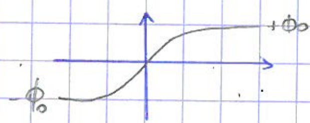
A flavor of the computation of the dynamic scaling:

$T=0 \to$ no noise: $\frac{\partial \phi}{\partial t}(\vec{r},t) = \nabla^2 \phi(\vec{r},t) - \frac{\delta V}{\delta \phi(\vec{r},t)}$ $\quad V = -\frac{\mu}{2}\phi^2 + \frac{g}{4}\phi^4$

* trivial solution $\phi(r,t) = \pm \phi_0 \ \forall r,t$.

* finite wall



$\phi(x) = \pm \phi_0 \, \mathrm{th}\sqrt{\frac{\mu}{2}}(x - \underbrace{0}_{\text{position}})$

ALLEN-CAHN expanded around this solution: velocity interior direction of the curvature

↳ only static if wall is vertical!

↳ rounds and shrinks domain:

tend to round

$\frac{dA}{dt} = \frac{d(\pi R^2)}{dt} = 2\pi R \frac{dR}{dt} = -\frac{2\pi \lambda^2(T)}{2\pi} R \underbrace{K}_{\text{curvature } = 1/R} = -\lambda(T)$

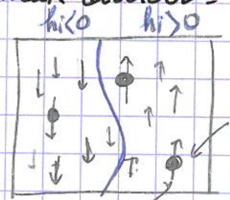
$\Rightarrow A(t) = A_0 - \lambda^2(T)t \to$ linearly vanishing (any form of domain) in 2d

↳ $R(t) \sim t^{1/2}$.

Collapse the domain wall

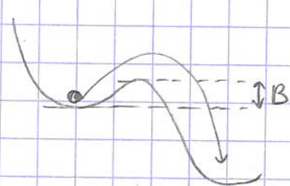
HULL ENCLOSED AREAS.

weak disorder.



fixed spins imposing random field.

$\to$ the domain wall is pinned by disorder $\to$ hard to move



Arrhenius time scale: $t_A \sim t_0 e^{B/k_BT}$

$B \sim$ length of the domain wall we would like to move $B(R)$.

↳ droplet argument: $\Delta F(R) \sim -\varepsilon R^d + \sigma R^{d-1}$

↳ $R^* = \max \Delta F(R)$ as barrier $\to R^* \sim \frac{\sigma}{\varepsilon} \to \Delta F(R^*) \sim \varepsilon R^{*d}$

$B(R) \sim \varepsilon R^{\psi}$ ← some power law

↳ $t \sim t_0 e^{\varepsilon R^{\psi}/k_BT} \Rightarrow R = \frac{k_BT}{\varepsilon}\left(\ln t/t_0\right)^{1/\psi}$ → much slower!

14/07/2017

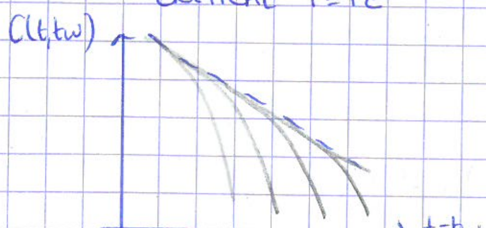
summary & discussions on coarsening systems.

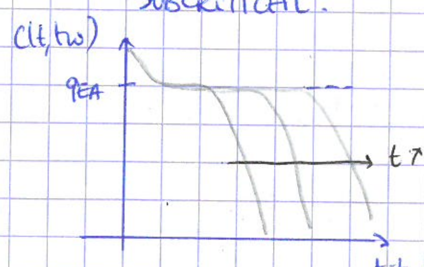
2.
3.

Instead of looking at $C(r,t)$ we can look at the 2 time self correlations $C(t,t_w) = \langle s_i(t) s_i(t_w)\rangle$ { local in space, non-local in time.

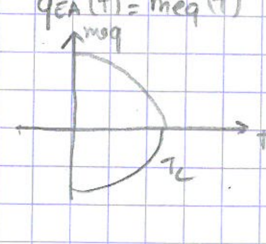
↳ the picture is equivalent

CRITICAL $T=T_c$ / SUBCRITICAL.





$q_{EA}(T) = m_{eq}(T)$



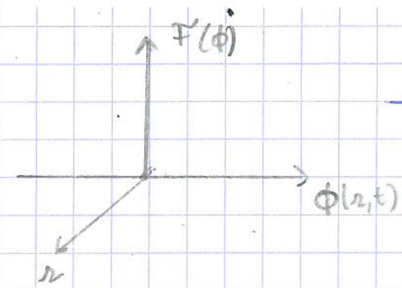
still looking at $t < t_{RELAX} \sim L^{z_d = 2}$

SEPARATION OF TIME SCALES

MULTIPLICATIVE

$f(0)$ ADDITIVE.

Now if looking at the whole field $\phi(\vec{r},t)$



→ non trivial interpretations here, beware!

