# Kicked rotor and Anderson localization

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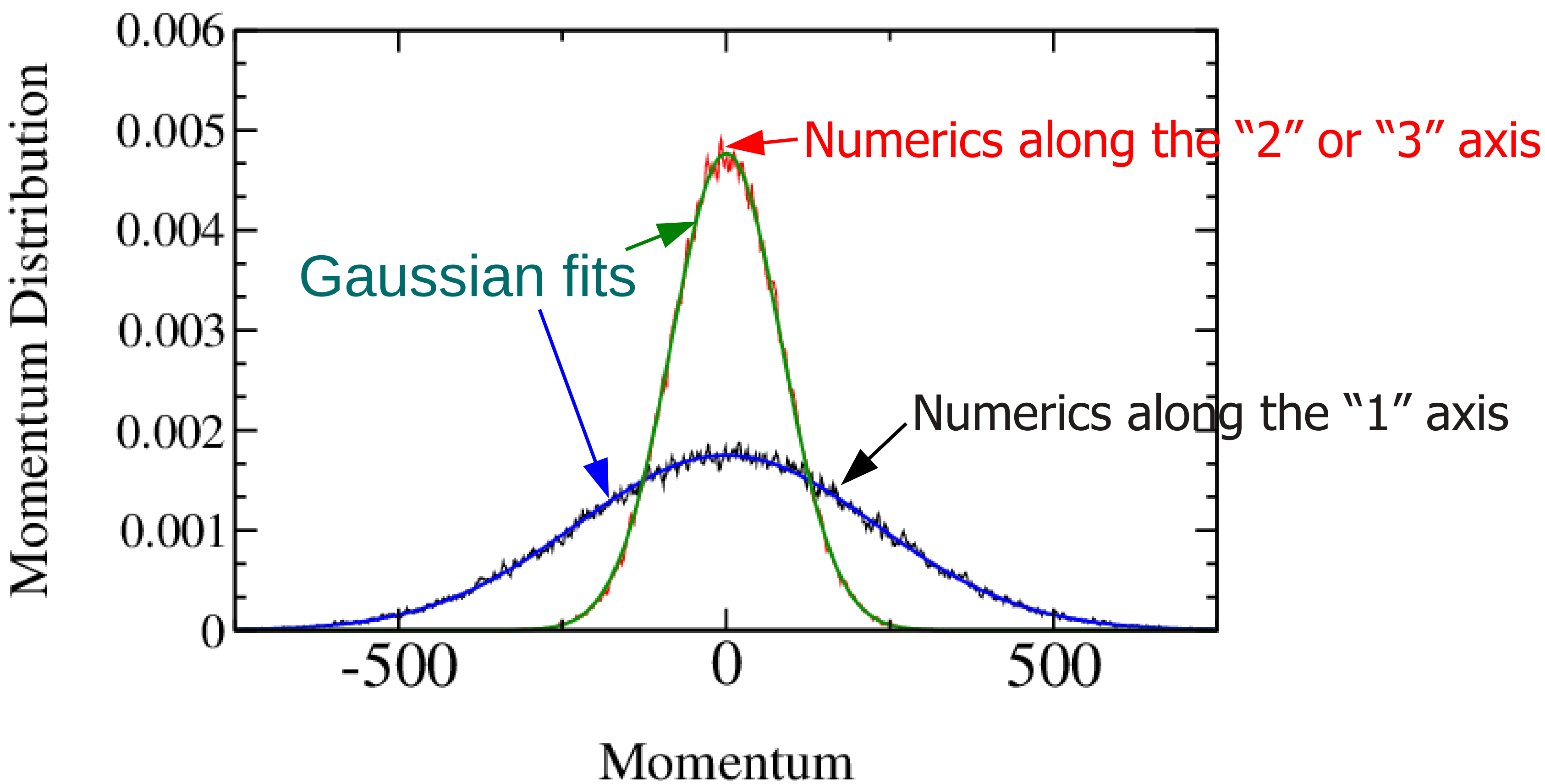
# Classical anisotropic diffusion

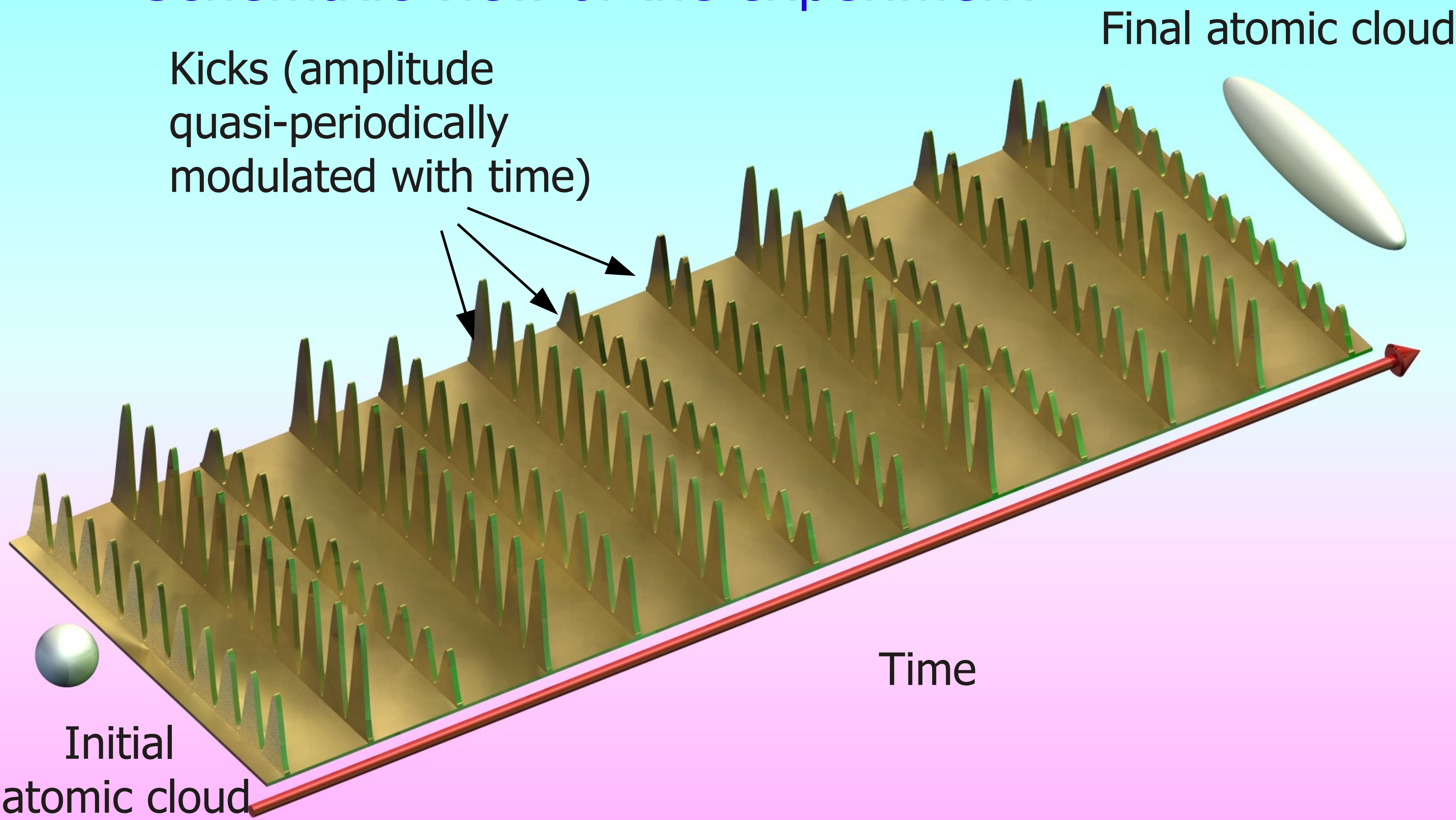



Diffusion tensor diagonal in the (1,2,3) axes: $D_{11} \neq D_{22} = D_{33}$

Approximate expressions: $$D_{11} = \frac{K^2}{4}\left(1 + \frac{\varepsilon^2}{4}\right)$$

$$D_{22} = D_{33} = \frac{K^2}{16}$$

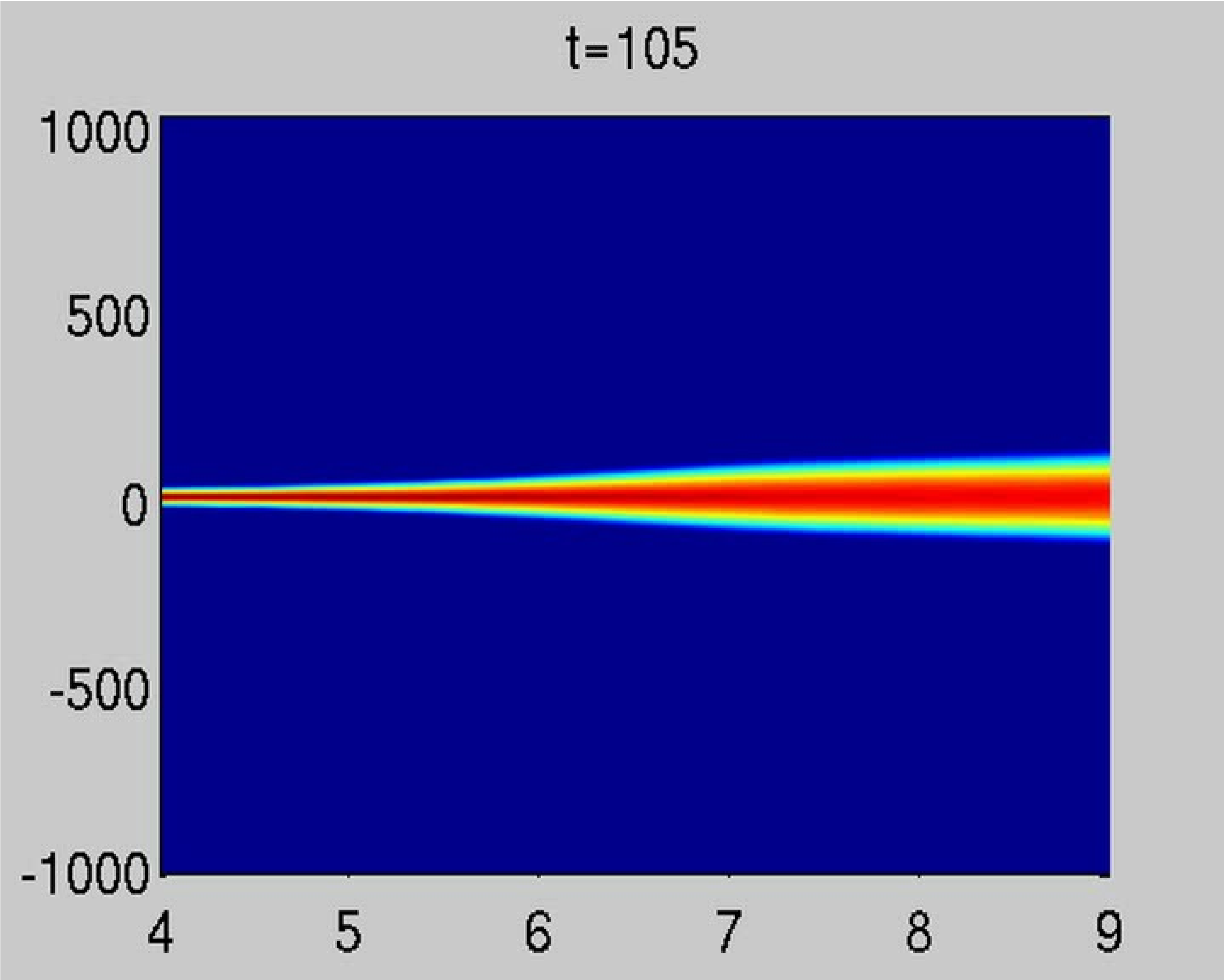
# Schematic view of the experiment




$$H = \frac{p^2}{2} + k \cos\theta \ (1 + \epsilon \cos\omega_2 t \ \cos\omega_3 t) \ \sum_n \delta(t - nT)$$

