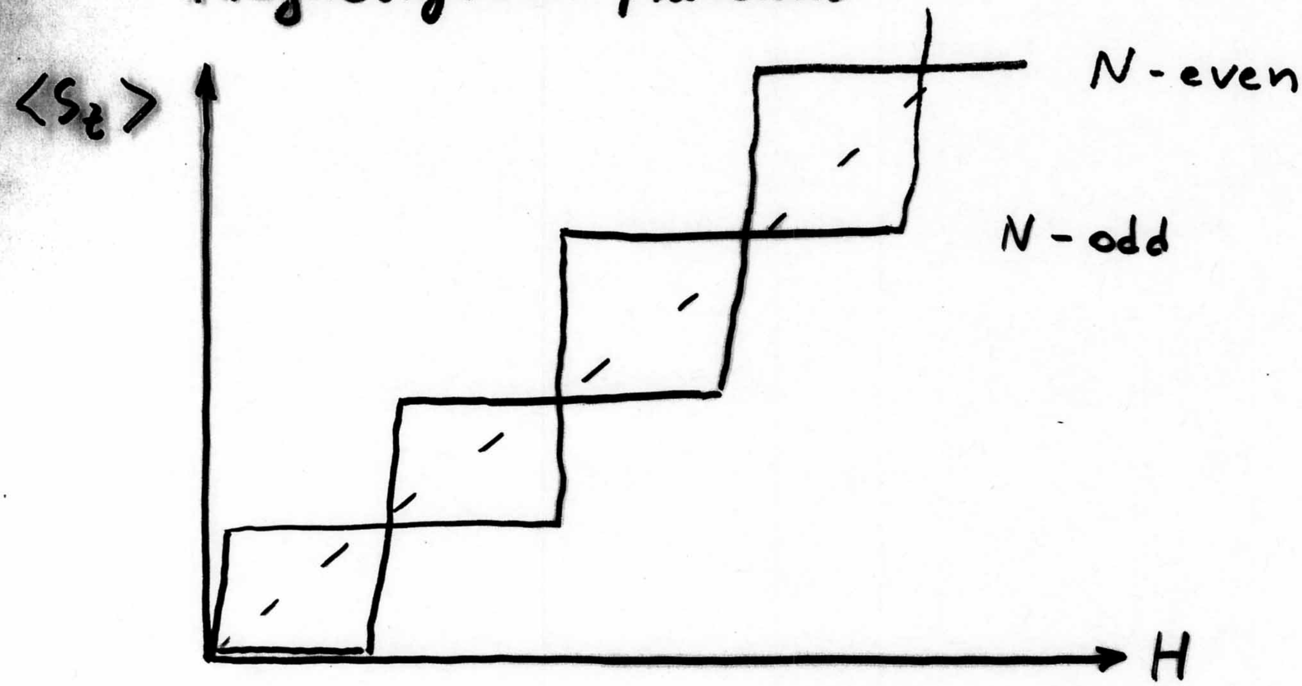
Even number of bosons per site



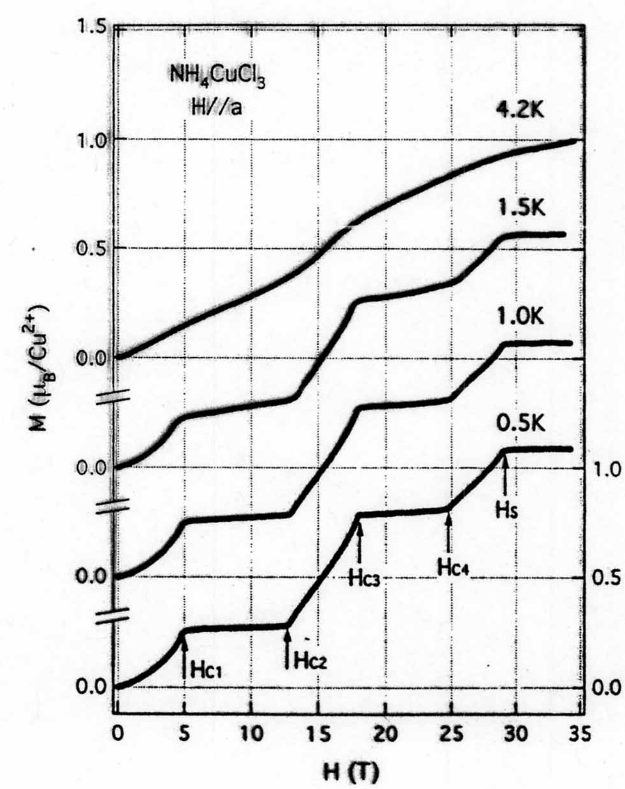
Odd number of bosons per site



- S=1 Bosons in optical lattice
  Magnetization plateaus



- Magnetization plateaus in solid state systems



Shiramura et. al.
J. Phys. Soc. Jpn.
67 : 1549 (1998)