Universty classes according to $R(t,T)$:

$R(t,T) \sim \quad \lambda(T)\, t^{1/2}$

↳ issue in the computation of the scaling functions describing aging: $f\left(\frac{R(t)}{R(t_w)}\right)$.

Response functions

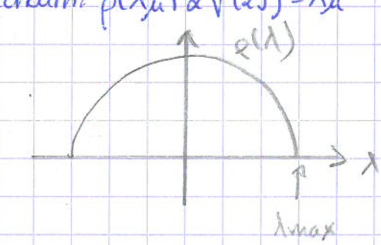
14/07/2017

mean field treatment of coarse ring.

$H = -\sum_{ij} J_{ij} s_i s_j + z\left(\sum_i s_i^2 - N\right)$

lagrange multiplier
spherical constrained
$s_i \in \mathbb{R}$

$J_{ij} \sim \mathcal{W}(0, \mathbb{I}) \rightarrow$ spectrum: $\rho(\lambda_\mu) \propto \sqrt{(2J)^2 - \lambda_\mu}$



$\equiv$ harmonic oscillators constrained
↳ to decouple them → go the eigen basis of J! $\{s_\mu\}$

$H = -\sum_\mu \lambda_\mu s_\mu^2 + z\left(\sum_\mu s_\mu^2 - N\right)$ → only coupled via the spherical constraint

$s_\mu(t) = \mathcal{F}(t, z(t))$ → perfectly solvable problem.

Yet this problem remains out of equilibrium because $s_{\lambda_{max}}(t)$ → random walk, diffusive

although all the others have a finite relaxation time.

p=2 N→∞ LFC & DEAN 99 SCHEHR #FYODOROV~

~ O(N) model ⇒ field formulation

↳ get ultimately to $\frac{\partial \tilde{\phi}_a}{\partial t}(\vec{k},t) = -k^2 \tilde{\phi}_a(\vec{k},t) + z(t)\tilde{\phi}_a(\vec{k},t) + \tilde{\xi}_a(\vec{k},t)$ → ~ spin model $\begin{cases} \phi = s \\ \vec{k} = \mu \\ k^2 = \lambda_\mu \end{cases}$

→ all a coordinates decouple
→ all behave like harmonic oscillator, except for $\vec{k}=0$: diffusive mode
$\equiv$ DOMAIN GROWTH

FORMALISM I

Langevin equations.

↳ Can be derived from scratch by considering a system coupled to the bath + Newton mechanics.

$$\mathcal{H} = \mathcal{H}_{SYST} + \mathcal{H}_{BATH} + \mathcal{H}_{INT} \left(+ \mathcal{H}_{COUNTERTERM} \right)$$

model BATH → oscillators $a = 1, \dots N \rightarrow \frac{m_a v_a^2}{2} + \frac{m_a w_a^2}{2} q_a^2$
model INTERACTIONS → linear couplings $\sum_{a=1}^{N} c_a q_a \cdot \nu(x)$ function of particle position
model SYSTEM → one particle p, x

We can solve Newton for oscillators, given some initial conditions $q_a(0), v_a(0)$.

↳ $\begin{cases} q_a(t;\, syst, q_a(0), v_a(0)) \\ v_a(t;\, syst, q_a(0), v_a(0)) \end{cases}$

↳ which yield for our system the following equations of motion:

$$m\ddot{x}(t) + \nu'(x(t)) \underbrace{\int_0^t dt' \, \Gamma(t-t') \, \nu'(x'(t)) \dot{x}(t)}_{\text{delayed friction}} = \underbrace{F(x(t))}_{\text{deterministic force}} + \nu'(x(t)) \, \xi(t)$$

DETERMINISTIC EQUATION!

$\Rightarrow \begin{cases} \xi(t) = \text{function } \{q_a(0), v_a(0), m_a \, w_a \, c_a\} \\ \Gamma(t-t') = \text{function of } \{m_a, w_a, c_a\} \end{cases}$

Stochasticity is introduced by assuming $q_a(0), v_a(0) \sim pdf = e^{-\beta(H_{BATH} + H_{int})}$ = assume oscillators are in eq. at temperature β at $t=0$.

$\Rightarrow \xi(t)$ gaussian noise: $\begin{cases} \langle \xi(t) \rangle = 0 \\ \langle \xi(t)\xi(t') \rangle = k_B T \, \Gamma(t-t') \end{cases}$

↳ The exact same as from the friction. Choice of $\{m_a, w_a, c_a\}$ → different $\Gamma(t-t')$ → memory kernel.

↳ $\nu'(x(t))\xi(t)$ = multiplicative noise term!

Assume furthermore:

i/ The time scale of the bath $t_{BATH} \ll$ all other time scales in the system, so that we have $\Gamma(t-t') = 2\,\delta(t-t')$ ⇒ WHITE NOISE CASE.
→ although still multiplicative. $p(w_a)$ flat.

ii/ $\nu(x) = x$ = linear coupling ⇒ ADDITIVE NOISE.

iii/ $t_I = \frac{m}{\gamma_0} \ll$ time all other scales in system ⇒ OVERDAMPED LIMIT $\gamma_0 \dot{x} = F(x) + \xi(t)$.

