A flavor of the computation of the dynamic scaling:

\[ \dot{x} - \frac{\partial \phi}{\partial x} = \nabla^2 \phi - \frac{\partial R}{\partial x} \frac{\partial \phi}{\partial t} \]

\[ \frac{\partial R}{\partial x} = -\frac{\partial \phi}{\partial t} \frac{\partial \phi}{\partial x} \]

\[ \phi(x, t) = \phi_0 \left( x - v_0 t \right) \]

**Weak disorder**

- The domain wall is pinned by disorder → hard to move.

\[ \text{Arhenius time scale:} \quad t_A \sim e^{B/R^d} \]

\[ B = \text{length of the domain wall we would like to move}. \]

\[ \Delta \phi(R) \sim R^{d-1} \]

\[ \Delta \phi(R) \sim e^{B/R^d} \]

\[ \Delta \phi(R) \sim \text{some much slower} \]

**Summary and discussions on coarsening systems**

Instead of looking at \((n, t)\) we can look at the 2 time self correlations \(C(l, t) = \langle s(l, t) s(l, t) \rangle \)

\(C(l, t) \sim \text{Critical} \quad T = T_c \)

\(C(l, t) \sim \text{Subcritical} \)

\(C(l, t) \sim \text{Multiplcative} \)

\(C(l, t) \sim \text{Separation of time scales} \)

\(C(l, t) \sim \text{Non-perturbative} \)

\(\Delta \phi(R) \sim \text{Log} (t/t_0) \)

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Universe classes according to $R(t)$:

$$R(t) \sim \lambda(t)^{1/4}$$

Issue in the computation of the scaling functions describing aging:

$$f\left(\frac{R(t)}{R(\infty)}\right)$$

Response functions

Mean field treatment of course:

$$H = \sum_{\mu} J_{\mu} \mathbb{S}_{\mu}^z + \frac{1}{2} \left( \sum_{\mu} s_{\mu}^z - N \right)$$

$$J_{\mu} \sim U(0, 1) \rightarrow \text{spectrum } p(\lambda_{\mu}) \propto \sqrt{\lambda_{\mu}^2 - \lambda^2}$$

$L$-harmonic oscillators constrained

Isolate them $\rightarrow$ go to eigen basis of $J$! $s_{\mu} \rightarrow$

$$H = -\sum_{\mu} \lambda_{\mu} s_{\mu}^z + \frac{1}{2} \left( \sum_{\mu} s_{\mu}^z - N \right) \rightarrow \text{only coupled via the spherical constraint}$$

$$s_{\mu}(t) = \mathbb{S}_{\mu}^z(t, t(0)) \rightarrow \text{perfectly solvable problem}$$

Yet this problem remains out of equilibrium because $s_{\mu}(t)$ $\rightarrow$ random walk, diffusive

although all the others have a finite relaxation time.

$O(N)$ model $\rightarrow$ field formulation

get ultimately to

$$\frac{\partial \phi_a(\mathbf{r}, t)}{\partial t} = -\nabla^2 \phi_a(\mathbf{r}, t) + \mathcal{Z}(t) \phi_a(\mathbf{r}, t) + \frac{\xi_0^2(\mathbf{r}, t)}{2}$$

all $\sigma$ commutes decouple

all behave like harmonic oscillators, except for $\phi = 0$: diffusive mode
Yangmien equations.

Can be derived from scratch by considering a system coupled to the bath + Newton mechanics:

\[ H = H_{\text{sys}} + H_{\text{bath}} + H_{\text{int}} + H_{\text{force}} \]

Model bath → oscillators \( a = 1, \ldots, N \) → mass \( m_a \) + mass \( m_a \) \( q_a^2 \)

Model interactions → linear coupling \( Z_a q_a \cdot l(x) \) function of particle position

Model system → one particle \( p, x \)

We can solve Newton for oscillators, open some initial conditions \( q_a(0), \dot{q}_a(0) \):

\[ C_1 q_a(t, \text{sys}, q_a(0), v_a(0)) \]

\[ L_2 v_a(t, \text{sys}, q_a(0), v_a(0)) \]

Which yield for our system the following equations of motion:

\[ m(x(t)) + m(x(t)) \int_0^t \left( \frac{\partial H}{\partial \dot{q}} \right) dt \cdot w(x(t)) p(x(t)) = \mathcal{F}(x(t)) + \eta(x(t)) \xi(t) \]

(Deterministic Equation)

\[ \xi(t) = \text{function of } q_0(t), \dot{r}_0(t), m, \text{mass on y} \]

\[ \eta(t) = \text{function of } m_1, \text{mass on x} \]

Stochasticity is introduced by assuming \( q_0(t), \dot{q}_0(t) \) \( \nu \) pdf = \( \mathcal{N} \)

\[ \mathcal{N} \]

\[ \text{Gaussian noise} \]

\[ \xi(t) = 0 \]

\[ \langle \xi(t) \xi(t') \rangle = \delta(t-t') \]

\[ \langle \eta(t) \eta(t') \rangle = \delta(t-t') \]

\( \eta(t) \) = multiplicative noise term!

\[ \dot{x}(t) = \text{multiplicative noise term} \]

Assume furthermore:

1/ The time scales of the bath + bath << all other time scales in the system, so that we have \( \eta(t-t') \) = WHITE NOISE CASE → although still multiplicative \( p(w_a) \) flat.

2/ \( l(x) = x \) = linear coupling → ADDITIVE NOISE

3/ \( \tau_s < \tau_2 \) << time all other scales in system → OVERDAMPED LIMIT \( \dot{x}_o = \mathcal{F}(x_o) + \eta(t) \)

The message here: Write Yangmien carefully to keep the assumption that the bath is in equilibrium and thereby keep the possibility of the system to be in equilibrium.

Stochastic calculus.

Why care about multiplicative noise? Makes the result depend on \( \xi(t) \) rather than \( \dot{x}(t) \).

\[ \text{for discrete time } t \rightarrow \delta t \]

\[ \mathcal{F}(x(t)) \rightarrow \mathcal{F}(x(t)) - (t-x(t)) \eta(t) \]

\[ \xi(t) \] equivalence formula \( \dot{x} = \dot{f}(x(t)) + g(x(t)) \eta(t) \)

\[ x = x(t) + \xi(t) \]

\[ \text{add term } \eta(t) \text{ to same results...} \]
WHY ADDITIVE NOISE DOES NOT HAVE SPATIAL PD?

\[ x(t_{k+1}) - x(t_k) = \mathcal{N}(x(t_k)) + g(x(t_k)) \Delta t \]

\[ \mathcal{N}(x(t_k)) \propto \frac{1}{\sqrt{\Delta t}} \]

\[ \Delta x \propto \frac{1}{\sqrt{\Delta t}} \]

\[ \Delta x = O(\Delta t^{1/2}) \]

and \( g(x) - g(x_0) \) is not negligible, and so because multiplied by \( H_{0,1} \)

17/07/2019

REF OKSENDAL

\[ \frac{dx}{dt} = f(x) + g(x) \]

the stochastic diff. eq. is properly defined only after \( \mathcal{N}(x) \) is specified by \( \delta(x) \).

\[ f(x) = \frac{d}{dx} \left( \left( 1 + x \right) x(t_k) + a x(t_{k+1}) \right) \]

\[ a = \frac{1}{2} \xi \frac{d}{dx} \]

\[ \xi = 0 \] or 0.

\[ \Delta x = \mathcal{N}(x) \]

\[ \Delta x = O(\Delta t^{1/2}) \]

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