# Numerical results for the three-color kicked rotor

Momentum distribution

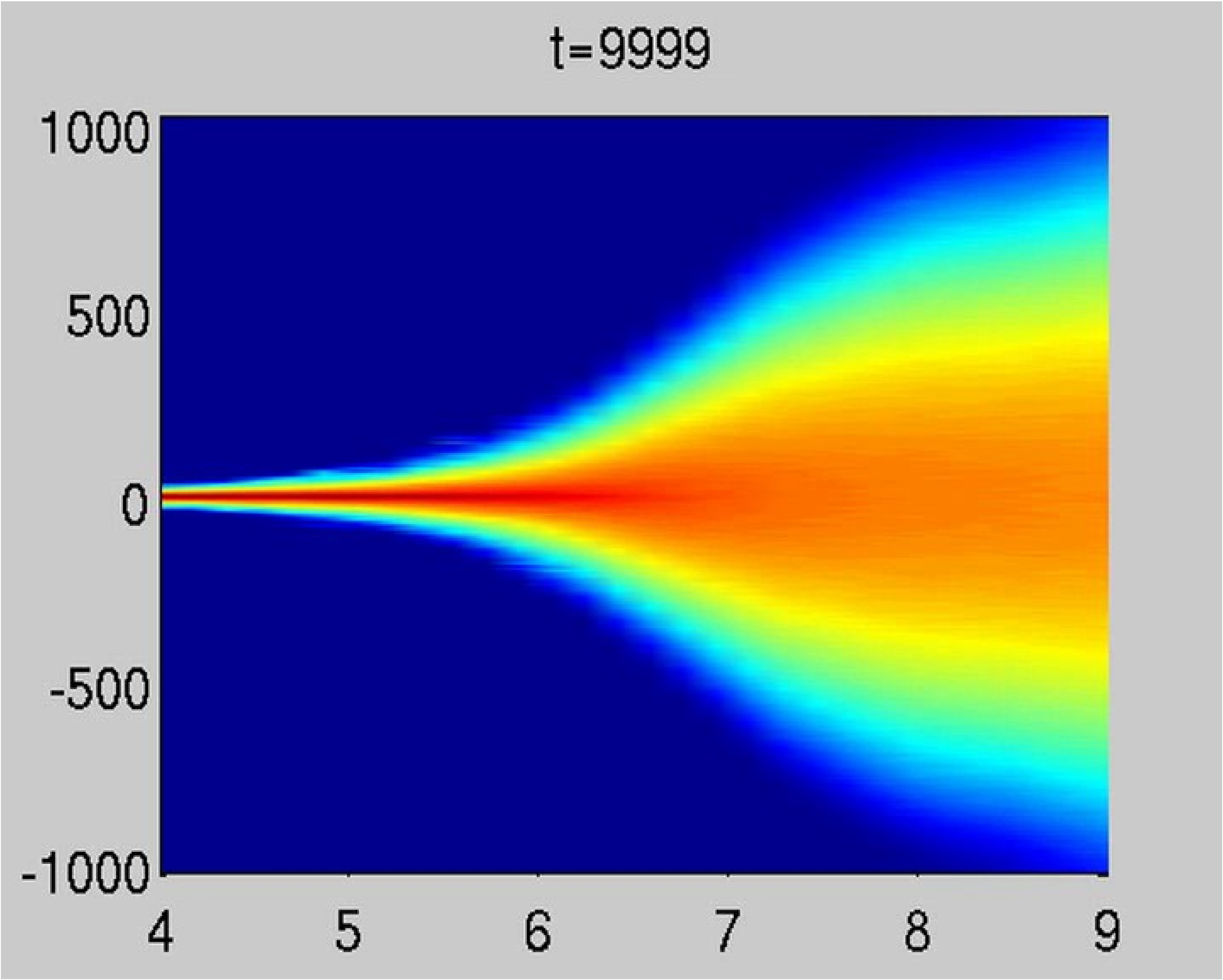



$K$ (kick strength)

# Numerical results for the three-color kicked rotor

Momentum distribution

Localized regime



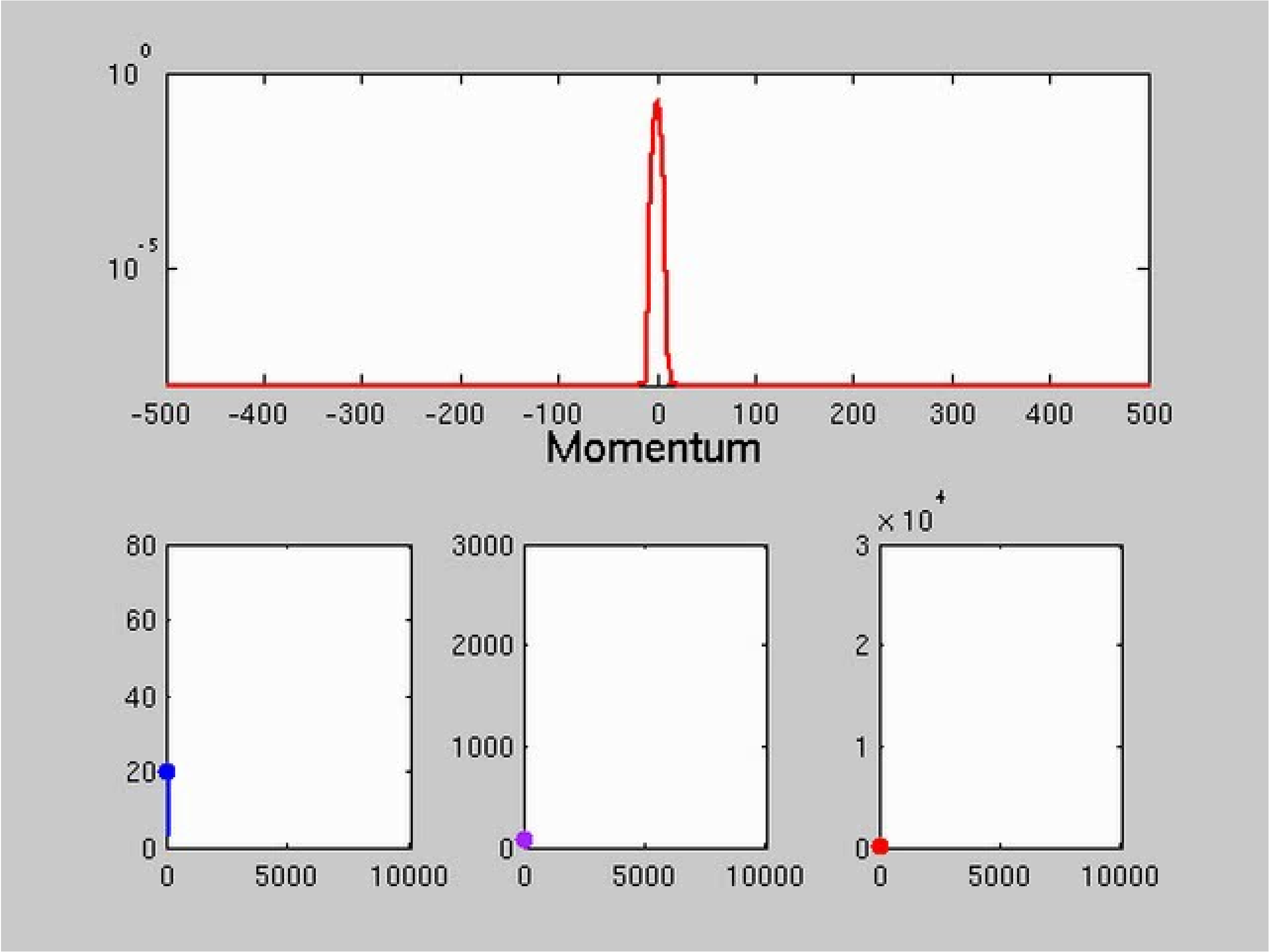

Diffusive regime

$K$ (kick strength)

# How to identify unambiguously the transition?

$|\psi(p)|^2$

$\langle p^2(t) \rangle$



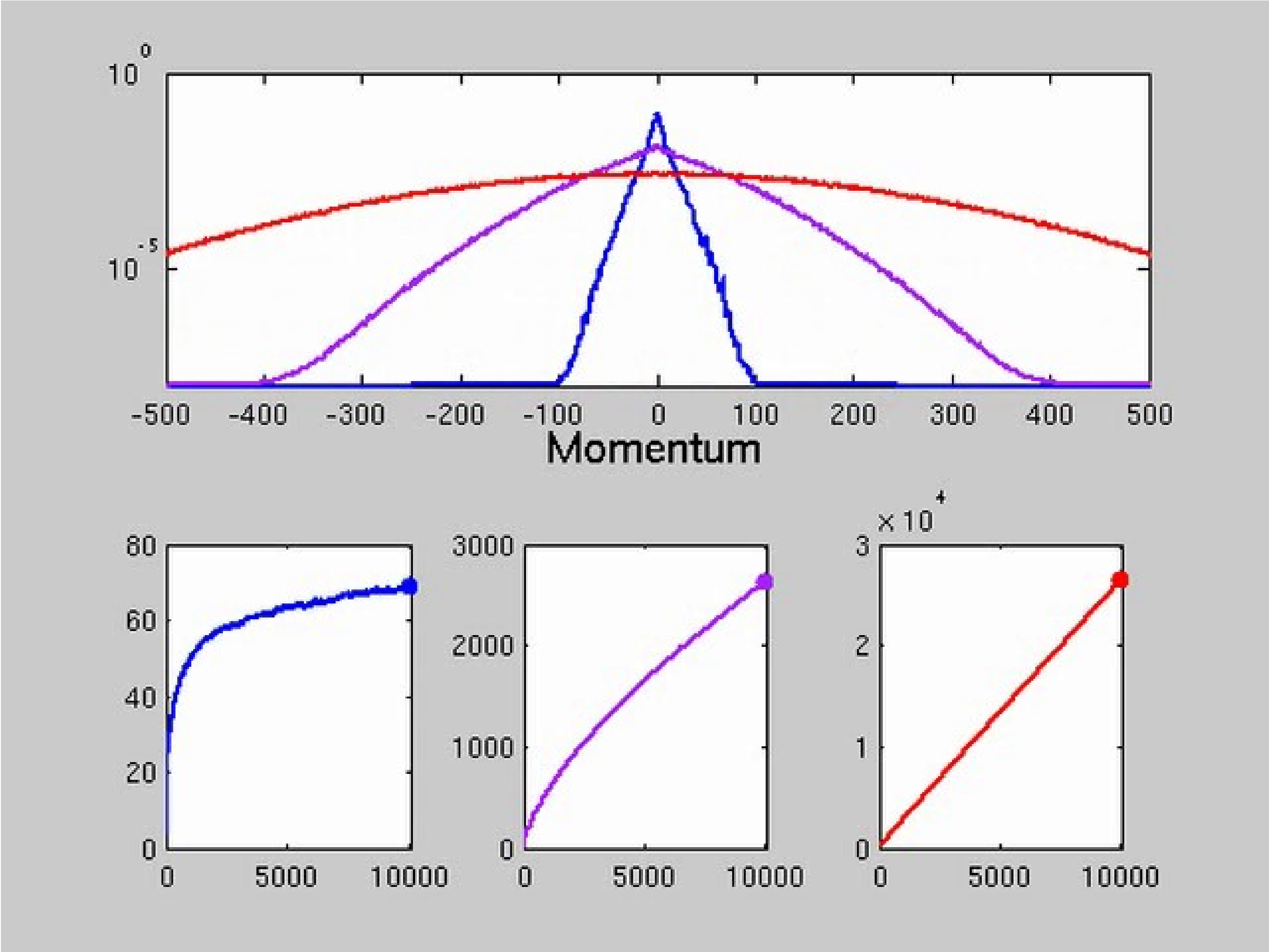

3 increasing $K$ values

Time (number of kicks)

