Kicked rotor and Anderson localization

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Classical dynamics of the kicked rotor

- Depends on a single parameter $K=kT$.
- Mainly regular for small K => momentum growth is bounded.

$$\begin{cases} I_{n+1} = I_n + K\sin\theta_n \\ \theta_{n+1} = \theta_n + I_{n+1} \end{cases}$$

$I=Tp$

θ

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- Chaos appears around $K=0.5$, then progressively invades the whole phase space.
- Unbounded momentum growth above $K=1$.

$I=Tp$

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$I = Tp$

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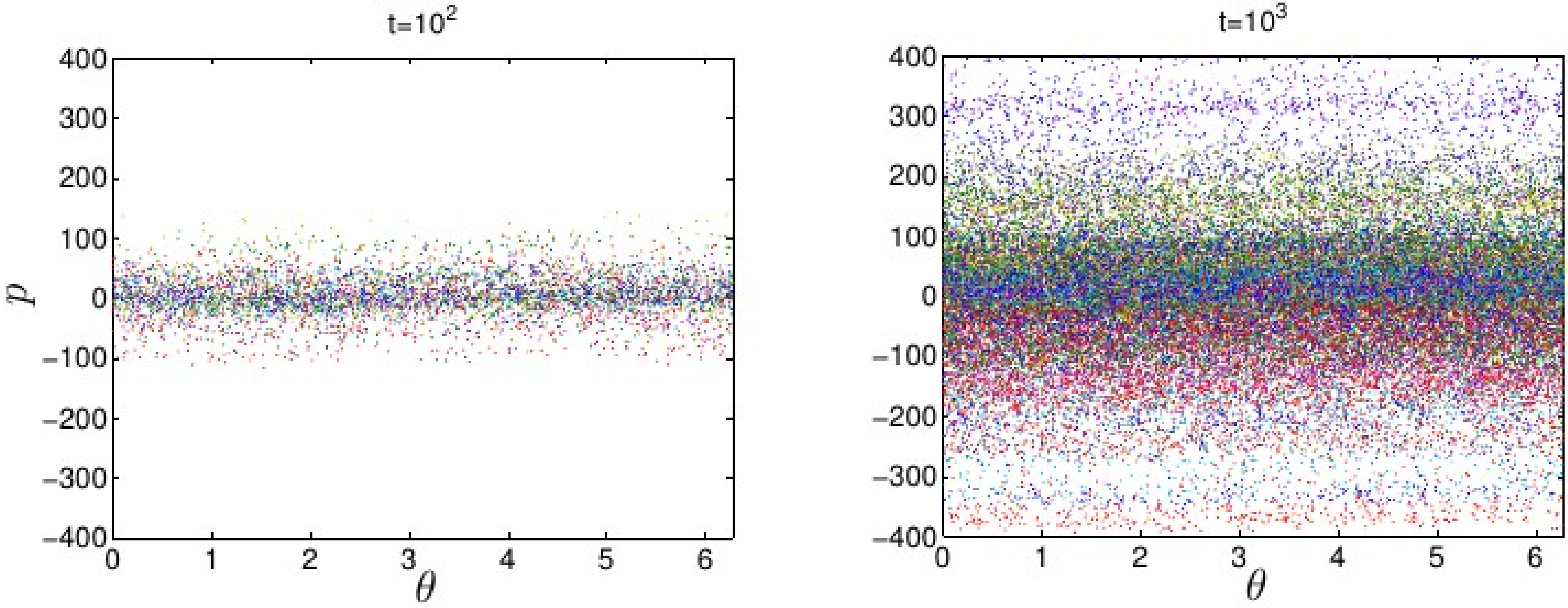
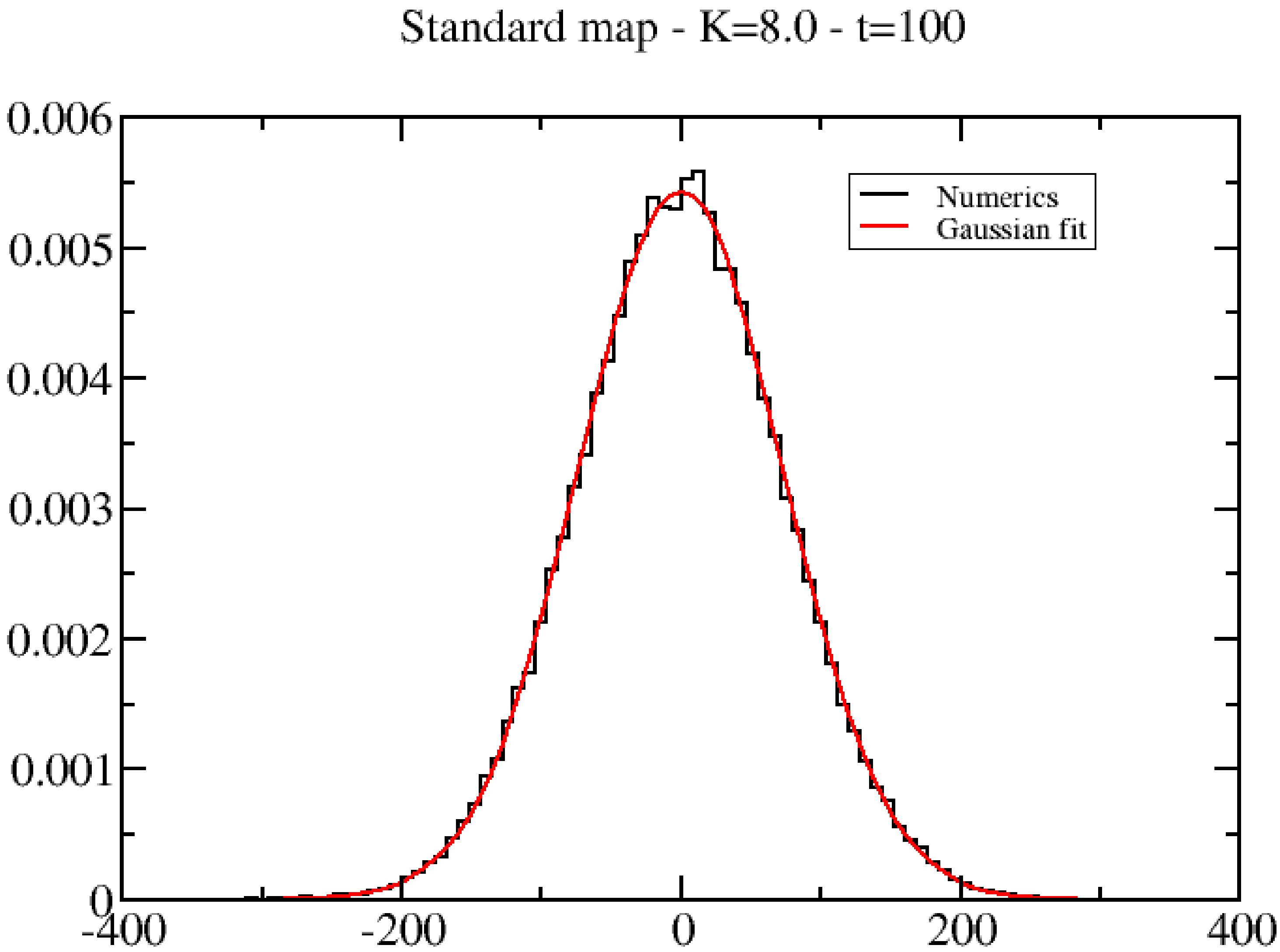
Chaotic diffusion for the standard map