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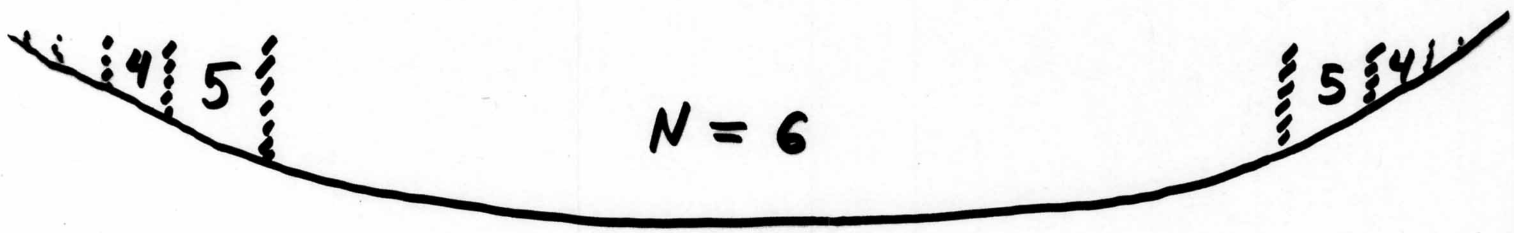
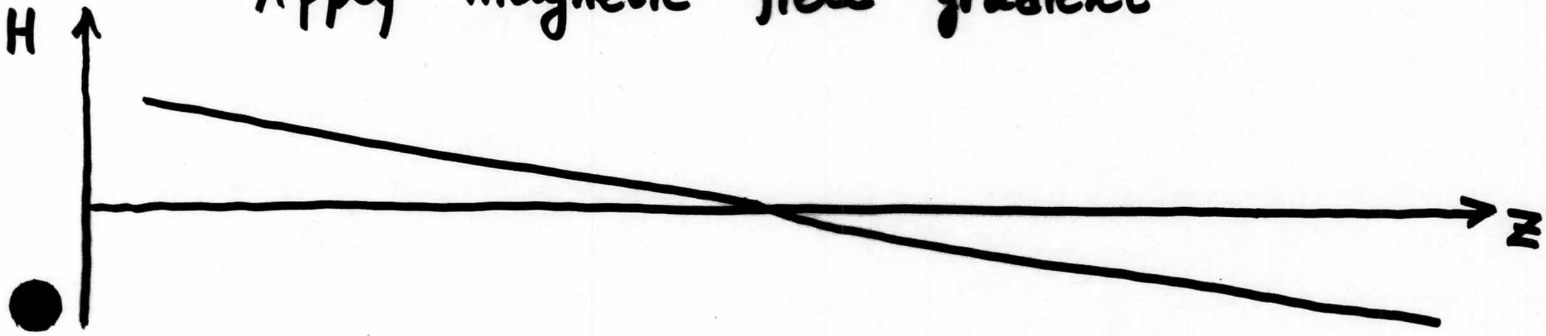
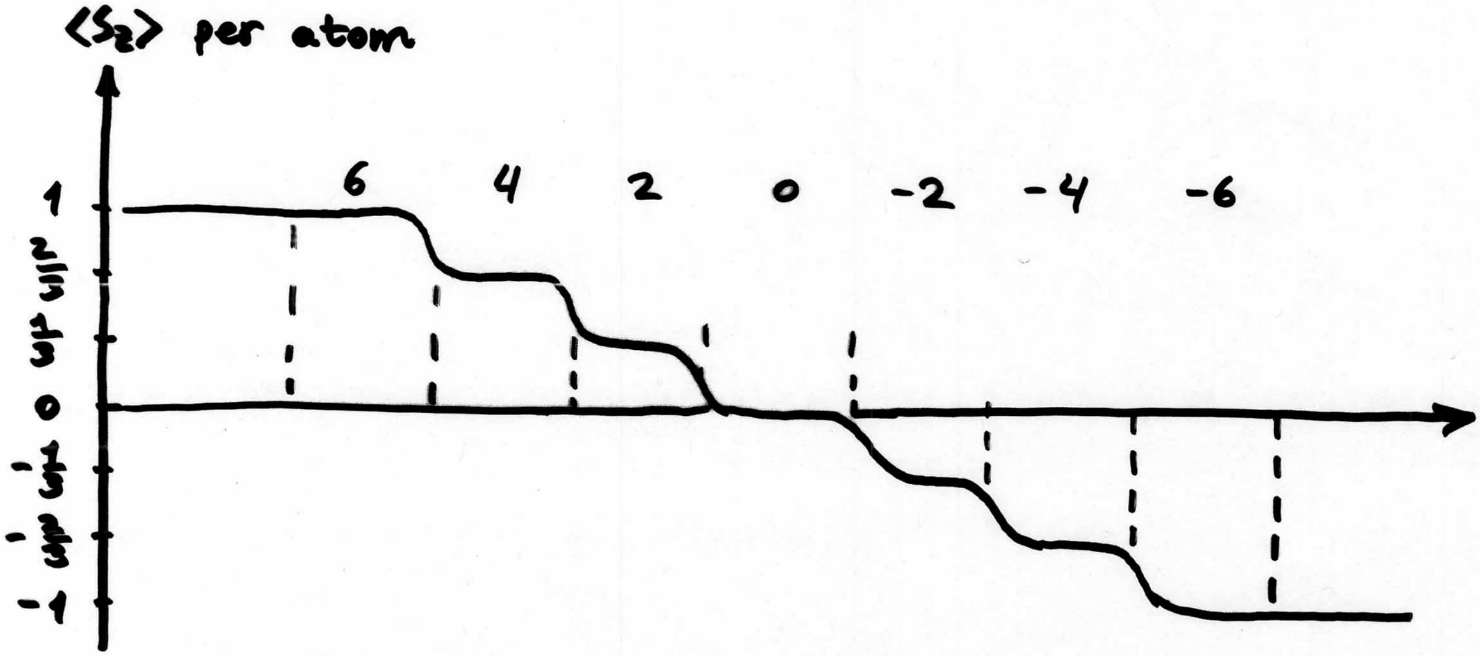
S=1 atoms in optical lattice
Stern-Gerlach experiments



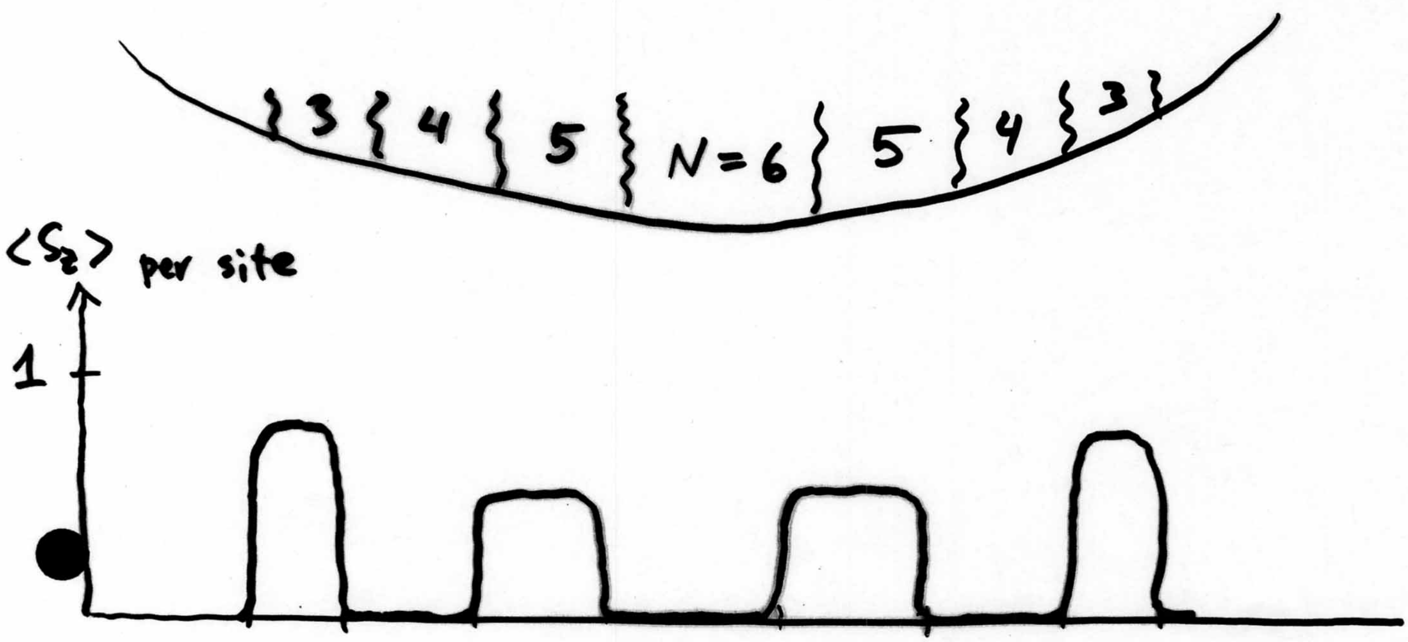
Apply magnetic field gradient





Compare to formation of spin domains for S=1 Na atom in a single optical trap with magnetic field gradient.
Stenger et. al., Nature 1998