THE MESSAGE HERE: write Langevin carefully to keep the assumption that the bath in equilibrium and thereby keep the possibility of the system to be in equilibrium.

Stochastic calculus.

Why care about multiplicative noise? → makes the result depend on ε rules of calculus for discrete time.
(Ito $\alpha=0$ vs Stratonovitch $\alpha=1/2$ ...)
$\Rightarrow \frac{dM}{dt} \longleftrightarrow (1-\alpha) M(t_k) + \alpha \, M(t_{k+1})$.

↳ equivalence formula $\dot{x} \overset{\alpha}{=} f(x(t)) + g(x(t))\,\xi(t)$

$\alpha' \neq \alpha \Leftrightarrow f \leftarrow f +$ drift term → same results...

WHY ADDITIVE NOISE DOES NOT HAVE SAME PB ?

$$x(t_{k+1}) - x(t_k) = \underbrace{f(\bar{x}(t_k))\,dt}_{o(dt)\ \ll} + \underbrace{g(\bar{x}(t_k))\,\xi(t_k)\,dt}_{o(dt^{1/2})} \longrightarrow \langle \xi(t)\xi(t')\rangle \propto \frac{1}{dt} \Rightarrow \xi(t_k) \sim \frac{1}{dt^{1/2}}$$

$\Rightarrow\ x(t_{k+1}) - x(t_k) \sim O(dt^{1/2})$

and $g(x) - g(\bar{x}) \to$ not negligeable, and so because multiplied by the noise

17/07/2017

REF OKSENDAL

$\frac{dx}{dt} = f(x) + g(x)\xi$ the stochastic diff. eq. is properly defined only after after the evaluation of the x_s in the r.h.s explicitely: $f(\bar{x}(t_k))\ g(\bar{x}(t_k))$

↳ $\bar{x}(t_k) = (1-\alpha)x(t_k) + \alpha\, x(t_{k+1})$ $\alpha = 1/2$ stratonovich, $\alpha = 0$ ito.

→ CHAIN RULE are fixed by the value of α chosen:

$$\frac{dY(x)}{dt} = \frac{dY}{dx}\frac{dx}{dt} + \underbrace{D(1-2\alpha)\frac{d^2Y}{dx^2}g^2(x)}_{\text{additional term}} \to \text{usual only if } \alpha = 1/2$$

⚠ easier to build the solution with $\alpha = 0$ but beware to add all necessary terms!

given by $\langle \xi(t_k)\xi(t_l)\rangle \propto \frac{D}{dt}\delta_{kl}$ ← comes from the $\delta(t_l - t_k) \to \frac{\delta_{kl}}{dt}$.

⚠ if integrating numerically with $\neq \alpha \Rightarrow$ different ensemble if g is not constant (multiplicative noise)

PROOF: $g(\bar{x}_\alpha(t_k))\xi(t_k) - g(\bar{x}_{\alpha'}(t_k))\xi(t_k) = g'(x(t_k))(\alpha-\alpha')\underbrace{dx\,\xi(t_k)}_{\Theta(dt^{1/2})\Theta(dt^{-1/2})} = \Theta(1)$

Fokker Planck equation $\frac{\partial P}{\partial t} = -\partial_x\left[\left(f(x) + 2D\alpha\, g(x)\partial_x g(x)\right)P(x,t)\right] + D\,\partial_x^2\left[g^2(x)P(x,t)\right]$

depends on g and α.

↳ Now choosing wisely the force, we can have equivalence between different αs:

Same process for: $f + 2D\alpha gg' = f_{\text{diff}} + 2D\alpha' gg'$

↳ to tune!

↳ steady state:

$$P_{st}(x) = \frac{1}{Z}e^{-\frac{1}{D}V_{\text{eff}}(x)} \quad \text{with } V_{\text{eff}}(x) = -\int^x dx'\frac{f(x')}{g^2(x')} + 2D(1-\alpha)\log g(x).$$

external potential $V(x)$, with $V_{\text{eff}}(x) = V(x)$ if $f(x) = -g^2(x)V'(x) + 2D(1-\alpha)g(x)g'(x)$.

e.g. Brownian motion experiment in small volume → feels the walls

friction $\gamma_0(\bar{x})$ → $\langle \xi(t)\xi(t')\rangle = 2\gamma_0(\bar{x})k_BT\,\delta(t-t')$ with spatial dependence

↳ MULTIPLICATIVE NOISE.

FORMALISM II

Functional formalism (Feynmann path integrals).

Well suited for:
- Perturbative expansion
- time dependent RG
- self consistent approximation ⇒ $p>2$ spin models, glasses
- dynamical symmetries ⇒ exact results for systems in eq (FDT) out eq FT

↳ SLIDES → REVIEW: LFC & Lecomte 1704.