Momentum distribution at long time
Fully chaotic regime



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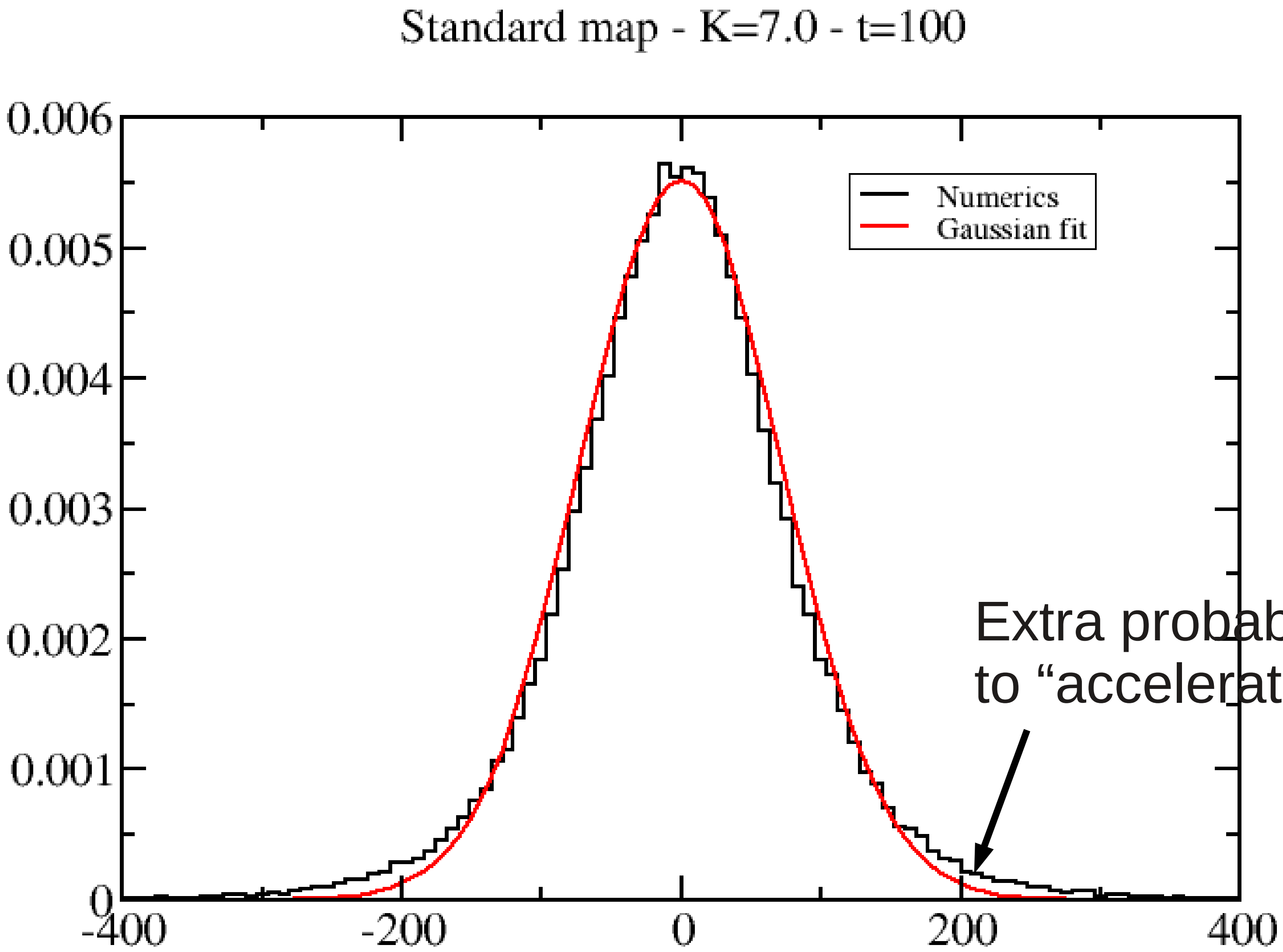
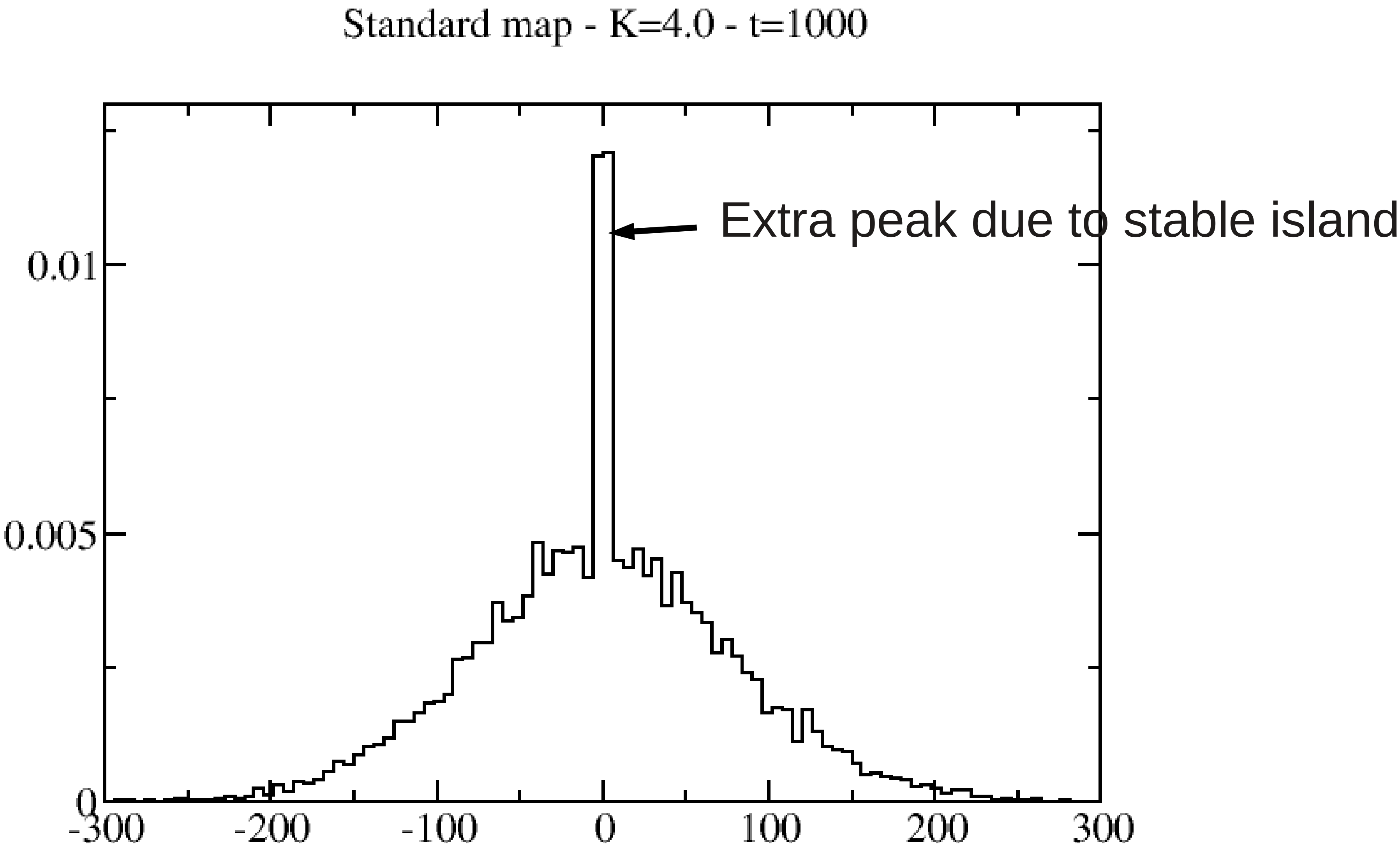
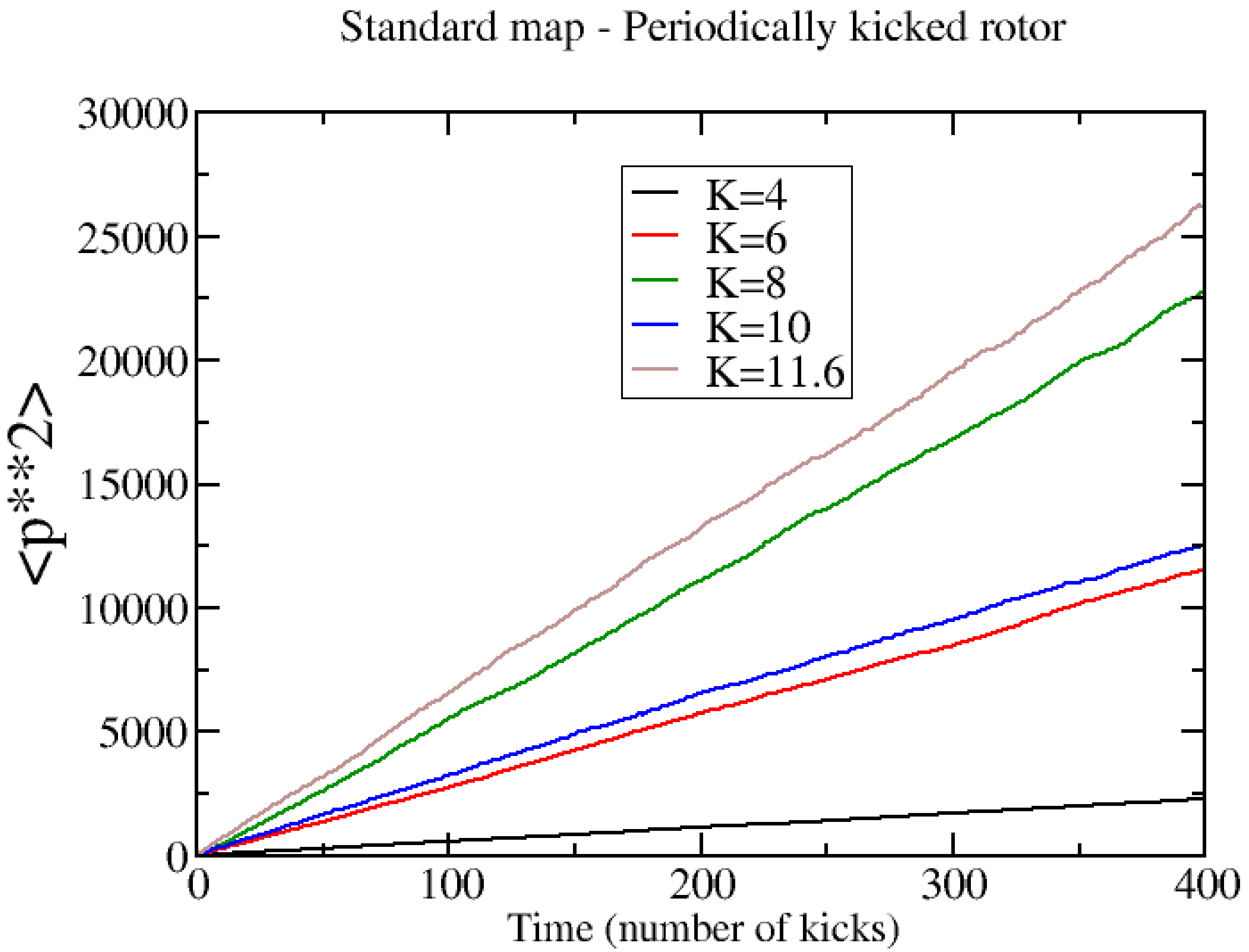


Extra probability due
to “accelerator” modes

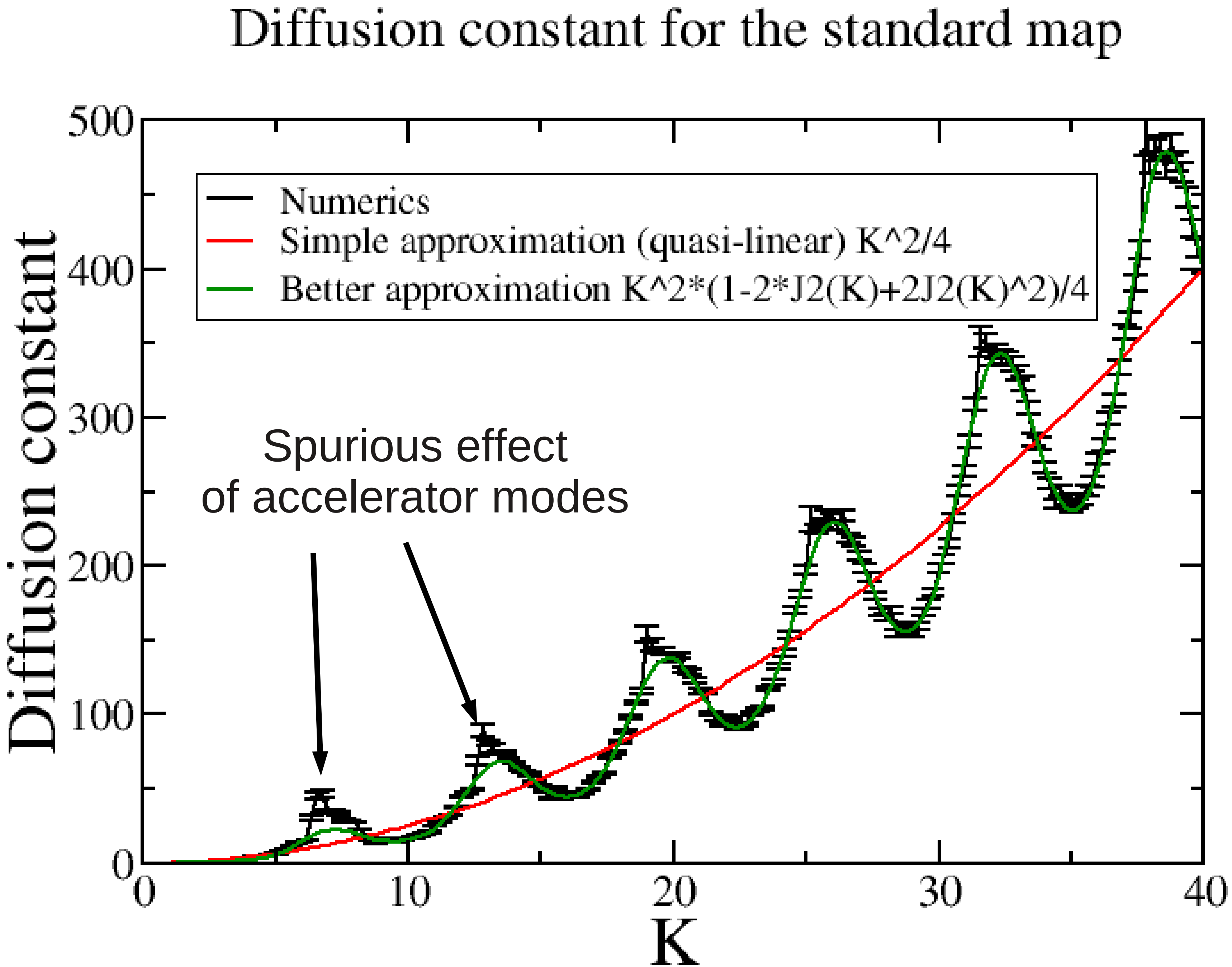
Momentum distribution at long time
Partially chaotic regime