S = 1 atoms in optical lattice
Spin decoration experiment



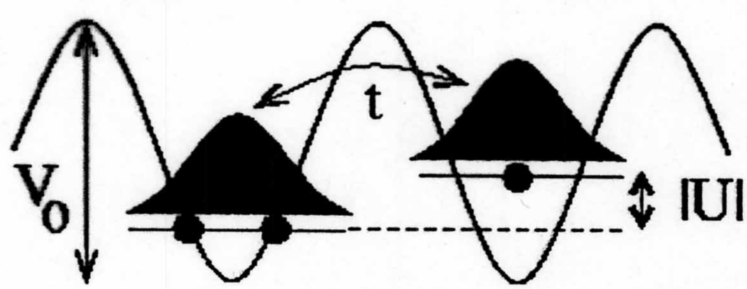
Enhancing superfluidity of fermionic atoms using optical lattices

Hofstetter, Cirac, Zoller, Lukin, Demler, PRL 89, 220407 (02)

Optical lattices enhance interactions and reduce kinetic energy of atoms.
Both enhance superfluidity.

Effective description:
Hubbard model, $U<0$

$$\mathcal{H} = -t \sum_{\langle ij \rangle \sigma} c^{+}_{i\sigma} c_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$



Polarized bosonic atoms in optical lattices:
superfluid - Mott insulator transition ($t \sim U \sim kHz$)

Theory: Jaksch et.al. PRL 81, 3108 (98)

Experiment: Greiner et.al. Nature 415, 39 (02)
Orzel et.al. Science 291, 2386 (01)

Enhancing superfluidity of fermionic atoms using optical lattices

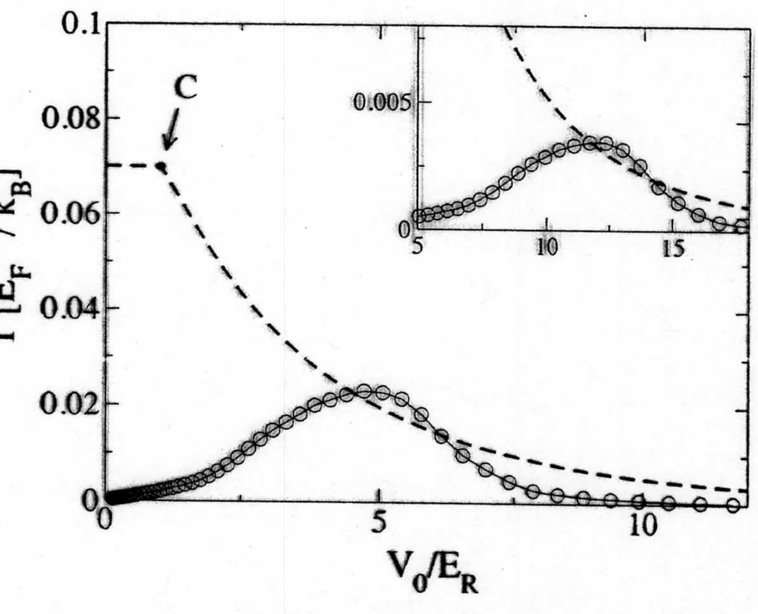
$$\mathcal{H} = -t \sum_{\langle ij \rangle \sigma} c_{i\sigma}^{+} c_{j\sigma} + U \sum n_{i\uparrow} n_{i\downarrow}$$

$t \gg |U|$ BCS regime $T_c \sim t\, e^{-7t/U}$

$t \ll |U|$ Condensation of composite bosons $T_c \sim t^2/U$

Highest transition temperature for $t \sim U$

$$T_c^{MAX} \propto T_F^{free} \cdot \sqrt[3]{n}\, |a_s|$$



In combination with effective atomic cooling due to turning on the optical lattice

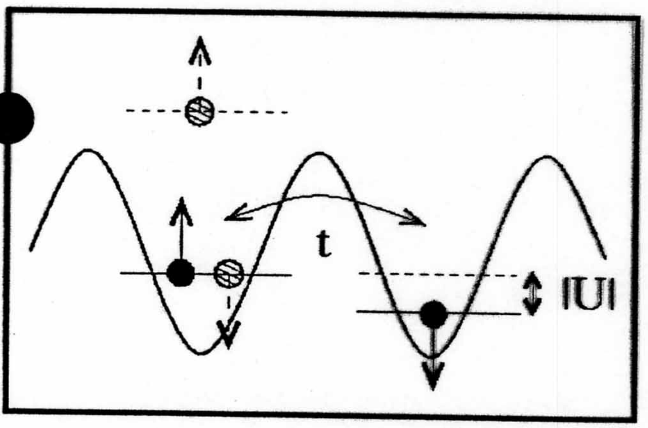
$$T_{in}^{MAX} \approx 0.1\, T_F^{free}$$

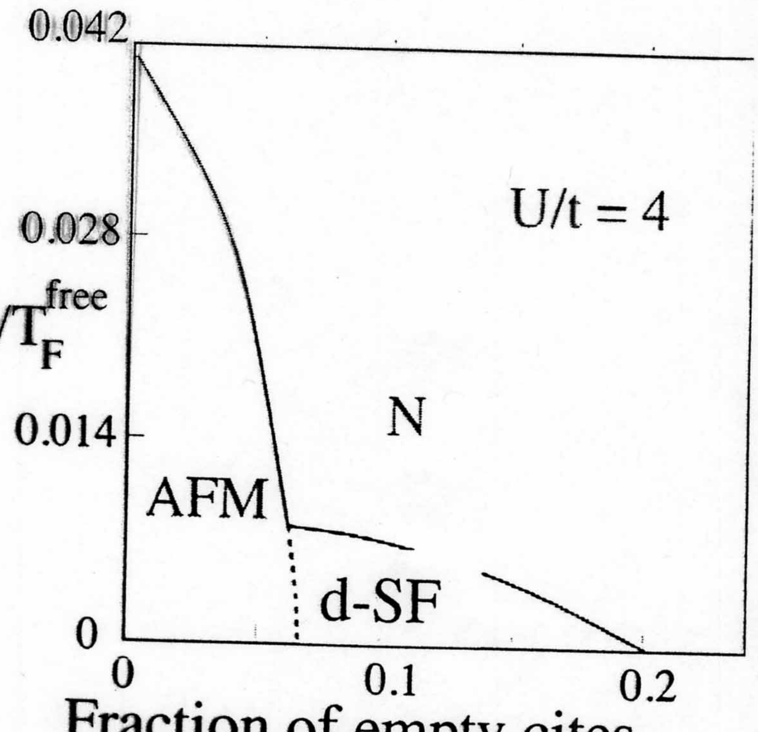
Cold atom test of high-Tc mechanism in cuprates