# How to identify unambiguously the transition?

$|\psi(p)|^2$

$\langle p^2(t) \rangle$




3 increasing $K$ values

Time (number of kicks)

- At criticality, one expects an anomalous diffusion with

$$\langle p^2(t) \rangle \simeq t^{\gamma} \quad \text{with} \quad \gamma = \frac{2}{3}$$

# Phase diagram of the Anderson transition

$$\alpha = \frac{d \log \langle p^2(t) \rangle}{d \log t}$$

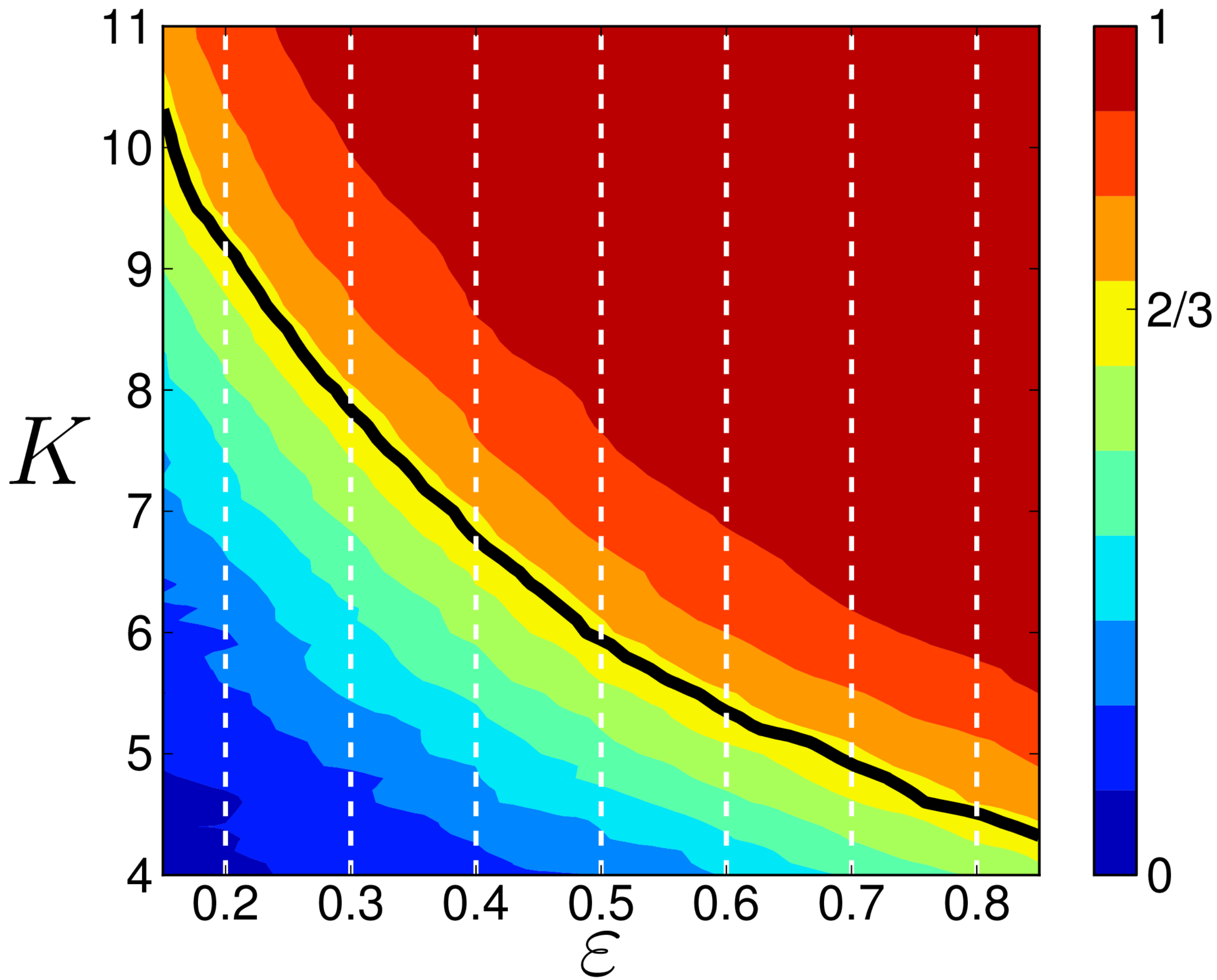
(from numerics)

$\alpha = 0$ Localized

$\alpha = 2/3$ Critical

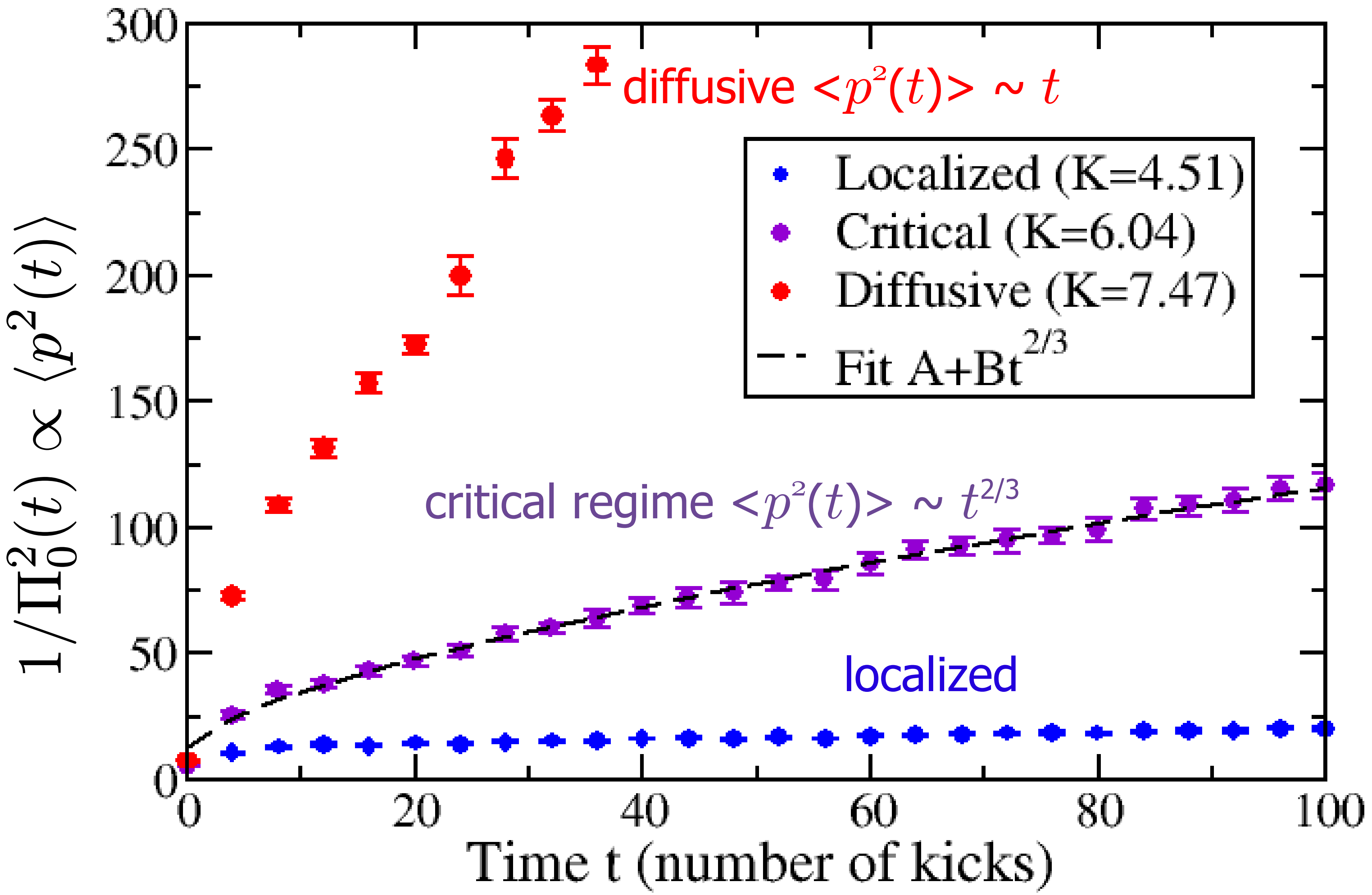
$\alpha = 1$ Diffusive

1000 kicks

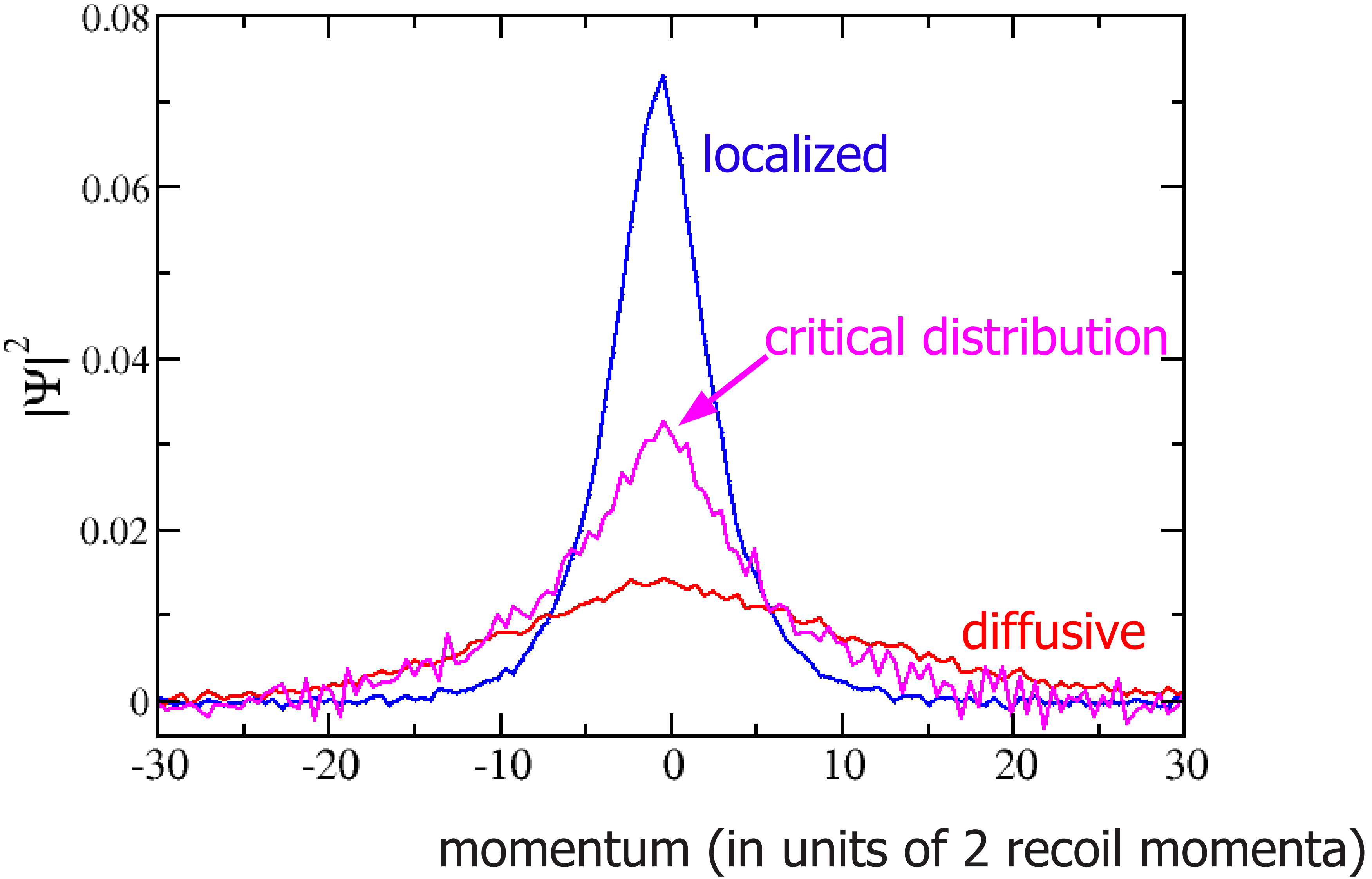



$$H = \frac{p^2}{2} + K \cos\theta \; (1 + \varepsilon \cos\omega_2 t \;\; \cos\omega_3 t) \; \sum_n \delta(t-n)$$