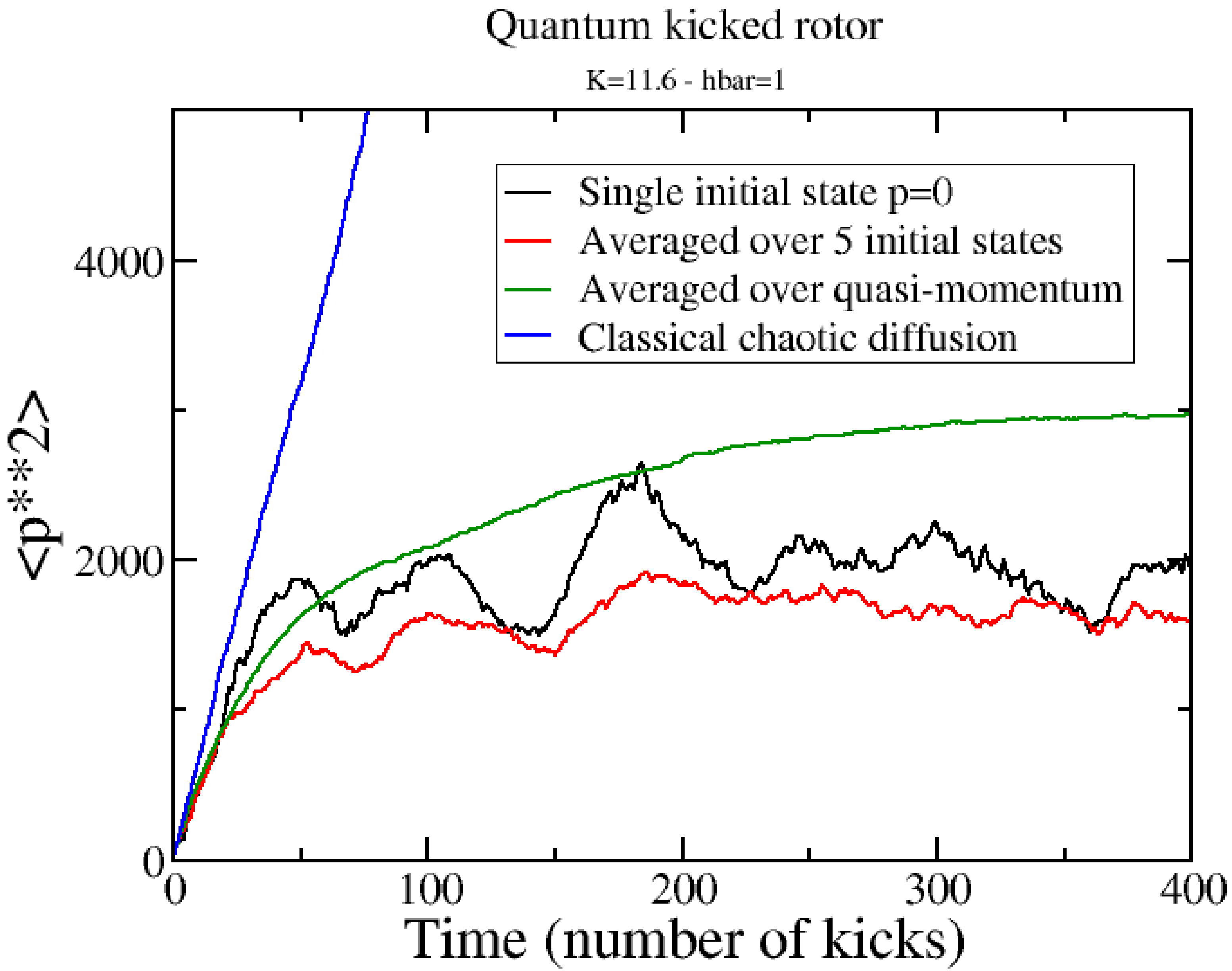
Chaotic diffusion for the classical kicked rotor



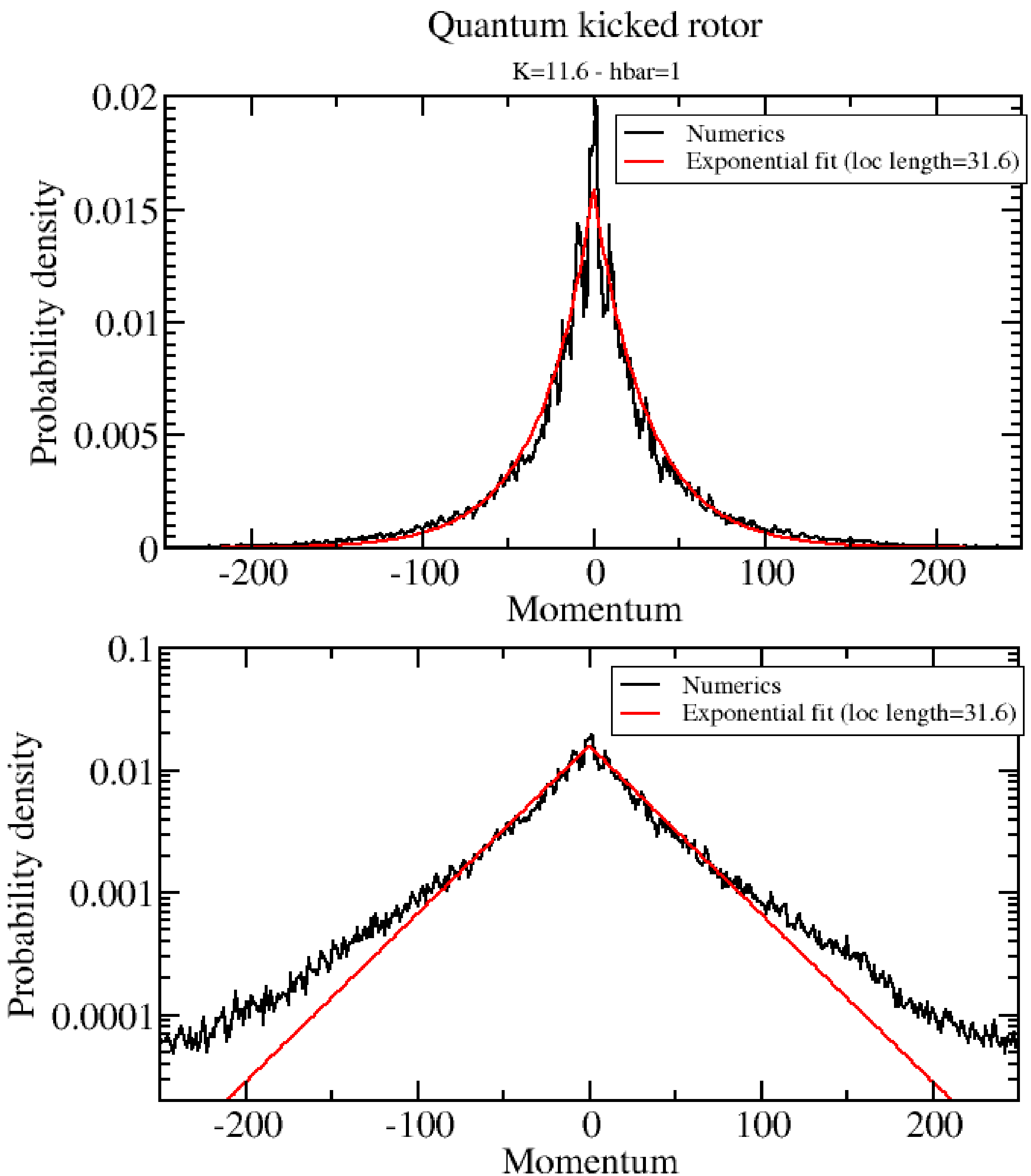
Diffusion constant for the classical kicked rotor



Quantum kicked rotor – Dynamical localization



Quantum kicked rotor – Dynamical localization



Quantum dynamics of the kicked rotor

- Numerical experiment: compare classical and quantum dynamics (averaged over an ensemble of initial conditions).
- Start from a state well localized in momentum space.

$|\psi(p)|^2$

(log scale)

$\langle p^2(t) \rangle$

Time t (number of kicks)

Quantum dynamics of the kicked rotor

- Numerical experiment: compare classical and quantum dynamics (averaged over an ensemble of initial conditions).
- Start from a state well localized in momentum space.
- At short time, the quantum and classical dynamics are equivalent.

$|\psi(p)|^2$

(log scale)

Classical dynamics

Quantum dynamics

$\langle p^2(t) \rangle$

Time t (number of kicks)

Quantum dynamics of the kicked rotor

- Numerical experiment: compare classical and quantum dynamics (averaged over an ensemble of initial conditions).
- Start from a state well localized in momentum space.
- At long time, the quantum dynamics freeze.
- **Equivalent to Anderson localization in momentum space (Fishman et al, 1982)**

$|\psi(p)|^2$

(log scale)