Cold repulsive fermions in a lattice

Effective description: Hubbard model, $U>0$

$$\mathcal{H} = -t \sum_{\langle ij\rangle \sigma} c^{\dagger}_{i\sigma} c_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$



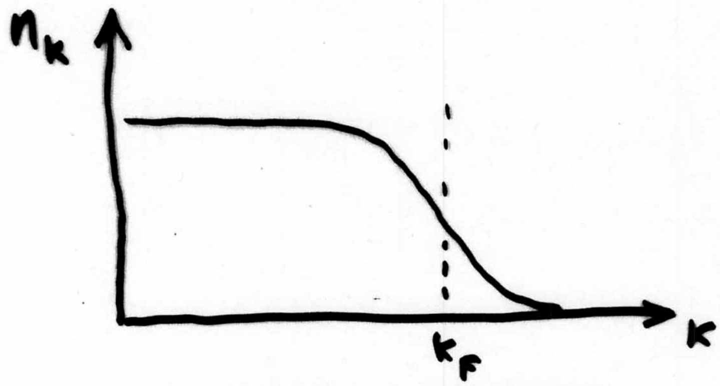


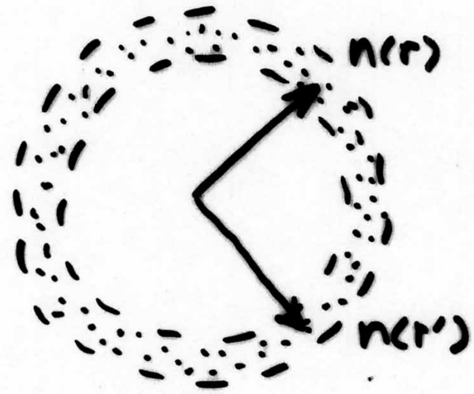
FLEX, Bickers, Scalapino '95

- Antiferromagnetic order when lattice is completely filled (1 atom per site)

- d-wave superfluidity at fractional fillings

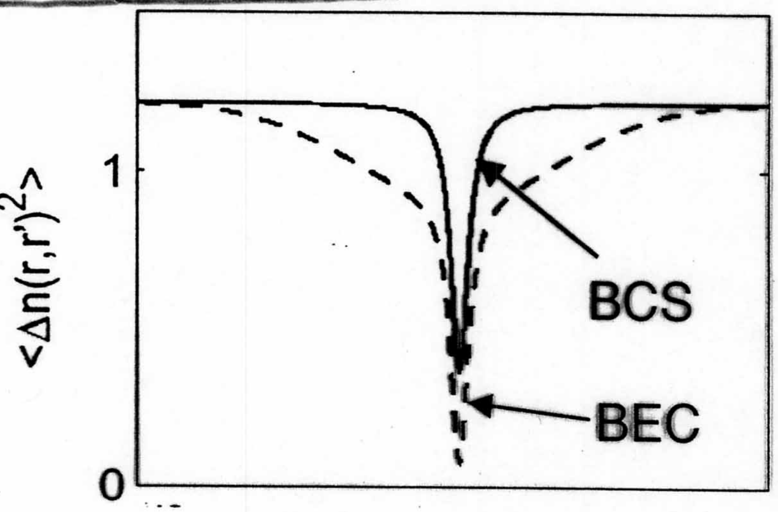
Second order interference from the BCS superfluid phase

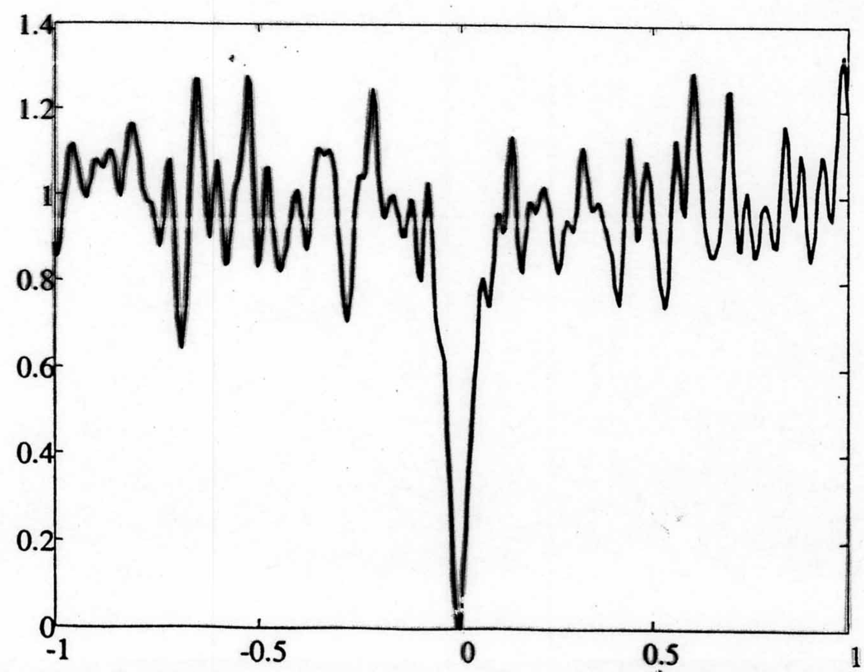




$\boxed{\Delta n(r,-r)\,|\psi_{BCS}\rangle = 0}$

$\Delta n(r,r') = n(r) - n(r')$

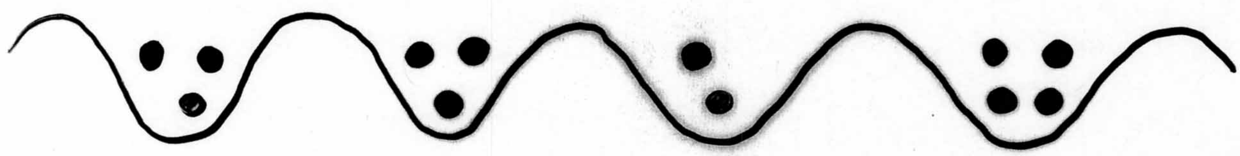




Boson-Fermion mixtures in 1d optical lattices

Mathey, Wang, Hofstetter, Lukin, Demler, quant-ph/040115

Bosonic atoms are in the superfluid phase (high density). They mediate interactions between fermions and provide cooling.