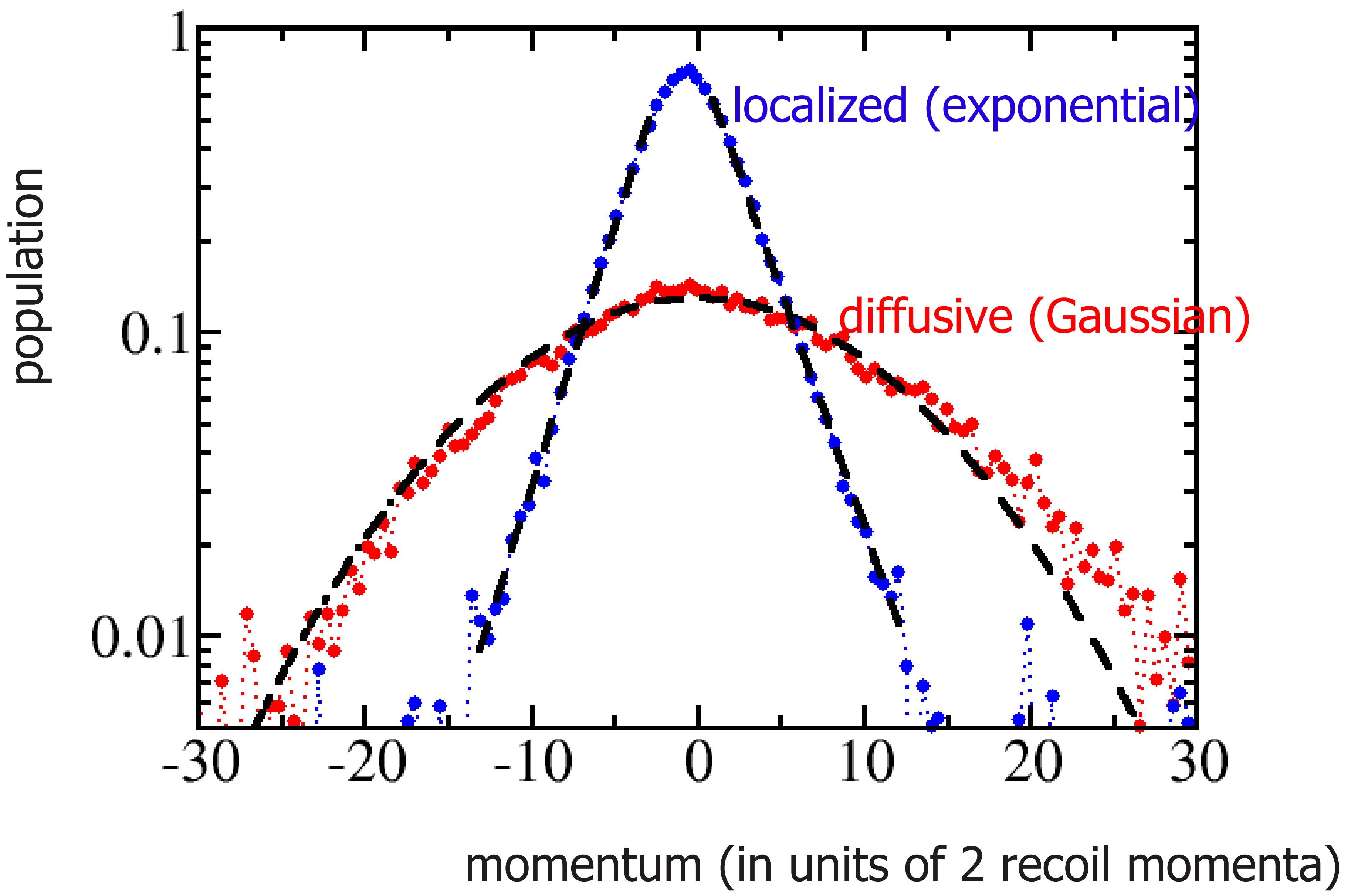
# From localization to diffusive regime: **experimental** results