Quantum dynamics: dynamical localization

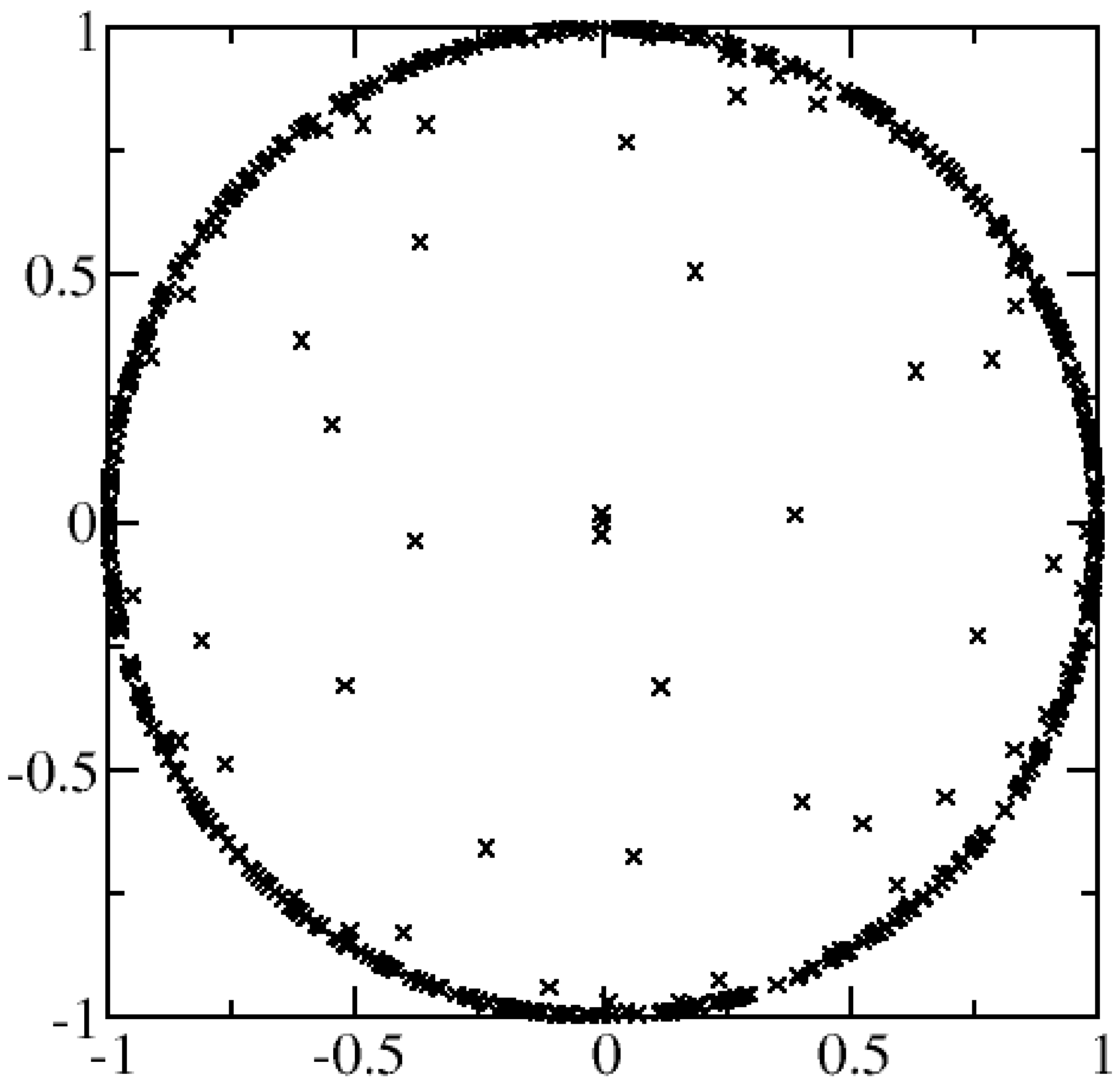
Casati et al. (1979)

$\langle p^2(t) \rangle$

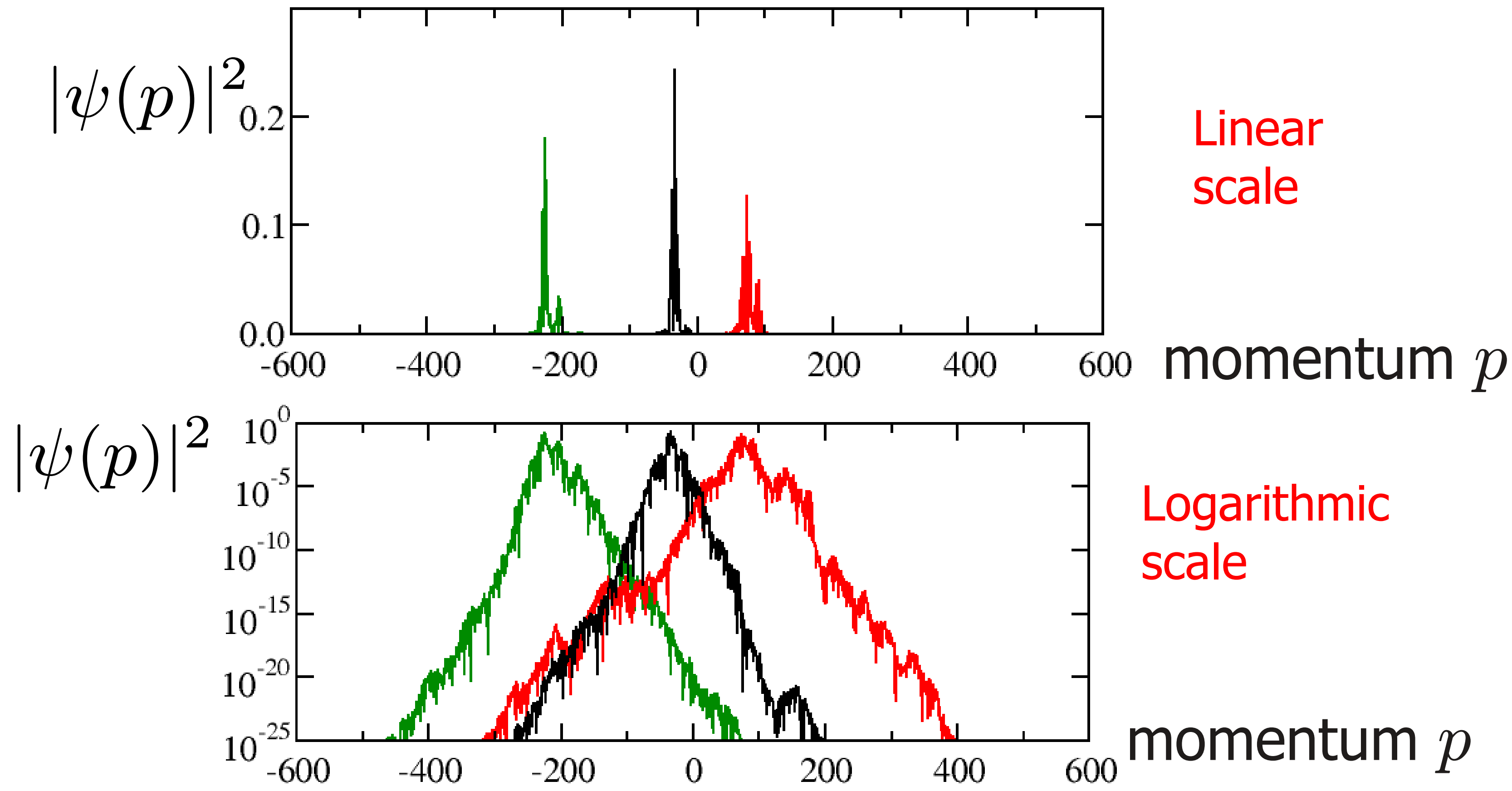
Classical dynamics: chaotic diffusion

Time t (number of kicks)

Floquet spectrum in the localized regime



Kicked rotor: Floquet eigenstates of the evolution operator

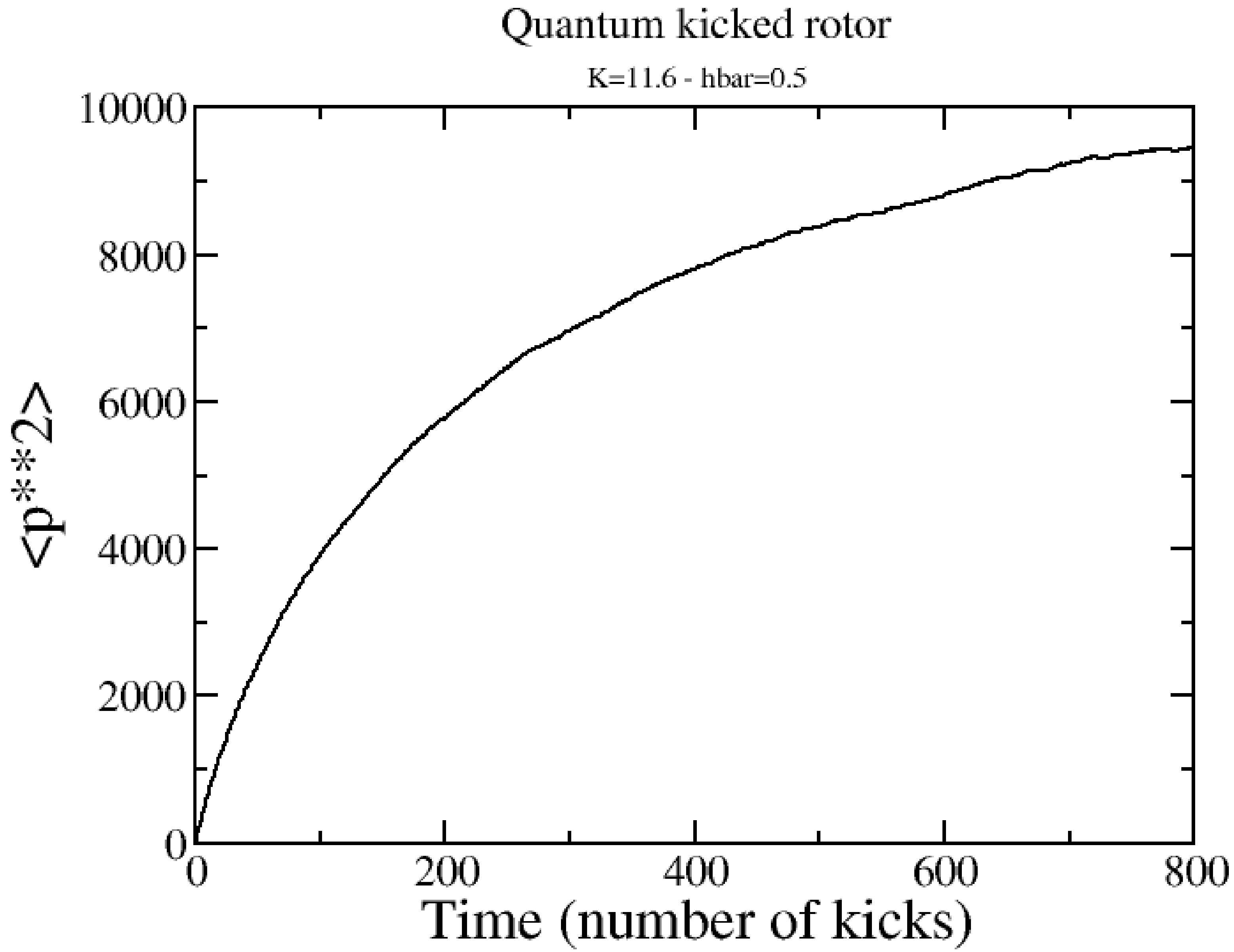


Three typical Floquet eigenstates of the chaotic quantum kicked rotor showing exponential localization in momentum space

$$|\psi(p)|^2 \simeq \exp\left(-\frac{|p-p_0|}{\ell}\right)$$

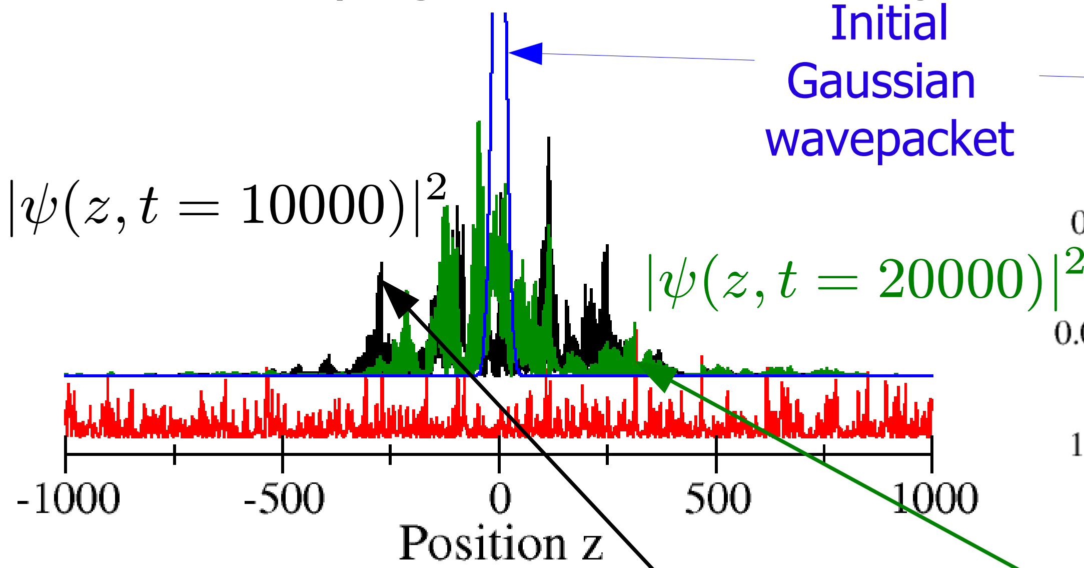
The localization length depends on K, but is similar for all states