- spin polarized fermions
- bosons

Boson-Fermion Hubbard Hamiltonian

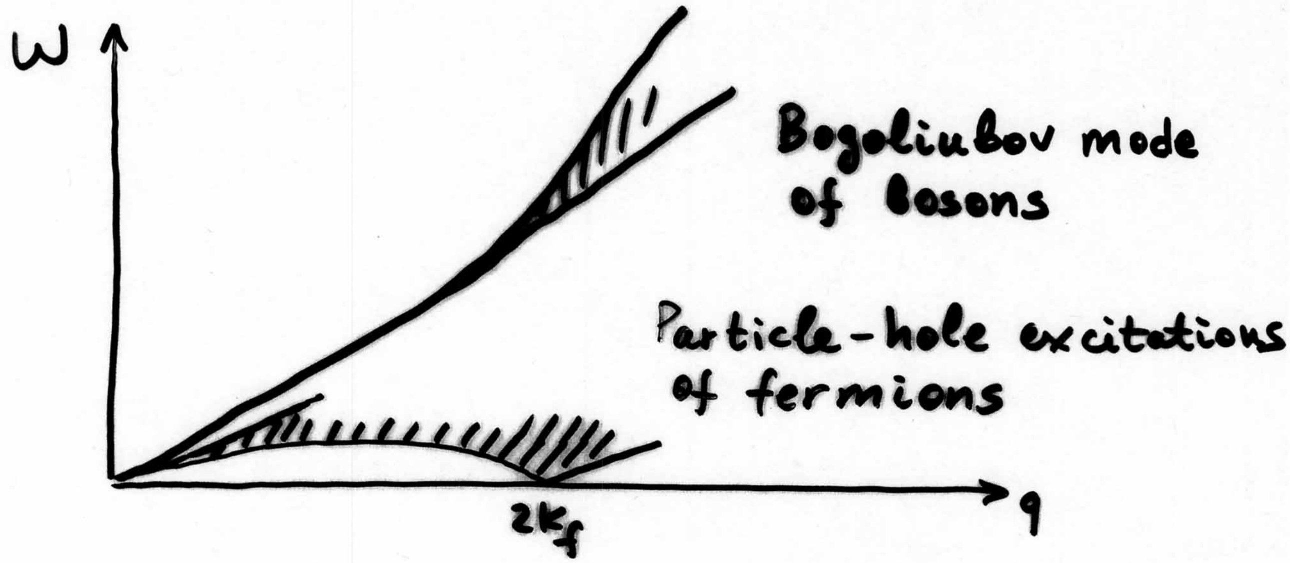
$$\mathcal{H} = -t_b \sum_{ij} b_i^\dagger b_j - t_f \sum_{ij} f_i^\dagger f_j - \mu_b \sum_i n_{bi} - \mu_f \sum_i n_{fi} + U_b \sum_i n_{bi}^2 + U_{bf} \sum_i n_{bi} n_{fi}$$

Instabilities

CDW: Periodic arrangement of fermions

Fermion pairing: Fermions bind into p-wave pairs

Polarons in Boson-Fermion mixtures

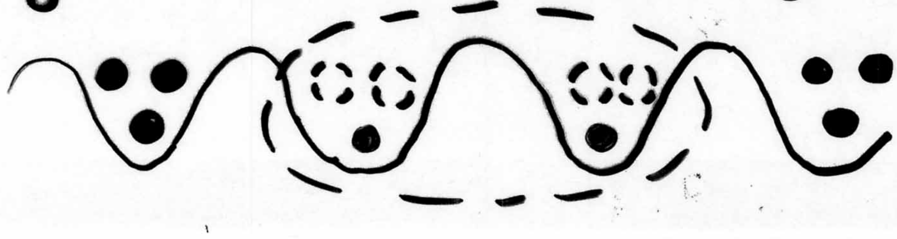


- Interaction of fermions with the Bogoliubov mode of fermions is similar to electron-phonon coupling in solids.

Different order of limits $V_B \gg V_f$

Fermionic polaron:
Fermion plus a screening cloud of Bosons

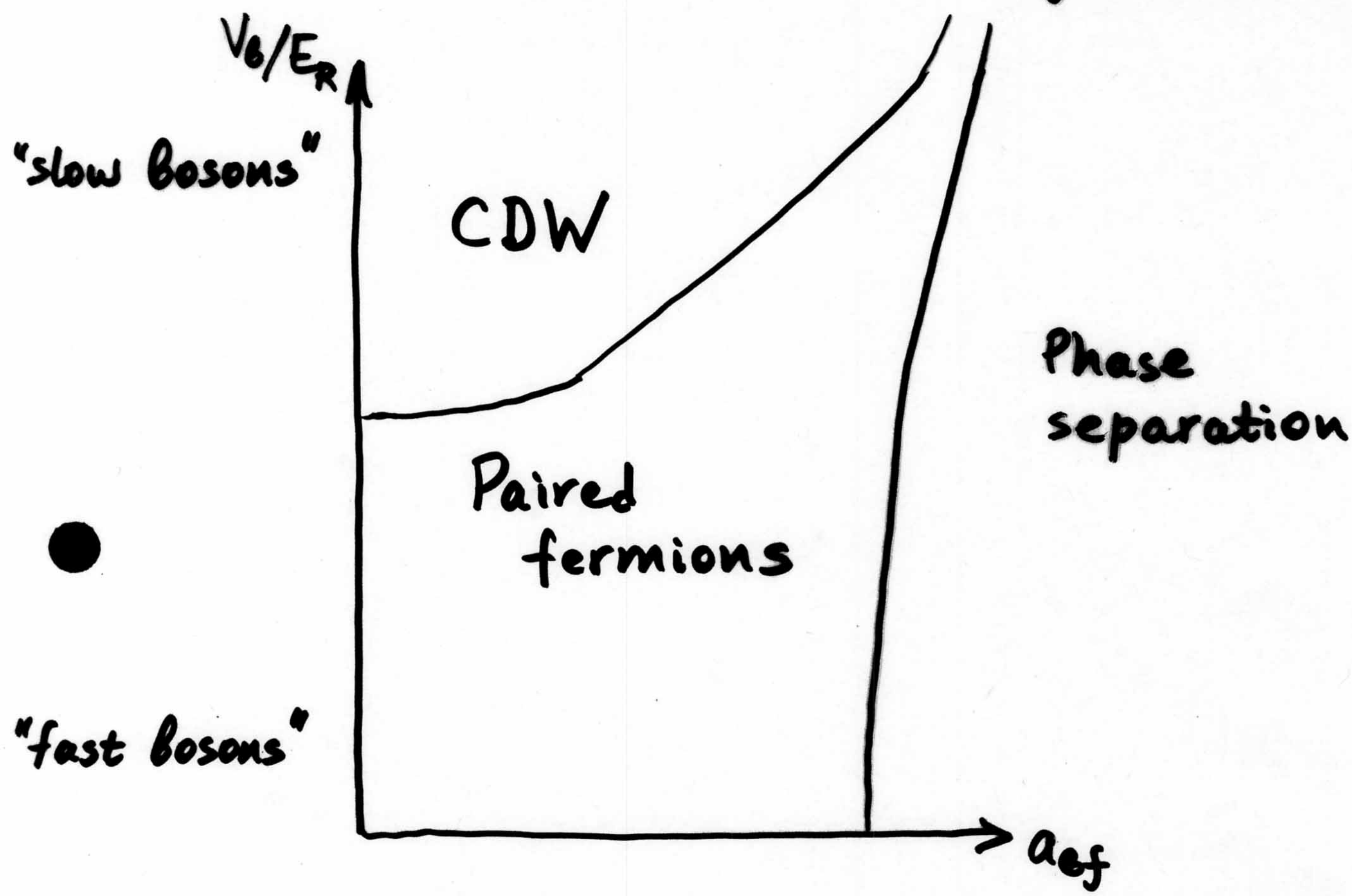
- Pairing in BF mixtures is pairing of polarons