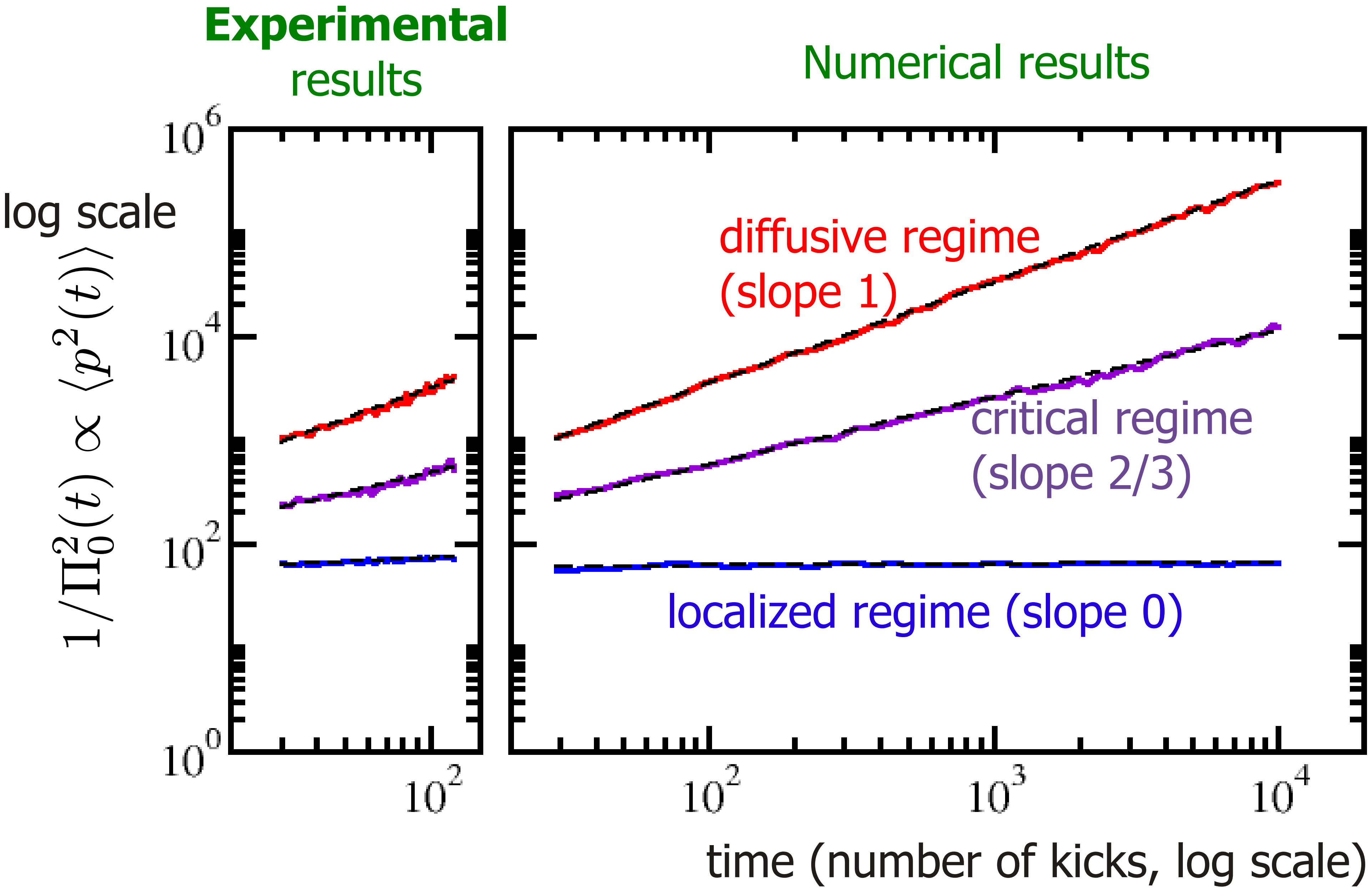
# Experimental momentum distributions



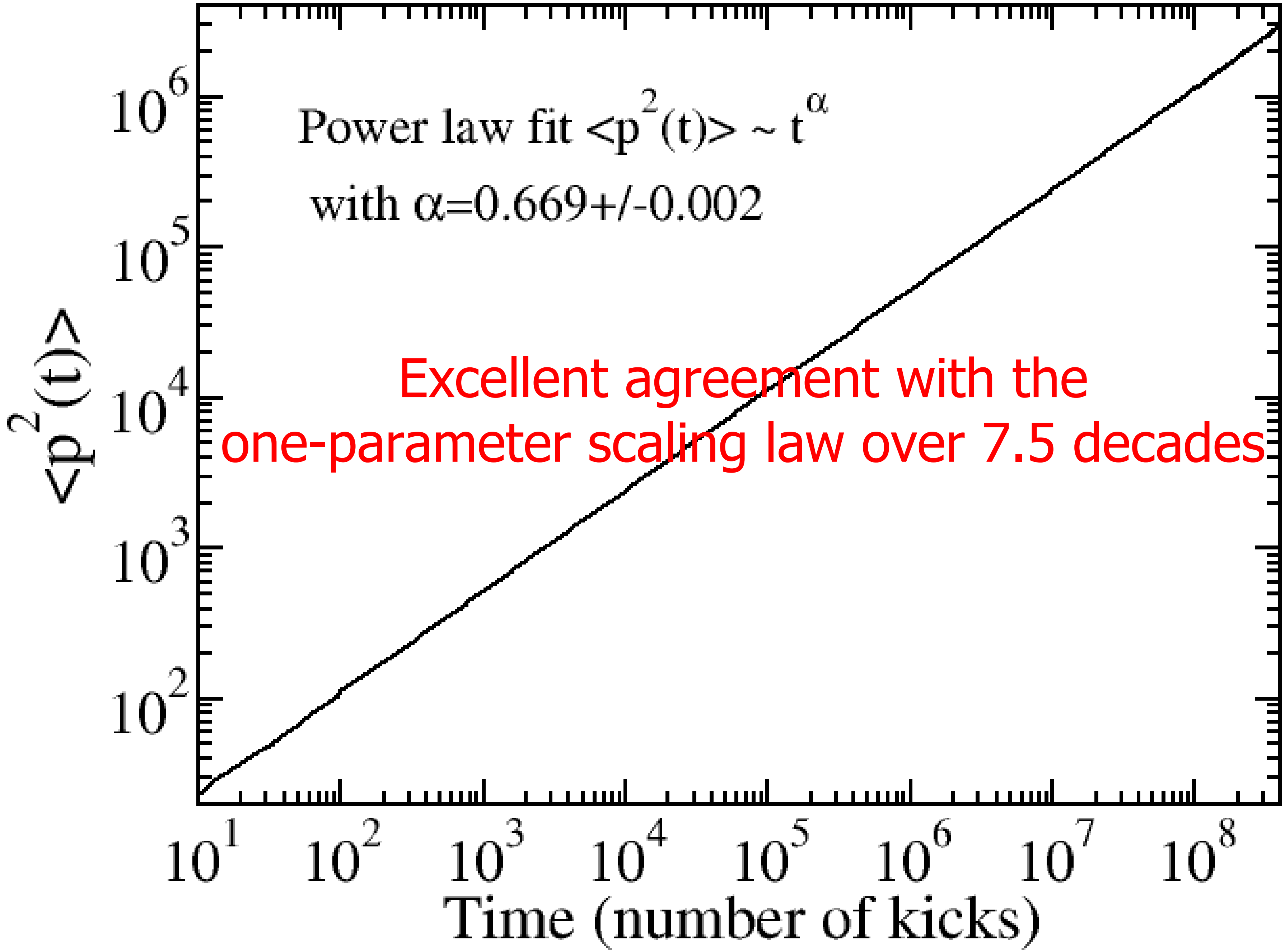
# Experimental momentum distributions



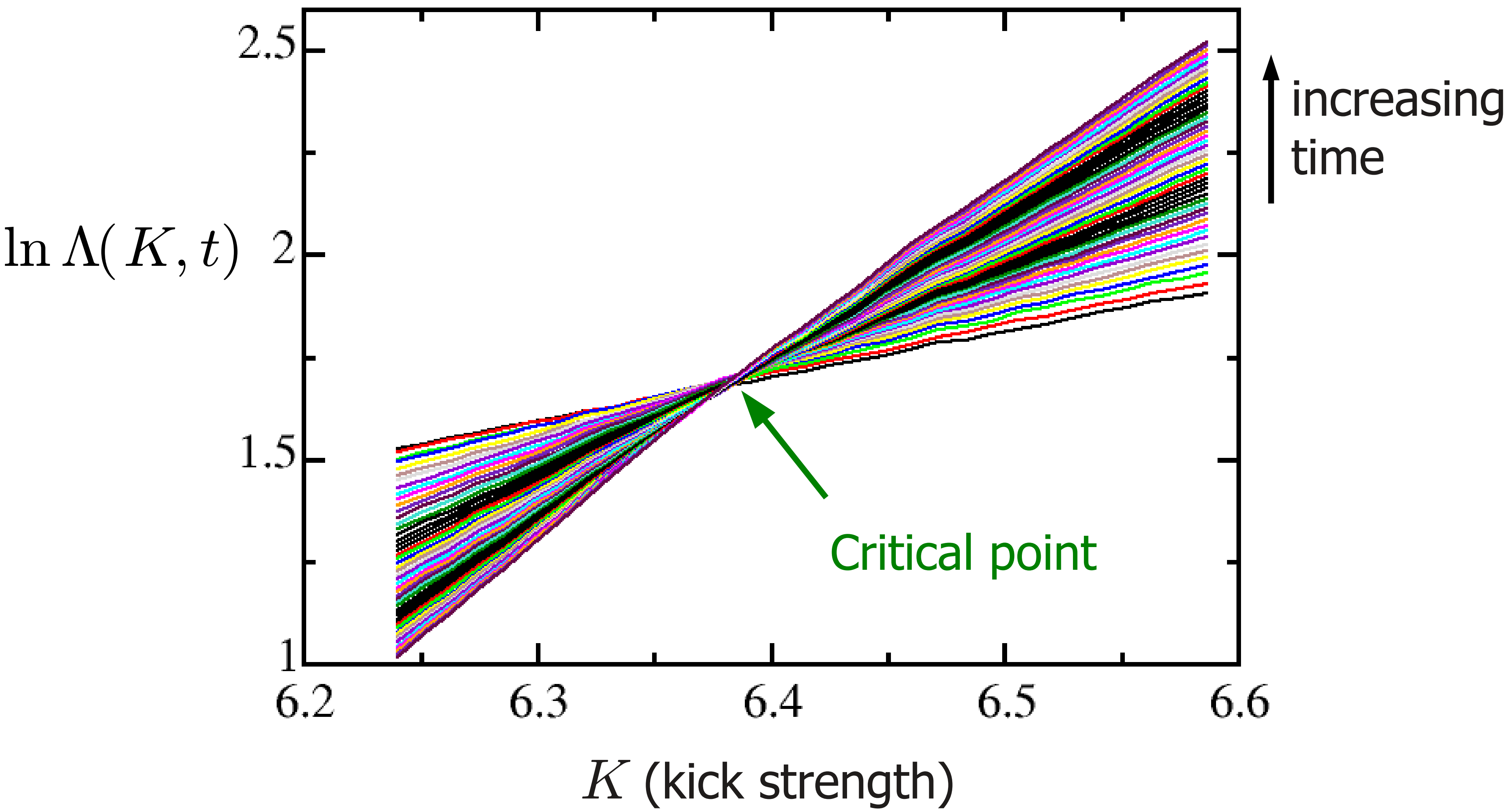
# From localized to diffusive regime



# Critical regime of the quasi-periodic kicked rotor

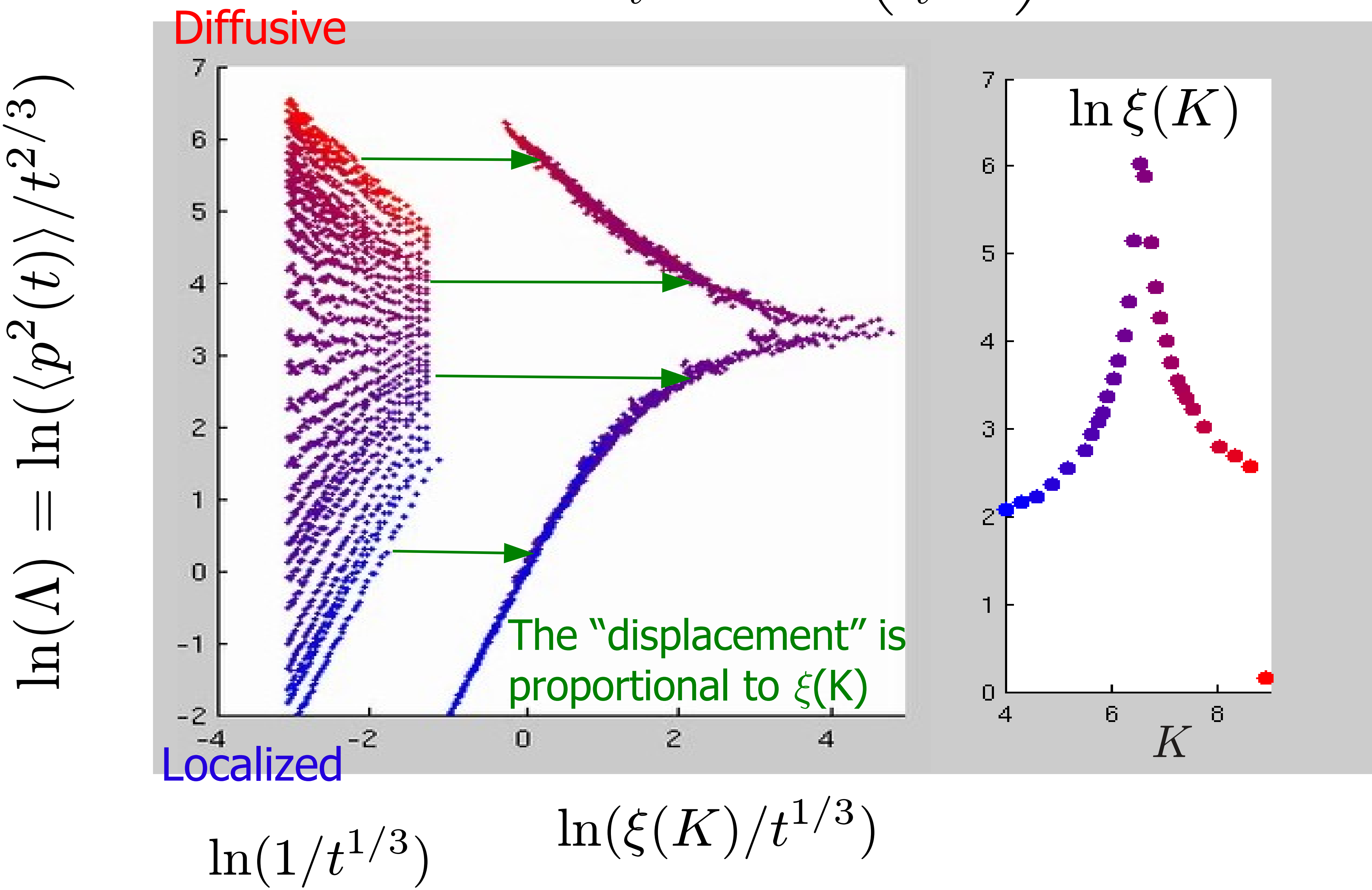


# Rescaled dynamics at various times (numerics)



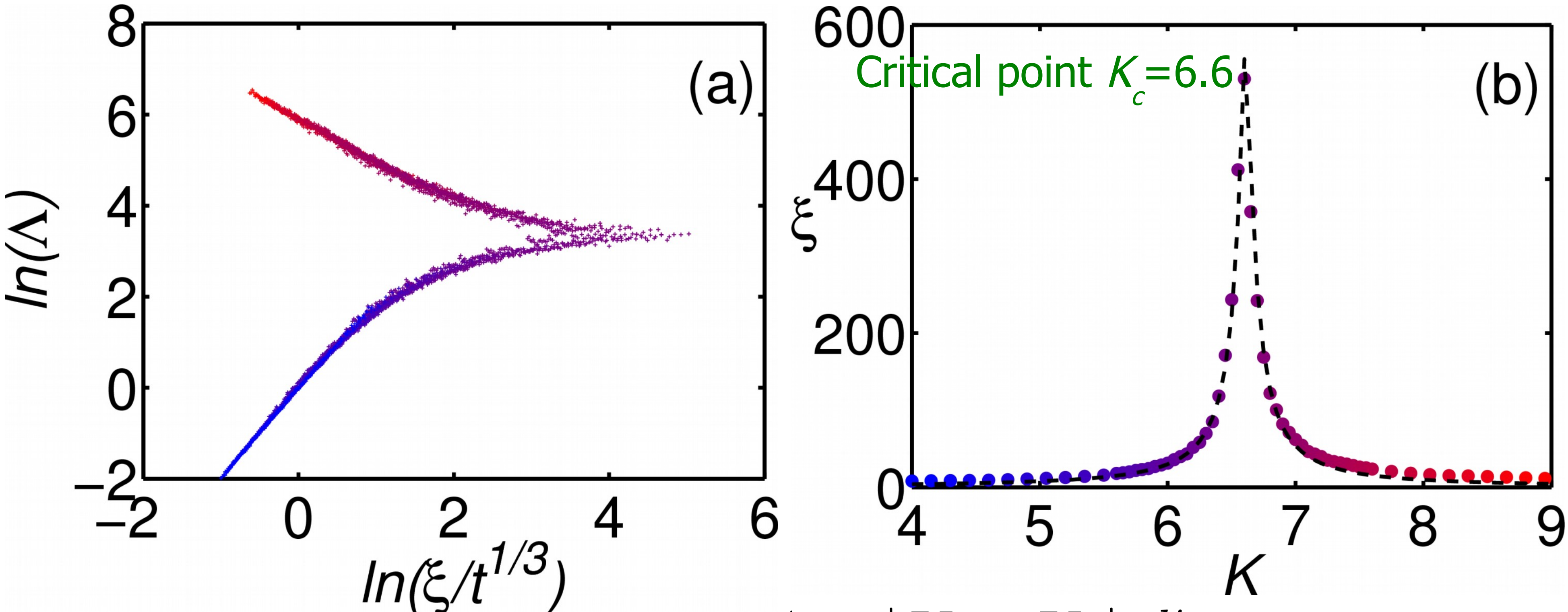
# Finite time scaling

$$\Lambda(t) = \frac{\langle p^2(t) \rangle}{t^{2/3}} = \mathcal{F}\left(\frac{\xi(K)}{t^{1/3}}\right)$$