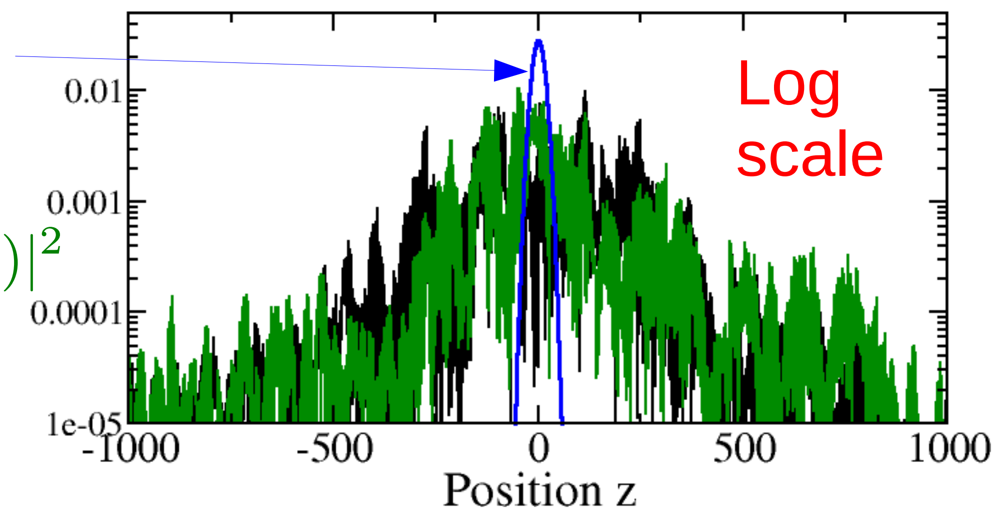
Quantum kicked rotor – Dynamical localization



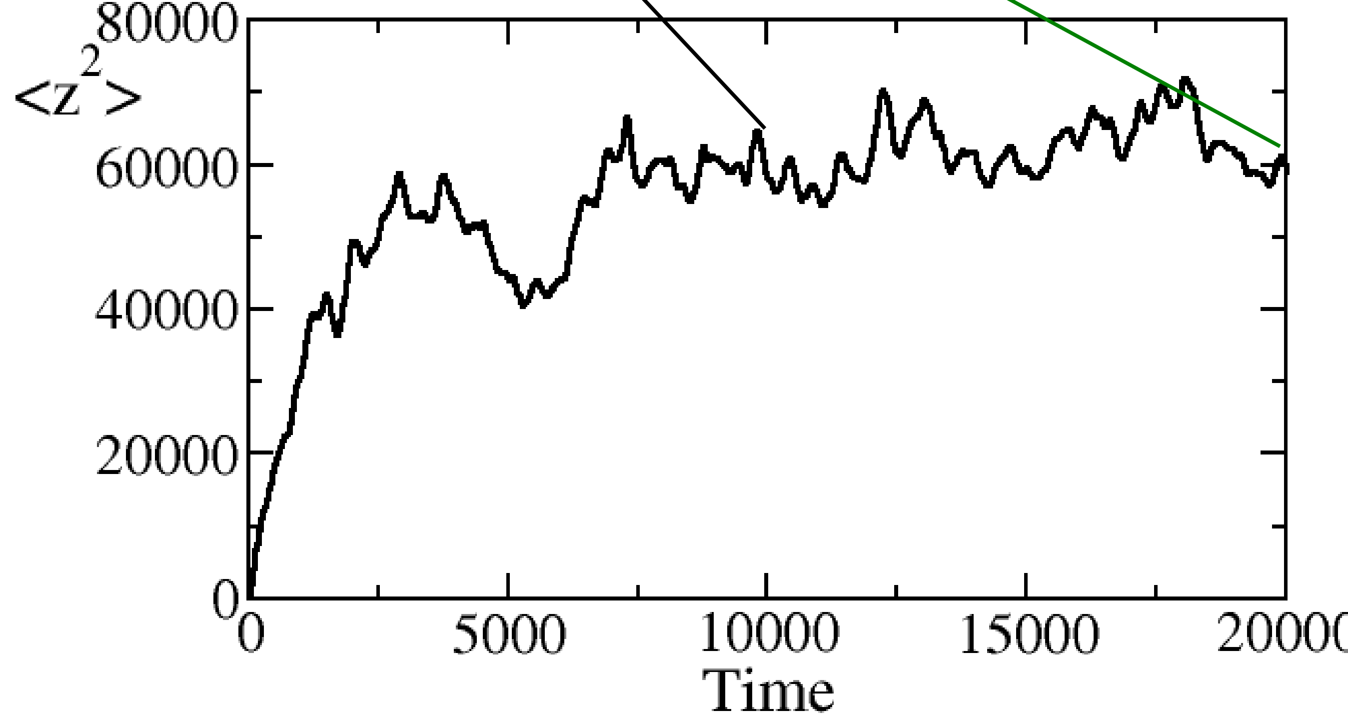
Anderson localization in 1d

- Propagation of an initially localized wave-packet:





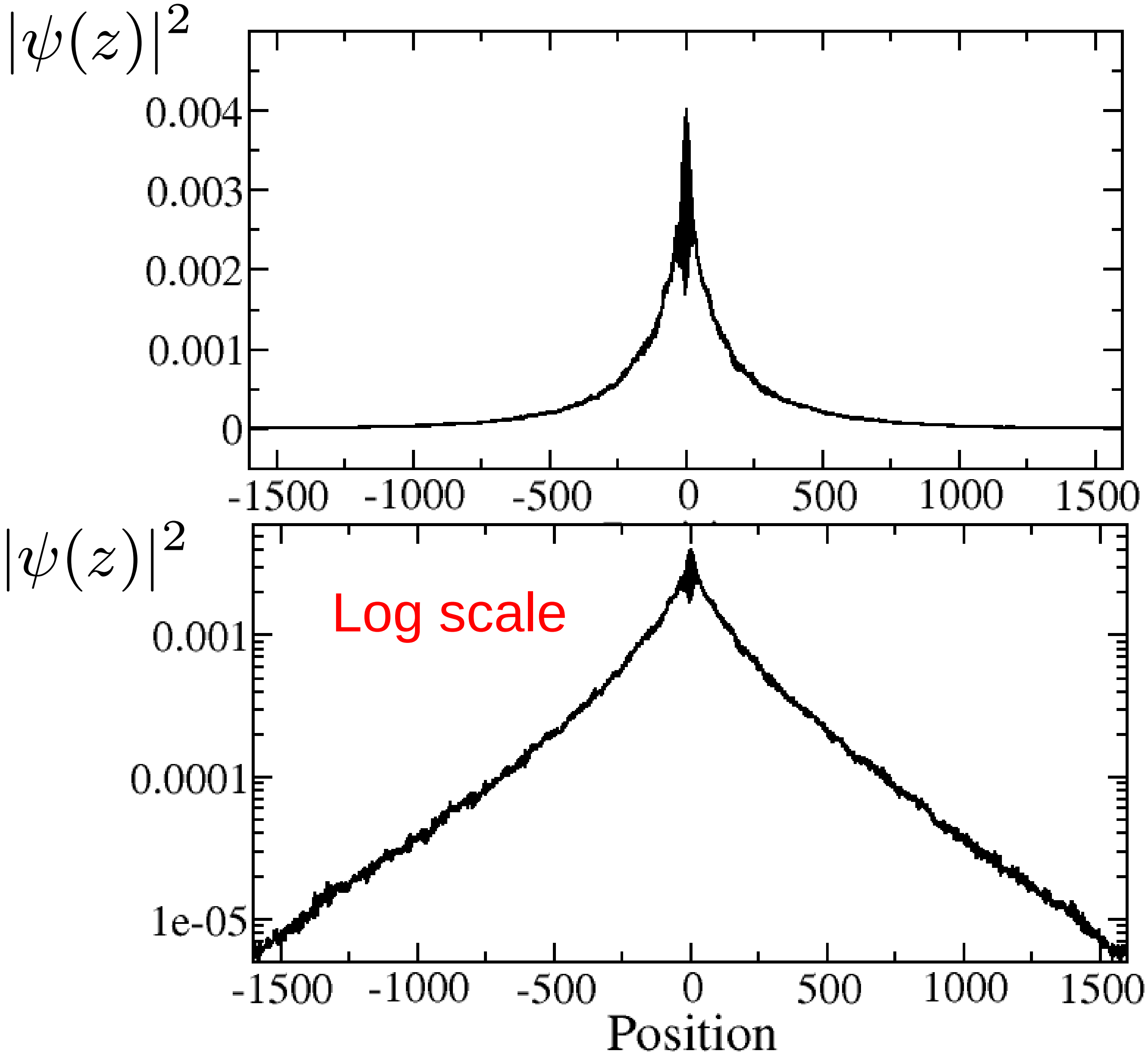
Exponential localization at long time



- Both $\langle z^2\rangle(t)$ and $|\psi(z,t)|^2$ display signatures of Anderson localization