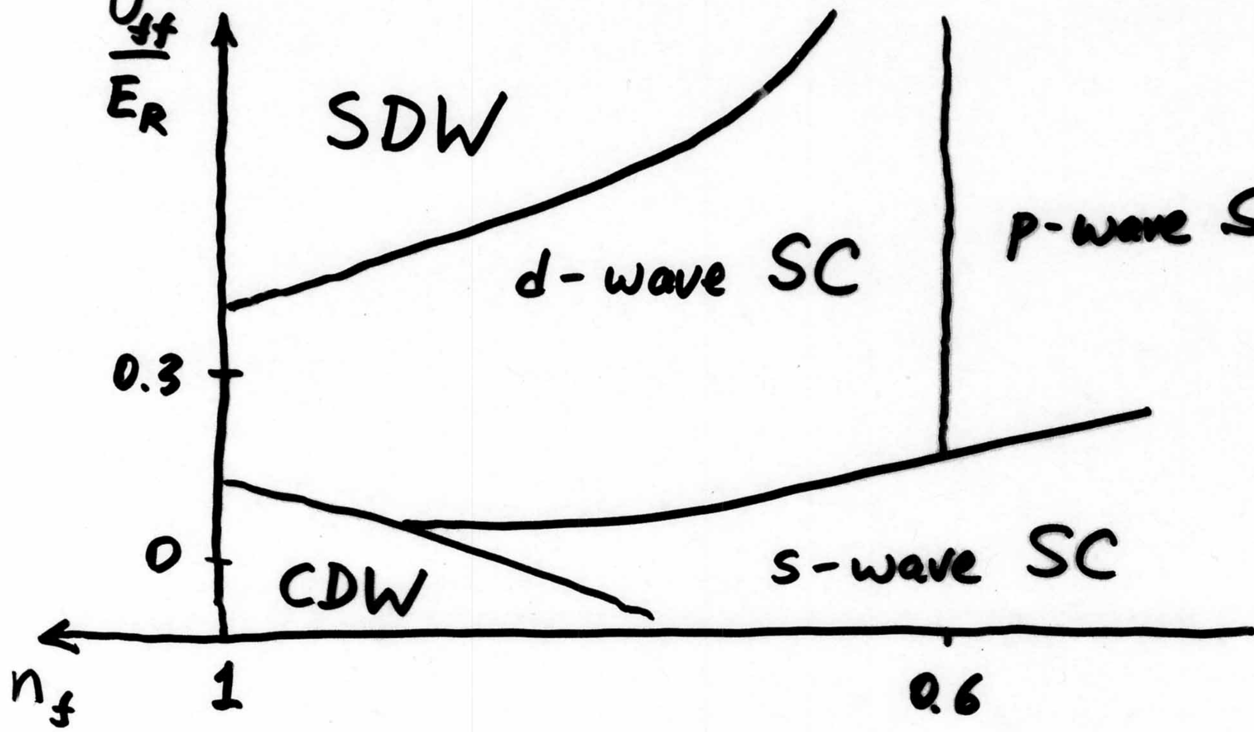
Boson-Fermion mixture in 1d optical lattice