# Finite time scaling analysis of numerical results

Scaling function $\Lambda(t) = \frac{1}{\Pi_0^2(t)\ t^{2/3}}$ Localization length




$\xi \sim |K - K_c|^{-\nu}$ Critical exponent

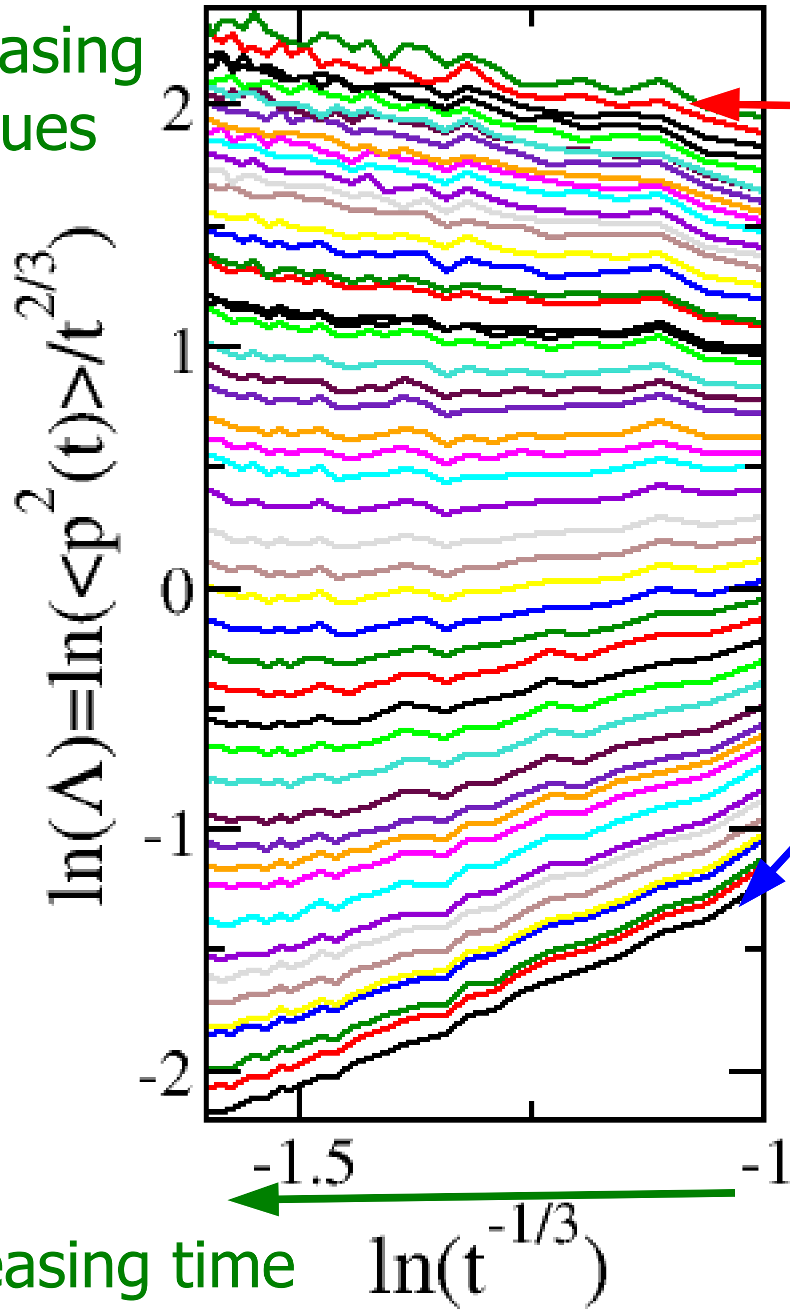
$\nu = 1.60 \pm 0.05$

Chabé et al, PRL, **101**, 255702 (2008)

Numerical data up $10^6$ kicks, latest result: $\nu = 1.58 \pm 0.02$

# Rescaled **experimental** results

increasing K values




increasing time

diffusive (slope -1)

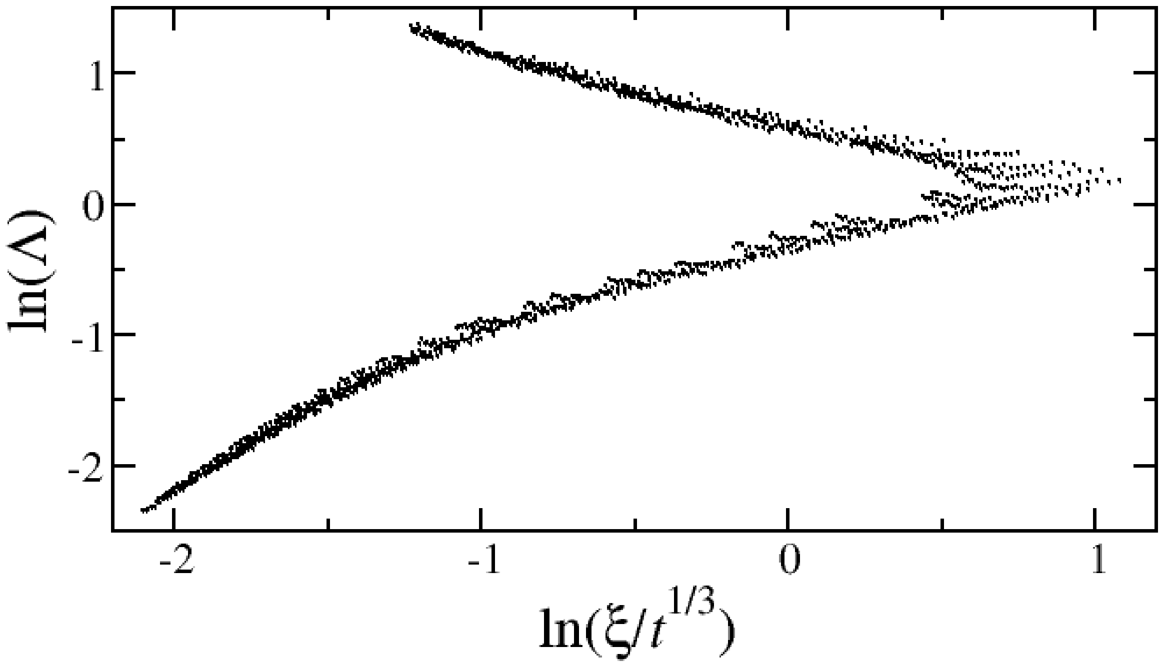
$$\Lambda(K,t) = \frac{\langle p^2(t)\rangle}{t^{2/3}} \approx \frac{1}{\Pi_0^2(t)\ t^{2/3}}$$

$\Pi_0(t)$ : Population in the zero-velocity class

localized (slope 2)

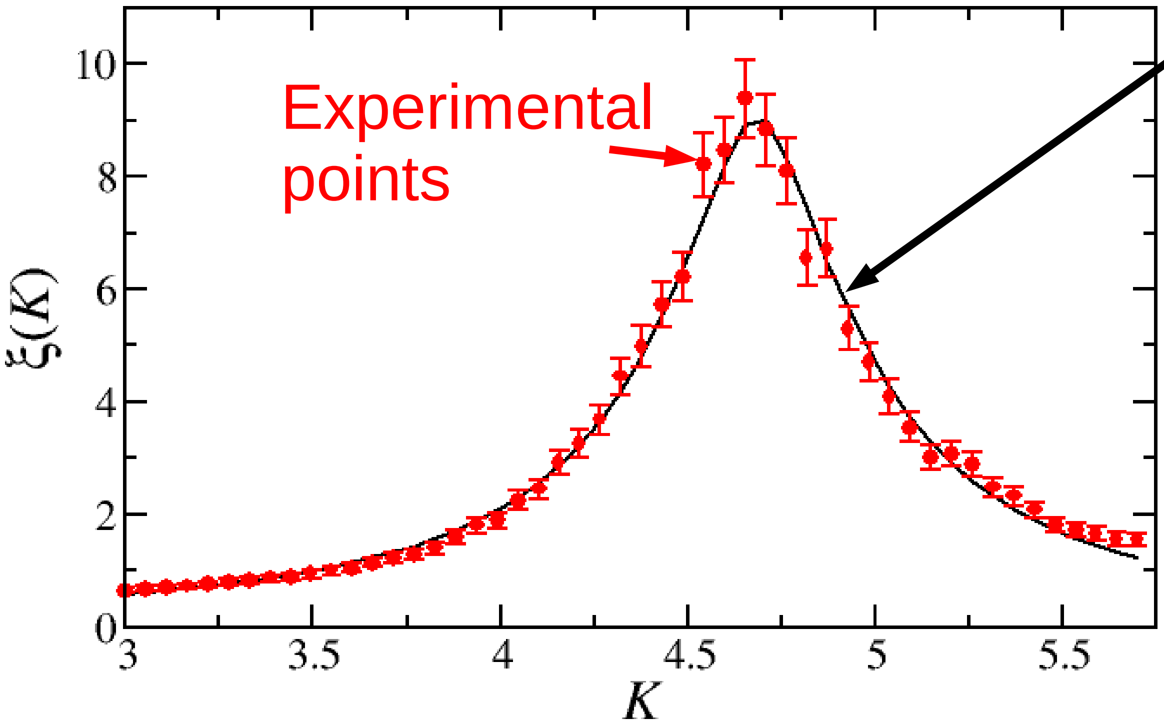
- The critical regime is the horizontal line.
- Problem: it requires very long times to accurately measure the position of the transition as well as the critical exponent.

# **Experimental** measurement of the critical exponent




Scaling function:

$$\Lambda = \frac{\langle p^2(t) \rangle}{t^{2/3}} = F\left(\frac{\xi(K)}{t^{1/3}}\right)$$




Fit using:

$$\frac{1}{\xi(K)} = \alpha(K - K_c)^{\nu} + \beta$$

$\beta$ : cut-off taking into account experimental imperfections

$$\nu = 1.64 \pm 0.08$$

M. Lopez et al, PRL, 108, 095701 (2012), arxiv:1108.0630

# Universality of the critical exponent: **experimental test**

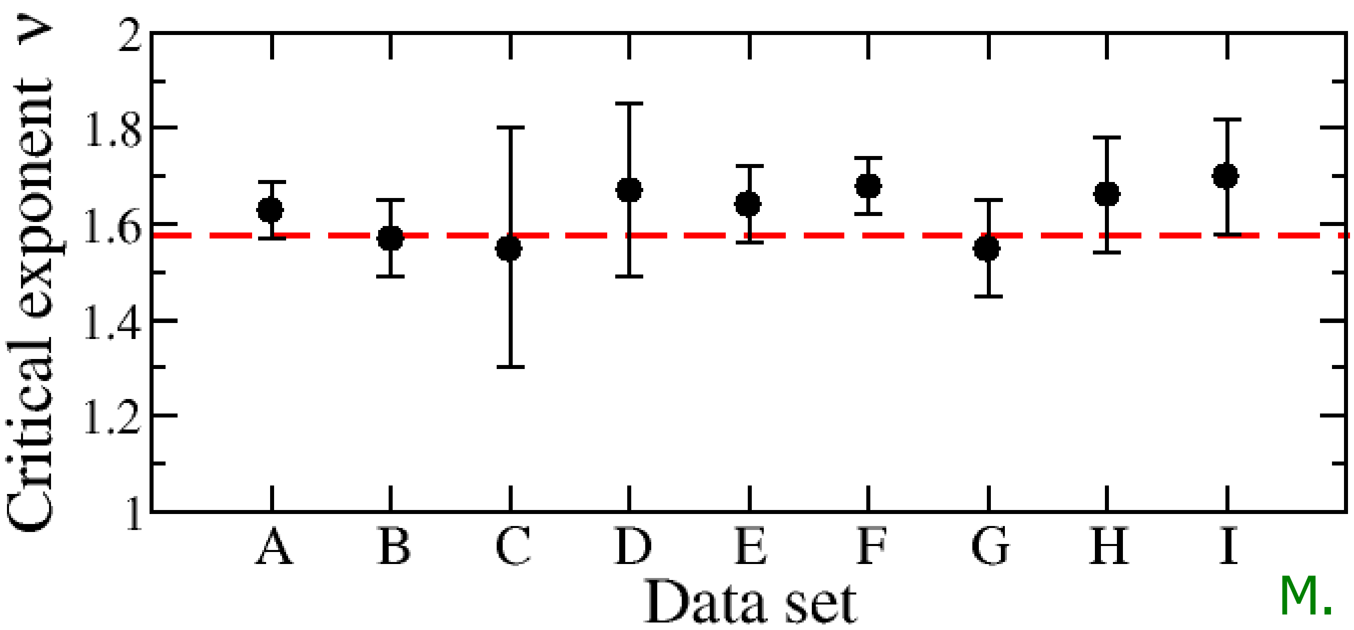
The critical exponent is universal

Weighted average:

$$\nu = 1.63 \pm 0.05$$

| | $\hbar$ | $\frac{\omega_2}{2\pi}$ | $\frac{\omega_3}{2\pi}$ | Path in $(K, \varepsilon)$ | $K_c$ | $\nu$ |
|---|---|---|---|---|---|---|
| A | 2.89 | $\sqrt{5}$ | $\sqrt{13}$ | 4,0.1 $\rightarrow$ 8,0.8 | 6.67 | 1.63±0.06 |
| B | 2.89 | $\sqrt{7}$ | $\sqrt{17}$ | 4,0.1 $\rightarrow$ 8,0.8 | 6.68 | 1.57±0.08 |
| C | 2.89 | $\sqrt{5}$ | $\sqrt{13}$ | 3,0.435 $\rightarrow$ 10,0.435 | 5.91 | 1.55±0.25 |
| D | 2.89 | $\sqrt{5}$ | $\sqrt{13}$ | 7.5,0 $\rightarrow$ 7.5,0.73 | $\varepsilon_c$=0.448 | 1.67±0.18 |
| E | 2.00 | $\sqrt{5}$ | $\sqrt{13}$ | 3,0.1 $\rightarrow$ 5.7,0.73 | 4.69 | 1.64±0.08 |
| F | 2.31 | $\sqrt{5}$ | $\sqrt{13}$ | 4,0.1 $\rightarrow$ 9,0.8 | 6.07 | 1.68±0.06 |
| G | 2.47 | $\sqrt{5}$ | $\sqrt{13}$ | 4,0.1 $\rightarrow$ 9,0.8 | 5.61 | 1.55±0.10 |
| H | 3.46 | $\sqrt{5}$ | $\sqrt{13}$ | 4,0.1 $\rightarrow$ 9,0.8 | 6.86 | 1.66±0.12 |
| I | 3.46 | $\sqrt{5}$ | $\sqrt{13}$ | 4,0.1 $\rightarrow$ 9,0.8 | 7.06 | 1.70±0.12 |

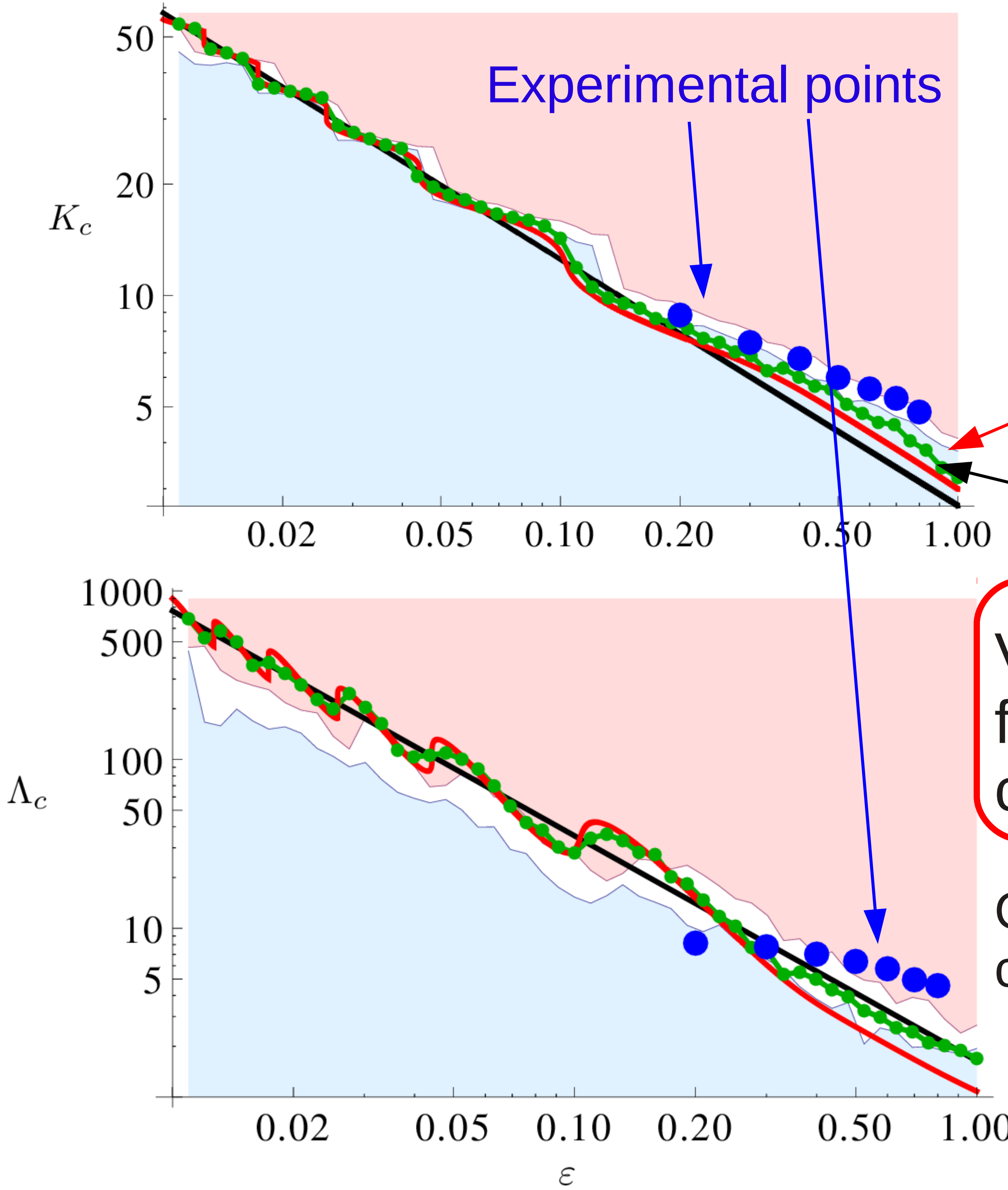
Table of data sets




$\nu = 1.58$

M. Lopez et al, PRL, 108, 095701 (2012), arxiv:1108.0630

# Prediction of the self-consistent theory of localization




M. Lopez et al, NJP 15, 065013 (June 2013) arXiv:1301.1615

improved predictions

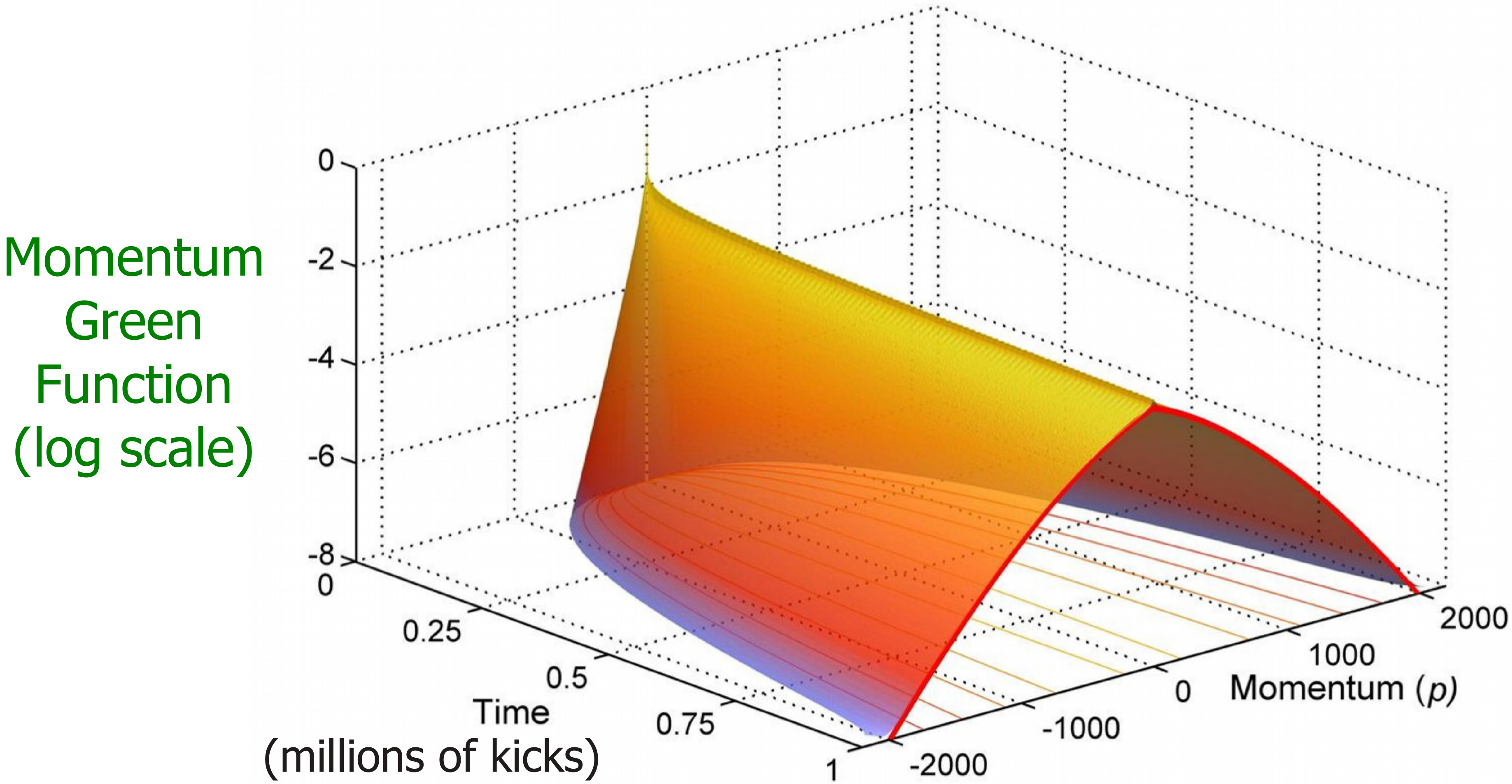
simple prediction

Very good agreement for the position of the critical point $K_c$

Only fair agreement for the critical "conductance" $\Lambda_c$

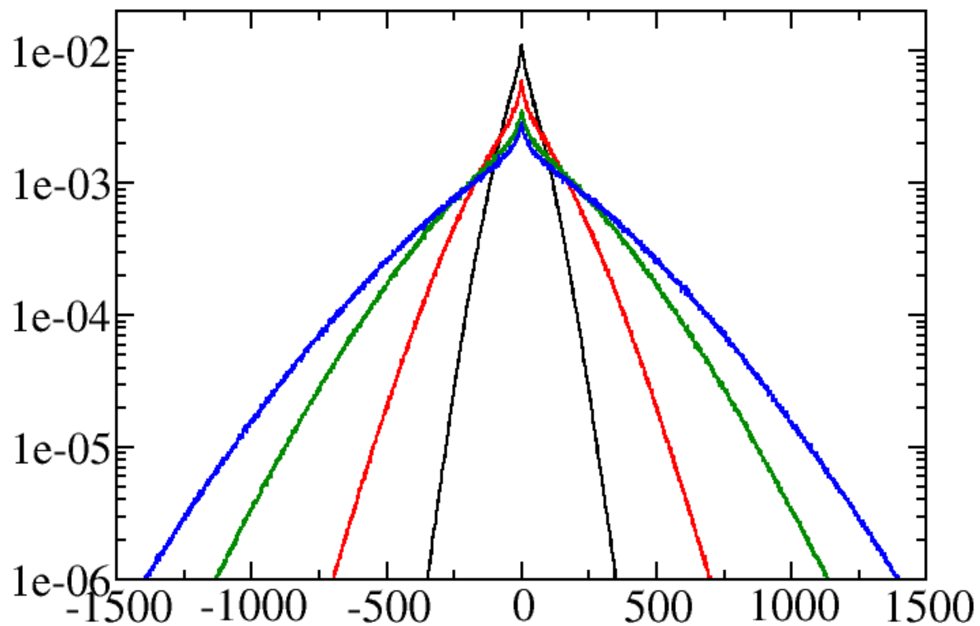
# Momentum distribution at the critical point