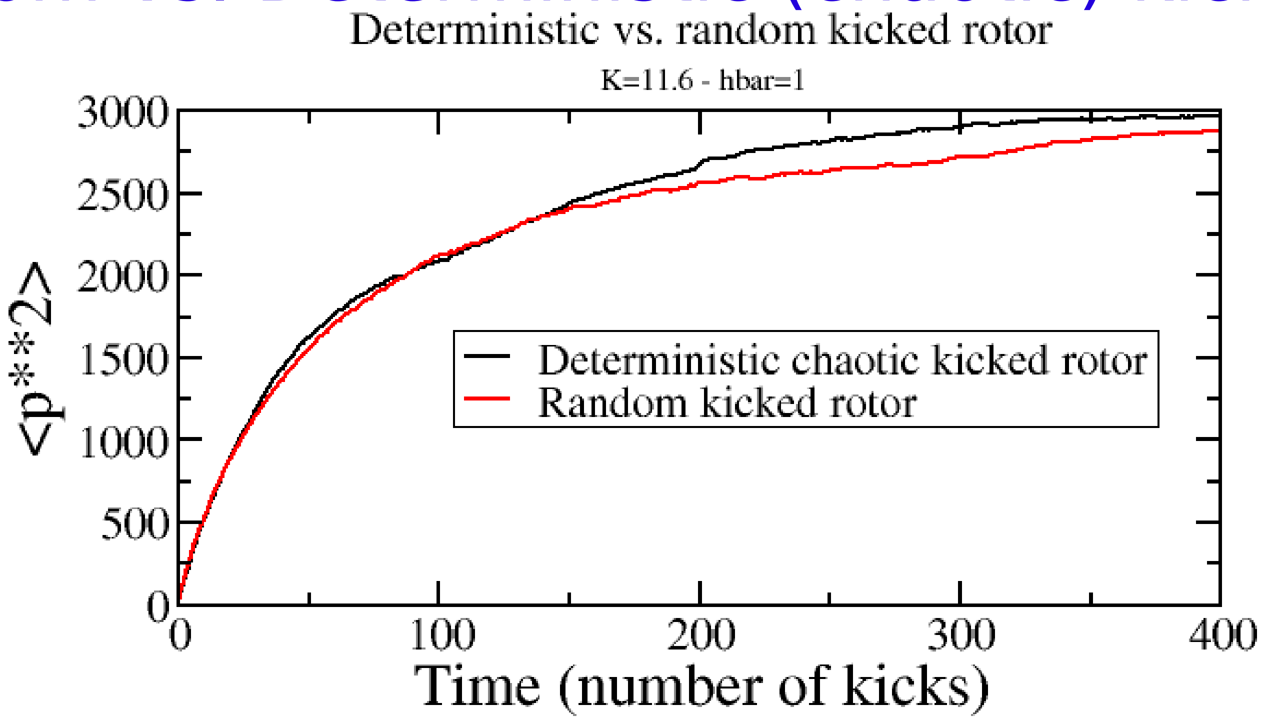
Anderson localization in 1d

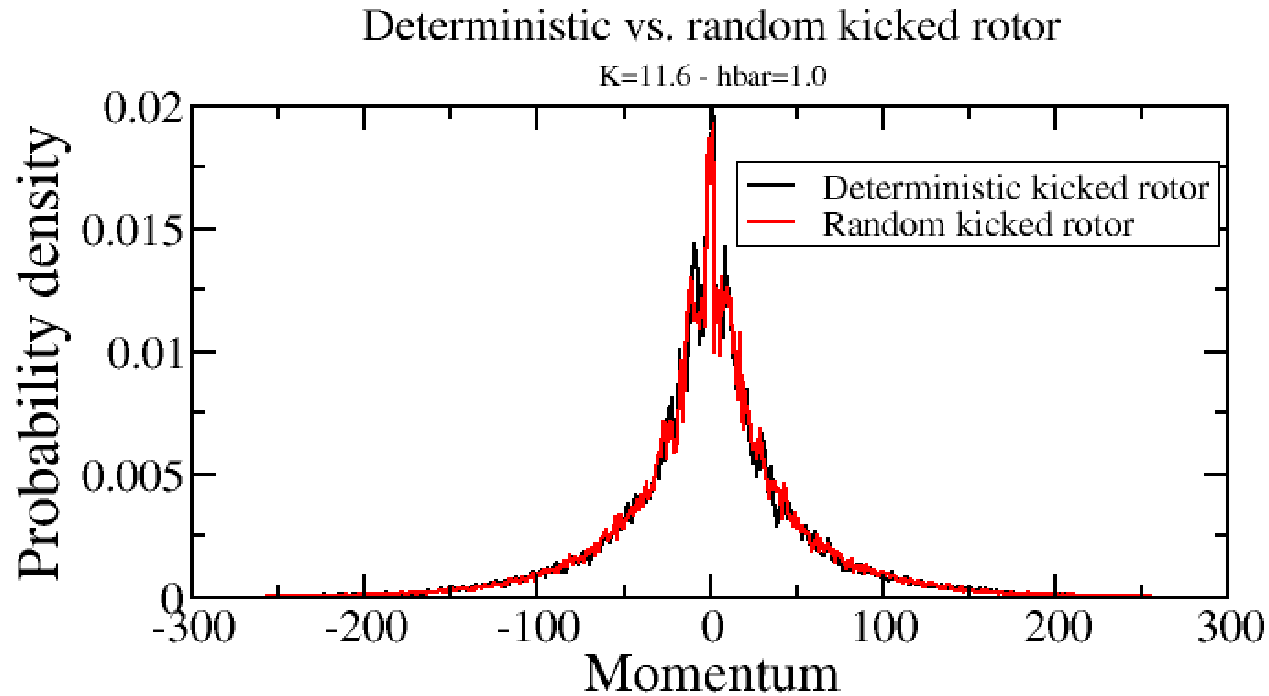
- When averaged over time and/or different realizations of the disorder, the fluctuations are smoothed out:



Averaged over 800 realizations of the disorder

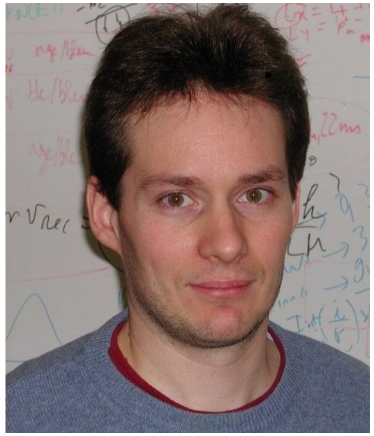
Random vs. Deterministic (chaotic) kicked rotor





The “kicked rotor” collaboration (Paris-Lille)


Dominique Delande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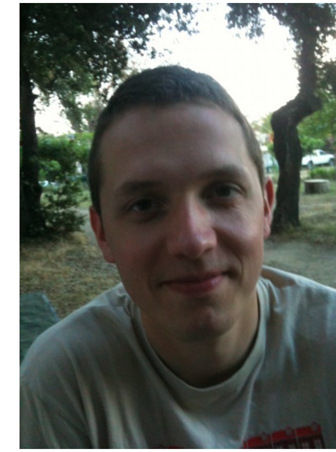
Pascal Szriftgiser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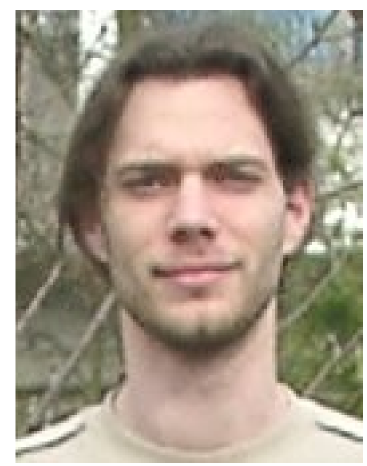
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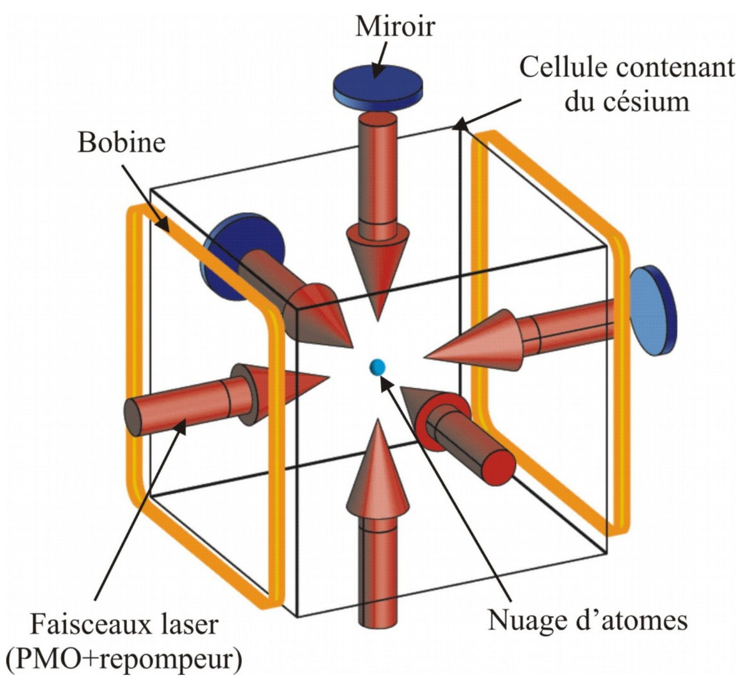
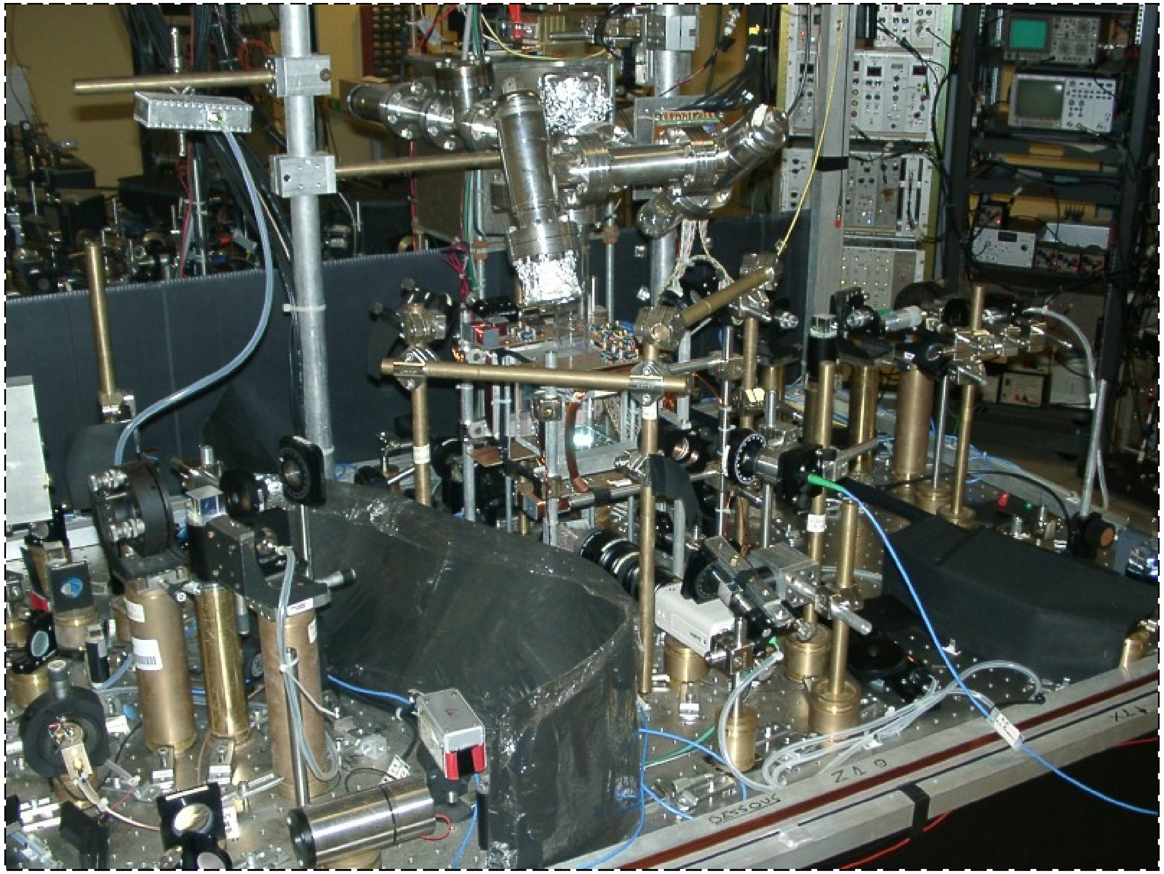
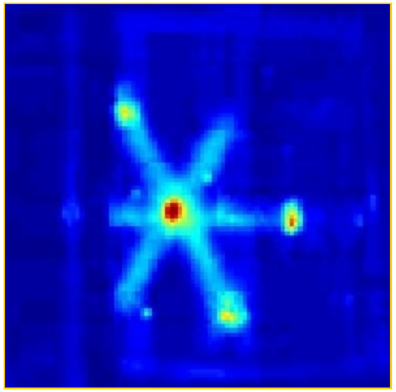
Université de Lille