Mathey, Wang, Hofstetter, Lukin, Demler

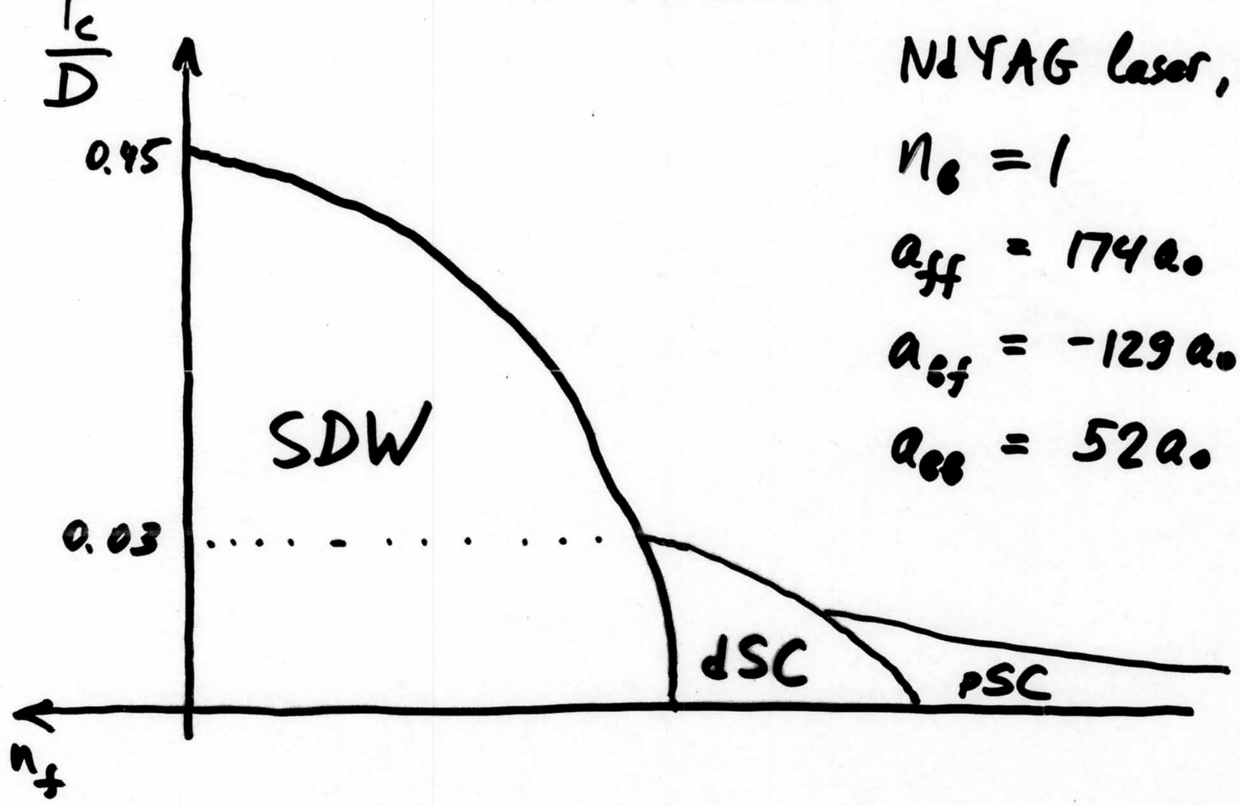
quant-ph/0401151



Boson-Fermion mixtures in 2d optical lattices



$^{40}K - ^{23}Na$ mixture



NdYAG laser, $\lambda = 1.06$,

$n_b = 1$

$a_{ff} = 174\, a_0$

$a_{bf} = -129\, a_0$

$a_{bb} = 52\, a_0$

$D = 8t_f$ bandwidth for fermions

Quantum Hall effect with ultracold atoms

Rotating condensates

N. Wilkin, J. Gunn, PRL 84, 6 (00)
N. Cooper, N. Wilkin, PRB 60, R16279 (99)
J. Reijnders et.al., PRL 89, 120401 (02)

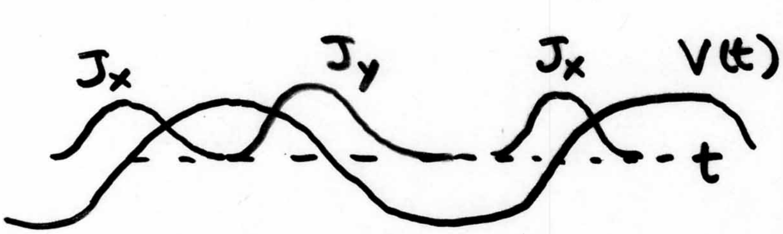
Creating effective magnetic field for neutral atoms in optical lattices

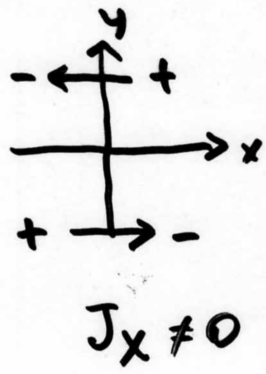
D. Jaksch, P. Zoller, New J. Phys. 5, 56 (03)
E. Mueller, cond-mat/0404306
A. Sorensen, E. Demler, M. Lukin, cond-mat/0405079

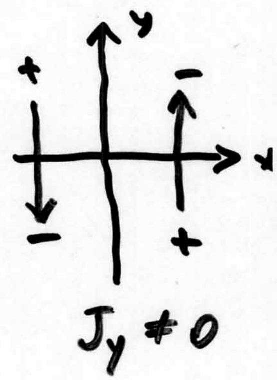
Rotating quadrupole potential + time dependent optical lattice

$$V(x,y,t) = A \sin\omega t \cdot x \cdot y$$

$$\mathcal{H}_0 = -J_x(t) \sum_i a_i^\dagger a_{i\pm x} - J_y(t) \sum_i a_i^\dagger a_{i\pm y}$$







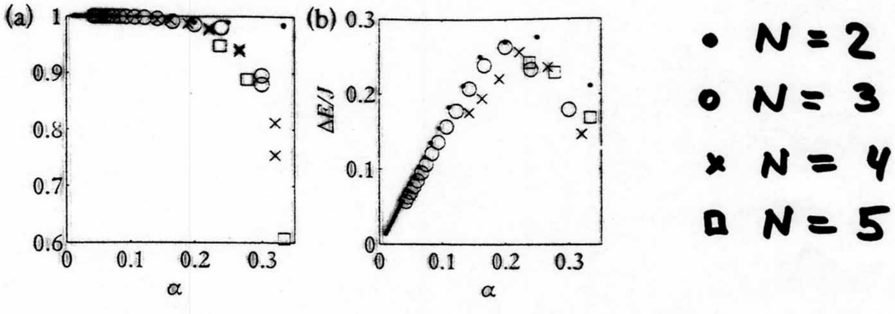
Fractional quantum Hall effect with ultracold atoms in optical lattices

Expect fractional quantum Hall phases

when $\nu = \frac{\#\text{ atoms}}{\#\text{ fluxes}} = \frac{1}{2m}$

Exact diagonalization for hard core bosons

We fix $\nu = 1/2$, α - flux density



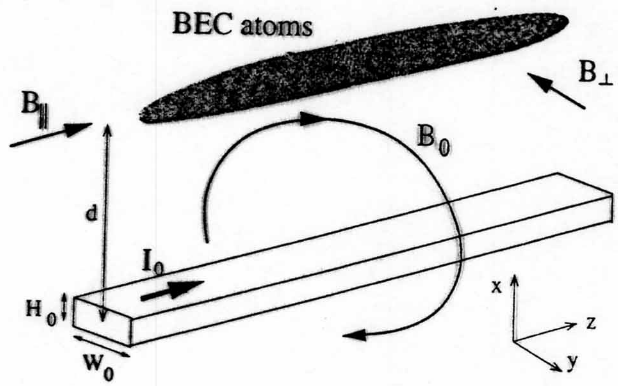
(a) - overlap with the Laughlin wavefunction

$$\psi = \prod_{i<j} (z_i - z_j)^2 \, e^{-\frac{1}{4}\sum_k |z_k|^2}$$

(b) - energy gap to the lowest excited state

BEC in quasi one-dimensional magnetic microtrap

Thywissen et.al., Eur. Phys. J. D <u>7</u>, 361 (1999)

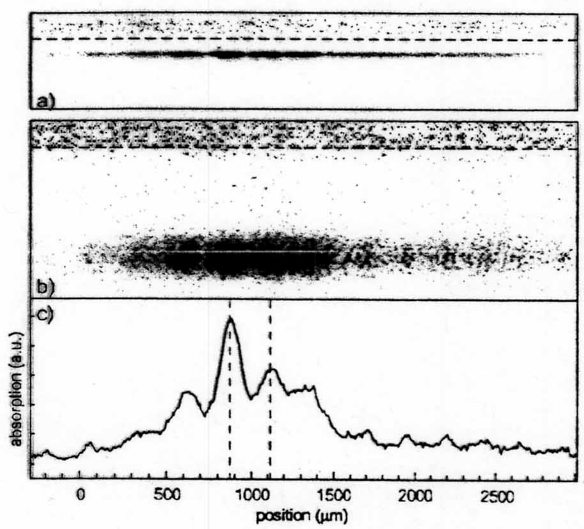


Condensate fragmentation in magnetic microtraps

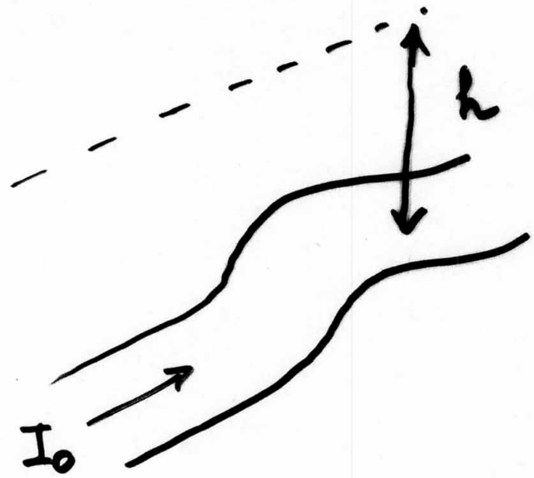
Kraft et.al., J. Phys. B <u>35</u>, L469 (2002