- Very localized initial state => $\langle|\psi(p,t)|^2\rangle$ is a direct measure of the average intensity Green function $G(0,p;t)$
- Numerical experiment at the critical point:

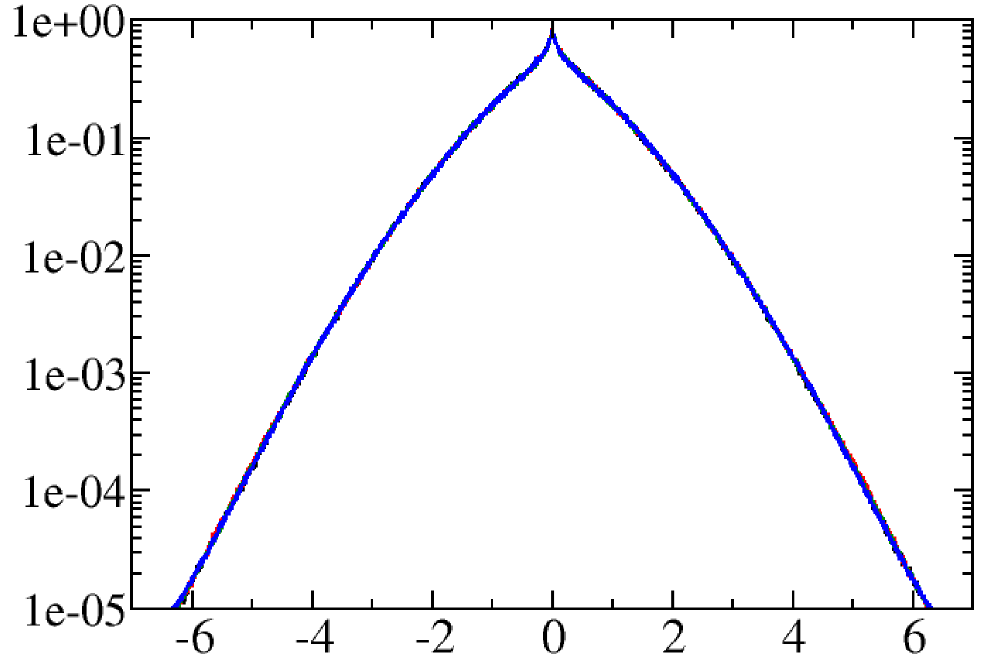



- Time invariant shape (neither Gaussian, nor exponential)

# Momentum distributions at criticality




Distributions at various times




Distributions at various times rescaled by the critical $t^{1/3}$ law

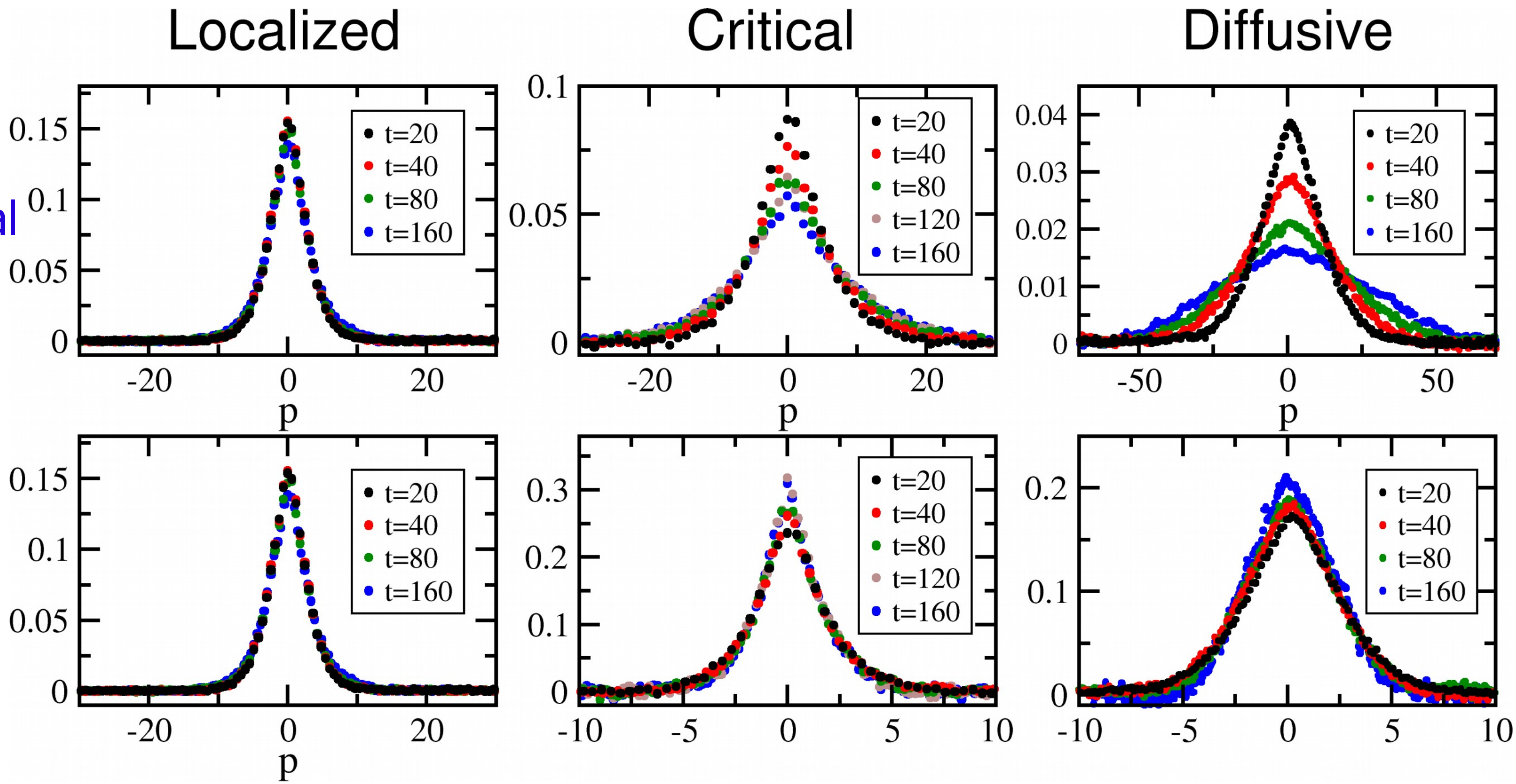
# Experimental measurements in the critical regime

- Characterized by a specific scaling: $p \propto t^{1/3}$

$p \propto t^0$ $\quad$ $p \propto t^{1/3}$ $\quad$ $p \propto t^{1/2}$

Raw experimental data

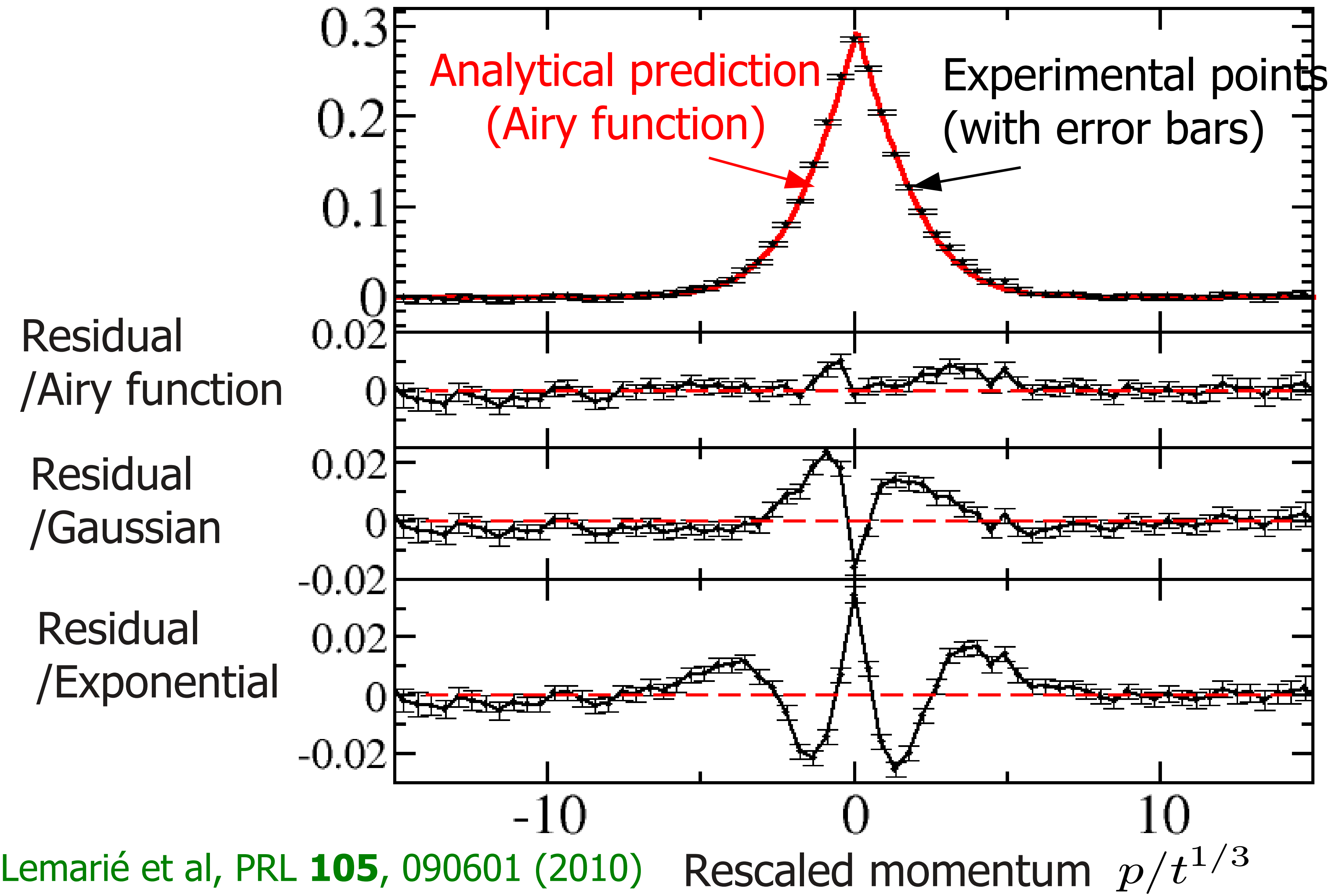
Rescaled data




$p/t^0$ $\quad$ $p/t^{1/3}$ $\quad$ $p/t^{1/2}$

Lemarié et al, Phys. Rev. Lett. **105**, 090601 (2010)

# Experimentally measured critical Green function




Lemarié et al, PRL **105**, 090601 (2010)