Experimental setup

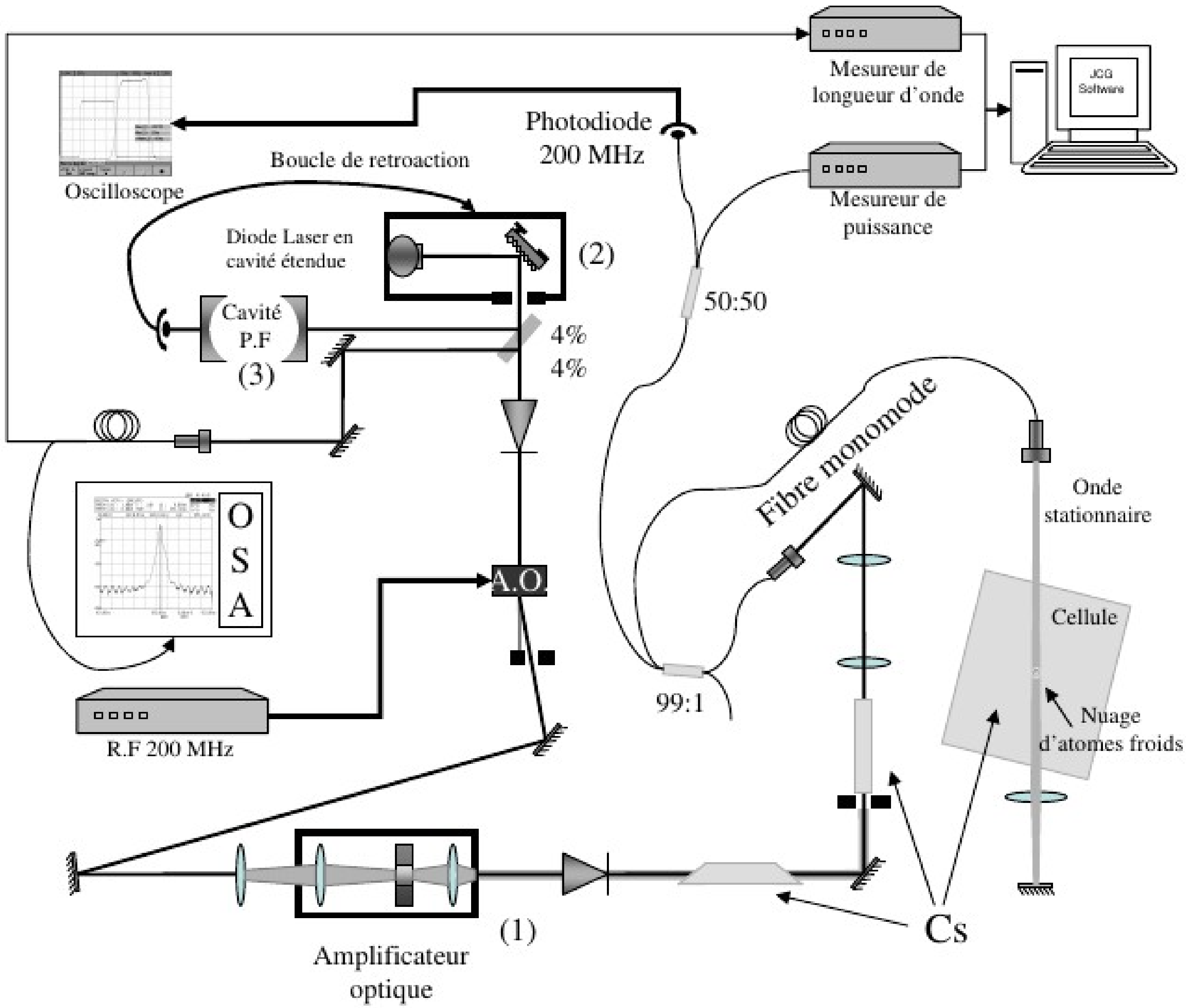




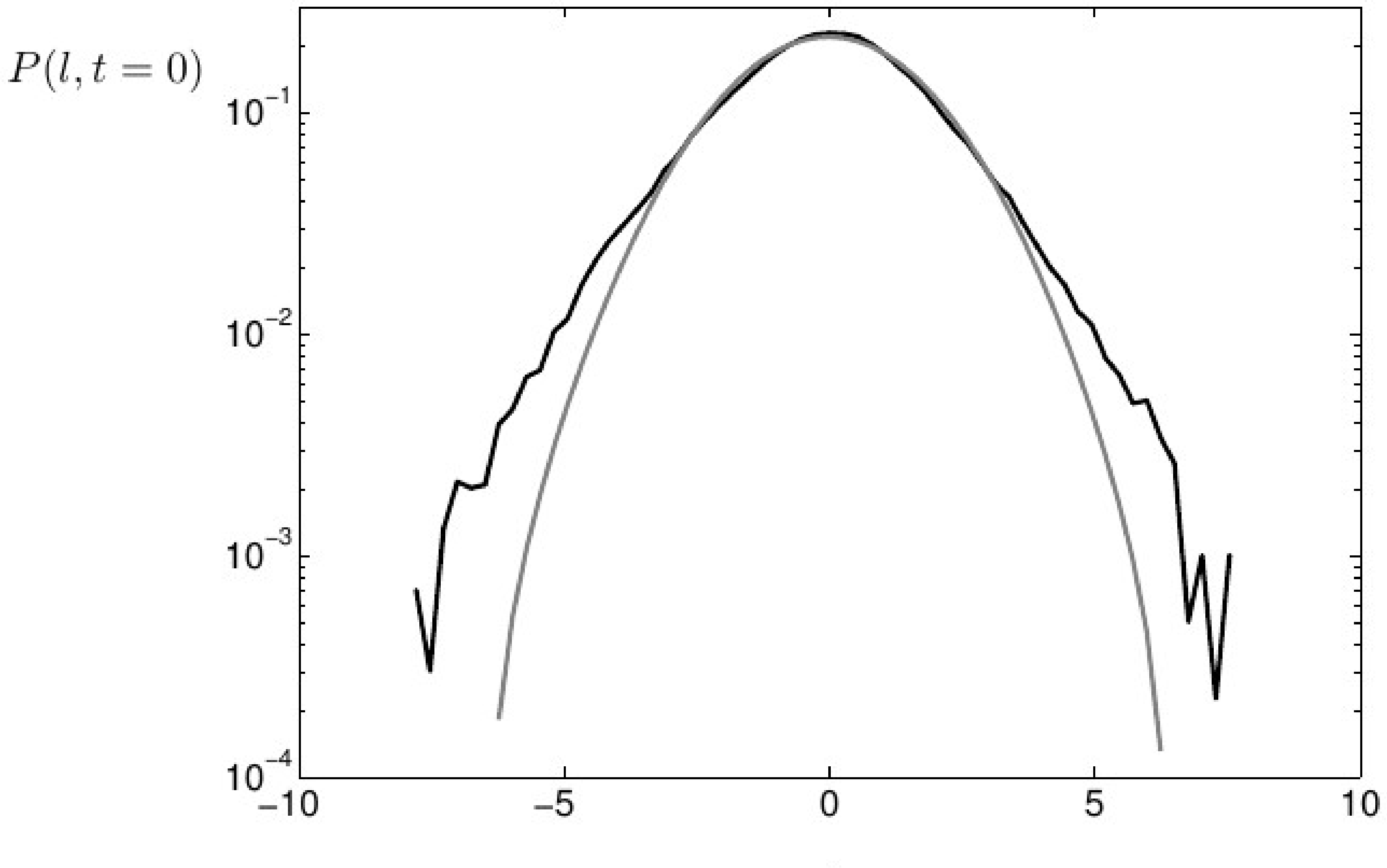


Standard Magneto-Optical Trap

Experimental setup (simplified)



Initial momentum distribution (experimental)

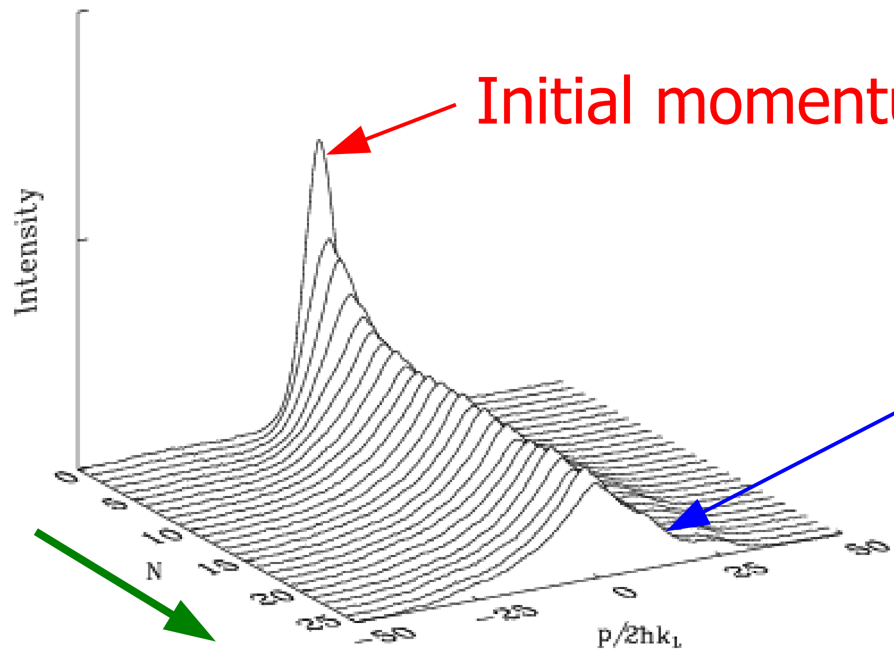


Momentum

Experimental observation of dynamical localization with cold atoms

Initial momentum distribution (Gaussian)