Leanhardt et.al., PRL <u>89</u>, 040401 (2002

Fortagh et.al., PRA <u>66</u>, 041604 (2002



Correlated random potential
in magnetic microtraps
D. W. Wang, M. Lukin, E. Demler, PRL 92, 76802 (04)

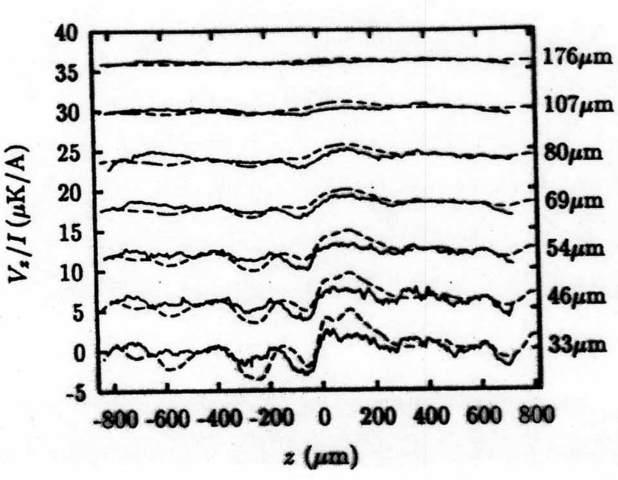


Random potential due to wire meandering

- Geometrical deformations at wavelengths smaller than h average out

- Wire width fluctuations and long wavelength deformations are not important By the Biot-Savarat law

- Lengthscale of the correlated random potential is set by the atom-wire separation, h

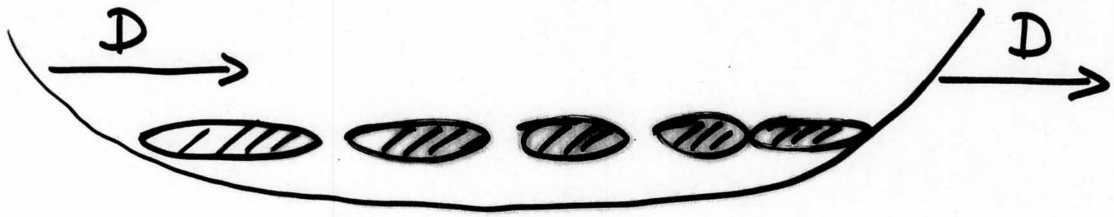


J. Estève et. al.
physics/0403020

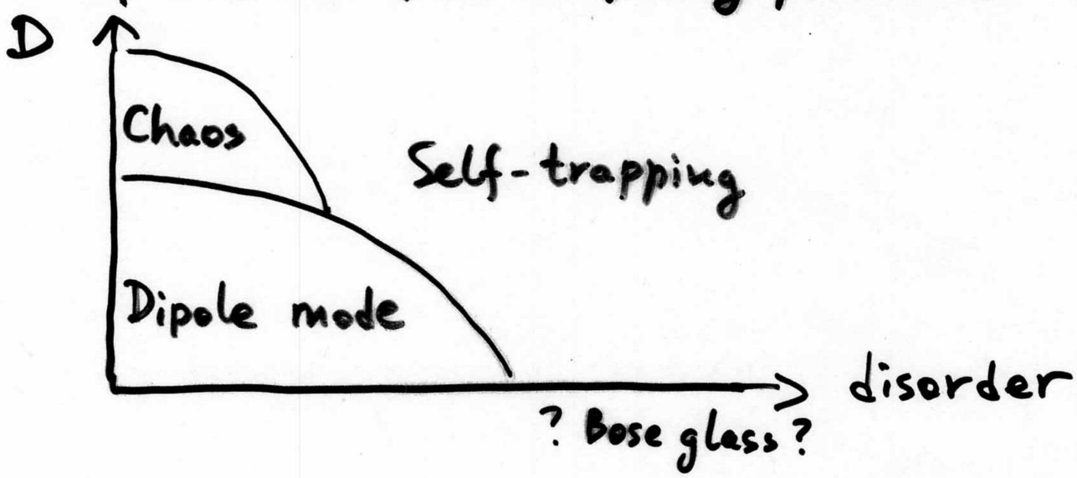
Probing fragmented condensates

Shaking experiments

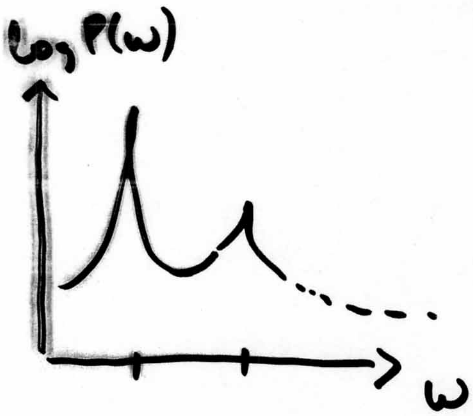
c-m/0307402, D.W. Wang, E.D., M. Lukin



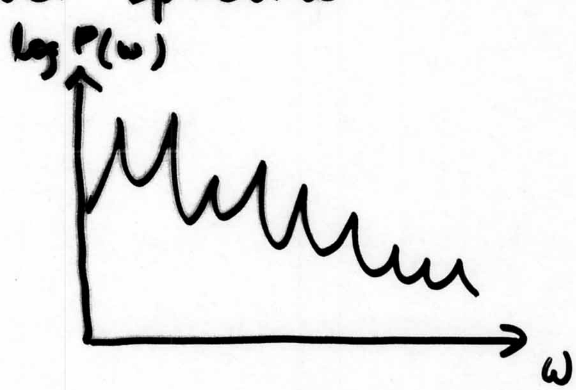
D - displacement of the confining potential



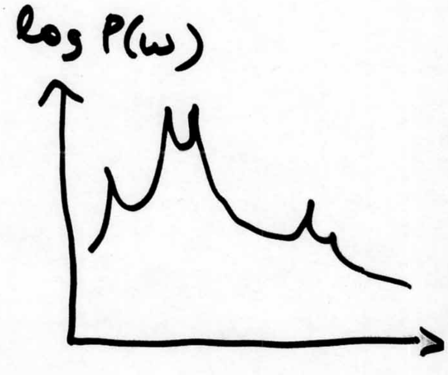
Power spectra



Dipole



chaos



Self-trapping

Summary

Two component Bose mixtures in optical lattices can be used to design spin 1/2 quantum systems. They can be used to study ferro and antiferromagnetism fractionalized spin states.

Spin 1 Bosons in optical lattices have a rich phase diagram with several insulating and superfluid phases

Optical lattices are an efficient tool for reaching superfluidity of fermionic atoms. Fermions with repulsive interactions can provide important insights into the origin of high temperature superconductivity (cuprates).

Boson-Fermion mixtures can be used to study formation of polarons and competition between superfluidity (BCS) and charge density wave order in 1d systems.

Fragmented condensates in magnetic microtraps can elucidate the role of disorder for interacting systems

A combination of oscillating quadrupole potential and time dependent optical lattices can be used to create fractional quantum Hall states of ultracold atoms.