Final momentum distribution (exponentially localized)

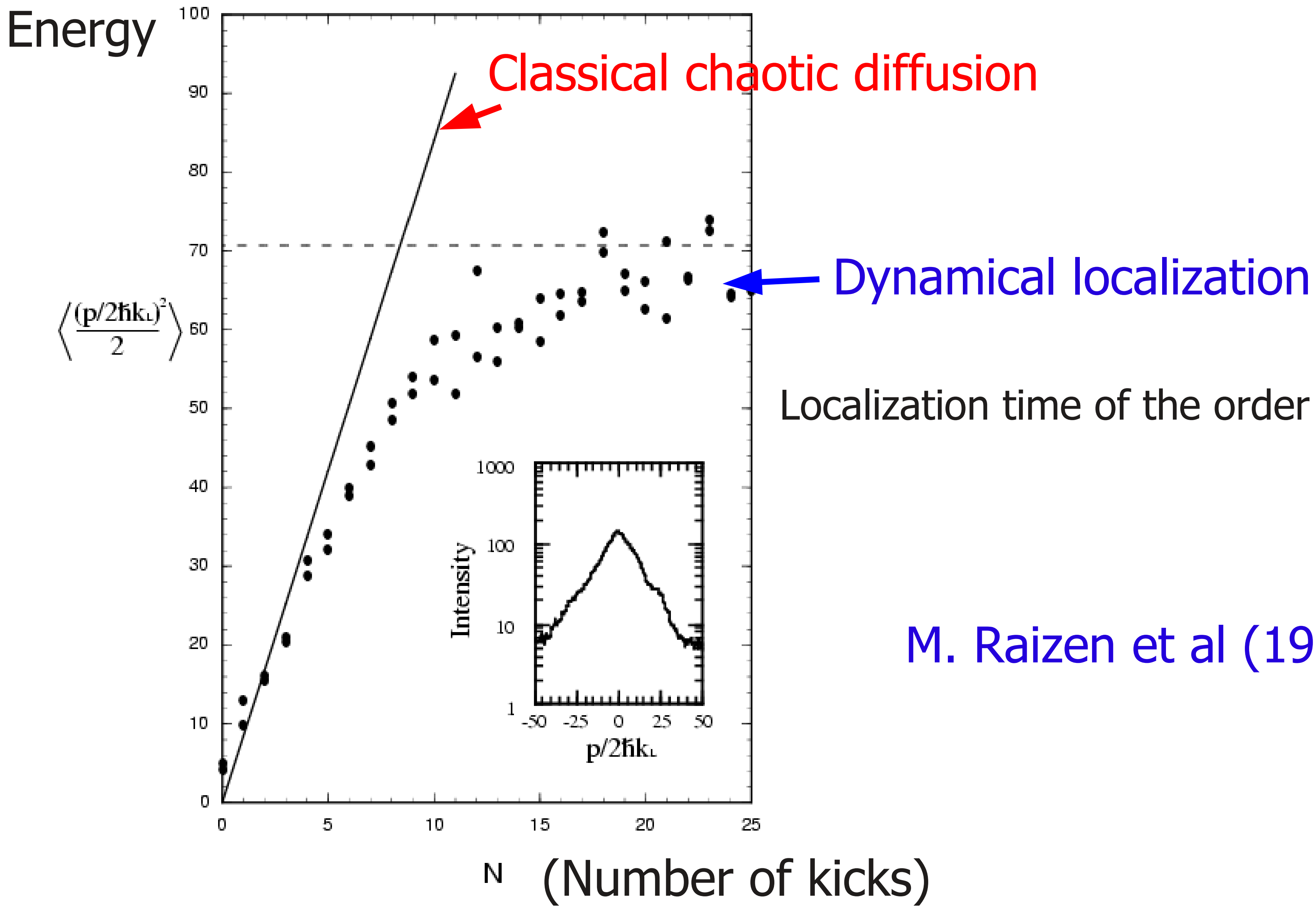


Time
(number of kicks)

Momentum (in units of
2 recoil momenta)

M. Raizen et al (1995)

Experimental observation of dynamical localization with cold atoms

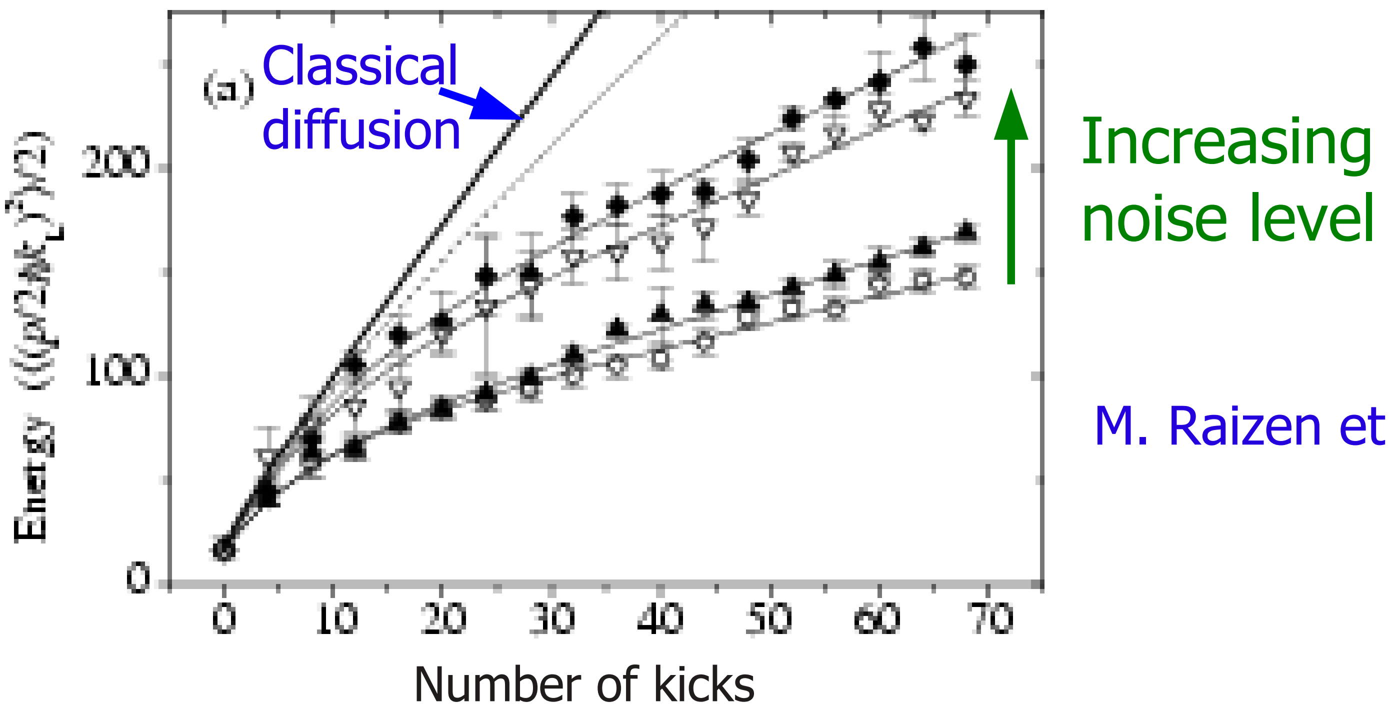


Localization time of the order of 10 kicks

M. Raizen et al (1995)

Destruction of dynamical localization by "noise"

- Change kick strength at each kick
  - The evolution is completely Hamiltonian, but the evolution operator over one kick varies.
  - Scrambles the phases and kills destructive interference effects => restoration of chaotic diffusion.
- Experimental observation using noise on the kick strength

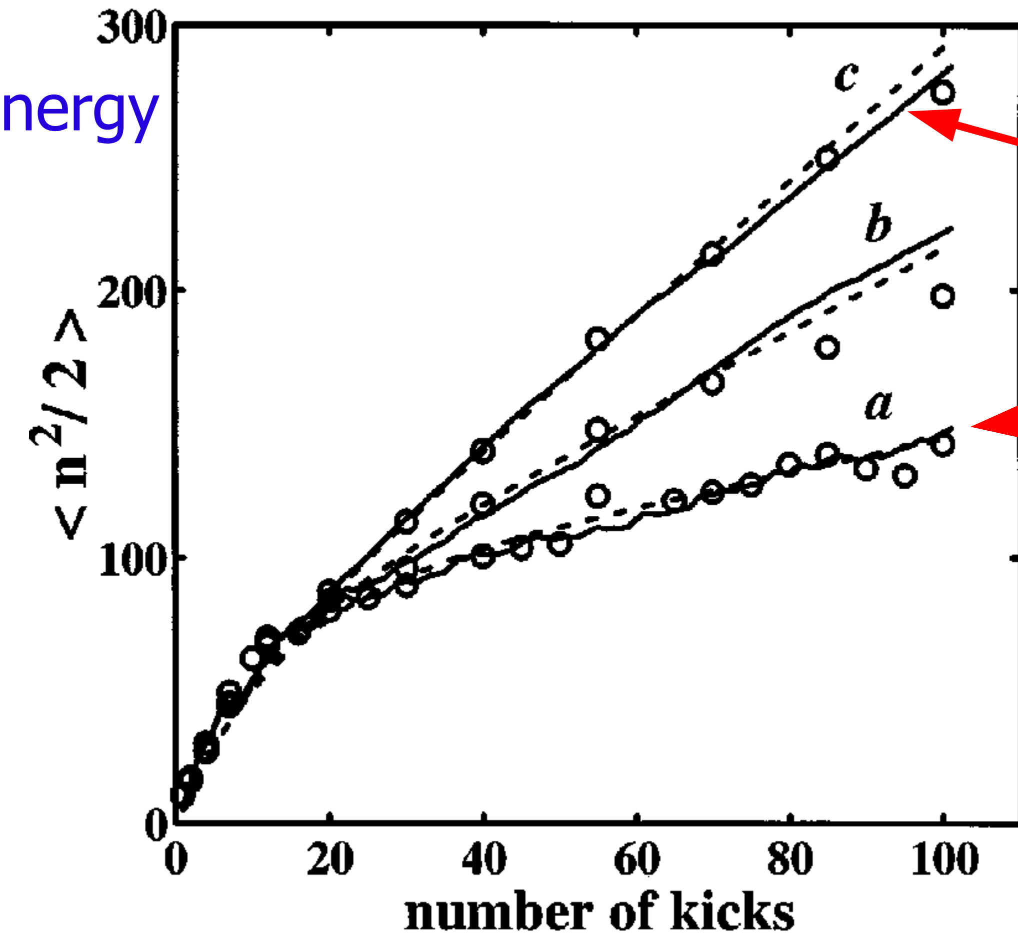


M. Raizen et al (1998)

Experimental observation of decoherence on kicked atoms

- Add some spontaneous emission events. One event is enough to kill the phase coherence of the atomic wavefunction (with negligible energy transfer) and destroy dynamical localization.

Energy



One spontaneous emission every 20 kicks

One spontaneous emission every 130 kicks

Ammann et al (1998)